Thinking Out Loud:

The Role of Discourse in Understanding the Derivative in Calculus I

by

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ABSTRACT

Studies of discourse are prevalent in mathematics education, as are investigations on facilitating change in instructional practices that impact student attitudes toward mathematics. However, the literature has not sufficiently addressed the operationalization of the commognitive framework in the context of Calculus I, nor considered the inevitable impact on students' attitudes of persistence, confidence, and enjoyment of mathematics. This study presents an innovation, founded, designed, and implemented, utilizing four frameworks. The overarching theory pivots to commognition, a theory that asserts communication is tantamount to thinking.

Students experienced a Calculus I class grounded on four frames: a theoretical, a conceptual, a design pattern, and an analytical framework, which combined, engaged students in discursive practices. Multiple activities invited specific student actions: uncover, play, apply, connect, question, and realize, prompting calculus discourse. The study exploited a mixed-methods action research design that aimed to explore how discursive activities impact students' understanding of the derivative and how and to what extent instructional practices, which prompt mathematical discourse, impact students' persistence, confidence, and enjoyment of calculus.

This study offers a potential solution to a problem of practice that has long challenged practitioners and researchers—the persistence of Calculus I as a gatekeeper for Science, Technology, Engineering, and Mathematics (STEM). In this investigation it is suggested that Good and Ambitious Teaching practices, including asking students to explain their thinking and assigning group projects, positively impact students' persistence, confidence, and enjoyment. Common calculus discourse among the experimental students, particularly discursive activities engaging word use and visual representations of the derivative, warrants further research for the pragmatic utility of the fine grain of a commognitive framework. For researchers the work provide a lens through which they can examine data resulting from the operationalization of multiple frameworks working in tandem. For practitioners, mathematical objects as discursive objects, allow for classrooms with readily observable outcomes.

Keywords: commognitive, commognition, discourse, communication, thinking, attitude, persistence, confidence, enjoyment, good teaching, ambitious teaching, discursive, discursive cognition, calculus, STEM

LOVE AND KNOW LIFE

The only way to know life is to love many things.

—Van Gogh

Love and know life in the company of people you most love. FAMFAM, the unforgettables of life are our journeys together. I can't imagine a brighter life than the life we hack and experience together. Thank you for savoring the *unimaginables* and navigating the *impossibles* the past three years—as always, equally resolute, next to me.

—*I love you more than words Masud, Yasmynn (momma), and Nasrynn (meems).*

Mom, you gave me life so I would know life.

—*I love you, Mom.*

Amma and Abba, you gave us a better life. From heaven, I feel your pride in us.

—*Tomake bhalobasi. Dekha habe, inshAllah, Amma and Abba.*

Dad, you are there for Mom. Your life with her matters.

—*Gracias Dad.*

Brothers and sisters let's stand united in and for peace. Mercy and grace for those whom peace comes with pain and suffering in life. Solace from those blessed with peace.

—*Salaam.*

You have my heart and you've touched my soul.

always,

—*madeleine*

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—WE ARE THE CHAMPIONS, my friends—

Sincerely, thanks for

Thinking Out Loud!

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CHAPTER 1

LEADERSHIP CONTEXT AND PURPOSE OF THE ACTION

Silence is the language of God, all else is poor translation.

—Rumi, (Kononenko & Kononenko, 2010, p. 134)

Personal Background

Prior to establishing the context for my work and the innovation that were central to the study, I have provided background that was crucial to setting that context. I grew up in south Texas, spoiled by the tropical oasis of Padre Island and cooled on hot summer nights by the Gulf Coast winds. In the wealth of support in my development in science, technology, engineering, and mathematics (STEM), I had not realized until eighth grade that I attended schools that were marginalized, reflecting in their dilapidated infrastructure, the impoverished neighborhoods in which we, its children of color, lived. What I remembered fondly were people in my school life, like Mrs. Thompson, who extended a warm welcome when I arrived in her classroom from the Philippines. She embraced me and raved about my mathematical abilities. Mrs. Neighbor, our sixth-grade teacher, who all my peers swore was wicked to everyone but me, goaded on and championed my mathematical and writing skills equally. In retrospect, this was serendipity—the right time and place combined with sincere teachers dedicated to their craft, who understood my desire to think, to understand, to communicate, and to learn unremittingly about the world around me. The only rule I thought was oddly practiced by my elementary school teachers was "*silence is golden*."

STEM Roots with Endearing Support

In eighth grade, I completed my own bussing forms to attend a prestigious "south side" school. Even at 13, I knew the school offered a better education, but I quickly realized I did not belong at this school. I returned to school in my community where dedicated teachers and a circle of friends embraced and nurtured our collective growth in STEM. I accepted education would be my only ticket out of a marginalized but happy life—a life that also engendered my STEM lifeline.

I had vivid images of our biology, chemistry, precalculus, and physics teachers. Nostalgia easily took me back to Mr. Gillespie's voice in our mathematics classes, "What does that have to do with the price of eggs in China?!" he chided us daily. I had no clue what he meant. I laughed to reciprocate his amusement. Mr. Gillespie was the kind of teacher who taught us to feed our curiosity. He taught courses that did not exist at our northeast-side high school —until I inquired. When I wanted to know what analytic geometry was, Mr. Gillespie told me the only way I would ever know was to "do" analytic geometry and "experience" analytic geometry. I was the only student in his analytic geometry class. Through the lens of educational research, I have come to know my STEM high school teachers engaged in the persistent practice of "good teaching" (Chickering & Gamson, 1999). "Good teaching" had a lasting positive effect on my attitudes—my persistence, confidence, and enjoyment, toward STEM disciplines, particularly mathematics, as studies indicated they would (Chickering & Gamson, 1999; Bressoud et al., 2015; Rasmussen et al., 2019). In contrast to my elementary teachers, my high school STEM teachers constantly prompted us for questions and answers. Our talk was a translation of our thoughts, they seemed to believe.

I had a counselor who acted as my guardrail, making sure I remained on course with my STEM aspirations. She ensured I did not fall off the precarious cliffs that were part of our community's landscape, figuratively not literally. In retrospect, I was certain her connections afforded many of us with opportunities unimaginable to others in the same socioeconomic status. For example, following 11th grade, I spent a summer in Phillips Academy Andover in a co-educational university-preparatory school, where I was exposed to a rich, diverse ecosystem that changed my perspective. I realized the social rift and the socioeconomic gaps between the "haves" and "have nots." However, this perception made me keenly aware of opportunities, rather than resentful.

Andover, unlike the south-side middle school in my hometown, gave me the opportunity to belong. Phillips Academy teachers invited us to engage our curiosity and to be essential in our Academy as well as the surrounding communities. The teachers, in addition to practicing "good teaching," welcomed us into their homes. At 16, this experience was significant to my life's trajectory, especially having grown up in a trailer park on the *wrong* side of town which was mostly ignored by many in our surrounding communities and leaders. Like my high school STEM teachers, to my Andover teachers, our verbal, written words, in my English class, and visual mediators, in my trigonometry class, were translations of our thoughts. Poor translations were as welcomed as silence.

A Supportive Academic Engineering Environment

I was a first-generation college student. Unfortunately, neither of my parents graduated from high school. As an engineering student at a university, as an engineering intern in industry, and a part-time employee in an engineering office when I was an undergraduate student, serendipity was again my fate. Like in grade school and high school, I was guided by mentors and kept on track by guardrails. My academic interest was shaped by the prowess of my professors who also practiced the characteristics of "good teaching." They communicated high expectations, encouraged direct contact with them, and provided prompt feedback with supportive advice. I recall my design professor making an analogy between problem solving and playing pool. He was steadfast in his advice that we had to understand the problem first, that we paused and thought in order to strategize, and that we set up the next shot. From my advocates, I learned to value the opportunity to communicate my thoughts and to understand poor translation was the root of human learning.

Challenges to Aspirations for Thinking and Understanding

At the center of my continuous development and growth was my keen interest in the processes of thinking, communicating, and understanding. For my first and second positions as a full-time engineer, at my interviews I was asked about the "A" that was struck from my undergraduate transcript. I explained that, against the counsel of my academic advisor and advice of my friends, I retook the class to better understand and have a better command of thinking about and communicating the course material. Not expecting such a question, I was convinced I missed the opportunities for these full-time positions, but to my surprise, I was hired.

I left my engineering career because as a mother I wanted to focus on my daughters' education and their development and growth in thinking, communicating, and understanding toward learning. My eldest daughter's first rebellion, at the age of five, raged as she asked me a question. She then added with cogency, "And don't ask me what I think, just tell me the answer, Momma!" Twenty years later, at a community college in the Southwest, I have continued to embrace and appreciate the opportunity to think about, communicate, and understand how people learn, particularly in calculus, with an increasingly diverse population of students, who have likely told me under their breath, "Don't ask me what I think, just tell me the answer."

Larger Context in STEM

Although in 2012 the United States produced approximately 300,000 college graduates with bachelor and associate degrees in STEM fields, fewer than 4 out of 10 students who go to college intending to major in a STEM field complete a STEM degree. With the industry demand far higher than the supply for specialized skills in STEM, in 2015, President Obama called for a national focus to accelerate production of one million additional college graduates in STEM (Olson & Riordan, 2012). The lack of inclusivity and the disproportionate participation in STEM fields, based on gender, race, and socioeconomic backgrounds, have been and have continued to be serious problems in our nation's efforts to navigate a 21st century economy that is increasingly dependent on STEM literacy.

A Vision for STEM Education in America

In June 2018, the White House Office of Science and Technology Policy (OSTP) joined the National Science Foundation (NSF) and sixteen other Federal agencies to inform the development of the federal government's five-year strategic plan for STEM education. Proponents envisioned "a future where all Americans will have lifelong access to high-quality STEM education and the United States will be the global leader in STEM literacy, innovation, and employment," which was required by the COMPETES Act of 2010 (National Science & Technology, 2018, p. v).

Proponents of the Federal report, *Charting a Course for Success: America's Strategy for STEM Education* (National Science & Technology, 2018) envisioned three aspirational goals: (a) build strong foundations for STEM literacy; (b) increase diversity, equity, and inclusion in STEM; and (c) prepare the STEM workforce for the future. One interesting observation and takeaway was the spotlight on mathematics. “Make mathematics a magnet” was one of three objectives under the pathway “Engage students where disciplines converge” (p. 15). No other STEM discipline was called out explicitly in the list of objectives in the Federal report.

Broadening Participation in STEM Through NSF Outreach

Broadening participation (BP) has been a long-standing NSF programmatic effort as well as a key component of NSF’s strategic plan. NSF engaged in extensive outreach from 2014-2016 with Dear Colleagues Letters (DCL) to recruit at two-year Hispanic-serving institutions (HSI) or at community colleges as well as universities to encourage the development of communities of practice to conduct high-impact innovative work to address student success in the first two years of mathematics courses (James & Singer, 2016). NSF projects like this, conducted to carry out the research on strategies for BP, were necessary to fully advance the NSF BP agenda, continuing to build on the knowledge base of effective strategies for high-quality STEM learning environments.

The Role of Calculus in STEM Learning at a National Level

In 2009-2014, the Mathematical Association of America (MAA) conducted an investigation of Calculus I at U.S. colleges and universities. The two-phase *Characteristics of Successful Programs in College Calculus* (CSPCC) Project culminated into the seminal report, *Insights and Recommendations from the MAA National Study of College Calculus* (Bressoud et al., 2015). CSPCC was the first nationwide project to integrate large-scale survey data with in-depth case study analysis. The innovation for this study drew from two pedagogically-related outcomes from the CSPCC project results. In measuring the impact of characteristics of calculus classes that influence student success, Sonnert and Sadler (2015), as part of the CSPCC team of researchers, identified two pedagogical factors, *Good Teaching* and *Ambitious Teaching*, from their analysis of the national survey data. In addition, Bressoud et al. recommended seven best practices based on their CSPCC project key findings.

Best Practices at Work in Community Colleges

In the context of teaching calculus at a community college, six of the seven recommended best practices were directly applicable to calculus courses at the community college level (Bressoud et al., 2015):

- construction of challenging and engaging courses;
- active learning strategies, e.g., whole classroom dialogues, peer-to-peer discussions, and in-class problem solving;
- regular use of local data to serve as curricular guidance and for structural modifications;
- use of proactive student support services, including the fostering of student academic and social integration;
- adaptive placement systems to place students in the highest course in which they could succeed; and
- coordination of instruction, including building of communities of practice.

The seventh practice, related to training of graduate teaching assistants, was not applicable to the community college setting.

In particular, the characteristics related to construction of challenging and engaging courses and active learning strategies, in conjunction with lectures, have been central to my instructional approach for more than two decades. Both characteristics make up two constructs identified by Sonnert and Sadler (2015) as *Good Teaching* and *Ambitious Teaching*. Both constructs were investigated further by Mesa et al. (2015) and Larsen et al. (2015) in Phase II of the CSPCC project, comprising an in-depth case study analysis at several universities.

In my precalculus and calculus classes, practicing *Good Teaching* was realized through intentional instructional actions (Chickering & Gamson, 1999): I encouraged direct contact with me, collaboration among students, and active learning. I emphasized time on task. I communicated high expectations. I respected diverse talents and ways of learning. As I increased the quality of *Good Teaching* characteristics, I have persistently incorporated *Ambitious Teaching* factors into my classes. My efforts were guided by more than my ambitions to teach effectively. However, I anticipated the needs: (a) to provide a theoretical foundation for my practice; (b) to address my course-content structure; (c) to address what it means for students to understand mathematical concepts; and lastly, (d) to assess my goals as they align to my instructional demand on my students. Multiple frameworks were essential for sustainable systemic improvement that efficiently and, more importantly, effectively, assisted students to succeed in calculus or any class I taught.

Local Research Context: The Need for Improvement

The District Level

My community college system is one of the largest community college systems in the nation with ten regionally accredited colleges serving the greater metropolitan area. Our community college district enrolled approximately 93,511 students in 2021-2022. In fall 2021, our female-male student proportions were 58-41%, with 1% undeclared. There were 29% full-time and 71% part-time students with an average age of 24 enrolled, an average of 8.3 credit hours, and an average class size of 15.4 students. About 90% are in-county residents. Our demographics by specific ethnic groups was the following: White 43%, Hispanic 37%, Black 6%, Two or More 5%, Asian 4%, Not Specified 3%, and American Indian 2%. Student intent ranged from 35% transfer to university, 24% high school students, 22% enter/advance in job market, 11% personal interest, 4% university students, and 3% undeclared. First generation college students comprise 49% of our district's student population.

In a period from February to April 2017, an ad hoc Transformation Task Force, created by our Chancellor, at the time, produced 42 recommendations categorized under student access and success, workforce responsiveness, resources, efficiency and collaboration, and leadership at the district and college level. The district transformation proposal was developed as a response to declining student enrollment and revenue. The overarching topics were leadership, guided pathways, student services to support success, marketing and outreach, accountability and performance, and reducing competition between sister district colleges. Three areas of focus emerged: Guided Pathways, Industry Partnership, and Enterprise Performance. Guided Pathways helped students to identify a clear and coherent educational pathway and to determine their career and educational goals toward timely completion. Industry Partnership provided students with valuable hands-on workplace experience. Finally, Enterprise Performance was a comprehensive talent management system with the goal to enhance our district's competitiveness.

The College Level

The specific setting of this study was conducted at one of the largest colleges in the community college district located in the Southwest. The mathematics department at our college found itself at a crossroad. Mathematics faculty members could have reinvented the teaching and learning opportunities the department offered to students or they could have continued on the same course with no change in student completion rates in our gateway courses. Concurrently, our mathematics department faculty members were required to consider the implications the district transformation proposal had at our college, in our department, and particularly on our students.

As a mathematics faculty member, my strategy was to engage in the Guided Pathway to Success (GPS) efforts. I was a member of multiple GPS Pathway Mapping teams for three years. My problem of practice targeted the GPS goal: close the achievement equity gap in student success. The directly relevant GPS goal and outcomes were: (a) improvement in equity and (b) an increase in completion of gateway mathematics and English courses in the first year.

Leadership with Intimate Contextual Knowledge

For 25 years, I have owned my responsibility as a teacher to our community college students and as an engaged and accountable member of our educational community—as did the individuals and communities that intentionally assisted me in transforming my narrative from a student to an engineer, to a mother, and finally to a teacher—from my arrival in the U.S. to decades that followed. My desire was to make a positive difference.

My intimate contextual mathematical proficiency was developed through my practice as an engineer, collaborating with colleagues to apply mathematics with observable implications and impact. My intimate contextual understanding of educational research began with a fellowship program at the district level that served as an institute for faculty learning. The experience ignited my continuing dedication to the scholarship of teaching and learning (SoTL). As a faculty developer for my college for three years, I had the opportunity to collaborate with other faculty, staff, and our Administration to pave directions for multiple initiatives for our Center for Teaching and Learning.

My collaboration with colleagues, students, artists, engineers, masons, contractors, and our college's facilities employees, to design, create, produce, and install four Islamic art murals and a free-standing ceramic mural, within a span of 10 years, empowered teams to realize creativity that transcended disciplines and rank. My experience as a member of NSF Review Panels for STEP, S-STEM, TUES, and CCIC offered me a glance at the incredible national research efforts in which our colleagues engaged to captivate our students' imaginations and engage them in scholarship, learning, leadership, and innovation. Through such experiences, I recognized to have a significant impact, I needed not only to understand my subject matter well, but I needed to intimately understand students and to possess a resolve perhaps beyond my ken, and often beyond my energy, but never beyond my effort.

A Change Effort Worth Pursuing

At the time I applied for the Leadership & Innovation program in fall 2018, I was an ad hoc member of our college mathematics developmental education transformation team. The team was charged with a directive to find ways to meet two district-level developmental education project goals. Those goals were: (a) all students should have the opportunity to complete college-level mathematics and English courses in their first year at our district; and (b) 80% of students should be placed into college-level mathematics and English with the appropriate support they needed to succeed within their first semester. For years, our college's success rates and completion rates in mathematics have been dismal, with a large percentage of our students placed in developmental courses, often at the arithmetic review level. I realized our students at our college were likely not different from students cited in the MAA studies (Bressoud et al., 2015; Rasmussen et al., 2019) who intended to be STEM majors but left the STEM discipline after their first calculus course. At the time I applied to this program, I stated I would look for direction and guidance to frame our mathematics department program to assist students in completing gateway mathematics courses within their first year in gateway courses such as Calculus I. It was a specific problem of practice statement and change effort worth pursuing because our students at our college and district, and truly all students, deserve nothing less.

Purpose Statement and Research Questions

With the innovation in my Calculus I class, my intention was to influence and affect student attitudes toward mathematics and their calculus knowledge, particularly their thinking about, and understanding of, the derivative concept through a systemic holistic approach to motivating discursive activity in a Calculus I class. I imagined my Calculus I students *mathematizing* and proud to call themselves *mathematists*—those who engage and revel in mathematical discourse.

The first purpose of my investigation was to engage students in mathematical discourse to motivate student thinking and understanding of calculus, particularly, the derivative concept. The second purpose was to assess the impact of the combination of two factors, *Good Teaching* and *Ambitious Teaching*, on a composite mathematics attitude comprising the dependent variables *persistence, confidence*, and *enjoyment* in mathematics, particularly calculus. Lastly, the third purpose was to determine if there was a significant and observable difference in performance of students in an experimental calculus class designed to motivate active engagement in mathematical discourse in contrast to students in a traditional calculus class. Given the purpose of the study, four research questions guided its conduct.

RQ1: How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?

RQ2: To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?

RQ3: How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?

RQ4: To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?

Organization of Dissertation

My action research was organized into five chapters. In this chapter, I presented a case for my investigation by situating my problem of practice in varied contexts spanning the spectrum from personal to national. In chapter 2, I provided a description of how my problem of practice has been conceptualized in the literature. Next, in chapter 3, the setting and participants, my role as practitioner and researcher, my innovation and frameworks that undergird the innovation, and the collection and analysis of data are provided. Following this, the analysis and results are presented in chapter 4. Finally, chapter 5 offers a discussion of the significance and relevance of this study and its results to practice and research.

CHAPTER 2

THEORETICAL PERSPECTIVES AND RESEARCH GUIDING THE PROJECT

It is the theory which decides what can be observed.

—Albert Einstein

The Calculus Reform: A Gateway

Calculus for a New Century: A Pump not a Filter (Steen, 1988) emerged from the growing discontent in the 1980s, revealing the immense complexity of a calculus reform and the diverse opinions about reshaping calculus for the new century. Well into the 21st century, calculus persisted as a gatekeeper course in STEM education as evident in *Charting a Course for Success: America's Strategy for STEM Education* (National Science & Technology, 2018). Calculus has persisted as a gatekeeper, not a gateway course, for STEM students (Bressoud, 2019; Bressoud et al., 2015; Rasmussen et al., 2019; Zorn, 2015). STEM-intending students consistently listed Calculus I as a factor for navigating away from STEM (Bressoud et al., 2015; Ellis et al., 2014; Olson & Riordan, 2012; Rasmussen et al., 2019; Seymour & Hewitt, 1997). Thirty years after the initial outcry for reform, Bressoud (2019) exhorted that the critical elements leading to a better understanding of undergraduate instruction continued to benefit from the amassed data from the Calculus Reform of the 1980s and 1990s.

Calculus Reform: What is different this time?

Bressoud (2019) asserted three reasons why the current reform agenda was not an iteration of the 1980s Calculus Reform effort: (a) the current agenda accelerated undergraduate mathematics research and improved undergraduate instruction based on accumulated data that support best practices; (b) the combined pressure of high failure rates and the workforce demanded for the 21st century weighed heavily on educational leaders; and (c) the focus was on developing the new generation of educators accompanied by an emphasis from what was taught, and more importantly, to how it was taught.

A collective effort and narration by both practitioners and researchers had the potential to recast Calculus I from its apparent gatekeeping ignominy to a primed pump, not a filter, for STEM students. The bridging of theory and research with classroom, assessment, and course design practices promoted effective teaching and learning that could provide equitable access to mathematics for all students and were forces for social change. In their *Mathematical Association of America* (MAA) *Instructional Guide*, Abell (2018) upheld these values and asserted our society deserved nothing less.

Grounding Practice in Research and Theory

Three foundational practices supported effective teaching: classroom practice, assessment practice, and course design practice. The *MAA Instructional Guide* ushered mathematics instruction and provided evidence-based teaching strategies along with these three foundational practices. A significant body of research and theory on effective teaching and learning grounded the three foundational practices proffered by Abell (2018). A brief on theories was offered and highlighted in the instructional guide, specifically in the discussion of instructional course design. The effort to recast Calculus I as a gateway, packaged with the curriculum and instruction in the first two years of college, was clearly a joint endeavor among researchers, practitioners, and policy makers as they cite each other's work as impetus for this charge they had embraced. Guiding this effort were literature such as the *Characteristics of Successful Programs in College*

Calculus (Bressoud et al., 2015); *MAA Instructional Practices Guide* (Abell, 2018); *2015 CUPM Guide to Majors in the Mathematical Sciences* (Zorn, 2015); and *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025* (Saxe & Braddy, 2015). A palpable synergy gave momentum to a collective effort to synthesize instructional practices with research and theory.

What is Theory and Why Do We Need It?

Sfard (2018) lamented the harbingers of indifference toward theory-less research evident by articles such as "Against Theory" (Knapp & Michaels, 1982) and "Big Data and the Death of the Theorist" (Steadman, 2013). Claiming theories emerged from research, Sfard offered a description of research as telling stories about aspects of reality (Sfard, 2018). Mathematics education researchers, for example, shared their stories about the realities of learning and teaching mathematics. However, by employing specially designed discursive tools, a researcher's unique discourse reached beyond the norm in a methodical, unequivocal, and precise way. Within a given discourse, for example mathematics education research, the set of exact and useful stories endorsed by a community of researchers was what Sfard called—a *theory*.

Four dimensions delineated research discourses like mathematics education research from other type of discourses: (a) keywords and their use, (b) visual mediators used by storytellers to make their stories vividly clear, (c) the research participants' routine actions, and finally (d) a smaller subset of narratives that determined the base properties of discourse's focal objects—that was, the theoretical assumptions (Sfard, 2008, 2018). Central to a theory was the key property of internal consistency, ensuring mutual endorsability with any pair of narratives. Furthermore, mathematics was defined by Sfard (2001, 2008) as a type of discourse. Albeit object-rules are fixed, Sfard asserted discourse is an autopoietic system that blooms

> by annexing its own metadiscourses, and this means, among others, that what counts as a metarule in one mathematical discourse will give rise to an object-level rule as soon as the present metadiscourse turns into a full-fledged part of the mathematics itself. (Sfard, 2008, pp. 201-202)

The autopoietic property and the ability for discourse to absorb endorsed narratives into a collective, I hypothesized, offered complex challenges for my action research. The Commognitive Framework (Sfard, 2008) was a theory of mathematics learning premised on the assumption thinking was tantamount to communication. Sfard (2008, 2020) blended communication and cognition and coined the term *commognition*. The pragmatic lean of the commognitive methodology, the methods, and the innovation, informed by published scholarship, fueled the trajectory of my investigation and supported the prospects of the success of my action to address my problem of practice and research questions from diverse perspectives of researchers and practitioners.

Achieving the STEM Imperative: A Collective Narrative

The STEM imperative, the need for more and better prepared STEM graduates to fill the anticipated gap between the estimated millions more STEM jobs than qualified workers available, continued to be a challenge (Adkins, 2012; Lazio & Ford, 2019; Rothwell, 2012; Smithsonian Science Education Center, n.d.). Trends showed the number of STEM majors decreased over the past decades (Lazio & Ford, 2019; Olson & Riordan, 2012; Rothwell, 2012). Inconsistent with common beliefs, students who switched away from STEM majors did not lack persistence nor academic preparation. Students cited poor instructional experiences in gateway first-year courses (Bressoud et al., 2015; Olson & Riordan, 2012; Rasmussen et al., 2019) as a primary reason for their departure from the STEM fields. Calculus I, supposedly a first-year gateway course, was a requisite for most STEM degrees. The scale of the problem was global (Rasmussen et al., 2019) and the correlated narratives were a collective continuing to fuel persistent extant quandaries in mathematics education. The *Common Vision* project (Saxe & Braddy, 2015) recently connected leaders of the five professional associations that represented undergraduate mathematical science programs, with partner STEM disciplines and other vested organizations and industry, to identify common research themes, forming—a collective narrative.

Galvanized and Guided by a Common Vision

The *Common Vision* project (Saxe & Braddy, 2015) reported on four categories to improve undergraduate learning, particularly in the first two years. My investigation was galvanized by the *Common Vision* project guide that addressed two of the four categories, curricula and course structure. In the category of curricula and course structure, they promoted: (1) a contemporary curriculum, (2) evidence-based teaching methods, and (3) interdisciplinary collaboration. Instructors were charged to motivate and illustrate key ideas and concepts via multiple perspectives using a wide range of subject matters in modern applications. The backdrop for the instructional practices guide were provided by MAA documents, the *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences* (Zorn et al., 2015) and *IMPACT—Improving Mathematical Prowess and College Teaching* (MAA, 2018), offering course recommendations for the mathematical sciences and sample syllabi. Literature from professional associations in mathematics and public scholarship provided knowledge to drive specific change efforts with a critical lens to shaping my overall action research.

Research Questions Inspired and Guided by MAA and ICME

My research questions were as follows:

RQ1: How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?

RQ2: To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?

RQ3: How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?

RQ4: To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?

These were questions inspired by calls to action and recommendations of opportunities for growth and improvement in continuing research in two MAA and one International Congress on Mathematical Education (ICME) publications. In *Insights and Recommendations from the MAA National Study of College Calculus*, Sonnet and Sadler (2015) identified two pedagogical factors of highly correlated attributes that influenced students' attitudes toward calculus at the classroom level—*Good Teaching* and *Ambitious Teaching*; this gave rise to my third and fourth research questions. The interest in my first two research questions was piqued by "The Calculus Sequence" (Zorn et al., 2015) which highlighted the contemporary trend and importance of covariation and accumulation with respect to the derivative topic. This current shift in thinking about the concept of the derivative, juxtaposed with the overview of the theoretical frameworks used and evolution of trends in the field of calculus education from the 1980s to the present, particularly regarding the concept of the derivative (Bressoud et al., 2016), supported the prospect of my first and second research questions. Bressoud et al. (2016) provided a global vision of learning and teaching calculus, including a discussion on the epistemological aspects of calculus concepts and an appraisal of well-defined theoretical frameworks in mathematics education. Their work motivated my interest in Sfard's Commognitive Framework, a theory based on the idea of thinking-as-communicating (Sfard, 2001, 2008).

Commognition: Thinking-as-Communicating

Communication: The Principal Outcome

In their study of the characteristics of successful calculus programs, Bressoud et al. (2015) asserted improving calculus can be accomplished by developing a common vision of specific skills and knowledge, or learning outcomes, and assessing at the end of the course to determine whether the learning outcomes were attained. Communication was clearly a primacy in *A Common Vision* (Saxe & Braddy, 2015). In the executive summary, one of the common themes from seven national curricular guides underscored "Students should learn to communicate complex ideas in ways understandable to collaborators, clients, employers, and other audiences" (p. 1).

Advancing assertions that communication was a primacy as a learning outcome, Sfard (2015), in "Why All This Talk About Talking Classrooms? Theorizing the Relation Between Talking and Learning," argued communication was indispensable to learning. That is, the process of learning was tantamount to changing and shaping ways of communicating by the commognitive approach—the approach of thinking-as-communicating.

> We look at the object of changes resulting from innovations in discursive practices of the classroom; we end up focusing on the activity of communication. Communication, rather than playing a secondary role as the means for learning, is in fact the centerpiece of the story— the very object of learning. [W]hen we change rules of interpersonal communication, it is not surprising thinking—the individualized form of communication— changes as well. If mathematics is a particular discourse with its own special ways of storytelling, there is no other way to learn mathematics than by adjusting the rules of classroom communication. (Sfard, 2015, pp. 239-240)

Evident to Sfard (2015) was the dichotomy of communication and thinking was strong and pervasive in the research community and thus likely in our students. She asserted this belief was based on a false dichotomy. The overall goal of my investigation was to determine which observable characteristics of and change in mathematical discourse, at the classroom level, lent itself to students understanding essential calculus concepts, particularly the derivative concept, through Sfard's (2001, 2008) approach of thinking-as-communicating.

