Projection Properties and Analysis Methods for Six to Fourteen Factor No Confounding Designs in 16 Runs

by

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ABSTRACT

During the initial stages of experimentation, there are usually a large number of factors to be investigated. Fractional factorial (2^{k-p}) designs are particularly useful during this initial phase of experimental work. These experiments often referred to as screening experiments help reduce the large number of factors to a smaller set. The 16 run regular fractional factorial designs for six, seven and eight factors are in common usage. These designs allow clear estimation of all main effects when the three-factor and higher order interactions are negligible, but all two-factor interactions are aliased with each other making estimation of these effects problematic without additional runs.

Alternatively, certain nonregular designs called no-confounding (NC) designs by Jones and Montgomery (Jones & Montgomery, Alternatives to resolution IV screening designs in 16 runs, 2010) partially confound the main effects with the two-factor interactions but do not completely confound any two-factor interactions with each other. The NC designs are useful for independently estimating main effects and two-factor interactions without additional runs. While several methods have been suggested for the analysis of data from nonregular designs, stepwise regression is familiar to practitioners, available in commercial software, and is widely used in practice. Given that an NC design has been run, the performance of stepwise regression for model selection is unknown. In this dissertation I present a comprehensive simulation study evaluating stepwise regression for analyzing both regular fractional factorial and NC designs.

Next, the projection properties of the six, seven and eight factor NC designs are studied. Studying the projection properties of these designs allows

the development of analysis methods to analyze these designs. Lastly the designs and projection properties of 9 to 14 factor NC designs onto three and four factors are presented. Certain recommendations are made on analysis methods for these designs as well.

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Chapter 1

INTRODUCTION

During the initial stages of experimentation, there are usually a large number of factors to be investigated. Two-level fractional factorial designs are particularly useful during this phase of experimental work. These experiments, called screening experiments, allow practitioners to reduce the large number of factors to a smaller set that can be studied more extensively. Regular fractional factorial designs are widely used for factor screening. Plackett-Burman designs are another class of screening design in common usage. The main difference between these two classes of designs is the aliasing structure. Effects in regular fractional factorial designs are either completely confounded or unaliased whereas the Plackett-Burman designs have a more complex partial aliasing pattern. A third set of designs recently proposed by Jones and Montgomery (2010) are the no confounding (NC) designs which like the Plackett-Burman designs do not completely confound any of the main effects and two-factor interactions. Plackett-Burman designs and the NC designs of Jones and Montgomery are examples of nonregular designs. Because the nonregular designs do not completely confound two-factor interactions and main effects, it may be possible to use these designs to identify active factors that could not be identified without additional follow-up experimentation when using regular designs.

Stepwise regression is a popular method for model selection because it is easy to use and widely available in standard software. Though it is widely used, there is no comprehensive study available documenting the effectiveness of using stepwise regression to analyze nonregular designs. Chapter one details the simulation study done to study the effectiveness of stepwise regression to analyze regular fractional factorial and NC designs.

The projection properties of fractional factorial designs and Plackett-Burman designs are well documented. Montgomery (2013) discusses the projection properties of the 2^{k-p} designs that collapse into either full factorial or a fractional factorial in any subset of $r \leq k - p$ of the original factors. The subsets that result in fractional factorials are subsets appearing as words in the complete defining relation. Lin and Draper (1992) and Box and Bisgaard (1993) showed that some of the Placket-Burman designs in fewer runs when projected onto three factors result in a complete 2^3 design and a half replicate of the 2^3 design. The projection properties of NC designs have not been studied. In chapter 2 I present the projection properties of NC designs for the six, seven and eight factor cases in 16 runs.

Johnson and Jones (2011) show that the six, seven and eight factor NC designs have a classical-type construction with a 2^4 or a replicated 2^3 starting point. These generating columns can be used to study the projection properties of the NC designs. Studying the projection properties of the NC designs can suggest possible analytical methods for these designs. Suggestions for analysis methods for these designs are also discussed in Chapter 2.

In chapter three the 9 – 14 factor NC designs are listed. A metric to evaluate these NC designs is presented, and it is used to obtain the choices for the nonregular 16-run fractional factorials through the use of a variation of the D-optimality criterion. I then present the projection properties of these designs when projected to three and four factors and discuss an analysis strategy for these designs. I also present an example that illustrates the potential usefulness of these designs and the effectiveness of the analysis method.

Chapter 2

ANALYSIS OF FRACTIONAL FACTORIAL DESIGNS USING STEPWISE REGRESSION

2.1. Background

Stepwise regression is a popular method for model selection because it is easy to use and widely available in standard software. Though it is widely used, there is no comprehensive study available documenting the effectiveness of using stepwise regression to analyze nonregular designs.

2.2. Literature Survey

A brief review of methods for analyzing nonregular designs is presented in this section. Hamada and Wu (1992) proposed a two-step method to analyze Plackett-Burman designs considering both the main effects and interactions. This paper sparked interest in analysis methods for nonregular fractional factorial designs. Box and Meyer (1993) suggested a Bayesian approach to identifying the active factors in screening experiments with complex aliasing. Chipman, Hamada and Wu (1997) proposed another Bayesian approach combining the Stochastic Search Variable Selection algorithm of George and McCulloch (1993) with priors for related predictors given by Chipman (1996). Hamada and Hamada (2010) proposed an all subsets regression method while imposing model heredity restrictions to dramatically reduce the number of models to consider.

Tyssedal and Samset (1997) suggested using contrast plots to use the aliasing structure of the nonregular designs to identify the significant effects. Samset and Tyssedal (1998) suggested certain modifications to the Bayesian approach introduced by Box and Meyer (1993) to overcome some of the limitations they observed while using the method. Samset (1999) discussed two variable selection methods to identify the active factors for nonregular designs. The first method is a best subsets regression procedure based on the effect heredity principle and the second one is based on the stepwise regression procedure. Lawson (2002) proposed a subsets regression method on a shortlisted set of candidates to identify the most significant main and interaction effects. The shortlist of candidates of candidate interactions is identified using an alias plot. Yuan et al (2007) propose extensions to a general purpose variable selection algorithm, Least Angle Regression (LARS), Li and Lin (2009) used penalized least squares with the SCAD penalty to identify the active factors in screening experiments.

Due to the accessibility and simplicity of use of stepwise regression, it is a popular method for model selection in the analysis of fractional factorial designs. Marley and Woods (2010) evaluated $E(s^2)$- Optimal and Bayesian D-optimal designs to compare three analysis strategies representing regression, shrinkage and a novel model-averaging procedure using simulated experiments. In this paper I evaluate the effectiveness of stepwise regression for model selection. The performance of stepwise regression is evaluated on the 16-run regular fractional factorial designs and the 16-run NC designs proposed by Jones and Montgomery (2010) for the six, seven and eight factor cases.

2.3. Preliminary Study and Results

Stepwise regression is the most commonly used analysis method to analyze the results from fractional factorial designs. There is no complete study available in the literature which studies how well stepwise regression actually works. JMP was used to run the simulations. The data was simulated assuming different true models to see how well stepwise regression performs when used to analyze the results from the three designs being studied. Since the experimenter will never know the form of the true model, simulations were run to test different true models. The true models tested are listed in Table 2-1. Three different coefficient / noise ratios were also tested. The next parameter that was varied was the number of true active terms. Depending on the true model and the number of variables in the model, this was varied over the entire possible range. The different settings of the simulation parameters are listed in Table 2-1.

Table 2-1 Preliminary Simulation Study

<table>
<tr><td colspan="2">Designs Used</td><td colspan="3">Fractional Factorial</td><td colspan="2">Plackett Burman</td><td colspan="3">No Confounding</td></tr>
<tr><td colspan="2">True Model</td><td colspan="3">Main Effects Only</td><td colspan="2">Main Effects + 1 hierarchical interaction</td><td colspan="3">Main Effects + 2 hierarchical interaction</td></tr>
<tr><td colspan="2">No. of variables</td><td colspan="3">6</td><td colspan="2">7</td><td colspan="3">8</td></tr>
<tr><td colspan="2">Coefficient / Noise Ratio</td><td colspan="3">2/0.667 = 3</td><td colspan="2">2/1 = 2</td><td colspan="3">2/2 = 1</td></tr>
<tr><td rowspan="3">No of Active Terms</td><td>Main Effects Only Model</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>Main Effects + 1 hierarchical Interaction Model</td><td>-</td><td>2+1</td><td>3+1</td><td>4+1</td><td>5+1</td><td>6+1</td><td>7+1</td><td>8+1</td></tr>
<tr><td>Main Effects + 2 hierarchical Interaction Model</td><td>-</td><td>-</td><td>3+2</td><td>4+2</td><td>5+2</td><td>6+2</td><td>7+2</td><td>8+2</td></tr>
</table>

Stepwise regression can be classified into three broad categories: (1) forward selection, (2) backward elimination and (3) stepwise (mixed) regression. Stepwise regression is a combination of the first two methods. Since the models being analyzed have more variables than the number of rows of data, backward elimination is not a feasible option. I ran the simulations using both stepwise regression and forward selection. Since the results were similar using both methods the simulations were continues using just the stepwise regression approach.

Since stepwise regression (forward and mixed) entail adding or adding and removing variables, there is a need for rules to add and remove the variables. For stepwise (mixed) regression two cutoff values are required. ▪in and ▪out needs to be selected for the entering and leaving variables. For these simulations $\alpha_{in} = 0.05$ & $\alpha_{out} = 0.10$ was chosen. Choosing an $\alpha_{in} < \alpha_{out}$ ensures that it will make it relatively more difficult to add a regressor than to delete one.

To maintain the hierarchy in the model, certain rules need to be followed. In JMP there are two different options to maintain hierarchy (1) Combine and (2) Restrict. The combine option groups a term with its precedent terms and calculates the group's significance probability for entry as a joint F-test. The restrict option restricts the terms that have precedents so that they cannot be entered until their precedents are entered. For the current simulation study Stepwise regression with the Combine option was used.

Initial experimentation showed that the results stabilized after 2000 runs therefore I ran 2000 runs for each combination of the simulation parameters. Figure 2.1 shows how the results stabilized after 2000 runs.

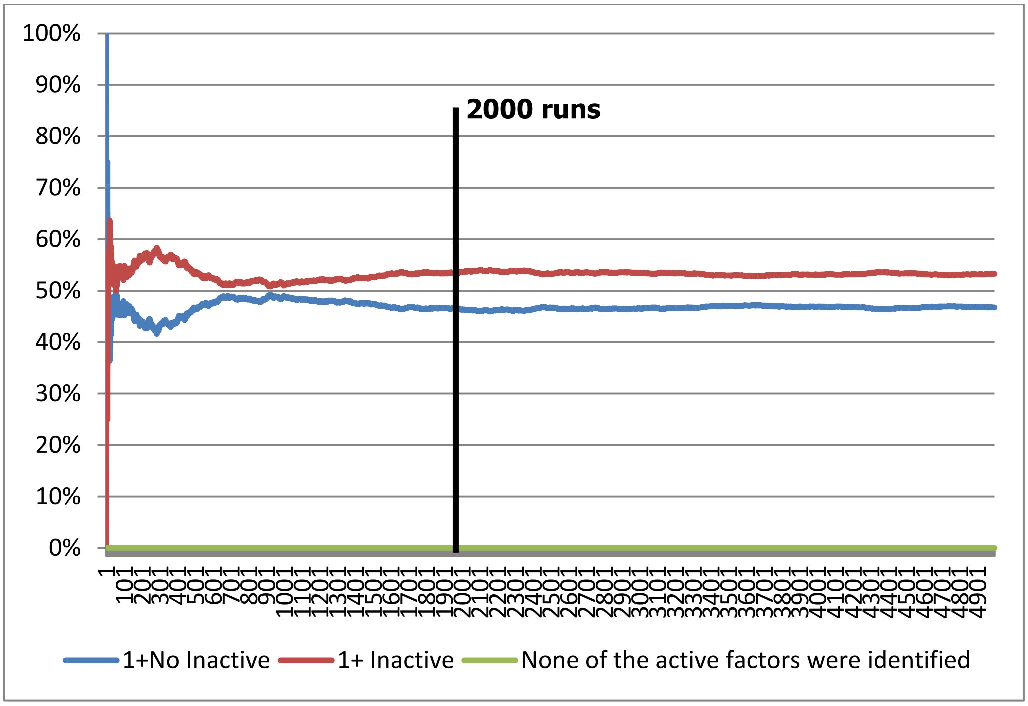


Figure 2.1 Simulation – Steady State

The results from each simulation run (one combination of factors) was evaluated by categorizing the runs into one of the following four categories.

1. Only Active terms identified as active
2. All Active terms identified + some inactive terms identified as active (Type I Error)
3. Missed some Active Terms (Type II Error)
4. Missed all Active terms (Type II Error)

When screening experiments are run, the experimenter is more tolerant to Type I error versus Type II error. You definitely do not want to miss the true active terms but false positive results can be eliminated in subsequent experiments. Therefore any analysis method utilized must have the ability to minimize the number of total errors particularly the Type II errors.

In the case of the Main Effects only models, the results for the three designs can be directly compared as there is no aliasing between the Main effects and any other terms. But in the cases where there are interactions in the true model, the results need to be adjusted for the Fractional Factorial designs. Since the aliasing between the two factor interactions in these designs is complete, the results need to incorporate the aliasing. Therefore whenever the analysis identifies a two factor interaction in the Fractional Factorial case, there is always Type I error due to the aliasing pattern. This is not the case when Plackett-Burman and No Confounding designs are used.

2.3.1. Six factor Designs

The results indicate that for the Main Effects Only model, the three designs behave very similarly. The results from using stepwise regression to analyze the data show that there is no difference when the coefficient/noise ratios are three or two. When the coefficient/noise ratio is 1, stepwise regression fails for all three designs and generates large Type I and Type II errors even when the number of active terms is just one. All three designs make no type II error when the number of active terms is one or two. Fractional Factorial designs make no Type II error even for the three and four active term cases. All three designs start making large type II errors (> 80%) when the number of active terms is five or more. One interesting observation about the No Confounding design is that it never misses all the active terms even when the number of active terms is six whereas the analysis of the Fractional Factorial designs totally breaks down.

For the Main Effects + 1 hierarchical interaction case, the results are very similar when the coefficient/noise ratio is three and two. When the coefficient/noise ratio is one, stepwise regression fails to give reasonable results. Once the true model includes hierarchical interactions, the fractional factorial starts generating Type I errors even when the number of active terms is three (two main effects + 1 hierarchical interaction) due to the complete aliasing in these designs. The NC and PB designs perform better as there is no complete confounding in these designs. All three designs have type II error in more than 50% of the cases when the number of active terms is two or more.

The Main Effects + 2 hierarchical interactions case the results are very similar when the PB and NC designs are used at the three and two coefficient/noise ratios. The FF designs totally break down (misses all Active terms) when the number of active terms is six (4 main effects + 2 hierarchical interactions). The NC designs are able to avoid missing all active terms even when there are 5 Main Effects + 2 hierarchical interactions in the true model. These results are shown in Table 2-2, Table 2-3 and Table 2-4.

Table 2-2 Results Summary – Six factor Main Effects Only Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	1AF	47.3	52.7	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	93.1	0	6.9	93.1	6.9
		5AF	0	14	46.1	39.9	14	86
		6AF	0	0	82.3	17.7	0	100
	FF	1AF	48.5	51.5	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	100	0	0	100	0
		5AF	0	5	0	95	5	95
		6AF	0	0	0	100	0	100
	NC	1AF	50.9	49.1	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	92.3	7.7	0	92.3	7.7
		4AF	0	71.3	28.7	0	71.3	28.7
		5AF	0	0	100	0	0	100
		6AF	0	0	100	0	0	100
2SD	PB	1AF	46.3	53.7	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	99.9	0.1	0	99.9	0.1
		4AF	0	86.8	3.2	10	86.8	13.2
		5AF	0	20.1	45.9	34	20.1	79.9
		6AF	0	0	62.1	37.9	0	100
	FF	1AF	46.8	53.2	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	100	0	0	100	0
		5AF	0	59.8	0.3	39.9	59.8	40.2
		6AF	0	3	6.8	90.2	3	97
	NC	1AF	51.9	48.1	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	94.6	5.4	0	94.6	5.4
		4AF	0	71	29	0	71	29
		5AF	0	1.5	98.5	0	1.5	98.5
		6AF	0	0	100	0	0	100
1SD	PB	1AF	12.4	37.6	0	50	50	50
		2AF	1.4	30.4	38.2	30	31.8	68.2
		3AF	0	19	54.2	26.8	19	81
		4AF	0	10.3	68.6	21.1	10.3	89.7
		5AF	0	4.4	72.7	22.9	4.4	95.6
		6AF	0	2.8	67.7	29.5	2.8	97.2
	FF	1AF	20.7	41.3	0	38	62	38
		2AF	2.9	40.1	34.5	22.5	43	57
		3AF	0	27.2	54.4	18.4	27.2	72.8
		4AF	0	20.9	63.3	15.8	20.9	79.1
		5AF	0	8.3	70.2	21.5	8.3	91.7
		6AF	0	3.8	69.1	27.1	3.8	96.2
	NC	1AF	20	38.2	0	41.8	58.2	41.8
		2AF	1.9	33.8	44.9	19.4	35.7	64.3
		3AF	0	21.8	64.2	14	21.8	78.2
		4AF	0	12.6	76.2	11.2	12.6	87.4
		5AF	0	5.1	86.3	8.6	5.1	94.9
		6AF	0	2.1	89.7	8.2	2.1	97.9

Table 2-3 Results Summary – Six Factor Main Effects + 1 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	3AF	50	50	0	0	100	0
		4AF	15.65	22.9	61.4	0.05	38.55	61.45
		5AF	0.5	27.6	65.45	6.45	28.1	71.9
		6AF	0	3.45	50.25	47.7	3.45	97.95
		7AF	0	0	50.2	48.4	0	98.6
	FF	3AF	0	100	0	0	100	0
		4AF	0	40.8	59.2	0	40.8	59.2
		5AF	0	41.2	58.8	0	41.2	58.8
		6AF	0	27.5	4.5	95.3	27.5	99.8
		7AF	0	0.2	40.5	32	0.2	72.5
	NC	3AF	54.05	45.95	0	0	100	0
		4AF	17.25	19.3	63.45	0	36.55	63.45
		5AF	0	33.35	66.65	0	33.35	66.65
		6AF	0	3.45	89.5	8.6	3.45	98.1
		7AF	0	0	89.85	8.6	0	98.45
2SD	PB	3AF	49.25	50.7	0.05	0	99.95	0.05
		4AF	15.55	24.15	59.5	0.8	39.7	60.3
		5AF	0.5	25.85	63.85	9.8	26.35	73.65
		6AF	0	5.45	70.05	24.5	5.45	94.55
		7AF	0	0.3	51.05	48.65	0.3	99.7
	FF	3AF	0	100	0	0	100	0
		4AF	0	42.5	57.5	0	42.5	57.5
		5AF	0	40	59.9	0.1	40	60
		6AF	0	29.1	51.1	19.8	29.1	70.9
		7AF	0	9.1	23.3	67.6	9.1	90.9
	NC	3AF	54.35	45.6	0.05	0	99.95	0.05
		4AF	18	19.8	62.2	0	37.8	62.2
		5AF	0	31.55	68.45	0	31.55	68.45
		6AF	0	6.55	92.9	0.55	6.55	93.45
		7AF	0	0	94.7	5.3	0	100
1SD	PB	3AF	9.55	10.75	52.05	27.65	20.3	79.7
		4AF	1.15	6.35	70.65	21.85	7.5	92.5
		5AF	0.05	3.4	73.3	23.25	3.45	96.55
		6AF	0	1.6	74.15	24.25	1.6	98.4
		7AF	0	0.55	70.95	28.5	0.55	99.45
	FF	3AF	0	28.9	48.5	22.6	28.9	71.1
		4AF	0	11.9	70.4	17.7	11.9	88.1
		5AF	0	6.9	74.7	18.4	6.9	93.1
		6AF	0	3.6	74.5	21.9	3.6	96.4
		7AF	0	2.3	69.8	27.9	2.3	97.7
	NC	3AF	11.9	11.25	54.45	22.4	23.15	76.85
		4AF	1.7	6.8	77.95	13.55	8.5	91.5
		5AF	0.15	4.65	86.75	8.45	4.8	95.2
		6AF	0	1.45	90.15	8.4	1.45	98.55
		7AF	0	0.85	90.55	8.6	0.85	99.15

Table 2-4 Results Summary – Six Factor Main Effects + 2 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	3AF	50	50	0	0	100	0
		4AF	15.65	22.9	61.4	0.05	38.55	61.45
		5AF	0.5	27.6	65.45	6.45	28.1	71.9
		6AF	0	3.45	50.25	47.7	3.45	97.95
		7AF	0	0	50.2	48.4	0	98.6
	FF	3AF	0	100	0	0	100	0
		4AF	0	40.8	59.2	0	40.8	59.2
		5AF	0	41.2	58.8	0	41.2	58.8
		6AF	0	27.5	4.5	95.3	27.5	99.8
		7AF	0	0.2	40.5	32	0.2	72.5
	NC	3AF	54.05	45.95	0	0	100	0
		4AF	17.25	19.3	63.45	0	36.55	63.45
		5AF	0	33.35	66.65	0	33.35	66.65
		6AF	0	3.45	89.5	8.6	3.45	98.1
		7AF	0	0	89.85	8.6	0	98.45
2SD	PB	3AF	49.25	50.7	0.05	0	99.95	0.05
		4AF	15.55	24.15	59.5	0.8	39.7	60.3
		5AF	0.5	25.85	63.85	9.8	26.35	73.65
		6AF	0	5.45	70.05	24.5	5.45	94.55
		7AF	0	0.3	51.05	48.65	0.3	99.7
	FF	3AF	0	100	0	0	100	0
		4AF	0	42.5	57.5	0	42.5	57.5
		5AF	0	40	59.9	0.1	40	60
		6AF	0	29.1	51.1	19.8	29.1	70.9
		7AF	0	9.1	23.3	67.6	9.1	90.9
	NC	3AF	54.35	45.6	0.05	0	99.95	0.05
		4AF	18	19.8	62.2	0	37.8	62.2
		5AF	0	31.55	68.45	0	31.55	68.45
		6AF	0	6.55	92.9	0.55	6.55	93.45
		7AF	0	0	94.7	5.3	0	100
1SD	PB	3AF	9.55	10.75	52.05	27.65	20.3	79.7
		4AF	1.15	6.35	70.65	21.85	7.5	92.5
		5AF	0.05	3.4	73.3	23.25	3.45	96.55
		6AF	0	1.6	74.15	24.25	1.6	98.4
		7AF	0	0.55	70.95	28.5	0.55	99.45
	FF	3AF	0	28.9	48.5	22.6	28.9	71.1
		4AF	0	11.9	70.4	17.7	11.9	88.1
		5AF	0	6.9	74.7	18.4	6.9	93.1
		6AF	0	3.6	74.5	21.9	3.6	96.4
		7AF	0	2.3	69.8	27.9	2.3	97.7
	NC	3AF	11.9	11.25	54.45	22.4	23.15	76.85
		4AF	1.7	6.8	77.95	13.55	8.5	91.5
		5AF	0.15	4.65	86.75	8.45	4.8	95.2
		6AF	0	1.45	90.15	8.4	1.45	98.55
		7AF	0	0.85	90.55	8.6	0.85	99.15

2.3.2. Seven factor Designs

When the number of terms in the design is seven, for the main effects only model, the NC designs performs better than the FF and PB designs. It is able to detect all active terms even when the number of active terms in the model is five or six. In fact the PB designs performs the worst as it starts deteriorating in performance and starts generating Type II errors even when the number of active terms is the model is four. All three designs perform similarly when the coefficient / noise ratio is 3 or 2. The only case where all three designs have some cases generate no type I or type II error is when the number of active terms is one.

