Holography in Rindler Space

by

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A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved November 2012 by the
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ARIZONA STATE UNIVERSITY

December 2012

ABSTRACT

This thesis addresses certain quantum aspects of the event horizon using the AdS/CFT correspondence. This correspondence is profound since it describes a quantum theory of gravity in $d+1$ dimensions from the perspective of a dual quantum field theory living in d dimensions. We begin by considering Rindler space which is the part of Minkowski space seen by an observer with a constant proper acceleration. Because it has an event horizon, Rindler space has been studied in great detail within the context of quantum field theory. However, a quantum gravitational treatment of Rindler space is handicapped by the fact that quantum gravity in flat space is poorly understood. By contrast, quantum gravity in anti-de Sitter space (AdS), is relatively well understood via the AdS/CFT correspondence.

Taking this cue, we construct Rindler coordinates for AdS (RAdS) space in $d+1$ spacetime dimensions. In three spacetime dimensions, we find novel one-parameter families of stationary vacua labeled by a rotation parameter β. The interesting thing about these rotating Rindler-AdS spaces is that they possess an observer-dependent ergoregion in addition to having an event horizon.

Turning next to the application of AdS/CFT correspondence to Rindler-AdS space, we posit that the two Rindler wedges in AdS_{d+1} are dual to an entangled conformal field theory (CFT) that lives on two boundaries with geometry $\mathbb{R} \times H_{d-1}$. Specializing to three spacetime dimensions, we derive the thermodynamics of Rindler-AdS space using the boundary CFT. We then probe the causal structure of the spacetime by sending in a time-like source and observe that the CFT "knows" when the source has fallen past the Rindler horizon. We conclude by proposing an alternate foliation of Rindler-AdS which is dual to a CFT living in de Sitter space.

Towards the end of this thesis, we consider the concept of weak measurements in quantum mechanics, wherein the measuring instrument is weakly coupled to the

system being measured. We consider such measurements in the context of two examples, viz. the decay of an excited atom, and the tunneling of a particle trapped in a well, and discuss the salient features of such measurements.

ACKNOWLEDGEMENTS

First and foremost, I want to thank my supervisor, Maulik Parikh, who has been an excellent mentor during my years as a graduate student. He is an extremely wise, enthusiastic, supportive, and tolerant person, and I owe him immeasurable thanks for making me realize the value of critical and independent thinking. To say that he has taught me how theoretical physics is done, would be an understatement. Maulik, it has been a pleasure working with you, and I am truly honored to be your first PhD student.

I would also like to thank Paul Davies, Tanmay Vachaspati, Damien Easson, and Ricardo Alarcon for serving on my dissertation committee. In my view, the cosmology/theory group at ASU is an extremely vibrant place to be a graduate student, and I would like to extend my thanks to all the faculty members, graduate students, postdocs, and administrative personnel. Special thanks to Araceli, Adriana, Katie, Rozela, Ixxy, Thomas, Andrew, Yi-Fu, Eray, Jay, Yao, Russ, Subir, Scott, Hiroyuki, Aaron, and Hank.

Many thanks to my collaborators, who have taught me a lot and were all a pleasure to work with: Paul Davies, Erik Verlinde, and Sara Walker.

Even though I will be graduating officially as an ASU student, the foundations of much of my knowledge was laid at IUCAA, where I spent close to five years as a graduate student. I owe many thanks to Ajit Kembhavi, Naresh Dadhich, P. P. Divakaran, Paddy, Kandu, Srianand, Chella, Santosh Khadilkar and the rest of the faculty members and administrative personnel, for being extremely supportive during my stay at IUCAA. However, life at IUCAA was made fulfilling largely due to the circle of friends I had there. In Momo, Jones, Vivek, Baba, Tuhin, and Shruti, I had great batchmates who made each day of my graduate school rewarding and

lively. Many thanks to Susmita di, Soummo da, Samir, Vinu, Sanil, Jayanti, Sudipta, Sharanya, Mudit, Dawood, Shubarthi da, Gaurav, Sowgat, Debil, Sanved, Aditya, Charles, Hasan, Main Pal, Nagendra, Sibhasish, Suprit, Krishna, and Santanu, for your friendship. In particular, my deepest gratitude goes to Nisha, Swara, Gaurang, and Rizwan for being the best of friends who contributed in a major way towards this thesis. I miss all of you, treasure your friendship, and for once, I wish I could turn the clock back.

"*Big things have small beginnings*" - This dialogue from the film "Prometheus" resonates within me as I look back in life, for I would not be doing physics today if I had not come in contact with one Mr. Kundu, my science tutor. As I now recall, it was like a revelation to me (I was just 14 at that time) when he used Newton's laws to prove that planets move around the sun in ellipses. God had suddenly become powerless, and I fell in love with physics that very moment. The rest of course is history. Mr. Kundu, a largely unknown and eccentric figure, now lives in utter oblivion due to his chronic health problems. I owe him my love for physics, and wish him the best in life.

I also take this opportunity to thank my professors at IIT-Madras who encouraged my interest in physics. Special thanks to Prof. V. Balakrishnan and Prof. Suresh Govindarajan for being the most awesome teachers at IIT. Not to forget the most awesome (and the smartest!) set of friends I had there: Priyank, Akash, Vivek Rawat, Hitesh, Pragdev, Madhu, Vinay, Sai, Anirudh, Mohit, Amresh, Anees, Vineeth and the rest of my hostel mates. Thank you all.

Special thanks to Acharya Shriram and Sudhir who were always supportive of me, and remain friends for life. I cannot thank you guys enough for the sweet memories.

Finally, I thank my fiance, Aditi, my relatives, and my parents - to whom I dedicate

this thesis - for their continual support and unconditional love. To you all, I owe my existence.

To all the above, and to the ones I might have missed, thank you.

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CHAPTER 1

INTRODUCTION

It has been thirty seven years since Hawking, in his historic paper [1], proved that black holes are not really black because they emit particles. This discovery, known as Hawking radiation, had a quantum origin and was the first succesful "partial" union between the two leading physical theories of our universe, viz. the general theory of relativity and quantum field theory. Using semi-classical techniques, Hawking confirmed Bekenstein's previous results [2, 3, 4] that black holes have entropy, and he also showed that black holes radiate with a pure black body spectrum dependent upon their charge, mass, and angular momentum. The entropy of the black hole is related to the area of its event horizon as

$$S_{BH} = \frac{A}{4G}.$$

(1.1)

The fact that black holes radiate and that the spectrum is purely thermal immediately leads to a serious paradox. Imagine forming a hypothetical black hole in the following two ways:

1) By radially collapsing a large number of cats.

2) By radially collapsing a large number of mice.

We of course can adjust the numbers such that the black holes formed have the same mass i.e. $M_{cats} = M_{mice}$. If left to evolve, both black holes would evaporate away via Hawking radiation having an identical thermal spectrum. This is along expected lines since black holes have no "hair". In other words, black holes are uniquely characterized by their total mass, charge, and angular momentum and not by the microscopic details of the infalling matter which make the hole. However, if all that is left after evaporation is just thermal radiation, then what happened to the initial information which contained cats and mice? Stated technically, a pure state of cats (or mice) has evolved to a mixed thermal state [5]. Such an evolution is

manifestly non-unitary since no unitary operator in quantum mechanics can evolve a pure state into a mixed state. Thus, the upshot of Hawking's result is that the existence of black holes causes *loss of information*, leading to a breakdown of quantum mechanics. For more than two decades, physicists have tried to understand the implications of such a result. If information is indeed lost, then the universe would be a chaotic and noisy place without any coherence or conservation of energy. This would spell doom for all existing theories based on quantum mechanics, and therefore is extremely unsatisfactory from the viewpoint of physics. On the other hand, if information is not destroyed by the black holes, then how do we retrieve it? The current general consensus is that a complete quantum gravitational treatment, starting from the formation of a black hole until its final moments of evaporation, will solve the information loss paradox. Currently, string theory is the most promising candidate for a quantum theory of gravity that has been used to calculate the entropy of certain class of extremal black holes [6]. A glance at the expression (1.1) indicates that the entropy in any gravitational theory should scale as the area rather than the volume of space. This fact highly suggests that a true theory of quantum gravity should have *holography* as one of its fundamental ingredients. In 1997, Maldacena [7] conjectured a duality between a certain type IIB string theory in anti de Sitter space (AdS) in 4+1 dimensions and a $\mathcal{N} = 4$ super Yang-Mills conformal field theory (CFT) living on its boundary. This AdS/CFT correspondence is remarkable as it is holographic in nature, i.e. we can study quantum gravity in $d+1$ dimensions using quantum field theory without gravity in d dimensions. Black holes have been modeled using the AdS/CFT correspondence previously, where the black hole in the bulk appears as a thermal state in the boundary CFT. The correspondence makes the Hawking process manifestly unitary, since the dual CFT is by definition unitary. However, to better understand the information loss puzzle, we need to have a quantum gravitational description for the process of black hole

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formation itself. A conceptually related toy model is to study the infalling matter into the black hole, from the dual CFT perspective. This requires sending probes past the event horizon of AdS-Schwarzschild black holes, and to date such calculations have not been easy. A major reason is that the singularity inside the black hole renders such calculations intractable and conceptually difficult to set up.

In this thesis, we aim to circumvent this roadblock by considering quantum gravity in Rindler space, which is the spacetime seen by an observer with constant proper acceleration. We currently understand quantum gravity in anti de Sitter (AdS) space via the AdS/CFT correspondence. Hence in order to describe quantum gravity in Rindler space, we consider a constantly accelerating observer not in flat space, but in AdS space. We refer to such a space as Rindler-AdS space. Rindler-AdS space has a Rindler horizon and does not have a singularity. The physical processes in it can be given a quantum gravitational description via the AdS/CFT correpondence. This provides us with a concrete and tractable handle to probe the event horizon.

The organisation of this thesis is as follows. In *chapter 2*, we essentially give a brief pedagogic review of some standard calculations of quantum fields in Rindler space. In *chapter 3*, we construct Rindler-AdS spaces in three space-time dimensions by considering the conjugacy class of the Lorentz group. We find novel one-parameter families of rotating Rindler vacua, which are related to the usual Rindler vacuum by non-trivial Bogolubov transformations. In particular, we observer that rotating Rindler-AdS space possesses an observer-dependent ergosphere. We also find rotating vacua in three-dimensional global AdS space as well as in de Sitter space, provided a certain region of spacetime is excluded.

In *chapter 4*, we review the celebrated AdS/CFT correspondence, and develop the necessary machinary to understand the application of this correspondence to Rindler-AdS spacetimes.

chapter 5, uses the results discussed in the preceeding chapters to synthesize a

novel approach to probe the event horizon. We study the geometry and thermody-namics of Rindler-AdS space. Applying the AdS/CFT correspondence, we rederive these themodynamical properties, and find the response of the dual boundary CFT as a test source falls into the Rindler horizon.

We conclude the thesis by considering quantum weak measurements in the con-text of 1) decay of an excited atom, and 2) a particle trapped in a potential barrier. Implications are discussed.

CHAPTER 2

QUANTUM FIELDS IN RINDLER SPACE

In 1975, Paul Davies realized that since, by the equivalence principle, the effects of a uniform gravitational field are equivalent to the effects observed in a non-inertial uniformly accelerated frame, there could be quantum effects in flat space similar to what Hawking derived in 1974 by studying black holes [8]. The following year, William Unruh [9] placed this correspondence on a more physical foundation by considering an accelerating model particle detector responding to the quantum vacuum. In this chapter we present a simple demonstration of the effect of acceleration on the quantum vacuum by closely following the references [10, 11]. In order to obtain the line element of a uniformly accelerating observer, consider uniform acceleration λ along the x-axis. The equation for such an observer with $c = 1$ is

$$\frac{d}{dt}\left(\frac{v}{\sqrt{1-v^2}}\right) = \lambda \tag{2.1}$$

where $v(t) = \frac{dx(t)}{dt}$. Solving the above equation with the initial conditions

$$x(0) = \lambda^{-1} \ ; \ v(0) = 0, \tag{2.2}$$

we obtain,

$$x(t) = \lambda^{-1}\left(1 + \lambda^2 t^2\right) \tag{2.3}$$

Using the above solution, the proper time s as measured by a clock carried by the observer is related to the Minkowski time t as

$$s(t) = \int_0^t dt \sqrt{1-v^2} = \lambda^{-1}\text{arcsinh}(\lambda t) \tag{2.4}$$

Using this relation and the equation (2.3), we can express the trajectory of a uniformly accelerating observer as

$$t = \lambda^{-1}\sinh(\lambda s) \ \text{ and } \ x = \lambda^{-1}\cosh(\lambda s) \tag{2.5}$$

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If we now choose $\lambda = ge^{-g\xi}$ and $s = \tau e^{g\xi}$ and substitute into the standard Cartesian Minkowski line element, we get the so-called Rindler line element as

$$ds^2 = e^{2g\xi} \left(-d\tau^2 + d\xi^2 \right) + dy^2 + dz^2 \tag{2.6}$$

The above line element also appears as the near-horizon limit of black holes. Con-
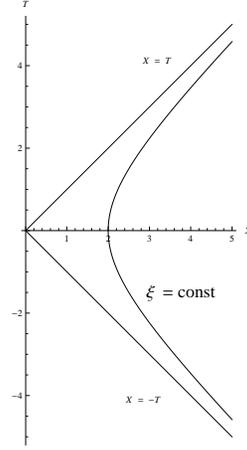
Figure 2.1: $\xi = const$ worldline describes an accelerating observer in 1+1 Minkowski space

sider the Schwarzschild metric in 3+1 dimensions ($c = G = \hbar = 1$)

$$ds^2 = - \left(1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} \right)} + r^2 d\Omega_2^2 \tag{2.7}$$

The horizon is at $r = 2M$. Expanding in Taylor series near the horizon, we get

$$ds^2 \approx - \left(\frac{r - 2M}{2M} \right) dt^2 + \frac{dr^2}{\left(\frac{r-2M}{2M} \right)} + 4M^2 d\Omega_2^2$$

Furthermore, we are interested in a small angular region of the horizon centered around $\theta = 0$, we set

$$
\begin{aligned}
r &= 2M + \frac{x^2}{8M} \\
y &= 2M\theta \cos\phi \\
z &= 2M\theta \sin\phi
\end{aligned}
\tag{2.8}
$$

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and get the Rindler line element in the form

$$ds^2 = -g^2 x^2 dt^2 + dx^2 + dy^2 + dz^2 \ \text{ where } \ g = \frac{1}{4M} \tag{2.9}$$

Doing a further transformation $x = \frac{1}{g}e^{g\xi}$, we get the line element (2.6). Thus studying the behavior of quantum fields near the horizon of black holes is equivalent to studying quantum fields in Rindler space.

2.1 Quantum field theory in Minkowski and Rindler coordinates

In this section, we show that the concept of particles is observer dependent. The essential idea is that in a theory with diffeomorphism invariance, any globally hyperbolic spacetime including Minkowski space, admits infinitely many choices of the time coordinate. These different time coordinates in turn are generated by different Hamiltonians and correspondingly the minimum energy state - the vacuum state can also be different. Thus a state that appears empty to an observer whose worldline traces one time coordinate need not appear so to a different observer following a different time coordinate. We consider a massless scalar field in 1+1 dimensions for mathematical simplicity. The action for a massless scalar field is

$$I[\phi] = \frac{1}{2} \int d^2 x \sqrt{-g} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi \tag{2.10}$$

The equation of motion for the scalar field ϕ is

$$\partial^2 \phi = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \right) \phi = 0 \tag{2.11}$$

We define a scalar product for any two solutions ϕ_1 and ϕ_2 for the scalar field ϕ as

$$(\phi_1, \phi_2) = -i \int d\Sigma^\mu \sqrt{-g_\Sigma} \left(\phi_1 \partial_\mu \phi_2^* - \phi_2^* \partial_\mu \phi_1 \right) \tag{2.12}$$

where $d\Sigma^\mu$ is the volume of the hypersurface Σ and is directed normal to the unit vector n^μ. This scalar product is called the Klein-Gordon inner product and it is motivated by the fact that it is independent of the surface Σ.

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2.2 Minkowski coordinates

The line element for Minkowski space in 1+1 dimensions is

$$ds^2 = -dt^2 + dx^2 \tag{2.13}$$

The equation of motion for ϕ from (2.11) is

$$\left(-\partial_t^2 + \partial_x^2\right)\phi(t,x) = 0 \tag{2.14}$$

The solutions to the above equation are plane waves. We write the solutions as

$$u_k(t,x) = \frac{1}{\sqrt{4\pi\omega}} e^{-i(\omega t - kx)} \tag{2.15}$$

where $\omega = |k|$ and $-\infty < k < \infty$. Since the metric is independent of the coordinate t, we can define positive frequency modes with respect to the time coordinate t. In this coordinate system, we can choose the hypersurface $d\Sigma^\mu$ to be a constant-t surface while evaluating the scalar product (2.12). We see that the modes u_k and their complex conjugate u_k^* satisfy the following orthonormality relations with respect to the scalar product (2.12)

$$(u_k, u_{k'}) = \delta\left(k - k'\right) \;\; ; \;\; (u_k^*, u_{k'}^*) = -\delta\left(k - k'\right) \;\text{and}\; (u_k, u_{k'}^*) = 0 \tag{2.16}$$

The quantization of the scalar field proceeds in the usual manner by treating ϕ as an operator and defining the equal time commutation relations (ETCR)

$$
\begin{aligned}
[\phi(t,x), \phi(t,x')] &= 0 \\
[\Pi(t,x), \Pi(t,x')] &= 0 \\
[\phi(t,x), \Pi(t,x')] &= i\delta\left(x - x'\right)
\end{aligned}
\tag{2.17}
$$

where Π is the conjugate momentum of the field defined as

$$\Pi = \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial_0 \phi \tag{2.18}$$

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Since the normalized modes u_k and u_k^* satisfy the relations (2.16), they form a complete basis so that the quantum field ϕ can be expanded as

$$\phi(t, x) = \int_{-\infty}^{\infty} dk \left(\hat{a}_k u_k + \hat{a}_k^\dagger u_k^* \right) \tag{2.19}$$

where $\hat{a}_k$ and $\hat{a}_k^\dagger$ are the annihilation and creation operators for the mode k respectively. In terms of these operators the ETCR (2.17) become

$$
\begin{aligned}
\left[\hat{a}_k, \hat{a}_{k'} \right] &= 0 \\
\left[\hat{a}_k^\dagger, \hat{a}_{k'}^\dagger \right] &= 0 \\
\left[\hat{a}_k, \hat{a}_{k'}^\dagger \right] &= \delta(k - k')
\end{aligned}
\tag{2.20}
$$

The Minkowski vacuum $|0_M\rangle$ is then defined as the state annhilated by $\hat{a}_k$

$$\hat{a}_k |0_M\rangle = 0 \quad \forall k \tag{2.21}$$

We can form multi-particle states by acting repeatedly with the creation operator $\hat{a}_k^\dagger$ on the Minkowski vacuum. One can also consider the usual number operator $\hat{N}_k = \left(\hat{a}_k^\dagger \hat{a}_k \right)$ for the mode k which has the property that for a state $|n_k\rangle$, it satisfies

$$\hat{N}_k |n_k\rangle = n_k |n_k\rangle \tag{2.22}$$

where n_k is the number of quanta for mode k. Thus for the Minkowski vacuum $|0_M\rangle$, we have

$$\langle 0_M | \hat{N}_k | 0_M \rangle = \langle 0_M | \hat{a}_k^\dagger \hat{a}_k | 0_M \rangle = 0 \quad \forall k. \tag{2.23}$$

This state is distinguished by the fact that it is invariant under Poincare transformations, Hence all inertial observers perceive this state to have no particles i.e. as the vacuum state.

2.3   Rindler coordinates

We consider the following transformations of the Minkowski coordinates (t, x) as

$$t = g^{-1} e^{g\xi} \sinh(g\tau) \quad \text{and} \quad x = g^{-1} e^{g\xi} \cosh(g\tau) \tag{2.24}$$

where g is a constant. We call (τ, ξ) Rindler coordinates. These cover only the $x > |t|$ in the (t, x) plane. In terms of Rindler coordinates (2.24), the Minkowski line (2.13) element becomes

$$ds^2 = e^{2g\xi} \left(-d\tau^2 + d\xi^2 \right) \tag{2.25}$$

From equation(2.24), it is obvious that

$$x^2 - t^2 = g^{-2} e^{2g\xi} \quad \text{and} \quad \tanh(g\tau) = \frac{t}{x} \tag{2.26}$$

These relations imply that curves of constant ξ are hyperbolae, while the curves of constant τ are straight lines through the origin in the (t, x) plane. As expected, each of these hyperbolae is the spacetime trajectory of a uniformly accelerating observer with a proper acceleration of $ge^{-g\xi}$. Different hyperbolae correspond to different uniform accelerations, with the acceleration decreasing as the Minkowski coordinate x increases. The null-lines $x = \pm t$ are the asymptotes to these hyperbolae and therefore act as horizons for the uniformly accelerating observers.

In order to quantize the scalar field ϕ in Rindler coordinates, we observe that the action (2.10) is conformally invariant as we are working in 1+1 dimensions and the transformations (2.24) are a class of conformal transformations of the Minkowski line element. Exploiting this symmetry, the equation of motion for ϕ in the Rindler coordinates becomes

$$\left(-\partial_\tau^2 + \partial_\xi^2 \right) \phi(\tau, \xi) = 0 \tag{2.27}$$

and just as in the case of Minkowski coordinates, we write the solution to the above equation as

$$v_l(\tau, \xi) = \frac{1}{\sqrt{4\pi\nu}} e^{-i(\nu t - l\xi)} \tag{2.28}$$

where $\nu = |l|$ and $-\infty < l < \infty$. Since the metric (2.6) is independent of the coordinate τ, we can define positive frequency modes with respect to the Rindler time coordinate τ. In the Rindler case, we can choose the hypersurface $d\Sigma^\mu$ to be a constant-τ surface while evaluating the scalar product (2.12). Again, we see that the modes v_l and its complex conjugate v_l^* satisfy the orthonormality relations with respect to the scalar product (2.12)

$$(v_l, v_{l'}) = \delta(l - l') \;\; ; \;\; (v_l^*, v_{l'}^*) = -\delta(l - l') \text{ and } (v_l, v_{l'}^*) = 0 \tag{2.29}$$

and hence the quantized field $\phi(\tau, \xi)$ can be expanded just like in the Minkowski coordinates as

$$\phi(\tau, \xi) = \int_{-\infty}^{\infty} dl \left(\hat{b}_l v_l + \hat{b}_l^\dagger v_l^* \right) \tag{2.30}$$

where $\hat{b}_l$ and $\hat{b}_l^\dagger$ are the annihilation and creation operators for the Rindler mode l. These operators follow the same commutation relations as the operators $\hat{a}_k$ and $\hat{a}_k^\dagger$ in (2.20). The vacuum state can analogously be defined as

$$\hat{b}_l |0_R\rangle = 0 \;\; \forall l. \tag{2.31}$$

In the next sub-section we will see that the Rindler observer perceives the Minkowski vacuum as a teeming thermal bath of particles unlike in (2.23).

2.4 The Bogolubov transformations

We carried out the program of quantizing the scalar field ϕ in two different coordinate systems. We found that the field ϕ can be expanded in terms of two different modes which are complete and orthornormal. These two decompositions lead to

two vacuum states $|0_M\rangle$ and $|0_R\rangle$. The question is : are these two vacua equivalent ?