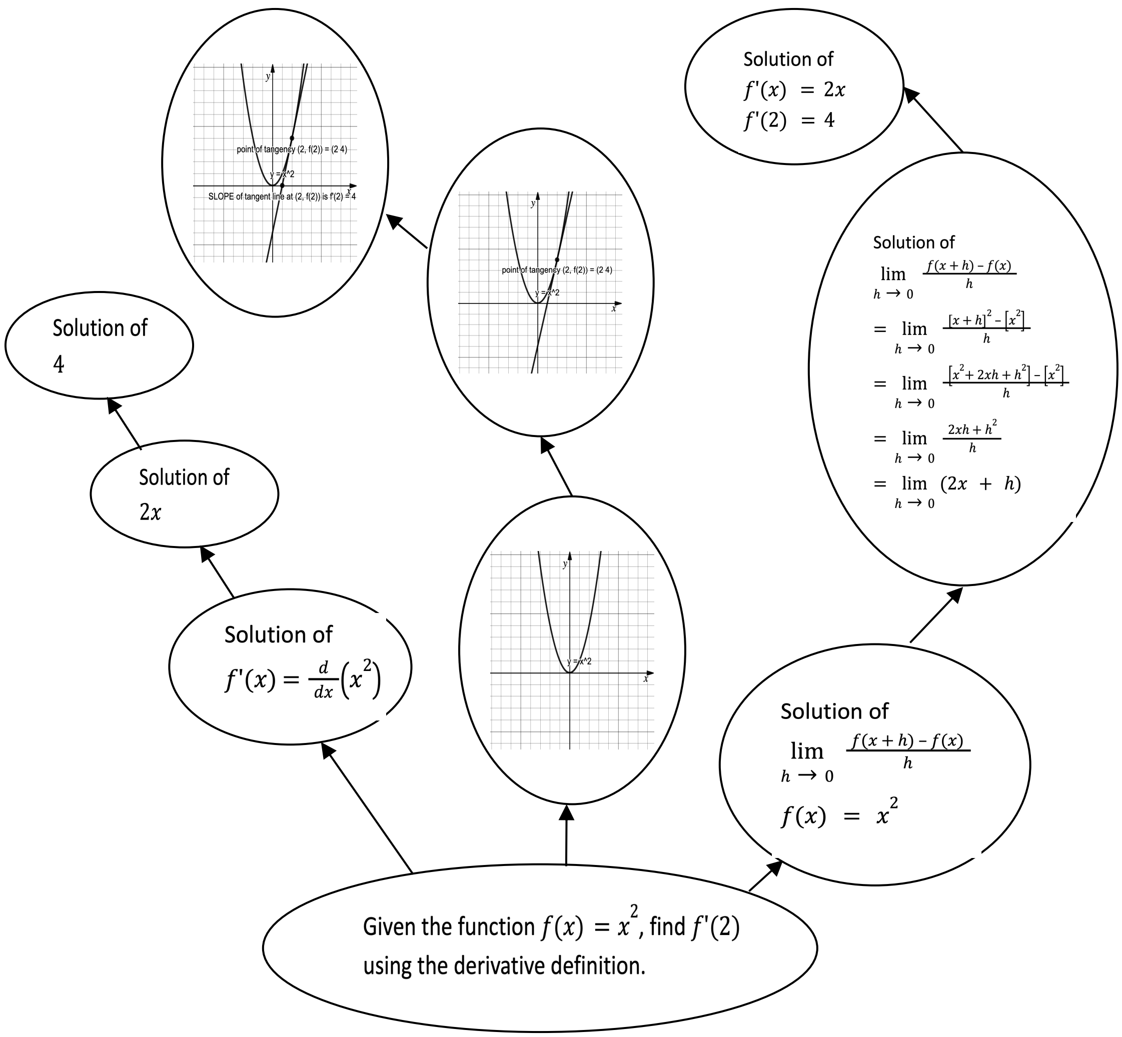
A Pragmatic Communicational Approach

In his review of Sfard's commognitive perspective, Wing (2011) underscored the pragmatism of an approach that deems communication as tantamount to thinking. Mathematical objects as discursive objects allowed practitioners to teach in the classrooms with readily observable outcomes and other learning phenomena. Sfard (2008) defined mathematical objects as abstract discursive objects with distinctly mathematical signifiers. In Figure 1 below, the signifier was the statement "Given $f'(x) = x^2$, determine $f'(2)$." The signifier served as a root for the branches that stem from the nodes of the realization tree. All the nodes that follow comprised the realization tree for this signifier. Each node was simultaneously a realization for the signifier node before it and a signifier for the node that followed it. This exemplified the iterative, autopoietic nature of a mathematical discourse. The combination of the initial signifier and its corresponding realization tree was defined as a discursive object that, in this case, was produced from a hypothetical discursive activity of three students.

The pragmatism of Sfard's Commognitive Framework guided my investigation at multiple levels. At the curriculum level, developing understanding of the derivative concept was imagined using the four characteristics of mathematical discourse: word use, visual mediators, routines, and endorsed narratives (Bressoud et al, 2016; Nardi et al., 2014; Park, 2011; Ryve, 2011; Tabach & Nachlieli, 2016). At the classroom level, the commognitive approach necessitated the instructor's facilitation of discursive shifts (Bressoud et al., 2016; Sfard, 2001, 2008), and necessarily involved discursive shifts for learners, or *mathematists*, (Sfard, 2008). Lastly, at the institutional level, the contribution of mathematical discourse to our community college students was the potential realization of the learning outcomes of critical thinking and communication. To act on this demand required a realization of multiple frameworks to guide the course structure design, to frame the derivative concept, and to frame the cognitive demand and the characteristics of the coursework in Calculus I.

Figure 1

A Realization Tree for a Derivative-Concept Signifier



The rest of this chapter is organized into three areas of research and theoretical perspectives that shape my overall action research followed by implications for my practice. First, to inform and further effective teaching and support deep student learning, I discuss two constructs or composite factors, *Good Teaching* and *Ambitious Teaching*.

Second, I outline commognition as the central theoretical framework I employed for my investigation, giving particular attention to fundamental tenets of Sfard's Commognitive Framework. To finalize the theoretical perspectives, I provide an overview of research in learning and understanding the concept of derivative and frameworks for describing the structure of the concept of derivative. Finally, I close the chapter with implications of my investigation for student success in calculus as related to my action research inquiry.

Good and Ambitious Teaching in Calculus I

In their seminal study, Chickering and Gamson (1999) offered seven principles based on their research on *Good Teaching*: student-faculty contact, cooperation among students, active learning, prompt feedback, time on task, high expectations, and respect for diverse talents and ways of learning. The utility of these principles paved the way for additional research on the use of active learning resulting in transformative and affective cognitive changes that increased students' confidence in their abilities to learn (Kuh, 2008; Kuh, et al., 2007). Prompt, individualized, supportive, and corrective feedback were shown to affect students' learning positively (Angelo & Cross, 1993). Other research work, consistent with these outcomes, were conducted on *Good Teaching* that promoted quality learning (Darling-Hammond & Baratz-Snowden, 2007; Dinham, 2006; Harris & Sass, 2009).

The Impact of Instructor Factors on Student Attitudes

In the *Characteristics of Successful Programs in College Calculus* (CSPCC) project (Bressoud et al., 2015), the first nationwide study of college-level calculus, student success in Calculus I was correlated to passing rates, persistence onto Calculus II, and changes in confidence, interest, and enjoyment of mathematics. Applying factor analysis to the project survey data, Sonnert and Sadler (2015) identified a dependent variable composite called *Mathematics Attitude* based on the students' mathematics *persistence, confidence*, and *enjoyment*. Sonnert and Sadler similarly identified three composites of pedagogical characteristics, independent variables or factors, in their study. The first factor had 22 survey components, including items such as "My calculus instructor acted as if I was capable of understanding key ideas in calculus" and "Assignments completed outside of class were challenging but doable." These items were traditionally regarded as good teaching practices; Sonnert and Sadler appropriately called this factor *Good Teaching*. Another factor had 14 survey items including, "Assignments completed outside of class time were submitted as a group project" and "How frequently did your instructor require you to explain your thinking on your homework?" The kind of pedagogical practices that spark such survey questions tended to lean toward reform and progressive approaches; Sonnert and Sadler suitably called the factor *Ambitious Teaching*. The third factor, not directly examined in this study, was related to technology; thus, labeled by Sonnert and Sadler, the *Technology Factor*.

Good Teaching Has a Positive Effect

Sonnert and Sadler's (2015) statistical analysis on CSPCC survey data asserted changes for *Mathematics Attitude* were all negative. Three effects of the variables were strong and pervasive: the students' initial attitude and the students' strong prior mathematical experience and preparation. *Good Teaching* had a positive effect and *Ambitious Teaching* had a small negative effect. Technology use had no influence on students' attitudes toward mathematics. With respect to institutional characteristics, Sonnert and Sadler analyzed the four composite factors including *Student Centeredness*, *TA Quality factor, Tutoring Center* provisions, and *Technology Use.* These eight composite factors for both instructional pedagogy and institutional characteristics made up the variables of interest. Sonnert and Sadler (2015) concluded tutoring centers had a bigger payoff than any of the other composite factors in their study of all the survey data from the CSPCC project, the first nationwide investigation correlating student success to characteristics of learning and teaching Calculus I.

Understanding Components of Good and Ambitious Teaching

Mesa et al. (2015) discussed the importance of *Good* and *Ambitious Teaching*, especially for women and minorities and posited inquiry-based learning of mathematics had a positive effect on affective gains for women and minority students (Laursen et al., 2014; Rasmussen & Kwon, 2007; Stephan & Rasmussen, 2002). This research informed the survey and interviews for the CSPCC study and resulted in the following three components being incorporated into the study: acknowledging students in classroom interaction, encouraging and available instructors, and providing fair assessments. Mesa et al. (2015) proposed mathematics departments support the practice of *Good Teaching* by maintaining a positive classroom environment with a positive attitude, pacing lectures appropriately, setting high standards, being available for students' questions, and responding to student needs.

Ambitious Teaching (Larsen et al., 2015) was defined as a composite of pedagogical characteristics including requirements for students to explain their answers, group projects, and the inclusion of unfamiliar problems both in homework and on exams. Further, *Ambitious Teaching* tended to decrease reliance on lectures. These characteristics were correlated and were independent of another composite pedagogical factor, *Good Teaching.*

In their effort to capture what children needed to learn mathematics successfully, Kilpatrick et al. (2009), posited five strands or components of mathematical proficiency to capture all aspects of competence, knowledge, and facility in mathematical learning. The five strands include:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations;
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification; and finally
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (Kilpatrick, et al., 2009, p. 116)

In their work on mathematics instructions, Lampert et al. (2010) defined *Ambitious Teaching* as teaching that incorporated these five components of mathematical proficiency.

In their study using MAA's study of CSPCC, Larsen et al. (2015) examined the relationships between *Ambitious Teaching* and retention and changes in attitudes and beliefs. Results indicated there was some promise of improving student persistence to continue to Calculus II with a combination of *Ambitious* and *Good Teaching* (Larsen et al., 2015). Larsen et al. asserted there were significant methodological challenges for research attempting to relate *Ambitious Teaching* to changes in student attitudes and beliefs. Additionally, they advised case studies indicated *Ambitious Teaching* practices required substantial institutional support and advanced knowledge, skills, and beliefs on the part of instructors.

Sonnert and Sadler (2015) determined the *Ambitious Teaching* composite factor had a small negative effect on student attitudes and beliefs. In terms of relative effect size, however, the positive effect was nearly three times as large for *Good Teaching* as for *Ambitious Teaching*. In the *Progress through Calculus* (PtC) project (Rasmussen et al., 2019), a follow-up study grounded on insight gained from CSPCC (Bressoud, 2015), it was found successful institutions encouraged the use of active learning strategies as a primary instructional approach and promoted challenging courses that required students to engage in conceptually-oriented content—both characteristics of *Ambitious Teaching*. Although the benefits of *Ambitious Teaching* were evident, instructors and institutions needed to be aware of the challenges of implementation. In their efforts to improve courses and course instruction through *Good* and *Ambitious Teaching*, Zazkis and Nuñez (2015) proposed faculty members hold a common vision of specific knowledge and skills goals which would include learning outcomes students should have attained from a course. In the case of the community college that served as the setting for this research, these student learning outcomes include communication and critical thinking.

The Commognitive Framework

Defining Commognition

Sfard (2020) offered an explanation or definition of commognition as follows: "*Commognition*, the portmanteau of *communication* and *cognition*, is the focal notion of the approach to learning grounded in the assumption that thinking can be usefully conceptualized as one's communication with oneself." Sfard (1998, 2001, 2007, 2008, 2009) critically reviewed the usefulness of the behaviorist, cognitivist, and acquisitionist traditions in the study of learning processes. Inspired by Wittgenstein (1953) and Vygotsky (1978), Sfard favored the participationist approach and its metaphorical nuances for her Commognitive Framework. Sfard claimed (2008, 2009) individual thinking and interpersonal communication were inextricable in describing the same phenomena.

I begin this section with a focus on learning informed by Sfard's (1998, 2001, 2008, 2009) bird's-eye view on the current conceptualization of learning which, although rich and diverse, can be divided into two broad but distinct categories, learning-as-acquisition and learning-as-participation. I follow by expounding on Sfard's claim that different types of communication are called discourses and her further assertion that mathematics is a discourse. Subsequently, I present the four characteristics that distinguish mathematical discourse, and finish the section by offering a discussion of critiques of the Sfard's Commognitive Framework.

Metaphors Underlying Theories of Learning

Sfard (1998, 2009) asserted a researcher's perspective and a teacher's practice depend on how they talk about teaching and learning in terms of the most fundamental level of thinking, implicit assumptions, values, and beliefs—the underlying metaphor for their way of viewing learning. Sfard discussed at length the primacy of metaphors citing Michael Reddy's seminal work, "The Conduit Metaphor," and posited that a metaphor was a conceptual transplant crossing disciplinary boundaries without notice and a conduit that grows new knowledge from old knowledge. She identified two metaphors for learning that conducted and gave guidance to learners, teachers, and researchers: the acquisition metaphor and the participation metaphor. With discourse, as an intervention for learning, having gained substantial basis among cognitive scientists and educational researchers, investigations were dominated by the participation metaphor, while the acquisition metaphor maintained its stance for the traditional choice.

Sfard (1998, 2009) proposed the two approaches to learning, learning-as-acquisition and learning-as-participation, could coexist and that the language of a participationist and that of an acquisitionist were not incompatible but were instead incommensurable. The participation and acquisition approaches were different in their unit of analysis, in their resolution, and in the roles attributed to direct contact with the nature or human agency through interactions with others. Learning in either metaphor translated to change; however, the quandary and the difference was rooted in the question, *in the process of learning, what was it exactly that changed?*

Based on the prominent work of Jean Piaget (1954), during a learning activity, mental entities known as schemes were constructed. In the acquisition metaphor, concepts were the basic unit of knowledge considered as private property, accumulated to form richer cognitive structures. The common image of the human mind was that of an empty vessel filled with cognitive structures such as schema. With the use of terms like “knowledge acquisition” and “concept development,” the learner took the role of the owner of their constructed entities or commodities within frameworks including moderate to radical constructivism, interactionism, or sociocultural theories. Research topics grounded on the acquisition metaphor have transformed from studies that focused on passive reception, active construction, and transfer of concept from a social to an individual plane; to studies that focused on internalization by the learner; and more recently, studies that focused ongoing process of self-regulation and interaction with peers, teachers, and texts (Sfard, 1998, 2009).

Against the backdrop of the prevalent tradition of learning-as-acquisition, a new metaphor emerged that signaled a change with use of terms such as *participation*, *communication*, *collective reflection*, *reflective discourse*, and *dialogue* by authors like Lave & Wenger (1991) and Rogoff (1990) who recognized peripheral participation and apprenticeship in thinking (Sfard, 1998). The existence of permanent entities gave way to the harbinger of action in terms like *knowing* and the image of permanence, indicated by the acquisitionist terms like *having*, was replaced by the force in constant flux of *doing*. Table 1 provides a schematic comparison of the acquisition and participation metaphors. The focus turned to learning activities considered and not separated from their context, culture, situation, and social mediation. The basic unit of analysis used in research grounded on the participation metaphor was—discourse.

Table 1

The Metaphorical Mappings

Acquisition metaphor		Participation metaphor
Individual Enrichment	Goal of learning	Community building
Acquisition of something	Learning	Becoming a participant
Recipient (consumer), (re-)constructor	Student	Peripheral participant, apprentice
Provider, facilitator, mediator	Teacher	Expert participant, preserver of practice/discourse
Property, possession, commodity (individual, public)	Knowledge, concept	Aspects of practice/discourse/activity
Having, possessing	Knowing	Belonging, participating, communicating

Note. Reprinted from "On Two Metaphors for Learning and the Dangers of Choosing Just One," by A. Sfard, 1998, *Educational Researcher, 27*(2), p. 7. Copyright 1998 by JSTOR.

Discourse as a Type of Communication

Inspired by Vygotsky (1978), Sfard's (2008) definition of thinking as a form of communication was derived from the participationist assertion that individual human development presupposed patterned collective activity. This basic commognitive tenet gave rise to an operational definition of communication as non-objectified, observable activities and behavior. There were many types of communication that vary with context differentiated by the objects they refer to, mediators used, and metarules employed by the community, participants, or actors. Central to the commognitive approach to learning was that discourses were dynamic and were time-dependent processes that were in constant flux with cyclic mechanisms of discursive change. Sfard (2008) described the recursive growth in complexity as a chain of expansion and compression. Although the term *development* under the traditional psychological lens referred to an inner change in the actor, the term *development*, under a commognitive lens, referred to a change in discourse, a modification of activity which can be public, such as communicating with others, or private, such as thinking (Sfard, 2012).

Mathematics as a Distinct Discourse

Different types of communication were referred to as discourses. *Discursive action* and *communicational* were terms used interchangeably by Sfard. Because of the specificity of this type of communication, a discourse can bring some actors together and exclude others. Objects, mediators, and metarules distinguished discourses from each other. Given the commognitive vision and the definition of discourse, as a well-defined form of communication, Sfard (2001, 2008, 2012, 2015) posited that mathematics is a form of discourse. Sfard offered four characteristics that distinguish mathematics as a discourse: word use, visual mediator, routine, and endorsed narrative (Ben-Yuhada et al., 2005; Sfard, 2008).

What Makes Mathematical Discourse Distinct?

Word use considered how participants used mathematical words. In mathematics, these were mainly, although not always, shapes and quantities (Sfard, 2008). This was inclusive of mathematical terminology like *negative number* or *derivative* or colloquial terms with specific meaning in mathematics, such as *open*, *continuous*, and *group*. Word use signified what the user uttered and saw in the world. However, what a speaker intended may not be equivalent to what the listener accepted. In the case of the word use of the term *derivative*, what the instructor intended to portray as the derivative may not be what the student accepted to be the derivative (Park, 2011). The four-stage model of the development of word use included passive, routine-driven, phrase-driven and object-driven word use.

Visual mediators were visual objects used as metadiscursive tools to identify the object of talk and to coordinate communication. Mathematical objects rendered by various symbolic, iconic, and visual representations mediated mathematical discourses (Sfard, 2008). Derivatives, for example, were visually represented as a dynamic object with icons of a vehicle or moving gears. The graph of a tangent to a graph of a nonlinear function, a graphical form, or the limit process of a different quotient, a symbolic form, were common visual mediators representing the derivative.

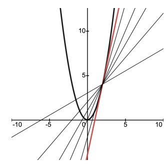
The nature of and the inner structure of mathematical *routines* were thought of as the "anatomy of mathematizing" (Sfard, 2008, p.220). Sfard used the term *mathematizing* to designate engagement in mathematical discourse. The actors in this discourse were referred to as *mathematists*. Object-level rules delineated regularities in the behavior of communicational objects. In contrast, the ordered, regular nature of the *mathematists*' actions were reflected by metarules. The recursive patterns in the mathematists' course of action were described by a set of metadiscursive rules called *routines*.

The three subsets that defined routines were: (a) the applicability conditions, (b) the course of action (procedure), and (c) the closing conditions. Routines were categorized into two kinds of metarules, when- and how-routines. The how-routine determined the course of action and the when-routine determined situations when action began (applicability condition) and when action came to closure (the closing conditions.) Depending on the task accomplished (the closing conditions or closure,) three types of discursive routines emerged: exploration, deeds, and rituals. Deeds were methods to change objects; explorations added to theory; and rituals were socially oriented. The how-routine was often individualized before the when-routine (Sfard, 2008).

Finally, the fourth characteristic of a mathematical discourse was *endorsed narrative.* These included texts describing processes and objects, and the relationships between them, subject to community endorsement (or rejection.) “In the case of scholarly mathematical discourse, consensually endorsed narratives are known as mathematical theories, and this includes such discursive constructs as definitions, proofs, and theorems” (Sfard, 2008, p. 134). However, in a class, students endorsed their narratives, and that of their peers, which were not necessarily endorsed by the mathematical community. Table 2 offers a brief description of the four characteristics with examples in the context of a discursive activity about the derivative concept.

Table 2

The Four Characteristics of Mathematical Discourse

Characteristic	Example in a Derivative Context
Word use includes the use of mathematical terms.	"the derivative is a limit;" "rate of change;" "secant and tangent lines;" "velocity"
Visual mediators include graphs, diagrams, or any visual representation of a mathematical object.	 ; $\frac{dy}{dx}$; $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$
Routines include well-defined practices or repetitive patterns in the discourse.	As she has done previously with similar problems, a student finds a derivative function by first graphing the basic function; then determining the derivative; and finally graphing the derivative.
Endorsed narratives include written or spoken text describing processes and objects and relationships, subject to approval or rejection (e.g., theorems and definitions in formal mathematics.)	"the derivative of a function is also a function," "the slope of a tangent line is the derivative function value at a point," $f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$

Critiques of Sfard's Commognitive Framework

Critiques of Sfard's Commognitive Framework tended to fall under two categories including critiques that had direct implications in the classroom for the learner and practicing teacher and critiques with reference to topics peripheral to mathematics education. Critiques of commognition related directly to student learning and the practice of teaching were immediately relevant to this investigation—thus the focus in the discussion that follows. The critiques spanned three points of view: (a) from the perspective of researchers who compared and contrasted their research to Sfard's central empirical analysis supporting the commognition perspective in her book (2008); (b) from the perspective of an educator who questioned the nonduality aspect and rote learning implication of commognition; and (c) from the perspective of a researcher who aimed to further analyze Sfard's Commognitive Framework in the context of Computer-Supported Collaborative Learning (CSCL) as well as sociocultural theory.

Necessity Principle vs. Commognitive Conflict

In their critique, Rabin et al. (2013) discussed the similarities and differences between Sfard's (2008) analysis, and their study, of the episode of a teacher introducing negative numbers to students. Sfard's commognition perspective, particularly commognitive conflict, was based on her in-depth work with young children's foundational concepts of mathematical cognition when the children were first introduced to negative numbers. Rabin et al. argued the use of commognitive analysis implicitly labeled learning difficulties as epistemological rather than didactic obstacles.

The participants in Rabin et al.'s study, in contrast to the episodes for Sfard's analysis, were 15-year-old students in an algebra class with their teacher, focused on the topic of negative integer powers in exponential functions. Rabin et al. established their argument on the Duality–Necessity–Repeated Reasoning (DNR) theoretical framework and disagreed with Sfard's assessment of children's discourses about negative numbers as an example of an "incommensurable discourse" (Sfard, 2007, p. 597). Based on what Sfard's termed as *commognitive conflict*, Sfard posited students adopted a new and foreign discourse and deferred understanding. To the contrary, Rabin et al. argued such instances provided students opportunities to address the epistemological challenges while minimizing didactic obstacles. They further argued toward pedagogical improvements as credible and productive interventions. Framing their argument within the Necessity Principle of DNR, Rabin et al. propounded teachers, being expert guides and translators, possessed knowledge of what necessitated a particular mathematical discourse historically and what necessitated the discourse pedagogically (Rabin et al., 2013).

Dualist–Unobservable vs. Nondualist-Observable

Sfard's nondualism position on cognition and communication and the value placed on rote learning, vis-à-vis commognitive conflict, were the focus of Wing's (2011) critique. Wing posited that in contrast to Sfard's nondualist realization tree and observable objects-processes, the dualist notion of concept image was useful as a reminder of the affective aspect of the developing individuality of a child's thinking—often not observable. However, Wing recognized Sfard's acknowledgment that subjectifying discourses was an exploration that remained unaddressed within the Commognitive Framework. Furthermore, the idea that students adopted a new and foreign discourse and deferred understanding, argued Wing, was contrary to his long-held position that rote learning had no value and was inimical to understanding. Nevertheless, Wing upheld Sfard's work on the inner mechanisms of discourse development as authentic and significant. He added that, from Sfard's commognition perspective, he realized situated learning was necessarily coupled with situated understanding, or lack of understanding, a necessary interim phase in learning.

Group vs. Dyadic Interaction

Stahl (2008), in his critique, situated Sfard's empirical analyses within the discourse of Computer-Supported Collaborative Learning (CSCL). This was of particular interest to my investigation due to the collaborative nature of the computer-supported discursive methods used in Stahl's work. Stahl argued that Sfard's commognition perspective, based on her book *Thinking as Communicating: Human Development*, *the Growth of Discourses and Mathematizing* (Sfard, 2008), did not account for the value of small group mechanism in shared discourse nor did the Commognitive Framework account for computer-mediated interactions in online environments. Contrary to Sfard's definition of the unit analysis as the discourse, Stahl asserted that current technologies allowed for scaling up to group interactional dynamics along a longitudinal timeline, while maintaining high quality data of multi-modal observations, in contrast to Sfard's dyadic interactions, and at times individual utterances as discourse, in a single session.

Another commentary Stahl (2008) offered related to Sfard's discussion of the perspective of a researcher guided by the Commognitive Framework. While Stahl agreed research necessitated a removed analytic perspective, he maintained the importance of differentiating this from the behaviorist or cognitivist perspective, which acknowledged only objects physically observable and hypothetical mental schemes. Further, Stahl posited that, by incorporating collaborative learning and computer-supported discourse, CSCL learning problematized mathematical discourse with explicit complexity and mediation. In order to explore discourse from an outsider's metadiscourse, an analyst, Stahl asserted, must possess both human understanding and familiarity with the "form of life" of students (p. 6) as well as competence in the specific discourse.

Topics Peripheral to Mathematics Education

Finally, in critiques related to topics peripheral to mathematics education, Sfard's Commognitive Framework was situated within the discourse of socio-cultural theory. Stahl (2008) contended that commognition did not account for phenomenon in the broader spectrum. In addition to scaling up to include complex group cognitive processes in mediated environments, Stahl proposed Sfard's unit analysis encompass "what activity theory calls the activity structure or actor-network theory identifies as the web of agency," (p. 367) including resources from culture, social institutions, collective rememberings, and power relationships. Felton and Nathan (2009) and Walker-Johnson (2009) argued the commognitive paradigm, with its broader utility, can lend insight about power relations and status. Sfard's reliance on classroom episodes involving children provided a limited window to mathematical discourse settings, Felton and Nathan contended, and ignored the argument that social markers such as race, class, and gender influence engagement in discourse. Walker-Johnson (2009) echoed Felton and Nathan's sentiment and further questioned Sfard's use of the terms *alienation*, *objectification*, and *subsumption* in the context of the historiographies of mathematics in terms of Marx's political economy. Stahl also questioned the use of the terms *alienation* and *reification*, borrowed he asserted, as Walker-Johnson also did, from social theory, specifically fetishism of commodities. Related to mathematics, Walker-Johnson questioned to what extent such pedagogy impacted mathematical curriculum and communications, particularly that of students. Next, I introduce a complex component of the Calculus I curriculum on which my study focuses—the derivative.

Framing Students' Thinking About the Derivative Concept

Bressoud et al. (2016) offered a survey of research dealing with cognitive development from the 1980s to the present focused on students' thinking about calculus concepts based on two frameworks: the Concept Image and Concept Definition (CID) and the Three Worlds of Mathematics (TWM). CID (EMS-Committee of Education, 2014; Tall & Vinner, 1981; Vinner & Hershkowitz, 1980) framed the student's individual cognitive representation related to the concept in contrast to the formal mathematical definition of the concept. All mental attributes linked to the concept were integral in the concept image. When conflicting elements of the concept were evoked at the same time, a cognitive conflict ensued. In comparison, TWM (Tall, 2004) was a useful categorization of three mathematical contexts: the conceptual-embodied, the proceptual-symbolic, and the formal-axiomatic. In the world of conceptual-embodied, students used their senses to construct mental conceptions using physical perceptions; in the proceptual-symbolic world, actions on mathematical symbols operated as process and concept (procepts); and in the third world, the use of verbal reason transformed to a formal language of mathematics for advanced mathematical thinking. In this section, I offer a brief survey of research (Bressoud et al., 2016) and discuss students' difficulty with and misconceptions about the derivative concept. Furthermore, I discuss what it means to understand the derivative and research on students' and an instructor's discourse on the derivative.

Students' Difficulty with Understanding the Derivative Concept

Orton's (1983) investigation was one of the early seminal qualitative studies to analyze and describe students' difficulty with derivatives. Orton outlined the errors and misconceptions associated with the students' understanding of derivatives and algebra in detail. The types of errors made by students almost 40 years ago, which Orton classified as arbitrary, executive, and structural, were the same or similar errors made and misconceptions held by our students in today's calculus classrooms. Furthermore, Orton's conclusions and implications for the curriculum and teaching methods resulting from his investigation, were consistent with those offered in recent mathematics education research on calculus.

Listed below are students' areas of difficulties in understanding derivatives evidenced by Orton almost 40 years ago. Orton's findings regarding student difficulties with derivatives are aligned with research included in Bressoud et al.'s survey. Contemporary researchers have studied and advanced similar or the same topics. Orton asserted the following difficulties that students had with derivatives:

- Students had difficulties with the fundamental idea of ratio and proportions, which is the basis for their difficulties with rate of change (Byerley et al. 2012; Confrey & Smith, 1994). Although Confrey and Smith argue that rate is different from ratio.

- Students had difficulty connecting the graph of a function with the graph of its derivative (Asiala et al., 2001; Aspinwall et al., 1997; Baker et al., 2000; Borgen & Manu, 2002, Ferrini-Mundy & Graham, 1994; Nemirovsky & Rubin, 1992).
- Students had difficulty transitioning from understanding the derivative at a point to the derivative as a function (Zandieh, 1997; Zandieh & Knapp, 2006; Habre & Abboud, 2006; Park, 2013).
- Students had difficulty applying computed derivatives to the determination of the equation of a tangent line (Bingolbali et al., 2007).
- Students had difficulty with making sense of symbolic representations (Park, 2011). Park (2013) is an extension of his dissertation (Park, 2011) which is not part of Bressoud's research survey.

There were trending topics implied but not emphasized directly by Orton. In "The Calculus Sequence" in the *Curriculum Guide to Majors in the Mathematical Science* (Zorn et al., 2015), the importance of covariation (Confrey & Smith, 1994; Thompson & Carlson, 2017; White & Mitchelmore, 1996) and the idea of accumulation being more readily cognitively accessible by students than differentiation (Apostol, 1967; Thompson et al. 2013) were addressed. The latter implied a non-traditional sequencing of topics taught in calculus with integration coming before differentiation. This change in sequence would have had substantive implication to my investigation, particularly because the commognitive approach was at the study's center.

Misconceptions Transcend Space and Time

The evident alignment of current research with Orton's (1983) research discoveries suggested the difficulties and lack of understanding students have persisted through time. Sfard's (2008) posited five quandaries: the quandaries of number, abstraction, misconceptions, learning disability, and understanding. Sfard asserted that misconceptions were among the most thought-provoking phenomena. Misconceptions were evident when a student used a concept, for example the derivative function concept, that was contrary to the way an expert would use the same concept. This action was interpreted by researchers as a tendency for students to create their own meanings that were not context appropriate.

What was intriguing was the same misconception, for example that high school students believe that functions, including derivative functions, are algorithmic, were held by students who live in different countries, speak different languages, and were taught by different teachers using different curricula and textbooks. Without understanding the function concept, students recited the universal definition of a mathematical function. Sfard queries, "How is it that misconceiving [students] agree among themselves about how to disagree with the definition?" (Sfard, 2008, p. 17). Misconception appeared to have transcended both space, time, and individuals.

What Does it Mean to Understand the Concept of Derivative?

Zandieh (1997) utilized multiple cognitive development frameworks, including Concept Image and Concept Definition (CID) and Sfard's (1992) process or operational to static structural conception, to posit what understanding the concept of derivative meant. Sfard (1992) framed three transition stages from operational to structural: interiorization, condensation, and reification. Interiorization was described as a familiarization process through which a student was able to step through key procedures. Condensation was described as a generalization process in which students imagined the procedures holistically, not relying on a step-by-step process. Reification was described as the transformation of the process as an object—objects students were able to again act upon.

Like the discussion regarding research discourse in this chapter, "What is Theory and Why Do We Need It?" the operational to structural conception was autopoietic. Both ideas originated from Sfard (1992, 2018). Each process was reified into an object acted upon by other processes to form a chain of process-object transitions. In Zandieh's framework, the derivative concept has three process-object layers, ratio, limit, and function, described in four representations, symbolic, numeric, graphic, and verbal. Zandieh also includes physical contexts such as velocity and acceleration.

Zandieh (1997) designed a concise visual coding process to document her research participants' utterances relevant to their understanding of derivative. Her desire gave form to a diagrammatic way to summarize the three-layers process-object structure of the derivative. Zandieh used concentric circles to visualize the three layers of the derivative concept. The meaning for each of the three layers of the circle diagram, circle-diagram examples, and an example of Zandieh's use of circle diagrams to code her interview transcript data have been provided in Appendices G, H, I, and J, respectively.