As in the six factor designs, once hierarchical interaction terms are added to the true model, fractional factorial designs are only able to detect the alias chains for the interactions and therefore always generate atleast one type I error. All three designs start generating close to 60% type II errors when the total number of active terms is four (3 main effects + 1 hierarchical interaction). Again the performance is very similar when the coefficient/noise ratio is two or three.

For the main effects + two hierarchical interactions case, the trend seen in the one hierarchical interaction case continues. All three designs start generating large type II errors (>60%) when the number of active terms (main effects + 2 hierarchical interactions) is four or more. For the one active term case, the PB design performs worse than the NC and FF designs. These results are shown in Table 2-5, Table 2-6 and Table 2-7.

Table 2-5 Results Summary – Seven Factor Main Effects Only Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	1AF	35.2	64.8	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	90.1	4.85	5.05	90.1	9.9
		5AF	0	9.4	55.9	34.7	9.4	90.6
		6AF	0	0	85	15	0	100
		7AF	0	0	63.95	36.05	0	100
	FF	1AF	42.85	57.15	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	100	0	0	100	0
		5AF	0	5.55	0	94.45	5.55	94.45
		6AF	0	0	0	100	0	100
		7AF	0	0	56.6	43.4	0	100
	NC	1AF	40.5	59.5	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	99.9	0.1	0	99.9	0.1
		5AF	0	75.95	0.35	23.7	75.95	24.05
		6AF	0	81.45	2.05	16.5	81.45	18.55
		7AF	0	0	59.3	40.7	0	100
2SD	PB	1AF	35.8	64.2	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	99.9	0.1	0	99.9	0.1
		4AF	0	81.1	9.1	9.8	81.1	18.9
		5AF	0	13.9	59.3	26.8	13.9	86.1
		6AF	0	0.05	64.05	35.9	0.05	99.95
		7AF	0	0	62.65	37.35	0	100
	FF	1AF	41.7	58.3	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	100	0	0	100	0
		5AF	0	59.5	0.3	40.2	59.5	40.5
		6AF	0	2.6	7.35	90.05	2.6	97.4
		7AF	0	0	58.7	41.3	0	100
	NC	1AF	39.60	60.40	0	0	100	0
		2AF	0	100.00	0	0	100	0
		3AF	0	100.00	0	0	100	0
		4AF	0	99.45	0.55	0	99.45	0.55
		5AF	0	90.60	1.90	7.50	90.6	9.4
		6AF	0	68.65	9.40	21.95	68.65	31.35
		7AF	0	0.00	59.50	40.50	0	100
1SD	PB	1AF	11.1	45.3	0	43.6	56.4	43.6
		2AF	0.6	33	41.35	25.05	33.6	66.4
		3AF	0	21.05	59.85	19.1	21.05	78.95
		4AF	0	11	71.15	17.85	11	89
		5AF	0	4.3	75.65	20.05	4.3	95.7
		6AF	0	1.8	73.25	24.95	1.8	98.2
		7AF	0	0	62.75	37.25	0	100
	FF	1AF	15.05	47.4	0	37.55	62.45	37.55
		2AF	1.7	42.35	35.25	20.7	44.05	55.95
		3AF	0.3	28.75	54.9	16.05	29.05	70.95
		4AF	0	21.95	61.1	16.95	21.95	78.05
		5AF	0	10.4	69.45	20.15	10.4	89.6
		6AF	0	4.55	70.8	24.65	4.55	95.45
		7AF	0	0	55.35	44.65	0	100
	NC	1AF	15.85	47.65	0	36.50	63.5	36.5
		2AF	0.95	43.60	38.05	17.40	44.55	55.45
		3AF	0.05	27.10	60.55	12.30	27.15	72.85
		4AF	0	18.05	69.90	12.05	18.05	81.95
		5AF	0	10.70	76.95	12.35	10.7	89.3
		6AF	0	6.60	78.85	14.55	6.6	93.4
		7AF	0	0	58.70	41.30	0	100

Table 2-6 Results Summary – Seven factor Main Effects + 1 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	3AF	47.75	52.25	0	0	100	0
		4AF	15.7	20.5	63.8	0	36.2	63.8
		5AF	1.85	34.5	63.65	0	36.35	63.65
		6AF	0	13.55	34.9	51.55	13.55	86.45
		7AF	0	0	3.5	96.5	0	100
		8AF	0	0	0	100	0	100
	FF	3AF	0	100	0	0	100	0
		4AF	0	39.45	60.55	0	39.45	60.55
		5AF	0	39.3	60.7	0	39.3	60.7
		6AF	0	18.8	29.65	51.55	18.8	81.2
		7AF	0	0.05	3.45	96.5	0.05	99.95
		8AF	0	0	0	100	0	100
	NC	3AF	42.55	57.45	0	0	100	0
		4AF	14.1	24.4	61.5	0	38.5	61.5
		5AF	0	36.65	63.35	0	36.65	63.35
		6AF	0	21.65	66.4	11.95	21.65	78.35
		7AF	0	9.3	66.45	24.25	9.3	90.7
		8AF	0	2.25	78.1	19.65	2.25	97.75
2SD	PB	3AF	35.55	64.4	0.05	0	99.95	0.05
		4AF	12.75	26.3	60.8	0.15	39.05	60.95
		5AF	0.1	28.35	62.8	8.75	28.45	71.55
		6AF	0	3.3	74.15	22.55	3.3	96.7
		7AF	0	0.15	61.15	38.7	0.15	99.85
		8AF	0	0	35.85	64.15	0	100
	FF	3AF	0	100	0	0	100	0
		4AF	0	38.9	61.1	0	38.9	61.1
		5AF	0	38.45	61.55	0	38.45	61.55
		6AF	0	24.95	46.6	28.45	24.95	75.05
		7AF	0	5.35	23.2	71.45	5.35	94.65
		8AF	0	0.05	3.3	96.65	0.05	99.95
	NC	3AF	43.3	56.7	0	0	100	0
		4AF	13.3	25.7	61	0	39	61
		5AF	0.1	35.35	64.55	0	35.45	64.55
		6AF	0	23.2	70.35	6.45	23.2	76.8
		7AF	0	11	69.05	19.95	11	89
		8AF	0	8.05	71.05	20.9	8.05	91.95
1SD	PB	3AF	6	13.05	55.1	25.85	19.05	80.95
		4AF	0.4	7.3	71.7	20.6	7.7	92.3
		5AF	0.05	4.05	78.15	17.75	4.1	95.9
		6AF	0	1.6	79.8	18.6	1.6	98.4
		7AF	0	0.5	76.7	22.8	0.5	99.5
		8AF	0	0.05	70.65	29.3	0.05	99.95
	FF	3AF	0	28.5	51.8	19.7	28.5	71.5
		4AF	0	12.8	71.05	16.15	12.8	87.2
		5AF	0	7.7	75.3	17	7.7	92.3
		6AF	0	3.35	75.3	21.35	3.35	96.65
		7AF	0	1.7	71.35	26.95	1.7	98.3
		8AF	0	1.05	60.7	38.25	1.05	98.95
	NC	3AF	8.75	14.85	57.15	19.25	23.6	76.4
		4AF	1.6	9	76.95	12.45	10.6	89.4
		5AF	0.05	6.9	81.95	11.1	6.95	93.05
		6AF	0	3.5	84.35	12.15	3.5	96.5
		7AF	0	1.25	85.55	13.2	1.25	98.75
		8AF	0	1.5	82.6	15.9	1.5	98.5

Table 2-7 Results Summary - Seven factor Main Effects + 2 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	5AF	16.75	45.9	5.45	31.9	62.65	37.35
		6AF	0	1.45	88.5	10.05	1.45	98.55
		7AF	0	0	36.3	63.7	0	100
		8AF	0	0	3.85	96.15	0	100
		9AF	0	0	0	100	0	100
	FF	5AF	0	94.05	5.95	0	94.05	5.95
		6AF	0	4.2	14.4	81.4	4.2	95.8
		7AF	0	0	0	100	0	100
		8AF	0	0	0	100	0	100
		9AF	0	0	0	100	0	100
	NC	5AF	26.8	59.15	12.4	1.65	85.95	14.05
		6AF	0.2	14.2	47.75	37.85	14.4	85.6
		7AF	0	0.9	57.1	42	0.9	99.1
		8AF	0	0.15	28.4	71.45	0.15	99.85
		9AF	0	0	0	100	0	100
2SD	PB	5AF	16.75	44.25	13.5	25.5	61	39
		6AF	0.1	0.85	84.75	14.3	0.95	99.05
		7AF	0	0	38.95	61.05	0	100
		8AF	0	0	4.2	95.8	0	100
		9AF	0	0	0	100	0	100
	FF	5AF	0	94	5.95	0.05	94	6
		6AF	0	5.1	14.95	79.95	5.1	94.9
		7AF	0	0	0.4	99.6	0	100
		8AF	0	0	0	100	0	100
		9AF	0	0	0	100	0	100
	NC	5AF	27.35	58.65	11.65	2.35	86	14
		6AF	0.05	12.7	49.5	37.75	12.75	87.25
		7AF	0	0.7	58.05	41.25	0.7	99.3
		8AF	0	0.05	31.15	68.8	0.05	99.95
		9AF	0	0	0	100	0	100
1SD	PB	5AF	1.95	3.5	69.05	25.5	5.45	94.55
		6AF	0.05	0.95	70.8	28.2	1	99
		7AF	0	0.05	48.65	51.3	0.05	99.95
		8AF	0	0	14.4	85.6	0	100
		9AF	0	0	0.2	99.8	0	100
	FF	5AF	0	15.5	62.1	22.4	15.5	84.5
		6AF	0	2.95	52.5	44.55	2.95	97.05
		7AF	0	0.2	28.1	71.7	0.2	99.8
		8AF	0	0	3.2	96.8	0	100
		9AF	0	0	0	100	0	100
	NC	5AF	5.3	7.95	67.75	19	13.25	86.75
		6AF	0.1	2	70.95	26.95	2.1	97.9
		7AF	0	0.3	62.75	36.95	0.3	99.7
		8AF	0	0.05	37.7	62.25	0.05	99.95
		9AF	0	0	8.35	91.65	0	100

2.3.3. Eight factor Designs

When the number of factors in the design is eight the results look very similar to the seven factor case for all three model types.

To summarize there is no difference in the results when the coefficient/noise ratio is either three or two. The fractional factorial designs generate type I error once the true model contains hierarchical terms. The Plackett-Burman and no confounding designs outperform the regular fractional factorial designs once interactions are present in the true model. When the true models contains four or more active terms, the analysis method starts breaking down irrespective of which design is used. In cases with interactions when the design does not break down completely, the FF design starts generating type I error for all cases and the NC design outperforms the PB design by generating the fewest type II errors. These results are shown in Table 2-8, Table 2-9 and Table 2-10.

Table 2-8 Results Summary – Eight factor Main Effects Only Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	1AF	22.6	77.4	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	78.15	16.05	5.8	78.15	21.85
		5AF	0	6.85	60.3	32.85	6.85	93.15
		6AF	0	0	89.05	10.95	0	100
		7AF	0	0	39.9	60.1	0	100
		8AF	0	0	0	100	0	100
	FF	1AF	33.65	66.35	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	98.6	0.9	0.5	98.6	1.4
		4AF	0	96.2	3.8	0	96.2	3.8
		5AF	0	79.4	17.15	3.45	79.4	20.6
		6AF	0	46.2	53.55	0.25	46.2	53.8
		7AF	0	2.9	55.8	41.3	2.9	97.1
		8AF	0	0	100	0	0	100
	NC	1AF	32.8	67.2	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	100	0	0	100	0
		5AF	0	89.05	0.25	10.7	89.05	10.95
		6AF	0	75.05	24.85	0.1	75.05	24.95
		7AF	0	0.6	24.15	75.25	0.6	99.4
		8AF	0	0	0	100	0	100
2SD	PB	1AF	24.35	75.65	0	0	100	0
		2AF	0.05	99.95	0	0	100	0
		3AF	0	99.85	0.15	0	99.85	0.15
		4AF	0	71.85	19.4	8.75	71.85	28.15
		5AF	0	10.3	64.75	24.95	10.3	89.7
		6AF	0	0	75.65	24.35	0	100
		7AF	0	0	37.35	62.65	0	100
		8AF	0	0	1.7	98.3	0	100
	FF	1AF	33.55	66.45	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	100	0	0	100	0
		4AF	0	100	0	0	100	0
		5AF	0	58.6	0.25	41.15	58.6	41.4
		6AF	0	3.35	6.65	90	3.35	96.65
		7AF	0	0	0.9	99.1	0	100
		8AF	0	0	0.05	99.95	0	100
	NC	1AF	31.9	68.1	0	0	100	0
		2AF	0	100	0	0	100	0
		3AF	0	99.7	0.3	0	99.7	0.3
		4AF	0	98.8	1.2	0	98.8	1.2
		5AF	0	92.45	4.15	3.4	92.45	7.55
		6AF	0	70.6	26.25	3.15	70.6	29.4
		7AF	0	7.75	68.7	23.55	7.75	92.25
		8AF	0	0	19.45	80.55	0	100

Table 2.8 (contd.) Results Summary – Eight factor Main Effects Only Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
1SD	PB	1AF	6.7	54.45	0	38.85	61.15	38.85
		2AF	0.2	38	40.85	20.95	38.2	61.8
		3AF	0	21.65	63.4	14.95	21.65	78.35
		4AF	0	11.1	74.7	14.2	11.1	88.9
		5AF	0	4.75	80.15	15.1	4.75	95.25
		6AF	0	1.2	81.15	17.65	1.2	98.8
		7AF	0	0.15	73.75	26.1	0.15	99.85
		8AF	0	0	66.7	33.3	0	100
	FF	1AF	11.6	55.25	0	33.15	66.85	33.15
		2AF	1	48.35	34.25	16.4	49.35	50.65
		3AF	0.05	34.85	51.45	13.65	34.9	65.1
		4AF	0	20.2	63.55	16.25	20.2	79.8
		5AF	0	12.65	69.4	17.95	12.65	87.35
		6AF	0	5.9	69.5	24.6	5.9	94.1
		7AF	0	2.55	64.8	32.65	2.55	97.45
		8AF	0	0.55	56	43.45	0.55	99.45
	NC	1AF	11.95	53	0	35.05	64.95	35.05
		2AF	1.05	40.85	38.8	19.3	41.9	58.1
		3AF	0.05	28.8	61	10.15	28.85	71.15
		4AF	0	16.9	74.35	8.75	16.9	83.1
		5AF	0	10.3	82.1	7.6	10.3	89.7
		6AF	0	5.3	88.1	6.6	5.3	94.7
		7AF	0	2.75	90.05	7.2	2.75	97.25
		8AF	0	1.05	90.85	8.1	1.05	98.95

Table 2-9 Results Summary – Eight factor Main Effects + 1 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	3AF	12.75	62.10	25.15	0.00	74.85	25.15
		4AF	3.30	27.75	68.95	0.00	31.05	68.95
		5AF	0.60	19.40	76.80	3.20	20	80
		6AF	0.00	7.95	77.40	14.65	7.95	92.05
		7AF	0.00	0.55	68.60	30.85	0.55	99.45
		8AF	0.00	0.05	55.40	44.55	0.05	99.95
		9AF	0.00	0.00	31.70	68.30	0	100
	FF	3AF	0.00	78.70	21.30	0.00	78.7	21.3
		4AF	0.00	37.75	62.25	0.00	37.75	62.25
		5AF	0.00	31.35	68.65	0.00	31.35	68.65
		6AF	0.00	31.40	66.85	1.75	31.4	68.6
		7AF	0.00	26.30	72.00	1.70	26.3	73.7
		8AF	0.00	14.40	80.55	5.05	14.4	85.6
		9AF	0.00	3.85	92.55	3.60	3.85	96.15
	NC	3AF	16.45	57.50	26.05	0.00	73.95	26.05
		4AF	5.20	26.45	68.35	0.00	31.65	68.35
		5AF	0.80	22.80	76.40	0.00	23.6	76.4
		6AF	0.00	15.65	78.35	6.00	15.65	84.35
		7AF	0.00	10.75	81.85	7.40	10.75	89.25
		8AF	0.00	5.35	85.25	9.40	5.35	94.65
		9AF	0.00	1.25	94.95	3.80	1.25	98.75

Table 2.9 (contd.) Results Summary – Eight factor Main Effects + 1 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
2SD	PB	3AF	12.45	62.25	25.30	0.00	74.7	25.3
		4AF	4.30	27.55	67.95	0.20	31.85	68.15
		5AF	1.35	19.30	75.80	3.55	20.65	79.35
		6AF	0.15	7.90	80.30	11.65	8.05	91.95
		7AF	0.00	1.20	76.25	22.55	1.2	98.8
		8AF	0.00	0.00	61.00	39.00	0	100
		9AF	0.00	0.00	35.95	64.05	0	100
	FF	3AF	0.00	79.35	20.65	0.00	79.35	20.65
		4AF	0.00	37.95	62.05	0.00	37.95	62.05
		5AF	0.00	32.45	67.55	0.00	32.45	67.55
		6AF	0.00	28.55	70.75	0.70	28.55	71.45
		7AF	0.00	23.30	75.25	1.45	23.3	76.7
		8AF	0.00	12.75	84.60	2.65	12.75	87.25
		9AF	0.00	4.00	92.40	3.60	4	96
	NC	3AF	20.35	57.85	21.80	0.00	78.2	21.8
		4AF	4.60	26.90	68.50	0.00	31.5	68.5
		5AF	0.90	23.00	76.10	0.00	23.9	76.1
		6AF	0.00	17.15	80.65	2.20	17.15	82.85
		7AF	0.00	12.85	82.75	4.40	12.85	87.15
		8AF	0.00	5.75	87.35	6.90	5.75	94.25
		9AF	0.00	4.25	91.05	4.70	4.25	95.75
1SD	PB	3AF	1.75	12.05	61.20	25.00	13.8	86.2
		4AF	0.50	6.90	75.65	16.95	7.4	92.6
		5AF	0.10	2.70	82.50	14.70	2.8	97.2
		6AF	0.00	1.25	83.90	14.85	1.25	98.75
		7AF	0.00	0.45	83.55	16.00	0.45	99.55
		8AF	0.00	0.30	80.95	18.75	0.3	99.7
		9AF	0.00	0.00	71.55	28.45	0	100
	FF	3AF	0.00	21.55	59.10	19.35	21.55	78.45
		4AF	0.00	9.75	77.95	12.30	9.75	90.25
		5AF	0.00	5.90	84.45	9.65	5.9	94.1
		6AF	0.00	3.35	90.10	6.55	3.35	96.65
		7AF	0.00	2.55	91.50	5.95	2.55	97.45
		8AF	0.00	1.70	91.95	6.35	1.7	98.3
		9AF	0.00	0.65	92.80	6.55	0.65	99.35
	NC	3AF	4.85	14.10	58.50	22.55	18.95	81.05
		4AF	0.60	7.70	79.30	12.40	8.3	91.7
		5AF	0.10	3.95	86.10	9.85	4.05	95.95
		6AF	0.00	3.35	88.20	8.45	3.35	96.65
		7AF	0.00	1.50	91.15	7.35	1.5	98.5
		8AF	0.00	0.90	91.30	7.80	0.9	99.1
		9AF	0.00	0.70	93.20	6.10	0.7	99.3

Table 2-10 Results Summary – Eight factor Main Effects + 2 Hierarchical Interaction Model