Since both sets of the normal modes u_k and v_l are complete, one set of mode can be expanded in terms of the other

$$v_l(\tau, \xi) = \int_{-\infty}^{\infty} dk \ (\alpha(l,k)u_k + \beta(l,k)u_k^*) \tag{2.32}$$

The inverse transformation is

$$u_k(t, x) = \int_{-\infty}^{\infty} dl \ (\alpha^*(l,k)u_k - \beta(l,k)v_l^*) \tag{2.33}$$

These relations are known as the Bogolubov transformations. Using (2.32) and the orthonormality relation (2.16), the Bogolubov coefficients α and β can be expressed as

$$\alpha(l,k) = (v_l, u_k) \ \text{ and } \ \beta(l,k) = -(v_l, u_k^*) \tag{2.34}$$

Making use of the orthonormality relations (2.16) and (2.29) for the modes u_k and v_l, it can be shown that

$$\hat{a}_k = \int_{-\infty}^{\infty} dl \ \left(\alpha(l,k)\hat{b}_l + \beta^*(l,k)\hat{b}_l^\dagger \right) \tag{2.35}$$

and

$$\hat{b}_l = \int_{-\infty}^{\infty} dk \ \left(\alpha^*(l,k)\hat{a}_k - \beta^*(l,k)\hat{a}_k^\dagger \right) \tag{2.36}$$

It can be seen from equations (2.32), (2.33) and (2.36) that unless the coefficient β is zero, the Minkowski vacuum $|0_M\rangle$ will not be annhilated by the Rindler annihilation operator b_l. Thus the vacua $|0_M\rangle$ and $|0_R\rangle$ are inequivalent if the coefficient β is non-zero. The Bogolubov coefficients can be evaluated using the relations (2.34) and choosing the hypersurface $\tau = 0$ for convenience. The coefficients turn out to be

$$\alpha(l,k) = \frac{1}{4\pi\sqrt{\omega\nu}} \int_{-\infty}^{\infty} d\xi \ \left(\omega e^{g\xi} + \nu \right) \ e^{il\xi} \exp{-i(kg^{-1}e^{g\xi})}$$

and

$$\beta(l,k) = \frac{1}{4\pi\sqrt{\omega\nu}} \int_{-\infty}^{\infty} d\xi \left(\omega e^{g\xi} - \nu\right) e^{il\xi} \exp i(kg^{-1}e^{g\xi})$$

Setting $z = e^{g\xi}$, these integrals become

$$\alpha(l,k) = \frac{g^{-1}}{4\pi\sqrt{\omega\nu}} \int_{0}^{\infty} \frac{dz}{z} \left(\omega z + \nu\right) z^{ilg^{-1}} e^{-ikzg^{-1}} \tag{2.37}$$

$$\beta(l,k) = \frac{g^{-1}}{4\pi\sqrt{\omega\nu}} \int_{0}^{\infty} \frac{dz}{z} \left(\omega z - \nu\right) z^{ilg^{-1}} e^{ikzg^{-1}} \tag{2.38}$$

One can recognize that the above integrals are related to the Gamma function $\Gamma(z) = \int_{0}^{\infty} t^{z-1} e^{-t}$. Evaluating these integrals (see reference [12]) by noting that $\Gamma(z+1) = z\Gamma(z)$, we get

$$\alpha(l,k) = \left(\frac{g^{-1}}{4\pi\sqrt{\omega\nu}}\right) \left(\omega l + k\nu\right) \left(kg^{-1}\right)^{-ilg^{-1}} \Gamma(ilg^{-1}) e^{\pi l/2g} \tag{2.39}$$

$$\beta(l,k) = -\alpha(l,k) e^{-\pi l/g} \tag{2.40}$$

The expectation value of the number operator defined in terms of $(\hat{b}, \hat{b}^{\dagger})$ operators in the vacuum $|0_M\rangle$ is given by

$$
\begin{aligned}
\langle 0_M | N_l | 0_M \rangle &= \langle 0_M | \hat{b}_l^{\dagger} \hat{b}_l | 0_M \rangle \\
&= \int_{-\infty}^{\infty} dk \, |\beta(l,k)|^2
\end{aligned}
\tag{2.41}
$$

This is a general result. The Bogolubov transformations relate the operators (both annihilation and creation) of the quantum field in two different coordinate systems in terms of a linear expansion involving the Bogolubov coefficients α and β. As previously mentioned, the vacuum defined by the time coordinate in one coordinate system will not in general be equivalent to the vacuum defined in the other coordinate system. In fact the vacuum in one coordinate system would appear as a state full of particles from the perspective of the other coordinate system. The mean number of such particles is given by (2.41).

Using (2.36), (2.39), (2.40) and (2.41), we see that the expectation value of the

Rindler number operator in the Minkowski vacuum state turns out to be thermal in nature i.e.

$$
\begin{aligned}
\langle 0_M | N_l | 0_M \rangle &= \langle 0_M | \hat{b}_l^\dagger \hat{b}_l | 0_M \rangle \\
&= \int_0^\infty \frac{dk}{2\pi k} \left(\frac{g^{-1}}{e^{(2\pi \nu g^{-1})} - 1} \right)
\end{aligned}
\tag{2.42}
$$

Hence the Rindler observer perceives the Minkowski vacuum as a thermal bath of massless particles obeying a Bose-Einstein distribution with the Unruh temperature

$$
T = \frac{g}{2\pi}
\tag{2.43}
$$

Therefore a uniformly accelerating observer perceiving thermality due to his or her acceleration. A physical interpretation for the effect is that the accelerated detector is coupled to the quantum vacuum fluctuations and these fluctuations act on the "detector" (a theoretical instrument to detect the particles) and excite it as if the detector were in a thermal bath with the Unruh temperature (2.43). For a more comprehensive review on the theory of detectors and the Unruh effect, the reader is urged to look up [11, 13, 8] and the references therein.

As mentioned previously, the Rindler spacetime gives the local properties of black holes and cosmological horizons. The Davies-Unruh effect would then be the near-horizon form of the Hawking radiation. This is one of the major reasons why this effect and Rindler space have been studied extensively [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25].

CHAPTER 3

STATIONARY VACUA IN ANTI DE SITTER SPACE

This chapter is based on my work [27], and [26] done with M. Parikh, and E. Verlinde.

3.1   Introduction

Previously, we derived Rindler space in two dimensional spacetime as a solution describing a uniformly accelerated observer in Minkowski space. Alternatively, we can also construct Rindler space in two spacetime dimensions by looking at the isometries of Minkowski space $ds^2 = -dT^2 + dX^2$ as follows. Poincaré invariance dictates that there are three isometries, viz. the translations ∂_T, ∂_X, and the Lorentz boost $T\partial_X + X\partial_T$. If we naively demand that the desired spacetime be stationary with its "time" translation generator ∂_t given by the Lorentz boost, then the resulting spacetime unambiguously turns out to be Rindler space. The unique parameterization which makes these conditions manifest is

$$X = \xi \cosh gt, \quad T = \xi \sinh gt$$

where g is just a parameter with mass dimension one. The generator $\partial_t = T\partial_X + X\partial_T$ is time-like everywhere since its norm $|\partial_t|^2 = -X^2 + T^2 < 0$ in the given metric signature. Therefore, ∂_t is a valid Hamiltonian. With the above parameterization, the metric becomes

$$ds^2 = -g^2\xi^2 dt^2 + d\xi^2 \;, \tag{3.1}$$

which is the usual Rindler space. Therefore, the study of symmetries provides a powerful tool to construct novel spacetimes. Concretely, the choice of time and the time-evolution operator, in a diffeomorphism invariant theory, is essentially arbitrary [11], and the Hamiltonian which is the generator of time translations depends on the definition of the time coordinate. Certain preferred class of Hamiltonions are those

that are the generators of time-like isometries. These Hamiltonions are special since conservation of energy is ensured and a quantum field can be decomposed in terms of positive and negative energy modes in such a spacetime. The definition of a particle in such a spacetime is therefore unambiguous. Minkowski space is an obvious example of such a spacetime. It has a timelike killing vector ∂_T generating the time evolution. One can now define a new time coordinate $T' = \frac{T - \beta X}{\sqrt{1 - \beta^2}}$ and ask the following question: is the vacuum defined by the new Hamiltonian $\partial_{T'}$ different from the one defined by ∂_T? The answer, of course is no since the Hamiltonia in this case are related by an isometry transformation of the Minkowski space, viz. the Lorentz transformation. Thus the Hamiltonia are said to be "group equivalent". Formally, vacua defined by different Hamiltonia are said to be inequivalent if there exists non-zero Bogolubov transformations between the two. In this case the corresponding Hamiltonia are said to be "particle inequivalent". Now, Group inequivalence does not necessarily imply particle inequivalence, but the converse always holds. Therefore, the Minkowski Hamiltonian $H_M = \partial_T$, and the Rindler Hamiltonian $H_R = T\partial_X + X\partial_T$ are not related to each other by any isometry transformation since there is a non-zero Bogolubov transformation between the respective vacua. However, since group inequivalence is a necessary condition, it serves as a starting point to classify all the probable inequivalent Hamiltonia of a space based on their conjugacy classes.

In oder to find the possible stationary vacua of a given spacetime, we require three conditions to hold:

- the candidate Hamiltonian should be a continuous isometry,

- there must exist a region of spacetime that admits a Cauchy surface such that the Hamiltonian is future-directed and timelike at the (possibly asymptotic) spatial boundary of the region, and

• the orbits of the Hamiltonian must not exit that region.

The motivation for these conditions is the following. The candidate Hamiltonian describes the time-evolution, and hence, in order to be able to do quantum field theory, one needs to be able to define energy as positive frequency modes with respect to the time coordinate. If the Hamiltonian is not an isometry, then particle number is not conserved, and therefore the definition of energy becomes ambiguous in such spacetimes. The Hamiltonian may also not be globally timelike, so that we may have to restrict our quantum field theory to some region of spacetime, such as the static patch of de Sitter space or the Rindler wedge in Minkowski space. That region of spacetime should be globally hyperbolic (i.e. admit a Cauchy surface) so that time evolution of quantum states can be defined. For the same reason, the orbits of the candidate Hamiltonian should not exit the region. Finally, for the isometry to even be considered a Hamiltonian, it must be timelike at least at the asymptotic boundary of the region; we do not require the stronger condition that the Hamiltonian be timelike everywhere within our region so as not to preclude the existence of an ergosphere.

There may be multiple Hamiltonians that satisfy the above conditions. If the different possible Hamiltonians are isometrically equivalent i.e. if they can be related by isometries (so that they are both elements of the same conjugacy class of the isometry group), then they lead to the same vacuum state. However, if the Hamiltonians are isometrically inequivalent (being part of different conjugacy classes), then, given some quantum field theory, they could correspond to different vacuum states.

In order to illustrate these ideas, let us list all the stationary vacua [28] of Minkowski space $ds^2 = -dX_0^2 + \sum_i dX_i^2$. The most general continuous isometry of Minkowski

space is generated by a linear combination of translations, boosts, and rotations:

$$\alpha^\mu P_\mu + \beta^i K_i + \omega^{ij} J_{ij} \ . \tag{3.2}$$

This must be timelike, at least in some suitable region, for the generator to be a candidate Hamiltonian. Choosing the Hamiltonian to be $P_0 = \partial_{X_0}$ yields the usual Poincaré-invariant vacuum. Next, we note that the boost generator, $K_i = X_0 \partial_i + X_i \partial_0$, squares to $X_0^2 - X_i^2$, which is timelike when restricted to the wedges $X_i^2 > X_0^2$ and is future-directed when further restricted to $X_i > 0$. This is of course the right Rindler wedge. Moreover, the orbit of K_i starting from a point in the right Rindler wedge remains in the wedge. Hence, K_i is a candidate Hamiltonian for a stationary vacuum; indeed, choosing the Hamiltonian to be K_i yields the Rindler vacuum for the right Rindler wedge, while choosing the Hamiltonian to be $-K_i$ gives the Rindler vacuum for the left Rindler wedge. It is straightforward to check that there are no other inequivalent isometric Hamiltonians. For example, the combination $P_0 + \omega J_{12}$, which generates the worldlines of observers rotating in the $X_1 - X_2$ plane with angular velocity ω, becomes spacelike outside the sphere $X_1^2 + X_2^2 = 1/\omega^2$ [29]. Restriction to the world-volume of the inside of the sphere fails because such a region does not admit a Cauchy surface. Another possibility, the combination $P_0 + \beta K_i$, generates the worldlines of Rindler observers in a translated Rindler wedge. Also, the combination $P_0 + \alpha^i P_i$ is timelike for $\alpha^i \alpha_i < 1$ but this is obviously isometrically-equivalent to the Poincaré Hamiltonian via a Lorentz boost. It is easy to check that there are no other inequivalent isometries that could be used as the Hamiltonian.

Thus, in Minkowski space, the only stationary vacua corresponding to isometric Hamiltonians are the Poincaré-invariant vacuum and the Rindler vacuum.

3.2 Conjugacy Classes of the Lorentz Group

The Lorentz group has an interesting structure. There are five types of Lorentz transformations; that is, group elements of $SO(1,3)$ fall into five conjugacy classes. One conjugacy class consists of the elliptic transformations. This is the set of Lorentz transformations conjugate to the pure rotations i.e. the elliptic transformations consist of all Lorentz transformations, $\Lambda J_i \Lambda^{-1}$, that can be obtained from pure rotations via Lorentz transformations. Another conjugacy class is that of the hyperbolic transformations; these consist of pure boosts and their conjugates, $\Lambda K_i \Lambda^{-1}$. There is also the class of parabolic transformations, whose representative elements are the so-called null rotations, generated by $J_i + K_j$ for $i \neq j$. Most interesting for our purposes is the class of loxodromic transformations.[1] These are Lorentz transformations generated by a commuting pair of a rotation and a boost, such as $K_z + \beta J_z$. These "rota-boosts" cannot be reduced to either pure rotations or pure boosts by Lorentz transformations because obviously those lie in different conjugacy classes. Indeed, strictly speaking, the number of conjugacy classes is infinite, with each class labeled by a different value of the rotation parameter β. These are all the nontrivial conjugacy classes of $SO(1,3)$. (There is also the trivial conjugacy class containing the identity transformation.)

There is also an electromagnetic analogy to the rota-boosts. The Lorentz generators $M_{\mu\nu}$, which are anti-symmetric, can be thought of as the electromagnetic field strength, $F_{\mu\nu}$; the boosts are then like the electric field and the rotations like the magnetic field. Then we know that there are Lorentz invariants of the type $F \wedge {}^*F \sim E^2 - B^2$ but also of the type $F \wedge F \sim E \cdot B$. If $E \cdot B \neq 0$, no Lorentz trans-

---

[1]The peculiar names of the conjugacy classes are derived from types of curves on a sphere, as named by maritime navigators. Because Lorentz transformations leave light cones invariant, the celestial sphere of an observer's night sky is mapped to itself. The orbits of the Lorentz transformations are curves on the sphere; a loxodrome (also known as a rhumb line) is a curve that spirals from one pole to the other while intersecting all longitudinal meridians at the same angle.

formation can transform the field into a configuration that is either a pure electric field ($E^2 - B^2 > 0$), a pure magnetic field ($E^2 - B^2 < 0$), or pure electromagnetic "radiation" ($E^2 - B^2 = 0$), since these all have $E \cdot B = 0$. Correspondingly, rota-boosts are generated by generators that have $J \cdot K \neq 0$.

Specifically, a generator of a rota-boost takes the form

$$M_{01} - \beta M_{23} \tag{3.3}$$

in Cartesian coordinates. The key property is that rota-boosts are linear combinations of the usual Lorentz generators with no shared indices. In higher dimensions, there are additional parameters. For example, in six spacetime dimensions, there are two-parameter generators of the form

$$M_{01} - \beta_1 M_{23} - \beta_2 M_{45} \tag{3.4}$$

The Lorentz-invariant Casimir which generalizes $J \cdot K$ is

$$\epsilon_{i_1 \ldots i_d} \omega^{i_1 i_2} \ldots \omega^{i_{d-1} i_d} \,, \tag{3.5}$$

where ω is the parameter for the most general generator $\frac{1}{2} \omega_{ij} M^{ij}$. For example, the invariant of the generator (3.3) is 2β. In odd dimensions no invariant can be formed using the Levi-Civita tensor but it is nevertheless possible to argue that linear combinations of Lorentz generators with no shared indices cannot be reduced to elliptic, hyperbolic, or parabolic transformations. We will see that taking the Hamiltonian to be a generator of rota-boosts leads to novel stationary vacua in three-dimensional anti-de Sitter space.

3.3 Anti-de Sitter β-Vacua and Rotating Rindler-AdS Space

As we shall show, anti-de Sitter space permits a richer set of possibilities for stationary vacua. AdS space can be viewed as a hyperboloid embedded in a flat

embedding space; the isometry group is therefore a higher-dimensional Lorentz group. In embedding coordinates, AdS$_{d+1}$ is the hypersurface

$$-X_0^2 + X_1^2 + \ldots + X_d^2 - X_{d+1}^2 = -L^2 \ , \tag{3.6}$$

embedded in flat $(d+2)$-dimensional Minkowski space with two time directions. The AdS isometry group $O(2,d)$ is the Lorentz group of the embedding space and contains spatial rotations M_{ij}, two types of boosts, M_{0i} and $M_{i(d+1)}$, as well as a rotation $M_{0(d+1)}$ in the time-time plane. Consider irreducible rota-boosts of the form $M_{01} - \beta_1 M_{23} + \ldots$. There are two types of such boosts: those in which X_0 and X_{d+1} are paired with X_i's, and those in which X_0 and X_{d+1} are paired with each other. In general a rota-boost of the first type with nonzero Casimir (3.5) can be written as

$$M_{01} - \beta_1 M_{23} - \beta_2 M_{45} - \ldots \ . \tag{3.7}$$

Its norm is

$$-X_1^2 + X_0^2 + \beta_1^2 \left(X_2^2 + X_3^2 \right) + \ldots + \beta_{d/2}^2 \left(X_d^2 - X_{d+1}^2 \right) \ . \tag{3.8}$$

Using the embedding equation (3.6), this is not, for $d > 2$, time-like at the AdS boundary. Therefore, in higher dimensions, the above rota-boost cannot be considered as a candidate Hamiltonian.

Now consider the isometry generated by $M_{01} - \beta M_{23}$ in three spacetime dimensions $(d = 2)$. This generator belongs to the loxodromic conjugacy class of rota-boosts. Technically, because we are dealing with the AdS_3 isometry group $SO(2,2)$, it is a combination not of a rotation and a boost but of two boosts in the embedding space:

$$\frac{\partial}{\partial t} = (X_1 \partial_0 + X_0 \partial_1) - \beta (X_3 \partial_2 + X_2 \partial_3) \ . \tag{3.9}$$

From the flat metric of the embedding space this has norm

$$-(X_1^2 - X_0^2)(1 - \beta^2) + \beta^2 \ , \tag{3.10}$$

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using the embedding equation. Restricted to the right Rindler wedge $X_1, X_1^2 - X_0^2 > 0$ we see that our candidate loxodromic generator is future-directed and timelike for $X_1^2 \gg X_0^2$ and has orbits that stay within the wedge. By construction, it is group-inequivalent to the usual (non-rotating) Rindler Hamiltonian $X_1 \partial_0 + X_0 \partial_1$, the invariant (3.5) for its conjugacy class being

$$\epsilon_{0123}\omega^{01}\omega^{23} = 2\beta \ . \tag{3.11}$$

The wedge admits a Cauchy surface on which one can define quantum states.

In 2+1 dimensions, rotating Rindler-AdS space can be coordinatized by

$$
\begin{aligned}
X_0 &= \xi \sinh\left(\frac{t}{L} - \beta\frac{\chi}{L}\right) \\
X_1 &= \xi \cosh\left(\frac{t}{L} - \beta\frac{\chi}{L}\right) \\
X_2 &= \sqrt{L^2 + \xi^2} \sinh\left(\frac{\chi}{L} - \beta\frac{t}{L}\right) \\
X_3 &= \sqrt{L^2 + \xi^2} \cosh\left(\frac{\chi}{L} - \beta\frac{t}{L}\right) \ ,
\end{aligned}
\tag{3.12}
$$

where β is the rotation parameter. Here $-\infty < t, \ \chi < +\infty$ and $\xi > 0$. The rotating Rindler metric is given by

$$ds^2 = -\left((\xi/L)^2(1-\beta^2) - \beta^2\right)dt^2 - 2\beta dt\, d\chi + \frac{d\xi^2}{1 + (\xi/L)^2} + \left(1 + (\xi/L)^2(1-\beta^2)\right)d\chi^2 \ . \tag{3.13}$$

The event horizon is at $\xi = 0$; the determinant of the metric vanishes there. Notice also that at $\xi = \frac{\beta L}{\sqrt{1-\beta^2}}$, the t-t component of the metric vanishes. This indicates the presence of an ergosphere. Presumably this means that there are super-radiance effects in this space.

For $\beta = 0$, we recover the metric for non-rotating Rindler-AdS space:

$$ds^2 = -(\xi/L)^2 dt^2 + \frac{d\xi^2}{1 + (\xi/L)^2} + \left(1 + (\xi/L)^2\right)d\chi^2 \ . \tag{3.14}$$

Note that in the limit that the AdS radius, L, goes to infinity, so that $\xi/L \ll 1$, the non-rotating metric gives ordinary (flat) Rindler space, where now $\frac{1}{L}$ is re-interpreted as the acceleration parameter of Rindler space instead of as the AdS scale. As a check we note that this limit is singular for the $\beta \neq 0$ metric, confirming that there is no rotating Rindler metric in flat space.

Both rotating and non-rotating Rindler-AdS space are of course a portion of anti-de Sitter space just as ordinary Rindler space is a piece of Minkowski space. In fact, even globally the portion of the spacetime covered by the coordinates above is identical to that covered by non-rotating Rindler coordinates. The diffeomorphism

$$t \Rightarrow t - \beta\chi \qquad \chi \Rightarrow \chi - \beta t \tag{3.15}$$

maps one spacetime to the other. In that sense, rotating Rindler space is classically the same spacetime as non-rotating Rindler space. However, the Hamiltonians for non-rotating and rotating Rindler space are isometrically-inequivalent and, as we shall see shortly, the corresponding vacuum states of scalar field theory are particle-inequivalent.

That rotating and non-rotating Rindler space describe the same part of spacetime may seem puzzling at first because one of them has an ergosphere and the other does not. This is because rotating Rindler-AdS space possesses an observer-dependent ergosphere, in addition to an observer-dependent event horizon. The existence of an observer-dependent ergosphere can be understood as follows. Recall the origin of the ergosphere for the Kerr black hole. In the two-dimensional subspace spanned by its time-translation and azimuthal Killing symmetries, the Kerr metric at large values of r along the equator ($\theta = \pi/2$) approaches $-dt^2 + r^2 d\phi^2$, because the Kerr spacetime is asymptotically flat. Therefore, for the Kerr black hole there is a unique choice of Killing vector that is timelike at infinity, namely $(d/dt)^a$; any other linear combination of $(d/dt)^a$ and $(d/d\phi)^a$ is spacelike at infinity. The

Killing vector corresponding to time translations is therefore fixed, and hence so is the place where it becomes null i.e. the ergosphere. The geometry ensures that the location of the ergosphere is unambiguous. Contrast this with the situation in AdS. The metric for the two-space spanned by the time-translation and azimuthal Killing vectors in Rindler-AdS approaches $(\xi/L)^2(-dt^2 + d\chi^2)$. The boundary metric is simply a re-scaled two-dimensional Minkowski metric. Any observer moving along a timelike linear combination of $(d/dt)^a$ and $(d/d\chi)^a$ can choose his or her worldline as the time-translation direction. Each such linear combination of Killing vectors becomes null in a different place. Consequently, the existence and location of the ergosphere are both observer-dependent.