An Instructor's and Student's Discourse on the Derivative

Park (2011, 2013) adapted Zandieh's (1997, 2000) matrix-form framework and adopted Sfard's Commognitive Framework to investigate a persistent student difficulty with the derivative concept: students have difficulty transitioning from understanding the derivative at a point to the derivative as a function. The results indicated that use of exact mathematical terms by the instructors, and purposeful discussions on areas of difficulty with regards to the derivative, opened opportunities for students to learn and understand the derivative concept. Zandieh's (1997) framework on the derivative concept and Park's (2011) conclusions, undergirded by Sfard's (2008) Commognitive Frameworks, highlighted theoretical perspectives and research guiding my project. Lastly, I discuss the realization of student success that these theoretical perspectives may have rendered possible as well as implications for my action research inquiry.

Implications for Student Success and my Action Research Inquiry

Buller (2015) offered, "Trust the people you work with, empower them, and recognize their efforts to be creative. [M]eaningful change is all about the culture and the culture is—all about the people" (Buller, p. 238). As I chose theories, concepts, and frameworks, I was mindful of the culture of the actors who shared ownership of the endeavor and those affected foremost. Students engaged in their roles as the *mathematists*, participants, and interlocutors. I realized my role as a practitioner, interlocutor, researcher, and participant, and observed from the inside and outside, my own action research. I strived to be effective in my role as a practitioner and researcher. I integrated and operationalized *Good Teaching* and *Ambitious Teaching* practices in my Calculus I class and shared my experiences and ideas with both students and colleagues.

Show Me the Learning

The commognitive or communicational approach, tempered with pragmatism, renders mathematical objects observable as discursive objects (Wing, 2011). Understanding of concepts—observable? That is an irresistible notion. Recall my statement-question combination posed earlier in this chapter:

> Learning in either the acquisitionist or participationist metaphor translated to change; however, the quandary and the difference was rooted in the question, *in the process of learning, what was it exactly that changed?*

This study embraced and adapted Sfard's Commognitive Framework (2001, 2008) with its promise of defining thinking in a way that is accessible not only practitioners and researchers but more importantly, to students. True to the participationist roots of commognition, Sfard defined thinking as a connection through learning or becoming a participant, acting on knowledge through practice of discourse, and demonstrating knowing by communicating (Wood, 2008). When Sfard defined thinking as communicating, she asserted that learning, as a result, can be defined as a change in discourse. For example, Nasrynn, a Calculus I student who experienced an intervention, founded on the development of calculus discourse realized a gradual change in her understanding of the derivative concept. Accordingly, Nasrynn's understanding and learning related to the derivative unit was observable to her peers and to me, her instructor, through a gradual change in her discourse related to the derivative.

Several studies have reported the significance of word use in children's thinking (Cobb et al., 1993; Sfard & Lavie, 2005; Sfard, 2008). However, there were relatively fewer studies investigating the role of word use, visual mediators, routines, and endorsed narratives in students' thinking about advanced mathematical concepts like the derivative (Park, 2011). There were even fewer studies in which the researcher–practitioner offered an innovation that aimed to motivate students to *mathematize*, through a commognitive approach, buttressed by design frameworks to ground the concept development, course content structure, and cognitive load for students. In the spirit of action research, this study offered a solution to a specific problem of practice in a particular context: increase completion of gateway mathematics, particularly Calculus I, and improve equity. This study also offered a potentially generalizable solution to the larger, more complex, problem of success in calculus or other gateway courses for STEM disciplines.

Show Equity: See and Listen to Their Stories

If our learning community in the classroom realized the autopoietic system of mathematical metadiscourses, I imagined the student narratives bloomed and blossomed giving rise to a symphony of endorsable narratives. If the autopoietic nature of a discursive mathematical classroom was indeed realizable, the most inviting implications were not the students' realizations of the three process-object layers, ratio, limit, and function toward understanding the derivative. Nor were the general applicability of frameworks that buttress the understanding of the derivative concept the most alluring aspect of this investigation. The most inviting and irrefutable implication was the transformation of the classroom from a sage monologue to a multivocal learning space—rooted in equity and mutual respect through discursive actions enlightened by commognition. This investigation offered an insight of how we—students, transformed into *mathematists*, and instructors alike—*mathematize*. That is, how we became participants in the discourse of mathematics. More importantly, this study provided a lens to observe how individualization of communication broadened, reinforced, and diversified one's bond with others (Sfard, 2008). As humans, if we were indeed "storytelling animals" (Fisher, 1984, p. 1), "mathematizing was just one special type of storytelling activity" (Sfard, 2008, p. 222). Next, in chapter 3, the setting and participants, my role as practitioner and researcher, my innovation and frameworks that undergird the innovation, and the collection and analysis of data are presented.

CHAPTER 3

METHOD

No research without action, no action without research.

—Kurt Lewin

What does the transformation of a student's understanding of the derivative concept look like? Nasrynn enrolled in a calculus class designed to immerse her in mathematical discourse. In contrast, Yasmynn enrolled in a traditional calculus class where her activities consisted primarily of traditional calculus assignments. The key difference was in the experienced activities that engaged Nasrynn and Yasmynn in mathematical discourse both verbal and nonverbal, individual and social. As a result of the difference in activities, were Nasrynn's and Yasmynn's performance in and attitude about calculus changed? Were Nasrynn's and Yasmynn's learning significantly different? Did the transformation of their understanding of the derivative concept look significantly different? The intervention for my experimental calculus class was grounded on and designed using multiple frameworks to immerse the experimental group in mathematical discourse—the core theoretical framework asserted that communication is tantamount to thinking.

Mixed Methods Design: Purpose and Research Questions

The purpose of this experimental mixed method study was threefold: (a) to determine the role of classroom communication, specifically mathematical discourse in a student's understanding of the derivative concept; (b) to investigate how *Good* and *Ambitious Teaching* impacted students' attitude in calculus; and (c) to determine if there was a significant and observable difference in performance and in attitude of students in an experimental calculus class designed to motivate active engagement in mathematical discourse in contrast to students in a traditional calculus class. Given the purpose of the study, four research questions guided its conduct.

RQ1: How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?

RQ2: To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?

RQ3: How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?

RQ4: To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?

If the purpose of action research was to learn through action, the following sections reflected on the acts of developing, planning, implementing, analyzing, and imagining what my experimental class looked and felt like to my calculus students in contrast to the control class. In this chapter, I introduced the setting, participants, innovation, the data collection and the instruments, and the data analysis. An overview for the research design has been presented along with a timeline for implementation. Next, I offer a view of the environment for this study.

Setting

This study was conducted in fall 2021 in three Calculus I classes at a community college located in the Southwest. Due to the unprecedented effects of the COVID-19 pandemic, all three classes were offered in a synchronous online environment.

The study was set against the background of features of our mathematics department operating in the largest of the sister colleges in our community college system. In fall 2021, the enrollment at our community college was 16,494 students, with 3,400 enrolled in classes offered through our mathematics department. The enrollment in Calculus I was 272 students and 26 of the 49 students initially enrolled in my three classes participated in this study. The Calculus I classes held during the prime daytime hours were taught by full-time residential faculty while the evening courses were often taught by adjunct faculty. Our college's calculus series included *Calculus I*, *Calculus II*, *Calculus III,* and *Differential Equations*. Mathematics departmental characteristics and practices dictated how classes were offered to our students.

Classroom

At our college, Calculus I was a five-credit course requiring five classroom hours per week. My first of three synchronous online calculus classes met Monday through Friday at 8:00-8:50 a.m.; the second class met on Monday, Wednesday, and Thursday at 5:35-7:00 p.m.; and the third class met on Tuesday and Thursday at 12:00-2:35 p.m. The 8:00-8:50 a.m. class served as the study control group. The 12:00-2:35 p.m. and 5:35-7:00 p.m. classes, combined, served as the experimental group. Next, I present course competencies and common course content, integral to the calculus environment.

Calculus Competencies and Course Content

Listed below were the Calculus I course competencies and content, common to all our district's community colleges, which directly related to the derivative concept.

- Analyze the behavior and continuity of functions using limits.
- State the definition and explain the significance of the derivative.
- Compute the derivative using the definition and associated formulas for differentiation.
- Solve application problems using differentiation.

The common content for all calculus courses is described in Table 3. As I have discussed in chapter 2 and in the *Innovation* section, in this study the concept of derivative was defined as a three-layer process-objects structure adapted from Zandieh's conceptual framework (1997) using multiple representations. The differentiation rules by-passed the three-layer structure. In addition, the application topics used the derivative as a tool in application problems; derivatives used as a tool is another instance where the three-layered structure was not used. In Table 3, therefore, section titles related to differentiation rules and applications were not separately provided. I presented only the four sections central to Zandieh's three-layer process-objects structure for the derivative concept definition.

Table 3

Content of the Derivative Unit in Calculus (Stewart, 2012)

Chapter	Section Title
Limits and Derivatives	The Tangent and Velocity Problems The Limit of a Function Derivatives and Rates of Change The Derivative as a Function
Differentiation Rules	All topics linked to differentiation rules including Derivatives of Polynomials and Exponential Functions, The Product and Quotient Rules, Derivatives of Trigonometric Functions, The Chain Rule, Implicit Differentiation, Derivatives of Logarithmic Functions, and Hyperbolic Functions
Differentiation Rules	All topics linked to differentiation rules that lean toward applications including Rates of Change in the Natural and Social Sciences, Exponential Growth and Decay, Related Rates, and Linear Approximations and Differentials
Applications of Differentiation	All topics linked to applications of differentiation including Maximum and Minimum Values, The Mean Value Theorem, How Derivatives Affect the Shape of a Graph, Indeterminate Forms and L'Hospital's rule, Summary of Curve Sketching, Graphing with Calculus and Technology, Optimization Problems, Newton’s Method, and Antiderivatives

Students were the key participants and their actions, related to this study, provided the primary source of research data. Participants are introduced in the section that follows.

Participants

An objective of this action research was to synthesize practice, research, and theory. To meet this objective, the primary actors were calculus students enrolled in classes I taught.

Calculus Students' Roles as Mathematists

The key participants in both the experimental and control group were my Calculus I students who were likely in their freshman or sophomore year, often aspiring to major in science, technology, engineering, or mathematics (STEM) disciplines. For a very few, this class was a terminal mathematics course. For most students, Calculus I was their first gateway mathematics course in the series of three calculus courses required for most STEM programs. Student participation provided both quantitative and qualitative data for this mixed-method study. Students in the experimental classes were expected to act as *mathematists* engaged consistently in mathematics discourse or in *mathematizing* (Sfard, 2008).

All students in the experimental and control groups were given the opportunity to participate in the quantitative data collection conducted through two instruments, the Derivative Concept Assessment, given as a pretest and posttest, as well as a pre- and post-Attitude Survey. All participants took the pre- and post-Derivative Concept Assessment in week 5 and at the end of the derivative unit, in week 7, respectively. All participants also took the pre- and post-Attitude Survey in week 3 or 4 and week 12, respectively. After the students completed the pre- and post-Derivative Concept Assessment, I recruited a sample size of 12 students, 4 control students and 8 experimental students, for the one-to-one interviews to collect qualitative data.

Recruitment details have been provided in the *Instruments* section of this chapter. The interviews were conducted in weeks 12, 13, and 15 of fall semester 2021.

Innovation

The innovation was premised on objectives that aimed at characterizing the change in the students' mathematical discourse that led to students understanding essential calculus concepts, particularly the derivative concept. Sfard's (2001, 2008) commognitive approach was central to the innovation. To achieve this goal, realizing what the calculus class looked and felt like to experimental students, in contrast to the control class, was my priority for the innovation.

In this section, I offer my innovation design process in an inverted fashion. First, I introduce the outcome of my design efforts, which started with the calculus experience I designed for the experimental group juxtaposed to the experience for the control group. Then, I follow with a brief overview of the four essential frameworks that undergird the design of my innovation. I present how I used the frameworks in the derivative unit of calculus. The frameworks and the process of designing this innovation for the derivative unit were generalizable to other units in calculus or in any academic course.

The Experimental vs. Control Group Experience

I implemented a time-proven and improved intervention in fall 2021 and collected and analyzed quantitative and qualitative data for the experimental and control groups. Tables 4 and 5 provide a summary of what the experimental and control group experienced in our calculus classes in fall 2021, respectively. The summaries depicted in the two tables include: (a) the experience label, explained later in the *Design Pattern* section; (b) a description of the activity; and (c) the expected deliverables from students as an observable outcome of the activity. In addition, the percentage weight of the student's final grade for each activity category was provided along with the percentage weight of each activity category of the student's final grade accounting for the derivative unit only.

Table 4

The Experimental Group Experience

Experience	Activity	Deliverable	% Class Grade | derivative unit only
Uncover	be informed: read and watch videos	timed asynchronous quiz	5 | 1.3[a]
Play	do *play* or basic procedural problems with reflections	*Play* packet	15 | 3.9[a]
Apply	do *apply* problems or applied problems with reflections	*Apply* packet	15 | 3.9[a]
Connect	meet team for *play/apply* discussions per specifications and specific *play* and *apply* problems to be discussed and specified prompts to be addressed	team discussion video YouTube URL	15 | 3.9[a]
Question	ask *play/apply*-related questions motivated by *connect* team discussion; questions will motivate whole-class Canvas discussion	1 *play*-related Q and 1 *apply*-related Q	5 | 1.3[a]
Realize	take team exams timed and asynchronous	parts 1 & 2 written *play*-focus and *apply*-focus, respectively; part 3 team oral validation for parts 1 and 2 written items YouTube URL; For final individual validation video	45 | 11.7[a]

Note. [a] This number indicates % of class grade based on point values of activities in the derivative unit only.

Table 5

The Control Group Experience

Experience	Activity	Deliverable	% Class Grade | derivative unit only
Uncover	be informed: read and watch videos	timed asynchronous quiz	10 | 2.6[a]
Play	do *play* or basic procedural problems	*Play* packet	17.5 | 4.55[a]
Apply	do *apply* problems or applied problems	*Apply* packet	17.5 | 4.55[a]
Question	ask *play*/*apply*-related questions motivated by *connect* team discussion; questions will motivate whole-class Canvas discussion	1 *play*-related Q and 1 *apply*-related Q	5 | 1.3[a]
Realize	take exams individually timed and asynchronous	parts 1 & 2 written *play*-focus and *apply*-focus, respectively	50 | 13[a]

Note. [a] Number indicates % of class grade based on point values of activities in the derivative unit only.

What is the Same and Different?

For both groups, there were five activities, *uncover*, *play*, *apply*, *question, and realize* which were either the same or had common elements for both control and experimental groups. The *connect* activity that engaged the experimental group in discourse development with a team of peers was experienced by the experimental group only. There were also discourse-specific tasks integrated in the common activities for the experimental group but were not required for the control group. The experimental group consistently participated in activities that motivated *mathematizing*, that is, in mathematical discourse, either individually or in teams, whereas the control group consistently engaged in traditional activities. However, the class discussions and lectures were consistent in all three classes for the control and experimental groups. Next, I discuss each activity experienced by the experimental and control groups.

Uncover

The *uncover* experience was the same for both experimental and control groups. The activity involved "uncovering" the course content by either reading our textbook, watching instructor videos, and/or leveraging other resources. Both experimental and control groups took a weekly untimed quiz given asynchronously online to demonstrate learning with respect to the most current topics of discussion. The pre- and post-Derivative Concept Assessments served as extra credit quizzes that were given in the same format as the weekly *uncover* quiz.

Play and Apply

Both groups solved exercises from our textbook (Stewart, 2012). Both experimental and control groups were required to solve and write out their solutions to the exercises. The exercises comprised both basic procedural *play* exercises and *apply* exercises. I provided a guideline that outlined what the students were expected to address when doing *play* and *apply* exercises. Both *play* and *apply*-type exercises were utilized by Reinholz (2014) in his intervention to promote explanations of basic to very challenging calculus ideas using the terms *opening problems* (OP) and *peer-assisted-reflection* (PAR) problems.

Both groups had to (1) write the problem statement; (2) declare variables and parameters with units; (3) show all relevant work; and (4) graph data appropriately. The experimental group was required to write a reflection for one *play* and one *apply* exercise per section covered. The control group did not have to engage in this reflective discourse activity. Our classes uncovered an average of three sections per week.

Connect and Question

The experimental students were placed in teams of four for the duration of the semester. The teams engaged in the *connect* and *question* activities. The control group did not engage in the *connect* activity but engaged in the *question* activity individually. The experimental group met their teams every other week for seven weeks throughout the semester outside of our synchronous online class meetings.

Two discussion meetings occurred during the derivative unit. There were guidelines provided for the *connect* activity. The guidelines described their deliverables and the actions they had to take as a team and as team members during their discussions. The teams were expected to deliver a discussion video and two questions related to one *play* and one *apply* exercise as an outcome of their team meeting. In addition, members shared links to their *play* and *apply* exercise sets with team members in a Canvas Discussion before their meetings. Criteria for eligibility to take team exams focused on the connect activities and team members sharing written work-in-progress on exercises with team members. While their team connect discussion was prompted by their individual effort on the *play* and *apply* exercise sets, the *question* activity was motivated by their team discussions.

The teams submitted their questions on a Canvas Discussion forum to be viewed by the entire class. The control group was also required to ask questions; however, they were not placed in teams. The control group students posted two questions, one for a *play* and one for an *apply* exercise, on a Canvas Discussion forum individually, while the experimental students were required to post two questions for the entire group. I created and posted a video in which I answered all questions posed by all my Calculus I classes. The questions served as signifiers, or prompts, for a whole-class Canvas Discussion when students opted to have a class discussion regarding students' questions and my responses.

Realize

The *realize* activity involved students taking midterm and final exams. The written parts, for the experimental and control group, were the same and were used to assess both control and experimental groups on their mathematical proficiency with basic procedural and applied problems. These assessments reflected the *play* and *apply* exercises given to both groups. The exams were given asynchronously. Individuals, as well as teams, had 60-hours to submit the exam after the exam was posted on Canvas. Both control and experimental groups took 2-part written exams that were consistent in difficulty to their assigned *play* and *apply* exercise sets.

In addition, for the experimental group, there was a mandatory part 3 exam-related activity that mirrored their *connect* experience. The experimental group participants were required to submit a video of their team discussion regarding selected items on their 2-part written exam. The list of validation items was provided after the deadline for submission of the written exam lapsed. The teams had 36 hours, after receiving the list of validation items, to submit a team oral validation video with similar specifications as their *connect* team discussion video. For their final exam, in addition to the team validation video, individual team members of the experimental group were required to produce an individual validation video.

A timeline for the derivative unit implementation, related to the module topics, and the actions of the experimental and control group, are provided in Table 6.

Table 6

Timeline for Derivative Unit Implementation

Module (M) Title and Sections	Timeframe fall 2021	Participants | Action/Experience E: experimental group C: control group PR: practitioner/researcher
M1: Welcome/orientation, and Precalculus review	week 1 Aug 23-Aug 29	E and PR | team check-ins, and review C | view orientation video, review
M2: Limits and Derivatives 2.1 The Tangent Line Problem 2.2 The Limit of a Function	week 2 Aug 30-Sep 5	E | *uncover*, *play*, *apply*, *connect*, and *question* C | *uncover*, *play*, *apply*, and *question* PR | instruct and observe
M3: Limits and Derivatives 2.3 Calculating Limits Using the Limit Laws 2.4 The Precise Definition of Limits *pre–Attitude Survey Sep 10*	week 3 Sep 6-Sep 12	E | *uncover*, *play*, *apply*, and take attitude survey C | *uncover*, *play*, and *apply* PR | instruct and observe, and collect qualitative data
M4: Limits and Derivatives 2.5 Continuity 2.6 Limits at Infinity	week 4 Sep 13-Sep 19	E | *uncover*, *play*, *apply*, *connect*, and *question* C | *uncover*, practice, and formative assessment, and take attitude survey practitioner-research | instruct, observe, and collect quantitative data
M5: Limits and Derivative 2.7 Derivatives and Rates of Change 2.8 The Derivative as a Function *pre–Derivative Assessment on Sep 21*	week 5 Sep 20-Sep 26	E | *uncover*, *play*, *apply*, and take pretest C | *uncover*, practice, and take pretest PR | instruct, observe, and collect quantitative data

Rebirths of an Innovation: Cycles 1, 2, and 2.5

Through my cycles 1 and 2 study, which were iterative cycles of action research integral to our EdD program, I initiated and continued the design of this innovation. In spring 2021 during cycle 2.5 study, I piloted components of the innovation in all my calculus and precalculus classes. The design was informed by results from cycles 1 and 2. My realization of the need to ground my research, in a theory and methodology in which discourse was central, literally fell in my lap as a centennial happenstance. In spring 2020, because of the COVID-19 pandemic, students and instructors rapidly adjusted to the new normal of synchronous online classes.

Because of this unfortunate pandemic, I shifted my research actions to carry on with my cycle 1 plan and timeline by appropriating rapid changes. For example, spring 2020 was the first time I gave students asynchronous exams with oral exam validations. Leveraging students' ease with video conferencing, I conducted check-ins, exam validations, and the interviews for my cycle 1 study—all online. My intervention for fall 2021 reflected the pilot intervention I implemented in spring 2021; however, in spring 2021, I developed a foundation of multiple frameworks consisting of tools and guiding processes to sustain the continuous improvement, utility, and generalizability of my intervention. Next, I introduce the four critical frameworks that gave form and shape to my innovation.

Framing an Innovation

Realizing what the experimental class looked and felt like to my calculus students, in contrast to the control class, was a priority for me as a practitioner-researcher. Realization required intimate contextual knowledge that prompted me to reframe my

Calculus I class iteratively through cycles 1, 2, and 2.5 with an eye toward effective and pragmatic frameworks that provided quality structure for the content and activities, necessary to incentivize learning content of a Calculus I class.

To design and undergird the structure of my Calculus I class for the study, I operationalized four frameworks simultaneously. The mainframe was Sfard's (2008) commognitive approach. The units for teaching the derivative were grounded in an object-process framework used to define what it means to understand the derivative (Zandieh, 1997). The online class was structurally framed by an architectural design approach (Hathaway & Norton, 2013) to organize and structure course content. Finally, an analytical frame was used to support students' opportunity to learn through given exercises and other tasks (White & Mesa, 2014).

Because Sfard's (2008) Commognitive Framework was the nucleus of this study, the details about the theory accompanied by the central tenets of commognition, formed a large part of chapter 2. A discussion of Zandieh's (1997, 2006) framework for the concept of derivative was also presented in chapter 2. The Commognitive Framework provided theoretical and methodological basis for this study, while Zandieh's three-layer process-objects: ratio, limit, and function, rendered the framework for the derivative concept. In the next sections I offer two frameworks that were not introduced in chapter 2. One framework was the design pattern approach to organize and structure course content. The other was an analytical framework to characterize the range of tasks that I chose for both my experimental and control classes.

Design Pattern and its Four Essential Elements

Indispensable in motivating participants to engage in mathematical discourse was content. Well organized and structured course content was a requisite to capture and sustain students' attention. In this study a design pattern approach (Hathaway & Norton, 2013), which focused on goals, objectives, and expectations for the learner, was adapted for the Calculus I course content structures, particularly the derivative unit. As a practitioner-researcher, my goal in using a design pattern was to conceptualize a generalized, reusable solution for other units uncovered in Calculus I, for example the integration unit. A generalized, reusable solution was also useful for consideration in my other classes, including Calculus II and III.

A design pattern was manifested through four key elements: the pattern name, the description of the problem, the core of the solution, and the pattern's sequence (Gamma et al., 1995). The *Pattern Name* alluded to the design problem, its solution, and consequences. The *Description of Problem* offered the context of the problem and explained the problem itself, often with a list of criteria satisfied before the pattern was applied. The *Core of the Solution* was articulated in the form of a solution, identifying the elements that made up the design and the relationships and collaboration between the elements of the design. Lastly, the fourth element, the *Pattern's Consequence*, was an account of applying the pattern in its use context. That is, this element enabled a designer to link design patterns to other patterns and other contextual ideas. Table 7 provides a summary of my design decisions and directly related the design pattern in the context of Sfard's Commognitive Framework and Zandieh's derivative concept framework for the innovation for this study.

Table 7

The Design Pattern Approach for Course Content Structure in Calculus I

Design Pattern	Calculus I Course Design
Pattern Name	*Process-Objects Nested Under the Derivative Concept*
Description of Problem	In the context of Calculus I, what was an appropriate content structure to motivate students to engage in mathematical discourse around Zandieh's (1997) derivative concept framework?
Core of the Solution	To enable learners to benefit from Zandieh's derivative concept framework, I chose Bloom's Revised Taxonomy, an established method of classifying thinking behaviors that also aligns with the commognitive approach. I used the experience labels *Uncover*, *Play*, *Apply*, *Connect*, *Question*, and *Realize* to align with Bloom's Revised Taxonomy and *Connect* to align with the interpersonal and intrapersonal communication core of Sfard's (2008) Commognitive Framework.
Pattern's Consequence	I had to consider the dimensions of online learning interactions. Hathaway and Norton (2013) considered learner-instructor interaction, learner-learner interaction, learner-interface interaction, facilitation, and presence. This component was more evident after the innovation was implemented.

The six intervention experiences designed for the experimental group: *uncover, play*, *apply*, *connect*, *question*, and *realize*, resulted from the use of the design pattern approach. These intervention activities offered a wide range of tasks for students in the experimental group as well as the control group. The five labels *Uncover, Play*, *Apply*, *Connect*, and *Question* were borrowed from activity labels used in our TEL 713

Advanced Qualitative class taught by Dr. Leigh Wolf and Dr. Lisa Yanez-Fox in fall 2020. Dr. Wolf generously offered and shared a group chat in which she and her colleagues discussed the development of these labels. The effectiveness of activities with these labels, coupled with factors influencing learning, was determined in the study.

To discern the differences in outcomes for the experimental and control groups, specifically during my analysis of the collected data, it was critical that I was aware of what potentially gave rise to these factors of influence. This was especially true for the calculus exercises, particularly those in the derivative unit, for this study. Therefore, a valid and reliable analytical framework, to determine the level of cognitive demand and to characterize the types of coursework in both experimental and control classes, was warranted.

Cognitive Demand and Characteristics of Coursework

White and Mesa (2014) posited that tasks given to students must be designed and presented with the characteristics and cognitive demand that align to instructional goals and expectations. The goal of my investigation was to determine what observable characteristics of and change in mathematical discourse lent itself to students understanding essential calculus concepts through Sfard's (2001, 2008) approach of thinking-as-communicating. Therefore, cognitive demand inherent in the calculus exercises was designed to be consistent. Characterizing the cognitive demand of the activities and tasks ensured a consistency in my instructional learning objectives built into the calculus exercises given to both the experimental and control groups. A table outlining the categories of task orientation, definitions, and examples, adopted from

White and Mesa (2014, p. 680) has been included in Appendix A, outlining the orientation with the definition guided my instructional expectations.

Additionally, the types of coursework were characterized for both the experimental and control groups. The coursework represented both the instructor's intentions and the students' opportunities to learn. These were potential factors of influence for learning that I kept in mind as I analyzed the difference in outcomes between the experimental and control groups. Two characteristics that White and Mesa used were grade weight, that is, percentage of the class grade, and the resources available or those that students were allowed to use while engaged in tasks. White and Mesa asserted that tasks provide students: (a) the opportunity to learn the content, and (2) the opportunity to demonstrate that they have learned the content. Matrices like Tables 4 and 5 were the outcomes and deliverables for my effort to outline the characteristics of all tasks for the experimental and control groups. Both have been provided in Appendix B and Appendix C, respectively.

Next, I offer descriptions of the instruments and the procedure followed by the quantitative data analysis, trustworthiness and credibility, and finish this section and chapter 3 with the timeline for the implementation.

Data Collection and Instruments

For this experimental mixed method design, first I collected quantitative data and followed by collecting qualitative data. I used two instruments, the Attitude Surveys and the Derivative Concept Assessment, to collect quantitative data. The qualitative data was collected using interviews.

Attitude Survey

Survey instruments, referred to as the Attitude Surveys, were adopted from the Mathematical Association of America (MAA) national survey (Bressoud, 2015) and served as an affective-scale measure for this study. The pre–Attitude Survey contained 56 questions. The aim of the survey was to identify students' attitudes and beliefs as they entered our Calculus I class. The students were surveyed about their beliefs relevant to their calculus prerequisite skills, previous preparation for calculus, and about the use of technology in mathematics classes. They were asked to identify their field of study and asked several questions about their anticipated time management for the semester. For example, the students were asked, "Approximately how many hours did you work at a job this semester?"

There were 19 survey questions that were crossover questions, appearing both on the pre- and post-Attitude Surveys. Three questions that explicitly asked about persistence to Calculus II, confidence, and enjoyment were intended to address research question three, "How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?" The other 16 questions were intended to measure the students' beliefs before and after our Calculus I class and provided additional data to inform research question three.

The post-Attitude Survey, containing 97 questions, addressed research question four, "To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?" Of the 97 questions administered through the survey, 22 related to the *Good*

Teaching Factor. The remaining 14 questions were related to the *Ambitious Teaching* Factor. The Attitude Survey questions have been provided in Appendix D.

The surveys were administered to all participants in both experimental and control groups. Participants took the pre-Attitude Survey at the start of the derivative unit, in weeks 3 and 4 of the semester and the post-Attitude Survey in week 12 of the semester. Two Likert-scale ratings from 0 (*Strongly Disagree* or *Not at all*) to 5 (*Strongly Agree* or *Very Often*) were used. The validity and reliability of this survey has been reported and/or relied on to directly advance research in Calculus I by multiple researchers through the *Characteristics of Successful Programs in College Calculus* (CSPCC) project report (Bressoud et al.) and the *Progress through Calculus* (PtC) project reports (Rasmussen et al., 2019).

Quantitative Data Collection: Derivative Concept Assessment

A pretest and posttest, which were the same test, referred to as the Derivative Concept Assessment, served as a performance-measure to assess the participants' ability to understand basic differential calculus. This measurement addressed research question one, "How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?" The test consisted of nine questions which were adopted from what Park (2011) refers to as his "survey." This study, like Park's study, assessed the same dependent variable; that is, the understanding of the derivative concept grounded on Zandieh's (1997) three-layer process-objects conceptual framework. Five of the nine questions were excerpts from Epstein's (2007, 2013) Calculus Concept Inventory (CCI). The nine-item

Derivative Concept Assessment has been presented in Appendix E. The reliability and validity of the CCI has been reported by Epstein (2013).

Qualitative Data Collection

I recruited and interviewed twelve participants following an interview protocol available in Appendix F. I interviewed twelve participants, eight from the experimental group and four from the control group, to collect qualitative data for this study. The data collected from interview transcripts served to inform the second and fourth research questions:

RQ2: To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?

RQ4: To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?

Due to the unprecedented and unpredictable circumstances presented by the COVID-19 pandemic, all participants were enrolled in my Calculus I class that met synchronously online in fall 2021. The positive effect of our new social norm of having all academic activities online was the relative ease that we, both students and instructors, adjusted to audiovisual technology. All interviews were conducted as synchronous online one-on-one interviews using video conferencing applications. The transcripts were conveniently downloaded from an online platform. Interview data comprised unstructured verbal or written data from the interviewee responding to open-ended questions in conversations.

Although there were distinct differences between the quantitative and qualitative data analysis I used, the general processes looked very similar for my study. I organized my data before analysis, focused on the questions I hoped to answer, and finally thought about the presentation of the results in the dissertation, focusing on the interpretation of the results (Creswell & Guetterman, 2019). In addition to quantitative and qualitative data analysis, in this section I have also included a discussion of trustworthiness and credibility, triangulation, and development and complementarity purposes in my mixed method study.

Procedure

This experimental mixed method design had two groups, the experimental and control groups, which were not randomly assigned but were formed from three sections of fall 2021 Calculus I classes. One class served as the control group and two classes combined served as the experimental group. The experimental group began the treatment at the start of the semester. For quantitative data, all participants took a pretest and an affective measure pre-survey at the start of the intervention. After the intervention, the participants took a posttest and post survey. During the intervention, audiovisual materials and activity documents were collected for qualitative data in case the data was needed. After the post survey was completed, 12 participants, who completed both pre- and post-measures for the derivative assessment and surveys, were recruited and interviewed.