Coefficient / Noise Ratio	Design	Active Factors in True Model	All AFs + No In active (I)	All AFs + Inactive (II)	Missed Some AF (III)	Missed All AF (IV)	No AFs Missed = (I) + (II)	AFs Missed = (III) + (IV)
3SD	PB	5AF	12.15	46.45	4.65	36.75	58.6	41.4
		6AF	0	1.2	87.25	11.55	1.2	98.8
		7AF	0	0.15	40.35	59.5	0.15	99.85
		8AF	0	0	8.65	91.35	0	100
		9AF	0	0	0.05	99.95	0	100
		10AF	0	0	0	100	0	100
	FF	5AF	0	72.9	21.6	5.5	72.9	27.1
		6AF	0	11.5	71.95	16.55	11.5	88.5
		7AF	0	2.55	80	17.45	2.55	97.45
		8AF	0	1.4	80.4	18.2	1.4	98.6
		9AF	0	0	82.8	17.2	0	100
		10AF	0	0	76.6	23.4	0	100
	NC	5AF	21.25	64.7	14.05	0	85.95	14.05
		6AF	0	10.45	68.3	21.25	10.45	89.55
		7AF	0	1.5	89.2	9.3	1.5	98.5
		8AF	0	0.1	83.8	16.1	0.1	99.9
		9AF	0	0	60.45	39.55	0	100
		10AF	0	0	15.55	84.45	0	100
2SD	PB	5AF	11.9	46.3	13.4	28.4	58.2	41.8
		6AF	0	0.95	84.6	14.45	0.95	99.05
		7AF	0	0.25	41.45	58.3	0.25	99.75
		8AF	0	0	9.25	90.75	0	100
		9AF	0	0	0	100	0	100
		10AF	0	0	0	100	0	100
	FF	5AF	0	75.15	18.9	5.95	75.15	24.85
		6AF	0	12.35	71.05	16.6	12.35	87.65
		7AF	0	1.95	80.5	17.55	1.95	98.05
		8AF	0	1.55	81.55	16.9	1.55	98.45
		9AF	0	0	81.45	18.55	0	100
		10AF	0	0	77.3	22.7	0	100
	NC	5AF	20.45	64.6	14.9	0.05	85.05	14.95
		6AF	0.4	9.55	68.4	21.65	9.95	90.05
		7AF	0	1	89.9	9.1	1	99
		8AF	0	0.25	84.4	15.35	0.25	99.75
		9AF	0	0	64.1	35.9	0	100
		10AF	0	0	23.65	76.35	0	100
1SD	PB	5AF	1.1	4.65	73.45	20.8	5.75	94.25
		6AF	0.05	1.15	74.8	24	1.2	98.8
		7AF	0.05	0.25	54.7	45	0.3	99.7
		8AF	0	0	23.1	76.9	0	100
		9AF	0	0	3.45	96.55	0	100
		10AF	0	0	0	100	0	100
	FF	5AF	0	12.4	72.55	15.05	12.4	87.6
		6AF	0	3.75	77.4	18.85	3.75	96.25
		7AF	0	0.8	78.3	20.9	0.8	99.2
		8AF	0	0.45	80.7	18.85	0.45	99.55
		9AF	0	0	80.4	19.6	0	100
		10AF	0	0	75.25	24.75	0	100
	NC	5AF	1.5	6.55	78.45	13.5	8.05	91.95
		6AF	0.05	1.5	85.9	12.55	1.55	98.45
		7AF	0	0.15	89.25	10.6	0.15	99.85
		8AF	0	0.25	91.1	8.65	0.25	99.75
		9AF	0	0	87.75	12.25	0	100
		10AF	0	0	81.05	18.95	0	100

2.3.4. Preliminary Results

After studying these results, the Plackett-Burman designs were dropped from the study since these designs only have 12 runs as compared to the 16 runs in the FF and NC designs. Also when the coefficient to noise ratio is less than two, the noise level is too high for any method to identify the active terms in the model. Therefore the next simulation study done only considers the NC and FF designs and considers only true models with coefficient to noise ratios greater than two.

2.4. Simulation Study

The results of an extensive simulation study on the effectiveness of stepwise regression to analyze the regular 16-run fractional factorial design with 6-8 factors and the 16-run NC designs with 6-8 factors are detailed in this section.

The other factors studied in the simulation are:

- True Model – This is unknown in real experiments, but controlled in the simulation study. For the purpose of this study the following models were studied; Main effects only, Main Effects + 1 interaction entering with strong heredity and Main Effects + 2 interactions entering with strong heredity.
- Number of design factors – six, seven and eight.
- Coefficient to Noise Ratio - Normally distributed with a mean of zero and standard deviation = 1. The β/σ ratio is varied as described subsequently.
- Type of Design – Fractional Factorial and No Confounding
- Number of Active Terms – This depends on the true model and the number of factors in the design and is listed in Table 2-11.
- Model Selection Method – Stepwise Regression with AICc criterion, Two Stage Stepwise Regression (include main effects in stage one and then

including the interactions with strong heredity in stage two) with AICc criterion (2004).

JMP was used to run the simulations. There are various options available in JMP to perform stepwise regression; in the case of p-value with stepwise regression, the combine option with mixed (stepwise) regression was used and for the AICc case the forward regression and combine was used. For the p-value case the Prob to Enter was set as 0.10 and Prob to Leave was set as 0.15.

The combine option groups a two-factor interaction term with its two associated main effects and calculates the group's significance probability for entry using a joint F-test. In each iteration, the active terms were randomly assigned to the columns of the model matrix. The coefficient of the inactive terms was set to zero. The β's of the active terms are randomly generated. The largest coefficient is varied from 3.8 to 4.2 and the smallest coefficient is varied from 2.0 to 2.2. The coefficients are varied following an exponential distribution from the largest coefficient value to the smallest coefficient value.

Table 2-11 Number of active terms

		Number of active terms		
		6	7	8
True Model	Main Effects Only	1, 2, 3, 4, 5, 6	1, 2, 3, 4, 5, 6, 7	1, 2, 3, 4, 5, 6, 7, 8
	Main Effects + 1 Interaction	2+1, 3+1, 4+1, 5+1, 6+1	2+1, 3+1, 4+1, 5+1, 6+1, 7+1	2+1, 3+1, 4+1, 5+1, 6+1, 7+1, 8+1
	Main Effects + 2 Interactions	3+2, 4+2, 5+2, 6+2	3+2, 4+2, 5+2, 6+2, 7+2	3+2, 4+2, 5+2, 6+2, 7+2, 8+2

There are three levels for three factors (true model, number of factors in the design, model selection method) and 6, 7 and 8 levels for the fourth factor (number of active terms). The full factorial design in the simulation therefore, required a total of 216 factor combinations.

2.5. Simulation Results

The results from each trial (one combination of simulation factors) were evaluated by calculating.

1. π_1: Percentage of runs where only active terms were identified as active
2. π_2: Percentage of runs where all active terms were identified, plus some inactive terms were identified as active (Type I Error)
3. π_3: Percentage of runs where some of the active terms were missed (Type II Error)
4. π_4: Percentage of runs where all the active terms were missed (Type II Error)

In an ideal scenario, π_1 would be close to 100 and π_2, π_3 and π_4 would be close to zero. Since it is a screening scenario, the experimenter would tolerate some Type I errors but would want to avoid Type II errors. This is because it is hard to recover from excluding important factors in the initial stage of experimentation. However, if some inactive factors are carried forward to the next stage of experimentation, these can usually be discovered and removed later. For a model selection method to be successful, π_3 & π_4 need to be close to zero. For the FF designs when the true model contains interactions, if the analysis identifies the alias chain correctly, I do not include that as a success. This is because there is no analytical way to correctly identify which interaction effect in the alias chain is active without running more experiments.

Graphical summaries of the results are shown in Figures 2.2 – 2.13. For the six factor FF designs, the model selection method does not affect the error rate. The designs performs well only when the true model only contains main effects and results in 0% type II error when the active terms in the model are five or less. Once the true model includes interactions, FF design results in 100% Type II error. In the case of the NC design, the stepwise regression with AICc as the model selection criterion works better than the 2-stage stepwise regression for models which contain interactions. The 2-stage stepwise regression method works better for the NC design when the true model contains only main effects. Once interactions are present in the true model, stepwise regression with AICc as the model selection criteria works better when the number of active terms are between two and four. As the number of active terms increases, the error rates for the two methods converge. The results for the eight factor NC and FF designs are similar to the results from the six factor designs.

In the case of the seven factor NC design, the results look better than those for the six and eight factor designs. The error rate in the cases where the true model contains interactions varies from 30% to 78% for the case with one interaction and between 70% and 94% for the case with two interactions. In the case of the six and eight factor NC designs, the error rate ranges between 19% – 100% and 55% - 98% for models with one interaction and from 55% - 100% and 88% - 100% for models with two interactions.