For each of the different possible time choices labeled by β, there is a corresponding stationary vacuum state annihilated by the Hamiltonian that generates that time evolution. We shall call this one-parameter family of vacuum states "β-vacua." Like the α-vacua of de Sitter space [30], these vacuum states are particle-inequivalent. The particle-inequivalence of the β-vacua to the usual Rindler vacuum (and to each other) can be verified explicitly by a Bogolubov transformation. Consider a positive-frequency ($\omega > 0$) mode of the Klein-Gordon equation:

$$u_{k,\omega}(t, \chi, \xi) = e^{-i\omega t + ik\chi} f_{\omega,k}(\xi) \ . \tag{3.16}$$

Demanding normalizability with respect to the Klein-Gordon inner product, one can show [31] that the value of ω does not constrain k. Now, under the transformation $t \to t - \beta\chi$ and $\chi \to \chi - \beta t$, the mode transforms as

$$u_{k,\omega} \to e^{-i(\omega+\beta k)t} e^{i(k+\beta\omega)\chi} f_{\omega,k}(\xi) \ . \tag{3.17}$$

We see that for $k < -\frac{\omega}{\beta}$, the mode has negative frequency. Hence there is a mixing between the negative and positive frequency modes under transformation from rotating to a non-rotating Rindler-AdS space. This fact can be formally demonstrated

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in terms of the Bogolubov coefficients. Consider a positive frequency mode with respect to one of the β rota-boosts:

$$v_{l,\nu} = e^{-i\nu t' + il\chi'} g_{l,\nu}(\xi) \ . \tag{3.18}$$

Since $t = t' - \beta\chi'$ and $\chi = \chi' - \beta t'$, (3.18) can be re-expressed in terms of the modes of the nonrotating vacuum:

$$u_{k',\omega'} = e^{-i\omega' t + ik'\chi} f_{k',\omega'}(\xi) \ , \tag{3.19}$$

where $\omega' = \frac{\nu - \beta l}{1 - \beta^2}$ and $k' = \frac{l - \beta\nu}{1 - \beta^2}$. If $\omega' < 0$, then the beta Bogolubov coefficient is nonzero between (3.16) and (3.18) and can be easily calculated as [11]

$$\beta(k, \omega; l, \nu) = i\Theta(-\nu + \beta l)\delta\left(\omega + \frac{\nu - \beta l}{1 - \beta^2}\right)\delta\left(k + \frac{l - \beta\nu}{1 - \beta^2}\right) \ , \tag{3.20}$$

while the Bogolubov alpha coefficient vanishes.

The expression for the beta coefficient implies that the nonrotating Rindler observer perceives any β-vacuum as filled with an infinite sea of particles for each positive frequency ω. Of course the global AdS vacuum appears thermal with a different temperature for each β-vacuum observer. Indeed, already from the metric it is clear that the temperature depends on the rotation parameter, β:

$$T = \frac{1 - \beta^2}{2\pi L} \ . \tag{3.21}$$

Interestingly, the limit $\beta \Rightarrow 1$ appears to correspond to an extremal vacuum state in Rindler-AdS space, with vanishing temperature.

3.4 Rotating Rindler-AdS and the BTZ black hole

When the cosmological constant is zero, it can be shown that there exists no black hole solution in three spacetime dimensions. However, a black hole solution can be shown to exist in three spacetime dimensions, in the presence of a negative

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cosmological constant, as shown by Maximo Banados, Claudio Teitelboim, and Jorge Zanelli [88]. The BTZ metric is given by

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{L^2 r^2} dt^2 + \frac{L^2 r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi - \frac{r_+ r_-}{L r^2} dt \right)^2$$

(3.22)

where r_+, r_- are the outer and inner radii, and L the AdS$_3$ scale. The mass and angular momentum is

$$M = \frac{r_+^2 + r_-^2}{L^2}, \quad J = \frac{2 r_+ r_-}{L}$$

(3.23)

The existence of an ergosphere in Rotating Rindler-AdS space recalls the BTZ black hole, and in fact it turns out that Rindler-AdS space is related to the BTZ black hole [88] via

$$\chi \sim \chi + 2\pi .$$

(3.24)

A change of coordinates

$$\xi = \sqrt{\frac{r^2 - 1}{1 - \beta^2}}$$

(3.25)

puts the metric in the familiar BTZ form:

$$ds^2 = -\frac{(r^2 - 1)(r^2 - \beta^2)}{r^2} dt^2 + \frac{r^2}{(r^2 - 1)(r^2 - \beta^2)} dr^2 + r^2 \left(d\chi - \frac{\beta}{r^2} dt \right)^2$$

where r is dimensionless in this case with $r_+ = 1$ and $r_- = \beta$. Therefore, Rindler-AdS (3.14) is the universal cover for the BTZ black hole [76, 77, 78, 79, 80]. The black hole solution is obtained by making an identification in a direction perpendicuar to ∂_t at the boundary. However, there is an important difference between Rindler-AdS space and the BTZ black hole. The identification breaks the symmetry group down from $SL(2, R) \times SL(2, R)$ to $SL(2, R) \times U(1)$. Consequently, the freedom of picking out the time direction is lost; neither the event horizon nor the ergosphere of the BTZ black hole is observer-dependent. Put another way, the identification $\chi \sim \chi + 2\pi$ gives the two-dimensional boundary Minkowski space a

cylinder topology. But special relativity on a cylinder has a preferred frame, singled out by the identification [89, 90]. Hence there is a preferred direction of time.

3.5 Rotating Global Vacua

Another type of loxodromic generator in AdS is

$$\frac{\partial}{\partial t} = (X_0 \partial_3 - X_3 \partial_0) - \beta (X_1 \partial_2 - X_2 \partial_1) \ . \tag{3.26}$$

This is a combination of a temporal and a spatial rotation. For comparison, the generator of global time, τ, is just the temporal rotation $(X_0 \partial_3 - X_3 \partial_0)$.

The embedding coordinates can be parameterized by

$$X_0 = \sqrt{\frac{r^2 + 1}{1 - \beta^2}} \cos(t - \beta\theta)$$

$$X_3 = \sqrt{\frac{r^2 + 1}{1 - \beta^2}} \sin(t - \beta\theta)$$

$$X_1 = \sqrt{\frac{r^2 + \beta^2}{1 - \beta^2}} \cos(\theta - \beta t)$$

$$X_2 = \sqrt{\frac{r^2 + \beta^2}{1 - \beta^2}} \sin(\theta - \beta t) \ . \tag{3.27}$$

Then, with the AdS scale set to unity, the line element reads

$$ds^2 = - \left(r^2 + 1 + \beta^2\right) dt^2 + \frac{r^2 dr^2}{(1 + r^2)(\beta^2 + r^2)} + r^2 d\theta^2 + 2\beta dt d\theta \ . \tag{3.28}$$

Here we have $0 \le \beta < 1$, $0 \le r < \infty$, and $\theta \sim \theta + 2\pi$. Clearly when $\beta = 0$ this reduces to the AdS metric in global coordinates, as it should.

For $\beta \neq 0$, there is however a subtlety with this solution. The generator of rotations $\frac{\partial}{\partial\theta}$ in embedding coordinates is

$$\frac{\partial}{\partial\theta} = -\beta (X_0 \partial_3 - X_3 \partial_0) + (X_1 \partial_2 - X_2 \partial_1) \ . \tag{3.29}$$

This has norm $-\beta^2 + (X_1^2 + X_2^2)(1 - \beta^2)$, which, however, becomes timelike for

$$X_1^2 + X_2^2 = \frac{r^2 + \beta^2}{1 - \beta^2} \le \frac{\beta^2}{1 - \beta^2} \ . \tag{3.30}$$

So that region cannot be covered by this coordinate system. To see where that region is, we note that the relation between the radius in global coordinates, ρ, and r is

$$\rho^2 = \frac{r^2 + \beta^2}{1 - \beta^2} \ . \tag{3.31}$$

For $\beta \neq 0$, $r = 0$ no longer corresponds to the center $\rho = 0$ of the AdS cylinder but is instead a surface of non-zero ρ. That is, we have effectively removed a concentric cylinder from within the AdS cylinder for the purposes of this coordinate system.

At AdS infinity, however, nothing has changed and so we can calculate the conserved charges of this space. The mass and angular momentum of rotating global AdS space can be evaluated using the prescription of [40, 41]. The result is

$$
\begin{aligned}
M &= -\frac{1}{8\pi G} \int_0^{2\pi} \frac{r^4}{2} \left(\frac{1 + \beta^2}{r^4} \right) d\theta = -\frac{1 + \beta^2}{8G} \\
|J| &= \frac{1}{8\pi G} \int_0^{2\pi} \beta d\theta = \frac{\beta}{4G} \ .
\end{aligned}
$$

Of course, excising the inner region leaves the resulting spacetime geodesically incomplete. However, we can still do quantum field theory in the region outside the inner cylinder using (3.26) for time evolution. The fact that the coordinates break down outside the inner region can be circumvented by defining the angular generator to be $\frac{\partial}{\partial \theta} = (X_1 \partial_2 - X_2 \partial_1)$. Unlike (3.13), the Killing vectors ∂_θ and ∂_t would then not be orthogonal to each other at the conformal boundary. The line element can be written as

$$- \left(1 + r^2(1 - \beta^2)\right) dt^2 + \frac{dr^2}{1 + r^2} + r^2 d\theta^2 - 2\beta r^2 dt\, d\theta \ . \tag{3.32}$$

To show that the β-vacua corresponding to the time choice (3.26) are distinct from the global AdS vacuum, we need to again show that positive and negative frequency modes mix. Normalizability conditions for fields in global AdS space were

investigated in [42]. Using the Ansatz $\Phi(r, t, \theta) = e^{-i\omega t} e^{im\theta} f(r)$ (where $m \in Z$) in global coordinates, Klein-Gordon normalizability implies that

$$\omega = \pm \left| 2h_+ + m + 2n \right| \ , \ \ n = 0, 1, 2, \dots \ , \tag{3.33}$$

where $h_+ = 1 + \sqrt{1 + M^2}$ with M the mass of the scalar field. Under the transformation $\theta \rightarrow \theta - \beta t$, which takes the global coordinates metric into (3.32), the mode solutions become

$$\Phi \rightarrow e^{-i(\omega + \beta m)t} e^{im\theta} f(r) \ . \tag{3.34}$$

Given any value of n, we see from (3.33) that we can always find sufficiently large values of negative m such that $\omega < |\beta m|$. A positive frequency mode can therefore become a negative frequency mode, and hence the rotating global AdS β-vacua are different from the global AdS vacuum. This can also be confirmed by calculating the Bogolubov coefficients directly as we did for rotating Rindler space. In higher dimensions, the quantization condition becomes [42]

$$\omega = \pm \left| 2h_+ + l + 2n \right| ; \ \ l, n = 0, 1, 2, \dots \ . \tag{3.35}$$

By the semi-positiveness of l, we always have $\omega > l$ and hence the positive frequency and negative frequency modes cannot mix under a transformation to rotating coordinates. β-vacua therefore do not exist in higher-dimensional global AdS space.

3.6   Rotating vacua in de Sitter space

The analysis in the previus section(s) suggest the existence of similar interesting vacua in de Sitter space as well. The fact that de Sitter space does not have a spatial boundary is a conceptual departure from AdS space, and therefore in order to obtain novel vacua for de Sitter space, we need to impose different conditions, which are the following:

- the candidate Hamiltonian should be a continuous isometry,

- the orbits of the Hamiltonian must not exit that region, and

- the Hamiltonian be space-like at future and null infinity $\mathcal{I}^{\pm}$.

The first two conditions are the same that were imposed on the AdS Hamiltonia, and have the same justification here. The last condition is justified as follows. De Sitter space has no global timelike Killing vector. Since we seek stationary vacua, the solutions will only cover a certain patch of the entire de Sitter space. Such a patch would admit a timelike vector, like in the case of the static patch. However, lack of any global timelike symmetry implies that this vector fails to remain timelike outside this patch. Since all stationary de Sitter vacua would be diffeomorphic to the vacuum of the static patch, it is natural to expect along the lines of the static patch that any timelike vector would become spacelike past the de Sitter horizon i.e. at future and null infinity. The last condition is also consistent with the holographic principle in de Sitter space [43]. The time translation generator of the boundary conformal field theory (living on $\mathcal{I}^{\pm}$) is dual to the Hamiltonian generator in the bulk de Sitter space which becomes spacelike at the future and null infinity. As in the case of AdS, de Sitter (dS) space also admits interesting solutions which describe non-trivial vacua. In $d+1$ dimensions, de Sitter space is described by a hyperboloid of constant positive curvature in $d+2$ dimensions

$$-X_0^2 + X_1^2 + ... + X_d^2 + X_{d+1}^2 = 1 \tag{3.36}$$

embedded in Minkowski M_{d+2}

$$ds^2 = -dX_0^2 + dX_1^2 + ... + dX_d^2 + dX_{d+1}^2 \tag{3.37}$$

As discussed in previous sections, the Hamiltonian which is the generator of time translations, defines the vacuum. Global de Sitter space has no time like killing

symmetry. Therefore in order to define a Hamiltonian, we restrict ourselves to a patch of the spacetime which enjoys this symmetry. Focussing on dS$_3$, a candidate Hamiltonian which satisfies all the necessary conditions is given by

$$H = M_{01} = \frac{\partial}{\partial t} = (X_1 \partial_0 + X_0 \partial_1) \tag{3.38}$$

This choice of Hamiltonian covers the static patch of the spacetime having time translation symmetry. The condition that the Hamiltonion be space-like at $\mathcal{I}^\pm$ yields the condition $-X_0^2 + X_1^2 > 0$. Therefore, the static patch essentially describes Rindler observers in global de Sitter and covers the causal diamond in the Penrose diagram [44]. Since the symmetry group of dS$_3$ is O(1,3), we can also consider Hamiltonia which belong to the loxodromic conjugacy class of the Lorentz group in M_4. Such a generator can be written as

$$H = M_{01} - \beta M_{23} = \frac{\partial}{\partial t} = (X_1 \partial_0 + X_0 \partial_1) - \beta \left(X_2 \partial_3 - X_3 \partial_2 \right) \tag{3.39}$$

where β is a parameter. The requirement that this generator be timelike yields

$$|H|^2 = -X_1^2 + X_0^2 + \beta^2 \left(X_2^2 + X_3^2 \right) < 0 \tag{3.40}$$

One can also see that $|H|^2$ becomes positive, i.e. the generator becomes spacelike for large values of X_0, which is one of the assumptions. The above generator cannot be reduced to (3.38) by any isometry transformation. This is gauranteed by the existence of a non-zero Casimir $\epsilon_{ijkl}\omega^{ij}\omega^{kl} = 2\beta$, where $\omega_{ij} = -\omega_{ji}$ are the usual parameters of the Lorentz generators in 3+1 dimensions. Therefore, the Hamiltonia (3.38) and (3.39) are group inequivalent. But for the vacua described by (3.38) and (3.39) to be inequivalent, there has to be a non-zero Bogolubov beta coefficient between the two, or in other words they have to be particle inequivalent. In order to calculate the Bogolubov coeficients, we coordinatize dS$_3$ described by

(3.39) as

$$X_0 = \sqrt{\frac{1-r^2}{1+\beta^2}} \sinh(t - \beta\phi)$$

$$X_1 = \sqrt{\frac{1-r^2}{1+\beta^2}} \cosh(t - \beta\phi)$$

$$X_2 = \sqrt{\frac{r^2+\beta^2}{1+\beta^2}} \cos(\phi + \beta t)$$

$$X_3 = \sqrt{\frac{r^2+\beta^2}{1+\beta^2}} \sin(\phi + \beta t) \tag{3.41}$$

The metric then reads

$$ds^2 = -\frac{(r^2+\beta^2)(1-r^2)}{r^2} dt^2 + \frac{r^2 dr^2}{(r^2+\beta^2)(1-r^2)} + r^2 (d\phi + \frac{\beta}{r^2} dt)^2 \tag{3.42}$$

where $\phi \sim \phi + 2\pi$. The above metric describes the Kerr de Sitter spacetime in 2+1 dimensions, without any point defect. [40] studied the above spacetime from a dS/CFT point of view, [45] derived it as a solution to the three dimensional Einstein's equation for a positive cosmological constant. The mass and angular momentum for the above metric is calculated as

$$M = \frac{1-\beta^2}{8G} \tag{3.43}$$

$$L = \frac{\beta}{4G} \tag{3.44}$$

Is the Kerr-de Sitter vacua labeled by the parameter β, the same as the vacuum described by the static coordinates for de Sitter space? Consider a positive-frequency ($\omega > 0$) mode of the Klein-Gordon equation in the usual static de Sitter spacetime:

$$u_{n,\omega}(t, \phi, r) = e^{-i\omega t + in\phi} f_{n,\omega}(r) , \tag{3.45}$$

where n is any integer. The transformation $t \to t - \beta\phi$ and $\phi \to \phi + \beta t$, the spacetime becomes Kerr de Sitter. Consider a positive energy mode ($\nu > 0$) in Kerr

de Sitter

$$v_{l,\nu}(t', \phi', r) = e^{-i\nu t' + il\phi'} g_{l,\nu}(r) \ , \tag{3.46}$$

The Bogolubov beta coefficient between the two modes can be easily calculated as

$$\beta(n, \omega; l, \nu) = i\Theta(-\nu - \beta l)\delta\left(\omega + \frac{\nu + \beta l}{1 + \beta^2}\right)\delta\left(n + \frac{l - \beta\nu}{1 + \beta^2}\right) \ , \tag{3.47}$$

while the Bogolubov alpha coefficient vanishes. This implies that the observer defined by the static vacua perceives any β-vacuum of Kerr-de Sitter as filled with an infinite sea of particles for each positive frequency ω.

At the boundaries $\mathcal{I}^\pm$, one can define another Killing vector which can be made orthogonal to (3.39) and is given by

$$J = \frac{\partial}{\partial \phi} = \beta \left(X_1 \partial_0 + X_0 \partial_1\right) + \left(X_2 \partial_3 - X_3 \partial_2\right) \tag{3.48}$$

as can be seen from the coordinatization (3.41). While the region covered by these coordinates demand that killing vector J be positive. This implies that

$$|J|^2 = X_2^2 + X_3^2 \geq \frac{\beta^2}{1 + \beta^2} \tag{3.49}$$

From conditions (3.40), (3.49) and the embedding equation, the region of validity for Kerr-de Sitter coordinates is given by

$$\frac{\beta^2}{1 + \beta^2} < X_2^2 + X_3^2 < \frac{1}{1 + \beta^2} \tag{3.50}$$

This region is best visualized when expressed in terms of familiar coordinates which describe the entire static patch, i.e.

$$X_0 = \sqrt{1 - R^2} \sinh(T) \tag{3.51}$$

$$X_1 = \sqrt{1 - R^2} \cosh(T) \tag{3.52}$$

$$X_2 = R\cos(\phi) \tag{3.53}$$

$$X_3 = R\sin(\phi) \tag{3.54}$$

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The metric is

$$ds^2 = -\left(1 - R^2\right) dT^2 + \frac{dR^2}{1 - R^2} + R^2 d\phi^2 \tag{3.55}$$

The condition (3.50) expressed in terms of coordinates (3.54) reads

$$\frac{\beta^2}{1 + \beta^2} < R^2 < \frac{1}{1 + \beta^2} \tag{3.56}$$

Therefore, we observe that the Kerr-de Sitter coordinates cover a region of concentric annulus. Another noteworthy point is that Kerr-de Sitter spacetime has an ergoregion where the norm of ∂_t goes null i.e. at $R = \sqrt{\frac{1}{1+\beta^2}}$. The horizon is at $R = 1$. Kerr de Sitter has been in studied in great detail, and recently there has been a renewed interest in this spacetime owing to the proposed dS/CFT correspondence [40], [45] since it serves as an excellent toy model to test the dS/CFT conjecture.

What about de Sitter vacua in higher dimensions? Can we construct inequivalent vacua using this group theoretic method for general dS$_{d+1}$? Consider dS$_4$, which has an embedding equation

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = +1 \tag{3.57}$$

The O(1,4) symmetry is manifest here. Taking the cue from previous analysis, we consider a candidate loxodromic Hamiltonian as

$$H = M_{01} - \beta M_{34} = \frac{\partial}{\partial t} = (X_1 \partial_0 + X_0 \partial_1) - \beta (X_3 \partial_2 - X_2 \partial_3) \tag{3.58}$$

Since the embedding space has odd dimensionality, we note that the above Hamiltonian generator doesn't include one of the coordinates (X_2 in this case). It is not at all obvious that the above generator and (3.38) belong to different conjugacy classes of the O(1,4) group since the Casimir $\epsilon_{abcd..}\omega^{ab}\omega^{cd}\omega^{..}\omega^{..}$ in odd dimensions does not exist. In the absence of a Casimir, proving the group inequivalence of the Hamiltonia (3.38) and (3.58) is non-trivial. The question is - do (3.38) and (3.58)

belong to different conjugacy classes of O(1,4)? To answer this question, it would therefore suffice to prove the particle inequivalence between the two Hamiltonia. We consider a massive scalar field operating in the static patch of dS_{d+1}, noting that this spacetime is described by the Hamiltonian (3.38).

Separating variables using spherical harmonics $Y_l(\Omega)$, we seek the solution for massive Klein-Gordon equation in the static coordinates as [46]

$$\Phi(t, r, \Omega) = \varphi(r)e^{-i\omega t}Y_l(\Omega) \,, \tag{3.59}$$

The general solution to the radial part of the wave equation has the form

$$\varphi = B\varphi_n + A\varphi_{n.n} \tag{3.60}$$

where

$$*\varphi_n = \left(1 - \frac{r^2}{\ell^2}\right)^{-i\omega/2} \left(\frac{r}{\ell}\right)^l {}_2F_1\left(a + h_-, a + h_+; \frac{d}{2} + l; \frac{r^2}{\ell^2}\right) \,, \tag{3.61}$$

$$\varphi_{n.n} = \left(1 - \frac{r^2}{\ell^2}\right)^{-i\omega/2} \left(\frac{r}{\ell}\right)^{2-d-l} {}_2F_1\left(b + h_-, b + h_+; \frac{4-d}{2} - l; \frac{r^2}{\ell^2}\right) \tag{3.62}$$

*

Here $a = (l - i\ell\omega)/2$, $b = (2 - d - l - i\ell\omega)/2$ and the weights

$$h_\pm = \frac{d}{4} \pm \frac{x}{2} \,. \tag{3.63}$$

where

$$\ell^2 m^2 = \frac{d^2}{4} - x^2 \,. \tag{3.64}$$

Based on the falloff behavior near the origin, we observe that φ_n is normalizable and $\varphi_{n.n}$ non-normalizable. Expanding the hypergeometric functions in the solutions (3.62) near the horizon, as $r \Rightarrow \ell$, one finds the two behaviors: $\varphi \sim (1 - r^2/\ell^2)^{\pm i\ell\omega/2}$. These are again a superposition of ingoing and out going plane waves if one defines a tortoise coordinate. These means that value of ω is independent of l. The spacetime described by the Hamiltonian (3.39) is related to

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the spacetime of the usual static patch by the simple transformation $\phi \rightarrow \phi + \beta t$, where ϕ is the azimuthal angle in Ω. In other words, for $\omega > 0$ and $n \in \mathbb{Z}$ where $-l \leq n \leq +l$, we can have in (3.59)

$$e^{-i\omega t}e^{in\phi} \rightarrow e^{-i(\omega-\beta n)t}e^{in\phi} \tag{3.65}$$

This, coupled with the fact that ω is not constrained by l implies that a positive energy mode in the vacuum described by the Hamiltonian (3.38) is not necessarily a positive energy mode in the vacuum described by the Hamiltonian (3.39), i.e $\omega - \beta n < 0$ for certain values of ω and n. Therefore the vacua are particle inequivalent, as there exists a non-zero Bogolubov coefficient β between the two spacetimes. This is a general argument and holds for all spacetime dimensions. This result also ensures that the corresponding Hamiltonia, (3.38) and (3.58) belong to different conjugacy classes of the O(1,4) group and are therefore group inequivalent. A suitable coordinatization which describes this vacua is

$$X_0 = \sqrt{1-r^2}\sinh(t-\beta\phi) \tag{3.66}$$

$$X_1 = \sqrt{1-r^2}\cosh(t-\beta\phi) \tag{3.67}$$

$$X_2 = r\sin(\theta)\cos(\phi-\beta t) \tag{3.68}$$

$$X_3 = r\sin(\theta)\sin(\phi-\beta t) \tag{3.69}$$

$$X_4 = r\cos(\theta) \tag{3.70}$$

The corresponding metric is

$$ds^2 = -\left[1-r^2-r^2\beta^2\sin^2\theta\right]dt^2 + \frac{dr^2}{1-r^2} + \left[r^2\sin^2\theta - \beta^2\left(1-r^2\right)\right]d\phi^2$$
$$+ 2\beta dtd\phi\left[1-r^2\left(1+\sin^2\theta\right)\right] \tag{3.71}$$

The horizon is at $r = 1$ as expected, and the ergosphere is given by the surface $r^2 = \frac{1}{1+\beta^2\sin^2\theta}$. This is essentially a four dimensional analogue of Kerr-de Sitter

solution in three spacetime dimensions. In principle, one can also construct higher

dimensional rotating de Sitter spaces using Hamiltonia of the form

$$
\begin{aligned}
H &= M_{01} - \beta_1 M_{34} - \beta_2 M_{56} \\
&= (X_1 \partial_0 + X_0 \partial_1) - \beta_1 (X_3 \partial_4 - X_4 \partial_3) - \beta_2 (X_5 \partial_6 - X_6 \partial_5) \,..\quad (3.72)
\end{aligned}
$$

These solutions are analogous to the topological black holes in anti-de Sitter space

[76], even though they are not black hole solutions by themselves. It would be

interesting to study the thermodynamics properties of such rotating vacuum solu-

tions which we reserve for future investigations. These solutions can potentially

serve as the testing ground for the dS/CFT correspondence in general spacetime

dimensions.