Data from the pre- and posttest and pre- and post-surveys provided quantitative data that was analyzed using descriptive and inferential statistics. For the qualitative data, I observed “The Bottom-Up Approach to the Process of Qualitative Analysis” (Plano

Clark and Creswell, 2015). I applied two coding approaches. I completed first cycle, after first cycle, second cycle, and after second cycle coding (Saldaña, 2021) for both approaches. Next, I present the quantitative and qualitative data analysis.

Quantitative Data Analysis

Quantitative data analysis served to answer my first and third research questions:

RQ1: How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?

RQ3: How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?

For both quantitative instruments in this study, I analyzed and interpreted the data using similar methods used in the original studies from which I have adopted or adapted the instruments. In general, for the pre- and post-Attitude Surveys and the Derivative Concept Assessment, I conducted descriptive analysis to report the data trends. The two categories of descriptive statistics, applied to the experimental and control group data, were measures of central tendency and measures of dispersion.

Because this study used a quasi-experiment, in which groups are not randomly assigned as experimental or control, to compare group performance or attitude, I used a group comparison design and inferential statistics methods for my quantitative data analysis. The null hypothesis was there was no difference in the performance measures with respect to understanding the derivative concept when comparing the means within or between the control and experimental groups.

The data was tested for normality to determine if I used a parametric or nonparametric test. For comparing means within groups, for a normal distribution, I applied a paired samples *t*-test and for data that was not normally distributed, I applied a Wilcoxon signed-rank test. For comparing means between groups, for a normal distribution, I applied an independent samples *t*-test and for data that was not normally distributed, I applied a nonparametric Mann-Whitney U test.

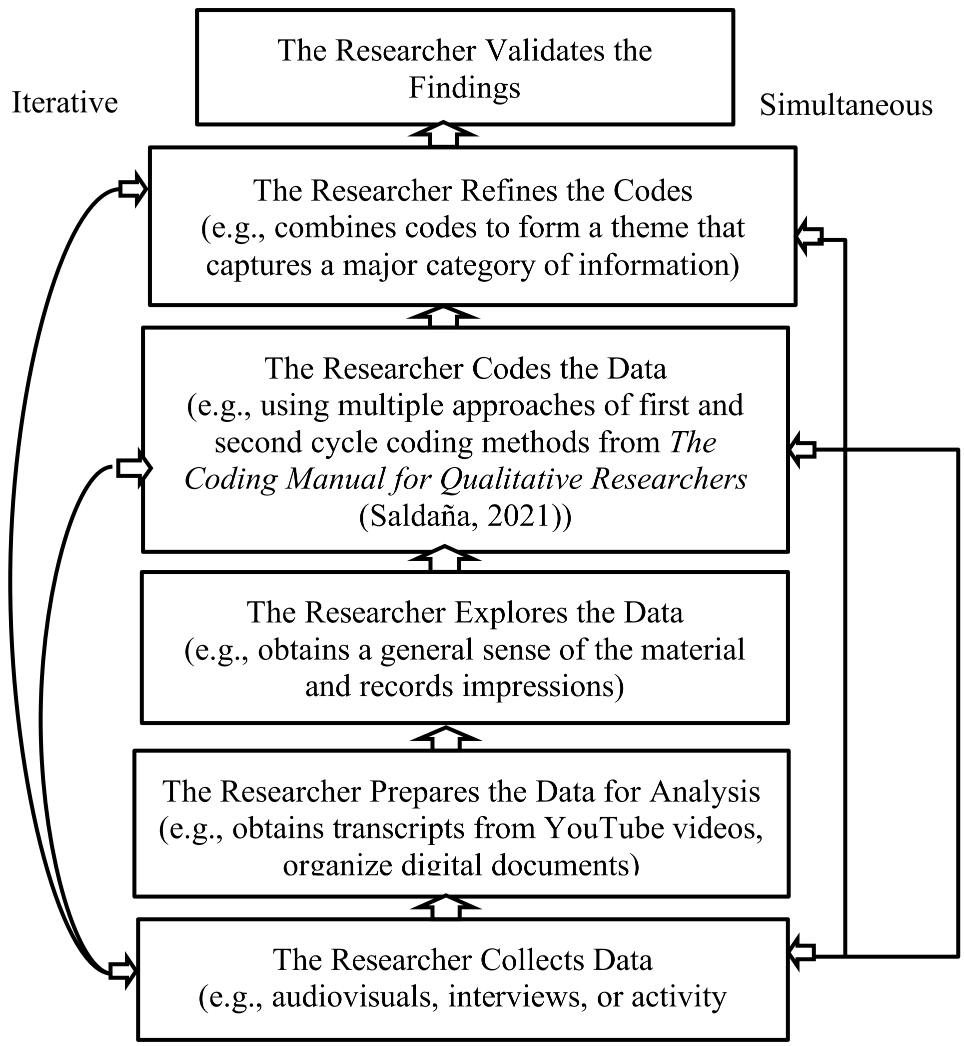
Qualitative Data Analysis

Plano Clark and Creswell (2015) asserted the process of qualitative data analysis must be systematic, rigorous, and thoughtful. I interpreted their bottom-up or inductive process and adapted the process to suit the commognitive methodology (Sfard, 2008). In addition, I embedded Zandieh's (1997) three-layer process-objects (ratio, limit, function) coupled with the three layers of the circle diagram for coding into the "The Bottom-Up Approach to the Process of Qualitative Analysis" (Plano Clark and Creswell, 2015, p. 356) in Figure 2 below.

Immediately after I collected data, I prepared the transcripts from the interviews. After reviewing the transcripts, I abandoned the use of a priori goals. I did not code by using the four characteristics of mathematical discourse outlined by Sfard (2008) or Zandieh's (1997) three-layer process-objects circle diagrams. I decided to use the purest bottom-up approach and compare the outcome of the analysis in my chapter 5 discussion. To analyze the interview transcripts, I applied two coding approaches. I completed first cycle, after first cycle, second cycle, and after second cycle coding (Saldaña, 2021) for both approaches. I initially organized the data using a spreadsheet, then I used a computer-assisted qualitative data analysis software (CAQDAS), and finally, I used a mind mapping tool. Details of the analysis have been provided in chapter 4.

Figure 2

Bottom-Up: Qualitative Data Analysis of Discourse with Calculus I Students



Trustworthiness and Credibility

I took several steps to validate findings to ensure accuracy and credibility and trustworthiness and dependability. Plano Clark and Creswell (2015) focus on two forms: bracketing and member checking.

Bracketing was a process of self-reflection for the researcher. I, for example, acknowledged my biases, wrote them down, and placed my biases aside or "bracketed" the biases during the analysis phase. Triangulation was the process of convergence, collaboration, or correspondence. I, for example, examined multiple information sources and verified that there was evidence that supported the theme I had surfaced. Member checking was requesting one of the participants to validate the accuracy of my findings. This was done casually by discussing and confirming findings during office hour visits. Only three of the interviewees, however, visited my office hours consistently after the interviews.

Development and Complementarity Purposes of Mixed Methods

The mixed method design approach served to develop and enhance the data. The results from the Derivative Concept Assessment informed and developed my sampling, measuring, and implementation decisions for the qualitative method in this study. Reciprocally, the use of the qualitative data from the interviews complemented the quantitative findings from the Attitude Survey and the Derivative Concept Assessment. Having discussed the innovation and the research design for the study, I conclude this chapter by presenting the timeline for implementation of the innovation and the research design.

Timeline for Implementation

The timeline for the implementation of this proposal is provided in Table 8. The research activity, the corresponding time frame for the activity, and the participants and their actions were summarized in the matrix. The activities dated from July 19 to December 13, 2021, for a total of 23 weeks. Details regarding the scheduling of the elements of the research design were presented in the implementation timeline. Following the timeline for implementation, the analysis and results are presented in chapter 4.

Table 8

Timeline for Implementation of Innovation and Research Design

Research Activity	**Timeframe** Summer & Fall 2021	**Participants | Action** E: experimental | C: control | PR: practitioner/researcher
Final Innovation design	July 19	PR | finalize innovation design
Produce orientation videos Course overview for experimental and control groups	July 26	C | video for course overview; E | video for orientation videos for course activities | *Uncover, Play*, *Apply*, *Connect*, *Question*, *Realize* PR | produce videos
Finalize Canvas courses C: traditional course E: course with innovation elements	Aug 9-15	PR | upload Canvas courses for C; upload Canvas course with all innovation elements for E; finalize Canvas course
Publish Canvas courses for C and E	Aug 16	PR | publish Canvas courses for C and E
Implement Innovation Details in Table 6 weeks 1-4	Aug 23-Sep 26	C students | homework and quizzes; E students | *play*, *apply*, *connect*, *question*; PR | instruct/collect quantitative data
Implement Innovation weeks 1-16	Aug 26	C and E | Engage in activities PR | Implement Innovation
Collect data: qualitative; team discussion video qualitative data *Connect* and *Question*	Aug 30-Dec 6	C students | quizzes; E students | team discussion PR | collect quantitative data
Analyze data: qualitative; team discussion video weeks 2-16 alternating weeks after team discussions	Aug 30-Dec 6,	PR | qualitative analysis coding
Collect data: qualitative; pre–Attitude Survey week 3 and 4	Sep 10-14	C and E students | take pre–Attitude Survey PR | collect qualitative data (survey)

Research Activity	**Timeframe** Summer & Fall 2021	**Participants | Action** E: experimental | C: control | PR: practitioner/researcher
Collect data: qualitative; pre–Derivative Assessment week 5	Sep 20, 21	C and E students | take pre–Derivative Assessment PR | collect quantitative data (assessment)
Collect data: quantitative; post Derivative Assessment week 7	Oct 7	C and E students | take post Derivative Assessment PR | collect quantitative data (assessment)
Analyze data: quantitative and qualitative scoring assessments and survey; weeks 7-11	Oct 8-Nov 5	PR | quantitative and qualitative data analysis scoring assessment and survey
Collect data: qualitative; post Attitude; Survey; week 12; 12 potential interviewees, 4 control students; 8 experimental students	Nov 9-Nov 13	C and E students | take post Attitude Survey PR | collect qualitative data (survey) total: 12 interviewees, 4 control students; 8 experimental students
Collect data: qualitative Interviews; weeks 12 and 13	Nov 13, 14	C and E students | Interviews PR | collect qualitative data (interviews)
Collect data: qualitative Interview; week 15	Dec 2	C and E students | Interview PR | collect qualitative data (interview)
Finish class; weeks 16-17; Final Exam	Dec 6-Dec 10	C and E students | final exam PR | finish class
End data collection and semester; week 17	Dec 13	PR | qualitative and quantitative analysis and report out

CHAPTER 4

DATA ANALYSIS AND RESULTS

Above all else show the data.

—Edward R. Tufte

The goal of my investigation was to engage students in mathematical discourse to motivate student thinking and understanding of calculus, particularly, the derivative concept. My goal was to assess the impact of the combination of two factors, *Good Teaching* and *Ambitious Teaching*, on a composite mathematics attitude comprising the dependent variables, *persistence*, *confidence*, and *enjoyment* in mathematics, particularly calculus. In addition, I aimed to determine if there was a significant and observable difference in performance of students in an experimental calculus class designed to motivate active engagement in mathematical discourse in contrast to students in a traditional calculus class. Four research questions guided the conduct of my study:

RQ1: How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?

RQ2: To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?

RQ3: How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?

RQ4: To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?

Results from data collection and analysis are provided in this chapter. In the section that follows, I report the data characterizing the participants as they entered the research setting, including

- their prerequisite grades,
- their attitudes toward and beliefs about mathematics and how they learn mathematics,
- their precalculus and calculus experience,
- their field of study, and
- their anticipated activities requiring time management.

The Entering Calculus Students

This study sought to understand the characteristics of the entering calculus student. Like all mathematics topics, each topic in calculus builds on previous topics. For students entering calculus, mastery of the prerequisite topics of algebra, trigonometry, and geometry was critical. At our community college system, the prerequisites for Calculus I were precalculus, which is a combination of college algebra and trigonometry, or college algebra and trigonometry, or placement into Calculus I using Accuplacer or EdReady scores combined with ACT or SAT scores or combined with grades of B or better in high school college algebra and trigonometry or precalculus. The prerequisite profiles of the students entering calculus in this study are given in Table 9.

Table 9

Number of Entering Calculus Students Who Met Given Prerequisite

	Control (n = 12)	Experimental (n = 16)	Experimental (n = 21)
MAT187 (A)			
MAT187 (B)	3	1	2
MAT187 (C)	1	1	2
MAT15X (A)	3	2	1
MAT15X (B)	2	1	6
MAT15X (C)	1	9	
MAT15X (P)			1
MAT182 (A)	2	3	1
MAT182 (B)	4	7	3
MAT182 (C)		3	3
Placement	2		5
None on record		1	1
Other			4
Repeating Calculus I	3	8	3

Note. MAT187 is Precalculus. MAT15X is College Algebra with "X" representing the number of credit hours. MAT182 is Trigonometry. Placement exams included Accuplacer and EdReady.

Using the Attitude Survey, this study sought to identify the students' academic background and confidence in their mathematics ability. At the start of the study, students were asked to self-assess on four skills they would need for calculus: *factoring*, *solving equations*, *solving inequalities*, and *solving word problems*. They were also asked if they understood the mathematics they studied before calculus and if they were ready for calculus. The results are reported in Table 10. A mid to high level of self-reported confidence in their prerequisite skills and confidence in prerequisite understanding of mathematics and thus, readiness for calculus, was evident from student responses depicted in Table 10. Although the experimental group reported higher self-confidence in three of the four skills, the control group reported higher self-confidence in understanding their prerequisite skills overall and in their readiness for calculus.

Table 10

Percentage of Students' Self-Assessment of High School Preparation

		Control (n = 7)	Experimental (n = 18)
Can factor expressions	Somewhat[c]	29	6
	Yes[d]	43	78
Can solve equations	Somewhat[c]	0	0
	Yes[d]	86	94
Can solve inequalities	Somewhat[c]	43	39
	Yes[d]	43	56
Can solve word problems	Somewhat[c]	29	22
	Yes[d]	57	50
Understand what I have studied[a]	Somewhat[c]	57	28
	Yes[d]	43	67
Ready for calculus[b]	Somewhat[c]	14	11
	Yes[d]	86	78

Note. For the first three questions, the prompts began "My mathematics courses in high school have prepared me to …," followed by "factor expressions," "solve equations," "solve inequalities," and "solve word problems."

[a] "I understand the mathematics that I have studied."

[b] "I believe I have the knowledge and abilities to succeed in this course."

[c] Combines *Slightly Disagree* and *Slightly Agree.*

[d] Combines *Agree* and *Strongly Agree.*

In Table 11, the percentage of students who completed precalculus at college or university and the percentage of students repeating calculus are reported. Approximately half of the students in both groups took precalculus in college. About one third were retaking calculus at the college level.

Table 11

Percentage of Students who Completed Precalculus and Calculus at College Level

	Control (n = 7)	Experimental (n = 18)
Took precalculus in college	43	50
Previously took calculus in college	29	28

The participants were also queried about their field of study (Table 12) and their anticipated time management during the study (Table 13). Both the control and experimental groups declared engineering and medical professions to be their primary choice fields of study. When the fields were combined, 72% of the control group and 72% of the experimental group chose engineering and medical fields.

Related to time management, the students were asked to report on the number of hours per week they worked at a job, participated in extracurricular activities, studied for all their classes, and studied exclusively for calculus. Observations regarding the participant time management included the following: when compared to the experimental group, the control group reported working less and, additionally, reported working fewer hours per week. The hours per week reported spent on extracurricular activities were comparable in the two groups. Finally, when compared to the experimental group, the control group anticipated spending more hours per week studying for all their classes as well as for calculus.

Table 12

Percentage of Students in Each Field

	Control (n = 7)	Experimental (n = 18)
Computer Scientist	0	6
Engineer	29	44
Life scientist	14	6
Medical professional	43	28
Physical scientist	14	11
Undecided	0	5

Table 13

Percentage of Time at Work, Extra Curricular, Prep for all Classes, Prep for Calculus

	Hours per Week Spent	Control (n = 7)	Experimental (n = 18)
Work	0	14	28
	1-5	29	0
	6-10	14	0
	11-15	0	0
	16-20	14	33
	21-30	0	11
	>30	29	28
Extracurricular	0	57	50
	1-5	14	17
	6-10	14	22
	11-15	0	0
	16-20	14	11
	21-30	0	0
	>30	0	0
Prep for all class (including calculus)	0	0	0
	1-5	0	0
	6-10	14	33
	11-15	0	0
	16-20	29	45
	21-30	57	11
	>30	0	11
Prep for calculus	0	0	0
	1-5	29	33
	6-10	29	44
	11-15	0	0
	16-20	29	17
	21-30	14	0
	>30	0	6

Effects of Discourse on Attitudes: Entering vs. Exiting Students (RQ3)

Results from the Attitude Surveys were used to address research question three: *How does Good Teaching and Ambitious Teaching impact students' attitudes toward mathematics, particularly persistence*, *confidence*, and *enjoyment in mathematics?* The

Attitude Surveys were administered prior to the intervention and several weeks after the implementation of part of the intervention focused on the understanding of the derivative concept.

The Attitude Survey

The pre-Attitude and post-Attitude Surveys were administered in the control and experimental classes in week three or four and week twelve of the semester with the aim of identifying students' attitudes and beliefs, particularly *persistence*, *confidence*, and *enjoyment*. Observable in Table 10 was that both the control and experimental groups entered the study with consistently high levels of self-confidence in their calculus prerequisite skills and previous preparation for calculus.

A comparison of students' sense of readiness for Calculus I at the start and end of the study is provided in Table 14. The entire control group reported they were somewhat or definitely ready for calculus, whereas 11% of the experimental group reported they were not prepared by previous courses. At the end of the study, in comparison to the mixed attitude of the control group, all students in the experimental group reported they were either somewhat prepared or definitely prepared for Calculus I. The experimental students who felt definitely prepared responded with *Strongly Agree* or *Agree* to the prompt, "My previous courses prepared me to succeed in this course." In contrast, at the end of the intervention, one of four of the control students disagreed with the statement, "I believe I have the knowledge and abilities to succeed in this course."

Table 14

Percentage of Students' Self-Assessment of High School Preparation, Start and End

			Control (n = 7)	Experimental (n = 18)
Confident about knowledge/abilities for calculus success[a]	Start	Somewhat[c]	14	11
		Yes[d]	86	78
			Control (n = 4)	Experimental (n = 13)
Confident about preparation for calculus success[b]	End	Somewhat[c]	50	31
		Yes[d]	25	69

Note. [a] "I believe I have the knowledge and abilities to succeed in this course."
[b] "My previous courses prepared me to succeed in this course."
[c] Combines *Slightly Disagree* and *Slightly Agree.*
[d] Combines *Agree* and *Strongly Agree.*

In particular, the pre-Attitude and post-Attitude surveys sought to identify student attitudes, defined as a composite of three dependent variables corresponding to confidence in mathematical ability, enjoyment of mathematics, and intention to persist to Calculus II. An overall decrease in *persistence*, *confidence*, and *enjoyment* was observed in comparing the aggregate results from pre- to post-Attitude survey data. However, an analysis focused only on data from students who completed both pre-Attitude and post-Attitude surveys produced different results. The results for students completing both surveys are reported in Table 15.

Overall, no changes in *confidence* were evident for either the control or experimental group. Although the affirmative responses (*Strongly Agree* and *Agree*—equivalent to a "Yes") from the control group decreased for *enjoyment*, it was evident by the data that overall, the control group continued to enjoy doing mathematics. In comparison, the affirmative responses for *enjoyment* or enjoying doing mathematics increased for the experimental group (Table 15).

Table 15

Percentage of Students Reporting Persistence, Confidence, and Enjoyment per Surveys

	Control (n = 4)		Experimental (n = 13)	
	Pre	Post	Pre	Post
I am confident in my mathematics abilities.	SW[a] 100	SW[a] 100	SW[a] 39	SW[a] 39
	Yes[b] 0	Yes[b] 0	Yes[b] 54	Yes[b] 54
I enjoy doing mathematics.	SW[a] 50	SW[a] 75	SW[a] 62	SW[a] 46
	Yes[b] 50	Yes[b] 25	Yes[b] 23	Yes[b] 39
Do you intend to take Calculus II?	NS 25	NS 50	NS 31	NS 31
	Yes 50	Yes 25	Yes 54	Yes 46

Note. Acronyms include the following: "Not sure" (NS); and "Somewhat" (SW).
[a] Combines *Slightly Disagree* and *Slightly Agree*.
[b] Combines *Agree* and *Strongly Agree*.

For the *persistence* to Calculus II variable, the pre- and post-responses for each student were compared (Tables 15). There were no changes in responses for three sets of pre- and post-responses for the control group (n = 4). When asked if they intended to take Calculus II, pre-survey responses from the control group of "Yes, No, I don't know yet" were paired with "Yes, No, I'm not sure" on the post survey. One student's pre- to post-response changed from "Yes" to "I'm not sure." It was evident from additional survey responses that the student was not sure if Calculus II was required for their intended major. The student did not intend to take Calculus II because, as the student stated, "I have too many other courses I need to complete."

Similar findings resulted from further analysis of the pre- and post-responses to Calculus II *persistence* for the experimental group. The pre- to post-responses from two students changed from "I don't know yet" and "Yes" to "No" and "I'm not sure," respectively. Their survey responses regarding why they did not intend to take Calculus II were "I changed my major" and "My grade in Calculus I was not high enough."

Based on results from only experimental students who completed both pre-Attitude and post-Attitude surveys (n = 13), there was no change in the composite measure of *attitude* toward mathematics in the area of *persistence*, *confidence*, and *enjoyment*. However, there was an increase in *enjoyment* for the experimental group. Two of the 13 students indicated a positive change in their enjoyment.

At the end of the study, students were additionally asked to self-assess on two skills introduced in calculus, limits and derivatives, which were the focus of the calculus units for this study. They were also asked if they were able to use ideas of calculus in word problems and if the course increased their interest in taking more mathematics courses. The results are reported in Table 16.

All responses from the control group were equivalent to "Somewhat" or "Yes." However, compared to the control group, the experimental group had higher levels of *confidence* in their abilities. For example, 46% of the experimental group responded either with *Strongly Agree* or *Agree* to the question, "I am able to use ideas of calculus (e.g., limits, differentiation) to solve word problems that I have not seen before." In comparison, no student in the control group reported this level of certainty.

The level of interest or *persistence* in mathematics were approximately the same for the control and experimental groups. However, 3 of the 13 experimental students responded with either *Strongly Disagree* or *Disagree* to the prompt, “This course has increased my interest in taking more mathematics.” In contrast, all the control group’s responses were equivalent to “Somewhat” or “Yes.”

Table 16

Percentage of Students’ Self-Assessment of Abilities and Interests, End of Study

		Control (n = 4)	Experimental (n = 13)
Can compute limits and derivatives[a]	Somewhat[d]	75	69
	Yes[e]	25	31
Can solve word problems[b]	Somewhat[d]	100	46
	Yes[e]	0	46
Course has increased interest in math[c]	Somewhat[d]	75	54
	Yes[e]	25	23

Note. [a] “I am good at computing limits and derivatives.”
[b] “I am able to use ideas of calculus (e.g., limits, differentiation) to solve word problems that I have not seen before.”
[c] “This course has increased my interest in taking more mathematics.”
[d] Combines *Slightly Disagree* and *Slightly Agree.*
[e] Combines *Agree* and *Strongly Agree.*

Measuring Good Teaching Practice with the Post-Attitude Survey

Students were asked to assess 36 questions relevant to instructor characteristics in the post-Attitude Survey given at the end of the study. These survey questions comprised 22 variables that identified traditionally accepted *Good Teaching* characteristics. For 15 of the 22 variables, a 6-point scale from *Strongly Disagree* to *Strongly Agree* was used. The other 7 of the 22 variables used a 6-point scale response from *Very Often* to *Not at all*. These results are provided in Table 17.

The control students reported they experienced *Good Teaching* practices (Table 17). Their responses were equivalent to “Somewhat” or “Yes” for 13 of the 15 variables. The two other variables, for which the students’ responses were also equivalent to “No,” were variables that loaded negatively on *Good Teaching*. That is, “Discouraged continuing calculus” and “Made students feel nervous in class” countered Good *Teaching*. The students' responses were also related to the frequency of practice of *Good Teaching* characteristics. The results were all equivalent to “Sometimes” or “Frequently” for *Good Teaching* characteristics. Similar results were evident for the experimental group. However, for “Encouraged students to enroll in Calculus II,” “Presented more than one method for solving problems,” and “Exams good assessment of what students learned,” it was evident that one experimental student’s response was equivalent to “No,” as evidenced in Table 17. Similarly, one experimental student’s response was equivalent to “Infrequently” to these prompts: “Provided explanations that were understandable,” “Frequently asked students questions,” “Frequently prepared extra material to help students understand,” and “Assignments challenging but doable.”

Table 17

Students' Assessment of Good Teaching Practices for Instructor (Percentages)

		Control (n = 4)	Experimental (n = 13)
Asked questions to determine students' understanding [a]	Somewhat[w]	0	15
	Yes[x]	100	85
Listened to students questions[b]	Somewhat[w]	0	8
	Yes[x]	100	92
Discussed applied problems[c]	Somewhat[w]	0	8
	Yes[x]	100	92
Helped students become better problem solvers[d]	Somewhat[w]	0	15
	Yes[x]	100	85
Discouraged continuing calculus[e]	Somewhat[w]	25	31
	Yes[x]	0	8
Made students feel nervous in class[f]	Somewhat[w]	25	31
	Yes[x]	25	15
Encouraged students to enroll in Calculus II[g]	Somewhat[w]	25	23
	Yes[x]	75	69
Acted as if I was capable of understanding key ideas[h]	Somewhat[w]	0	39
	Yes[x]	100	61
Made me feel comfortable asking questions[i]	Somewhat[w]	50	23
	Yes[x]	50	77
Encouraged students to seek help during office hours[j]	Somewhat[w]	0	0
	Yes[x]	100	100
Presented more than one method for solving problems[k]	Somewhat[w]	50	23
	Yes[x]	50	69
Made class interesting[l]	Somewhat[w]	50	15
	Yes[x]	50	85
Exams were good assessment of what students learned[m]	Somewhat[w]	25	31
	Yes[x]	75	62
Exams were graded fairly[n]	Somewhat[w]	0	15
	Yes[x]	100	85

		Control ($n = 4$)	Experimental ($n = 13$)
Homework was graded fairly[o]	Somewhat[w]	0	8
	Yes[x]	100	92
Allowed students time to understand difficult ideas[p]	Sometimes[y]	50	23
	Frequently[z]	50	77
Provided explanations that were understandable[q]	Sometimes[y]	0	23
	Frequently[z]	100	69
Was available for appointments outside office hours[r]	Sometimes[y]	25	23
	Frequently[z]	75	77
Frequently showed students how to work on specific problems[s]	Sometimes[y]	0	0
	Frequently[z]	100	100
Frequently asked students questions[t]	Sometimes[y]	0	23
	Frequently[z]	100	69
Frequently prepared extra material to help students understand[u]	Sometimes[y]	50	39
	Frequently[z]	50	53
Assignments were challenging but doable[v]	Sometimes[y]	25	31
	Frequently[z]	75	61

Note. Two Likert-scale rated from 0 (*Strongly Disagree* or *Not at all*) to 5 (*Strongly Agree* or *Very Often*).

[a-v] Refer to Appendix K for all 22 *Good Teaching* survey questions.

[w] Combines *Slightly Disagree* and *Slightly Agree.*

[x] Combines *Agree* and *Strongly Agree.*

[y] Combines *Occasionally* and *Seldom.*

[z] Combines *Very Often* and *Often.*

Measuring Ambitious Teaching Practice with the Post-Attitude Survey

In addition to the 22 variables related to *Good Teaching* (Table 17), the post-Attitude Survey comprised 14 variables that identified traditionally accepted *Ambitious Teaching* characteristics. All 14 variables used a 6-point scale response from *Very often* to *Not at all*. The results are given in Table 18.

It was evident that the instructor's practice of *Ambitious Teaching* was not as consistent as their practice of *Good Teaching* (Table 18). For many of the *Ambitious*

Teaching variables, the results were all equivalent to “Sometimes” or “Frequently” for both the control and experimental groups. These included the following variables: “Frequently asked students to explain thinking during class,” “Frequently required students to explain thinking on homework,” “Gave assignments outside of class that included word problems,” and “Gave exams requiring students to solve word problems.” The other queries were met with mixed responses, including what was equivalent to “Infrequently.” Neither the control group nor the experimental group were required to give presentations during class. Nor were there assigned readings given. This was consistent with the survey responses (Table 18).

Table 18

Students' Assessment of Ambitious Teaching Practices for Instructor (Percentages)

		Control (n = 4)	Experimental (n = 13)
Had students work together during class[a]	Sometimes[o]	0	62
	Frequently[p]	0	8
Held whole-class discussions during class[b]	Sometimes[o]	0	15
	Frequently[p]	100	31
Had students give presentations during class[c]	Sometimes[o]	25	15
	Frequently[p]	0	8
Frequently lectured[d]	Sometimes[o]	25	15
	Frequently[p]	75	69
Frequently asked students to explain thinking during class[e]	Sometimes[o]	25	54
	Frequently[p]	75	46
Frequently required students to explain thinking on homework[f]	Sometimes[o]	50	23
	Frequently[p]	50	77
Frequently required students to explain thinking on exams[g]	Sometimes[o]	25	8
	Frequently[p]	50	92
Assigned reading in textbook for reading before coming to class[h]	Sometimes[o]	0	46
	Frequently[p]	25	8
Returned assignments with helpful feedback/comments[i]	Sometimes[o]	25	15
	Frequently[p]	75	77
Gave assignments outside of class that were group projects [j]	Sometimes[o]	0	54
	Frequently[p]	0	39
Gave assignments outside of class that included word problems[k]	Sometimes[o]	0	8
	Frequently[p]	100	92
Gave word problems unlike those in textbook[l]	Sometimes[o]	25	69
	Frequently[p]	50	23
Gave exams requiring students to solve word problems[m]	Sometimes[o]	0	8
	Frequently[p]	100	92

		Control (n = 4)	Experimental (n = 13)
Gave exams requiring students to solve word problems unlike those in textbook[n]	Sometimes[o] Frequently[p]	25 50	54 15

Note. One Likert-scale rated from 0 (*Not at all*) to 5 (*Strongly Agree* or *Very Often*).
[a-n] Refer to Appendix K for all 14 *Ambitious Teaching* survey questions.
[o] Combines *Occasionally* and *Seldom.*
[p] Combines *Very Often* and *Often.*

Summary: The Entering and Exiting Calculus Student

All but two entering students met the official prerequisites for Calculus I. Both were given permission to take the course. About one fifth of the entering students were repeating Calculus I. In one of the two experimental classes that comprised the aggregate experimental group, half of the students were repeating Calculus I. Of the students who took the pre–Attitude Survey, about one third reported taking calculus again. No student taking Precalculus as a prerequisite made an A in the class. Entering calculus, the control and experimental students reported they understood the mathematics that they had studied and believed they had the knowledge and abilities to succeed in Calculus I. At the start of the study, the control students reported being more confident in their readiness for calculus versus the experimental students. At the end of the study, the control students had mixed feelings about their readiness for calculus, whereas the experimental students reconfirmed their readiness.

The students' career aspirations leaned predominantly toward the engineering and medical fields. Approximately 30% of the students worked more than 30 hours per week while approximately 30% anticipated working 6-10 hours per week and 30% anticipated working 1-5 hours per week to prepare for calculus.

Summary of RQ3 Results

How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence*, *confidence*, and *enjoyment* in mathematics? On the post-Attitude Survey both control and experimental groups reported experiencing *Good Teaching* in and out of the classroom setting. For the most part, this was also the case for *Ambitious Teaching*. Part of the aim of the Attitude survey was to confirm if the students reported experiencing accepted characteristics of *Good Teaching* and *Ambitious Teaching* practices.

An observation of the percentages reporting *persistence* in taking Calculus II, *confidence*, and *enjoyment* indicated that there was no change in the variable composites, *persistence*, *confidence*, and *enjoyment*, of students' attitude towards Calculus I. These participants entered Calculus I with a mid to high self-reported confidence in their prerequisite skills and readiness for calculus. The effects of mathematical discourse on the understanding of the derivative concept in a Calculus I course are addressed in the following section.