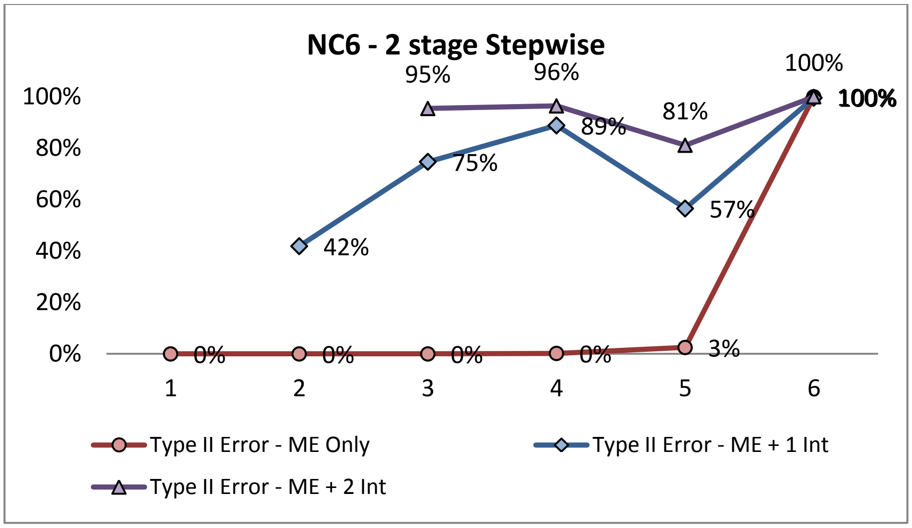


Figure 2.2 2 stage stepwise AICc – NC Six Factor Design

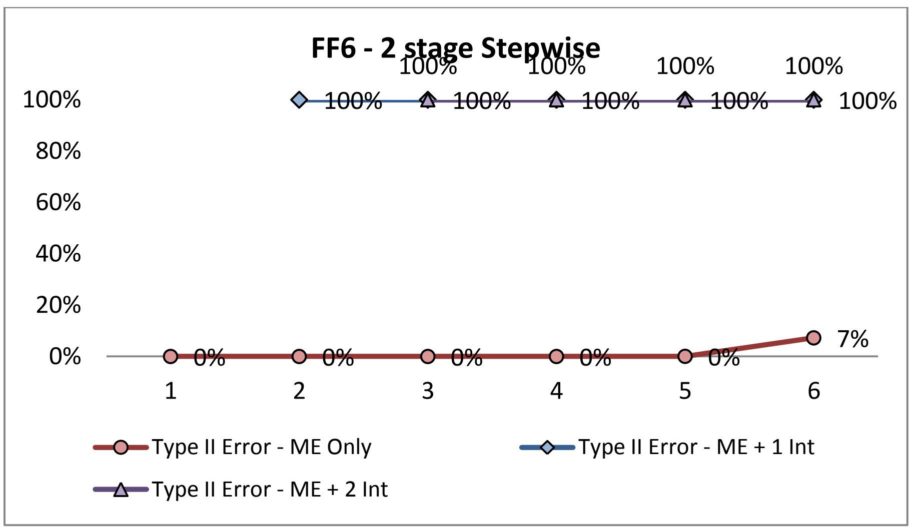


Figure 2.3 2 stage stepwise AICc – FF Six Factor Design

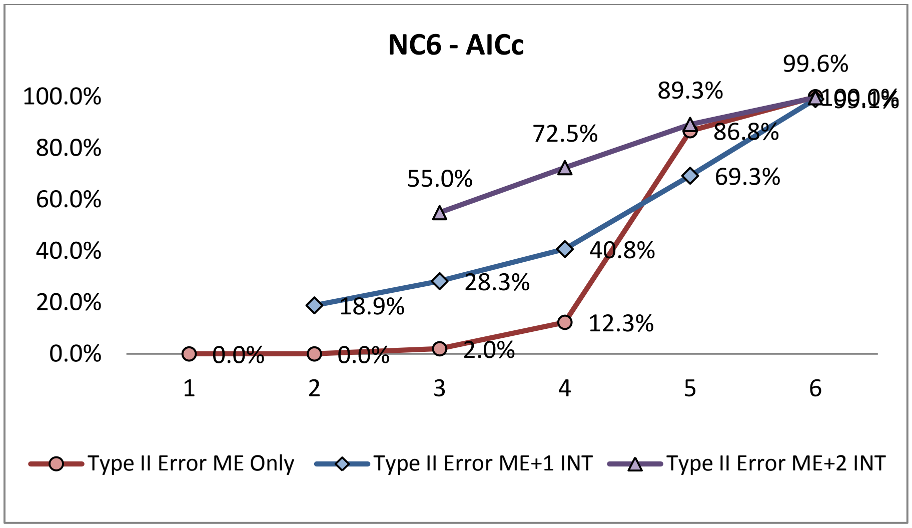


Figure 2.4 Stepwise AICc – NC Six Factor Design

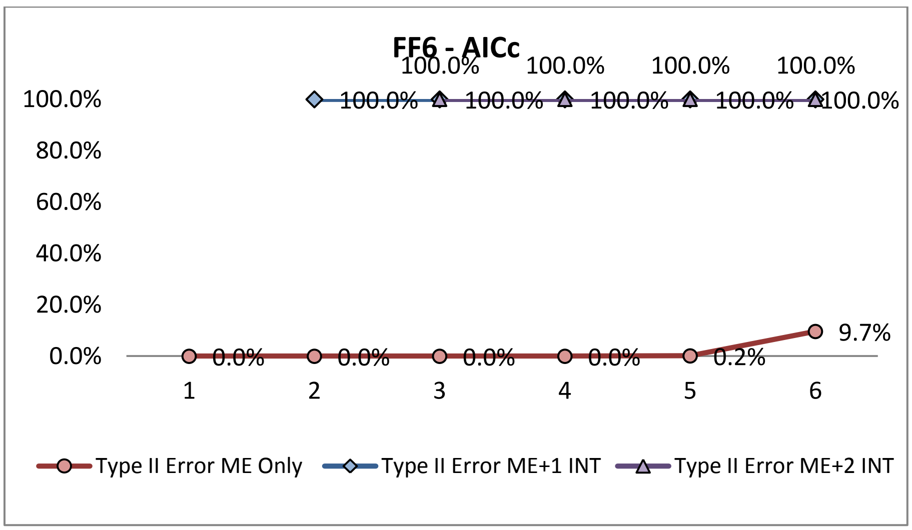


Figure 2.5 Stepwise AICc – FF Six Factor Design

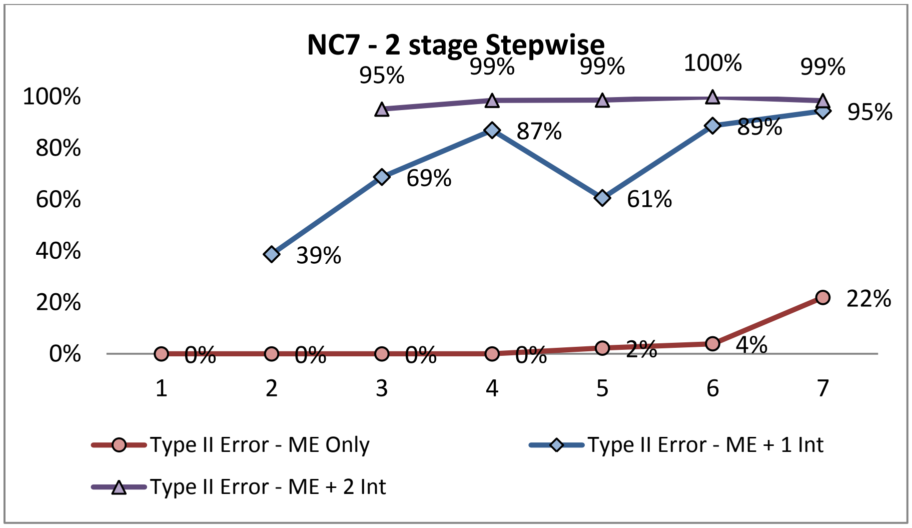


Figure 2.6 2 stage stepwise AICc – NC Seven Factor Design

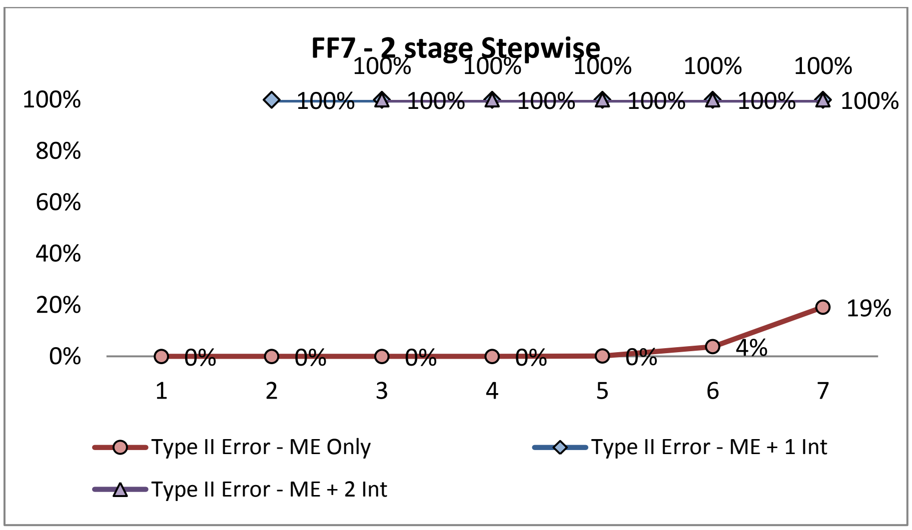


Figure 2.7 2 stage stepwise AICc – FF Seven Factor Design

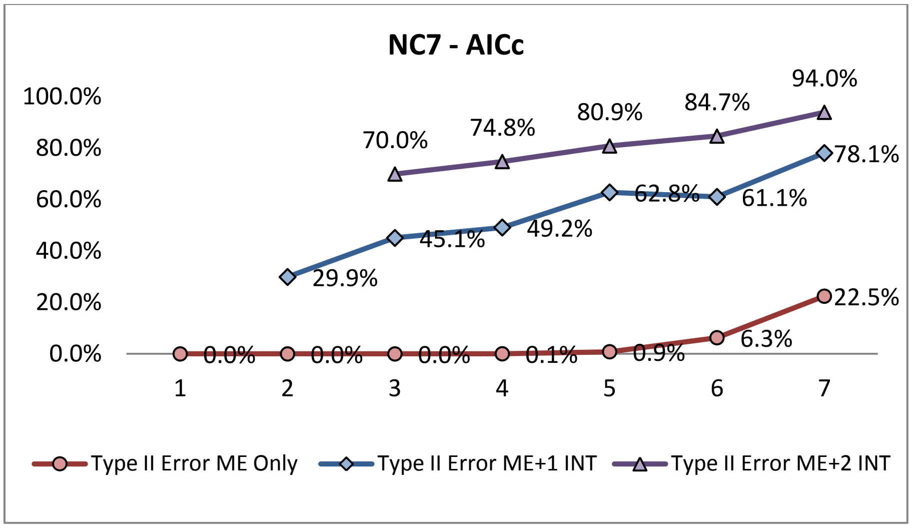


Figure 2.8 Stepwise AICc – NC Seven Factor Design

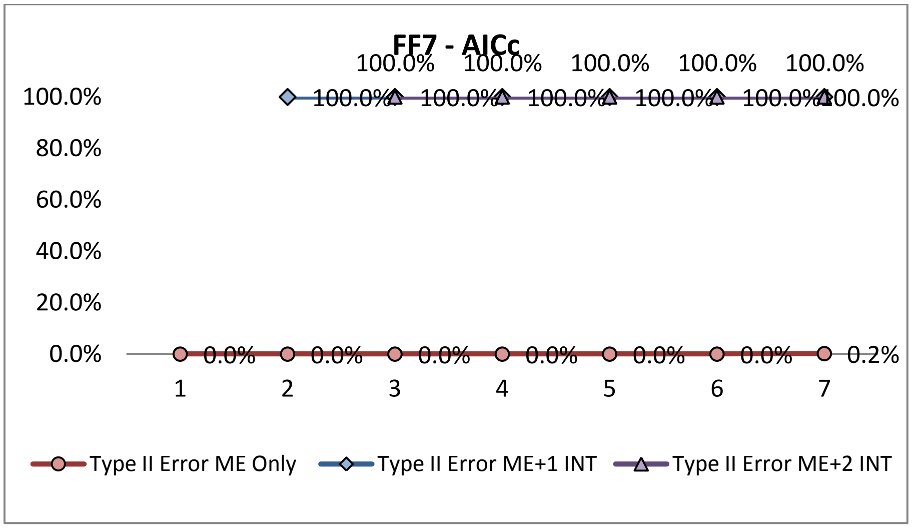


Figure 2.9 Stepwise AICc – FF Seven Factor Design

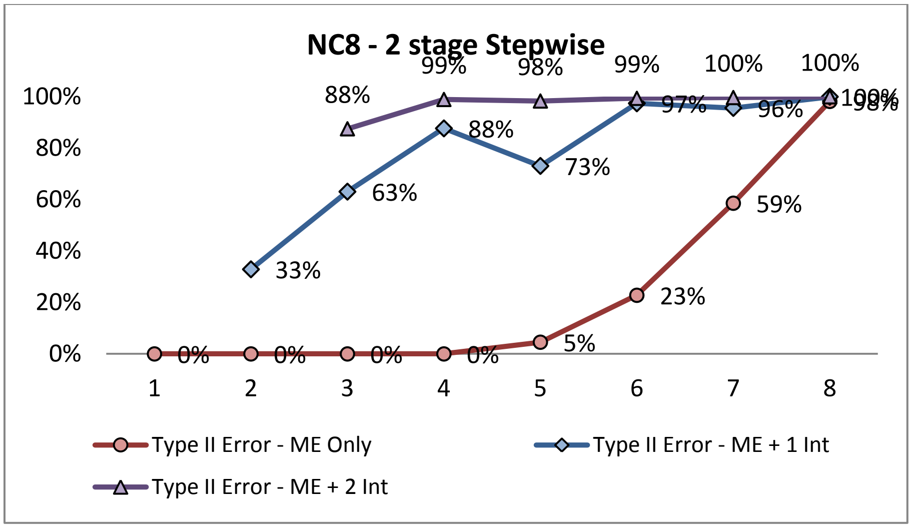


Figure 2.10 2 stage stepwise AICc – NC Eight Factor Design

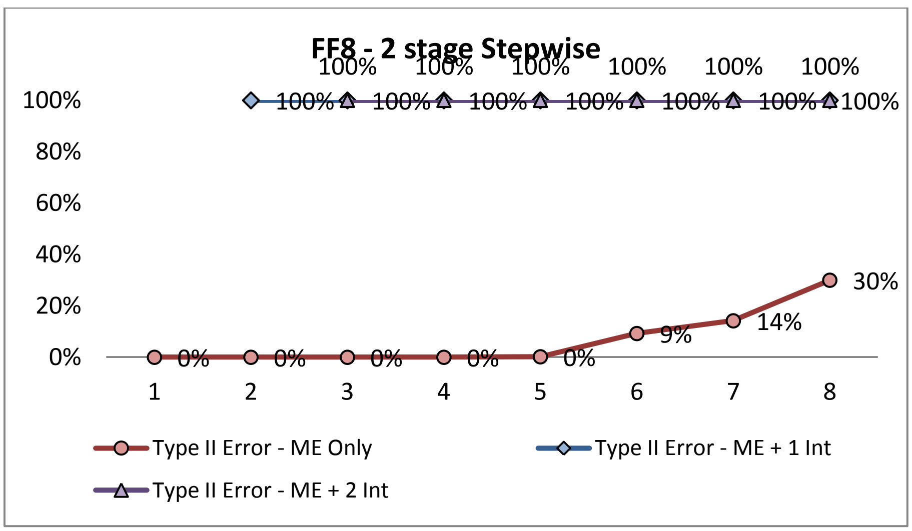


Figure 2.11 2 stage stepwise AICc – FF Eight Factor Design

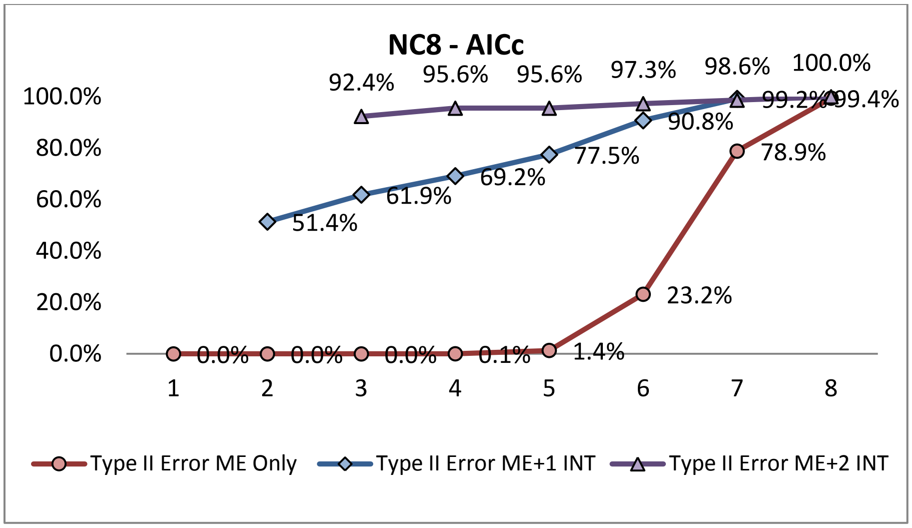


Figure 2.12 Stepwise AICc – NC Eight Factor Design

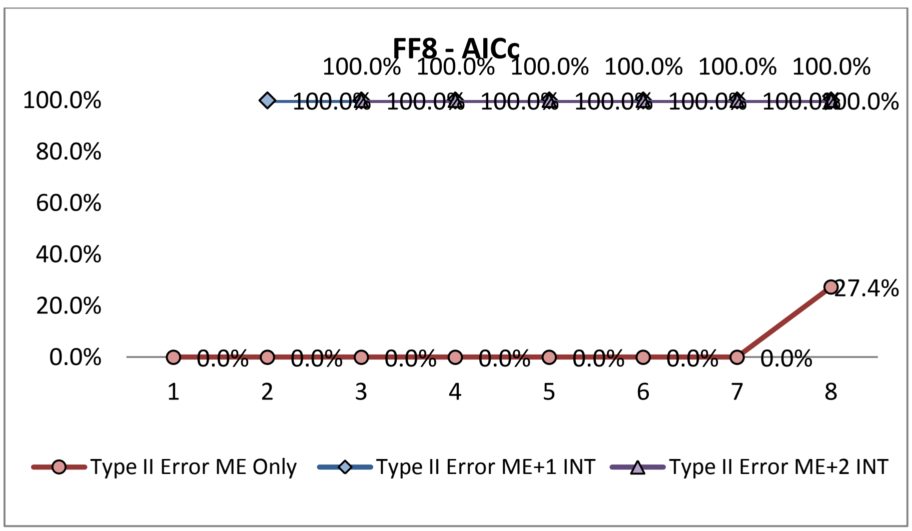


Figure 2.13 Stepwise AICc – FF Eight Factor Design

2.6. Simulation Output Analysis

The response, $\pi_3 + \pi_4$ (Type II Error) is regressed on the five variables varied in the simulation study. The ANOVA results from this analysis are shown in Table 2-12. The results show that the model predicted R-Squared is 0.89 and the significant effects include three of the main effects (B: True Model, D: No. of Design factors and E: No. of Active terms) and two 2 - factor interactions; BC and CE. The main effect C: Design is included in the analysis to maintain hierarchy.

The interaction plots shown in Figure 2.14 and Figure 2.15 clearly show the interactions between the main factors. These plots are interpreted below and summarizes the effect the different factors have on the error rate.

- The True Model * Design Interaction shows that both NC and FF designs have fairly small error rates when the true model consists of only main effects though the FF design has a lower error rate than the NC design in this case. But once the true model has interactions present in it, the NC design has a lower error rate as compared to the FF designs. For the FF designs the error rate goes to a 100% when any interaction is present in the true model whereas in the case of the NC designs, the error rate gradually increases as the number of interactions in the model increases.
- The Design * No. of Active factors interaction shows that although the performance deteriorates as the No. of Active factors increases, the performance deteriorates from an error rate of 20% to an error rate of almost 100% in the NC design case and varies from 43% to 71% for the FF design case. The NC design actually performs better when the number of active terms in the model is lower and starts deteriorating faster than in the case of the FF design as the number of active terms in the model increases.

This leads to the conclusion that if the number of active terms is relatively small, 4 or less, that is, sparsity of effects prevails, the NC designs are good alternatives to the FF designs.

Table 2-12 Simulation ANOVA results

Response: $\pi_3+\pi_4$ *Transform: Square root* *Constant: 0.01*

Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob>F
Model	30.195	21	1.438	95.898	< 0.0001
B-True Model	21.272	2	10.636	709.381	< 0.0001
C-Design	0.021	1	0.021	1.404	0.2374
D-No. of Design Factors	0.086	2	0.043	2.874	0.0589
E-No. of Active Factors	1.852	7	0.265	17.648	< 0.0001
BC	1.253	2	0.626	41.773	< 0.0001
CE	0.736	7	0.105	7.014	< 0.0001
Residual	2.909	194	0.015		
Cor Total	33.104	215			

Std. Dev.	0.122	**R-Squared**	0.912
Mean	0.665	**Adj R-Squared**	0.903
C.V. %	18.423	**Pred R-Squared**	0.889
PRESS	3.688	**Adeq Precision**	29.581

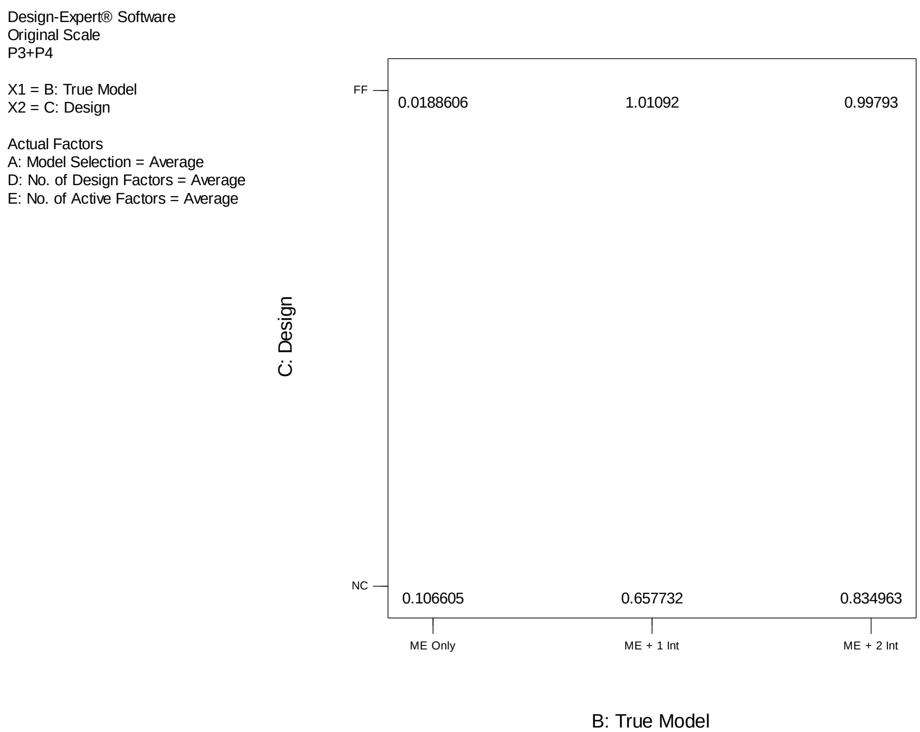


Figure 2.14 True Model * Design Interaction

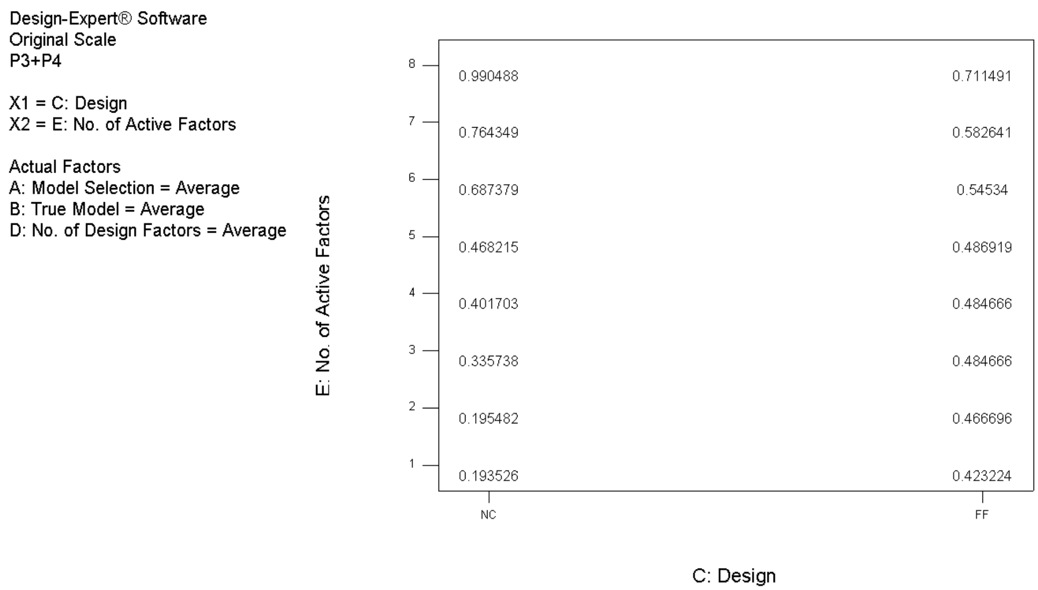


Figure 2.15 Design * No. of Active factors Interaction

2.7. Conclusion

The regular fractional factorial designs with six, seven or eight factors in 16 runs are widely used. However due to the complete confounding of the two-factor interactions with one another, these designs often require the experimenter to perform runs to resolve ambiguities whenever any of the two-factor interactions are identified as being active. The NC designs allow for the estimation of all main effects along with some of the two-factor interactions since there is no complete confounding in these designs.

The simulation study confirmed that stepwise regression does not work well once the total number of active terms exceeds four. However the study also showed that NC designs allow for estimation of two factor interactions without the need to run additional runs. Furthermore, once the true model contains interactions, regular fractional factorial designs are unable to compete with the nonregular designs due to the complete confounding of the two-factor interactions.

The simulation study shows that although stepwise regression may not be the best method to use for the analysis of nonregular designs, it is reasonably effective if the number of active terms (main effects and interactions included) is not more than four. There is no statistically significant difference between using a 2-stage stepwise regression method and a stepwise regression method. Both model selection methods used the AICc criterion.

I believe that the NC designs are good alternatives to the FF designs specially when running another set of experiments is not an alternative. With the NC designs, the experimenter would be able to study both the main effects and the interactions from the initial 16 runs of the experiment when the effect sparsity principle holds true.

Chapter 3

PROJECTION PROPERTIES OF NO-CONFOUNDING DESIGNS FOR SIX, SEVEN AND EIGHT FACTORS IN SIXTEEN RUNS

3.1. Introduction

The NC designs do not completely confound any of the main effects and two-factor interactions. Plackett-Burman designs and the NC designs of Jones and Montgomery are examples of nonregular designs. The projection properties of fractional factorial designs and Plackett-Burman designs are well documented. Montgomery (2013) discusses the projection properties of the 2^{k-p} designs that collapse into either full factorial or a fractional factorial in any subset of $r \leq k - p$ of the original factors. The subsets that result in fractional factorials are subsets appearing as words in the complete defining relation. Lin and Draper (1992) and Box and Bisgaard (1993) showed that some of the Placket-Burman designs in fewer runs when projected onto three factors result in a complete 2^3 design and a half replicate of the 2^3 design. The projection properties of NC designs have not been studied. In this paper the projection properties of NC designs for the six, seven and eight factor cases in 16 runs are presented.

The principle of effect sparsity in designed experiments allows experimenters to study a larger number of factors under the assumption that only a few of them will have a significant effect on the response/s being studied. Once the design is collapsed to a smaller number of factors, the resulting design may have properties that allow for easier analysis of these designs. Studying the projection properties of the NC designs can suggest possible analytical methods for these designs. Here I present the three factor and four factor projections of the six, seven and eight factor NC designs.

Johnson and Jones (2011) show that the six, seven and eight factor NC designs have a classical-type construction with a 2^4 or a replicated 2^3 starting point. These generating columns can be used to study the projection properties of the NC designs. Sections 2, 3 and 4 describe the projection properties of six, seven and eight factor NC designs. Section 5 describes how these projections are related to the generating columns described in Johnson and Jones (2011). Section 6 suggests two potential analysis methods for NC designs. Sections 7 and 8 illustrates the analysis methods for two example experiments from the literature.

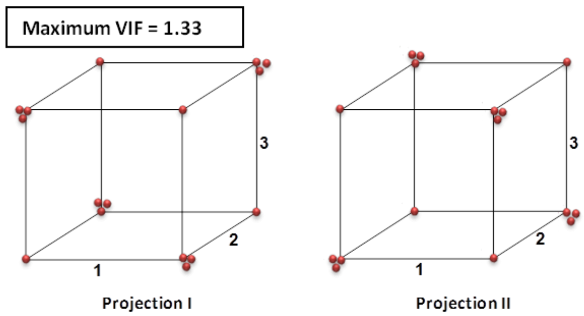
3.2. Projection properties of six factor NC design

Box (1996), Cheng (1995), Cheng (1998), Dey (2005) and Evangelaras (2005) talk about projection properties of orthogonal arrays. There are a few other papers that discuss the projection properties of screening designs, Placket - Burman designs and nonregular designs such as Box & Tyssedal (2001), Bulutoglu et al (2003), Lin & Draper (1992), Loeppky et al (2007), Tsai et al (2000) and Xu et al (2005). All these papers talk about different projection properties of the designs and how they can be used to the experimenter's benefit during both the design phase and the analysis phase of experiments. Studying the projection properties of the designs gives valuable insight into possible analysis methods. The following sections discuss the projection properties of the NC designs and provides valuable insight into how these properties can be used to develop analysis methods for these designs.

The six factor NC design has 20 different three factor projections. 12 of these projections result in replicated full factorial designs in three factors. These projections can therefore be analyzed like a full factorial design. The other eight projections result in two different projection types which are isomorphic in nature. The two projection types are shown in Figure 3.1. The projections show that there are eight distinct design points for the three factor projections of the six factor NC design. This allows for the estimation of the three main effects and the three two-factor interactions. The maximum VIF for any term for any of these projections is 1.33.

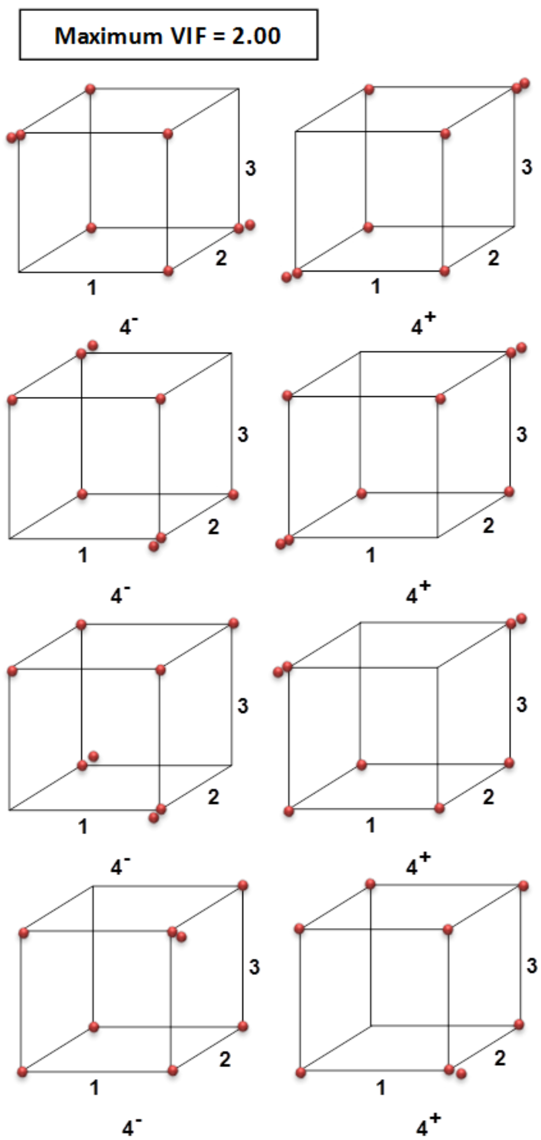
There are 15 possible four factor projections of the six factor NC design. Three of these projections, ABCD, ABEF and CDEF, result in full factorial projections whereas the remaining 12 projections result in nine different projection types. The nine different types of projections are shown in Figure 3.2. Projections 1, 2, 3, 6, 7, 8 & 9 have 12 distinct design points while projections 4 & 5 have 14 distinct design points.

This allows for estimation of all 10 terms (main effects and two-factor interactions). The correlation patterns for all three and four factor projections of the six factor NC design show that the maximum correlation between any two effects is ± 0.5. The maximum VIF for any term for any of these projections is 2.