CHAPTER 4

HOLOGRAPHY AND THE ADS/CFT CORRESPONDENCE

4.1   Introduction

Ever since Einstein made space and time dynamical in his general theory of relativity, it is widely believed that quantization of gravity will necessitate an even more radical departure in our view of spacetime. Currently superstring theory appears to be the most appealing and consistent candidate for a quantum theory of gravity. What makes superstring theory unique is that the basic ingredients of this theory are one dimensional objects called strings. This is in contrast with the other theories of physics which have the point particle as their basic component. These strings have tension and therefore the theory has a fundamental length scale given by the string length l_s. Unlike point particles, strings possess internal degrees of freedom or vibrations which not only gives the entire spectrum of particles but also fixes the dimensionality (D) of spacetime. In the bosonic theory D turns out to be 26 and in the superstring version to be 10. Of course, the extra dimensions have to compactified in order to make contact with our usual four dimensional universe. Upon quantization, the free relativistic string yields an infinite tower of excitations, which can be interpreted as different particles of which the massless excitations correspond to the photon field A_μ for an open string, and the graviton field $g_{\mu\nu}$ for the closed string. This is remarkable as it shows that string theory naturally incorporates general relativity. Particle interactions can be understood as joining and splitting of strings, and the strength of these interactions is governed by a dimensionless coupling constant g_s in the superstring theory. In fact, even Newton's constant G is given by $G \sim g_s^2 l_s^8$ in the ten dimensional superstring theory.

In the last couple of decades it has become clear that superstring theory has even more structure to it than previously thought. In addition to strings, the theory also contains exotic objects called branes. These are extended (generally p dimen-

sional) dynamical objects floating in space, which, like strings, have tension built into them. Open strings can have their endpoints attached to such branes, in which case they are called D-branes [47], where the D stands for "Dirichlet". Branes are non-perturbative objects in the sense that their tension scales inversely as the coupling constant g_s, and therefore these objects do not show up in the string perturbative series when $g_s << 1$. As $g_s \to 0$, these branes appear heavy. However, their gravitational field is proportional to $GT_b \sim g_s$, where T_b is the brane tension, and therefore these objects have a flat space description in this limit.

Even though string theory is poorly understood at a non-perturbative level, it has indicated a radical new view of spacetime called holography. Pedagogically stated, holography imples that the physics in a region of spacetime can be described by the degrees of freedom residing on the boundary of such a region. This idea dates back to the work of 't Hooft [48] and Susskind [49] who were in turn motivated by the result that the entropy of a black hole scaled as the horizon area. The most concrete realization of holography came up in 1997 when Maldacena, by looking at certain class of D-branes, conjectured that the type IIB string theory (one of the five versions of the existing superstring theories) on $AdS_5 \times S^5$ space is dual to $\mathcal{N}$=4 supersymmetric Yang-Mills conformal field theory on the 3+1 dimensional boundary of AdS_5 [7]. In this chapter, we shall introduce this correspondence and demonstrate its applicability using a toy example. For introduction to string theory, the reader is directed to the references [50, 51, 52, 53]. However, as a precursor let us first have a brief look at the preliminaries which will be useful in stating and understanding the correspondence.

4.2 Anti-de Sitter Space

Anti de Sitter was very briefly introduced in Chapter 3. It is space of $d + 1$ dimensions, AdS$_{d+1}$, is a space of constant negative curvature that can be taken as a hyperboloid in a larger $d + 2$ dimensional flat embedding space with coordinates $(X_0, X_1, ..., X_d, X_{d+1})$ and satisfying

$$-X_0^2 + X_1^2 + ... + X_d^2 - X_{d+1}^2 = -L^2 \tag{4.1}$$

where L is the AdS curvature scale. Anti-de Sitter space is maximally symmetric, and naturally appears as a solution to Einstein's field equation with a negative cosmological constant. The global coordinate system is constructed by defining

$$X_0 = L \sec \rho \cos \tau$$

$$X_i = L \tan \rho \, \Omega_i$$

$$X_{d+1} = L \sec \rho \sin \tau$$

where the ranges of the coordinates are $\sum_{i=1}^{d} \Omega_i^2 = 1$, $0 \leq \rho < \pi/2$, $0 \leq \tau < 2\pi$

The time variable τ is compact as seen from the above parameterization. We must "unwrap" the time coordinate by actually considering the AdS covering space and let $-\infty < \tau < \infty$. This represents the global coordinate system for AdS as it covers the entire manifold. For our purposes though, the global coordinate system isn't very useful. The AdS/CFT correspondence is usually demonstrated using the so-called Poincare coordinate system which can be constructed by defining coordinates $(z\,,\,x^i\,,\,t)$ (with $z \geq 0$)

In order to satisfy the embedding equation (4.1), we define

$$
\begin{aligned}
X_0 &= \frac{1}{2z}\left(z^2 + L^2 + \vec{x}^2 - t^2 \right) \\
X_i &= \frac{Lx^i}{z} \\
X_d &= -\frac{1}{2z}\left(z^2 - L^2 + \vec{x}^2 - t^2 \right) \\
X_{d+1} &= \frac{Lt}{z}
\end{aligned}
$$

and the metric becomes

$$
ds^2 = \frac{L^2}{z^2}\left(dz^2 + d\vec{x}^2 - dt^2 \right)
$$

The AdS boundary in this coordinate system is at $z \to 0$. Since the Poincare coordinate system suffices for our purpose in this chapter, we now briefly turn to a discussion of conformal field theory which is the other critical component required to understand the AdS/CFT correspondence.

4.3   Conformal Field Theory

This section essentially follows the excellent review by Ginsparg [54]. We consider a space with metric $g_{\mu\nu} = \eta_{\mu\nu}$ on $\mathbf{R}^d$. The line element is given by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$. The conformal group is defined as the set of transformations which leave the metric invariant upto a scale change. Under such a coordinate transformation $x \to x'$, the metric tensor transforms under the tensor transformation law as

$$
g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \Omega(x)g_{\mu\nu}(x) \tag{4.2}
$$

These transformations can be regarded as angle preserving. It is obvious that the usual Poincare and Lorentz groups(with $\Omega(x) = 1$) are subgroups of the much larger conformal group.

In order to determine the infinitesimal generators of the conformal group, we write $x^\mu \to x^\mu + \epsilon^\mu$. The metric under such a transformation becomes

$$ds^2 \to ds^2 + (\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu)dx^\mu dx^\nu \tag{4.3}$$

In order to satisfy (4.2), it is required that

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu \propto \eta_{\mu\nu} \tag{4.4}$$

or by tracing both sides with $\eta^{\mu\nu}$,

$$\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu = \frac{2}{d}(\partial.\epsilon)\eta_{\mu\nu} \tag{4.5}$$

From (4.2), we deduce that $\Omega(x) = 1 + \frac{2}{d}(\partial.\epsilon)$. We also note from (4.5) that

$$(\eta_{\mu\nu}\partial^2 + (d-2)\partial_\mu\partial_\nu)\partial.\epsilon = 0 \tag{4.6}$$

For $d \geq 2$, we observe from (4.5) and (4.6) that ϵ is at most quadratic in x. We have

1) $\epsilon^\mu = a^\mu$ which are the ordinary translations.

2) $\epsilon^\mu = \omega^\mu_\nu x^\nu$ (ω antisymmetric) are the rotations.

3) $\epsilon^\mu = \lambda x^\mu$ are scale transformations.

and finally,

4) $\epsilon^\mu = b^\mu x^2 - 2x^\mu(b.x)$ the so called special conformal transformations.

We integrate to obtain the finite form of the transformations as

1) $x \to x' = x + a$

2) $x \to x' = \Lambda x$ i.e. Lorentz transformations where (Λ^μ_ν belongs to $SO(1, d-1)$)

3) $x \to x' = \lambda x$, the dilatations where λ is a scalar

4) $x \to x' = \frac{x + bx^2}{1 + 2b.x + b^2 x^2}$ are the special conformal transformations.

We remark that for the special case where $d = 2$, (4.5) becomes the Cauchy-Riemann equation

$$\partial_1 \epsilon_1 = \partial_2 \epsilon_2, \quad \partial_1 \epsilon_2 = -\partial_2 \epsilon_1 \tag{4.7}$$

We can then form the complex coordinates $z, \bar{z} = x^1 \pm ix^2$ and write $\epsilon(z), \bar{\epsilon}(\bar{z}) = \epsilon^1 \pm i\epsilon^2$. Hence for $d = 2$, the conformal transformations become the analytic coordinate transformations

$$z \to f(z), \;\; \bar{z} \to \bar{f}(\bar{z}) \tag{4.8}$$

for which the local algebra is infinite-dimensional. The two-dimensional conformal field theory has been studied extensively owing to its relevance in mathematics and theoretical physics. For a more exhaustive review on two-dimensional conformal field theory, the reader can consult [55].

We shall now consider the form of the two-point functions in a conformal field theory constrained by the conformal group. Under global conformal transformation,$x \to x'$, we define the "quasi-primary fields" or "conformal fields" in any conformal field theory to transform as

$$\phi_i(x) \to \left| \frac{\partial x'}{\partial x} \right|^\Delta \phi_i(x') \tag{4.9}$$

where Δ is the conformal dimension of ϕ_i. We then have a covariant theory under (4.9) in the sense that the correlation functions satisfy

$$\langle \phi_1(x_1)...\phi_n(x_n) \rangle = \left| \frac{\partial x'}{\partial x} \right|^\Delta_{x=x_1} .... \left| \frac{\partial x'}{\partial x} \right|^\Delta_{x=x_n} \langle \phi_1(x_1')...\phi_n(x_n') \rangle \tag{4.10}$$

We also demand that there is a vacuum $|0\rangle$, invariant under the global conformal group.

The condition (4.10) is very restrictive on the form of two-point functions of primary/conformal fields. It can be shown (refer to [55]) that under the global conformal group and the condition (4.10), the two-point function is of the form

$$\langle \phi(x_1)\phi(x_2) \rangle \propto \frac{1}{|x_1 - x_2|^{2\Delta}} \tag{4.11}$$

This is precisely the form of the two-point function we will obtain when we demonstrate the AdS/CFT correspondence later.

4.4 Maldacena's Conjecture and the AdS/CFT Correspondence

As mentioned previously, superstring theory contains non-perturbative objects called branes. A Dp-brane, for example is a Dirichlet brane extending along p spatial dimensions on which the end points of an open string can end. If there are two such branes, the string end points can attach itself to these branes in 2×2 ways. Consider a stack of N such Dp-branes. In this case, there are $N \times N$ possible configurations for the end points of an open string. The lowest order quantum excitations of such a string-brane system, when the N branes become coincident, is shown to be a gauge theory with the symmetry group $U(N)$ [56]. Since superstring theory has supersymmetry built into it, the resulting gauge theory on the brane world volume turns out to be the supersymmetric version of Yang Mills (SYM) theory. Maldacena considered a stack of N D3-branes, described by the solution

$$
\begin{aligned}
ds^2 &= H^{-1/2}(r)\eta_{\nu\mu}dx^\mu dx^\nu + H^{1/2}(r)dx^i dx^i \,, \quad \mu,\nu = 0,1,2,3, \quad i = 4,..,9 \\
H &= 1 + \frac{4\pi g_s N \alpha'^2}{r^4}, \quad r^2 = x^i x^i
\end{aligned}
\tag{4.12}
$$

where α' is the square of the fundamental string length i.e. $\alpha' = l_s^2$. Clearly the function H is singular as $r \to 0$, but the metric behaves nicely in the small r limit,

$$
ds^2 \to \frac{r^2}{L^2}\eta_{\nu\mu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + L^2 d\Omega_5^2
\tag{4.13}
$$

The first couple of terms represent AdS$_5$ spacetime with a curvature radius L given by $L^4 = 4\pi g_s N \alpha'^2$. The last term describes S^5. When $g_s N >> 1$, The AdS curvature scale is large compared to the string length l_s. In this regime, general relativity (or supergravity) is a valid effective description. However, in the complementary regime i.e. $g_s N << 1$, string perturbation series is a valid description where D-branes act as boundary conditions for strings. Therefore by varying the parameter $\lambda = g_s N$, one can adiabatically go between different descriptions.

Now consider the limit $\alpha' \to 0$ in the D-brane picture. This is called the decoupling limit, since the SYM theory on the branes decouples from the supergravity theory in the bulk. Maldacena [7] conjectured that these two decoupled theories are in fact the same. Thus technically stated, the AdS/CFT correspondence is that (type IIB) superstring theory on $AdS_5 \times S^5$ is dual to $\mathcal{N}{=}4$ SYM theory in 3+1 dimensions with gauge group U(N). This particular SYM theory is conformally invariant (CFT) and lives on the boundary of AdS. The coupling constants of the SYM theory and the string theory are related by $g_{YM}^2 = g_s$. The parameter $\lambda = g_s N = g_{YM}^2 N$ is called the 't Hooft coupling. It should be noted that when $\lambda >> 1$, the field theory side is strongly coupled whereas the string theory side in the bulk is weakly coupled, and hence supergravity is an effective description. When $\lambda << 1$, the field theory side is weakly coupled (hence perturbation series can be performed). However, in this regime the quantum string corrections become important and supergravity ceases to be a valid description. Therefore, in a certain sense this correspondence can be interpreted as a "weak/strong" duality. The unique feature about this duality is that it is manifestly holographic and it implies that quantum gravity in higher dimensions could be described by quantum field theories without gravity in lower dimensions.

In the initial days of the AdS/CFT correspondence, it was widely believed that the correspondence was heavily dependent on the tools of string theory. However, now the AdS/CFT correspondence is studied in its own right and is considered part of the more general principle viz. the gauge/gravity duality (see the references towards the end of the chapter). The correspondence has now been put on a more concrete mathematical footing by the work of Witten, Gubser, Polyakov et al in [84, 58] in which computational tools were developed. The correspondence formally reads

$$Z_{AdS}(\phi) = Z_{CFT}(\phi_0) \tag{4.14}$$
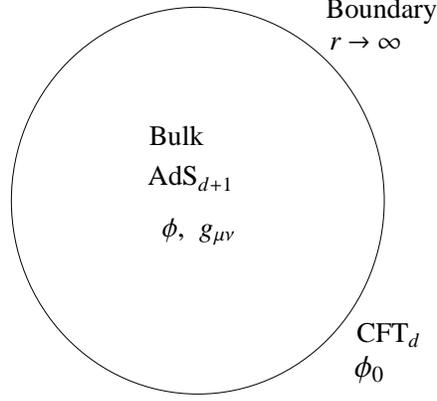
Figure 4.2: A schematic description of the AdS/CFT correspondence. The boundary CFT lives on the surface defined by large r in the metric (4.13)

where ϕ is the bulk field living in the AdS spacetime with $d+1$ dimensions and ϕ_0 is the boundary value of the bulk field at the d dimensional boundary of the AdS spacetime. Therefore, the correspondence equates the partition functions of the two dual theories. For our purpose, we consider the saddle-point approximation of the correspondence which reads

$$e^{-I_{AdS}(\phi)} = \left\langle e^{\int_{\partial B} \phi_0 \mathcal{O}} \right\rangle \tag{4.15}$$

where $\phi(z, x^i, t) \rightarrow \phi_0(x^i, t)$ as $z \rightarrow 0$ and the expectation value on the field theory side is taken with respect to the path integral in the CFT vacuum, living on the boundary ∂B. $I_{AdS}(\phi)$ is euclidean action in the bulk AdS$_{d+1}$. The saddle-point approximation is justified as long as we assume that the 't Hooft coupling $\lambda >> 1$ so that quantum corrections in the bulk are supressed. Taking ϕ_0 as the boundary value of the field which acts as a source for the conformal operator $\mathcal{O}$ on the boundary, the two-point function of the CFT is computed using the standard field theory result i.e.

$$\frac{\delta}{\delta\phi_0(x_1)}\frac{\delta}{\delta\phi_0(x_2)}Z_{CFT}(\phi_0) = \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \tag{4.16}$$

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We illustrate this fact by considering a massive scalar field propagating in AdS$_{d+1}$ and employ the Poincare coordinate system (4.2) for convenience. The bulk side calculation proceeds by trying to find the so called bulk-boundary propagator $K(z, x^i, t;\ x'^i, t')$, a function that satisfies the equations of motion for a massive scalar field with the property that $K(z, x^i, t) \to \delta(x^i - x'^i, t - t')$ as $z \to 0$. Witten in his classic paper [58], found such a solution given by

$$K(z, x^i, t;\ x'^i, t') = \left(\frac{z}{z^2 + |x - x'|^2 - (t - t')^2} \right)^\Delta \tag{4.17}$$

where $\Delta = \frac{1}{2}(d + \sqrt{d^2 + 4m^2})$. We see that the above bulk-boundary propagator satisfies the equations of motion, is well behaved as $z \to \infty$ (regularity in the bulk for precisely this value of Δ) and also has the desired property at the boundary i.e.

$$\lim_{z \to 0} \left(\frac{z}{z^2 + |x - x'|^2 - (t - t')^2} \right)^\Delta \propto \delta(x^i - x'^i, t - t') \tag{4.18}$$

Using the above bulk-boundary propagator we write the bulk field ϕ as

$$\phi(z, x^i, t) = \int K(z, x^i, t; x'^i, t') \phi_0(x'^i, t')\, dx' dt' \tag{4.19}$$

In order to evaluate the correlation functions in the CFT, we compute the classical action using (4.19). Only the surface term of the action survives as the bulk term vanishes because ϕ in (4.19) satisfies the equations of motion. Hence the action for a massive scalar field in AdS$_{d+1}$ upto a normalization reads

$$I(\phi) = \left[\int \sqrt{g} g^{zz} \phi \partial_z \phi\, dt\, dx^1 .. dx^{d-1} \right]_{z \to 0}^{z \to \infty} \tag{4.20}$$

From (4.17),(4.19) and (4.20), the action upto a normalization turns out be

$$I(\phi) = \int dx dt dx' dt' \frac{\phi_0(x, t) \phi_0(x', t')}{\left[|x - x'|^2 - (t - t')^2 \right]^\Delta} \tag{4.21}$$

Using (4.15), we get the two-point function of the conformal field $\mathcal{O}$ by differentiating the action twice with respect to ϕ_0

$$\langle \mathcal{O}(x_1\, t_1) \mathcal{O}(x_2\, t_2) \rangle = \frac{1}{\left[(x_1 - x_2)^2 - (t_1 - t_2)^2 \right]^\Delta} \tag{4.22}$$

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Comparing with (4.11), we see that $\Delta = \frac{1}{2}(d + \sqrt{d^2 + 4m^2})$ is the conformal weight of the operator $\mathcal{O}$.

We have thus demonstrated the AdS/CFT correspondence using the simplest of examples. For related reviews, the reader is directed to [59, 60, 61, 62, 63, 64, 65, 66, 67, 68]. The correspondence can be applied to understand certain aspects of field theories since many calculations involving quantities at very strong coupling can be reduced to a pure gravity calculation. For example, progress has been made in understanding QCD at strong coupling using AdS/CFT [69]. Though technically the gravity dual of QCD is yet to be found. AdS/CFT has also been applied to condensed matter systems for which the usual analytical tools may fail [70, 71, 72]. Therefore, the AdS/CFT correspondence is a potentially powerful tool which can help us understand strongly coupled quantum field theories and the quantum aspects of gravity.

With this chapter, we have laid down the necessary foundation to discuss the application of AdS/CFT to Rindler-AdS space to synthesize a formulation which will try to probe event horizons more quantitavely as compared to just studying quantum fields in Rindler/black hole spacetimes.

CHAPTER 5

RINDLER-ADS/CFT

In anti-de Sitter space a highly (compared to the AdS scale) accelerating observer admits a Rindler horizon. The two Rindler wedges in AdS_{d+1} are holographically dual to an entangled conformal field theory that lives on two boundaries with geometry $\mathbb{R} \times H_{d-1}$. For AdS$_3$, the holographic duality is especially tractable, allowing quantum-gravitational aspects of Rindler horizons to be probed. We recover the thermodynamics of Rindler-AdS space directly from the boundary conformal field theory. We derive the temperature from the two-point function and obtain the Rindler entropy density precisely, including numerical factors, using the Cardy formula. We also probe the causal structure of the spacetime, and find from the behavior of the one-point function that the CFT "knows" when a source has fallen across the Rindler horizon. This is so even though, from the bulk point of view, there are no local signifiers of the presence of the horizon. Finally, we discuss an alternate foliation of Rindler-AdS which is dual to a CFT living in de Sitter space. The discussion in this chapter is based on my work [73] with M. Parikh.

5.1   Introduction and Motivation

Rindler space, the portion of Minkowski space with which an observer undergoing constant acceleration can interact, is perhaps the simplest spacetime with a horizon. As the near-horizon limit of all nonextremal black holes and an example of a spacetime with an observer-dependent horizon, Rindler space has been much studied. Nevertheless, most of the literature on the subject has treated Rindler space using the techniques of quantum field theory in curved spacetime; however it is now recognized that many of the most interesting problems of horizon physics are not accessible with those techniques. Instead one would like to be able to study Rindler space in a theory of quantum gravity. This has not been done for the simple

reason that a tractable theory of quantum gravity in asymptotically flat space does not presently exist.

Fortunately, a tractable theory of quantum gravity in anti-de Sitter space does exist via the AdS/CFT correspondence. This motivates us to consider accelerating observers not in Minkowski space but in AdS space. Observers in anti-de Sitter space with suitably high proper acceleration (compared with the AdS length scale) have acceleration horizons; Rindler-AdS space is thus the portion of anti-de Sitter space that such observers can interact with. The purpose of this chapter then is to investigate quantum-gravitational aspects of Rindler-AdS space via its dual CFT. It is worth emphasizing that Rindler-AdS space is a particularly advantageous spacetime for studying the quantum gravity of horizons. Unlike eternal black holes in AdS, Rindler-AdS has no singularities where known physics breaks down. And unlike flat Rindler space, the existence of a dual conformal field theory is assured; indeed, in the case of AdS$_5$ it is known to be $\mathcal{N} = 4$ super Yang-Mills theory. Thus in principle one has all the necessary tools to study event horizons in quantum gravity.

While Rindler-AdS space in general dimensions has been described and studied previously, the real power of the AdS/CFT correspondence can be brought to bear when the (bulk) spacetime dimension is three. For that special case, the boundary theory becomes a two-dimensional CFT, with all the ensuing advantages. In particular, the two-point function can be calculated explicitly and the Rindler entropy density can be derived from the Cardy formula. The result matches the Bekenstein-Hawking entropy density of the Rindler horizon precisely, including numerical factors. Even more interestingly, one can test questions of information loss within this context. For example, we find that the boundary theory "knows" when a source has fallen past the Rindler horizon even though, from a bulk point of view, there are no local invariants that mark the presence of the event horizon.

This chapter is organized as follows. In Section 5.2, we present the classical geometry of Rindler-AdS space. In Section 5.3, we quickly review Rindler-AdS thermodynamics. Section 5.4 describes the boundary theory and contains our main results. We calculate the bulk-boundary propagator and the two-point correlation function of operators in the boundary theory. Specializing to AdS_3, we show that the Cardy formula precisely reproduces the Bekenstein-Hawking entropy density. We then discuss the relation between Rindler-AdS space and AdS black holes. Next, we turn to perhaps our most interesting derivation. We consider a source that falls freely into the Rindler horizon. By calculating the one-point function of a boundary operator, we show that a "boundary theorist" can tell whether the source has fallen across the horizon. In Section 5.5, we consider an alternate foliation of Rindler-AdS in which the boundary conformal field theory lives in de Sitter space. We briefly discuss some subtleties of this variant of Rindler-AdS/CFT. We summarize and conclude in Section 5.6 with some remarks about directions and puzzles suggested by Rindler-AdS/CFT.