Effects of Discourse on Understanding of Derivative Concept (RQ1)

Results from the Derivative Concept pre- and post-Assessments were used to address research question one: *How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?* The Derivative Concept Assessment was administered prior to and after the implementation of the intervention. The intervention was implemented in course modules focused on the limit-based definition of the derivative concept.

Comparing Means within Groups

The pretest and posttest were administered in both control and experimental classes in weeks five and seven of the semester, respectively. Participants completed and submitted both pretest and posttest on Canvas during their synchronous online class meeting. The scores of students who completed both pre- and posttests were considered in the statistical analyses that follow.

Normality of the pretest and posttest scores for each group were checked visually by inspecting the frequency histograms and the Q-Q Plots. The null hypothesis for the Shapiro-Wilk test was that the data were normally distributed. Results of Shapiro-Wilk tests for the for the control group, pretest ($n = 8$, $p = 0.506$) and posttest ($n = 8$, $p = 0.083$), and for the experimental group, pretest ($n = 18$, $p = 0.656$) and posttest ($n = 18$, $p = 0.185$), indicated that the pretest and posttest data for both groups were normally distributed.

Essentially, the difference in the control and experimental groups was the added teamwork for the experimental group. Both groups engaged in the same or similar in-class discussions to develop calculus discourse. Thus, inferential statistics was conducted on the aggregate to determine if there was a difference between the pretest and posttest within the aggregate group. For the aggregate, results of Shapiro-Wilk tests indicated the pretest scores were normally distributed ($n = 26$, $p = 0.626$), while the posttest scores were not normally distributed ($n = 26$, $p = 0.048$). For consistency, the nonparametric test, specifically the Wilcoxon signed-rank test, was applied to compare the means of both the pretest and posttest within the control group, experimental group, both of which had normally distributed data, and the aggregate group with the non-normal data.

The results are provided in Table 19. The null hypothesis was the difference in group means was zero. There were no significant statistical differences between the pretest and posttest within either the control ($p = 0.141$) or the experimental group ($p = 0.162$). There was a significant statistical difference between the pretest and posttest within the aggregate group ($p = 0.043$).

Table 19

Wilcoxon Test Results, Derivative Concept Assessment

	Control ($n = 8$) Wilcoxon Test	Experimental ($n = 18$) Wilcoxon Test	Combined ($n = 26$) Wilcoxon Test
Pretest			
Mean	5.031	4.611	4.7404
SD	2.512	2.292	2.319
Posttest			
Mean	6.875	5.125	5.664
SD	3.404	2.462	2.836
p	0.141	0.162	0.043

Note. The total-point value for the Derivative Assessment was 11.5 points.

Comparing Growth between Groups

For the aggregate (a combination of all students in the control and the experimental group), results of Shapiro-Wilk tests indicated the pretest scores were normally distributed ($n = 26$, $p = 0.625$), while the posttest scores were not ($n = 26$, $p = 0.048$). A nonparametric Mann-Whitney U Test was applied to assess if there were any significant differences in growth, from pre- to post-assessment, between the experimental and control groups. The Mann-Whitney U Test results are provided in Table 20. The null hypothesis was the difference in group means was zero. There were no significant statistical differences between the control and experimental group when comparing the pretest ($p = 0.807$), posttest ($p = 0.160$), and the growth ($p = 0.311$).

Table 20

Mann-Whitney U Test Results, Derivative Concept Assessment

	Pretest (n = 26)	Posttest (n = 26)	Difference (n = 26) (post – pre)
Mean	4.740	5.664	0.923
SD	2.319	2.836	2.172
p	0.807	0.160	0.311

Summary of RQ1 Results

The aim in administering the Derivative Assessment instrument was to determine how the development of mathematical discourse affected the understanding of the derivative concept (RQ1). For consistency, a Wilcoxon signed-rank test was applied to determine if there was a statistically significant difference between the pretest and posttest within the experimental, control group, and the aggregate. Within the control group and within the experimental group, the difference was not statistically significant. Within the aggregate, however, there was a statistically significant difference between the pretest and posttest when considering all students in the study.

To compare the growth between the control and experimental group, the differences between the posttest and pretest results were examined using the Mann-Whitney U Test. The results indicated no statistically significant difference between the performance of the control group and the experimental group when comparing their pretests and when comparing their posttests. Results also indicate no significant growth when comparing the differences between the posttest and pretest between the groups.

Effects of Discourse on Understanding the Derivative Concept (RQ2)

The interviews of twelve students, four control students and eight experimental students, provided data for further exploration of students' understanding of the derivative concept. In particular, the analysis and synthesis of the interview data addressed research question two: *To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?*

The interviews were conducted individually using video conferencing. The individual interviews, which were task-based and semi-structured, lasted a period of 26 to 74 minutes, depending on the student. The interview protocol is provided in Appendix F. The first set of ten interview questions were relevant to the derivative concept and thus addressed RQ2.

Interviews were administered in weeks twelve and thirteen of the semester with a sample of students from both the control group and experimental group. The IRB for the study was revised to obtain approval for a minor to participate in the study. Following the IRB approval, the twelfth interview was administered in week fifteen. Table 21 provides demographic and academic information about the interviewees.

Table 21

The Twelve Interviewees: Demographics and Academics

Name	Group [a]	Gender	Major [b]	Year [c]	Prereq	Repeating Calculus	Pre and Posttest Score out of 15
Lambda	1	F	Bio	So	MAT151 MAT182	N	3 | 2.5
Mu	1	F	Bio	Fr	MAT152 MAT182	N	1 | 2.5
Iota	0	M	Sci	So	MAT187	N	3.25 | 2.25
Rho	0	F	Sci	So	MAT151 MAT182	Y	4.5 | 7.75
Alpha	1	F	Bio	Fr	Placement	N	4.25 | 3
Sigma	0	F	Sci	So	MAT151 MAT182	N	2 | 4
Beta	1	M	Sci	So	MAT151 MAT182	Y	4.5 | 4
Epsilon	1	M	Engr	Fr	Placement	N	6 | 7.5
Pi	1	M	CS	So	MAT151 MAT182	Y	7 | 4
Delta	1	F	Engr	So	MAT187	N	4.5 | 7.75
Zeta	1	M	Bio	Fr	Placement	N	2.5 | 5
Kappa	0	M	Und	Fr	MAT187	N	5.75 | 10.5

Note. MAT187 is Precalculus. MAT151 and MAT152 are College Algebra. MAT182 is Trigonometry. Placement exams included Accuplacer and EdReady. Majors and year in college were obtained from MCCCD Student Information System (SIS).

[a] 0: Control Group and 1: Experimental Group

[b] Majors: Bio: Biology, Sci: Science, Engr: Engineering, CS: Computer Science, and Und: Undeclared.

[c] Academic Years: Fr: Freshman, and So: Sophomore.

Discourse Analysis

To analyze the interview transcripts, I applied two coding approaches. I completed first cycle, after first cycle, second cycle, and after second cycle coding (Saldaña, 2021) for both approaches. I initially organized the data using a spreadsheet, then I used a computer-assisted qualitative data analysis software (CAQDAS), and finally, I used a mind mapping tool. Table 22 depicts the coding methods I used for all cycles in Approaches #1 and #2.

Table 22

Discourse Analysis Applied to Transcript Data (Saldaña, 2021)

	First Cycle Coding Using Spreadsheet	After First Cycle Using CAQDAS	Second Cycle Using CAQDAS	After First and Second Cycle Using Mind Mapping Tool
Approach #1	In Vivo and Initial Coding	Process Coding	Pattern Coding	Categories of Categories
Approach #2	Structural Coding	Code Landscaping	Focus Coding	Categories of Categories

Using the discourse analysis depicted in Table 22 for two interview transcripts resulted in seven categories. For the other ten transcripts, I completed the first cycle coding using In Vivo and Initial Coding. Then I used the seven categories to code the students' responses manually and created additional categories when necessary. Three additional categories emerged in the analysis of the other ten interviewee transcripts.

The ten categories condensed into four categories after applying an after first and second cycle coding, *categories of categories*, using a mind mapping tool. The four condensed categories were *application*, *rate of change*, *symbolic representation, and* *visual representation*. *Evaluating*, *interpreting*, *limiting*, *mathematical rule*, *mathematical modeling*, *processing*, and *relating* became subcategories of *symbolic representation*. *Application* subsumed *unit analysis* and *modeling* as subcategories. The context provided by the students in their responses determined whether *modeling* was a subcategory of *application* or *symbolic representation*. The ten categories, example related codes, that is, students' utterances, and the four condensed categories are presented in Figure 3.

Figure 3

Participants' Derivative Discourse: Initial and Condensed Categories

Initial Categories	Condensed Categories
1. Evaluating: direct substitution, evaluate, plug in, put in, value at	
2. Interpreting: mathematical notation, this means, almost like	
3. Limiting: *limiting value*, *like the limit*, *take the limit*	
4. Relating: *how variables relate to each other*, *related to each other*, *this is a function of*	Symbolic representation
5. Processing: *steps*, *over and over again*, *for every x value*, *keep finding the derivative at each point*, *how much more is it compared to the previous*, *plug in for x*	Note. Modeling or Mathematical Rule, depending on context, may be subsumed by Symbolic representation or the Application category
6. Modeling or Mathematical Rule: *original function*, *whole derivative function*, *function behaving*	
7. Applying: position, velocity, acceleration, car moving, cost per mile	Application
8. Unit Analysis: *dollar per miles*, *miles*, *miles per dollar, miles per hour*	
9. Rate of Change: *average of change*, *change*, *rate*, *ratio*, *rate of change*	Rate of change
10. Visualizing: *slope*, *shape*, *derivative at a point*, *slope of tangent line*	Visual representation

Students' Descriptions of the Derivative

The first five interview questions were to elicit the students' description or definition of a function, derivative, the derivative function, and the derivative at a point. The students' understanding of the relationship between these terms and of the use of the derivative function were also prompted within the first five interview questions. These interview questions (IQ1-IQ5) were as follows: What is the derivative? What is the derivative function? What is the derivative at a point? Is there any relationship between these two terms, derivative function and derivative at a point? How about the term function? Is the term function related to the derivative function and the derivative at a point? How?

Interview questions six to ten (IQ6-IQ10) were taken directly from the Derivative Assessment administered in weeks five and seven of the semester. One instrument was used for the pre- and post-assessment. The students were provided the original problem statement along with their original answers to their pretest and posttest. Two questions involved interpreting a function and derivative function in an application context. Two questions required students to state the unit of the input value for a function and its derivative function. The last question required students to evaluate a derivative function to elicit students' understanding of a derivative function as another function. The questions are in Appendix F. Students who cited three of four correct answers and were able to explain their answers, for IQ6-IQ10, were determined to understand the application of derivatives as depicted in Table 23.

The ten interview questions, IQ1-IQ10, taken comprehensively, addressed research question two: *To what extent does participation in classroom discourse in*

Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course? Table 23 shows the condensed response from each student to the first ten interview questions. Although responses have been condensed, the students' actual word use during the interviews are reflected in the responses in Table 23. When students expressed derivative-concept terms such as *instantaneous rate of change* or *average rate of change*, I followed up with prompts for clarifications to determine their understanding of these terms.

Table 23 depicts my evaluation of the students' understanding of the derivative concept using the four condensed categories, application (APP), rate of change (ROC), symbolic representation (SYM), and visual representation (VIZ). The Ys in Table 23 indicates "yes" the student was able to articulate the representation of the derivative concept either as an application, rate of change, symbolic representation, or a visual representation. Ns indicates "no" the student was not able to articulate the representation.

Table 23

Summary of Students' Understanding of the Derivative Concept

	Condensed Response for IQ1–IQ5	C[a] or E[b]	APP[c]	ROC	SYM	VIZ
Lambda	derivative is the inverse function of the original function the derivative at a point is one less point of the original function we're using the same equation to find the derivative and the points	E	N	N	Y	N
Mu	derivative is the slope of a line that involves the tangent to the curve derivative function is the rate of change that involves respecting a variable at a point is the instantaneous rate of change	E	N	Y	N	Y
Iota	the rate of change would be the derivative derivative function is just a math problem	C	N	N	N	N
Rho[d]	derivative representation of slope and shape models change in slope of original function derivative at a point shows you how the function is behaving at that point the slope of the tangent line the derivative is a function	C	Y	Y	Y	Y
Alpha	the graph of all the slopes like changing slope of the original function I picture what the slope is doing and which direction it's going you plug something in like 2 and then you get the derivative at that specific point it has a method to do that and has a certain order and steps you have to follow	E	N	N	Y	Y
Sigma	taking a derivative it's a change in something so a change in *x* or a change in *y* would have to do with limits? I have a feeling it has to do with limits because a slope is a change the tangent line a slope is an adjustment from one point to another then the derivative point is the slope	C	N	N	N	Y
Beta	rate of change or like the slope of the tangent line like a function if you have your input and then you have your output limit at a point of what the secant line or the slope line your function is your input and output the derivative at a point is whatever the slope is at that point	E	N	Y	Y	Y

	Condensed Response for IQ1–IQ5	C[a] or E[b]	APP[c]	ROC	SYM	VIZ
Epsilon	derivative is the rate of change with respect to a variable *y* equals *x* and the derivative is like the function in respect to other function limit at that point like the secant line at a certain point along the regular function you plug in *x*.	E	N	Y	Y	Y
Pi	find the slope of an equation at a point or on a graph the function that allows you to find the slope at any point slope of the graph or equation at that point by plugging in any number into the equation you're looking for a point the derivative function is the equation and then you use that equation to find the derivative at a point	E	N	N	Y	Y
Delta	derivative is the reduced form of the equation or the function look for the slope for the rise over the run the derivative function would have tangents everywhere in any graph there's a lot of points in that graph you will have slope or tangents in the graph the derivative at a point just pick a point wherever the in the graph and just plug it in and look for the slope or the tangent	E	N	N	Y	Y
Zeta[d]	a derivative is the rate of change of a function at a given *x* value at a given *x* value you can find the slope of the line at that point the derivative at a point is byproduct of the derivative function we use the derivative function to find the derivative at a point slopes at those points and the relationship between the different points	E	Y	Y	Y	Y
Kappa[d]	derivative of a function means the slope of a function the rate of change of a function –either at a specific point or of a function in general take the derivative of a function and then find a specific point on that derivative function and then it would be the instantaneous rate of change at that point graphically it would look like a slope like if you started with a position function you'd have a velocity function	C	Y	Y	Y	Y

Note. [a] C: control group
[b] E: experimental group
[c] Students who cited three of four correct answers and were able to explain their answers, for IQ6-IQ10, were determined to understand the application of derivatives.
[d] Student correctly answered and explained all IQ6-10 on Derivative Concept Assessment.

Understanding the Derivative: The Control Group

The students interviewed from the control group were Iota, Kappa, Rho, and Sigma. Results reported in Table 23 indicate the understanding of the derivative concept ranged from near nil to early and advanced stages of development. Iota was unable to utter any words that indicated he possessed clear understanding in terms of any of the four representations: application, rate of change, symbolic, and visualization of the derivative concept. He expressed the term *rate of change*, but when prompted to elaborate, Iota was unable to explain or give examples of the term *rate of change*.

Sigma was in early development of her visualization capabilities in understanding the derivative concept. Interestingly, Sigma was one of three of all interviewees that uttered the term *limit* during the interviews.

Rho’s and Kappa’s understanding of the derivative concept encompassed all four representations. Their understandings were very detailed. Rho’s strength was in her ability to explain and apply different aspects of the term *rate of change* and her ability to engage mathematical processing in applied problems. Kappa's understanding of the derivative concept was extensive. His agility to clarify and provide examples for any of the four representations, with or without prompting, indicated his development, that is, Kappa’s ability to think about and act on the concept of derivative, were very advanced in comparison to his peers in either his group, the control group, or the experimental group. From the analysis of interview data, there was no evident commonality in the control group’s calculus discourse relevant to understanding the derivative concept.

Understanding the Derivative: The Experimental Group

Eight of the interviewees, Alpha, Beta, Delta, Epsilon, Lambda, Mu, Pi, and Zeta, were from the experimental group. Results reported in Table 23 indicate that the understanding of the derivative concept ranged from early to advanced stages of development for the group. One of the eight, Lambda, demonstrated early stages of development. She was able to explain the derivative concept using symbolic representation only. Seven experimental students were able to demonstrate their understanding of derivative concept with at least two representations. Two of the students, Beta and Epsilon, used three representations and one, Zeta, was able to use all four representations to demonstrate his understanding of the derivative when prompted or not prompted. Beta and Epsilon were the only students that uttered the term *limit* from the experimental group.

All eight experimental students explained their understanding of the derivative concept using either a symbolic or visual representation. Six of eight demonstrated both symbolic and visual understanding of derivative. Evidently common to the discourse of the experimental group was use of terms such as *slope*, *shape*, *derivative at a point*, *slope of a tangent line*, *evaluate*, and *plug in*. Evidently common to the experimental group was their ability to demonstrate their understanding of the manipulation and visualization of the term *function*, both the original and the derivative functions.

Summary of RQ2 Results

The purpose of the interviews was to determine to what extent participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affected the understanding of the derivative concept in a Calculus I course. Eleven of the twelve students interviewed were able to demonstrate early to advanced development of the derivative concept in at least one of the four condensed categories of representations of the derivative concept. The following is the number of students who demonstrated their understanding of the derivative in the given representation: 3 in application, 6 in rate of change, 9 in symbolic representation, and 10 in visual representation.

When comparing the control to the experimental group, it is evident from analysis of the raw data that the experimental group understood the concept of the derivative at worst, marginally, and at best, holistically. Six of the eight understood the concept using two representations, symbolic and visual. Two of four control students, however, in contrast to only one of eight of the experimental students, understood the derivative concept exceptionally in all four representations.

Rising from the raw qualitative data was a commonality in the experimental group that was not evident in the control group. Common to the discourse of the experimental group was use of terms related to a symbolic and visual understanding of the derivative concept. Also common to the experimental group was their capacity as an aggregate to algebraically manipulate and visualize functions—the original or the derivative function.

The Extent of Effects of Instructor Practice on Attitude (RQ4)

The interviews provided data to further examine how pedagogical strategies influenced students' attitudes toward Calculus I. This study specifically explored attitudes related to two constructs, *Good Teaching and Ambitious Teaching*. Discourse analysis and synthesis of the interview data addressed research question four: *To what extent does*

Good Teaching and Ambitious Teaching impact students' attitudes toward mathematics, particularly persistence, confidence, and enjoyment in Calculus I?

The interview was divided into three parts and was guided by research questions two and four. The first part of the interview focused on addressing to what extent mathematical discourse affected the development of knowledge and skills relevant to the derivative concept. The first part was discussed in the previous section of this chapter. While the first part of the interview followed up on the Derivative Concept Assessment, the second and third parts of the interview followed up on the post-Attitude Survey questions relevant specifically to *Good Teaching* and *Ambitious Teaching*, respectively. The 22 variables related to *Good Teaching* and the 14 variables related to *Ambitious Teaching* characteristics were provided in Tables 17 and 18, respectively.

I assumed the students may not have been able to recall the clusters of 36 Attitude Survey questions related to *Good Teaching* (22 questions) and *Ambitious Teaching* (14 questions) practices. Therefore, I provided the Attitude Survey questions to serve as references. I divided the survey questions into five sets, 3 sets for *Good Teaching* and 2 sets for *Ambitious Teaching*. I also provided the list of interview questions.

At the beginning of the interview, the students were asked eight questions aimed at determining to what extent *Good Teaching* impacted their attitudes toward Calculus I. Then, for the third part of the interview, I followed by asking the same eight questions but referred them to the survey questions related to *Ambitious Teaching*. Specifically, students were asked how and to what extent, if at all, did any of the cluster of 36 Attitude Survey questions affect their attitudes with respect to *persistence*, *confidence*, and *enjoyment*. They were also asked which one of the instructor practices, if any, had the most impact on their overall attitude toward Calculus I. The last question prompted students to think of any other instructor practices, in any classes they have taken, that affected or could have affected their attitudes with respect to *persistence*, *confidence*, and *enjoyment*. The interview protocol is provided in Appendix F.

Effects of Good Teaching Practices on Attitude (RQ4)

Table 24 offers a summary of the results, presenting the five most frequently cited *Good Teaching* practices out of the cluster of 22 *Good Teaching* practices. During the interview, the students were given the option, but were not required, to choose from the 22 provided Attitude Survey questions related to *Good Teaching*. However, most students opted to choose from the cluster of 22 *Good Teaching* practices that impacted their attitude because they felt they had experienced the instructor practice(s) in our class.

There were several instances when students stated multiple *Good Teaching* practices that impacted their persistence, confidence, or enjoyment in our class. There were 6 of 12, 4 of 12, and 5 of 12 students that cited multiple *Good Teaching* practices that impacted their *persistence*, *confidence*, or *enjoyment*, respectively. Each student's recollection of an instructor practice was tallied for the total for that instructor practice. That is, if Delta stated that all five *Good Teaching* practices affected her *persistence*, all five would be credited once for impacting *persistence* in Table 24. There were also instances when students did not choose from the 22 *Good Teaching* practices, but instead offered their own thoughts about *Good Teaching* practices. All students chose from the given cluster of 22 *Good Teaching* practices when asked what practices impacted their *persistence*. There were 3 of 12 students that offered other practices that impacted their *confidence* and 1 of 12 offered other practices that impacted their *enjoyment*. These numbers are not observable in Table 24. Finally, when students reported the practice(s) that most impacted their composite attitude, if they cited more than one instructor practice, only the first one they stated was reported in Table 24.

Table 24

Most Cited Good Teaching Practices Students Reported Impacted Their Attitude and the Number of Students Reporting the Practice[a] *(n = 12)*

Good Teaching (GT) practice chosen of 22 GT practices	Asked questions to determine students' understanding[a]	Encouraged attendance of office hours[a]	Provided understandable explanations[a]	Showed specific work[a]	Made class interesting[a]
Persistence	2	4	1	4	0
Confidence	4	1	2	1	0
Enjoyment	5	1	2	0	4
Composite *Attitude*[b]	11	6	5	5	4
Most Impactful	2	1	2	0	4

Note. [a] Refer to Table 17 *Students' Assessment of Good Teaching Practices for Instructor* for all 22 *Good Teaching* variables.
[b] Composite Attitude total is a sum of *persistence, confidence,* and *enjoyment.*

The following observations were evident in Table 24:

- The *persistence of one-third* of the students was impacted by the instructor encouraging students to seek help during office hours and frequently showing students how to work specific problems during class.
- The *confidence of* one-third of the students was impacted by the instructor asking questions to determine if students understood what was being discussed.
- The *enjoyment of* about one-third of the students was impacted by the instructor asking questions to determine if students understood what was being discussed and making class interesting.

- The combination or *composite attitude* was impacted by the instructor asking questions to determine if students understood what was being discussed.
- The instructor making class interesting is the most influential *Good Teaching* practice and has the most impact on the overall composite *attitude* of students.

To offer context to observations from Table 24, examples of student responses are provided in Table 25. The qualitative analysis discussed in a previous section on discourse analysis was applied to the students' interview transcripts. The analysis provided data detailing students' expressions of the features of the five most cited *Good Teaching* practices students experienced in and out of our classroom. The five initial *Good Teaching* practices were condensed into two categories after applying an after first and second cycle coding, *categories of categories*, using a mind mapping tool. The two condensed categories were *interactions validating students and available and encouraging instructor*.

- The condensed category *interactions validating students* included the instructor asking questions to determine if students understood what was being discussed and providing explanations that were understandable. The category was cited most to impact *confidence* or *enjoyment*, as well as the composite *attitude*.
- The condensed category *available and encouraging instructor* included the instructor: (a) encouraging students to seek help during office hours; (b) showing students how to work specific problems; and (c) making class interesting. The category was cited most to impact *persistence*.

Table 25

Student Responses for Good Teaching Practices that Impact Attitude—Most Cited per Table 24

Good Teaching Practice	Asked questions to determine students' understanding[a]	Encouraged attendance of office hours[a]	Provided understandable explanations[a]	Showed specific work[a]	Made class interesting[a]
Persistence	Lambda (E) you ask questions and waiting for an answer not giving it to us allowing time to get a hold of what's being asked	Kappa (C) office hours were really helpful i liked that was an option and i feel like it help my persistence to know if i need help there's a source i can go to because i don't know very many other people who actually know calculus very well gave me more motivation	Pi (E) i think they all do because it helps alleviate a lot of the mental burden number 29 My calculus instructor provide explanations that were understandable	Rho (C) you did show us how to work specific problems they weren't the most advanced problems wasn't like you were just trying to throw out a freebie for the test you weren't teaching to the test	[not cited for impact on *persistence*]
Confidence	Zeta (E) even when you're asking questions you're doing examples and we get them right that's definitely a confidence builder it shows us that we know what's going on and we're not totally lost	Kappa (C) [cont. from above] when i'm feeling really good about things i feel motivated then that helps me to be more persistent because i have more of a desire and more confidence that i can do it if i keep trying	Pi (E) [cont. from above] i'm still kind of the same when it comes to my enjoyment if i get it right then i'm having a good time it helps alleviate a lot of the mental burden	Zeta (E) especially with when we're able to ask questions in class and you go over the problems and like provide specific work	[not cited for impact on *confidence*]

Good Teaching Practice	Asked questions to determine students' understanding[a]	Encouraged attendance of office hours[a]	Provided understandable explanations[a]	Showed specific work[a]	Made class interesting[a]
Enjoyment	Alpha (E) you like saying those funny so i enjoy that and i also enjoy you're like guys come on you gotta answer the question come on guys i know you guys know this	Epsilon (E) where you're encouraging us to get help if we need it or question we're comfortable like asking questions and getting help on like problems we were confused with during class	Delta (E) it really is satisfying to have a problem answered and be able to understand it so after that problem is answered i will i just go on to the next second hard question	[not cited for impact on *enjoyment*]	Beta (E) how you relate problems like to real world i said arch i mean that was cool because you put it on the graph and then i googled what the gateway arch looked like and that's pretty cool and gave that enjoyment

Good Teaching Practice	Asked questions to determine students' understanding[a]	Encouraged attendance of office hours[a]	Provided understandable explanations[a]	Showed specific work[a]	Made class interesting[a]
Most Impactful	[not cited for most impactful]	Sigma (C) i didn't go to your office hours as much as i wanted to because i work nights but i feel like office hours are because you're different in office hours than you are even you're teaching you're teaching me in a huge class and you're having to spread your knowledge through all of them	Zeta (E) providing explanations that are understandable because it's easy for a teacher to give an answer that is quick but when you really go into depth and like make sure we understand what's going on and like have a real understanding of like how to repeat the process beyond this one problem	[not cited for most impactful]	Lambda (E) you're motivational you're okay with accepting wrong answers and people to keep on trying you just engage with everybody even if it's not about calculus that's what impacts my enjoyment to want to stay in this class and proceed forward with taking another one of your classes

Note. [a] Refer to Table 17 *Students' Assessment of Good Teaching Practices for Instructor* for all 22 *Good Teaching* variables.

Effects of Ambitious Teaching Practices on Attitude (RQ4)

Table 26 offers a summary of the results, presenting the two most frequently cited *Ambitious Teaching* practices out of the cluster of 14 *Ambitious Teaching* practices. During the interview, the students were given the option, but were not required, to choose from the 14 provided Attitude Survey questions related to *Ambitious Teaching*. However, most students opted to choose from the cluster of 14 *Ambitious Teaching* practices that impacted their attitude because they felt they had experienced the instructor practice(s) in our class.

There were several instances when students stated multiple *Ambitious Teaching* practices impacted their persistence, confidence, or enjoyment in our class. There were 1 of 12, 7 of 12, and 5 of 12 students that cited multiple *Good Teaching* practices that impacted their *persistence*, *confidence*, or *enjoyment*, respectively. Each student's recollection of an instructor practice was tallied for the total for that instructor practice. That is, if Delta stated that both *Ambitious Teaching* practices affected her *persistence*, both would be credited once for impacting *persistence* in Table 26. There were also instances when students did not choose from the 14 *Ambitious Teaching* practices, but instead offered their own thoughts about *Ambitious Teaching* practices. There were 1 of 12, 4 of 12, and 2 of 12 students that offered other practices that impacted their *persistence*, *confidence*, and *enjoyment*, respectively. These numbers are not observable in Table 26. Finally, when students reported the practice(s) that most impacted their composite attitude, if they cited more than one instructor practice, only the first one they stated was reported in Table 26.

Table 26

Most Cited Ambitious Teaching Practices Students Reported Impacted Their Attitude and the Number of Students Reporting the Practice[a] *(n = 12)*

Ambitious Teaching (AT) practice chosen of 14 AT practices	Asked students to explain their thinking[a]	Assigned group projects[a]
Persistence	3	5
Confidence	6	3
Enjoyment	5	3
Composite *Attitude*[b]	14	11
Most Impactful	4	5

Note. [a] Refer to Table 18 *Students' Assessment of Ambitious Teaching Practices for Instructor* for all 14 *Ambitious Teaching* variables.
[b] Composite Attitude total is a sum of *persistence, confidence, and enjoyment.*

The following observations were evident in Table 26:

- The *persistence of about two-fifths* of the students was impacted by engagement in group projects outside of class.
- The *confidence of* one-half of the students was impacted by the instructor asking students to explain their thinking.
- The *enjoyment of about two-fifths* of the students was impacted by the instructor asking students to explain their thinking.
- Asking students to explain their thinking was the most influential *Ambitious Teaching* practice and had the most impact on the overall composite attitude of students.

Examples of student interview responses are provided in Table 27 to offer context for the observations from Table 26. The qualitative analysis discussed in a previous section on discourse analysis was applied to the students' interview transcripts. The analysis provided data detailing students' expressions of the features of the two most cited *Ambitious Teaching* practices students experienced in and out of our classroom.

Although the control group did not experience the *connect* activity that involved teamwork, all four control students commented on group work. The question from the Attitude Survey, "Assignments completed outside of class time were submitted as a group project," provided during the interview, prompted the students to comment on what they felt about group work. Iota, Kappa, and Sigma expressed their desire to experience group work in class. Iota, because he was a student in my precalculus class, lamented not taking the other Calculus I section that experienced teamwork. Rho, who experienced group work in her previous Calculus I class, expressed that her experience left her with a negative impression of the effectiveness and efficiency of teamwork.

The discourse analysis revealed three additional recurring cited instructor practices that impacted *persistence, confidence, or enjoyment*: (a) frequently requiring students to explain their thinking on homework; (b) frequently holding whole-class discussions; and (c) frequently requiring students to explain their thinking on exams. Each of these practices was also cited as an *Ambitious Teaching* practice that most impacted their overall attitude. The recurrent citing resulted in a total tally for the composite *attitude* of 8 of 12 students for practice (a) and 5 of 12 students for practices (b) and (c) each.

Table 27

Student Responses for Ambitious Teaching Practices That Impact Attitude —Most Cited per Table 26

Ambitious Teaching Practice	Asked students to explain their thinking[a]	Assigned group projects[a]
Persistence	Zeta (E) our class is like a constant live discussion where everyone can have their input whether it's with chat or saying something on live i think that especially is a positive impact on everyone's persistence because we always know that we can give our own input and see if like especially with problems that you do in class if we're on the right track with how we're going to it's how like we're going to solve it going through our own head	Alpha (E) i think for all three of these i'm going to answer my math group because i like them and they've helped me persevere through things because like if we're confused we work through it together or like we refer back to an information sheet or something and it helps us like build that knowledge that we didn't have before which is really nice
Confidence	Delta (E) honestly being able to explain what i'm thinking and being able to explain how i did this problem how i what was i thinking and yeah you know that also helped me in some other ways not only in the math class	Epsilon (E) especially like the group work it helps you like you get different opinions on like your work and like how things like how things uh are like could be solved different perspectives of like how you could solve like specific problems in different ways and like seeing how someone else solved it and how it might have been like a little bit better than how you solved it
Enjoyment	Rho (C) because you ask us so often to explain our thinking and you know where is this answer coming from not just oh i memorized it from the sheet it's a lot more enjoyable to tell you what i'm thinking and to look back at it and go yeah that kind of makes sense than it is to just say well it was in the book because then you just feel like a robot and are you really learning?	Lambda (E) the enjoyment of almost all of it is like the group projects that we have because if a team member is struggling with the question and it happens to be something that i'm good at they could come to me and ask me and i kind of love teaching regardless if it's anything in general if i know it i'll share it so i think it that gives me like a boost of confidence and it keeps me in the class because i get to use other people's brain on top of mine to solve something

Ambitious Teaching Practice	My calculus instructor asked questions to determine if I understood what was being discussed[a]	My calculus instructor encouraged students to seek help during office hours[a]
Most Impactful	Epsilon (E) asking students to explain their thinking because it helps us get a better knowledge or and better understanding of what we know and what we're struggling with it also helps the teacher like realize what the students struggling with and it helps them make it easier for them to help the student learn it because they know like what they're actually struggling with	Pi (E) probably question 66 yeah because you have that accountability but you also have like people to always hang out with

Note. [a] Refer to Table 18 *Students' Assessment of Ambitious Teaching Practices for Instructor* for all 22 *Ambitious Teaching* variables

Summary of RQ4 Results

To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence*, *confidence*, and *enjoyment* in mathematics? Discourse analysis of twelve student interviews revealed 5 of 22 *Good Teaching* and 2 of 14 *Ambitious Teaching* practices were considered by students as practices that effectively impact their attitudes in our Calculus I class.