Projection	Axis / Column 1	2	3
I	A	C	E
	A	D	E
	A	D	F
	B	C	E
	B	C	F
	B	D	F
II	A	C	F
	B	D	E

Figure 3.1 Three factor Projections for the Six Factor NC design



Axis / Column 1	2	3	4
A	B	C	E
A	B	D	F

Projection 1

Axis / Column 1	2	3	4
A	C	D	E
B	C	D	F

Projection 2

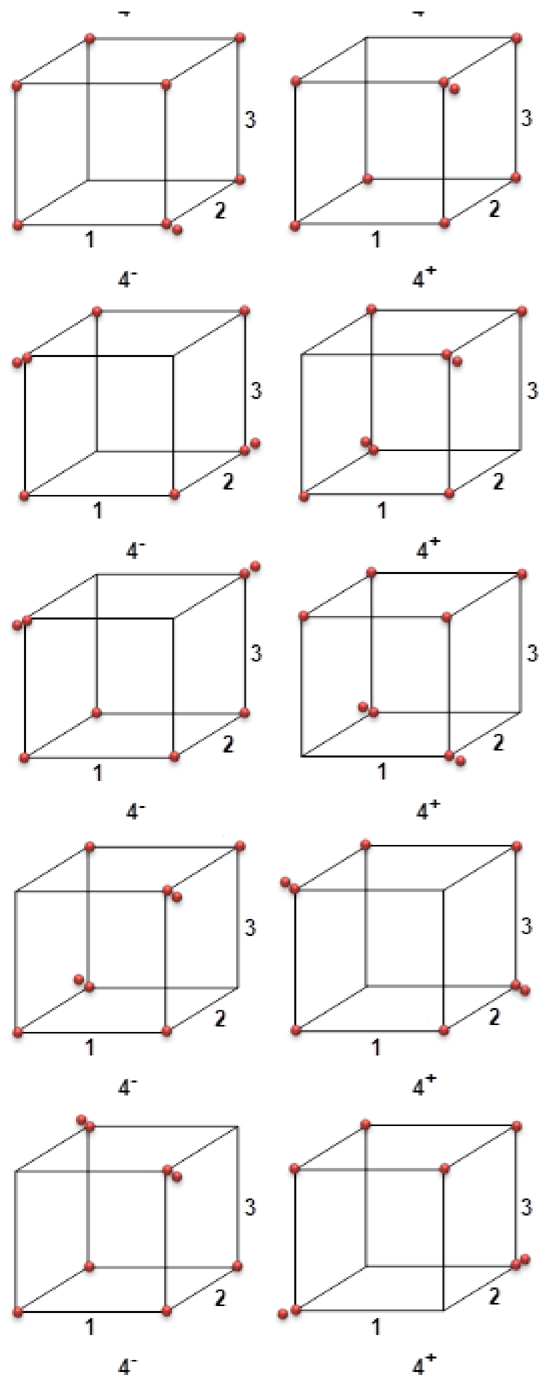
Axis / Column 1	2	3	4
A	D	E	F
B	C	E	F

Projection 3

Axis / Column 1	2	3	4
A	B	C	F

Projection 4

Figure 3.2 Four factor projections of the six factor NC design



Axis / Column			
1	2	3	4
A	B	D	E

Projection 5

Axis / Column			
1	2	3	4
A	C	D	F

Projection 6

Axis / Column			
1	2	3	4
A	C	E	F

Projection 7

Axis / Column			
1	2	3	4
B	C	D	E

Projection 8

Axis / Column			
1	2	3	4
B	D	E	F

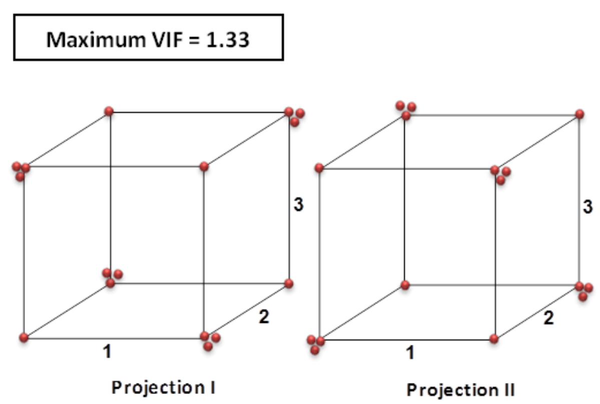
Projection 9

Figure 3.2 (contd.) Four factor projections of the six factor NC design

3.3. Projection properties of seven factor NC design

Out of the 35 possible three factor projections for the seven factor design, 27 projections result in a full factorial design with two replicates. The other eight projections result in the main effects being partially aliased (0.5 or -0.5) with the two factor interaction not involving itself. The generating equations can be used to study the projection properties. The projections show that there are eight distinct design points for the three factor projections of the 7 variable no confounding design. This allows for the estimation of the three significant main effects and their three two-factor interactions. The three factor projections of the seven factor NC design that are not replicated full factorials are shown in Figure 3.3. The maximum VIF for any of the terms for any of the projections is 1.33.

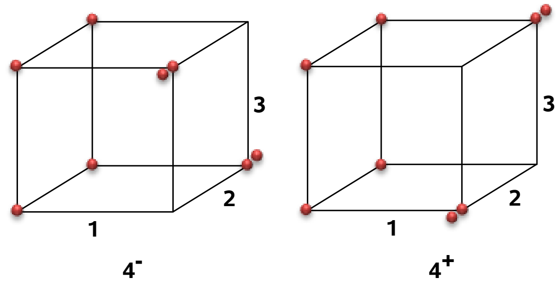
12 of the possible 35 four factor projections result in a full factorial design; ABCD, ABCE, ABCF, ABCG, ABDG, ABEF, ACDE, ACFG, ADEF, ADEG, ADFG, AEFG and DEFG are the projections that results in a full factorial design. The remaining 23 four factor projections result in 13 different types of projections. These four factor projections are shown in Figure 3.4. All projections have 12 distinct design points. This allows for estimation of all 10 terms (main effects and two-factor interactions). The maximum VIF for any term for any of the projections is 2.



Projection	Axis / Column 1	2	3
I	B	D	E
	B	D	F
	B	F	G
	C	D	F
	C	E	F
	C	E	G
II	B	E	G
	C	D	G

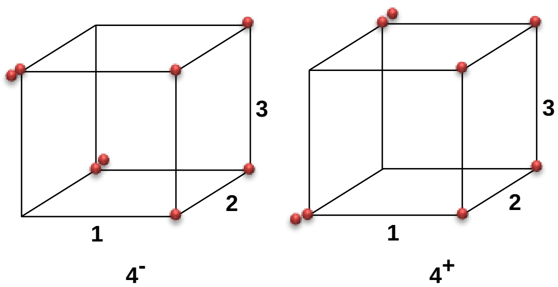
Figure 3.3 Three factor projections for the seven factor NC design

Maximum VIF = 2.00



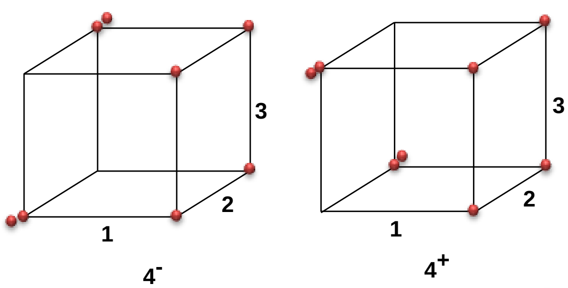
Axis / Column			
1	2	3	4
A	B	D	E
A	B	F	G
A	C	D	F
A	C	E	G
B	C	E	F

Projection 1



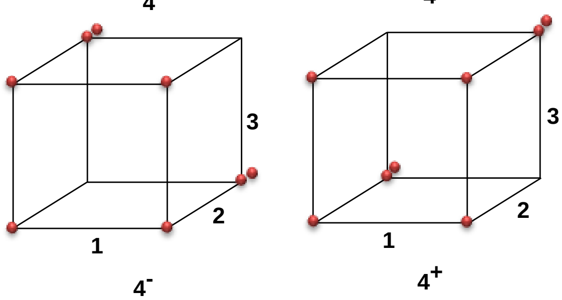
Axis / Column			
1	2	3	4
A	B	D	F
A	C	E	F

Projection 2



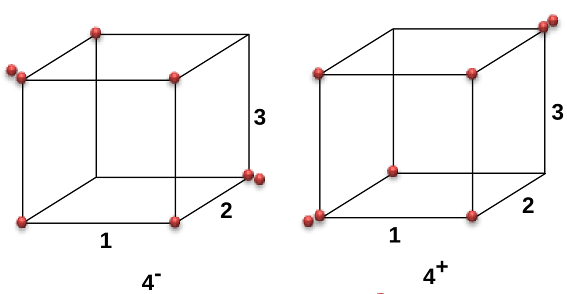
Axis / Column			
1	2	3	4
A	B	E	G
A	C	D	G
B	C	D	G

Projection 3



Axis / Column			
1	2	3	4
B	C	D	E

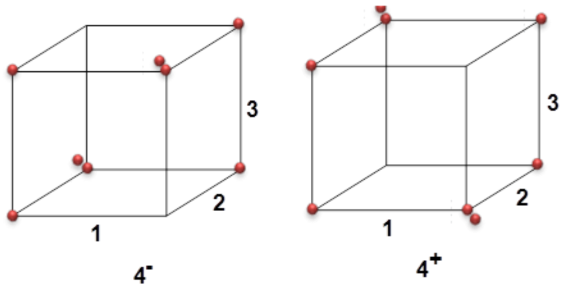
Projection 4



Axis / Column			
1	2	3	4
B	C	D	F

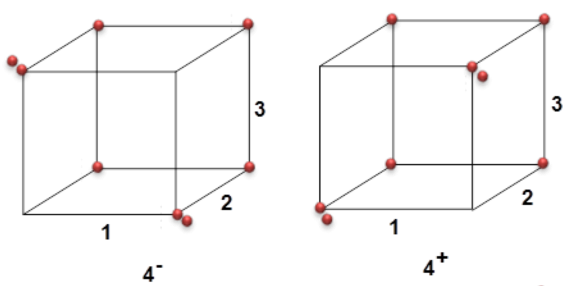
Projection 5

Figure 3.4 Four factor projections of the seven factor NC design



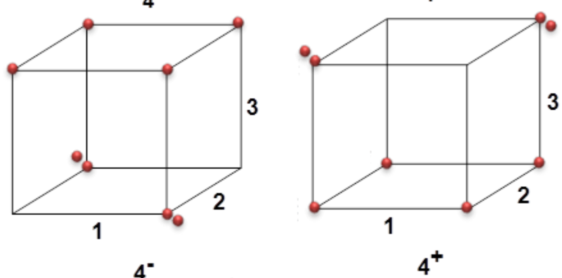
Axis / Column			
1	2	3	4
B	C	E	G

Projection 6



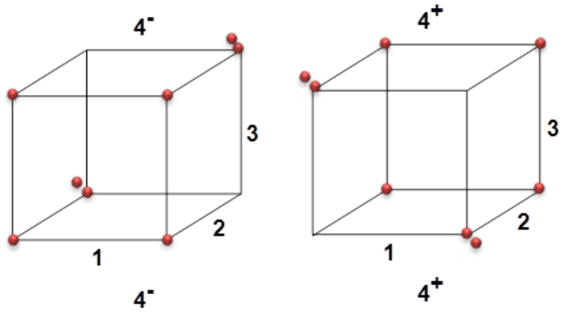
Axis / Column			
1	2	3	4
B	C	F	G

Projection 7



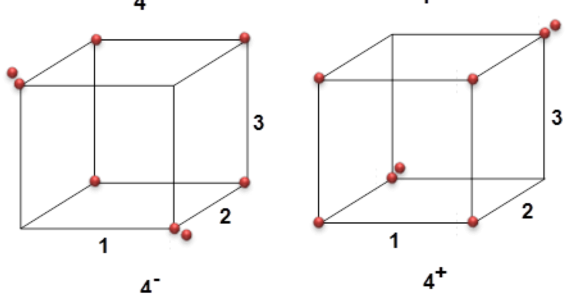
Axis / Column			
1	2	3	4
B	D	E	F
C	E	F	G

Projection 8



Axis / Column			
1	2	3	4
B	D	E	G

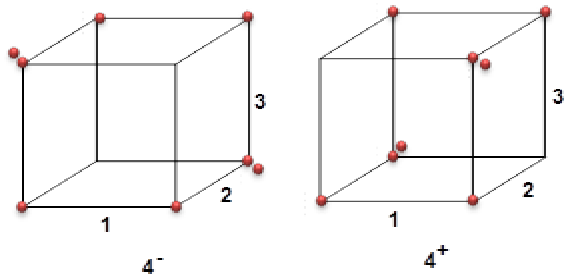
Projection 9



Axis / Column			
1	2	3	4
B	D	F	G

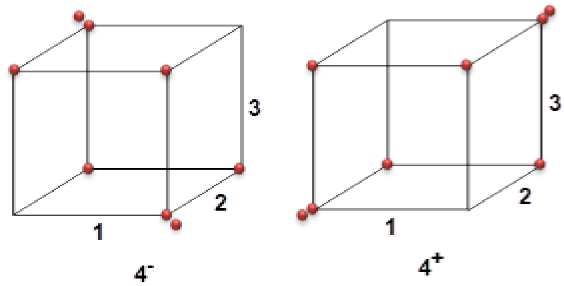
Projection 10

Figure 3.4 (contd.) Four factor projections of the seven factor NC design



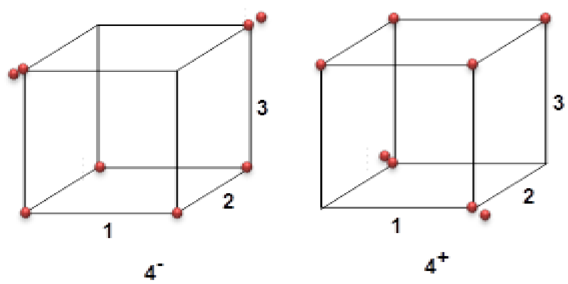
Axis / Column			
1	2	3	4
B	E	F	G
C	D	E	G

Projection 11



Axis / Column			
1	2	3	4
C	D	E	F

Projection 12



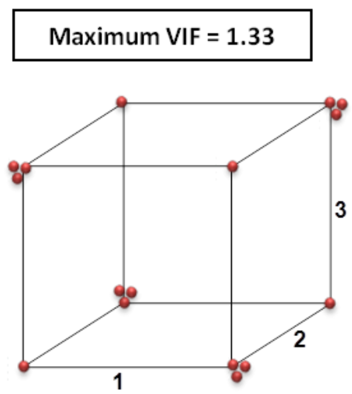
Axis / Column			
1	2	3	4
C	D	F	G

Projection 13

Figure 3.4 (contd.) Four factor projections of the seven factor NC design

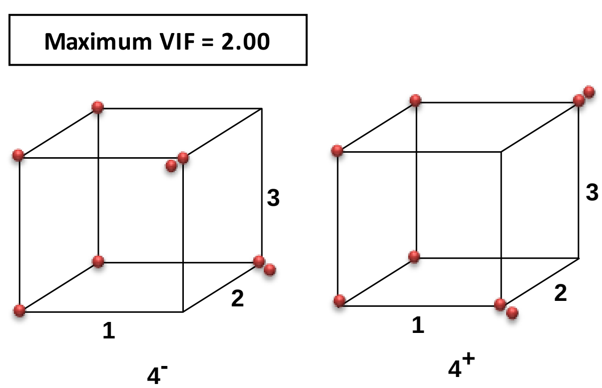
3.4. Projection properties of eight factor NC design

Out of the 56 possible three factor projections, 42 of the projections result in a full factorial design with two replicates, and 14 projections result in the main effects being partially aliased (0.5) with the two factor interaction not involving itself. These projections result in the display shown in Figure 3.5. The projection shows that there are eight distinct design points for the three factor projections of the eight factor NC design. This allows for the estimation of the three main effects and their three two-factor interactions. The maximum VIF for any of the terms for any of the projections is 1.33. In the case of the four factor projections, 21 of the possible 70 four factor projections result in a full factorial design. There are 11 projection types for the 49 projections that do not result in a full factorial design. These projections are illustrated in Figure 3.6. All 49 of these projections have 12 distinct design points. This allows for estimation of all 10 terms (main effects and two-factor interactions). The maximum VIF for any term for any of the four factor projections is 2.



Axis / Column		
1	2	3
B	C	D
B	C	E
B	D	G
B	E	F
B	F	H
B	G	H
C	D	H
C	E	G
C	F	G
C	F	H
D	E	F
D	E	H
D	F	G
E	G	H

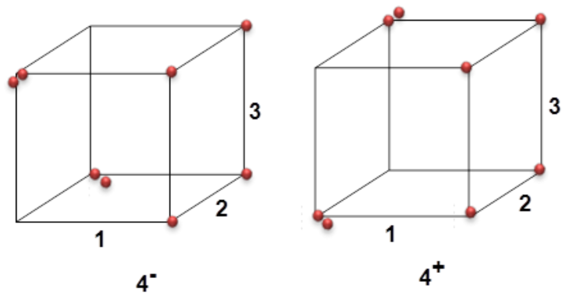
Figure 3.5 Three factor projections of the eight factor NC design



Axis / Column			
1	2	3	4
A	B	C	D
A	B	E	F
A	B	G	H
A	C	E	G
A	C	F	H
A	D	E	H
A	D	F	G
B	C	F	G
B	D	E	H
C	D	E	F

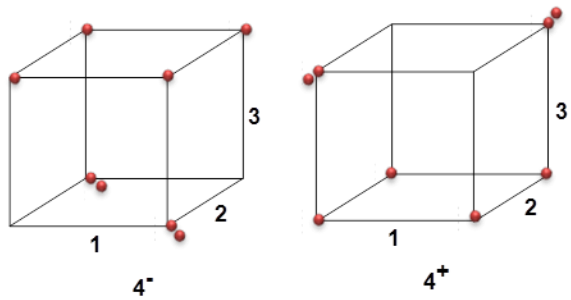
Projection 1

Figure 3.6 Four factor projections of the eight factor NC design



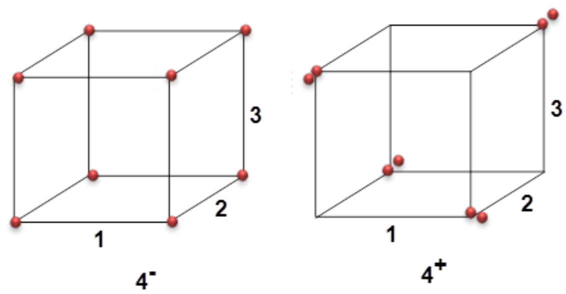
Axis / Column			
1	2	3	4
A	B	C	E
A	B	D	G
A	B	F	H
A	C	D	H
A	C	F	G
A	D	E	F
A	E	G	H

Projection 2



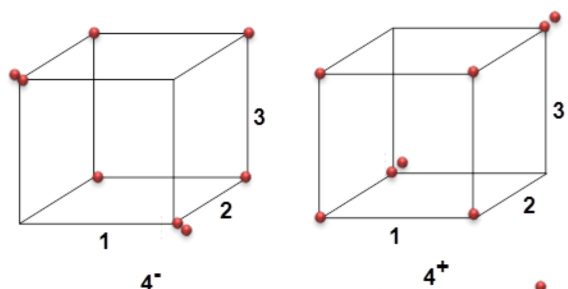
Axis / Column			
1	2	3	4
B	C	D	E
C	F	G	H
D	E	F	H

Projection 3



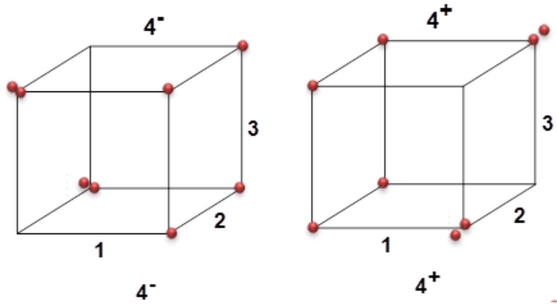
Axis / Column			
1	2	3	4
B	C	D	F
B	C	E	H
B	E	F	G
D	F	G	H

Projection 4



Axis / Column			
1	2	3	4
B	C	D	G
B	C	E	F
B	D	G	H
B	E	F	H
D	E	F	G

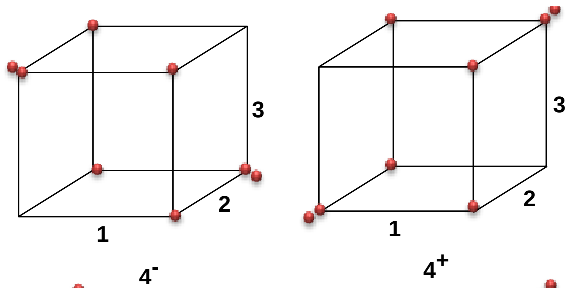
Projection 5



Axis / Column			
1	2	3	4
B	C	D	H
B	C	E	G
C	E	G	H

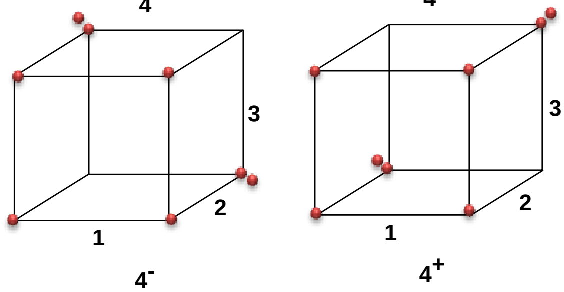
Projection 6

Figure 3.6 (contd.) Four factor projections of the eight factor NC design



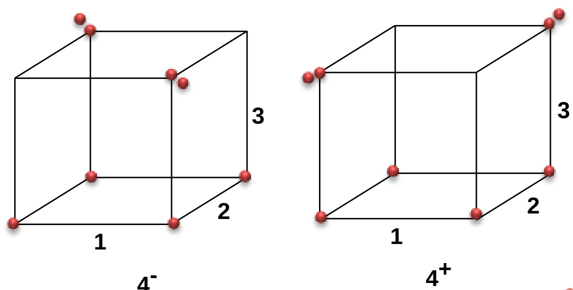
Axis / Column			
1	2	3	4
B	C	F	H
B	D	E	F
B	E	G	H
C	D	F	G

Projection 7



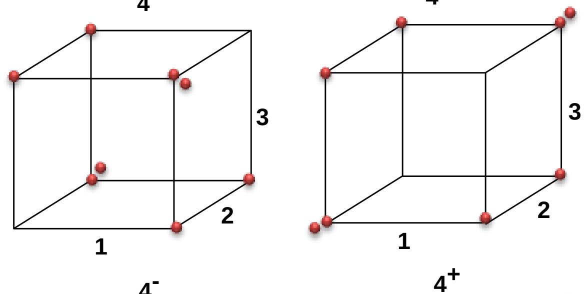
Axis / Column			
1	2	3	4
B	C	G	H
B	D	F	H
C	D	E	G
C	E	F	H
E	F	G	H

Projection 8



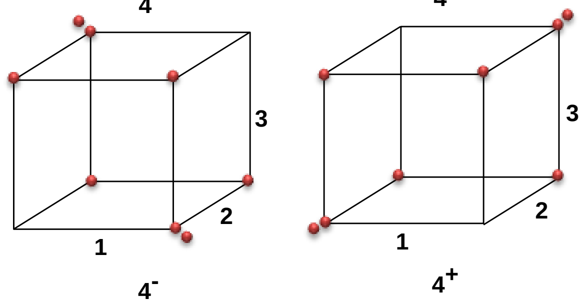
Axis / Column			
1	2	3	4
B	D	E	G
C	D	G	H

Projection 9



Axis / Column			
1	2	3	4
B	D	F	G
C	D	E	H
D	E	G	H

Projection 10



Axis / Column			
1	2	3	4
B	F	G	H
C	D	F	H
C	E	F	G

Projection 11

Figure 3.6 (contd.) Four factor projections of the eight factor NC design

3.5. Projections using the generating columns

Johnson and Jones (2011) present the generating columns for the NC designs for the six to eight factor cases. In the case of the fractional factorial designs, the defining relation and the subsets of the words appearing in the relation gives us an indication of which projections result in full factorials and which ones result in fractional factorials. In the case of the NC designs, the generating columns presented in Johnson and Jones (2011) works in a similar manner.