5.2   The Geometry of Rindler-AdS

We would like to cover anti-de Sitter space in the Rindler coordinates natural to an accelerating observer. AdS_{d+1} can conveniently be described using embedding coordinates of $d+2$-dimensional Minkowski space with two time-like directions:

$$- \left(X^0\right)^2 + \left(X^1\right)^2 + ... + \left(X^d\right)^2 - \left(X^{d+1}\right)^2 = -L^2 \ . \tag{5.1}$$

Here the AdS curvature scale is L and the $O(2,d)$ isometry group is manifest. In the embedding space, a Rindler observer is one whose Hamiltonian is a boost generator. It was shown in an elegant paper by Deser and Levin [74] that both acceleration and "true" horizons in an Einstein space (such as say Schwarzschild, de Sitter, or anti-de Sitter) can be regarded as Rindler horizons in a higher-dimensional flat embedding space. The Hawking or Unruh temperature detected by observers in the

lower-dimensional space can be obtained directly from accelerating trajectories in the embedding space.[2] In particular, Rindler observers in AdS are also Rindler observers in the embedding Minkowski space [75].

Consider then a Rindler observer in $d+2$-dimensional Minkowski space (with two time directions) uniformly accelerating in the X^1 direction:

$$X^0 = \xi \sinh(t/L) \qquad X^1 = \xi \cosh(t/L) \ . \tag{5.2}$$

To view these observers as part of AdS, define also

$$X^2 = \sqrt{L^2 + \xi^2} \sinh(\chi/L) \cos\theta_1$$

$$...$$

$$X^{d-2} = \sqrt{L^2 + \xi^2} \sinh(\chi/L) \sin\theta_1 ... \sin\theta_{d-3} \cos\theta_{d-2}$$

$$X^{d-1} = \sqrt{L^2 + \xi^2} \sinh(\chi/L) \sin\theta_1 ... \sin\theta_{d-2} \cos\phi$$

$$X^d = \sqrt{L^2 + \xi^2} \sinh(\chi/L) \sin\theta_1 ... \sin\theta_{d-2} \sin\phi$$

$$X^{d+1} = \sqrt{L^2 + \xi^2} \cosh(\chi/L) \ . \tag{5.3}$$

This satisfies the AdS embedding equation (5.1). The ranges of the coordinates are

$$0 < \xi \qquad -\infty < t < \infty \qquad -\infty < \chi < \infty \qquad 0 \leq \theta_i \leq \pi \qquad 0 \leq \phi < 2\pi \ . \tag{5.4}$$

These cover the part of the hypersurface (5.1) with $\left(X^1\right)^2 - \left(X^0\right)^2 > 0$ and $X^1, X^{d+1} > 0$. With this parameterization, the AdS metric becomes:

$$ds^2 = -(\xi/L)^2 dt^2 + \frac{d\xi^2}{1 + (\xi/L)^2} + (1 + (\xi/L)^2)\left[d\chi^2 + L^2 \sinh^2(\chi/L)d\Omega_{d-2}^2\right] \ . \tag{5.5}$$

A few remarks are in order: 1) Note that the constant-ξ hypersurfaces are of the form $\mathbb{R} \times H_{d-1}$. These are the hypersurfaces on which the boundary CFT will be

---

[2]This is because the response of Unruh detectors depends on the Wightman function which in turn depends only on geometric invariants constructed out of bi-vectors, and these can just as well be computed in the embedding space.

defined.

2) Indeed, as the AdS curvature scale diverges, so that $\frac{\xi}{L} \to 0$ and $L^2 \sinh^2(\chi/L) \to \chi^2$, we recover

$$ds^2 = -(\xi/L)^2 dt^2 + d\xi^2 + d\chi^2 + \chi^2 d\Omega_{d-2}^2 , \tag{5.6}$$

which is just the line element of standard (flat) $d + 1$-dimensional Rindler space.

3) In three spacetime dimensions, the metric reduces to the $\beta = 0$ form for the rotating Rindler-AdS metric (3.13).

The above metric was also discussed in [76, 77, 78, 79, 80] in various other contexts. To better understand the global properties of Rindler-AdS space, it is useful to consider AdS$_{d+1}$ in global coordinates for which the line element is

$$ds^2 = -(1 + (\rho/L)^2)d\tau^2 + \frac{d\rho^2}{1 + (\rho/L)^2} + \rho^2 d\Omega_{d-1}^2 . \tag{5.7}$$

For completeness, we also list how global coordinates are related to embedding coordinates:

$$
\begin{aligned}
X^0 &= \sqrt{L^2 + \rho^2}\sin(\tau/L) \\
X^1 &= \rho\cos\psi \\
X^2 &= \rho\sin\psi\cos\theta_1 \\
&\quad\ldots \\
X^{d-2} &= \rho\sin\psi\sin\theta_1\ldots\sin\theta_{d-3}\cos\theta_{d-2} \\
X^{d-1} &= \rho\sin\psi\sin\theta_1\ldots\sin\theta_{d-2}\cos\phi \\
X^d &= \rho\sin\psi\sin\theta_1\ldots\sin\theta_{d-2}\sin\phi \\
X^{d+1} &= \sqrt{L^2 + R^2}\cos(\tau/L) .
\end{aligned}
\tag{5.8}
$$

Here ψ is the polar angle on the S^{d-1}, which we have explicitly separated from the angles on the S^{d-2}. Comparison with (5.3) yields the transformation between Rindler and global coordinates in the chart where Rindler coordinates apply. The angles θ_i, ϕ on the S^{d-2} are the same in both coordinate systems. The remaining global coordinates can be expressed in Rindler coordinates by

$$
\begin{aligned}
\rho^2 &= \xi^2 \left[\cosh^2(\chi/L) + \sinh^2(t/L) \right] + L^2 \sinh^2(\chi/L) \\
\tan \psi &= \frac{\sqrt{\xi^2 + L^2} \; \sinh(\chi/L)}{\xi \cosh(t/L)} \\
\cos^2(\tau/L) &= \frac{(\xi^2 + L^2) \cosh^2(\chi/L)}{\xi^2 \left[\cosh^2(\chi/L) + \sinh^2(t/L) \right] + L^2 \cosh^2(\chi/L)} \; .
\end{aligned} \tag{5.9}
$$

In particular, at $t = 0$, we see that $\tau = 0$. At other times, the constant-time slices of t are tilted with respect to the constant-time slices of τ. We also see from the last of the above equations that, with the other coordinates held fixed, $\tau \to \pm \frac{\pi}{2}$ as $t \to \pm \infty$. Our Rindler coordinates therefore cover a finite interval of global time. This is illustrated in Fig. 5.3.

Since many of our calculations will be done in three dimensions, let us briefly mention that special case. The metric for Rindler-AdS$_3$ is

$$
ds^2 = -\frac{\xi^2}{L^2} dt^2 + \frac{d\xi^2}{1 + \frac{\xi^2}{L^2}} + \left(1 + \frac{\xi^2}{L^2} \right) d\chi^2 \; . \tag{5.10}
$$

Its asymptotic behavior near the AdS boundary is given by

$$
ds^2 \to \frac{L^2 d\xi^2}{\xi^2} + \frac{\xi^2}{L^2} \left(-dt^2 + d\chi^2 \right) \; . \tag{5.11}
$$

We see that, unlike in higher dimensions, the metric on a constant-ξ hypersurface is conformal to Minkowski space. Moreover, as $\xi \Rightarrow \infty$, the transformation $\xi \to \gamma \xi$ and $(\chi, t) \to \gamma^{-1}(\chi, t)$ is the usual scale-radius duality, and is manifestly an isometry of the asymptotic metric.
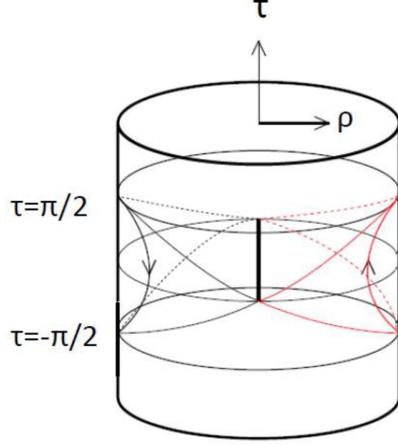
Figure 5.3: Geometry of Rindler-AdS$_{d+1}$ space. A surface of constant ξ is a $\mathbb{R} \times H_{d-1}$ hypersurface. τ and ρ are the time and radius in global coordinates; except at $\rho = 0$ each point in the interior corresponds to a S^{d-2}. The Rindler-AdS region extends only up to $\tau = \pm\pi/2$ at the boundary of AdS. The arrow on the right points in the direction of ∂_t, whose orbits are a Rindler observer's worldline; the arrow is reversed for the antipodal observer. One copy of the CFT lives on the boundary within the region shown in red.

5.3 Thermodynamics of Rindler-AdS

Contrary to the situation in flat space, the temperature seen by an observer moving with constant acceleration in curved spacetime is not always proportional to the proper acceleration. Rather, the general formula relating proper acceleration a and local temperature in $(A)dS_{d+1}$ from [75] is

$$T_{local} = \frac{1}{2\pi}\sqrt{\frac{2\Lambda}{d(d-1)} + a^2} = \frac{1}{2\pi}a_{embed} , \qquad (5.12)$$

where a_{embed} is the proper acceleration of the Rindler observer in the flat embedding space. This agrees for example with the fact that even a geodesic observer ($a = 0$) in de Sitter space sees a temperature. In AdS, there is a critical acceleration ($a_c = 1/L$) before the observer detects thermality. Observers at the critical acceleration see zero-temperature extremal horizons. Observers with lower acceleration do not have horizons. For example, an observer at a constant nonzero global radial

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coordinate ρ, moving in the direction of ∂_τ, has a constant nonzero acceleration but nevertheless does not measure a temperature. Such an observer moves vertically up the Penrose diagram and has no horizons. From the embedding point of view, sub-critical acceleration trajectories correspond to spacelike trajectories in the higher-dimensional space and therefore do not give an Unruh temperature.

Consider then a Rindler-AdS observer at constant ξ. The proper acceleration of such an observer is

$$a^2 = \frac{1}{\xi^2} + \frac{1}{L^2} \; . \tag{5.13}$$

Inserting (5.13) into (5.12) we get

$$T_{local} = \frac{1}{2\pi\xi} \; . \tag{5.14}$$

Therefore our coordinates manifestly describe observers having acceleration greater than the critical value. This can also be seen directly from the coordinates. The $SO(2,d)$-invariant vacuum state (analogous to the Poincaré-invariant vacuum in Minkowski space) is the state annihilated by the modes that have positive frequency with respect to the global time coordinate, τ. Being global, τ can be assigned to each point on the entire space, (5.1), in a single-valued manner. But (5.2) then implies that the Rindler time t must have an imaginary period of $2\pi L$. Thus the Green's function of the $SO(2,d)$-invariant vacuum, when expressed in Rindler coordinates is similarly periodic in imaginary time, indicating that an Unruh detector carried by the Rindler observer will record a temperature. Finally, the proper time of the Rindler observer has an extra factor of $\sqrt{-g_{tt}}$, giving precisely (5.14). Later, we will derive this temperature from the two-point correlation function in the boundary theory.

Next consider the entropy. The horizon is at $\xi = 0$. As in flat Rindler space,

the area of the horizon in Rindler-AdS space is infinite:

$$A_H(\mathrm{AdS}_{d+1}) \sim L^{d-2} \int_0^\infty \sinh^{d-2}(\chi/L) d\chi \ . \tag{5.15}$$

However, the entropy density, s, is finite and obeys the universal relation:

$$s = \frac{1}{4G_{d+1}} \ . \tag{5.16}$$

For the case of three-dimensional rotating Rindler space (3.13) described in chapter 3, the temperature and entropy are

$$T = \frac{1-\beta^2}{2\pi L} \qquad S = \frac{1}{4G_3} \int (1-\beta^2) d\chi_r \ , \tag{5.17}$$

where β is the rotation parameter, $-1 \leq \beta \leq 1$. The event horizon is still at $\xi = 0$ and the entropy is of course infinite. We would like to remind the reader that the non-rotating Rindler-AdS space is related to its rotating counterpart (3.13) by the simple diffeomorphism

$$t \to t - \beta\chi \ ; \quad \chi \to \chi - \beta t \tag{5.18}$$

5.4   The Boundary Theory

We are now interested in the holographically dual theory, which defines quantum gravity in Rindler-AdS space. As emphasized earlier, Rindler-AdS is simpler to study than eternal AdS black holes. Rindler-AdS space does not have singularities and the precise form of the boundary CFT is known in certain cases. Now, as usual in AdS/CFT [7], in the limit of large N and large 't Hooft coupling, the string partition function can be approximated at saddle point by the exponential of the classical supergravity action:

$$Z[\phi_0(x)]_{CFT} = \langle e^{i \int_{\partial AdS} \phi_0(x)\mathcal{O}(x)} \rangle \approx e^{iS_{sugra}[\phi(z,x)]} \tag{5.19}$$

where the bulk field $\phi(z,x)$ takes the value $\phi_0(x)$ on the boundary ∂AdS. In the Euclidean formulation, $\phi_0(x)$ acts as a source term in the CFT, and specification of the

boundary field $\phi_0(x)$ (along with the assumption of regularity in the interior) uniquely determines the bulk field, which can be determined using the bulk-boundary propagator. Thus bulk fields are dual to boundary sources. However, there are additional subtleties in the Lorentzian version of the correspondence [81, 82] because of the existence of normalizable modes in the bulk. These are bulk excitations that do not change the leading (in z) contribution to the boundary value of the field, $\phi_0(x)$. The normalizable modes are dual to states in the boundary theory. For our present purpose, we will ignore the contribution of the normalizable modes and just analytically continue the bulk-boundary propagators defined in Euclidean signature in order to study the various boundary correlation functions in Lorentzian signature. We will also focus on AdS$_3$ for computational convenience; most of the results can be extended without loss of generality to higher dimensions. Below we will first recover the thermodynamics from the CFT. Then we will perform a calculation that indicates how the boundary theorist could perceive the horizon. Remarkably, the calculation indicates that at least partial information is available to the CFT about events that are across the Rindler horizon.

Temperature, Two-Point Correlators and Entropy

We take the complete Hilbert space of conformal operators to be given by a direct product of two Hilbert spaces, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. We also take the complete state to be an entangled state of the two CFTs, as studied in [83]:

$$|\Psi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2}|E_n\rangle_1 \times |E_n\rangle_2 \ . \tag{5.20}$$

All expectation values of the conformal operators are taken with respect to the entangled state given by (5.20). In order to compute the correlation functions in the boundary theory, one needs the explicit form of the bulk-boundary propagator

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$K(\xi, \chi, t; \chi_0, t_0)$ defined by

$$\phi(\xi, \chi, t) = \int K(\xi, \chi, t; \chi_0, t_0) \phi_0(\chi_0, t_0) \, d\chi_0 dt_0 . \tag{5.21}$$

Here the point (χ_0, t_0) acts as a source on the boundary while the bulk point (ξ, χ, t) is the sink. In AdS$_3$, the bulk-boundary propagator for a minimally coupled massive scalar field, upto normalization, is

$$K(\xi, \chi, t; \chi_0, t_0) = \frac{1}{\left[\sqrt{1 + \frac{\xi^2}{L^2}} \cosh(\frac{\chi - \chi_0}{L}) - \frac{\xi}{L} \cosh(\frac{t - t_0}{L}) \right]^\Delta} . \tag{5.22}$$

Here $\Delta = 1 + \sqrt{1 + m^2}$ is the conformal dimension of the boundary operator dual to a bulk scalar of mass m. The bulk-boundary propagator satisfies the massive wave equation in Rindler-AdS coordinates and is valid as long as both the source and sink happen to be on the same side of the Rindler horizon i.e. when the conformal operaters are inserted on the same boundary. As $\xi \to \infty$, K becomes a delta function supported at $\chi = \chi_0$ and $t = t_0$. Using the standard rules for AdS/CFT [84, 85], the two-point function between conformal operators inserted on the same boundary is

$$\langle \mathcal{O}(\chi_1, t_1) \mathcal{O}(\chi_2, t_2) \rangle = \frac{1}{\left[\cosh(\frac{\chi_1 - \chi_2}{L}) - \cosh(\frac{t_1 - t_2}{L}) \right]^{1 + \sqrt{1 + m^2}}} \tag{5.23}$$

The two-point functions has a periodicity of $2\pi L$ in imaginary time; evidently the boundary CFT is thermal in nature. This is in agreement with the fact that the temperature of the Rindler horizon is indeed $T_H = \frac{1}{2\pi L}$. Hence the boundary theory gives the correct horizon temperature.

To evaluate the bulk-boundary propagator when the sink is on the other side of the horizon, we analytically continue the time as $t \to t - i\pi L$, as can be seen from (5.3). The bulk-boundary propagator then becomes

$$K(\xi, \chi, t; \chi_0, t_0) = \frac{1}{\left[\sqrt{1 + \frac{\xi^2}{L^2}} \cosh(\frac{\chi - \chi_0}{L}) + \frac{\xi}{L} \cosh(\frac{t - t_0}{L}) \right]^{1 + \sqrt{1 + m^2}}} \tag{5.24}$$

Using the above bulk-boundary propagator and the rules of AdS/CFT we arrive at the two-point function of operators inserted on the opposite boundaries

$$\langle \mathcal{O}_1(\chi_1, t_1)\mathcal{O}_2(\chi_2, t_2)\rangle = \frac{1}{\left[\cosh(\frac{\chi_1 - \chi_2}{L}) + \cosh(\frac{t_1 - t_2}{L})\right]^{1+\sqrt{1+m^2}}} \tag{5.25}$$

The two-point function is nonsingular because the operators are always spacelike separated. The reason the expectation value does not vanish even though the operators on opposite boundaries commute is that the CFTs are entangled. Given the explicit forms of both (5.23) and (5.25), it is suggestive that we should have complete knowledge of the entire causal structure of the Rindler-AdS spacetime. This suggests that one may be able to use the present framework to address certain issues about the information loss puzzle.

Entropy

What about entropy? First let us consider the entropy in higher dimensions. Specializing to AdS$_5$, the Rindler horizon has entropy

$$S_{Rindler} = \lim_{\chi_0 \to \infty} \frac{\pi L^2}{G_5} \int_0^{\chi_0} \sinh^2(\chi/L)d\chi \ , \tag{5.26}$$

which diverges as expected. The coordinate ξ scales the boundary theory. Specifically, for AdS_5, the dual theory is $\mathcal{N} = 4$ SYM theory, with a gauge field, four Weyl spinors and six conformally coupled scalars, all in the adjoint of $SU(N)$. The number of degrees of freedom is thus $15N^2$. The size of the gauge group is related to the AdS radius by the usual dictionary

$$N^2 = \frac{\pi L^3}{2G_5} \tag{5.27}$$

A priori, there are now two ways of calculating the entropy from the dual theory: as the entropy of a gas of thermal free fields, and as entanglement entropy. The free field entropy computation for a thermal CFT is done using the standard result

$$S_{CFT} = \frac{2}{3}\pi^2 N^2 V_{CFT} T_{CFT}^3 \tag{5.28}$$

Evaluating this "holographically" implies substituting boundary data into the above

expression. At fixed $\xi = \xi_0 \gg L$, the boundary metric is

$$ds^2 = \xi_0^2 \left[\frac{-dt^2}{L^2} + \frac{d\chi^2}{L^2} + \sinh^2\left(\frac{\chi}{L}\right) d\Omega_2^2 \right]$$ (5.29)

The horizon temperature is given by $T_H = \frac{1}{2\pi L}$ and the physical temperature at the

boundary is

$$T_{CFT} = \frac{T_H}{\sqrt{-g_{tt}}} = \frac{1}{2\pi\xi_0}$$ (5.30)

and V_{CFT} is given by

$$V_{CFT} = \lim_{\chi_0 \to \infty} \frac{4\pi\xi_0^3}{L} \int_0^{\chi_0} \sinh^2(\chi/L)d\chi$$ (5.31)

Using (5.27), (5.30), (5.31) and inserting them into (5.28),we see that the free field

CFT entropy scales in the same manner as (5.26), albeit with

$$S_{CFT} = \frac{1}{6} S_{Rindler}$$ (5.32)

This familiar numerical disagreement is presumably because of the fact that we

have assumed the large N limit and large 't Hooft coupling. In this approximation,

the entropy of the boundary theory is computed using the results for a free field

CFT. In the exact case however, the CFT could be a fully interacting field theory; we

do not yet understand how to calculate the entropy for such a theory directly.

So far this is all mostly familiar. We can do much better for Rindler-AdS$_3$.

For (5.10), the Bekenstein-Hawking entropy is given by

$$S_{BH} = \frac{A}{4G_3} = \frac{\int d\chi}{4G_3}$$

The Euclideanized boundary metric for (5.10) is given by

$$ds_{boundary}^2 = d\tau^2 + d\chi^2 \,,$$

where $\tau \sim \tau + \beta = \tau + 2\pi L$, and the last equality follows from the fact that the

boundary two-point function (5.23) is periodic in imaginary time with period $\beta =$

$2\pi L$. Since by the AdS/CFT correspondence $Z_{AdS} = Z_{CFT}$, we can now use the Cardy formula [86] which gives the entropy of a two dimensional CFT to calculate:

$$S_{CFT} = \frac{\pi}{3\beta} c \; \text{Volume} = \frac{\pi}{3} \frac{3L}{2G_3} \frac{1}{2\pi L} \int d\chi = S_{BH} \; , \tag{5.33}$$

where $c = \frac{3L}{2G_3}$ is the central charge of the unitary CFT as calculated by Brown and Henneaux [87]. Of course the entropy of the Rindler horizon is infinite, but it is very interesting that the entropy densities are now in precise agreement.

We can also use the Cardy formula for the CFT dual to the rotating Rindler-AdS space (3.13):

$$S_{CFT} = \frac{\pi}{3} cT \; \text{Volume} = \frac{\pi}{3} \frac{3L}{2G_3} \frac{1 - \beta^2}{2\pi L} \int d\chi_r = S_{BH} \tag{5.34}$$

Once again the CFT entropy density and the Bekenstein-Hawking entropy density are in precise agreement, including the numerical factor. Under the diffeomorphism (5.18), the volume element transforms as $d\chi \to (1 - \beta^2)d\chi$, and therefore (5.33) and (5.34) both have the universal entropy density $1/4G$.

Let us pause here to comment briefly on the relation between Rindler-AdS space and black holes in anti-de Sitter space. First, it is important to clarify that Rindler-AdS space is not the near-horizon limit of black holes in AdS; the near-horizon limit of all non-extremal black holes, including AdS black holes, is flat Rindler space.

The existence of an ergosphere in rotating Rindler-AdS space recalls the rotating BTZ black hole. Indeed, rotating Rindler-AdS space is related to the rotating BTZ black hole [88] via

$$\chi_r \sim \chi_r + 2\pi \; . \tag{5.35}$$

A change of coordinates

$$\xi = \sqrt{\frac{r^2 - 1}{1 - \beta^2}} \tag{5.36}$$

puts the metric in the familiar BTZ form:

$$ds^2 = -\frac{(r^2-1)(r^2-\beta^2)}{r^2}dt_r^2 + \frac{r^2}{(r^2-1)(r^2-\beta^2)}dr^2 + r^2\left(d\chi_r - \frac{\beta}{r^2}dt_r\right)^2 \ . \quad (5.37)$$

Rindler-AdS is thus the universal cover for the BTZ black hole [76, 77, 78, 79, 80]. The black hole solution is obtained by making an identification in a direction perpendicuar to ∂_t at the boundary. However, there is an important difference between Rindler-AdS space and the BTZ black hole. The identification breaks the symmetry group down from $SL(2,R) \times SL(2,R)$ to $SL(2,R) \times U(1)$. Consequently, the freedom of picking out the time direction is lost; neither the event horizon nor the ergosphere of the BTZ black hole is observer-dependent. Put another way, the identification $\chi_r \sim \chi_r + 2\pi$ gives the two-dimensional boundary Minkowski space a cylinder topology. But special relativity on a cylinder has a preferred frame, singled out by the identification [89, 90]. Hence there is a preferred direction of time.