Two condensed categories of *Good Teaching* practices*, interactions validating students* and *available and encouraging instructor*, were cited most to impact students' attitude in Calculus I. *Interactions validating students* was cited most to impact *confidence* or *enjoyment*, as well as the composite *attitude*, comprising *persistence, confidence*, and *enjoyment*. *Available and encouraging instructor* was cited most to impact *persistence*. *Ambitious Teaching* practices such as assigning students group projects outside of class also impacted *persistence*.

Most students conveyed that *Good* or *Ambitious Teaching* practices, in which instructors asked questions to determine what students understood or asked students to explain their thinking, impacted their *confidence* or *enjoyment* of Calculus I. The same practices were revealed to be the most effective in impacting students' composite attitude comprising *persistence, confidence*, and *enjoyment*. Assigning students group projects outside of class was cited 11 times, second highest, for the most impactful *Ambitious Teaching* practice.

Finally, although *making class interesting* was cited to impact only *enjoyment*, the *Good Teaching* practice was cited 4 times as the most influential and impactful of all instructor practices. That is, students conveyed, if instructors only had time to act on one practice to impact student attitude, instructors should—*make class interesting*.

Conclusion

Learning in either the acquisitionist or participationist metaphor translated to change; however, the quandary and the difference is rooted in the question, *in the process of learning, what was it exactly that changed?* What was conveyed by the data analysis and results?

With respect to RQ1, did the transformation of classroom communication change their understanding of the derivative concept? When comparing the pretest to posttest means within the control and experimental groups, there was no significant statistical difference. The results for the aggregate, however, showed that there was a significant statistical difference between the pretest and posttest for the aggregate. When comparing the growth between the control and experimental group, results indicated no statistically significant difference in pretest, posttest, and difference between the pretest and posttest.

With respect to RQ3, were the students' attitudes impacted by *Good Teaching* and *Ambitious Teaching*, in particular *persistence*, *confidence*, and *enjoyment*? An observation of the percentages reporting *persistence*, *confidence*, and *enjoyment* in taking Calculus II indicated that there were no changes in students' attitudes.

With respect to RQ2, to what extent did participation in classroom discourse affect the understanding of the derivative concept for the experimental group compared to the control group? Common to the discourse of the experimental group was use of terms related to a symbolic and visual understanding of the derivative concept and a capacity, as an aggregate, to algebraically manipulate and visualize derivative functions. This was not observable in the control group.

Lastly, with respect to RQ4, to what extent did *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence*, *confidence*, and *enjoyment* in mathematics? Rising from the raw qualitative data emerged seven instructor practices, five *Good Teaching* and two *Ambitious Teaching*, that students reported impacted their attitude in our Calculus I class.

The data analysis and results may have been black or white. However, the reality was our students, our classrooms, and the connections that occurred in the process of playing with and applying instructional strategies to develop mathematical discourse in Calculus I, were not monochrome. The process of learning and teaching was as vibrant, robust, and as captivating as the northern lights. In the next chapter, the discussion pivots back to the question I posed at the start of my dissertation journey:

> Learning in either the acquisitionist or participationist metaphor translates to change; however, the quandary and the difference is rooted in the question, *in the process of learning, what was it exactly that changed?*

Next, I conclude my dissertation, in the fifth chapter, with a discussion of the significance and relevance of outcomes from this study to practitioners and researchers.

CHAPTER 5

DISCUSSION

To think is to forget differences.

—Jorge Luis Borges

In fall 2018, colleges in our district were charged with the directive to achieve two outcomes: improve equity and increase completion of gateway English and mathematics courses. *Calculus for a New Century: A Pump not a Filter* (Steen, 1988) emerged from the growing discontent in the 1980s, revealing the immense complexity of a calculus reform. It is now 2022 and calculus persists as a gatekeeper, not a gateway course, for STEM students (Bressoud et al., 2015, 2019; National Science & Technology, 2018; Olson & Riordan, 2012; Seymour & Hewitt, 1997; Zorn, 2015). The scale of the problem is global (Rasmussen et al., 2019). The lack of inclusivity and the disproportionate participation in STEM fields, based on gender, race, and socioeconomic backgrounds, have been and have continued to be serious problems in our nation's efforts to navigate a 21st century economy that is increasingly dependent on STEM literacy (Olson & Riordan, 2012). I have taught calculus for 25 years. This problem of practice was and likely remains a pervasive problem of practice, a problem that has permeated all levels: our classrooms, our colleges and universities, our nation—our world.

In the section that follows, I offer a final argument for Sfard's commognitive solution for my problem of practice—the gatekeeping ignominy of Calculus I. The purpose and research questions are reintroduced. Next, the interpretations, accompanied by the examination of the complementarity between quantitative and qualitative data are presented. I then frame the first purpose of the study and follow by providing the implications for practice and recommendations for research. Finally, I share the limitations and offer a conclusion and closing thoughts. Woven into my final discussion is a contextualization of findings threaded with previous research and theory supporting my intention to address the following question one final time in this study.

> Learning in either the acquisitionist or participationist metaphor translates to change; however, the quandary and the difference is rooted in the question, *in the process of learning, what was it exactly that changed?*

Communication: The Pass for the Gatekeeper

In their study of the characteristics of successful calculus programs, Bressoud et al. (2015) asserted a common vision of knowledge and skills, or learning outcomes, could improve calculus. As a learning outcome, communication was clearly a priority in *A Common Vision* (Saxe & Braddy, 2015). In the executive summary, one of the common themes from seven national curriculum guides underscored "Students should learn to communicate complex ideas in ways understandable to … audiences" (Saxe & Braddy, 2015, p. 1). Sfard (2015) argued the process of learning was tantamount to changing and shaping ways of communicating by the commognitive approach—the approach of thinking-as-communicating.

> We look at the object of changes resulting from innovations in discursive practices of the classroom; we end up focusing on the activity of communication. Communication, rather than playing a secondary role as the means for learning, is in fact the centerpiece of the story—the very object of learning. [W]hen we change rules of interpersonal communication, it is not surprising that thinking—the individualized form of communication— changes as well. If mathematics is

a particular discourse with its own special ways of storytelling, there is no other way to learn mathematics than by adjusting the rules of classroom communication. (Sfard, 2015, pp. 239-240)

This was my inspiration to take another jab at this pervasive yet elusive problem of practice. Communication was the centerpiece of my calculus students' story—the very object of learning. We, my students and I, changed the rules of classroom discourse, changed the rule of interpersonal communication, and in the process likely changed our thinking and attitudes toward success in Calculus I.

Purpose and Research Questions

The first purpose of my investigation was to engage students in mathematical discourse to motivate student thinking and understanding of calculus, particularly, the derivative concept. The second purpose was to assess the impact of the combination of two factors, *Good Teaching* and *Ambitious Teaching*, on *persistence, confidence*, and *enjoyment* in mathematics, particularly calculus. Lastly, the third purpose was to determine if there was a significant and observable difference in performance of students in an experimental calculus class designed to motivate active engagement in mathematical discourse in contrast to students in a traditional calculus class. Given the purpose of the study, four research questions guided its conduct.

RQ1: How does the transformation of classroom communication, specifically mathematical discourse, affect the understanding of the derivative concept in a Calculus I course?

RQ2: To what extent does participation in classroom discourse in Calculus I, as compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?

RQ3: How does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in mathematics?

RQ4: To what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I?

Effects of Discourse on Understanding the Derivative (RQ1 and RQ2)

What does it mean to understand the concept of derivative? I addressed this question extensively in chapter 2. The Derivative Concept Assessment, the instrument used to collect data to inform RQ1 and RQ2, measured how the development of mathematical discourse affected the understanding of the derivative concept. The Derivative Concept Assessment was brief in length but extensive in its reach for evaluating students' understanding of the derivative concept; therefore, it proved to be a challenging assessment for students. The statistical analysis results from the Derivative Concept Assessment, when observed from only a quantitative lens, however, did not provide the full scope of the students' developing understanding of the derivative.

Interpretations of Results for RQ1, RQ2, and the Third Purpose

The third purpose of this study was to determine if there was a significant and observable difference in performance of students in an experimental class in contrast to students in the control class. This purpose was guided by research questions one and two: *How and to what extent does participation in classroom discourse in Calculus I, as* *compared to traditional pedagogy, affect the understanding of the derivative concept in a Calculus I course?*

Inferential statistics were applied to analyze the pretest and posttest data within and between the control and experimental group. There were no significant statistical differences within each group; nor were there significant statistical differences between the groups. Two factors kept my dismay at bay when considering these statistical results. First, the Derivative Concept Assessment proved to be a challenging assessment for students. Although the reliability of the instrument has been reported, for future use, the reliability and validity of the instrument should be confirmed for community colleges. In addition, the 2-week period afforded the students to experience the growth, was likely too short. These two non-sampling errors, questioning problems and/or fatigue and time period bias may have skewed the results.

Regardless of the possible non-sampling errors introduced, based on the statistical results, there was no significant and observable difference in performance of students in an experimental versus control class. A valid concern was the sample size, which was18 for the experimental and 8 for the control group. Of the 49 total number of students enrolled in all three classes participating in this study, 53% (26 students) participated in the study. This situation was not under the control of the researcher. At times the situation of non-participation in the study was due to COVID-19. Either the student or a family member was infected. The attrition rate was also a legitimate concern in one of the experimental classes in which half of the students were repeating Calculus I students.

In his review of Sfard’s commognitive perspective, Wing (2011) underscored the pragmatism of an approach that deems communication as tantamount to thinking.

Mathematical objects as discursive objects would allow practitioners to teach in the classrooms with readily observable outcomes and other learning phenomena. In the argument that follows, I present an interpretation of the qualitative data as observable outcomes.

Complementarity and Integration of Results for RQ1 and RQ2

Complementarity between quantitative and qualitative data was posited by Greene et al. (1989) as a reason for mixing methods. Complementarity seeks clarification, illustration, and elaboration of results from the other method (Greene et al., 1989). The examination of complementarity between quantitative and qualitative data was optimized in this mixed method study. Although the within and between group statistical analyses resulted in no significant statistical growth; the growth of the aggregate, that is, the difference between the pretest to posttest within the aggregate group, was determined to be statistically significant. The results from the quantitative analysis of the aggregate group were juxtaposed with the qualitative analysis of the transcripts from the student interviews that focused on the students' understanding of the derivative concept. However, as I discovered, the students' story regarding their developing understanding of the derivative concept was left untold by the statistical analysis results from the Derivative Concept Assessment outcomes.

The purpose of the interviews was to determine to what extent participation in classroom discourse in Calculus I affected the understanding of the derivative concept in a Calculus I course. Eleven of the twelve students interviewed were able to demonstrate early to advanced development of the derivative concept in at least one of the four condensed categories of representations of the derivative concept. When comparing the control to the experimental group, it was evident the experimental group had developed impressive calculus discourse about the idea of the derivative. Six of the eight experimental students understood the derivative concept using two representations, symbolic and visual. Two of four control students understood the derivative concept exceptionally in all four condensed categories or representations of the derivative, in comparison to one of eight of the experimental students.

Common to the discourse of the experimental students was their use of terms related to a symbolic and visual understanding of the derivative concept as well as their capacity as an aggregate to algebraically manipulate and visualize functions. When the quantitative results are juxtaposed with the qualitative results, the implication was that students' understanding of the derivative concept changed—learning occurred. Growth, that is, change in their understanding of the derivative, appeared to have been experienced by both the control and experimental groups.

Effects of Good and Ambitious Teaching on Attitude (RQ3 and RQ4)

The innovation for this study drew from two pedagogically related outcomes from the *Characteristics of Successful Programs in College Calculus* (CSPCC) Project results from analysis of their national survey data. Survey instruments, referred to as the Attitude Surveys, were adapted from the Mathematical Association of America (MAA) national survey (Bressoud, 2015) and served as an affective-scale measure for this study. The pre- and post-Attitude Surveys addressed RQ3 and RQ4.

Interpretations of Results for RQ3, RQ4, and the Second Purpose

How and to what extent does *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence*, *confidence*, and *enjoyment* in mathematics? Descriptive statistics were used to analyze the pretest and posttest data for the control and experimental groups. The participants entered Calculus I with a mid to high self-reported confidence in their prerequisite skills and readiness for calculus. An observation of the percentages reporting *persistence* in taking Calculus II, *confidence*, and *enjoyment* indicated that there was no change in the variable composites, *persistence*, *confidence*, and *enjoyment*, of students' attitude towards Calculus I.

The second purpose of this study was to assess the impact of the combination of two factors, *Good Teaching* and *Ambitious Teaching*, on a composite mathematics attitude comprising the dependent variables *persistence, confidence*, and *enjoyment* in mathematics, particularly calculus. Based on statistical results discussed above and in chapter 4, the answer is that there was no effect. On a positive note, the researcher can report that the *Ambitious Teaching* the students experienced in our class, based on statistical analysis, did not affect the students' attitudes toward Calculus I negatively.

These null quantitative-analysis results were contrary to the results reported by Sonnert and Sadler (2015) for the first nationwide study of college-level calculus, the CSPCC project. Sonnert and Sadler's (2015) statistical analysis on CSPCC survey data asserted changes for *Mathematics Attitude* were all negative. In the CSPCC project, three effects of the variables were strong and pervasive: the students' initial attitude and the students' strong prior mathematical experience and preparation. In the CSPCC project, *Good Teaching* had a positive effect and *Ambitious Teaching* had a small negative effect. For this study, the descriptive statistics resulting from analyzing the students' entering characteristics indicated the students had mid to high self-reported confidence in their prerequisite skills and readiness for calculus. However, contrary to the CSPCC project results, in this study, *Good Teaching* had no effect on attitude.

Complementarity and Integration of Results for RQ3 and RQ4

Based on the statistical analysis of collected survey data to inform RQ3, the intervention, at best, did no harm to the students' attitudes toward Calculus I as they exited the class. However, I argue that their exit attitude, if there was indeed no change in their attitudes, must be the same as the level of the attitudes with which they entered our calculus class. The descriptive statistics resulting from analyzing the students' entering characteristics indicated the students had mid to high self-reported confidence in their prerequisite skills and readiness for calculus. Therefore, the analysis suggests the students exited our class with the same mid to high self-reported confidence with which they entered.

The qualitative analysis results provide a positive story relevant to how and to what extent *Good Teaching* and *Ambitious Teaching* impact students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I.

Discourse analysis of twelve student interviews revealed 5 of 22 *Good Teaching* and 2 of 14 *Ambitious Teaching* practices were considered by students as practices that effectively impacted their attitudes in our Calculus I class. Two condensed categories of *Good Teaching* practices*, interactions validating students* and *available and encouraging instructor*, were cited most to impact students' attitude in Calculus I. Most students conveyed that *Good* or *Ambitious Teaching* practices, in which instructors asked questions to determine what students understood or asked students to explain their thinking, impacted their *confidence* or *enjoyment* of Calculus I. Finally, students conveyed that, if instructors only had time to act on one practice to impact student attitude, the students would ask the instructor to—*make class interesting*.

These results from the qualitative analysis are in line with the proposal of Mesa et al. (2015) that mathematics departments support the practice of *Good Teaching* by acknowledging students in classroom interaction, encouraging and available instructors, and providing fair assessments. In their investigation using MAA's study of CSPCC, Larsen et al. (2015) examined the relationships between *Ambitious Teaching* and retention and changes in attitudes and beliefs. Results indicated there was some promise of improving student persistence to continue to Calculus II with a combination of *Ambitious* and *Good Teaching* (Larsen et al., 2015). My findings from the student interviews were consistent with their findings. Regarding the frequency of students citings, for the *Good Teaching* variable, "My calculus instructor encouraged students to enroll in Calculus II," several students expressed wanting to continue to Calculus II or the next mathematics class in the context of discussing their persistence in our Calculus I class. One of my control students, Rho, a female student who was taking Calculus I for the second time, expressed her thoughts as follows:

> [You] encourage[d] me to enroll [in] calculus 2. Okay you never once said oh don't bother with Calculus 2. You were very clear about you know yeah this is hard. Calculus 2 gets harder but if you're even remotely interested you should try and that yeah that had a really big impact on confidence.

When the quantitative results are juxtaposed with the qualitative results, the implication was that *Good Teaching* and *Ambitious Teaching* impacted students' attitudes toward mathematics, particularly *persistence, confidence*, and *enjoyment* in Calculus I.

The implication is that their attitudes changed. Based on the qualitative data and discourse analysis, both the control and experimental groups appeared to have experienced change; that is, change in attitude toward mathematics which impacted their persistence, confidence, and enjoyment. Next, I frame the first purpose of the study.

Framing the First Purpose of This Study

The first purpose of my investigation was to engage students in mathematical discourse to motivate student thinking and understanding of calculus, particularly, the derivative concept. Realizing what the experimental class looked and felt like to my calculus students, in contrast to the control class, was a priority for me as a practitioner-researcher. To design and undergird the structure of my Calculus I class for the study, four frameworks were operationalized simultaneously. The mainframe was Sfard's (2008) commognitive approach. The units for teaching the derivative were grounded in an object-process framework used to define what it means to understand the derivative (Zandieh, 1997). The online class was structurally framed by an architectural design approach (Hathaway & Norton, 2013) to organize and structure course content. Finally, an analytical frame was used to support students' opportunity to learn through given exercises and other tasks (White & Mesa, 2014). By far the most important aspects of the innovation for the study, with regards to the course design, were the verbs that captured the essence of the impact of the four frameworks when operationalized in tandem. The student actions paved the path for their success in our Calculus I class —*uncover*, *play*, *apply*, *connect*, *question*, and *realize*—framing an innovation to engage students in mathematical discourse, fulfilling the first purpose of this study.

Although there is a section that discusses limitations, I believe it is appropriate to discuss the possible concern that surfaces regarding ensuring the only difference between the control and experimental is that the innovation is administered in the experimental group only. The two courses were not extremely different. One course was designed using the four frameworks. That one course was then published on Canvas and copied to all courses; that is, for both the control and experimental group. Then for the experimental group classes, I added the *connect* team activities for every even week. I also revised the *realize* activity for the experimental group, so they had the option to take exams with their team. In addition, for the *realize* activity, there was a third part which was a videoed team discussion of selected items on an exam for the teams who took an exam together. Essentially, the difference in the control and experimental groups was the added teamwork for the experimental group that aligned with the underlying participation metaphor. The basic unit of analysis used in research grounded on the participation metaphor was—discourse (Sfard, 1998, 2008).

In the next sections I discuss the implications for practice and recommendations for research. I also share the limitations of the results. Then I offer the implications for practice and recommendations. I then close our discussion by sharing the effect of my study on my development as a researcher and offering my concluding thoughts.

Implications for Practice and Recommendations for Research

Findings and outcomes from this study, particularly the design and implementation of the innovation, encourage multiple implications for practice. I present three implications here: (a) the need for emphasis from what is taught, to more importantly, how it is taught; (b) the need to incorporate multiple *Good Teaching* practices; and (c) the need to engage the simplest *Ambitious Teaching* practices.

Give Unrelenting Focus on How Rather Than What Students Learn

Bressoud (2019) asserted three reasons why the current calculus reform agenda is not an iteration of the 1980s Calculus Reform effort. One reason is the focus was on developing the new generation of educators accompanied by an emphasis from *what* is taught, to more importantly, *how* it is taught. The frameworks that undergird the design of my innovation supported how I delivered the learning material to my students. My purpose was to frame the calculus class for efficient, and more importantly, effective delivery. This operationalization of weaving calculus discourse within the existing curriculum took precedence over *Good Teaching and Ambitious Teaching* practices. The dividends on my investments to appropriately operationalize, in Calculus I, how the students engaged were observable, measurable, and sustainable. I am using the same Canvas shell this spring semester 2022 with many improvements.

As practitioners begin to focus on how to teach or how students learn, rather than what they learn, I anticipate the area of study will warrant additional research. When I began my literature review on Sfard's (2008) commognitive approach three years ago, research studies in calculus using the Commognitive Framework were being conducted by mostly PhD candidates. Sfard was often a member of the candidate's committee. More research is warranted in the area of study relevant to how students develop and how practitioners teach effective discourse in Calculus I.

Practice Good Teaching: Ask Questions to Determine Understanding

The five most frequently cited *Good Teaching* practices in my Calculus I classes were: (a) asking questions to determine students' understanding; (b) encouraging attendance of office hours; (c) providing understandable explanations; (d) showing specific work; and (e) making class interesting. All of these can be incorporated into daily practices in most, if not all, classrooms for all disciplines: gateway English, gateway mathematics, chemistry, or an art class, are a few examples. I asked my students, “What do you think is the most influential instructor practice that impacted your attitude towards calculus in terms of persistence, confidence and enjoyment?” The most frequently cited practice was to make class more interesting. Our students desire to get to know us and the world around them through our lens as teachers; however, we must come down to their level without sacrificing our professionalism or the respect for our discipline. My students often implied that I tried to relate to them and came down to their level as Cox posited in her book *The College Fear Factor* (Cox, 2009). I tried to invite my students to our classroom by easing their fears, particularly in a Calculus I class. Lambda, a female STEM major who experienced a traumatic event that undermined her confidence during our class, expressed her relevant thoughts:

> You're motivational, you’re okay with accepting wrong answers. [P]eople keep on trying. [Y]ou just engage everybody even if it’s not about calculus. [T]hat’s what impacts my enjoyment to want to stay in class and proceed forward taking another one of your classes.

Research on *Good Teaching* practices is abundant. *Good Teaching* was discussed in chapter 2. However, further research is needed to establish ways of operationalizing the combination of *Good Teaching* practices with the development of discourse in calculus, particularly engaging the triad of calculus concepts: limit, derivative, and integral.

In their discussion of results for their study, Bressoud et al. (2015) acknowledged that the outcomes they discussed are “only half of the story.” The other half of the story that Bressoud et al. (2015) alluded to are student performance data. The performance study was conducted in this study using the Derivative Assessment to measure the understanding of the derivative concept in both control and experimental groups. A comprehensive study in Calculus I, conducted by a practitioner-researcher in their own classroom, is uncommon and thus warrants attention from the mathematical research community.

Practice Ambitious Teaching: Ask Students What They Think

Larsen et al. (2015) asserted there were significant methodological challenges for research attempting to relate *Ambitious Teaching* to changes in student attitudes and beliefs. Additionally, they advised case studies indicated that *Ambitious Teaching* practices required substantial institutional support and advanced knowledge, skills, and beliefs on the part of instructors. After having experienced the changes that I have created in my classes, I realized the immense shift in attitude, particularly in persistence, confidence, and enjoyment that were necessary to move forward with both *Good Teaching* and *Ambitious Teaching* for both students and for the practitioner. The actions: *uncover*, *play*, *apply*, *connect*, *question*, and *realize* are habits of practice and of mind for both students and practitioners alike during the development of discourse in our classes.

Through interviews, students insisted, however, the seemingly simplest *Ambitious Teaching* practices were often the most effective. The qualitative analysis conducted on the twelve interview transcripts indicated that asking students to explain their thinking was the most influential *Ambitious Teaching* practice and had the most impact on the overall composite attitude of students. Delta, a female engineering student, appeared to be the most positively impacted by being asked to explain her thinking. Zeta, a male engineering student, was also positively impacted by being asked to explain his thinking. They expressed their sentiments as follows:

> Delta: [H]onestly being able to explain what I'm thinking and being able to explain how I did this problem how I was I thinking and yeah you know that also helped me in some other ways not only in the math class.
>
> Zeta: [O]ur class is like a constant live discussion where everyone can have their input whether it's with chat or saying something on live. [I] think that especially is a positive impact on everyone's persistence because we always know that we can give our own input and see if like especially with problems that you do in class if we're on the right track with how we're going. [I]t's how like we're going to solve it going through our own head.

Frequently asking students to explain their thinking is the simplest *Ambitious Teaching* practice. Of course, the instructor's response to students thinking out loud is critical to the effectiveness of the practice. For example, during my classes, I needed to remind myself to try to invite my students to our classroom by easing their fears about thinking out loud. There is a fast-paced interaction form known as IRE/F, Interaction, Response, and Evaluation/Feedback (Hicks, 1995). I practiced less verbal emphasis on the evaluation so the students sharing their thoughts out loud did not feel they were being judged when they shared their developing thoughts.

Research on *Ambitious Teaching* practices is abundant. *Ambitious Teaching* was discussed in chapter 2. However, like *Good Teaching* practices, further research is needed to establish ways of operationalizing the combination of *Ambitious Teaching* practices with development of discourse in calculus. In addition, as in *Good Teaching* practices, further studies should consider how we teach the triad of calculus concepts: limit, derivative, and integral such that we engage students in relevant discourse while capitalizing on the effectiveness of *Ambitious Teaching* practices such as asking students what they think in class, on homework, and on exams. Further studies should also consider students working in teams and the development of discourse longitudinally between the teammates and the individual student.

Limitations

Relevant to my research purpose, the limitations or features of the investigation that may potentially raise concern regarding the validity/credibility and reliability and decrease the confidence in the data include: (a) my positionality as an insider or researcher studying my own practice; (b) the small sample size; and (c) a time period bias.

I engaged in every aspect of this action research. I designed and implemented the innovation and taught Calculus I to both the control and experimental groups. I collected both the quantitative and qualitative data and conducted the analysis to determine the results. I was aware my positionality may jeopardize all purposes of the study from the start of this investigation. My intimate involvement may have affected the findings and/or outcomes of the investigation, if for example, the participants felt uncomfortable either participating or not participating in the investigation in their roles as my students. However, the investigation afforded me the opportunity to develop reflective practice (Dewey, 1989).

During the summer before implementing the innovation and collecting data, I met a reflective practice coach for several weeks. During the research, I consistently engaged reflexivity and criticality in order to avoid introducing bias into the study. Reflexivity was particularly relevant when I was collecting and analyzing qualitative data. The two different approaches and multiple stages and steps with multiple coding methods was an effective way to minimize or ideally eliminate introducing bias into the qualitative analysis and findings.

The generalizability of the results applied primarily to the Calculus I classes I taught. The study had a 53% participation rate (26 out of 49 students). If the small sample size did not affect the determination of the difference in performance within and between the experimental versus control groups, then the generalizability of the results would be applicable but limited to my class. My positionality in this study would likely cause concern if the generalizability was extended to our department's Calculus I population in fall 2021. The time period bias discussed in a previous section was my concern about the short two-week period between the pre- and post-Derivative Assessment that may have introduced a non-sampling error in the study.

Conclusion and Closing Thoughts

Conclusion

Are you glad that the scientists that discovered COVID-19 and those who produced the vaccine passed Calculus I? But do you know that fewer than 4 out of 10 STEM-intending majors complete a STEM degree? Trends show that the number of STEM majors decreased over the past decades. After more than four decades of calculus reform, Calculus I persists as a gatekeeper, not a gateway course, to STEM. The problem is no longer just a problem of practice, it is an international concern.

The STEM industry requires more and better prepared STEM students. The industry demand is far higher than the supply for specialized skills in STEM. The lack of inclusivity and the disproportionate participation in STEM fields, based on gender, race, and socioeconomic backgrounds, have been and have continued to be serious problems in our nation's efforts to navigate a 21st century economy that is increasingly dependent on STEM literacy. Inconsistent with common beliefs, students who switched away from STEM majors did not lack persistence nor academic preparation. Students cited poor instructional experiences in gateway first-year courses.

The findings of this action research study propose further investigation of how we teach Calculus I, particularly focusing on two constructs referred to as *Good Teaching* and *Ambitious Teaching* practices in literature. This investigation suggests that further investigation of how and to what extent the variables of *Good Teaching* and *Ambitious Teaching* impact students' composite *Mathematics Attitude* comprising persistence, confidence, and enjoyment. This research also suggests the practices that a teacher can adapt are general enough to use in STEM courses beyond calculus, including disciplines outside of STEM.

This study further suggests seven practices that can be adapted into STEM or non-STEM classes to improve students' persistence, confidence, and enjoyment. These practices include:

- ask questions to determine if students understand what is being discussed;
- encourage students to attend office hours;
- provide explanations that are understandable;
- show how to work specific problems;

- make class interesting;
- ask students to explain their thinking; and
- assign group projects.

Results from this study proposes further investigation of the Commognitive Framework and its role in engaging students in mathematical discourse to motivate student thinking and understanding of calculus, particularly, the derivative concept. This study suggests the process of learning is tantamount to changing and shaping ways of communicating by the commognitive approach—the approach of thinking-as-communicating. That is, as calculus students develop and change their discourse, their individualized form of communication, that is thinking, will also transform. This investigation suggests investigating how and to what extent students can understand a concept, such as the derivative concept, when they are fully engaged in discourse of the discipline, or in this study, Calculus I.

Closing Thoughts

This program and my research have been a remarkable journey. To think, to perform, and to act with integrity (Shulman, 2005) was a pleasure. I savored most moments of the experience. A year ago, I wistfully titled my dissertation. What a delightful surprise the outcome of this study is. The instructor practice that impacts my students' persistence, confidence, and enjoyment most is indeed—thinking out loud!

Below is my abstract for my sabbatical proposal for the academic year 2022-2023. The sabbatical title is *Beyond Thinking Out Loud: Innovating the Role of Discourse in Calculus I*. I look forward to continuing my study and this action research.

Abstract for Sabbatical

The title for my current dissertation research is *Thinking Out Loud: The Role of Discourse in Understanding the Derivative in Calculus I*. To extend my dissertation, I will disseminate my research findings, through presentations and publications, sharing my students' and my journey to realize the power of discourse in calculus. I will reconsider the critiques of and gaps in Sfard's Commognitive Framework, my current dissertation's theoretical center. The Commognitive Framework defines thinking as communicating, asserting that learning, as a result, is a change in discourse–an observable, measurable outcome. After delving deeper into the complexities of the Commognitive Framework, I will innovate further and deliver a redesigned Calculus I course grounded in theoretical, conceptual, analytical, and design frameworks which organize and structure course content and support students' opportunities to learn through discourse. A communicational approach rendering abstract concepts observable is significant for student success in Calculus I, a gateway STEM course.

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APPENDIX A

TASK ORIENTATION, DEFINITIONS, AND EXAMPLES

Orientation	Definition	Example
Remember	Students are prompted to retrieve knowledge from long-term memory.	Write the definition of the derivative of f at $x = a$.
Recall and Apply Procedure	Students must recall the algorithms for applying certain procedures and carry them out.	Find the derivative of f at $x = a$.
Recognize and Apply Procedure	Students must recognize what knowledge or procedures to recall without being directly prompted. Conceptual knowledge plays a plausible role in this venture, but some students may be able to answer the question with memorized procedures or formulas. Students may have to string together several procedures.	At what value of x does f attain a minimum?
Understand	Students are prompted to make interpretations, provide explanations, make comparisons or make inferences that require an understanding of a mathematics concept.	Is this graph a plausible graph for f given the table for values of $f(0)$? Explain why or why not?
Apply Understanding	Students must recognize when to use (or apply) a concept when responding to a question or when working on a problem. To recognize the need to apply, execute or implement a concept in the context of working a problem requires an understanding of the concept.	Given a flask shaped like an inverted cone, write a function rule that expresses the relationship between the height of the water in the flask and number of ounces of water in the flask.