For the six factor NC design, the columns A, B, C & D form a full factorial in 16 runs and the columns E and F can be generated using the following equations:

$$E=1/2\ [AC+BC+AD-BD]$$

$$F=1/2\ [-AC+BC+AD+BD]$$

These equations can be used to study the projection properties for both the three factor and the four factor projections. Since the columns A, B, C and D form a full factorial in 16 runs, any projections which contain only, A, B, C and D columns will result in full factorial projections. If the projection contains E or F, then the generating equations can be used to identify the correlation structure and hence the projection. Table 3-1 illustrates how to identify the correlation structure and the projections for the three factor projections of the six factor NC design.

A similar method can be applied to generating the four factor projections. Table 3-2 illustrates some examples for the four factor projections of the six factor NC design. Projections ABCE and ABDF are the same because the correlation structure is the same for the two projections.

For the seven factor NC design, columns A, B, C and D form a full factorial in 16 runs and the columns E, F and G can be generated using the following equations.

$$E=1/2\ [BD+ABD+BCD-ABCD]$$

F=1/2 [BD+CD-ABD+ACD]

G=1/2 [-CD+ACD+BCD+ABCD]

The projections can be studied using these generating equations in a similar manner as was done for the six factor NC design.

Table 3-1 Three factor projections based on generating columns for the six factor NC design

Proj $X_1X_2X_3$	Equation	Drop columns that are not part of the projection subset	Projection Type
ABC	Columns A, B, C, D form a full factorial	-	Full Factorial with 2 replicates
ABE	$E=\frac{1}{2}[AC+BC+AD-BD]$	$E=\frac{1}{2}[A\cancel{C}+B\cancel{C}+A\cancel{D}-B\cancel{D}]$	Full Factorial with 2 replicates
ACE	$E=\frac{1}{2}[AC+BC+AD-BD]$	$E=\frac{1}{2}[AC+\cancel{B}C+A\cancel{D}-\cancel{BD}]$ $E=\frac{1}{2}[AC]\rightarrow \boldsymbol{X_3}=\frac{\mathbf{1}}{\mathbf{2}}[\boldsymbol{X_1X_2}]$	Projection I
ADF	$F=\frac{1}{2}[-AC+BC+AD+BD]$	$F=\frac{1}{2}[-A\cancel{C}+\cancel{BC}+AD+\cancel{B}D]$ $F=\frac{1}{2}[AD]\rightarrow \boldsymbol{X_3}=\frac{\mathbf{1}}{\mathbf{2}}[\boldsymbol{X_1X_2}]$	Projection I
ACF	$F=\frac{1}{2}[-AC+BC+AD+BD]$	$F=\frac{1}{2}[-AC+\cancel{B}C+A\cancel{D}+\cancel{BD}]$ $F=\frac{1}{2}[-AC]\rightarrow \boldsymbol{X_3}=\frac{\mathbf{1}}{\mathbf{2}}[-\boldsymbol{X_1X_2}]$	Projection II
BDE	$E=\frac{1}{2}[AC+BC+AD-BD]$	$E=\frac{1}{2}[\cancel{AC}+B\cancel{C}+\cancel{A}D-BD]$ $E=\frac{1}{2}[-BD]\rightarrow \boldsymbol{X_3}=\frac{\mathbf{1}}{\mathbf{2}}[-\boldsymbol{X_1X_2}]$	Projection II

Table 3-2 Four factor projections based on generating columns for the six factor NC design

Proj $X_1X_2X_3X_4$	Equation/s	Drop columns that are not part of the projection	Calculating other correlations (ignore three factor and higher interactions)	Projection Type
$ABCD$	Columns A, B, C, D form a full factorial	-	-	Full Factorial
$ABEF$	$E = \frac{1}{2}[AC + BC + AD - BD]$ $F = 1/2\,[-AC + BC + AD + BD]$	$E = \frac{1}{2}[A\cancel{C} + B\cancel{C} + A\cancel{D} - B\cancel{D}]$ $F = \frac{1}{2}[-A\cancel{C} + B\cancel{C} + A\cancel{D} + B\cancel{D}]$	-	Full Factorial
$ABCE$	$E = \frac{1}{2}[AC + BC + AD - BD]$	$E = \frac{1}{2}[AC + BC + A\cancel{D} - B\cancel{D}]$ $E = \frac{1}{2}[AC + BC]$ $\rightarrow \boldsymbol{X_4 = \frac{1}{2}[X_1X_3 + X_2X_3]}$	$AE = \frac{1}{2}[AAC + ABC] = \frac{1}{2}[C + ABC]$ $\rightarrow \boldsymbol{X_1X_4 = \frac{1}{2}[X_3]}$ $AEE = A = \frac{1}{2}[CE + ABCE]$ $\rightarrow \boldsymbol{X_1 = \frac{1}{2}[X_3X_4]}$ $AB = \frac{1}{2}[BCE + ABBCE] = \frac{1}{2}[BCE + ACE]$ $\rightarrow \boldsymbol{X_1X_2 = Not\ correlated}$ $AAB = B = \frac{1}{2}[ABCE + AACE] = \frac{1}{2}[ABCE + CE]$ $\rightarrow \boldsymbol{X_2 = \frac{1}{2}[X_3X_4]}$ $BE = \frac{1}{2}[ABCEE + CEE] = \frac{1}{2}[ABC + C]$ $\rightarrow \boldsymbol{X_2X_4 = \frac{1}{2}[X_3]}$	Projection I

Table 3.2 (contd.) Four factor projections based on generating columns for the six factor NC design

Proj $X_1X_2X_3X_4$	Equation/s	Drop columns that are not part of the projection	Calculating other correlations (ignore three factor and higher interactions)	Projection Type
$ABDF$	$F = \frac{1}{2}[-AC + BC + AD + BD]$	$F = \frac{1}{2}[-A\cancel{C} + B\cancel{C} + AD + BD]$ $F = \frac{1}{2}[AD + BD]$ $\rightarrow \boldsymbol{X_4 = \frac{1}{2}[X_1X_3 + X_2X_3]}$	$AF = \frac{1}{2}[AAD + ABD] = \frac{1}{2}[D + ABD]$ $\rightarrow \boldsymbol{X_1X_4 = \frac{1}{2}[X_3]}$ $AFF = A = \frac{1}{2}[DF + ABDF]$ $\rightarrow \boldsymbol{X_1 = \frac{1}{2}[X_3X_4]}$ $AB = \frac{1}{2}[BDF + ABBDF] = \frac{1}{2}[BDF + ADF]$ $\rightarrow \boldsymbol{X_1X_2 = Not\ correlated}$ $AAB = B = \frac{1}{2}[ABDF + AADF] = \frac{1}{2}[ABDF + DF]$ $\rightarrow \boldsymbol{X_2 = \frac{1}{2}[X_3X_4]}$ $BF = \frac{1}{2}[ABDFF + DFF] = \frac{1}{2}[ABD + D]$ $\rightarrow \boldsymbol{X_2X_4 = \frac{1}{2}[X_3]}$	Projection I

The A, B, C and G columns in the eight factor NC design form a full factorial in 16 runs. The columns D, E, F and H can be generated using the following equations.

$$D = \frac{1}{2}[BC + BG + ABC - ABG]$$

$$E = \frac{1}{2}[BC + CG - ABC + ACG]$$

$$F = \frac{1}{2}[CG - ACG + BCG + ABCG]$$

$$H = \frac{1}{2}[BG + ABG + BCG - ABCG]$$

The projections for the eight factor NC design can be studied using these generating equations in a similar manner as was done for the six factor NC design.

3.6. Analysis of NC designs based on projection properties

The projection properties of the NC designs clearly show that all main effects and their interaction can be estimated if the number of active main effects is four or less. Li, Sudarsanam and Frey (2006) confirm three key ideas, effect sparsity, hierarchy and heredity associated with design of experiments through a meta-analysis of 113 datasets from published factorial experiments. But they also saw that exceptions to these ideas are more likely than previously thought. The meta-analysis showed that about 33% of the main effects were active while about 7.4% of the two factor interactions and about 2.2% of the three factor interactions were found to be active. They state in their paper that the data presented suggest that a system with four factors is more likely than not to contain a significant interaction given that $7.4\% \binom{4}{2} + 2.2\% \binom{4}{3} > 50\%$.

If you are able to project the NC designs to four factors you can essentially estimate 70% of the main effects for the six factor case, 60% of the main effects for the seven factor case and 50% of the factors for the eight factor case. It also allows for estimation of 40% of the two factor interactions for the six factor case, 29% of the interactions for the seven factor case and 21% of the interactions for the eight factor case. Therefore one logical approach to analyzing NC designs would be to fit all possible main effects and two-factor interaction models up to 10 terms and evaluate these models using criteria such as R-Sq, R-Sq Adj, Root Mean Square Error (RMSE), the corrected Akaike Information Criteria (AICc) and the Bayesian Information Criteria (BIC). Once the best models are shortlisted the top few can be evaluated using ordinary least squares. Table 3.3 lists the steps involved in this approach.

A second approach to analyzing these designs would be fitting all possible models with up to only four main effects, then evaluating these models using criteria such as R-Sq, R-Sq Adj, RMSE, AICc and BIC and choose the best model(s) from those. The second step would be to include the two-factor interactions into the best main effects models chosen in step one. One way to add the two-factor interactions would be to fit a model with the main effects and all interactions using ordinary least squares. The insignificant terms can then be eliminated using p-values for the effects. Another approach to add the two-factor interactions would be to use stepwise regression on the top models by forcing the main effects into the model and then using the stepwise algorithm to add the two-factor interactions to get the best fit. Table 3.3 lists the steps to be followed in this approach.

Table 3-3 Analysis Methods

Analysis Method 1	Analysis Method 2
• Fit all possible one to ten term models (both main effects and two factor interactions) • Evaluate the models using R^2, R^2-Adj, RMSE, AICc or BIC • Choose the top few models for further analysis • Use ordinary Least Squares (OLS) to fit the model and pick the best one	• Fit all possible main effects only models up to four factors • Evaluate the models using R^2, R^2-Adj, RMSE, AICc or BIC • Choose the top few models for further analysis • Use OLS to fit the model and pick the best one **OR** • Use stepwise regression to fit the model by forcing the main effects and then using the stepwise algorithm to add the two factor interactions

3.7. Example 1

These approaches are illustrated using an example. Montgomery (2012) presents an example of the regular 2^{6-2} resolution IV design applied to a photoresist application process. The response variable is thickness and the design factors are X_1 = speed RPM, X_2 = acceleration, X_3 = volume, X_4 = time, X_5 = resist viscosity, and X_6 = exhaust rate. Montgomery (2013) found that the main effects X_1, X_2, X_3 and X_5 along with the alias chain involving the X_3X_5 interaction were active. He used a fold-over technique to identify the significant X_3X_5 interaction. Jones and Montgomery (2010) simulated the response variable for this experiment using the six variable NC design. This simulated data is used as an example to illustrate the two methods described above. In constructing the simulation X_1, X_2, X_3 and X_5 were assumed to be the significant main effects and the X_3X_5 interaction was assumed to be the active interaction. The experiment is shown in Table 3-4.

3.7.1. Analysis Method I

This experiment was analyzed using JMP. All possible one to ten term models (excluding and including interactions) were fit using ordinary least squares regression and evaluated using the AICc criterion and the RSquare values. The top three models for the one term model to the ten term model cases are listed in Table 3-5. The top two models based on the AICc criterion and the R-Square value are model numbers 13 (X_1, X_2, X_3, X_5, X_3X_5) and 16 (X_1, X_2, X_3, X_5, X_2X_5, X_3X_5). These two models were selected for further analysis. This indicated that the interaction X_2X_5 in model number 16 has a p-value of 0.1133 and can be removed from the model resulting in model 13. The two model fits are shown in Figure 3.7 and Figure 3.8. The results clearly show that the better model using this analysis methodology is the one which includes the main effects X_1, X_2, X_3 and X_5 along with the interaction X_3X_5 which is the correct model.

Table 3-4 The no-confounding design for the photoresist application experiment

Run	**X_1**	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**	**Thickness**
1	1	1	1	1	1	1	4,494
2	1	1	-1	-1	-1	-1	4,592
3	-1	-1	1	1	-1	-1	4,357
4	-1	-1	-1	-1	1	1	4,489
5	1	1	1	-1	1	-1	4,513
6	1	1	-1	1	-1	1	4,483
7	-1	-1	1	-1	-1	1	4,288
8	-1	-1	-1	1	1	-1	4,448
9	1	-1	1	1	1	-1	4,691
10	1	-1	-1	-1	-1	1	4,671
11	-1	1	1	1	-1	1	4,219
12	-1	1	-1	-1	1	-1	4,271
13	1	-1	1	-1	-1	-1	4,530
14	1	-1	-1	1	1	1	4,632
15	-1	1	1	-1	1	1	4,337
16	-1	1	-1	1	-1	-1	4,391

Table 3-5 All Possible term Models up to ten terms (main effects and two-factor interaction) comparison

No.	Model	No of terms	RSq	RMSE	AICc	BIC
1	X1	1	0.66	86.67	194.06	194.38
2	X2	1	0.13	138.47	209.05	209.37
3	X3	1	0.06	143.99	210.30	210.62
4	X1, X2	2	0.79	70.47	189.89	189.34
5	X1, X3	2	0.72	81.52	194.55	194.00
6	X1, X5	2	0.68	86.72	196.53	195.98
7	X1, X2, X3	3	0.85	61.78	188.76	186.62
8	X1, X2, X5	3	0.81	69.02	192.31	190.17
9	X1, X2, X6	3	0.80	72.19	193.74	191.61
10	X1, X2, X3, X5	4	0.88	59.09	191.27	186.57
11	X1, X2, X3, X1*X3	4	0.86	61.76	192.69	187.99
12	X1, X2, X5, X2*X5	4	0.86	62.18	192.91	188.21
13	**X1, X2, X3, X5, X3*X5**	**5**	**0.95**	**37.98**	**182.27**	**173.68**
14	X1, X3, X4, X6, X4*X6	5	0.92	50.40	191.32	182.73
15	X1, X2, X3, X5, X2*X5	5	0.89	58.93	196.33	187.74
16	**X1, X2, X3, X5, X2*X5, X3*X5**	**6**	**0.97**	**34.56**	**186.13**	**171.74**
17	X1, X2, X3, X5, X6, X3*X5	6	0.96	37.11	188.42	174.03
18	X1, X2, X3, X5, X1*X3, X3*X5	6	0.95	39.36	190.30	175.91
19	X1, X2, X3, X5, X6, X2*X5, X3*X5	7	0.97	33.02	194.22	171.18
20	X1, X2, X3, X5, X6, X3*X5, X5*X6	7	0.97	33.27	194.47	171.42
21	X1, X2, X3, X4, X5, X2*X5, X3*X5	7	0.97	34.43	195.56	172.52
22	X1, X2,X3,X5 X6, X2*X5, X3*X5, X5*X6	8	0.98	27.21	201.89	165.62
23	X1,X2,X3,X4, X5, X1*X4, X2*X5, X3*X5	8	0.98	28.98	203.91	167.64
24	X1,X2,X3,X5,X6, X1*X6, X3*X5, X5*X6	8	0.98	29.21	204.17	167.89
25	X1,X2,X3,X5,X1*X4,X2*X5,X3*X4, X3*X5, X5*X6	9	0.99	18.08	210.34	152.84
26	X1, X2, X3, X5, X1*X4, X2*X5, X3*X4, X4*X6, X5*X6	9	0.99	18.08	210.34	152.84
27	X1, X3, X5, X1*X4, X2*X5, X3*X4, X3*X5, X4*X6, X5*X6	9	0.99	18.08	210.34	152.84
28	X1, X2, X3, X4, X5, X1*X4, X1*X6, X3*X4, X3*X5, X5*X6	10	0.99	11.83	233.85	139.12
29	X1, X2, X3, X4, X5, X1*X4, X1*X6, X3*X4, X4*X6, X5*X6	10	0.99	11.83	233.85	139.12
30	X1, X2, X3, X4, X5, X1*X4, X2*X5, X3*X4, X3*X5, X5*X6	10	0.99	11.83	233.85	139.12

Response Y

Actual by Predicted Plot

Summary of Fit

RSquare	0.965223
RSquare Adj	0.942039
Root Mean Square Error	34.55551
Mean of Response	4462.875
Observations (or Sum Wgts)	16

AICc	BIC
186.1338	171.7431

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	298275.00	49712.5	41.6324
Error	9	10746.75	1194.1	**Prob > F**
C. Total	15	309021.75		<.0001*

Lack Of Fit

Parameter Estimates

Effect Tests

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
X1	85.5	10.58042	8.08	<.0001*
X2	-77.75	10.58042	-7.35	<.0001*
X3*X5	54.75	12.21722	4.48	0.0015*
X3	-25.5	9.975317	-2.56	0.0309*
X5	21.5	8.638878	2.49	0.0345*
X2*X5	-17.5	9.975317	-1.75	0.1133

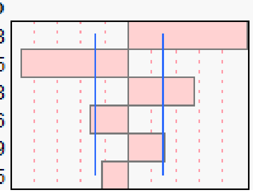
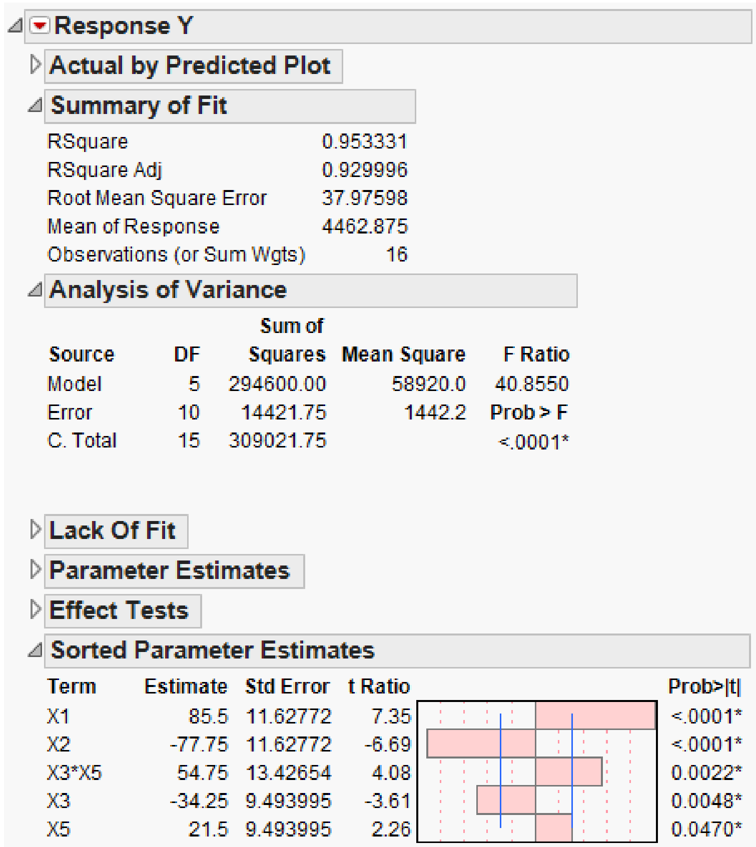


Figure 3.7 Model fit for X_1, X_2, X_3, X_5, X_2X_5, X_3X_5



Response Y

Actual by Predicted Plot

Summary of Fit

RSquare	0.953331
RSquare Adj	0.929996
Root Mean Square Error	37.97598
Mean of Response	4462.875
Observations (or Sum Wgts)	16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	294600.00	58920.0	40.8550
Error	10	14421.75	1442.2	Prob > F
C. Total	15	309021.75		<.0001*

Lack Of Fit

Parameter Estimates

Effect Tests

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
X1	85.5	11.62772	7.35	<.0001*
X2	-77.75	11.62772	-6.69	<.0001*
X3*X5	54.75	13.42654	4.08	0.0022*
X3	-34.25	9.493995	-3.61	0.0048*
X5	21.5	9.493995	2.26	0.0470*

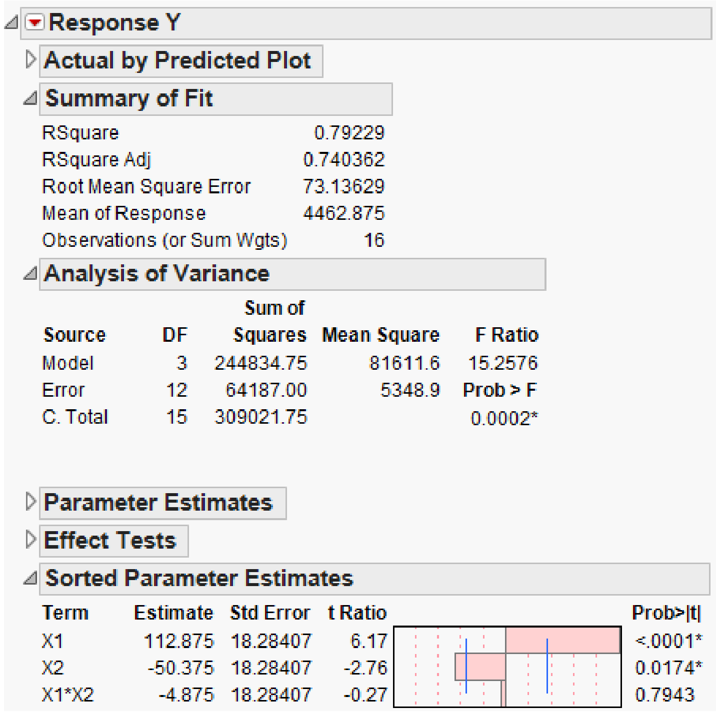
Figure 3.8 Model fit for X_1, X_2, X_3, X_5, X_3X_5

3.7.2. Analysis Method II

The same experiment was analyzed using the second method which entails fitting all the main effects only models and then adding the two factor interactions using one of two methods (OLS or stepwise regression). The results from fitting all the main effects models with up to four factors and their R-Sq, RMSE, AICc and BIC values are listed in Table 3-6. The top three models with one to four main effects are listed in Table 3-6. The top three models were chosen and then the two factor interactions were added using both OLS and stepwise regression. The result using both approaches is the same. For the two term model with X_1, X_2 when the interaction is added the model fit is not improved as the interaction effect is not significant. In the case of the three term model with X_1, X_2, X_3, when the interactions X_1X_2, X_1X_3 and X_2X_3 are added to the model, none of the interactions are significant and therefore does not lead to a better fit then the main effects only model. Whereas in the four factor model case when the strong heredity interactions are included in the model, the model fit is improved when the X_3X_5 interaction is added to the original X_1, X_2, X_3, X_5 main effects only model. The results from fitting these three models are shown in Figure 3.9, Figure 3.10 and Figure 3.11. The final model chosen based on the above analysis is again the one with X_1, X_2, X_3, X_5 and X_3X_5 interaction. Both analysis methods lead to the same result.