That Rindler-AdS$_3$ is the universal cover of the BTZ black hole also means that two-point functions in the CFT for BTZ black holes are infinite sums of Rindler-AdS two-point functions summed over all image points. For example, for operators inserted on opposite boundaries, the BTZ two-point correlator is

$$\langle \mathcal{O}_1(\chi_1,t_1)\mathcal{O}_2(\chi_2,t_2)\rangle_{BTZ} \ \sim \ \sum_{n=-\infty}^{n=+\infty} \frac{1}{\left[\cosh(\frac{\chi_1-\chi_2+2\pi n}{L}) + \cosh(\frac{t_1-t_2}{L})\right]^{1+\sqrt{1+m^2}}}$$

$$\sim \ \sum_{n=-\infty}^{n=+\infty} \langle \mathcal{O}_1(\chi_1+2\pi n,t_1)\mathcal{O}_2(\chi_2,t_2)\rangle_{\mathrm{Rindler}} \quad (5.38)$$

The relative simplicity of the two-point function in Rindler-AdS is, as we shall see below, another one of the advantages of Rindler-AdS as a model spacetime in the study of horizons.

The Omniscient CFT

It is now widely believed, if not proven, that the process of black hole formation and evaporation is unitary. The existence of a unitary conformal field theory dual to

anti-de Sitter space lends support to this belief, as the formation and evaporation of AdS black holes is presumably a process that has a dual description within a unitary theory. Nevertheless, a detailed account of how information emerges from a black hole is far from clear. Here we will take a step in that direction by showing that the dual CFT can tell whether an infalling source has crossed the horizon. In fact, the CFT even has partial information about events that happen across the horizon. This is promising because, from the local bulk point of view, the horizon is a nondescript place; by contrast, gauge/gravity duality is nonlocal and it is precisely in a theory with nonlocality that one expects to be able to evade the paradoxes of black holes.

There are of course several different ways to probe the horizon [91, 92, 93]. In particular, Shenker et al. in [94] probed the singularity structure of the BTZ black hole using spacelike geodesics. Even though Rindler-AdS spacetime does not have any singularity, it is instructive to carry out a similar analysis and study its implications in our case. The basic premise is to study geodesics which start at the boundary, cross the horizon in finite proper length and end at the opposite boundary of the spacetime.

In the case of Rindler-AdS in three dimensions (5.10), spacelike geodesics for $\Delta\chi = 0$ trajectories are given by

$$\frac{1}{1 + \frac{\xi^2}{L^2}} \left(\frac{d\xi}{ds} \right)^2 - \frac{E^2\lambda^2}{\xi^2} = 1, \tag{5.39}$$

where E denotes the conserved energy per unit mass and is given by

$$E = \frac{\xi^2}{\lambda^2} \frac{dt}{ds} \tag{5.40}$$

and "s" denotes the proper length. Integrating, we find

$$\xi(s) = L \sqrt{\left(\cosh\left(\frac{s}{L}\right) + \frac{E\lambda}{L} \sinh\left(\frac{s}{L}\right) \right)^2 - 1} \quad ; \quad E^2 > 0, \quad \frac{E^2\lambda^2}{L^2} < 1, \tag{5.41}$$

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where we have set $\xi(0) = 0$. The proper length required for the trajectory to begin at a bulk coordinate $|\frac{\xi_0}{L}| \gg 1$ (but from behind the horizon), and reach the horizon is given by (5.41)

$$s_- \approx -\ln\left(\frac{2\xi_0}{L - E\lambda}\right) \tag{5.42}$$

The proper length required to reach the same bulk coordinate on the other side of the horizon is

$$s_+ \approx \ln\left(\frac{2\xi_0}{L + E\lambda}\right) \tag{5.43}$$

Hence the total proper length is given by

$$s_+ - s_- \approx \ln\left(\frac{4\xi_0^2}{L^2 - E^2\lambda^2}\right) \tag{5.44}$$

The action for this process is $I = m\Delta s = 2m\ln\left(\frac{2\xi_0}{\sqrt{L^2 - E^2\lambda^2}}\right)$. In order to connect E to Δt, we solve for (5.40) using (5.41) and get

$$\tanh\left(\frac{-t}{\lambda}\right) = 1 + \left(\frac{L^2 + E^2\lambda^2}{LE\lambda}\right)\tanh(\frac{s}{L}) \tag{5.45}$$

Hence

$$\Delta t = t(+\infty) - t(-\infty) = \lambda\ln\left(\frac{L - E\lambda}{L + E\lambda}\right) - i\pi\lambda \tag{5.46}$$

As seen from the above equation, $i\pi\lambda$ is the required jump in imaginary time to go across the Rindler horizon. The real part, Δt_r, is related to E by

$$E\lambda = -L\tanh\left(\frac{\Delta t_r}{2\lambda}\right) \tag{5.47}$$

Using the above expression, the action for the geodesic is given by

$$I = m\Delta s = 2m\ln\left(\frac{2\xi_0}{\sqrt{L^2 - E^2\lambda^2}}\right) = 2m\ln\left(2\frac{\xi_0}{L}\cosh(\frac{\Delta t_r}{2\lambda})\right) \tag{5.48}$$

As the infrared cutoff $\frac{\xi_0}{L} \to \infty$, the above action diverges. Subtracting the infinite contribution, we arrive at the renormalized action for the geodesic given by

$$I_{ren} = 2m \ln \left(\cosh(\frac{\Delta t_r}{2\lambda}) \right) \tag{5.49}$$

The contribution of this geodesic to the two-point function given by the WKB approximation upto a normalization is

$$e^{-I_{ren}} = \frac{1}{\left(\cosh(\frac{\Delta t_r}{2\lambda})\right)^{2m}} \tag{5.50}$$

We see that for large m (when the WKB approximation is indeed valid),$1+\sqrt{1+m^2} \simeq m$, the leading term of (5.25) agrees with (5.50) as $\Delta t \to \infty$. This is analogous to the fact that although classically a particle is always confined inside the light-cone, quantum mechanically there is a small but finite amplitude for the particle to "leak" outside the light-cone and the amplitude to do so is given by e^{-I} where $I = m|\Delta \vec{x}|$.

The above calculation is encouraging and is highly suggestive of the fact that the CFT indeed has access to information across the horizon. However, the formation of an actual blackhole is through collapse and that the surface of the collapsing matter follows a timelike trajectory. Therefore, in order to better understand the causal structure of the event horizon we need to study infalling matter following timelike geodesics from the dual CFT perspective. This can be done as follows. we will consider "switching on" a source which freely falls into the Rindler horizon, before being "switched off" after the passage of some finite interval of proper time. The source couples to a bulk field which, for simplicity, we will take to be a free scalar field. The boundary value of the bulk field in turn plays the role of a coupling constant in the boundary CFT. The motivation for choosing such an

infalling source as a probe is both physical and technical. Consider, as an analogy, a Reissner-Nordstrom black hole. The bulk field here would be the electromagnetic field and a source would be any charge or current configuration. For the purpose of understanding information retrieval, one might like to send in a source that carries no coarse-grained hair (i.e. no mass, charge, or angular momentum) such as, say, an electric dipole, to test whether the CFT can determine what was thrown in. The alternative to throwing in a source would be to send in some excitation of the field itself; this would be analogous to probing our Reissner-Nordstrom black hole by sending in an electromagnetic wave. Technically, the problem with sending in a wave is that it is not localized even in the bulk; by contrast, the source can be localized. We would have to send in a wave packet and deal with issues of the spreading of the packet. Also, if the bulk field is massless, waves of this field will propagate on null trajectories. Hence in light-cone or Eddington-type coordinates, the wave would have a constant ingoing null coordinate and we would not be able to distinguish the moment the packet crossed the horizon from any earlier moment. The advantage of sending in a source is that it can travel on a timelike trajectory, for which the ingoing null coordinate time varies along the trajectory. And by considering the signatures of the "switching on" and "switching off" processes of our infalling source, we will see that the CFT can tell whether the source is switched on or off even after it crosses the Rindler horizon.

In order to describe an infalling source, we need to define the Rindler coordinates beyond the horizon i.e. into the region $(X^1)^2 - (X^0)^2 < 0$. To that end, we transition to ingoing Eddington-Finkelstein (EF) coordinates by defining

$$
\begin{aligned}
r &\equiv \frac{\xi^2}{2L} \\
v &\equiv t + \int \frac{dr}{\frac{2r}{L}\sqrt{1 + \frac{2r}{L}}} = t + \frac{L}{2}\ln\left[\frac{\sqrt{1 + \frac{2r}{L}} - 1}{\sqrt{1 + \frac{2r}{L}} + 1}\right] .
\end{aligned}
\tag{5.51}
$$

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With this, the Rindler-AdS$_3$ metric in EF coordinates becomes

$$ds^2 = -\frac{2r}{L}dv^2 + \frac{2dvdr}{\sqrt{\left(1+\frac{2r}{L}\right)}} + \left(1+\frac{2r}{L}\right)d\chi^2 \ . \tag{5.52}$$

The ranges of the coordinates is $-L/2 < r < \infty$ and $-\infty < v < \infty$, with the region outside the horizon being $0 < r < \infty$. In particular, these coordinates are perfectly smooth at the future horizon $r = 0$. These coordinates span one patch of the Rindler-AdS space time ($-\frac{L}{2} < r < \infty$). In the Penrose diagram, the entire space time can be viewed as an infinite concatenation of such identical patches, in the direction of the global time coordinate. The boundary metric at large r is

$$ds_b^2 = \frac{2r}{L}\left(-dv^2 + d\chi^2\right) \ , \tag{5.53}$$

which is conformally flat, an advantage of working in three dimensions.

In order to describe a source falling into the Rindler horizon, we consider timelike radially ingoing geodesics in Rindler-AdS$_3$. Since the metric is invariant under translations of the v coordinate, the momentum component p_v is conserved along geodesics. Since $p_v = mu_v$ (where u^a is the velocity vector), and setting $m \equiv 1$, we have that u_v is conserved. For simplicity, let the conserved value of u_v be -1. Then setting χ=const so that $u^\chi = 0$ (which corresponds to radial infall) we have

$$(u^r)^2 + \frac{4r^2}{L^2} = 1 \ . \tag{5.54}$$

Choosing the initial condition $r(0) = L/2$ and using $u_v = -1$, we find that the source's geodesic trajectory is given by

$$
\begin{aligned}
r_J(\tau) &= \frac{L}{2}\cos\left(\frac{2\tau}{L}\right) \\
v_J(\tau) &= \frac{L}{2}\ln\left[\frac{1+\sin\left(\frac{2\tau}{L}\right)}{(\sqrt{2}\cos\left(\frac{\tau}{L}\right)+1)^2}\right] \ ,
\end{aligned}
\tag{5.55}
$$

where τ is the proper time. The conditions are chosen such that, at $\tau = 0$, we have $r = L/2$ and $v = -L\ln(1+\sqrt{2})$. The source exits the patch covered by Eddington

coordinates at $\tau_{max} = L\pi/2$ for which $v_{max} = 0$. In particular, the source crosses the Rindler horizon at

$$\tau_h = L\frac{\pi}{4} \quad , \quad r_h = 0 \quad , \quad v_h = -\frac{L}{2}\ln 2 \; . \tag{5.56}$$

We now consider a bulk scalar field, ϕ, sourced by a freely falling localized source, J, which we model as

$$J = \int_{\tau_i}^{\tau_f} d\tau \; \delta(r - r_J(\tau))\delta(v - v_J(\tau))\delta(\chi - \chi_J(\tau)) \; , \tag{5.57}$$

where $r_J(\tau)$ and $v_J(\tau)$ are given by (5.55), and $\chi_J(\tau) = 0$ for simplicity. In addition, we require the source to get "switched on" at a certain instant with proper time $\tau_i \geq 0$, then traverse the geodesic path (5.55) before getting "switched off" or terminated at a later proper time, τ_f.

In order to describe the infall of the source into the horizon from the boundary perspective, we use the basic AdS/CFT tool

$$\int_{\mathrm{bulk}} \mathcal{D}\phi \, e^{iI[\phi]} = \left\langle e^{\int \phi_0 \mathcal{O}} \right\rangle_{CFT} \; , \tag{5.58}$$

where ϕ_0 is the boundary value of the bulk field ϕ. Using the SUGRA approximation, we can approximate the bulk path integral by its saddle-point

$$\int_{\mathrm{bulk}} \mathcal{D}\phi \, e^{iI[\phi]} \sim e^{iI[\phi_{cl}]} \; , \tag{5.59}$$

where $I[\phi_{cl}]$ is the action for the classical field configuration. In order to evaluate the bulk action, we need to first find ϕ_{cl}. Given J, we can solve for the bulk scalar field as

$$\phi_{cl}(r, \chi, v) = \int G(r, \chi, v; r', \chi', v')J(r', \chi', v')dr'd\chi'dv' \; , \tag{5.60}$$

where $G(r, \chi, v; r', \chi', v')$ is the bulk-bulk propagator. For our source (5.57) we have

$$\phi_{cl}(r, \chi, v) = \int_{\tau_i}^{\tau_f} G(r, \chi, v; r_J(\tau), \chi_J(\tau), v_J(\tau))d\tau \; . \tag{5.61}$$

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An important point to note is that the propagators that arise in path integrals, such as on the left-hand side of (5.58), are Feynman propagators; Feynman's $i\epsilon$ prescription is necessary for path integrals to converge. Hence we must use the Feynman propagator to evaluate $\phi_{\rm cl}$ in order to be consistent with our setup. This is very important since the Feynman propagator, which crucially does not vanish at spacelike separation, can yield signatures about across-horizon physics.

The boundary value, $\phi_0(\chi, v)$, of the scalar field can be obtained by taking $\lim\limits_{r \to \infty} \phi_{cl}(r, \chi, v) = \phi_0(\chi, v)$. The explicit form for the bulk-bulk Feynman propagator for AdS$_3$ was derived in [95] and is given by

$$G(r_1, \chi_1, v_1; r_2, \chi_2, v_2) \sim \gamma^\Delta \; {}_2F_1\left(\frac{\Delta}{2}, \frac{\Delta}{2} + \frac{1}{2}, \Delta, \gamma^2\right) , \tag{5.62}$$

where $\Delta = 1 + \sqrt{1 + m^2}$. Here γ is related to the AdS invariant geodesic distance,

$$\gamma = \frac{L^2}{X_1^a X_2^b \eta_{ab}} , \tag{5.63}$$

for any two vectors X_1^a and X_2^a, where η_{ab} is the Minkowski metric in the embedding space (i.e. with two time directions). In EF coordinates (see appendix), we find that

$$\gamma = \frac{L^2}{+\sqrt{+\frac{4r_1 r_2}{L^2}} \cosh\left(\frac{v_2 - v_1 - f(r_2) + f(r_1)}{L}\right) - \sqrt{\left(1 + \frac{2r_1}{L}\right)\left(1 + \frac{2r_2}{L}\right)} \cosh\left(\frac{\chi_2 - \chi_1}{L}\right)} . \tag{5.64}$$

According to the AdS/CFT correspondence, at large N and large 't Hooft coupling, the one-point function is given by

$$\langle \mathcal{O}(v, \chi) \rangle = \lim_{r \to \infty} \frac{1}{\sqrt{-h}} \frac{\delta I}{\delta \phi_0(v, \chi)} , \tag{5.65}$$

Here h is the determinant for the boundary metric (5.29). Let us first evaluate the action. The action for the field ϕ is

$$I[\phi] = \int \left(-\frac{1}{2}(\partial \phi)^2 - \frac{1}{2}m^2\phi^2 + J\phi\right) d\chi dv dr . \tag{5.66}$$

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Integrating (5.66) by parts, and separating the bulk and the surface terms, we get for the variation of the action

$$\delta I[\phi_{\mathrm{cl}}] \sim \int g^{\mu\nu} \delta\phi_{\mathrm{cl}} \, \partial_\mu \phi_{\mathrm{cl}} \, d\Sigma_\nu \ , \tag{5.67}$$

where $d\Sigma_\nu$ is the surface normal to the v coordinate and the variation of the bulk term vanishes on-shell. Since we wish to evauate this action at the boundary, i.e. at $r \to \infty$, using the above expression and (5.65), the one-point function is

$$\langle \mathcal{O} \rangle \sim \lim_{r\to\infty} \frac{\sqrt{-g}}{\sqrt{-h}} g^{r\mu} \partial_\mu \phi_{\mathrm{cl}} \ , \tag{5.68}$$

as one power of ϕ is pulled down by differentiation. We now plug in (5.61) to get

$$\langle \mathcal{O} \rangle \sim \lim_{r\to\infty} \frac{\sqrt{-g}}{\sqrt{-h}} g^{r\mu} \partial_\mu \int_{\tau_i}^{\tau_f} G(r,\chi,v;r_J(\tau),\chi_J(\tau),v_J(\tau)) d\tau \ . \tag{5.69}$$

Finally, we assume a massless scalar field $m = 0 \Rightarrow \Delta = 2$ for ease of calculation, $\chi_J = 0$, and insert (5.55), (5.62), and (5.64) into the above expression. Next, we notice from (5.64) that γ goes to zero as $r \Rightarrow \infty$. We can therefore perform a power series expansion of the hypergeometric function for small γ in terms of Pochhammer symbols. We then get

$$\begin{aligned}
\lim_{r\to\infty} \partial_r \left[\gamma^2 \, {}_2F_1\left(1,3/2,2,\gamma^2\right) \right] &= \lim_{r\to\infty} \frac{\partial}{\partial\gamma^2} \left[\gamma^2 \left(1 + \frac{3\gamma^2}{4} + \ldots \right) \right] \frac{\partial\gamma^2}{\partial r} \\
&= \lim_{r\to\infty} \left[1 + \frac{3\gamma^2}{2} + \ldots \right] \frac{\partial\gamma^2}{\partial r} \ , \tag{5.70}
\end{aligned}$$

where, from (5.64), we have

$$\lim_{r\to\infty} \frac{\partial\gamma^2}{\partial r} = \frac{-1}{r_\infty^2 \left[\sqrt{1 + \cos\left(\frac{2\tau}{L}\right)} \cosh(\chi) - \sqrt{\cos\left(\frac{2\tau}{L}\right)} \cosh\left(\frac{v}{L} - g(\tau)\right) \right]^2} \ . \tag{5.71}$$

Here r_∞ is the infrared cutoff that marks the surface on which the CFT lives. Therefore in the large $r = r_\infty$ limit, only the first term in (5.70) contributes. Noting that in the large r limit, $\sqrt{-h} \to \frac{2r_\infty}{L}$, $g^{rr} \to \frac{4r_\infty^2}{L^2}$, we have for the one-point function

$$\langle \mathcal{O}(v,\chi) \rangle \sim \int_{\tau_i}^{\tau_f} \frac{d\tau}{r_\infty \left[\sqrt{1 + \cos\left(\frac{2\tau}{L}\right)} \cosh(\chi) - \sqrt{\cos\left(\frac{2\tau}{L}\right)} \cosh\left(\frac{v}{L} - g(\tau)\right) \right]^2} \ , \tag{5.72}$$

where $g(\tau) = v_J(\tau) - \frac{L}{2} \ln \left[\frac{\sqrt{1+\frac{2r_J(\tau)}{L}} - 1}{\sqrt{1+\frac{2r_J(\tau)}{L}} + 1} \right] = \frac{1}{2} \ln \left[\frac{1+\sin\frac{2\tau}{L}}{\cos\frac{2\tau}{L}} \right]$. The appearance of the $\frac{1}{r_\infty}$ factor is consistent with the scaling dimensions of the operator $\mathcal{O}$. The above integral for $\langle \mathcal{O}(v,\chi) \rangle$ can be further simplified to yield

$$\int_{\tau_i}^{\tau_f} \frac{4 \, d\tau}{r_\infty \left[2\sqrt{1+\cos\left(2\tau/L\right)} \cosh \chi - e^{-v/L}\sqrt{1+\sin\left(2\tau/L\right)} - e^{v/L}\frac{\cos(2\tau/L)}{\sqrt{1+\sin(2\tau/L)}} \right]^2} \,. \tag{5.73}$$

Signatures of Across-Horizon Physics

First, let us consider the one-point function when the source is both switched on and switched off outside the horizon. For instance, we could take $\tau_i = 0$ and $\tau_f = L\pi/6 < \tau_h$. Setting $\chi = 0$ and performing the integral (5.73), we obtain

$$\langle \mathcal{O}(v,0) \rangle \sim \frac{1}{r_\infty \left(\sqrt{2} - \cosh(v/L) \right) \left(\sqrt{6} - \sqrt{3}\cosh(v/L) + \sinh(v/L) \right)} \,. \tag{5.74}$$

Notice that the one-point function has *four* poles at

$$u_i = L \ln(\sqrt{2}+1) \quad , \quad v_i = L \ln(\sqrt{2}-1)$$
$$u_f = \frac{L}{2} \ln(2+\sqrt{3})(5+2\sqrt{6}) \quad , \quad v_f = \frac{L}{2} \ln(2+\sqrt{3})(5-2\sqrt{6}) \,. \tag{5.75}$$

Here u and v are ingoing and outgoing Eddington-Finkelstein coordinates; u is re-lated to the v-coordinate by $u = v - 2f(r)$, where $f(r)$ is given by the log term in (5.51). We have expressed two of the poles in terms of u coordinates for reasons that will be clear soon.

Now, consider the case where the source switches off only after it crosses the horizon. For example, choose $\tau_i = 0$ and $\tau_f = L\pi/2 > \tau_h$. Evaluating the integral, we find

$$\langle \mathcal{O}(v,0) \rangle \sim \frac{1}{r_\infty \left(\sqrt{2} - \cosh(v/L) \right) \sinh(v/L)} \,. \tag{5.76}$$

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In this case the one-point function has only *three* poles. They are at

$$u_i = L \ln(\sqrt{2} + 1) \quad , \quad v_i = L \ln(\sqrt{2} - 1)$$

$$v_f = 0 . \tag{5.77}$$

The appearance of poles in the one-point function is easy to understand. We considered an idealized source which is nonzero only for a finite interval of proper time, $\tau_i \leq \tau \leq \tau_f$. As a result, the field ϕ_{cl} is discontinuous at the endpoints (τ_i, τ_f) since at these points we abruptly switch the source on and off. But the one-point function is related to the derivative of the field (5.69). The poles therefore come from taking the derivative of a discontinuous field. The discontinuity in the field propagates towards the AdS boundary along light-like trajectories. Moreover, since we are using the Feynman propagator, the propagation of these signals occur via the retarded (the u poles) as well as the advanced component (the v poles) of the propagator. In a certain sense, these poles indicate the creation and annihilation of the source from a boundary theory perspective. There are also poles in the χ (spatial) direction on the boundary. That is because the locus of poles is the intersection of the constant r hypersurface where the CFT lives with the past/future light cone emanating from the endpoint. See Figure 5.4.

Now the crucial point is that, once the source crosses the horizon, there is no pole corresponding to the outgoing Eddington coordinate u when the source switches off at τ_f. This is because once past the horizon, retarded signals from the source do not reach the surface where the CFT lives. This is schematically illustrated in Figure 5.5.