Note. Reprinted from "Describing Cognitive Orientation of Calculus I Tasks Across Different Types of Coursework," by N. White and V. Mesa, 2014, *ZDM Mathematics Education, 46*, p. 680. Copyright 2014 by FIZ Karlsruh

Orientation	Definition	Example
Analyze	Students are prompted to break material into constituent parts and determine how parts relate to one another and to an overall structure or purpose. Differentiating, organizing, and attributing are characteristic cognitive processes at this level.	Graph the given functions on a common screen. How are these graphs related? $y = 3^x$, $y = 10^x$, $y = \left(\frac{1}{3}\right)^x$, and $y = \left(\frac{1}{10}\right)^x$. What general conclusions can you draw?
Evaluate	Students are prompted to make judgments based on criteria and standards. Checking and critiquing are characteristic cognitive processes at this level.	Is it reasonable to use this model to predict the winning height [of the high jump] at the 2100 Olympics (Stewart, 2012, p. 35).
Create	Students are prompted to put elements together to form a coherent or functional whole; reorganize elements into a new pattern or structure. Generating, planning, and producing are characteristic cognitive processes at this level.	A student in our class wants to estimate the integral of a function using a Riemann sum with 5 partitions. Give a function and domain of integration such that the right-hand Riemann sum will be off of the true integral by more than 200 % but the left-hand Riemann sum will be exact.

Note. Reprinted from “Describing Cognitive Orientation of Calculus I Tasks Across Different Types of Coursework,” by N. White and V. Mesa, 2014, *ZDM Mathematics Education, 46*, p. 680. Copyright 2014 by FIZ Karlsruhe.

APPENDIX B

TYPES OF COURSEWORK FOR EXPERIMENTAL GROUP

White and Mesa (2014, p. 681), posited that coursework can be characterized using the factors of grade weight, time and resources available to students and as fulfilling two major roles:

(1) to engage students in learning the content (opportunities to learn), and
(2) to demonstrate that students have indeed learned the content (opportunities to demonstrate that learning has occurred).

Experience	Activity	Characteristic of Coursework
Uncover	be informed: read and watch videos	opportunity to learn and demonstrate learning has occurred
Play	do *play* or basic procedural problems	opportunity to learn and demonstrate learning has occurred
Apply	do *Apply* problems or applied problems	opportunity to learn and demonstrate learning has occurred
Connect	meet team for *play*/*apply* discussions per specifications and specific *play* and *apply* problems to be discussed and specified prompts to be addressed	opportunity to learn and demonstrate learning has occurred
Question	ask *play*/*apply*-related questions motivated by connect team discussion; questions will motivate whole-class Canvas discussion	opportunity to learn and demonstrate learning has occurred
Realize	take team exams timed and asynchronous Connect with team discussion on exam exercises; create team and solo exam validation videos	opportunity to learn and demonstrate learning has occurred

Note. The difference between the control and experimental students' experience was the *connect* experience. The experimental student's activities included the *connect* experience and the control student's activities did not. A reflection activity to develop calculus discourse was integrated into the *play* and *apply* activities for the experimental group. In addition, the experimental students had an option to take the exams with teammates. Team and solo validation videos on instructor-selected exam items were requirements for the experimental students.

APPENDIX C

TYPES OF COURSEWORK FOR CONTROL GROUP

White and Mesa (2014, p. 681), posited that coursework can be characterized using the factors of grade weight, time and resources available to students and as fulfilling two major roles:

(1) to engage students in learning the content (opportunities to learn), and
(2) to demonstrate that students have indeed learned the content (opportunities to demonstrate that learning has occurred).

Experience	Activity	Characteristic of Coursework
Uncover	be informed: read and watch videos	opportunity to learn and demonstrate learning has occurred
Play	do *play* or basic procedural problems	opportunity to learn and demonstrate learning has occurred
Apply	do *apply* problems or applied problems	opportunity to learn and demonstrate learning has occurred
Question	ask *play*/*apply*-related questions motivated by *connect* team discussion; questions will motivate whole-class Canvas discussion	opportunity to learn and demonstrate learning has occurred
Realize	take exams individually timed and asynchronous; no validation videos required	opportunity to learn and demonstrate learning has occurred

Note. The difference between the control and experimental students' experience was the *connect* experience. The experimental student's activities included the *connect* experience and the control student's activities did not. A reflection activity to develop calculus discourse was integrated into the *play* and *apply* activities for the experimental group. In addition, the experimental students had an option to take the exams with teammates. Team and solo validation videos on instructor-selected exam items were requirements for the experimental students.

APPENDIX D

STUDENT PRE- AND POST-ATTITUDE SURVEYS

This is a survey used to inform the improvement of Calculus I and inform the potential design of calculus programs to promote student success. The survey is administered via Google Forms.

You will need to enter a STUDY ID. To create your STUDY ID, please pick the first three letters of your mother's name and the last 3 digits of your phone number. For example, if your mother's name is Samantha and your phone number is 123-456-7890, your STUDY ID will be "sam890".

Reference: Bressoud, D. M., Mesa, V., & Rasmussen, C. (Eds.). (2015). *Insights and recommendations from the MAA national study of college calculus*. Mathematical Association of America.

*** Required**
1.
Enter your STUDY ID *
2.
Enter your class number*
3.
Q1 My placement in calculus was determined by: *
Check all that apply.
My ACT or SAT score
My score on a placement exam
My successful completion of prerequisite courses
My AP exam score
Don't know
4.
Q2 Did you take the SAT exam? *
Mark only one oval.
No
Yes
5.
Q3 Did you take the ACT exam? *
Mark only one oval.
No
Yes
6.
Q4 My math courses in high school have prepared me to solve word problems. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

7.
Q5 My math courses in high school have prepared me to complete calculations without a calculator. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
8.
Q6 My math courses in high school have prepared me to factor expressions. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
9.
Q7 My math courses in high school have prepared me to solve equations. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
10.
Q8 My math courses in high school have prepared me to solve inequalities. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
11.
Q9 The teacher of my last mathematics course lectured most of the time. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree

Strongly agree
12.
Q10 The teacher of my last mathematics course primarily showed us how to get answers to specific questions. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
13.
Q11 The teacher of my last mathematics course frequently had us work in groups. *
Mark only one oval.
Strongly disagree
Slightly Disagree
Disagree
Slightly Agree
Agree
Strongly agree
14.
Q12. The teacher of my last mathematics course frequently had us solve challenging problems.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
15.
Q13 The teacher of my last mathematics, high school or college, course allowed the use of graphing calculators on exams. *
Mark only one oval.
Never
Sometimes
Always
16.
Q14 The teacher of my last mathematics course, high school or college, allowed the use of calculators that performed symbolic operations on exams (e.g., TI-89, TI-92). *
Mark only one oval.
Never
Sometimes
Always

17.
Q15 The teacher of my last mathematics course, high school or college, showed students how mathematics is relevant. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
18.
Q16 The teacher of my last mathematics course cared that I was successful in the course. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
19.
Q17. I am comfortable using a graphing calculator and/or using a computer algebra system (e.g., Maple, MATLAB). *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
20.
Q18 The teacher of my last mathematics course, high school or college, used an electronic response system (such as clickers or Google Forms) to poll students during class. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
21.
Q19 Did you take a precalculus course in COLLEGE before this course? *
Mark only one oval.
No

Yes
22.
Q20 Where was your previous college precalculus course taken? *
Mark only one oval.
At my high school as a concurrent enrollment course
At this college
At another 2-year college
At a 4-year college or university
23.
Q21 What was the delivery mode of the precalculus (College Algebra, Trigonometry, or Precalculus) course you completed prior to this one? *
Mark only one oval.
Synchronous online
Asynchronous online
Face-to-face with an instructor
24.
Q22 What was the delivery mode of the last mathematics course (precalculus or another mathematics course) you completed prior to this one? *
Mark only one oval.
Synchronous online
Asynchronous online
Face-to-face with an instructor
25.
Q23 Did you take a calculus course in COLLEGE before this course? *
Mark only one oval.
No
Yes
26.
Q24 What was the delivery mode of the calculus course you completed prior to this one? *
Mark only one oval.
I did not take a calculus course before this one.
Synchronous online
Asynchronous online
Face-to-face with an instructor
27.
Q25 Are you taking this course again? *
Mark only one oval.
No
Yes
28.
Q26 Why are you taking this course again? *
Check all that apply.
This is my first time taking this course.
It did not count toward the credits I need

I passed, but I need/want a higher grade (e.g., for my major)
I did not pass the course
I dropped the class
I wanted to get a better grade
I wanted to improve my understanding of calculus
My college advisor told me to
29.
Q27 What grade do you expect in this calculus course? *
Mark only one oval.
A
B
C
D
F
Note for PI: Repeated question. This question is repeated in the post-survey Q1.
30.
Q28 Do you intend to take Calculus II? *
Mark only one oval.
No
Yes
I don't know yet
Note for PI: Repeated question. This question is repeated in the post-survey Q3.
31.
Q29 How important is a good grade in this course in influencing your decision whether or not to take Calculus II? *
Mark only one oval.
Not important at all
Unimportant
Slightly unimportant
Slightly important
Important
Very important
32.
Q30 Is Calculus II required for your major? *
Mark only one oval.
No
Yes
Don't know
Note for PI: Repeated question. This question is repeated in the post-survey Q2.
33.
Q31 I believe I have the knowledge and abilities to succeed in this course.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree

Slightly Agree
Agree
Strongly agree
34.
Q32 I understand the mathematics I have studied.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
35.
Q33 I am confident in my mathematics abilities.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
Note for PI: Repeated question. This question is repeated in the post-survey Q11.
36.
Q34 I enjoy doing mathematics.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
Note for PI: Repeated question. This question is repeated in the post-survey Q12.
37.
Q35 If I take another calculus course after this one, it will be because
Check all that apply.
It is required
I want to
38.
Note for PI: Repeated questions below Q36-Q44. These questions are repeated in the post-survey Q13-Q21.
Q36 How certain are you in what you intend to do after college? *
Mark only one oval.
Option A Not at all certain
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A

Option B Very certain
39.
Q37 When experiencing a difficulty in my math class, *
Mark only one oval.
Option A I try to figure it out on my own
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B I quickly seek help or give up trying
40.
Q38 For me, making unsuccessful attempts when solving a mathematics problem is *
Mark only one oval.
Option A a natural part of solving the problem
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B an indication of my weakness in mathematics
41.
Q39 My success in mathematics PRIMARILY relies on my ability to *
Mark only one oval.
Option A solve specific kinds of problems
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B make connections and form logical arguments
42.
Q40 My score on my mathematics exam is a measure of how well *
Mark only one oval.
Option A I understand the covered material
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B I can do things the way the teacher wants
43.
Q41 If I had a choice, *
Mark only one oval.
Option A I would never take another mathematics course
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B I would continue to take mathematics
44.
Q42 When studying Calculus I in a textbook or in course materials, I tend to *
Mark only one oval.
Option A memorize it the way it is presented
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B make sense of the material, so that I understand it
45.
Q43 When solving mathematics problems, graphing calculators or computers help

me to *
Mark only one oval.
Option A understand underlying mathematical ideas
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B find answers to problems
46.
Q44 The primary role of a mathematics instructor is to *
Mark only one oval.
Option A work problems so students know how to do them
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B help students learn to reason through problems on their own
Note for PI: Repeated questions above Q36-Q44. These questions are repeated in the post-survey Q13-Q21.
47.
Q45 Mathematics instructors should show students how mathematics is relevant *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
48.
Q46 If I am unable to solve a problem within a few minutes, it is an indication of my weakness in mathematics. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
49.
Q47 Mathematics is about getting exact answers to specific problems. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
50.

Q48 The process of solving a problem that involves mathematical reasoning is a satisfying experience. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
51.
Q49 In order to succeed in calculus at a college or university, I must have taken calculus before.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
52.
Q50 Approximately how many hours per week do you expect to work at a job this semester? *
Mark only one oval.
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the post-survey Q91.
53.
Q51 Approximately how many hours per week do you expect to participate in organized extracurricular activities such as sports, college paper, or clubs? *
Mark only one oval.
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the post-survey Q92.
54.
Q52 Approximately how many hours per week do you expect to spend preparing for

all classes(studying, reading, writing, doing homework or lab work, analyzing data, or other academic activities) this semester? *
Mark only one oval.
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the post-survey Q93.
55.
Q53 Approximately how many hours per week do you expect to spend preparing for calculus (studying, reading, doing homework or lab work) this semester? *
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the post-survey Q94.
56.
Q54 Which of the following BEST describes your current career goal?*
Check all that apply.
Medical professional (e.g., doctor, dentist, vet.)
Other health professional (e.g., nurse, medical technician)
Life scientist (e.g., biologist, medical researcher)
Earth/Environmental scientist (e.g., geologist, meteorologist)
Physical Scientist (e.g., chemist, physicist, astronomer)
Engineer
Computer Scientist
Mathematician,
Science/Math teacher
Other teacher
Social Scientist (e.g., psychologist, sociologist)
Business administration
Lawyer
English/Language Arts specialist
Other non-science related career
Undecided
Note for PI: Repeated question. This question is repeated in the post-survey Q95.

_End of Student Pre-Survey (Beginning of the Semester) _

This is a survey used to inform the improvement of Calculus I and inform the potential design of calculus programs to promote student success. The survey is administered via Google Forms.

You will need to enter a STUDY ID. To create your STUDY ID, please pick the first three letters of your mother's name and the last 3 digits of your phone number. For example, if your mother's name is Samantha and your phone number is 123-456-7890, your STUDY ID will be "sam890".

Reference: Bressoud, D. M., Mesa, V., & Rasmussen, C. (Eds.). (2015). *Insights and recommendations from the MAA national study of college calculus*. Mathematical Association of America.

*** Required**
1.
Enter your STUDY ID *
2.
Enter your class number*
3.
Q1 What grade do you expect (or did you receive) in this calculus course? *
Mark only one oval.
A
B
C
D
F
Note for PI: Repeated question. This question is repeated in the pre-survey Q27.
4.
Q2 Is Calculus II required for your intended major? *
Mark only one oval.
No
Yes
I don't know.
Note for PI: Repeated question. This question is repeated in the pre-survey Q30.
5.
Q3 Do you intend to take Calculus II? *
Mark only one oval.
No
Yes
I'm not sure
Note for PI: Repeated question. This question is repeated in the pre-survey Q28.
6.
Q4 If you are not intending to take Calculus II, check all the reasons that apply. *
Check all that apply.
I never intended to take Calculus II

I changed my major and now do not need to take Calculus II
My experience in Calculus I made me decide not to take Calculus II
I have too many other courses I need to complete
To do well in Calculus II, I would need to spend more time and effort than I can afford
My grade in Calculus I was not good enough for me to continue to Calculus II
I do not believe I understand the ideas of Calculus I well enough to take Calculus II

7.

Q5 When you started this class, did you intend to take Calculus II? *
Mark only one oval.
No
Yes
I wasn't sure

8.

Q6 This course has increased my interest in taking more mathematics.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

9.

Q7 I am good at computing limits and derivatives.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

10.

Q8 I am able to use ideas of calculus (e.g., limits, differentiation) to solve word problems that I have not seen before.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

11.

Q9 My previous math courses prepared me to succeed in this course.*
Mark only one oval.
Strongly disagree

Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
12.
Q10 Mathematics is about getting exact answers to specific problems.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
13.
Q11 I am confident in my mathematics abilities.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
Note for PI: Repeated question. This question is repeated in the pre-survey Q33.
14.
Q12 I enjoy doing mathematics. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
Note for PI: Repeated question. This question is repeated in the pre-survey Q34.
Note for PI: Repeated questions below Q13-Q21. These questions are repeated in the pre-survey Q36-Q44.
15.
Q13 How certain are you in what you intend to do after college? *
Mark only one oval.
Option A Not at all certain
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B Very certain
16.
Q14 When experiencing a difficulty in my math class, *

Mark only one oval.
Option A I try to figure it out on my own
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B I quickly seek help or give up trying

17.

Q15 For me, making unsuccessful attempts when solving a mathematics problem is *
Mark only one oval.
Option A a natural part of solving the problem
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B an indication of my weakness in mathematics

18.

Q16 My success in mathematics PRIMARILY relies on my ability to *
Mark only one oval.
Option A solve specific kinds of problems
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B make connections and form logical arguments

19.

Q17 My score on my mathematics exam is a measure of how well *
Mark only one oval.
Option A I understand the covered material
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B I can do things the way the teacher wants

20.

Q18 If I had a choice, *
Mark only one oval.
Option A I would never take another mathematics course
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B I would continue to take mathematics

21.

Q19 When studying Calculus I in a textbook or in course materials, I tend to *
Mark only one oval.
Option A memorize it the way it is presented
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B make sense of the material, so that I understand it

22.

Q20 When solving mathematics problems, graphing calculators or computers help me to *
Mark only one oval.
Option A understand underlying mathematical ideas

I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B find answers to problems

23.

Q21 The primary role of a mathematics instructor is to *
Mark only one oval.
Option A work problems so students know how to do them
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B help students learn to reason through problems on their own
Note for PI: Repeated questions above Q13-Q21. These questions are repeated in the pre-survey Q36-Q44.

24.

Q22 When my calculus instructor asked a question addressed to the whole class, s/he *
Mark only one oval.
Option A waited for a student to answer
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B answered the question if no one responded quickly

25.

Q23 When I asked a question about a problem I was having difficulty solving, my instructor *
Mark only one oval.
Option A solved the problem for me
I do not completely agree with option A, but agree with option A more than option B
I do not completely agree with option B, but agree with option B more than option A
Option B helped me figure out how to solve the problem

26.

Q24 My calculus instructor asked questions to determine if I understood what was being discussed. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

27.

Q25 My calculus instructor listened carefully to my questions and comments. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree

Strongly agree
28.
Q26 My calculus instructor discussed applications of calculus. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
29.
Q27 My calculus instructor allowed time for me to understand difficult ideas. *
Mark only one oval.
Very often
Often
Occasionally
Seldom
Rarely
Not all
30.
Q28 My calculus instructor helped me become a better problem solver.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
31.
Q29 My calculus instructor provided explanations that were understandable. *
Mark only one oval.
Very often
Often
Occasionally
Seldom
Rarely
Not all
32.
Q30 My calculus instructor was available to make appointments outside of office hours, if needed. *
Mark only one oval.
Very often
Often
Occasionally
Seldom

Rarely
Not all
33.
Q31 My calculus instructor discouraged me from wanting to continue taking calculus. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
34.
Q32 During class time, how frequently did your instructor show how to work specific problems?* Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
35.
Q33 During class time, how frequently did your instructor have students work with one another?*
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
36.
Q34 During class time, how frequently did your instructor hold whole-class discussion? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
37.
Q35 During class time, how frequently did your instructor have students give presentations? *
Mark only one oval.

Not all
Rarely
Seldom
Occasionally
Often
Very often
38.
Q36 During class time, how frequently did your instructor have students work individually on problems or tasks? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
39.
Q37 During class time, how frequently did your instructor lecture? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
40.
Q38 During class time, how frequently did your instructor ask questions? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
41.
Q39 During class time, how frequently did your instructor ask students to explain their thinking?*
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often
42.

Q40 How frequently did your instructor prepare extra material to help students understand calculus concepts or procedures? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often

43.

Q41 How frequently did your instructor require you to explain your thinking on your homework?*
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often

44.

Q42 How frequently did your instructor require you to explain your thinking on exams? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often

45.

Q43 How frequently did your instructor assign sections in your textbook for you to read before coming to class? *
Mark only one oval.
Not all
Rarely
Seldom
Occasionally
Often
Very often

46.

Q44 My calculus instructor made students feel nervous during class. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree

Slightly Agree
Agree
Strongly agree
47.
Q45 My calculus instructor encouraged students to enroll in Calculus II. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
48.
Q46 My calculus instructor acted as if I was capable of understanding the key ideas of calculus.*
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
49.
Q47 My calculus instructor made me feel comfortable asking questions during class. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
50.
Q48 My calculus instructor encouraged students to seek help during office hours. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
51.
Q49 My calculus instructor presented more than one method for solving problems. *
Mark only one oval.
Strongly disagree

Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
52.
Q50 My calculus instructor did not speak English very well. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
53.
Q51 My calculus instructor made class interesting. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
54.
Q52 Indicate how often your instructor assigned homework. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
55.
Q53 Indicate how often homework was collected (either hard copy or online). *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
56.
Q54 Indicate how often your instructor gave a short quiz. *
Mark only one oval.
never
some class sessions
about half the class sessions

most class sessions
every class session
57.
Q55 Indicate how often your instructor used technology. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
58.
Q56 Indicate how often your instructor demonstrated mathematics with a graphing calculator or graphing apps during class. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
59.
Q57 Indicate how often you used a graphing calculator or graphing apps during class. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
60.
Q58 Which of the following computing technologies did you use during your calculus class? *
Check all that apply.
None
Graph calculator, graphing apps, etc.
Computers, tablets, etc.
Clickers, polling apps, etc.
61.
Q59 Indicate how your instructor used technology during your class. *
Check all that apply.
To illustrate ideas
To find answers to problems
To check answers after we worked them out by hand
62.
Q60 Indicate how you used technology during your class. *
Check all that apply.

To find answers to problems
To understand underlying mathematical ideas
To check written answers after I worked them out by hand
63.
Q61 Do you have access to a computer algebra system (CAS), online or calculator, that has the capability to find the symbolic derivatives of a function? *
Mark only one oval.
No
Yes
I don't use a CAS calculator and/or a CAS online and/or CAS apps
64.
Q62 Were you allowed to use a graphing calculator and/or apps on exams? *
Mark only one oval.
No
Yes
65.
Q63 Assignments completed outside of class time were completed and graded online. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often
66.
Q64 Assignments completed outside of class time were graded and returned to me. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often
67.
Q65 Assignments completed outside of class time were returned with helpful feedback/comments. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often

68.
Q66 Assignments completed outside of class time were submitted as a group project. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often
69.
Q67 Assignments completed outside of class time were challenging but doable. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often
70.
Q68 Assignments completed *outside of class time* required that I solve word problems. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often
71.
Q69 Assignments completed *outside of class time* required that I solve problems unlike those done in class or in the book. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often
72.
Q70 Assignments completed outside of class time required that I use technology to understand ideas. *
Mark only one oval.
Not at all
Rarely

Seldom
Occasionally
Often
Very often

73.

Q71 The exam questions required that I solve word problems. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often

74.

Q72 The exam questions required that I solve problems unlike those done in class or in the book. *
Mark only one oval.
Not at all
Rarely
Seldom
Occasionally
Often
Very often

75.

Q73 My calculus exams were a good assessment of what I learned. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

76.

Q74 My calculus exams were graded fairly. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree

77.

Q75 My homework was graded fairly. *
Mark only one oval.
Strongly disagree

Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
78.
Q76 During class I contributed to class discussions. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
79.
Q77 During class I was lost and unable to follow the lecture or discussion. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
80.
Q78 During class I asked questions. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
81.
Q79 During class I simply copied whatever was written on the digital whiteboard. *
Mark only one oval.
never
some class sessions
about half the class sessions
most class sessions
every class session
82.
Q80 How often did you read the textbook prior to coming to class? *
never
some class sessions
about half the class sessions
most class sessions
every class session
83.

Q81 How often did you visit your instructor's office hours? *
never
some class sessions
about half the class sessions
most class sessions
every class session
84.
Q82 How often did you use online tutoring? *
never
some class sessions
about half the class sessions
most class sessions
every class session
85.
Q83 How often did you visit a tutor to assist with this course? *
never
some class sessions
about half the class sessions
most class sessions
every class session
86.
Q84 The homework for the course helped me learn the material. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
87.
Q85 The textbook and/or class materials helped me learn the material. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
88.
Q86 The textbook or reading materials for the course were readable. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree

Agree
Strongly agree
89.
Q87 I completed all my assigned homework. *
Mark only one oval.
Strongly disagree
Disagree
Slightly Disagree
Slightly Agree
Agree
Strongly agree
90.
Q88 Did you meet with other students to study or complete homework outside of class? *
No
Yes
91.
Q89 Did you belong to a calculus study group organized by your instructor or department? *
No
Yes
92.
Q90 Does your math department or university provide a walk-in tutor center for mathematics? *
No
Yes
93.
Q91 Approximately how many hours per week did you work at a job this semester? *
Mark only one oval.
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the pre-survey Q50.
94.
Q92 Approximately how many hours per week did you participate in organized extracurricular activities such as sports, college paper, or clubs? *
Mark only one oval.
0
1-5
6-10

11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the pre-survey Q51.
95.
Q93 Approximately how many hours per week did you spend preparing for all classes (studying, reading, writing, doing homework or lab work, analyzing data, or other academic activities) this semester? *
Mark only one oval.
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the pre-survey Q52.
96.
Q94 Approximately how many hours per week did you spend preparing for calculus (studying, reading, doing homework or lab work) this semester? *
0
1-5
6-10
11-15
16-20
21-30
More than 30
Note for PI: Repeated question. This question is repeated in the pre-survey Q53.
97.
Q95 Which of the following BEST describes your current career goal?*
Check all that apply.
Medical professional (e.g., doctor, dentist, vet.)
Other health professional (e.g., nurse, medical technician)
Life scientist (e.g., biologist, medical researcher)
Earth/Environmental scientist (e.g., geologist, meteorologist)
Physical Scientist (e.g., chemist, physicist, astronomer)
Engineer
Computer Scientist
Mathematician,
Science/Math teacher
Other teacher
Social Scientist (e.g., psychologist, sociologist)
Business administration
Lawyer

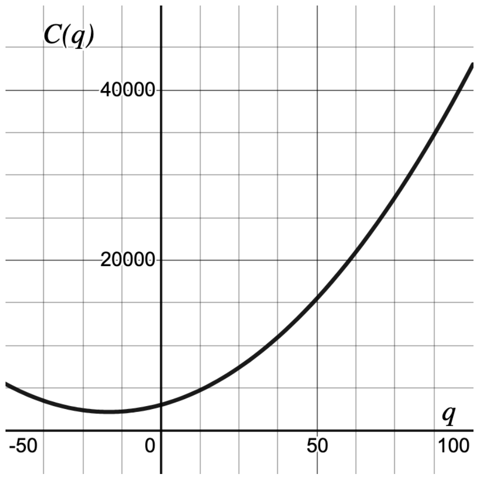
English/Language Arts specialist
Other non-science related career
Undecided
Note for PI: Repeated question. This question is repeated in the pre-survey Q54.

_End of Student Post-Survey (End of Intervention/Semester) _

APPENDIX E

DERIVATIVE CONCEPT ASSESSMENT

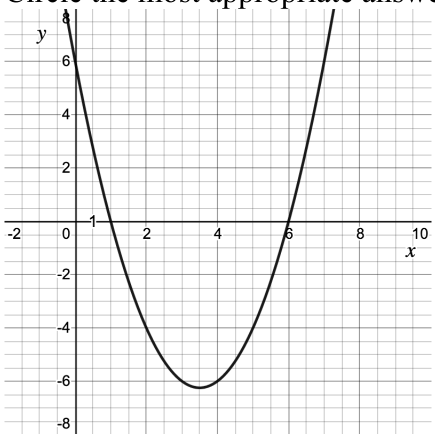
Item 1 $C(q)$is the total cost (in dollars) required to set up a new rope factory and produce q miles of the rope. The total cost, $C(q)$, satisfies the equation $C(q) = 3000 + 3q + 3q^2$ and the graph is given as follows



(a) Find the value of $C(2)$.
(b) What are the units of 2 in part (a)?
(c) What are the units of $C(2)$?
(d) What is the meaning of $C(2)$ in the context of the problem context?
(e) Find the value of $C'(2)$.
(f) What are the units of 2 in part (a)?
(g) What are the units of $C'(2)$?
(h) What is the meaning of $C'(2)$ in the context of the problem context?

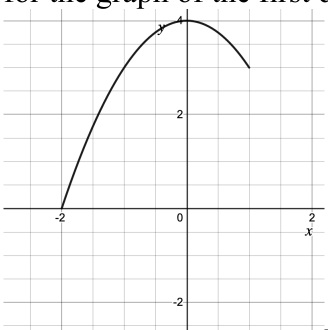
Item 2 The derivative of a function f is given as $f'(x) = x^2 - 7x + 6$.
What is the value of
$f'(2)$?

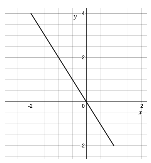
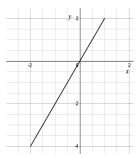
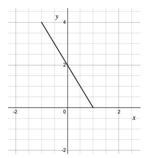
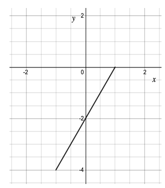
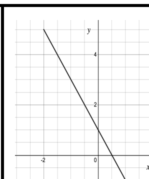
Item 3 The graph of the derivative $g'(x)$ of function g is given as follows. Circle the most appropriate answer for the value of $g'(2)$.



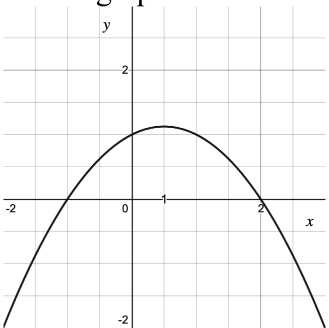
(a) -4	(b) -2	(c) 0	(d) 2	(e) 4

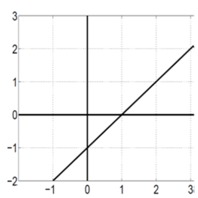
Item 4 Below is the graph of a function $f(x)$. Circle the most appropriate answer for the graph of the first derivative, $f'(x)$.



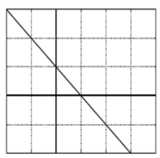
 (a)	 (b)	 (c)	 (d)	 (e)

Item 5 Below is the graph of a function $f'(x)$. Circle the most appropriate answer for the graph of the function $f(x)$.

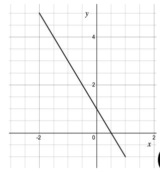




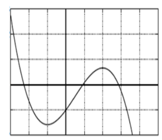
(a)



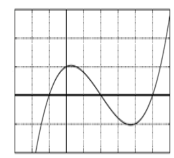
(b)



(c)



(d)



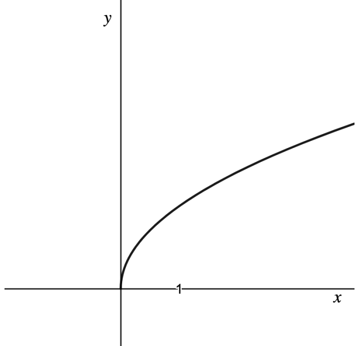
(e)

Item 6 If a function is always positive, then what must be true about its derivative function?

a) The derivative function is always positive.
b) The derivative function is never negative.
c) The derivative function is increasing.
d) The derivative function is decreasing.
e) You can't conclude anything about the derivative function.

Item 7 The derivative of a function $f(x)$ is negative on the interval $x = 2$ to $x = 3$. What is true for the function $f(x)$?
a) The function $f(x)$ is positive on this interval.
b) The function $f(x)$ is negative on this interval.
c) The maximum value of the function $f(x)$ over the interval occurs at $x = 2$.
d) The maximum value of the function $f(x)$ over the interval occurs at $x = 3$.
e) We cannot tell any of the above.

Item 8 The tangent line to this graph of $f(x)$ at $x = 1$ is given by $y = \frac{1}{2}x + \frac{1}{2}$. Which of the following statements is true and why?



(a) $\frac{1}{2}x + \frac{1}{2} = f(x)$	(b) $\frac{1}{2}x + \frac{1}{2} \leq f(x)$	(c) $\frac{1}{2}x + \frac{1}{2} \geq f(x)$	(d) $\frac{1}{2}x = \frac{1}{2}f(x)$	(e) None

Item 9 The derivative of a function, $f(x)$, is $f(x) = ax^2 + b$. What is required of the values of a and a so that the slope of the tangent line to the function f will be positive at $x = 0$.
a) a and b must both be positive numbers.
b) a must be positive, while b can be any real number.
c) a can be any real number, while b must be positive.
d) a and b can be any real number.
e) None of these

End of Derivative Concept Assessment

APPENDIX F

INTERVIEW PROTOCOL

Project Title: Thinking Out Loud: The Role of Discourse in Understanding the Derivative in Calculus I

Student's pseudonym

Time of Interview:

Date: Weeks of 12 and 13, Nov 13 and Nov 14

Place: Video Conference on Webex

Interviewer: Madeleine Chowdhury, Co-Principal Investigator

Interviewee/Position of Interviewee: Student's pseudonym/current Calculus I student in (class section) majoring in (major).