Table 3-6 All Main Effects only Models comparison

No.	Model	Number of terms in model	RSquare	RMSE	AICc	BIC
1	X1	1	0.66	86.67	194.06	194.38
2	X2	1	0.13	138.47	209.05	209.37
3	X3	1	0.06	143.99	210.30	210.62
4	X1,X2	2	0.79	70.47	189.89	189.34
5	X1,X3	2	0.72	81.52	194.55	194.00
6	X1,X5	2	0.68	86.72	196.53	195.98
7	X1,X2,X3	3	0.85	61.78	188.76	186.62
8	X1,X2,X5	3	0.81	69.02	192.31	190.17
9	X1,X2,X6	3	0.80	72.19	193.74	191.61
10	X1,X2,X3,X5	4	0.88	59.09	191.27	186.57
11	X1,X2,X3,X6	4	0.86	63.08	193.37	188.67
12	X1,X2,X3,X4	4	0.85	64.50	194.08	189.38



Response Y

Actual by Predicted Plot

Summary of Fit

RSquare	0.79229
RSquare Adj	0.740362
Root Mean Square Error	73.13629
Mean of Response	4462.875
Observations (or Sum Wgts)	16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	244834.75	81611.6	15.2576
Error	12	64187.00	5348.9	Prob > F
C. Total	15	309021.75		0.0002*

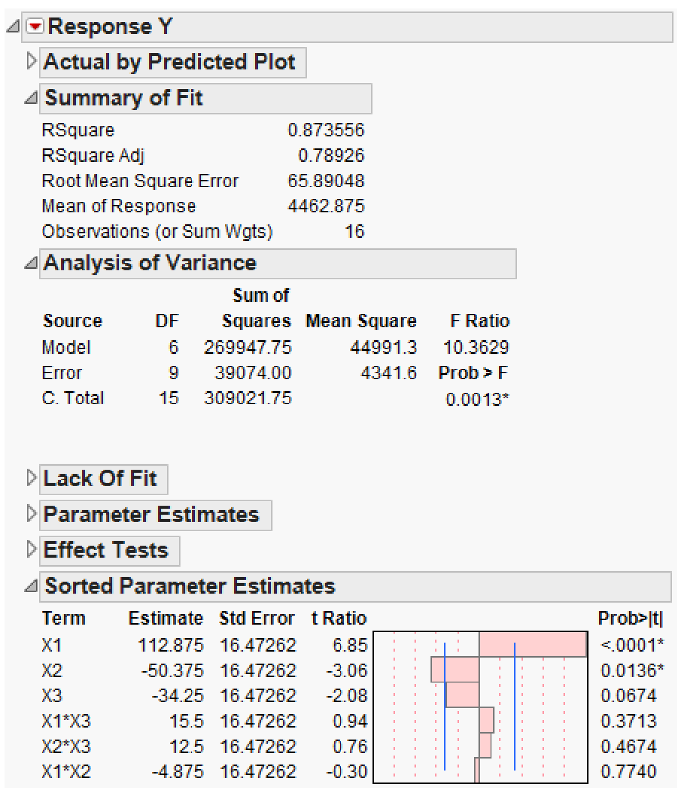
Parameter Estimates

Effect Tests

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
X1	112.875	18.28407	6.17	<.0001*
X2	-50.375	18.28407	-2.76	0.0174*
X1*X2	-4.875	18.28407	-0.27	0.7943

Figure 3.9 Two factor main effects model with interactions included



Response Y

Actual by Predicted Plot

Summary of Fit

RSquare	0.873556
RSquare Adj	0.78926
Root Mean Square Error	65.89048
Mean of Response	4462.875
Observations (or Sum Wgts)	16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	6	269947.75	44991.3	10.3629
Error	9	39074.00	4341.6	Prob > F
C. Total	15	309021.75		0.0013*

Lack Of Fit

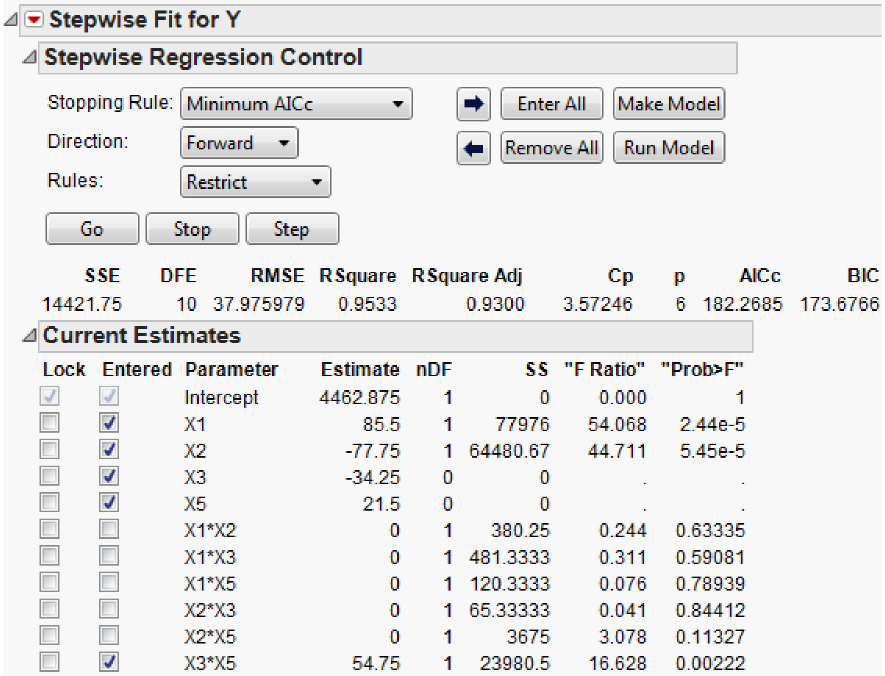
Parameter Estimates

Effect Tests

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
X1	112.875	16.47262	6.85	<.0001*
X2	-50.375	16.47262	-3.06	0.0136*
X3	-34.25	16.47262	-2.08	0.0674
X1*X3	15.5	16.47262	0.94	0.3713
X2*X3	12.5	16.47262	0.76	0.4674
X1*X2	-4.875	16.47262	-0.30	0.7740

Figure 3.10 Three factor main effects model with interactions included



SSE	DFE	RMSE	RSquare	RSquare Adj	Cp	p	AICc	BIC
14421.75	10	37.975979	0.9533	0.9300	3.57246	6	182.2685	173.6766

Lock	Entered	Parameter	Estimate	nDF	SS	"F Ratio"	"Prob>F"
☑	☑	Intercept	4462.875	1	0	0.000	1
☐	☑	X1	85.5	1	77976	54.068	2.44e-5
☐	☑	X2	-77.75	1	64480.67	44.711	5.45e-5
☐	☑	X3	-34.25	0	0	.	.
☐	☑	X5	21.5	0	0	.	.
☐	☐	X1*X2	0	1	380.25	0.244	0.63335
☐	☐	X1*X3	0	1	481.3333	0.311	0.59081
☐	☐	X1*X5	0	1	120.3333	0.076	0.78939
☐	☐	X2*X3	0	1	65.33333	0.041	0.84412
☐	☐	X2*X5	0	1	3675	3.078	0.11327
☐	☑	X3*X5	54.75	1	23980.5	16.628	0.00222

Figure 3.11 Four factor main effects model with interactions included

3.8. Example 2

The second example that demonstrated here is from Junqua, Duran, Gancet and Goulas (1997), where they study microbial transglutaminase production using a designed experiment approach. In the example they study five factors casein (X1), glycerol (X2), peptones (X3), yeast extract (X4) and oligoelements (X5). Two dummy variables were added to extend the design to a seven variable design. The original experiment was run as a 32 run full factorial experiment with five center runs. The results from the original experiment was used to simulate data for the NC seven factor design in 16 runs with the same coefficients and RMSE as the original experiment. The analysis of the original experiment showed that X1, X2, X4 and X1X2 are the significant effects. The two analysis methods described in section 6 are used to analyze this simulated experiment. The simulated dataset is shown in Table 3.7.

Table 3-7 The NC design for microbial transglutaminase production experiment

Run	X_1	X_2	X_3	X_4	X_5	X_6	X_7	Growth
1	1	1	1	1	1	1	1	0.1694404385
2	1	1	1	-1	-1	-1	-1	0.1557483244
3	1	1	-1	1	1	-1	-1	0.1694441029
4	1	1	-1	-1	-1	1	1	0.1556024284
5	1	-1	1	1	-1	1	-1	0.0459173274
6	1	-1	1	-1	1	-1	1	0.0320189712
7	1	-1	-1	1	-1	-1	1	0.0455156956
8	1	-1	-1	-1	1	1	-1	0.0318232652
9	-1	1	1	1	1	1	-1	0.0812673458
10	-1	1	1	-1	-1	-1	1	0.0679025994
11	-1	1	-1	1	-1	1	1	0.0817517700
12	-1	1	-1	-1	1	-1	-1	0.0672482974
13	-1	-1	1	1	-1	-1	-1	0.0177728542
14	-1	-1	1	-1	1	1	1	0.0031316223
15	-1	-1	-1	1	1	-1	1	0.0176315712
16	-1	-1	-1	-1	-1	1	-1	0.0033230487

3.8.1. Analysis Method I

JMP was used to analyze this experiment. All possible one to ten term models (excluding and including interactions) were fit using ordinary least squares regression and evaluated using the AICc criterion and the RSquare values. The top three models for the one term model to the ten term model cases are listed in Table 3-8.

The top two models based on the AICc criterion and the R-Sq values are model numbers 7 (X_1, X_2, X_1X_2) and 10 (X_1, X_2, X_4, X_1X_2). These two models were selected for further analysis. This indicated that the main effect X_4 has a p-value of 0.0349 and is added to the final model resulting in model 10. The final model fit is shown in Figure 3.12. The model fit identified using analysis method 1 is identical to the true model.

Table 3-8 All Possible subsets Models up to ten terms (main effects only and main effects and two-factor interaction) comparison

No.	Model	No of terms	R - Sq	RMSE	AICc	BIC
1	X2	1	0.66	0.04	-55.34	-55.0
2	X1	1	0.25	0.05	-42.61	-42.3
3	X4	1	0.01	0.06	-38.14	-37.8
4	X1,X2	2	0.92	0.02	-74.34	-74.9
5	X2,X4	2	0.68	0.04	-52.42	-52.9
6	X2,X5	2	0.66	0.04	-51.70	-52.3
7	**X1,X2,X1*X2**	**3**	**0.99**	**0.01**	**-97.37**	**-99.5**
8	X1,X2,X4	3	0.93	0.02	-73.16	-75.3
9	X1,X2,X5	3	0.92	0.02	-69.98	-72.1
10	**X1,X2,X4,X1*X2**	**4**	**1.00**	**0.00**	**-202.32**	**-207.0**
11	X1,X2,X5,X1*X2	4	0.99	0.01	-92.04	-96.7
12	X1,X2,X6,X1*X2	4	0.99	0.01	-92.04	-96.7
13	X1,X2,X4,X1*X2,X1*X4	5	1.00	0.00	-199.74	-208.3
14	X1,X2,X4,X5,X1*X2	5	1.00	0.00	-198.90	-207.5
15	X1,X2,X4,X1*X2,X2*X4	5	1.00	0.00	-197.37	-205.9
16	X1,X2,X4,X5,X1*X2,X2*X5	6	1.00	0.00	-197.22	-211.6
17	X1,X2,X4,X5,X1*X2,X1*X4	6	1.00	0.00	-195.49	-209.9
18	X1,X2,X4,X5,X1*X2,X1*X5	6	1.00	0.00	-193.74	-208.1
19	X1,X2,X3,X4,X6,X1*X2,X3*X6	7	1.00	0.00	-195.08	-218.1
20	X1,X2,X4,X5,X1*X2,X1*X5,X2*X5	7	1.00	0.00	-191.41	-214.5
21	X1,X2,X4,X5,X1*X2,X1*X4,X1*X5	7	1.00	0.00	-189.00	-212.0
22	X1,X2,X4,X5,X1*X2,X1*X5,X2*X4, X2*X5	8	1.00	0.00	-186.24	-222.5
23	X1,X2,X3,X4,X6,X1*X2,X1*X6,X3*X6	8	1.00	0.00	-185.79	-222.1
24	X1,X2,X4,X6,X7,X1*X2,X1*X4,X2*X7	8	1.00	0.00	-182.45	-218.7
25	X1,X2,X3,X4,X7,X1*X2,X3*X5,X3*X6, X5*X6	9	1.00	0.00	-205.21	-262.7
26	X1,X2,X4,X7,X1*X2,X1*X3,X3*X5, X3*X6,X4*X7	9	1.00	0.00	-198.19	-255.7
27	X1,X2,X4,X7,X1*X2,X3*X5,X3*X6, X4*X6,X4*X7	9	1.00	0.00	-197.96	-255.5
28	X1,X2,X3,X4,X7,X1*X2,X2*X3, X3*X5,X3*X6,X5*X6	10	1.00	0.00	-183.83	-278.6
29	X1,X2,X4,X7,X1*X2,X1*X3,X2*X3, X3*X5,X3*X6, X4*X7	10	1.00	0.00	-179.63	-274.4
30	X1,X2,X3,X4,X7,X1*X2,X3*X5,X3*X6, X4*X5, X5*X6	10	1.00	0.00	-179.37	-274.1

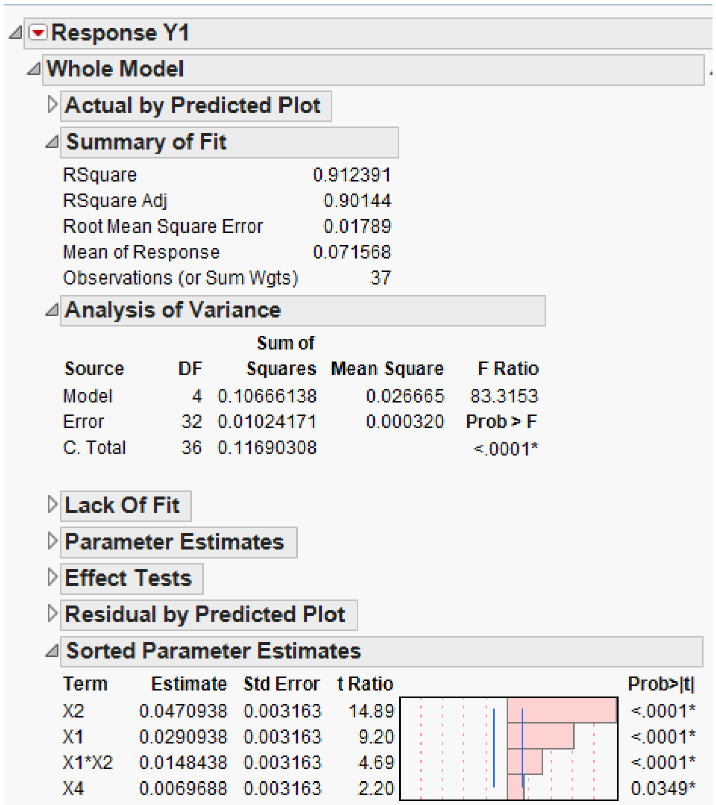
3.8.2. Analysis Method II

JMP was used to analyze this experiment using analysis method II. All possible one to four term main effects models were fit using the all possible regressions method. The next step was to include all the interaction effects for the significant main effects. The results from using this method are listed in Table 3-9.

The top two models were chosen and then the two factor interactions were added to the models. In the case of the two main effects model, the model fit improves to an RSquare Adj value of 0.981. But when the three term model (X1, X2, X4) is used with interactions, the model with the terms X1, X2, X4 & X1X2 results in a better model with an RSq - Adj value of 0.99 and a lower AICc value. This again matches with the true model. Using both analysis methods, the results match the simulated true model thereby showing that both these analysis methods work well when the NC design is used to identify the main effects and then any interactions effects involving these main effects that are significant.

3.9. Additional steps to consider for analysis

From the projection properties of the six, seven and eight factor NC designs, it can be seen that up to 11 term models can be fit as there are 12 distinct designs points when the designs are projected to four factors. Therefore once the initial models are fit using the above analysis methods, you can add up to a total of 11 terms (including one interaction term) to the final model if it improves the fit of the model. This can be done by fixing the terms identified from the previous steps in the model and then adding more terms if it does improve the fit. In the case of the above two examples, adding terms to the existing models did not result in a better fit. The best models were still the ones identified using the analysis methods 1 and 2 detailed in the previous sections.



Response Y1

Whole Model

Actual by Predicted Plot

Summary of Fit

RSquare	0.912391
RSquare Adj	0.90144
Root Mean Square Error	0.01789
Mean of Response	0.071568
Observations (or Sum Wgts)	37

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	0.10666138	0.026665	83.3153
Error	32	0.01024171	0.000320	Prob > F
C. Total	36	0.11690308		<.0001*

Lack Of Fit

Parameter Estimates

Effect Tests

Residual by Predicted Plot

Sorted Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
X2	0.0470938	0.003163	14.89	<.0001*
X1	0.0290938	0.003163	9.20	<.0001*
X1*X2	0.0148438	0.003163	4.69	<.0001*
X4	0.0069688	0.003163	2.20	0.0349*

Figure 3.12 Model fit for X_1, X_2, X_4, X_1X_2

Table 3-9 All Main Effects only Models comparison

No.	Model	Number of terms in model	RSquare	RMSE	AICc	BIC
1	X2	1	0.6636	0.0357	-55.3355	-55.0177
2	X1	1	0.2547	0.0532	-42.6105	-42.2927
3	X4	1	0.0147	0.0612	-38.1437	-37.8259
4	**X1,X2**	**2**	**0.9183**	**0.0183**	**-74.3439**	**-74.8899**
5	X2,X4	2	0.6783	0.0363	-52.4156	-52.9616
6	X2,X5	2	0.6636	0.0371	-51.6993	-52.2453
7	**X1,X2,X4**	**3**	**0.9330**	**0.0172**	**-73.1616**	**-75.2986**
8	X1,X2,X5	3	0.9183	0.0190	-69.9808	-72.1179
9	X1,X2,X6	3	0.9183	0.0190	-69.9805	-72.1176
10	X1,X2,X4,X5	4	0.9330	0.0180	-67.8289	-72.5267
11	X1,X2,X4,X6	4	0.9330	0.0180	-67.8285	-72.5263
12	X1,X2,X3,X4	4	0.9330	0.0180	-67.8285	-72.5263

3.10. Conclusion

The regular fractional factorial designs with six, seven or eight factors in 16 runs are widely used. However due to the complete confounding of the two-factor interactions with one another, these designs often require the experimenter to perform runs to resolve ambiguities whenever any of the two-factor interactions are identified as being active. The projection properties of the NC designs show that these designs allow for the estimation of all main effects along with some of the two-factor interactions since there is no complete confounding in these designs.

Two intuitive approaches to analyzing these designs based on the projection properties are presented. Systems with four active terms (main effects) are likely to have a significant interaction. Therefore being able to estimate the two-factor interactions without the need for design augmentation is a desirable characteristic. Based on the projection properties of the NC designs all the main effects and their interactions can be estimated for up to four active factors or in other words models with up to 11 terms (including the intercept) can be fit as there are 12 distinct designs points for the four factor projections of these designs. The two examples illustrate that both these methods identify the correct active terms.

Chapter 4

DESIGN, PROJECTION PROPERTIES AND ANALYSIS OF NO-CONFOUNDING ALTERNATIVES TO RESOLUTION III SCREENING DESIGNS FOR 9 – 14 FACTORS IN 16 RUNS

4.1. Introduction

For between 9 and 14 factors the regular minimum aberration resolution III designs are widely used. Montgomery (2013) gives the generators for these designs; for example, if there are k = 9 factors the generators are E = ABC, F = BCD, G = ACD, H = ABD, and J = ABCD. This produces a 16-run design with nine single-degree-of-freedom alias chains composed of a single main effect and one or more two-factor interactions and seven single-degree-of-freedom alias chains composed entirely of two-factor interactions assuming that all interaction of order three and higher are negligible. For example the alias chain for factor A is A = FJ, for J it is J = DE = AF = BG = CH, and for AB it is AB = CE = FG = DH. These are all regular designs; that is, the effects in any alias chain are completely confounded (the constants multiplying each effect are ± 1).

Because the regular resolution III designs are completely confounded, experimenters often end up with ambiguous conclusions about which main effects and two-factor interactions are important. Resolving these ambiguities requires either additional experimentation (such as use of a fold-over design to augment the original fraction) or assumptions about which effects are important or external process knowledge. None of these alternatives are entirely satisfactory and experimenters would like to avoid either the need to expend resources for a follow-up study, or make assumptions, or rely on experienced-based process knowledge.