Evidently, the poles of the one-point function, $\langle \mathcal{O} \rangle$, allows the boundary theorist to determine whether the source was annihilated before or after crossing the horizon. If there are four poles, the source switched off before it reached the Rindler

73
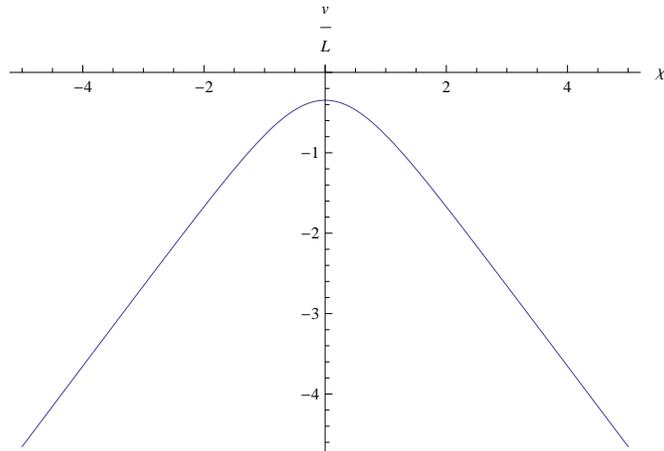
Figure 5.4: The locus of points on the boundary where there are poles coming from one endpoint of the source trajectory. The specific values plotted are for the case where the source switches off precisely on the horizon, for which there are only v poles coming from the intersection of the past light cone of the endpoint with the hypersurface on which the CFT lives.
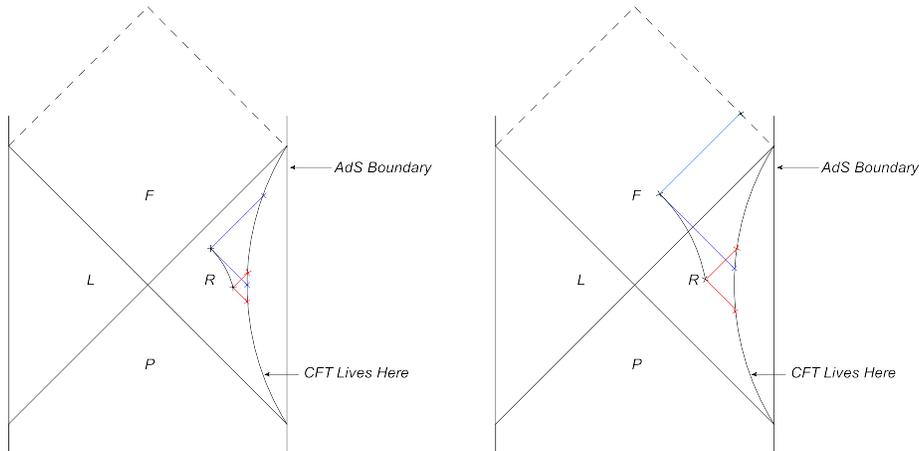

Figure 5.5: a) The left figure illustrates when the source is active for a certain time period outside the horizon in the right Rindler wedge (R). The red and blue lines indicate signals propagating towards the AdS boundary which correspond to the creation and annihilation of the source respectively. The four poles are indicated on the boundary where the CFT lives. b) The right figure shows a source that crosses the horizon. It is evident that the retarded signal from the annihilation (or switching off) of the source no longer reaches the CFT boundary, and therefore the CFT perceives just three poles as shown. The dashed lines indicate the boundary of the Eddington-Finkelstein coordinates.

horizon; if there are only three poles, it means that the source switched off after horizon-crossing. But in order to determine whether the source switches off before or after the horizon, the boundary theorist has to observe the one-point function for all time. For example, a source that is switched off just infinitesimally before crossing the horizon will contribute a future-light-cone (u) pole in the near-infinite future. So the boundary theorist has to wait till future infinity to determine whether there are three poles or four.

In fact, the boundary theorist even acquires partial information about the location of the switching off event, even if that event was across the horizon. In our radial infall scenario, we have effectively suppressed the χ coordinate and the location of a switching on/off event is characterized by its u and v coordinates. If the source switches off before it traverses the horizon, the CFT pole structure records both the u and the v values of the event so that its precise location can be identified. Even if the source switches off after it crosses the horizon, the CFT still knows about the v value of the event. So partial information is obtained even about events that happen across the event horizon. For complete information, note that the past light cone of a switching off event in the upper Rindler wedge (F) (see Figure 5.5) also intersects the antipodal CFT (associated with a hypersurface in region (L)). The missing fourth pole is actually in the antipodal CFT; complete knowledge of the pole structure of both CFTs is therefore sufficient to reconstruct switching off events in the upper Rindler wedge.

5.5   De Sitter space as the boundary of Rindler-AdS

In this section, we touch upon an alternate formulation of Rindler-AdS with a potentially wide spectrum of applications. Consider again a Rindler observer in $d + 2$-dimensional Minkowski space (with two time directions) uniformly accelerating in

the X^1 direction:

$$X^0 = \tilde{r}\sinh(t/L) \qquad X^1 = \tilde{r}\cosh(t/L) \tag{5.78}$$

This turns the flat space line element into

$$ds^2 = -\left(\frac{\tilde{r}}{L}\right)^2 dt^2 + d\tilde{r}^2 + dX_2^2 + ... + dX_d^2 - dX_{d+1}^2 \tag{5.79}$$

which, indeed, is Rindler space (albeit with two time directions). Rindler observers at constant $\tilde{r}$ have proper acceleration $1/\tilde{r}$. We foliate AdS as

$$
\begin{aligned}
X^0 &= R\cos\chi\sinh(t/L) \\
X^1 &= R\cos\chi\cosh(t/L) \\
X^2 &= R\sin\chi\cos\theta_1 \\
&\quad... \\
X^{d-2} &= R\sin\chi\sin\theta_1...\sin\theta_{d-3}\cos\theta_{d-2} \\
X^{d-1} &= R\sin\chi\sin\theta_1...\sin\theta_{d-2}\cos\phi \\
X^d &= R\sin\chi\sin\theta_1...\sin\theta_{d-2}\sin\phi \\
X^{d+1} &= \sqrt{L^2 + R^2}
\end{aligned}
\tag{5.80}
$$

This satisfies the AdS embedding equation (5.1). The first two coordinates are of the form (5.78) with what we called $\tilde{r}$ now being $R\cos\chi$. Defining $r = L\sin\chi$, we finally obtain

$$ds^2 = \frac{dR^2}{1+(R/L)^2} + (R/L)^2\left[-(1-(r/L)^2)dt^2 + \frac{dr^2}{1-(r/L)^2} + r^2 d\Omega_{d-2}^2\right] \tag{5.81}$$

We see that Rindler-AdS can also be foliated in slices that are conformal to static de Sitter space with de Sitter radius L [96, 97]. The ranges of the coordinates are

$$0 \le R \qquad -\infty < t < \infty \qquad 0 \le r < L \qquad 0 \le \theta_i \le \pi \qquad 0 \le \phi < 2\pi \tag{5.82}$$

The coordinate r is related to the polar angle on the S^{d-1} by $r = L \sin \chi$ in the region $0 \leq \chi < \pi/2$. The range $\pi/2 < \chi \leq \pi$ covers the static patch of the antipodal observer. Note that, since $\tilde{r} = \cos \chi$, the relation (5.78) between ∂_{X^0} and ∂_t is reversed for this observer.

Incidentally, the spatial geometry at constant t is given by

$$ds^2 = \frac{dR^2}{1 + (R/L)^2} + R^2 \left(d\chi^2 + \sin^2 \chi d\Omega_{d-2}^2 \right) \tag{5.83}$$

which is locally Euclidean AdS_d i.e. the hyperbolic space H_d. For the region $0 \leq \chi < \pi/2$ (corresponding to $0 \leq r < L$), the spatial part of AdS that corresponds to a Rindler observer is really H_d/Z_2 whose topology is B^d/Z_2. The geometry of Rindler-AdS space is depicted in Fig. 5.6
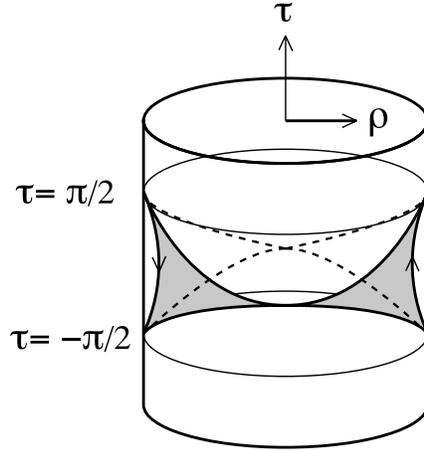
Figure 5.6: Geometry of Rindler-AdS$_{d+1}$ space. The shaded region is a surface of constant R, which covers the static patches of a pair of antipodal de Sitter observers. τ and ρ are the time and radius in global coordinates. The Rindler-AdS region extends only up to $\tau = \pm\pi/2$ at the boundary of AdS. The arrow in the right shaded region points in the direction of ∂_t, whose orbits are a Rindler/de Sitter observer's worldline; the arrow is reversed for the antipodal observer. Except at $\rho = 0$ each point in the interior corresponds to a S^{d-2}.

To compute the temperature of the Rindler horizon, consider a Rindler observer at constant R and constant r. The proper acceleration of such an observer

is

$$a = \frac{1}{R}\sqrt{(R/L)^2 + \frac{1}{1 - (r/L)^2}} \tag{5.84}$$

Inserting (5.84) into (5.12) we get

$$T_{local} = \frac{1}{2\pi R}\sqrt{\frac{1}{1 - (r/L)^2}} \tag{5.85}$$

and the horizon temperature is

$$T_H = \sqrt{-g_{tt}}T_{local} \tag{5.86}$$

From the boundary point of view the Rindler observer is an accelerating observer at fixed r in static de Sitter space. To obtain the de Sitter temperature, we define $t = \hat{t}/(R/L)^2$ which puts the constant R part of the metric in the form:

$$ds^2 = -f(r)d\hat{t}^2 + \frac{dr^2}{f(r)} + (R/L)^2 \, r^2 d\Omega^2_{d-2} \tag{5.87}$$

Then the de Sitter temperature is

$$T = \frac{f'(r_H)}{4\pi} = \frac{l}{2\pi R^2} \tag{5.88}$$

and the local temperature at constant r is

$$T_{boundary} = \frac{1}{2\pi R}\frac{1}{\sqrt{1 - (r/L)^2}} \tag{5.89}$$

which is again the physically-measured Rindler temperature.

The entropy of the Rindler horizon is calculated using the standard area formula. The horizon is at $r = \lambda$. Specializing to AdS_5, the Rindler horizon has entropy

$$S_{Rindler} = \frac{\pi}{G_5}\int_0^{R_0} \frac{R^2 dR}{\sqrt{1 + (R/L)^2}} = \frac{\pi L^2}{2G_5}\left(R_0\sqrt{1 + (R_0/L)^2} - L\sinh^{-1}(R_0/L)\right) \tag{5.90}$$

where R_0 is a cut-off radius which acts in the bulk as an infrared regulator. We see that for large R_0 the entropy scales like R_0^2:

$$S_{Rindler} \approx \frac{\pi L R_0^2}{2G_5} \tag{5.91}$$

The coordinate R scales the boundary theory in this parameterization. At fixed $R = R_0$, therefore, the theory is a UV cut-off CFT in static de Sitter space. The R_0^2 scaling of the entropy, (5.91), seems to indicate, perhaps surprisingly, that a free field computation for a thermal CFT will not give the right result either. A free field calculation, quite apart from being off by numerical factors, would be expected to yield an extensive entropy that scales like R_0^3 though oddly the entropy in this case is precisely $(R_0/L)^2 N^2$ using (5.27). The actual R_0^2 scaling strongly suggests that the correct boundary interpretation of Rindler entropy could be as entanglement entropy [98, 99, 100, 101]; the de Sitter horizon acts as a surface across which the conformal fields are entangled with the fields in the static de Sitter patch of the antipodal observer.

To calculate the two-point correlator consider a massive scalar field in Rindler-AdS$_{d+1}$. The easiest way to calculate the boundary correlation functions is to Wick-rotate the time coordinate as $t \to iL\psi$; the CFT then lives on an S^d. The two-point function of the dual operator can now be easily calculated as

$$\langle \mathcal{O}(\theta_1, \psi_1)\mathcal{O}(\theta_2, \psi_2)\rangle = \frac{1}{(1 - \cos D)^\Delta} \tag{5.92}$$

where $\Delta = 1 + \sqrt{1 + m^2}$, is the conformal dimension of the dual operator, and D is the de Sitter invariant distance in d dimensions, which in two dimensions becomes $\cos D = (\sin \theta_1 \sin \theta_2 \cos (\psi_1 - \psi_2) + \cos \theta_1 \cos \theta_2)$. We observe that (5.92) has the required periodicity in the imaginary time coordinate, ψ, and yields the correct Rindler temperature (5.86).

5.6 Conclusion

In this Chapter, we have presented a holographic duality for acceleration horizons. The key idea was to consider acceleration horizons in AdS, rather than in flat space, so as to be able to exploit the AdS/CFT correspondence. We then used the dual picture to holographically probe properties of the Rindler horizon. We recovered the horizon thermodynamics including the precise entropy density for the case of Rindler-AdS$_3$. We also showed that physics beyond the horizon can be probed from the perspective of the boundary theory by calculating the response of the boundary theory to an infalling horizon-crossing source. Evidently, Rindler-AdS/CFT holds much promise for studying the quantum gravity of horizons and, moreover, it is considerably more tractable than the holography of AdS-Schwarzschild black holes; we have surely only scratched the surface of this rich subject.

Among the obvious directions for future study are to work out two-point and higher correlation functions for infalling sources and to look at other more realistic scenarios that might probe the horizon. It would be particularly interesting to set up a problem in which information fell into the Rindler horizon, to see whether our intuition about information return is borne out. Another obvious direction is to perform calculations using Rindler-AdS/CFT and then finally make a global identification in the χ direction to learn about the holography of BTZ black holes.

Also, as mentioned earlier, there are subtleties in the Lorentzian version of AdS/CFT because of the presence of normalizable modes. We ignored in this work but it would be interesting to work out mode solutions for (5.5) and map them to the boundary theory. One can also determine the spectrum of normalizable modes and study the quantization conditions. This will throw more light on the dictionary between the bulk and the boundary descriptions in Rindler-AdS/CFT.

It should be noted that what we have done was, in some sense, still quantum field theory in curved spacetime. The boundary theory learned about the bulk from the boundary value of the bulk field which in turn was determined using a propagator over a fixed background geometry. By considering graviton fluctuations, we might be able to take a step beyond QFT in curved spacetime.

More speculatively, we could try to implement some kind of observer complementarity [102, 103]. For example, in our scenario we know that complete information about the switching off event in the upper Rindler wedge was provided by the pole structure in both CFTs. In order for all this information to be available to one observer, it might be necessary to perform some kind of antipodal identification [103] or to map the antipodal CFT to some other surface in the original wedge, such as at the stretched horizon [102, 104]. It might also be, however, that complete information is not provided even by both CFTs. In particular, the points where the two antipodal Rindler wedges intersect cannot be attributed unambiguously to either Rindler wedge. Correspondingly, operator insertions on the boundary of global AdS at precisely the points where it touches that intersection surface cannot obviously be thought of as insertions in either of the two CFTs.

Still more speculatively, there might be connections to the Hagedorn transition. In quantum field theory, acceleration and temperature are linearly related, but in string theory it is possible that something nontrivial happens when the temperature reaches the Hagedorn temperature. Perhaps the existence of a Rindler-AdS/CFT correspondence might provide a new angle from which to examine this old issue.

That a certain foliation of AdS has de Sitter space as its boundary is also very interesting. One can try to understand the vacuum states in de Sitter space using this setup [30]. It may allow us to use the AdS/CFT correspondence in the

reverse way: by using gravity in Rindler-AdS space to learn about strongly-coupled field theories in de Sitter space.

<center>Appendix</center>
<center>*Eddington-Finkelstein coordinates for Rindler-AdS$_3$*</center>

Ingoing Eddington-Finkelstein coordinates are related to AdS embedding coordinates through

$$
\begin{aligned}
X^0 &= \sqrt{2rL}\sinh\left(\frac{1}{L}(v - f(r))\right) \\
&= \frac{1}{2}\left[e^{v/L}\sqrt{\frac{2rL(\sqrt{1+2r/L}+1)}{\sqrt{1+2r/L}-1}} - e^{-v/L}\sqrt{\frac{2rL(\sqrt{1+2r/L}-1)}{\sqrt{1+2r/L}+1}}\right]
\end{aligned}
$$
(5.93)

$$
\begin{aligned}
X^1 &= \sqrt{2rL}\cosh\left(\frac{1}{L}(v - f(r))\right) \\
&= \frac{1}{2}\left[e^{v/L}\sqrt{\frac{2rL(\sqrt{1+2r/L}+1)}{\sqrt{1+2r/L}-1}} + e^{-v/L}\sqrt{\frac{2rL(\sqrt{1+2r/L}-1)}{\sqrt{1+2r/L}+1}}\right]
\end{aligned}
$$
(5.94)

$$
\begin{aligned}
X^2 &= \sqrt{L^2+2rL}\sinh\left(\frac{\chi}{L}\right) \\
X^3 &= \sqrt{L^2+2rL}\cosh\left(\frac{\chi}{L}\right) \ ,
\end{aligned}
$$
(5.95)

where $f(r) = \frac{L}{2}\ln\left[\frac{\sqrt{1+\frac{2r}{L}}-1}{\sqrt{1+\frac{2r}{L}}+1}\right]$ as given by (5.51). These coordinates are nonsingular at the Rindler horizon $r = 0$.

CHAPTER 6

WEAK MEASUREMENTS IN QUANTUM MECHANICS

This chapter is based on my work [105] with Y. Aharanov, P. C. W. Davies, and S. Walker.

6.1 Introduction

In quantum mechanics, we have two kinds of time evolution: the usual unitary evolution, and the sudden, irreversible, and nonunitary collapse of the wavefunction projected onto an eigenstate. The latter describes the "measurement" process according to Von Neumann [106], where it is understood that the system is strongly coupled to the measuring device. However, if one considers the measurement process carried out on an ensemble of such systems, certain novel features are observed.

Following the arguments in [107], we consider a large collection of quantum systems represented by the product state

$$|\Psi\rangle_N = |\psi\rangle_1 |\psi\rangle_2 ... |\psi\rangle_N \tag{6.1}$$

where the systems described by ψ are non-interacting. Consider the set of identical observables $(A_1, A_2, ..A_N)$, where A_i acts on the ith wavefunction of the ensemble. Let us now define the operator

$$\hat{A}_N = \frac{1}{N} \sum_{i=1}^{N} A_i \tag{6.2}$$

which can be interpreted as the ensemble average operator. Imposing the initial condition that all the N quantum systems were initially prepared in the same identical state, the action of the ensemble average operator in the limit that $N \to \infty$, on $|\Psi\rangle_N$ yields

$$\hat{A}_N |\Psi\rangle_N = \langle A \rangle |\Psi\rangle_N \tag{6.3}$$

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where $\langle A \rangle$ is the quantum expectation value for any single system in the ensemble (6.1). The interesting feature emerging out of this construction is that in the limit $N \to \infty$, a measurement of $\hat{A}_N$ does not disturb the ensemble, or any of its individual members. This can be understood by realizing that $\langle A \rangle$ is in fact the eigenvalue of the operator $\hat{A}_N$ in (6.3), and its known that repeated measurements of an eigenstate does not disturb the system. From the perspective of the measuring device, this can be understood as follows. If the measuring device is coupled to the whole ensemble (comprised of N non-interacting quantum systems) with certain fixed strength, then it is fair to assume that its coupling strength to each individual members is rescaled by $1/N$. Therefore, in the large N limit, the coupling to individual members approaches zero. Such measurements are called "weak" since the measuring device is weakly coupled to the system being measured, and therefore does not disturb the system. Nevertheless, the ensemble average is acquired in the large N limit.

A natural question that arises is what would be the outcome if a strong measurement is performed on the ith member of the ensemble, after a weak measurement has already been performed? Even though all the members of the ensemble were initially prepared in the same quantum state, it is not mandatory that subsequent strong measurements on the ith member would yield the same eigenvalue. Therefore, the initial ensemble (6.1) can be split into various new subensembles, with each subensemble satisfying the condition that a strong measurement at a later time yields a specific eigenvalue for its members. Depending on the outcome, we are then free to focus on any subensemble of our interest. Thus, in addition to *pre-selecting* the initial ensemble (N identical, non-interacting quantum systems), we can also *post-select* a subensemble ($\leq N$ identical, non-interacting quantum systems) which satisfy a specific final condition.

Let us denote the initial state of the whole ensemble as $|\Psi_i\rangle$, and the final state as

$|\Psi_f\rangle$. Correspondingly, let us denote the initial state of the kth member as $|\psi_i\rangle_k$, and the final state as $|\psi_f\rangle_k$. Consider the expectation value of the operator $\hat{A}_k$ (i.e. action of $\hat{A}$ on the kth member) with respect to $|\Psi_i\rangle$,

$$\langle\Psi_i|\hat{A}_k|\Psi_i\rangle = \sum_j |\langle\Psi_i|\psi_{f,j}\rangle_k|^2 \langle\psi_{f,j_k}|\hat{A}_k|\Psi_i\rangle/\langle\psi_{f,j_k}|\Psi_i\rangle \qquad (6.4)$$

Here we have inserted a complete single-member final states $\{|\psi_{f,j}\rangle_k\}$. The first term in the right hand side of the above equation is the probability that the kth member of the ensemble yields a final eigenstate j upon a strong measurement. This term in the large N limit represents the fraction of the whole ensemble that satisfies the initial pre-selection and also the specific post-selection (to be in the eigenstate j). Therefore, this term defines the pre and post-selected subensemble, and the expression (6.4) gives the expectation value $\hat{A}_k$ of a single member, expressed as a sum of all possible subensembles (i.e. all post-selections) weighted by the quantities

$$\langle\psi_{f,j_k}|\hat{A}_k|\Psi_i\rangle/\langle\psi_{f,j_k}|\Psi_i\rangle = \langle\psi_{f,j_k}|\hat{A}_k|\psi_i\rangle_k/\langle\psi_{f,j_k}\psi_i\rangle_k \qquad (6.5)$$

We drop the redundant index k, since all the members are identical. Therefore the quantity

$$\langle\psi_{f,j}|\hat{A}|\psi_i\rangle/\langle\psi_{f,j}|\psi_i\rangle \qquad (6.6)$$

is called the *weak value* of the operator $\hat{A}$ subject to the specific post-selected eigenstate j. Schematically, the weak value can be written as

$$w = \langle final|\hat{A}|initial\rangle/\langle final|initial\rangle \qquad (6.7)$$

where $|initial\rangle$ is the pre-selected state, and $|final\rangle$ is the post-selected state. In the case of time-dependent systems, the weak value at time $t_i \leq t \leq t_f$, can be expressed as

$$w = \frac{\langle\psi_f|U^\dagger(t-t_f)\hat{A}U(t-t_i)|\psi_i\rangle}{\langle\psi_f|U^\dagger(t-t_f)U(t-t_i)|\psi_i\rangle}, \qquad (6.8)$$

where U is the time-evolution operator $U(t - t_0) = e^{-iH(t-t_0)}$, and $|\psi_i\rangle$ is the ensemble of systems that has been pre-selected at time $t = t_i$. $|\psi_f\rangle$ then represents a subensemble post-selected at a time $t = t_f$.

These weak values are peculiar in the sense that they can be arbitrarily large, and may lie outside the range of eigenvalues. They may be even negative or complex valued. For more review on this subject, the reader is directed to the references [106, 108, 109, 110, 111, 112, 113, 114].

In this chapter, we will consider weak measurements performed on certain quantum systems subjected to specific pre and post-selections.

6.2   Weak values in "quiescent" regions

Consider an atom, coupled to the electromagnetic field, and prepared at time t_i in an excited state. Suppose a measurement made at a later time t_f finds the atom to still be in the initial, excited, state. What can one say about the electromagnetic field in the interval $[t_i, t_f]$? In recent years, problems of this sort have been tackled by considering weak measurements conducted at times in the interval between pre- and post-selected states. In the case of an excited atom coupled to the electromagnetic field, weak values of the field observables in the interval $[t_i, t_f]$ will generally be non-zero, *even when the atom is found to have not decayed* at time t_f. We are familiar with the fact that the decay of an atom excites the electromagnetic field around it. Here we show that the excitation energy of an atom that does *not* decay can nevertheless still create measurable effects in the surrounding field.