Description of Project: The purpose of this project is to collect qualitative data for PI's (my) action research, particularly regarding the understanding of the derivative concept, with students' use of appropriate mathematical discourse.

Opening statement: Hello. The purpose of this interview is for me to attain a better understanding of what students think and understand about the derivative concept. I will ask a total of # questions. The first five are general questions about the derivative. The rest of the questions are follow-up questions on items that were embedded in your last midterm exam. Please be mindful that I may ask you to refer back to your first five answers for the general questions when I start asking follow-up questions on those items from your last midterm. Before we get started, do you have any questions?

For Interviewer only: This is a 3-part interview with 3 sets of questions and a set of Survey-Question references for the interviewees.

Part 1: Derivative Concept Assessment follow-up. Ask general Qs about their understanding of the derivative (Park, 2011 and Zandieh, 1997) the ask specific Qs from Derivative Assessment results their pre- and post-Derivative Concept Assessment. Outcomes from the quiz taken by students will be available to them during the interview.

Part 2: Attitude Survey follow-up on *Good Teaching (GT)*

Part 3: Attitude Survey follow-up on *Ambitious Teaching (AT)*

Sets of survey questions relevant to GT and AT will be available to students.

There are 3 sets of 22 total Qs for GT and 2 sets of 14 total Qs for AT.

The protocol contains all of this information *with the exception of the students outcomes from the Derivative Concept Assessment*. Outcomes will be shared with student via shared screen on Webex. I will also give students temporary access to the Google Sheet. I'll turn the link on before the interview begins and turn the link off right after the interview.

Interview Questions Park (2011)

Q1. What is the derivative? Can you make a sentence with the word, "derivative"?
Follow-up 1. You can explain your concept of the derivative with your own words. Your answer
doesn't have to be mathematical. What does the derivative tell us?
Follow-up 2. There are more than one use of the term *derivative*. Can you specify them?
Follow-up 3. If they do not come up with the derivative function and the derivative at a point, then I will ask them about these two specific ideas. *Note. I'm not sure what the purpose of this follow-up is since Q4 serves this same purpose it seems.*

Q2. What is the derivative function?
Follow-up 1. Can you give me an example of the derivative function with any kind of mathematical objects such as an equation, graph, or table?

Q3. What is the derivative at a point?
Follow-up 1. Can you give me an example of the derivative function with any kind of mathematical objects such as equation, graph, or table?

Q4. Is there any relationship between these two terms, *derivative function* and *derivative at a point*?
Case 1. When the interviewee's answer is yes,
Follow-up 1.What is the relationship? Please describe their relationship. You may use your answers on the previous questions. You can use examples for example, equations or graphs.
Case 2.When the interviewee's answer is no, there is no relationship between these two terms, or they are totally different concepts?

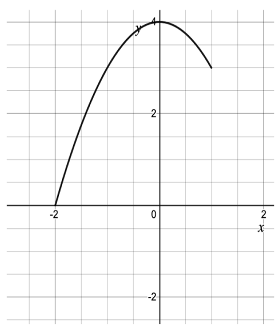
Follow-up 1. Can you explain why? You may use your answers on the previous questions.
Follow-up 2. As you can see, we use the same term *derivative* for these two concepts. What do you think about this?

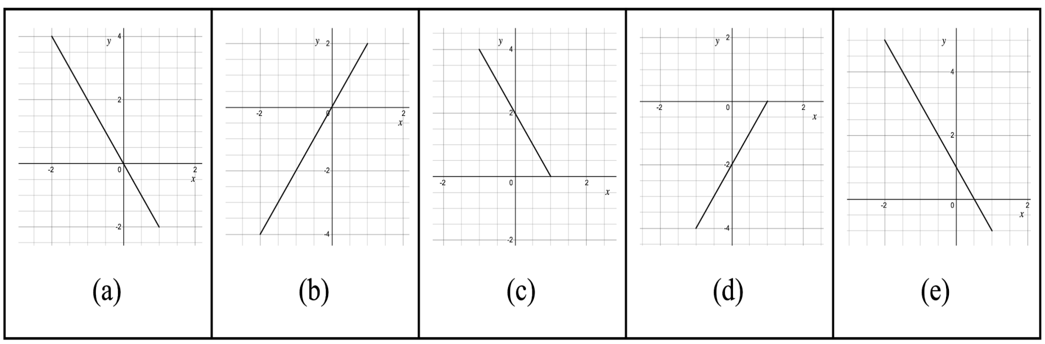
Q5. How about the term *function*? Is the term *function* related to the derivative function and the derivative at a point?
Follow-up 1. If so, how are they related? You can use examples.
Follow-up 2. *Note.* Students will be asked to explain and justify their answers.
Follow-up 2a. How did you come up with this example or problem?
Follow-up 2b. Could you explain what you wrote (drew) here?
Follow-up 3. *Note.* In the process of explaining, I will ask some clarifying questions.
Follow-up 3a. You just referred to the derivative as a function or the derivative at a point or does it matter?

Follow-up question regarding Derivative Concept Assessment
The following is a list of example questions to follow-up a student's responses to the assessment.

Q6. Let's look at your answer for Item 4. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.

Item 4 Below is the graph of a function $f(x)$. Circle the most appropriate answer for the graph of the first derivative, $f'(x)$.





Follow-up 1. Why did you choose (answer student chose)?
Case 1. When the interviewee's answer is because it is decreasing.
Follow-up 2. What is decreasing? That is, what is "it?"
Follow-up 3. What in the problem statement or signifier, tells you that "it" should be decreasing?

Follow-up 4. Why is d) not the answer?
Case 1. When the interviewee's answer is because it's negative.
Follow-up 5. Are you referring to the graph for your answer or the original graph?
Follow-up 6. Why can't the graph for your answer be negative?

Q7. Let's look at your answer for Item 6. Please have a look at your answer,

your work, and explanation. Please try to recall what you were thinking
and what you did.

Item 6 If a function is always positive, then what must be true about its derivative function?

a) The derivative function is always positive.

b) The derivative function is never negative.

c) The derivative function is increasing.

d) The derivative function is decreasing.

e) You can't conclude anything about the derivative function.

Case 1. c either a) or b)
Follow-up 1. Why? Is there any relationship between a function and its derivative?
Anticipated answers: If its derivative is positive, the function is increasing.
Follow-up 2: Can you explain why using your definitions in warm-up questions?
Case 2. When the interviewee's answer is either c) or d)
Follow-up 3: Why?
Anticipated interviewee's answer: Because a function and its derivative act similarly.
Follow-up 4: Always?
Anticipated interviewee's answers: Yes / No
Follow-up 5: If the answer is yes, consider the example of $y = -x$, what is its derivative? Do they act similarly? If the answer is no, why do you think this is a case of similarity of a function and its derivative. Or could you give me an example of a function which acts differently from its derivative?
Case 3. When the interviewee's answer is e)
Follow-up 6: Why? Can you explain this with your previous explanations?

Q8. Let's look at your answer for Item 7. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.
Item 7 The derivative of a function $f(x)$ is negative on the interval $x = 2$ to $x = 3$. What is true for the function $f(x)$?

a) The function $f(x)$ is positive on this interval.

b) The function $f(x)$ is negative on this interval.

c) The maximum value of the function $f(x)$ over the interval occurs at $x = 2$.

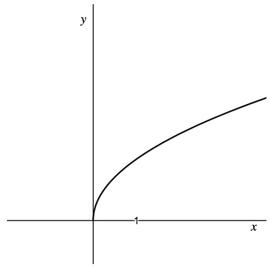
d) The maximum value of the function $f(x)$ over the interval occurs at $x = 3$.

e) We cannot tell any of the above.

Q8 continued. Use the same questions for the previous problem because this problem is about the relation between a function and its derivative, too.

Q9. Let's look at your answer for Item 8. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.

Item 8 The tangent line to this graph of $f(x)$ at $x = 1$ is given by $y = \frac{1}{2}x + \frac{1}{2}$. Which of the following statements is true and why?



(a)	(b)	(c)	(d)	(e) None
$\frac{1}{2}x + \frac{1}{2} = f(x)$	$\frac{1}{2}x + \frac{1}{2} \leq f(x)$	$\frac{1}{2}x + \frac{1}{2} \geq f(x)$	$\frac{1}{2}x = \frac{1}{2}f(x)$	

Case 1. When the interviewee's answer is: b) or c)
Follow-up 1: Why? Can you explain your answer with the graph of $f(x)$? Where would the tangent line be at?
Anticipated interviewee's answers: Draw the tangent line.
Follow-up 2: Can you explain how you got this inequality from this picture?
Anticipated interviewee's answers: The curve is below the line or above the line.
Case 2. When the interviewee's answer is: a) or d)
Follow-up 1: Are they the same? Why?
Follow-up 2: Can you explain your answer with the graph of $f(x)$? Where would the tangent line be at?
Follow-up 3: In the case of d), how did you get this equation?
Where is the $\frac{1}{2}x$ come from?
Anticipated interviewee's answers:
In the case of a), that's because the curve and the tangent line meet at this point.
In the case of d), that's because the curve and the tangent line meet at this point, and derivative at one point is the slope.
Follow-up 4: In this problem, should we look at only where the curve and tangent meet? Why?
Anticipated interviewee's answers: Yes because it is a tangent line at one point.
No, I changed the answer.
Follow-up 5: Then, we don't have to consider the other points beside the intersection point, right?
Anticipated interviewee's answers: No
Q10. Let's look at your answer for Item 9. Please have a look at your answer, your

work, and explanation. Please try to recall what you were thinking and what you did.

Item 9 The derivative of a function, $f(x)$, is $f(x) = ax^2 + b$.

What is required of the values of a and a so that the slope of the

tangent line to the function f will be positive at $x = 0$.

a) a and b must both be positive numbers.

b) a must be positive, while b can be any real number.

c) a can be any real number, while b must be positive.

d) a and b can be any real numbers.

e) None of these

Q10. How did you get this answer?
Case 1. When the interviewee's answer is: a) or b) because a is the slope and b is the y-intercept.
Follow-up 1. Why a is the slope and b the y-intercept?
Anticipated answer: because it has slope-intercept form.
Follow-up 2. Although the equation an x squared in the equation?
Case 2. When the interviewee's answer is: a, b, c, d, or e (chooses one of them)
Follow-up 3. Why did you choose that one?
Follow-up 4. Can you explain with the graph of function $f(x)$?
Follow-up 6. How did you guess the shape of the derivative?
Follow-up 7. How did you find the x-intercept in the graph of the derivative?
Follow-up 8. Can you explain how you use your definitions or explanations about the derivative at one point and the derivative function you gave in Task 1?
Follow-up 9. Why are the graphs of a function and of its derivative function similar?
Follow-up 10. Can you give me an example for the similarity?

Reference: Park, J. (2011). *Calculus instructors' and students' discourses on the derivative*. ProQuest Dissertations Publishing.

1. What is the derivative? Can you make a sentence with the word, “derivative?”
2. What is the derivative function?
3. What is the derivative at a point?
4. Is there any relationship between these two terms, *derivative function* and *derivative at a point*?
5. How about the term *function*? Is the term *function* related to the derivative function and the derivative at a point?
6. Let’s look at your answer for Item 4. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.
7. Let’s look at your answer for Item 6. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.
8. Let’s look at your answer for Item 7. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.
9. Let’s look at your answer for Item 8. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.
10. Let’s look at your answer for Item 9. Please have a look at your answer, your work, and explanation. Please try to recall what you were thinking and what you did.
11. What questions do you have for me?

_End of INTERVIEW QUESTIONS related
to the Derivative Concept Assessment_

—Note for PI only.
These survey questions are linked in research (Bressoud et al., 2015)
to *Good Teaching*—

1. Referring to the survey questions in Sets 1, 2, and 3, how do you believe these instructor practices impact your persistence in Calculus I?

2. Referring to the survey questions in Sets 1, 2, and 3, to what extent do you believe these instructor practices impact your persistence in Calculus I?

3. Referring to the survey questions in Sets 1, 2, and 3, how do you believe these instructor practices impact your confidence in Calculus I?

4. Referring to the survey questions in Sets 1, 2, and 3, to what extent do you believe these instructor practices impact your confidence in Calculus I?

5. Referring to the survey questions in Sets 1, 2, and 3, how do you believe these instructor practices impact your enjoyment of Calculus I?

6. Referring to the survey questions in Sets 1, 2, and 3, to what extent do you believe these instructor practices impact your enjoyment of Calculus I?

7. What do you believe are the most influential instructor practices with respect to impacting your persistence, confidence, and/or enjoyment of Calculus I from sets 1, 2, and 3?

8. Can you think of any other instructor practices that would impact your persistence, confidence, and/or enjoyment of Calculus I?

End of INTERVIEW QUESTIONS
related to Attitude Survey for *Good Teaching*

—Note for PI only.
These survey questions are linked in research (Bressoud et al., 2015)
to *Ambitious Teaching*—

9. Referring to the survey questions in Sets 4 and 5, how do you believe these instructor practices impact your persistence in Calculus I?

10. Referring to the survey questions in Sets 4 and 5, to what extent do you believe these instructor practices impact your persistence in Calculus I?

11. Referring to the survey questions in Sets 4 and 5, how do you believe these instructor practices impact your confidence in Calculus I?

12. Referring to the survey questions in Sets 4 and 5, to what extent do you believe these instructor practices impact your confidence in Calculus I?

13. Referring to the survey questions in Sets 4 and 5, how do you believe these instructor practices impact your enjoyment of Calculus I?

14. Referring to the survey questions in Sets 4 and 5, to what extent do you believe these instructor practices impact your enjoyment of Calculus I?

15. What do you believe are the most influential instructor practices with respect to impacting your persistence, confidence, and/or enjoyment of Calculus I from Sets 4 and 5?

16. Can you think of any other instructor practices that would impact your persistence, confidence, and/or enjoyment of Calculus I?

_End of INTERVIEW QUESTIONS related
to Attitude Survey for *Ambitious Teaching*_

—Note for PI only.
These survey questions are linked in research (Bressoud et al., 2015)
to Good Teaching—

From Post-**Survey Questions Set 1** {24, 25, 26, 27, 28, 29, 30, 31}

Q24 My calculus instructor asked questions to determine if I understood what was being discussed.

Q25 My calculus instructor listened carefully to my questions and comments.

Q26 My calculus instructor discussed applications of calculus.

Q27 My calculus instructor allowed time for me to understand difficult ideas.

Q28 My calculus instructor helped me become a better problem solver.

Q29 My calculus instructor provided explanations that were understandable.

Q30 My calculus instructor was available to make appointments outside of office hours, if needed.

Q31 My calculus instructor discouraged me from wanting to continue taking calculus.

From Post-**Survey Questions Set 2** {32, 38, 40, 44, 45, 46, 47, 48, 49, 51}

Q32 During class time, how frequently did your instructor show how to work specific problems?

Q38 During class time, how frequently did your instructor ask questions?

Q40 How frequently did your instructor prepare extra material to help students understand calculus concepts or procedures?

Q44 My calculus instructor made students feel nervous during class.

Q45 My calculus instructor encouraged students to enroll in Calculus II.

Q46 My calculus instructor acted as if I was capable of understanding the key ideas of calculus.

Q47 My calculus instructor made me feel comfortable asking questions during class.

Q48 My calculus instructor encouraged students to seek help during office hours.

Q49 My calculus instructor presented more than one method for solving problems.

Q51 My calculus instructor made class interesting.

From Post-**Survey Questions Set 3** {67, 73, 74, 75}

Q67 Assignments completed outside of class time were challenging but doable.

Q73 My calculus exams were a good assessment of what I learned.

Q74 My calculus exams were graded fairly.

Q75 My homework was graded fairly.

—Note for PI only.
These survey questions are linked in research (Bressoud et al., 2015)
to Good Teaching—

—Note for PI only.
These survey questions are linked in research (Bressoud et al., 2015)
to Ambitious Teaching—

From Post-**Survey Questions Set 4** {33, 34, 35, 37, 39, 41, 42, 43}
Q33 During class time, how frequently did your instructor have students work with one another?

Q34 During class time, how frequently did your instructor hold whole-class discussion?

Q35 During class time, how frequently did your instructor have students give presentations?

Q37 During class time, how frequently did your instructor lecture?

Q39 During class time, how frequently did your instructor ask students to explain their thinking?

Q41 How frequently did your instructor require you to explain your thinking on your homework?

Q42 How frequently did your instructor require you to explain your thinking on exams?

Q43 How frequently did your instructor assign sections in your textbook for you to read before coming to class?

From Post-**Survey Questions Set 5** {65, 66, 68, 69, 71, 72}
Q65 Assignments completed outside of class time were returned with helpful feedback/comments.

Q66 Assignments completed outside of class time were submitted as a group project.

Q68 Assignments completed *outside of class time* required that I solve word problems.

Q69 Assignments completed *outside of class time* required that I solve problems unlike those done in class or in the book.

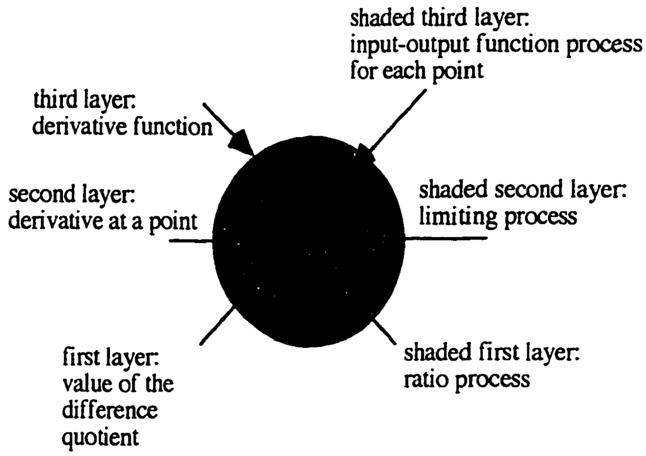
Q71 The exam questions required that I solve word problems.

Q72 The exam questions required that I solve problems unlike those done in class or in the book.

—Note for PI only.
These survey questions are linked in research (Bressoud et al., 2015)
to Ambitious Teaching—

APPENDIX G

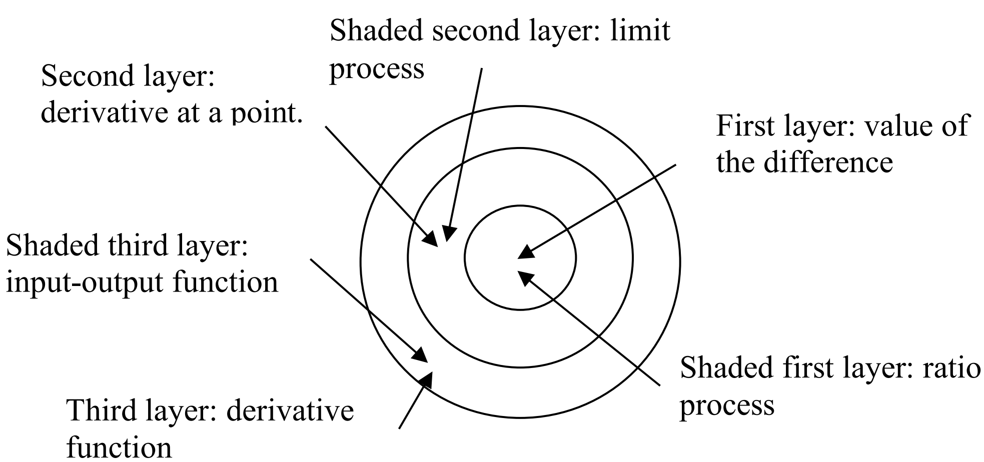
MEANING FOR THREE LAYERS OF CIRCLE DIAGRAM



Note. Reprinted from *The Evolution of Student Understanding of the Concept of Derivative* by M. Zandieh, 1997, p. 104. Copyright 1997 by ProQuest. Permission from Dr. Michelle Zandieh was requested and granted by email on March 21, 2021.

Below is a diagram that represents the three layers. Unfortunately, a clear depiction of the original diagram was not available.

This is my rendering of the three layers.



APPENDIX H

CIRCLE DIAGRAM EXAMPLES WITH ONE OR TWO CIRCLES

Circle Diagrams	Meaning of the circle diagrams in at least one context.
o	Slope or rate or velocity.
•	Slope given as rise over run or velocity described as distance over time or $\frac{\Delta y}{\Delta x}$.
○	A description of the instantaneous nature of the derivative without a further description of what the instantaneous value represents (e.g. slope, rate, velocity).
●	A description of a limiting process where the values in the process are not made clear; e.g. "It's when you get closer and closer."
◎	Slope at a point or instantaneous rate or velocity at a given time.
◙	The limit of the slopes of secant lines approaching the slope of a tangent line or the limit of average velocities converging on an instantaneous velocity.
◉	Slope at a point where the slope has been specifically identified as the rise over run or instantaneous rate where the rate has been identified as the change in y over the change in x.
●	The situation described for the immediately preceding icon with the addition that the limiting process has been described as well, or symbolically $\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$.
◯	A pseudostructural conception of a derivative function as a symbolic expression or a curve with no further structure described, or simply a statement that the derivative is a function without elaboration.
⬤	The derivative as a function where each input value has a certain output value, but the nature of these output values is left unspecified.

Note. Reprinted from *The Evolution of Student Understanding of the Concept of Derivative* by M. Zandieh, 1997, p. 104. Copyright 1997 by ProQuest. Permission from Dr. Michelle Zandieh was requested and granted by email on March 21, 2021.

APPENDIX I

CIRCLE DIAGRAM EXAMPLES WITH TWO OR THREE CIRCLES

Circle diagram	Meaning of the circle diagram in at least one context.
	Slope function, velocity function, or rate of change function where the nature of the descriptors as the output values has not been made explicit.
	The covariation of input values to a function with slope values as outputs. The slope values are not specified to be instantaneous slope values. They may be specifically described as slopes of particular secant lines at each point.
	The covariation of input values to a function with slope values as outputs where the nature of the slope as rise over run is specified. Here again the slope values are not specified to be instantaneous slope values. For a symbolic example, $\frac{f(x+.001)-f(x)}{.001}$.
	The covariation of input values to a function with instantaneous slope values as outputs.
	The covariation of input values to a function with instantaneous slope values as outputs where the limiting process for obtaining the instantaneous slope values is described.
	The covariation of input values to a function with instantaneous slope values as outputs where the nature of the slope as rise over run is described.
	The covariation of input values to a function with instantaneous slope values as outputs where both the limiting process and the nature of slope as rise over run are described. For a symbolic example, $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

Note. Reprinted from *The Evolution of Student Understanding of the Concept of Derivative* by M. Zandieh, 1997, p. 104. Copyright 1997 by ProQuest. Permission from Dr. Michelle Zandieh was requested and granted by email on March 21, 2021.

APPENDIX J

CIRCLE DIAGRAM FOR STUDENT INTERVIEW (ZANDIEH, 1997)

ID	Statement	Slope	Rate	Vel.	Sym.	
A	A measure of the slope of a function.	○				
B	The derivative of a function is when you take the function whatever it is -- say if it was x squared. Take x squared and take the exponent and move it down to the coefficient and you subtract 1 to it so it would be $2x$. And you would use that to determine zero points of the function. So wherever the function crosses zero usually that's maximum and minimum point.				↦	tool misstatement
C	... I know what it is but it's hard to explain. You take the function and you just take the derivative of it. What is it? I don't know. I can't think - I just know what it is.				↦	
D	It's the slope of a function.	●				
E	A derivative is - if you're going from A to B let's say it would be your function at F of (A+B) minus F of A all over B as B is going to zero ... It's the average slope.	●			●	misstatement misstatement
F	... a way to find the instantaneous growth rate of a function or a point of a line or it's the slope of a line at a certain point.	●	◎			misstatement
G	Opposite of integral ... First derivative defines the slope of the equation at whatever point. It's the opposite of an integral.	◎			↦	
H	A derivative is the slope of the tangent line to the curve. Or its the velocity. If you have the distance you can find the velocity.	◎		○		
J	...Well I guess the first thing that comes to mind is the slope function meaning that if you have a function f of x and you take the derivative using the power rule, chain rule, product rule and so forth you end up with the function f prime of x from which if you plug in a value of x you get the slope of the original function at that point.	◎ ◎			↦ ○	

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APPENDIX K

SURVEY QUESTIONS USED FOR TABLES 17 AND 18

Table 17

Students' Assessment of Good Teaching Characteristics for Instructor

[a] "My calculus instructor asked questions to determine if I understood what was being discussed."

[b] "My calculus instructor listened carefully to my questions and comments."

[c] "My calculus instructor discussed applications of calculus."

[d] "My calculus instructor helped me become a better problem solver."

[e] "My calculus instructor discouraged me from wanting to continue taking calculus. "

[f] "My calculus instructor made students feel nervous during class."

[g] "My calculus instructor encouraged students to enroll in Calculus II."

[h] "My calculus instructor acted as if I was capable of understanding the key ideas of calculus"

[i] "My calculus instructor made me feel comfortable asking questions during class."

[j] "My calculus instructor encouraged students to seek help during office hours."

[k] "My calculus instructor presented more than one method for solving problems."

[l] "My calculus instructor made class interesting."

[m] "My calculus exams were a good assessment of what I learned."

[n] "My calculus exams were graded fairly."

[o] "My homework was graded fairly."

[p] "My calculus instructor allowed time for me to understand difficult ideas."

[q] "My calculus instructor provided explanations that were understandable."

[r] "My calculus instructor was available to make appointments outside of office hours, if needed."

[s] “During class time, how frequently did your instructor show how to work specific problems?”

[t] “During class time, how frequently did your instructor ask questions?”

[u] “How frequently did your instructor prepare extra material to help students understand calculus
concepts or procedures?”

[v] “Assignments completed outside of class time were challenging but doable.”

Table 18

Students' Assessment of Ambitious Teaching Characteristics for Instructor

[a] "During class time, how frequently did your instructor have students work with one another?"

[b] "During class time, how frequently did your instructor hold whole-class discussion?"

[c] "During class time, how frequently did your instructor have students give presentations?"

[d] "During class time, how frequently did your instructor lecture?"

[e] "During class time, how frequently did your instructor ask students to explain their thinking?"

[f] "How frequently did your instructor require you to explain your thinking on your homework?"

[g] "How frequently did your instructor require you to explain your thinking on exams?"

[h] "How frequently did your instructor assign sections in your textbook for you to read before coming to class?"

[i] "Assignments completed outside of class time were returned with helpful feedback/comments."

[j] "Assignments completed outside of class time were submitted as a group project."

[k] "Assignments completed outside of class time required that I solve word problems."

[l] "Assignments completed outside of class time required that I solve problems unlike those done in class or in the book."

[m] "The exam questions required that I solve word problems."

[n] "The exam questions required that I solve problems unlike those done in class

or in the book. ”

[o] Combines *Occasionally* and *Seldom.*

[p] Combines *Very Often* and *Often.*

APPENDIX L

IRB APPROVAL NOTIFICATIONS ASU & MCCCD

EXEMPTION GRANTED

Eugene Judson
Division of Educational Leadership and Innovation - Tempe
480/727-5216
Eugene.Judson@asu.edu

Dear Eugene Judson:

On 8/18/2021 the ASU IRB reviewed the following protocol:

Type of Review:	Initial Study
Title:	Thinking Out Loud: The Role of Discourse in Understanding the Derivative in Calculus
Investigator:	Eugene Judson
IRB ID:	STUDY00014378
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• consent_letter_16-08-2021, Category: Consent Form; • IRB Protocol 13-08-2021_v2, Category: IRB Protocol; • supporting documents 14-08-2021_v3, Category: Measures (Survey questions/Interview questions /interview guides/focus group questions);

The IRB determined that the protocol is considered exempt pursuant to Federal Regulations 45CFR46 (2) Tests, surveys, interviews, or observation on 8/18/2021.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

If any changes are made to the study, the IRB must be notified at research.integrity@asu.edu to determine if additional reviews/approvals are required.

Changes may include but not limited to revisions to data collection, survey and/or interview questions, and vulnerable populations, etc.

REMINDER - All in-person interactions with human subjects require the completion of the ASU Daily Health Check by the ASU members prior to the interaction and the use of face coverings by researchers, research teams and research participants during the interaction. These requirements will minimize risk, protect health and support a safe research environment. These requirements apply both on- and off-campus.

The above change is effective as of July 29th 2021 until further notice and replaces all previously published guidance. Thank you for your continued commitment to ensuring a healthy and productive ASU community.

Sincerely,

IRB Administrator

cc: Madeleine Chowdhury

APPROVAL: EXPEDITED REVIEW

Eugene Judson
Division of Educational Leadership and Innovation - Tempe
480/727-5216
Eugene.Judson@asu.edu

Dear Eugene Judson:

On 11/19/2021 the ASU IRB reviewed the following protocol:

Type of Review:	Modification / Update
Title:	Thinking Out Loud: The Role of Discourse in Understanding the Derivative in Calculus
Investigator:	Eugene Judson
IRB ID:	STUDY00014378
Category of review:	Expedited (7)
Funding:	None
Grant Title:	None
Grant ID:	None
Documents Reviewed:	• child_assent_16-17_19-11-2021, Category: Consent Form; • consent_letter_16-08-2021, Category: Consent Form; • IRB Protocol 19-11-2021_chowdhury_v5, Category: IRB Protocol; • parental_permission_19-11-2021, Category: Consent Form; • supporting documents 14-08-2021_v3, Category: Measures (Survey questions/Interview questions /interview guides/focus group questions);

The IRB approved the protocol from 8/18/2021 to 11/18/2026 inclusive. Three weeks before you are to submit a completed Continuing Review application and required attachments to request continuing approval or closure.

If continuing review approval is not granted before the expiration date of approval of this protocol expires on that date. When consent is appropriate, you must use final, watermarked versions available under the "Documents" tab in ERA-IRB.

In conducting this protocol you are required to follow the requirements listed in the INVESTIGATOR MANUAL (HRP-103).

REMINDER - All in-person interactions with human subjects require the completion of the ASU Daily Health Check by the ASU members prior to the interaction and the use of face coverings by researchers, research teams and research participants during the interaction. These requirements will minimize risk, protect health and support a safe research environment. These requirements apply both on- and off-campus.

The above change is effective as of July 29th 2021 until further notice and replaces all previously published guidance. Thank you for your continued commitment to ensuring a healthy and productive ASU community.

Sincerely,

IRB Administrator

cc: Madeleine Chowdhury
Madeleine Chowdhury

MARICOPA
COMMUNITY COLLEGES

Institutional Review Board
2411 W. 14th St., Tempe, AZ 85281
(480) 731-8701

August 27, 2021

MCCCD IRB Authorization: 2021-08-PI-Chowdhury-M

Dear Madeleine,

The IRB of the Maricopa County Community College District (MCCCD) in Arizona has conducted an Administrative Review of the research study below and decided to authorize it at the site(s) indicated.

Name of Principal Investigator (PI)	Madeleine Chowdhury
Title of Research Study	Thinking Out Loud: The Role of Discourse in Understanding the Derivative in Calculus
Site(s) of Study in MCCCD	Mesa Community College

Name of Approving IRB	Arizona State University IRB
Protocol Number	STUDY00014378
Date of Approval	08/18/2021

The MCCCD Administrative Review process does not include review under 45 CFR 46 Protection of Human Subjects. The PI remains responsible for ensuring compliance with the determinations of the approving IRB. If changes or modifications are made to the study, the MCCCD IRB must be notified by the PI.

Additionally, by undertaking this study at MCCCD, the PI acknowledges and agrees that authorization does not commit MCCCD to provide resources or data collection for the investigator(s), nor recruit for or participate in the project.

Institutional Review Board
2411 W. 14th St., Tempe, AZ 85281
(480) 731-8701

Cordially,

Lutfi Hussein

Lutfi Hussein, PhD
MCCCD IRB Chair