Special interest attaches to cases where $\langle in|out \rangle \ll 1$, because *w* can then be very large, leading to potentially large physical effects. We predict that the weak values of the electromagnetic field in the vicinity of an un-decayed atom will

become exponentially large as a function of time, with an e-folding time that approaches the atom's expectation time for decay. To pursue this claim, we consider a simplified model in which a two-level atom is coupled to an infinite bath of other two-level atoms n with identical ground states and upper levels distributed as follows:

$$E_n - E_0 = n\Delta E; \;\; N \leq n \leq N \tag{6.9}$$

i.e. equispaced and distributed symmetrically about the excited state of atom "0". If for simplicity one assumes a constant identical interaction Hamiltonian H (H is assumed to be a real number) between 0 and each atom in the bath, then the evolution operator U of the system can be written down explicitly [107]. Here we wish to focus on the case that the atom 0 is both pre-selected (at time t_i) and post-selected (at time t_f) to be in the excited state, with all the bath atoms initially set in their ground states, and focus on the subsequent behavior of the bath atoms in the interval $[t_i, t_f]$. Intuitively one might imagine that because the atom has not decayed at time t_f then there will be no disturbance to the bath atoms, but this is not the case. Let the weak value for the projection operator onto the excited state of atom n be denoted w_n, and the bra vector for the initial state of the total system be denoted as (1,0,0,0,..), the first entry corresponding to atom 0 in its excited states and the remaining entries to the n bath atoms in their ground states. The projection operator P_n onto the excited state of atom n will then be, in this notation, a square matrix with all elements 0 except the entry for row n, column n, which will be 1. The Schrödinger equation for this system is a set of coupled differential equations

$$\begin{aligned} \dot{a}_0 &= -i\sum_n H a_n e^{-in\Delta Et} \\ \dot{a}_n &= -iH a_0 e^{in\Delta Et} \end{aligned} \tag{6.10}$$

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where a_n is the probability amplitude that the atom labeled by n is in the excited state. We set $\hbar = 1$ for convenience. The above set of equations (6.10) can be solved exactly using Laplace transforms, in the limit that $N \Rightarrow \infty, \Delta E \Rightarrow 0$, $H \Rightarrow 0$, and $\frac{H^2\pi}{\Delta E} \Rightarrow \gamma$, where γ is defined as the decay constant. The evolution operator (which in this case is a $2N + 1 \times 2N + 1$ matrix) can be written down as:

$$
\begin{aligned}
U_{00}(t) &= \exp\left[-\gamma|t| - iE_0t\right] \\
U_{n0}(t) &= H\exp\left(-iE_nt\right)\frac{\left[\exp\left[-\gamma|t| - iE_nt - 1\right]\right]}{\gamma - in\Delta E} \\
U_{0n}(t) &= H\exp\left(-iE_nt\right)\frac{\left[\exp\left[-\gamma|t| - iE_nt - 1\right]\right]}{\gamma + in\Delta E}
\end{aligned}
\tag{6.11}
$$

The elements U_{nm} are not required for what follows. It may be readily verified that the above operator satisfies the unitarity constraint $UU^\dagger = 1$ (for the elements given), and the evolution condition $U(t_f - t)U(t - t_i) = U(t_f - t_i)$. Using (6.8), the weak values of interest are given by

$$
w_n = \frac{[1, 0, 0, 0, ..]^T U(t_f - t)P_n U(t - t_i)[1, 0, 0, 0..]}{[1, 0, 0, 0, ..]^T U(t_f - t_i)[1, 0, 0, 0..]}
\tag{6.12}
$$

where $[1, 0, 0, 0, ..]^T$ is the transpose of the column vector $[1, 0, 0, 0, ..]$, and use has been made of the relation $U^\dagger(t - t_f) = U(t_f - t)$. The matrix multiplications are straightforward, and using (6.11) we find

$$
\begin{aligned}
w_n &= \left(\frac{H^2}{\gamma^2 + n^2\Delta E}\right)\exp\left[\gamma(t_f - t_i)\right]\left(\exp\left[-\gamma(t_f - t_i)\right] + \exp\left[-in\Delta E(t_f - t_i)\right]\right) \\
&\quad - \left(\frac{H^2}{\gamma^2 + n^2\Delta E}\right)\exp\left[\gamma(t_f - t_i)\right]\left(\exp\left[-\gamma(t - t_i) - in\Delta E(t_f - t)\right]\right) \\
&\quad - \left(\frac{H^2}{\gamma^2 + n^2\Delta E}\right)\exp\left[\gamma(t_f - t_i)\right]\left(\exp\left[-\gamma(t_f - t) - in\Delta E(t - t_i)\right]\right)
\end{aligned}
\tag{6.13}
$$

Thus, this gives the weak value that the nth atom is in an excited state, subject to our specific pre and post-selection.

6.3   Particle tunneling through a barrier

As a second example, let us consider the quantum tunneling of a particle trapped inside a potential well. This can be considered as a stylized model of alpha-decay,

in which a particle tunnels through a potential barrier with small probability. Let us model the system by the potential:

$$
\begin{aligned}
V(x) &= \frac{\hbar^2 \kappa}{m}\delta(x) \\
&= \infty \;\; ; \;\; x \leq -2L
\end{aligned}
\tag{6.14}
$$

where $\kappa > 0$, i.e. it describes a potential barrier centered at $x = 0$. The problem is set up as follows : A particle, modeled by a wave packet is confined to the region, $-2L < x < 0$, under the influence of a positive delta function potential at $x = 0$. The wave packet is prepared such that initially at time t = 0, it is confined to the well, $-2L < x < 0$, and is incident on the barrier with some speed. Therefore, the particle goes back and forth between the infinite wall at $x = -2L$ and the potential barrier at $x = 0$. Everytime the wave packet hits the potential, a part of it gets transmitted and the rest, reflected. Usually if we wait for a sufficiently long time, we will find that the wave packet has completely tunneled through the potential. We are interested in weak values at an intermediate time $0 < t < T$ of the projection operator $\delta(x - a)$ outside the well, subject to the post-selection that the particle is still confined to the well at time $t = T$.

For simplicity, let us model the trapped particle by the Gaussian wave packet

$$
\Phi(x, t) = \frac{b}{\sqrt{b^2 + \frac{i\hbar t}{m}}} \exp\left[ik_0\left(x - x_0 - \frac{vt}{2}\right) - \frac{(x - x_0 - vt)^2}{2(b^2 + \frac{i\hbar t}{m})}\right]
\tag{6.15}
$$

where m is the particle mass, and x_0 is the position of maximal $|\Phi(x, t)|^2$ at $t = 0$. To avoid the complications associated with wave packet spreading, we assume the following conditions:

1) the mass m is very large such that $b^2 \gg \frac{\hbar t}{m}$ in the range of time that we are interested in, i.e neglect spreading

2) the initial width b to be extremely small, and

3) the dominant wave number k_0 to be very large such that $v = \frac{\hbar k_0}{m}$ is finite.

With the above conditions, the wave packet can be approximated as

$$\Phi(x,t) = \exp\left[ik_0\left(x - x_0 - \frac{vt}{2}\right) - \frac{(x - x_0 - vt)^2}{2b^2}\right] \tag{6.16}$$

which would suffice for our calculation. The pre-selection condition at $t = 0$ is

$$\Phi_{pre}(x,0) = \exp\left[ik_0(x + L) - \frac{(x + L)^2}{2b^2}\right] \tag{6.17}$$

and we post-select the state at a later time $t = T$ to be

$$\Phi_{post}(x,T) = \exp\left[-ik_0(x + L) - \frac{(x + L)^2}{2b^2}\right] \tag{6.18}$$

We have chosen the post-selected state to be the same as the pre-selected state, except for the fact that the post-selected wave is traveling in the opposite direction. These choices of pre and post selections are chosen for convenience without any loss of generality. In order to compute the weak value, one requires an exact time dependent solution to the system (6.14), which for an incident Gaussian wave packet is intractable. However, one can derive the reflection(ρ) and transmission(τ) coefficients for the time-independent Schroedinger equation in the presence of a delta function barrier [115, 116, 117, 118]. Therefore, without introducing much error, it can be safely assumed that every time a narrow Gaussian wave packet is incident upon a delta barrier, the reflected component (assumed to be a Gaussian) has its amplitude reduced by a factor ρ. The rest of the wave packet is transmitted with an amplitude τ times the amplitude of the incident wave packet.

To include the effects of an infinite wall to the existing delta barrier is non-trivial. For our purpose, it would suffice to assume that the wall is at a large distance from the barrier, i.e. $\frac{L}{b} \gg 1$ and acts as a mirror. Also $b^2 \gg \frac{\hbar t}{m}$, and the typical time scale will be given by $t \sim L/v$ from dimensional grounds. However, since $mv = \hbar k_0$, we require that

$$bk_0 \gg \frac{L}{b} \gg 1 \tag{6.19}$$

With these approximations, we construct the forward time evolution $\Phi_{pre}(x,t)$ of the pre-selected wave packet (6.17), in the region $-2L < x < 0$ as

$$\rho^N \exp\left[ik_0\left(x + (4N+1)L - \frac{vt}{2}\right) - \frac{(x + (4N+1)L - vt)^2}{2b^2}\right] -$$

$$\rho^N \exp\left[-ik_0\left(x - (4N-3)L + \frac{vt}{2}\right) - \frac{(x - (4N-3)L + vt)^2}{2b^2}\right]$$

and in the region $0 < x < \infty$, as

$$\sum_{N=1}^{N} \tau\rho^{N-1} \exp\left[ik_0\left(x + (4N-3)L - \frac{vt}{2}\right) - \frac{(x + (4N-3)L - vt)^2}{2b^2}\right] \quad (6.20)$$

Here N is the number of interactions with the delta barrier. As required the solution satisfies the boundary condition at the wall $\Phi(-2L, t) = 0$. It can be seen that with increasing time, the amplitude of the oscillating wave packet decreases by a factor $\rho = \frac{-i\kappa}{k_0 + i\kappa}$. Also, N such interactions with the barrier creates a train of N transmitted wave packets in the region $x > 0$ as intuitively expected. Following similar reasoning, the backward time evolution $\Phi_{post}(x,t)$ of the post-selected wave packet, in the region $-2L < x < 0$, can be expressed as

$$\rho^S \exp\left[-ik_0\left(x + (4S+1)L + \frac{vt'}{2}\right) - \frac{(x + (4S+1)L + vt')^2}{2b^2}\right]$$

$$-\rho^S \exp\left[ik_0\left(x - (4S-3)L - \frac{vt'}{2}\right) - \frac{(x - (4S-3)L - vt')^2}{2b^2}\right]$$

and in the region $0 < x < \infty$, it is given by

$$\sum_{S=1}^{S} \tau\rho^{S-1} \exp\left[-ik_0\left(x + (4S-3)L + \frac{vt'}{2}\right) - \frac{(x + (4S-3)L + vt')^2}{2b^2}\right] \quad (6.21)$$

where $t' = t - T$, and S is the number of interactions with the delta barrier. These forward and backward evolving wavepackets overlap at certain specific points in the spacetime diagram (see figure), where the weak values are non-zero. A wise choice of post-selection time vastly simplifies calculations without compromising important qualitative features.
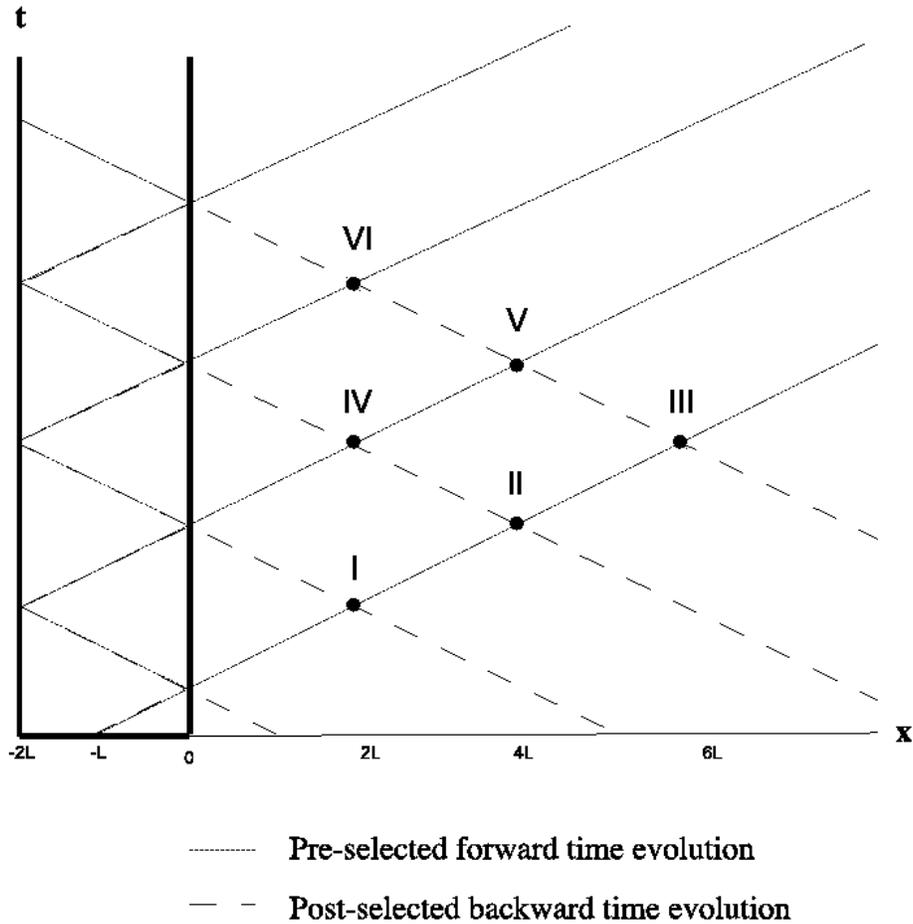
Figure 6.7: Spacetime diagram showing all "sweet spots" outside the well, for the post-selected time $T = 14L/v$. For this choice of T, the spacetime diagram is extremely symmetric. The wall is at $x = -2L$ and the delta barrier at $x = 0$

These "sweet spots" can be interpreted as follows. Consider the point in the figure represented by I. This is the intersection point of the forward and backward evolving packets, where the forward wave packet has interacted once with the barrier and the backward evolving wave packet has had three interactions. Therefore this point in spacetime corresponds to ($N = 1, S = 3$). Now consider the points III and IV. These points are on the same time slice (at $t = 7L/v$ in the figure). The point III is where both the forward and backward evolving wave packets have had just one interaction with the barrier. Therefore, this point

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corresponds to ($N = S = 1$). The point IV corresponds to ($N = S = 2$) since this intersection point is reached after the forward and backward evolving wave packets have had two interactions each with the barrier.

We will now proceed to evaluate weak values at these sweet spots for particular choices of the post-selection time T. The simplest non-trivial choice would be a post-selection at time $T = 6L/v$. In this case, there are two points of overlap between the forward and backward evolving wavepackets, viz. at $x = +2L$ and $x = -2L$ at time $t = 3L/v$. Using (6.20), (6.21), and the formula (6.8), the weak values at the time slice $t = 3L/v$ are evaluated to be

$$w(x, T = 6L/v)_{outside} = \frac{\sqrt{\frac{1}{b^2}} e^{\frac{\left(b^2 k_0 - i(x - 2L)\right)^2}{b^2}} k_0^2}{\sqrt{\pi} \left(k_0^2 + \left(-1 + e^{b^2 k_0^2}\right) \kappa^2\right)}, \quad x > 0 \qquad (6.22)$$

for outside the well (weak value for the transmitted packet overlaps), and

$$w(x, T = 6L/v)_{inside} = -\frac{\sqrt{\frac{1}{b^2}} e^{\frac{\left(b^2 k_0 - i(2L + x)\right)^2}{b^2}} \left(-1 + e^{2ik_0(2L + x)}\right)^2 \kappa^2}{\sqrt{\pi} \left(k_0^2 + \left(-1 + e^{b^2 k_0^2}\right) \kappa^2\right)} \qquad (6.23)$$

for inside the well, i.e. $-2L < x < 0$ (weak value for the reflected packet overlaps) Plots of the real and imaginary parts of these expressions are consistent with the expectation that the weak value oscillates rapidly outside the well, and therefore, the mean of the weak value should average out to zero. This is so because upon a strong projective measurement at time $T = 6L/v$, the particle is still found inside the well. In fact, as a check, it can be mathematically verified that $\int_{-2L}^{\infty} w(x)_{inside} dx + \int_0^{\infty} w(x)_{outside} dx = 1$. This is a generic feature of weak measurements.

Weak values can also be measured for other choices of post-selection time T. It turns out that if the post-selection time is chosen to be of the form $T = (4i + 2)L/v$

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where $i = 1, 2, 3, 4...$, the resulting spacetime diagram is symmetric, and the calculations simplify considerably. The spacetime diagram shown before corresponds to the choice $i = 3$. For completeness, we tabulate the weak values evaluated at time slices corresponding to all the sweet spots outside the well, for this choice of post-selected time:

$$w(x)_I = \frac{\sqrt{\frac{1}{b^2}} e^{\frac{\left(b^2 k_0 - 6iL + ix\right)\left(b^2 k_0 + i(2L + x)\right)}{b^2}} k_0^2}{\sqrt{\pi}\left(k_0^2 + \left(-1 + e^{b^2 k_0^2 + \frac{16L^2}{b^2}}\right)\kappa^2\right)} \tag{6.24}$$

$$w(x, T = 14L/v)_{II} = \frac{\sqrt{\frac{1}{b^2}} e^{b^2 k_0^2 - 2ik_0(2L - x) - \frac{(-4L + x)^2}{b^2}} k_0^2}{\sqrt{\pi}\left(e^{b^2 k_0^2}\kappa^2 + e^{4ik_0 L}(k_0 - \kappa)(k_0 + \kappa)\right)} \tag{6.25}$$

$$w(x, T = 14L/v)_{III} = \frac{\sqrt{\frac{1}{b^2}} e^{\frac{\left(b^2 k_0 - (4+6i)L + ix\right)\left(b^2 k_0 + (4-6i)L + ix\right)}{b^2}} k_0^2(k_0 - i\kappa)^2}{\sqrt{\pi}\left(k_0^4 - 2ik_0^3\kappa - 2k_0^2\kappa^2 - \left(-1 + e^{b^2 k_0^2}\right)\kappa^4\right)} \tag{6.26}$$

$$w(x, T = 14L/v)_{IV} = \frac{\sqrt{\frac{1}{b^2}} e^{\frac{\left(b^2 k_0 - (4+2i)L + ix\right)\left(b^2 k_0 + (4-2i)L + ix\right)}{b^2}} k_0^2\kappa^2}{\sqrt{\pi}\left(-k_0^4 + 2ik_0^3\kappa + 2k_0^2\kappa^2 + \left(-1 + e^{b^2 k_0^2}\right)\kappa^4\right)} \tag{6.27}$$

$$w(x, T = 14L/v)_V = \frac{\sqrt{\frac{1}{b^2}} e^{\frac{b^4 k_0^2 - 48L^2 - 2ib^2 k_0(2L - x) + 8Lx - x^2}{b^2}} k_0^2}{\sqrt{\pi}\left(e^{b^2 k_0^2}\kappa^2 + e^{4ik_0 L}(k_0 - \kappa)(k_0 + \kappa)\right)} \tag{6.28}$$

$$w(x, T = 14L/v)_{VI} = \frac{\sqrt{\frac{1}{b^2}} e^{\frac{b^4 k_0^2 - 52L^2 - 2ib^2 k_0(2L - x) + 4Lx - x^2}{b^2}} k_0^2}{\sqrt{\pi}\left(k_0^2 + \left(-1 + e^{b^2 k_0^2 + \frac{16L^2}{b^2}}\right)\kappa^2\right)} \tag{6.29}$$

6.4   Conclusion

In the case of an excited atom coupled to a bath of atoms, we observe that the exponential pre-factor dominates, and w_n rises exponentially on time scales greater than the decay time $1/\gamma$. That is, the weak values of the projection operators onto excited states of the bath atoms grow exponentially, and the later the initially excited atom 0 is post-selected to have remained in the excited state, the larger the values w_n become. Of course, for times $t_f \gg 1/\gamma$, the probability of finding the atom 0 un-decayed falls exponentially, so the sub-ensemble becomes exponentially small, but for a sufficiently large ensemble there will always be systems that satisfy the final condition. It may easily be checked that for $t = t_i$ and $t = t_f$ then $w_n = 0$, as required. For the case that $n = 0$, (6.13) is real, and the exponential growth is manifest:

$$w_0 = \frac{H^2}{\gamma^2} \left(1 + \exp\left[\gamma(t_f - t_i)\right] - \exp\left[\gamma(t_f - t)\right] - \exp\left[\gamma(t - t_i)\right]\right) \qquad (6.30)$$

Although w_n can grow exponentially large, the phase factors in (6.13) imply that the sign of the real part can be both positive and negative. Indeed, one may explicitly sum (6.13) over all n (in the limit $N \to \infty, \Delta E \to 0$), to find $\sum w_n = 0$.

In the second example of a trapped particle in a well, even though the solutions were an approximation, certain salient features can be deduced. we observe that the weak values oscillate rapidly outside the well. Of interest is the ratio between the weak values (6.22) and (6.24) at the sweet spot I, i.e at $x = 2L$. This ratio essentially indicates the effect of the post-selection time (T) on the real part of the weak value. Noting that the condition (6.19) holds, this ratio is given by

$$\frac{w(2L, T = 14L/v)}{w(2L, T = 6L/v)} \sim \exp[16L^2/b^2] \gg 1 \qquad (6.31)$$

Therefore, we see that the weak value is enhanced when the post-selection happens at a later time. Though, unlike in the case of an excited atom, the weak

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value in this case does not grow exponentially with the post selected time. In principle, weak measurements are amenable to experiments, and therefore it would be interesting to come up with experimental models to test the examples discussed in this chapter.
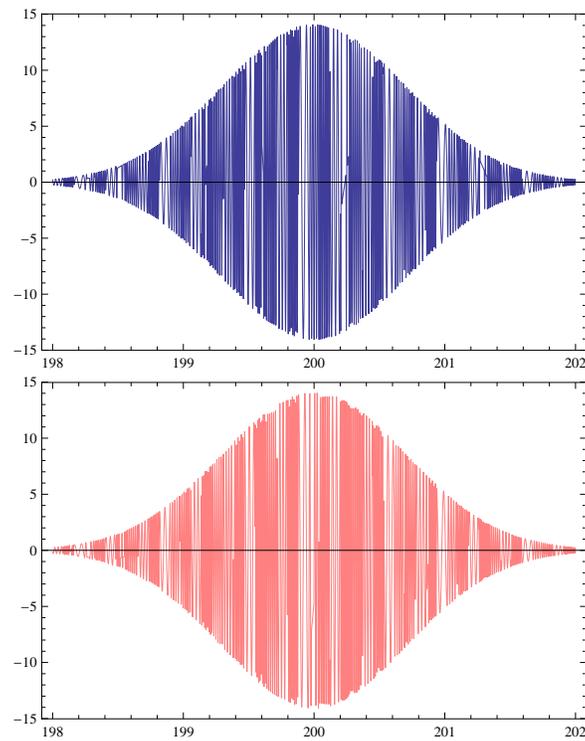
Figure 6.8: Shown are the real (blue) and imaginary (pink) components of the weak value outside the well for post-selection at $T = 6L/v$. Constants are set as $b = 1$; $m = 1,000$; $\kappa = 1,000$; $k_0 = 5,000$; and $L = 100$.
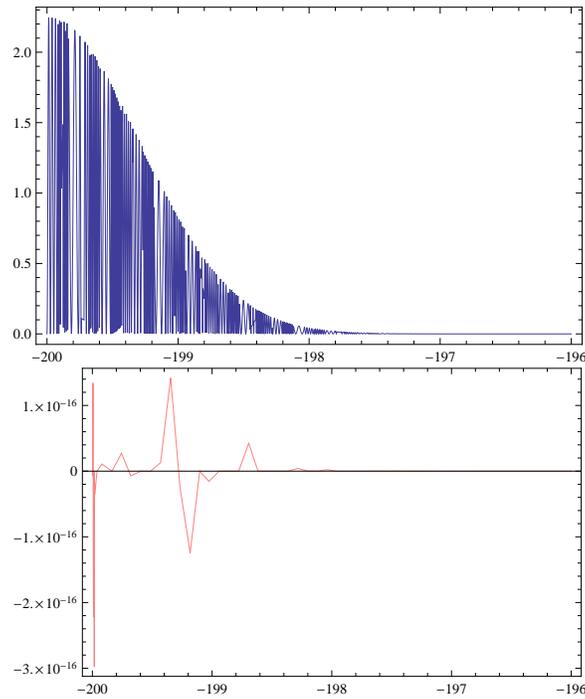
Figure 6.9: Shown are the real (blue) and imaginary (pink) components of the weak value inside the well for post-selection at $T = 6L/v$. Constants are set as $b = 1$; $m = 1,000$; $\kappa = 1,000$; $k_0 = 5,000$; and $L = 100$.

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