It is possible to reduce the risk of analytical ambiguity by using a specific orthogonal but nonregular fractional factorial design. Our proposed designs for 9 – 14 factors in 16 runs have no complete confounding between main effects and two-factor interactions and pairs of two-factor interactions. These designs are preferred and are recommended as alternatives for the usual regular minimum aberration resolution III fractional factorials. In subsequent sections, a metric to evaluate these fractional factorial designs is presented, and it is used to obtain the choices for the nonregular 16-run fractional factorials through the use of a variation of the D-optimality criterion. The projection properties of these designs are presented when projected to three and four factors and discuss analysis strategy for these designs. An example is also presented that illustrates the potential usefulness of these designs and the effectiveness of the analysis method.

4.2. Literature review

Plackett and Burman (1946) introduced nonregular orthogonal designs for sample sizes that are a multiple of four but not powers of two. Hall (1961) identified five non-isomorphic orthogonal designs for 15 factors in 16 runs. Contemporaneously with Hall's work, Box and Hunter (1961) introduced the regular fractional factorial designs that became the standard tools for factor screening. Sun et al. (2002) catalogued all the non-isomorphic projections of the Hall designs. Li et al. (2003) used this catalogue to identify the best designs to use in case there is a need for a foldover. For each of these designs they provide the columns to use for folding and the resulting resolution of the combined design. Loeppky et al. (2007) also used this catalogue to identify the best designs to use assuming that a small number of factors are active and the experimenter wished to fit a model including the active main effects and all two-factor interactions involving factors having active main effects.

Jones and Montgomery (2010) proposed 16-run nonregular orthogonal designs for 6 – 8 factors that are alternatives to the usual regular resolution IV minimum aberration fractions. These designs are projections of the Hall designs created by selecting specific sets of columns. Because there is no complete confounding of two-factor interactions, the authors referred to these designs as no-confounding designs. Johnson and Jones (2011) show how these designs can be found by using a column generator approach that is similar to that used for regular designs. The work in Jones and Montgomery (2010) is presented by developing no-confounding designs for 9 – 14 factors in 16 runs that are good alternatives to the usual minimum aberration resolution III designs when there are only a few main effects and two-factor interactions that are important.

4.3. Design Evaluation and Construction

The alias matrix is a generalization of the confounding pattern that is useful for comparing nonregular designs. Suppose that we plan to fit the model

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}$$

where $\mathbf{X}_1$ is the design matrix for the experiment that has been conducted expanded to model form, $\boldsymbol{\beta}_1$ is the corresponding vector of model parameters, and $\boldsymbol{\varepsilon}$ is the usual vector of NID(0, σ^2) random errors. However, the true model is

$$\mathbf{y} = \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}$$

where the columns of $\mathbf{X}_2$ contain additional factors not included in the original model (such as interactions) and $\boldsymbol{\beta}_2$ is the corresponding vector of model parameters. It is straightforward to show that the expected value of $\hat{\boldsymbol{\beta}}_1$, the least squares estimate of $\boldsymbol{\beta}_1$, is

$$E(\hat{\boldsymbol{\beta}}_1) = \boldsymbol{\beta}_1 + (\mathbf{X}_1\mathbf{X}_1)^{-1}(\mathbf{X}_1\mathbf{X}_2)\boldsymbol{\beta}_2 = \boldsymbol{\beta}_1 + \mathbf{A}\boldsymbol{\beta}_2$$

The alias matrix $\mathbf{A} = (\mathbf{X}_1\mathbf{X}_1)^{-1}(\mathbf{X}_1\mathbf{X}_2)$ shows how estimates of terms in the fitted model are biased by active terms that are not in the fitted model. Each row of $\mathbf{A}$ is associated with a parameter in the fitted model. Non-zero elements in a row of $\mathbf{A}$ show the degree of biasing of the fitted model parameter due to terms associated with the columns of $\mathbf{X}_2$.

In a regular design, an arbitrary entry in the alias matrix, A_{ij}, is either 0 or ±1. If A_{ij} is 0 then the ith column of $\mathbf{X}_1$ is orthogonal to the jth column of $\mathbf{X}_2$. Otherwise if A_{ij} is ±1, then the ith column of $\mathbf{X}_1$ and the jth column of $\mathbf{X}_2$ are perfectly correlated.

For nonregular designs the aliasing is more complex. If $\mathbf{X}_1$ is the design matrix for the main effects model and $\mathbf{X}_2$ is the design matrix for the two-factor interactions, then the entries of the alias matrix for orthogonal nonregular designs for 16 runs take the values 0, ±1 or ±0.5. A small subset of these designs have no entries of ±1.

Table 4-1 Number of Non-isomorphic Nonregular 16-run Designs

Number of Factors	Number of Non-isomorphic Designs
6	27
7	55
8	80
9	87
10	78
11	58
12	36
13	18
14	10

Bursztyn and Steinberg (2006) propose using the trace of $\mathbf{AA}'$ as a scalar measure of the total bias in a design. This is just the sum of squares of all of the elements of the alias matrix. They use this as a means for comparing designs for computer simulations but this measure works equally well for ranking competitive screening designs. The no-confounding designs in Jones and Montgomery (2010) all minimize the trace of $\mathbf{AA}'$. They found these designs by enumeration of all of the non-isomorphic nonregular 16-run designs. By non-isomorphic, I mean that one cannot obtain one of these designs from another one by permuting the rows or columns or by changing the labels of the factor. Table 4-1 shows the number of these designs for 6 – 14 factors.

Jones and Nachtsheim (2011) have proposed a design optimality criterion that effectively minimizes the aliasing in a design. They propose minimizing the trace of $\mathbf{AA}'$ subject to a lower bound on the D-efficiency of the design. They use a modification of the coordinate exchange algorithm for design construction. Formally, they choose a design to solve the following problem:

$$\begin{gathered}\text{Min } trace(\mathbf{AA}') \\ \text{subject to:} \\ D_{Eff} \geq l_D\end{gathered}$$

where D_{Eff} is the D-efficiency of the design and l_D is the lower bound on D-efficiency.

All of the designs in Jones and Montgomery (2010) satisfy this criterion with D_{Eff} of 100%, because all of their designs are orthogonal for the first-order model. This method also produces the recommended designs for 9 – 14 factors given in the next section. These designs are also first-order orthogonal so they are 100% D-efficient.

4.4. Recommended Designs

I now provide the recommended no-confounding designs. For nine factors the recommended design is in Table 4-2. Jones and Montgomery (2010) introduce the correlation matrix or cell plot as a convenient display of the alias relationships in a fractional factorial design. Figure 4.1 shows the correlation matrix for the principal fraction of this design. Notice that the design is orthogonal for the main effects and that no main effect is completely confounded with a two-factor interaction. All of the correlations between main effects and two-factor interactions are ± 0.5. Tables 4.3 through 4.7 present the design matrices for 10 – 14 factors and Figure 4.2 through Figure 4.6 present the associated correlation matrices. All designs are first-order orthogonal (100% D-efficient) and the correlations between main effects and two-factor interactions are ± 0.5.

Table 4-2 Recommended 16-run 9-factor no-confounding design

Run	A	B	C	D	E	F	G	H	J
1	-1	-1	-1	-1	-1	-1	1	-1	1
2	-1	-1	-1	1	-1	1	-1	1	-1
3	-1	-1	1	-1	1	1	1	1	-1
4	-1	-1	1	1	1	-1	-1	-1	1
5	-1	1	-1	-1	1	1	-1	1	1
6	-1	1	-1	1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	-1	-1	1	-1
8	-1	1	1	1	-1	1	1	-1	1
9	1	-1	-1	-1	1	-1	-1	-1	-1
10	1	-1	-1	1	1	1	1	1	1
11	1	-1	1	-1	-1	1	-1	-1	1
12	1	-1	1	1	-1	-1	1	1	-1
13	1	1	-1	-1	-1	1	1	-1	-1
14	1	1	-1	1	-1	-1	-1	1	1
15	1	1	1	-1	1	-1	1	1	1
16	1	1	1	1	1	1	-1	-1	-1

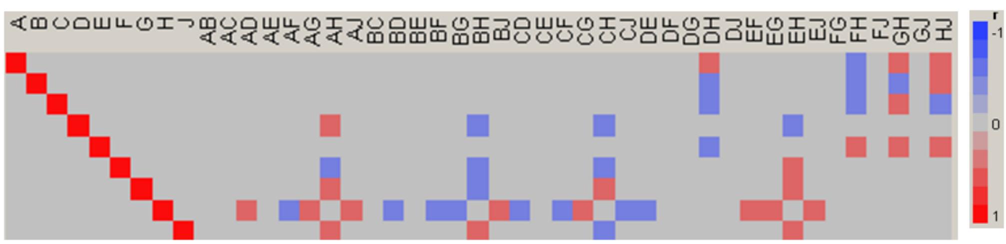


Figure 4.1 Correlations of Main Effects and Two-Factor Interactions, NC Design

for 9 Factors in 16 Runs

Table 4-3: Recommended 16-run 10-factor no-confounding design

Run	A	B	C	D	E	F	G	H	J	K
1	-1	-1	-1	-1	1	-1	-1	1	-1	1
2	-1	-1	-1	1	1	1	-1	-1	1	1
3	-1	-1	1	-1	-1	1	1	1	1	1
4	-1	-1	1	-1	1	-1	1	-1	1	-1
5	-1	1	-1	1	-1	1	1	1	-1	1
6	-1	1	-1	1	1	-1	1	-1	-1	-1
7	-1	1	1	-1	-1	-1	-1	1	-1	-1
8	-1	1	1	1	-1	1	-1	-1	1	-1
9	1	-1	-1	-1	-1	1	1	-1	-1	-1
10	1	-1	-1	1	-1	-1	-1	1	1	-1
11	1	-1	1	1	-1	-1	1	-1	-1	1
12	1	-1	1	1	1	1	-1	1	-1	-1
13	1	1	-1	-1	-1	-1	-1	-1	1	1
14	1	1	-1	-1	1	1	1	1	1	-1
15	1	1	1	-1	1	1	-1	-1	-1	1
16	1	1	1	1	1	-1	1	1	1	1

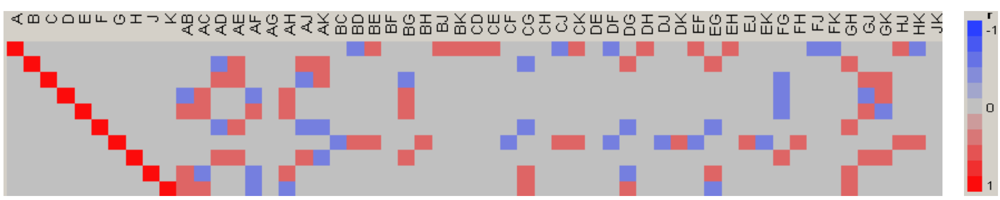


Figure 4.2 Correlations of Main Effects and Two-Factor Interactions, no-confounding Design for 10 Factors in 16 Runs

Table 4-4 Recommended 16-run 11-factor no-confounding design

Run	A	B	C	D	E	F	G	H	J	K	L
1	-1	-1	-1	1	1	-1	-1	-1	1	-1	1
2	-1	-1	1	-1	-1	-1	1	-1	1	-1	-1
3	-1	-1	1	-1	1	1	-1	1	-1	-1	-1
4	-1	-1	1	1	-1	1	1	1	-1	1	1
5	-1	1	-1	-1	-1	-1	-1	-1	-1	1	1
6	-1	1	-1	1	-1	1	1	-1	-1	-1	-1
7	-1	1	-1	1	1	1	-1	1	1	1	-1
8	-1	1	1	-1	1	-1	1	1	1	1	1
9	1	-1	-1	-1	-1	1	-1	1	1	-1	1
10	1	-1	-1	-1	1	1	1	-1	-1	1	1
11	1	-1	-1	1	-1	-1	1	1	1	1	-1
12	1	-1	1	1	1	-1	-1	-1	-1	1	-1
13	1	1	-1	-1	1	-1	1	1	-1	-1	-1
14	1	1	1	-1	-1	1	-1	-1	1	1	-1
15	1	1	1	1	-1	-1	-1	1	-1	-1	1
16	1	1	1	1	1	1	1	-1	1	-1	1

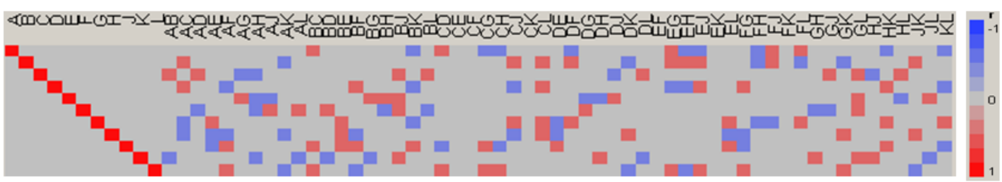


Figure 4.3 Correlations of Main Effects and Two-Factor Interactions, no-confounding Design for 11 Factors in 16 Runs

Table 4-5 Recommended 16-run 12-factor no-confounding design

Run	A	B	C	D	E	F	G	H	J	K	L	M
1	-1	-1	-1	-1	1	-1	-1	1	1	-1	1	1
2	-1	-1	-1	1	-1	1	1	1	-1	-1	1	-1
3	-1	-1	1	-1	-1	-1	1	-1	1	1	-1	1
4	-1	-1	1	1	1	1	-1	-1	-1	1	-1	-1
5	-1	1	-1	1	-1	-1	-1	-1	-1	-1	-1	1
6	-1	1	-1	1	1	1	1	-1	1	1	1	1
7	-1	1	1	-1	-1	1	-1	1	1	-1	-1	-1
8	-1	1	1	-1	1	-1	1	1	-1	1	1	-1
9	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1
10	1	-1	-1	-1	1	-1	1	-1	-1	-1	-1	-1
11	1	-1	1	1	-1	-1	-1	1	-1	1	1	1
12	1	-1	1	1	1	1	1	1	1	-1	-1	1
13	1	1	-1	-1	-1	1	1	1	-1	1	-1	1
14	1	1	-1	1	1	-1	-1	1	1	1	-1	-1
15	1	1	1	-1	1	1	-1	-1	-1	-1	1	1
16	1	1	1	1	-1	-1	1	-1	1	-1	1	-1

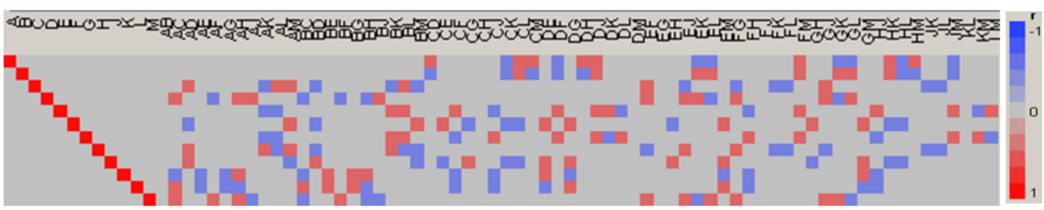


Figure 4.4 Correlations of Main Effects and Two-Factor Interactions, no-confounding Design for 12 Factors in 16 Runs

Table 4-6 Recommended 16-run 13-factor no-confounding design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N
1	-1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	1
2	-1	-1	1	-1	-1	-1	-1	-1	1	1	-1	-1	1
3	-1	-1	1	-1	1	1	1	1	1	-1	1	-1	-1
4	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	-1
5	-1	1	-1	-1	-1	1	-1	-1	-1	-1	1	-1	-1
6	-1	1	-1	-1	1	1	1	-1	-1	1	-1	1	1
7	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1
8	-1	1	1	1	1	-1	-1	-1	1	-1	-1	1	-1
9	1	-1	-1	-1	-1	-1	1	-1	1	1	1	1	-1
10	1	-1	-1	-1	1	-1	-1	1	-1	-1	-1	1	-1
11	1	-1	-1	1	1	1	1	-1	1	-1	-1	-1	1
12	1	-1	1	1	-1	1	-1	-1	-1	-1	1	1	1
13	1	1	-1	1	-1	1	-1	1	1	1	-1	-1	-1
14	1	1	1	-1	-1	-1	1	1	-1	-1	-1	-1	1
15	1	1	1	-1	1	1	-1	1	1	1	1	1	1
16	1	1	1	1	1	-1	1	-1	-1	1	1	-1	-1

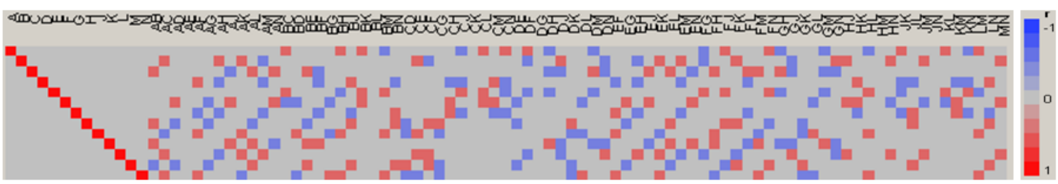


Figure 4.5 Correlations of Main Effects and Two-Factor Interactions, no-confounding Design for 13 Factors in 16 Runs

Table 4-7 Recommended 16-run 14-factor no-confounding design

Run	A	B	C	D	E	F	G	H	J	K	L	M	N	P
1	-1	-1	-1	-1	1	-1	1	1	-1	1	1	-1	-1	1
2	-1	-1	-1	1	-1	-1	1	-1	1	1	-1	1	1	-1
3	-1	-1	1	-1	-1	1	-1	1	1	1	-1	-1	1	1
4	-1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	-1
5	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	1
6	-1	1	-1	1	1	1	-1	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	-1	-1	1	-1	-1	1	1	1	-1
8	-1	1	1	1	1	-1	-1	-1	1	-1	-1	-1	-1	1
9	1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	1	1	1
10	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	-1
11	1	-1	1	-1	1	-1	-1	-1	1	1	1	1	-1	-1
12	1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	1
13	1	1	-1	-1	1	-1	1	1	1	-1	-1	-1	1	-1
14	1	1	-1	1	-1	-1	-1	-1	-1	1	1	-1	1	1
15	1	1	1	-1	-1	1	1	-1	-1	1	-1	-1	-1	-1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1

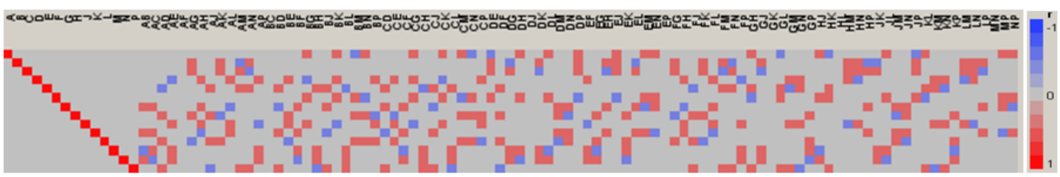
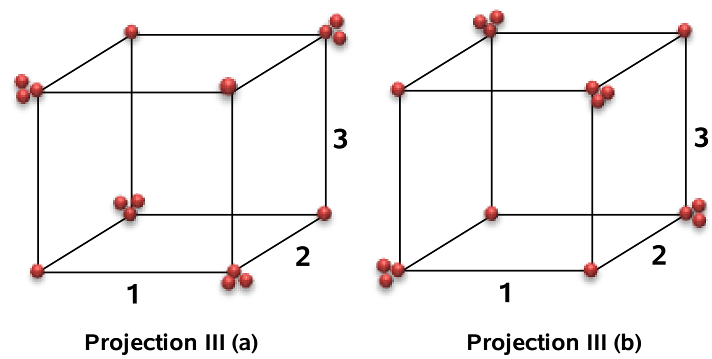


Figure 4.6 Correlations of Main Effects and Two-Factor Interactions, no-confounding Design for 14 Factors in 16 Runs

4.5. Projection Properties

The projection properties of these designs allow us to better understand the possible analysis methods. This section details the projection properties of the nine to 14 factor NC designs. I study both the three and four factor projections of the NC designs. There are three possible types of three factor projections for the 9 – 14 factor NC designs. One of the three factor projections results in a full factorial projection in eight distinct design points with two replicates. The other two projections result in eight distinct design points with four of the points replicated thrice and the other four points not replicated. Consequently I can fit the complete three-factor model (main effects and interactions) and have eight degrees of freedom for error. For the 9 – 14 factor NC designs, there are 43 different four factor projections possible. One of these 43 projections is a full factorial projection (Projection type I). Two of these projections result in a projection (Projection type II (i) and II (ii)) with eight distinct design points. The other 40 projections result in projections with 12 distinct design points. Figure 4.7 and Figure 4.8 lists the different three and four factor projections for the 9 – 14 factor NC designs. Tables 4.8 – 4.25 describe the different types of projections for the 9-14 factor NC designs.



Rotating projection III (a) on axis 3 results in projection III (b)

Figure 4.7 Three factor projections for 9 - 14 NC designs

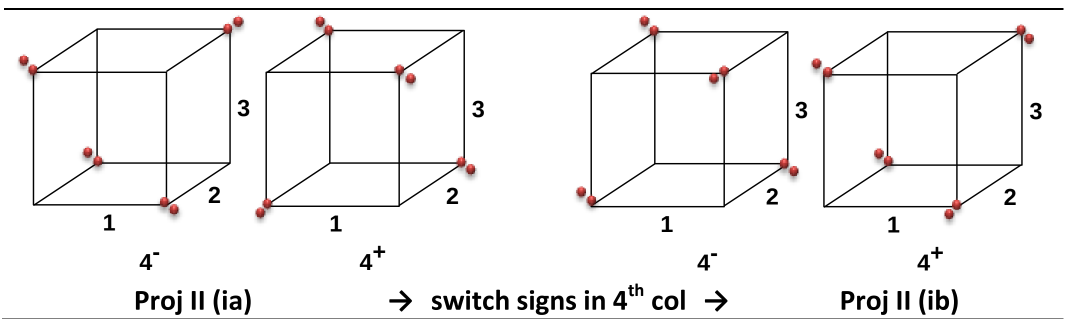


Figure 4.8 Four factor projections for 9 - 14 NC designs

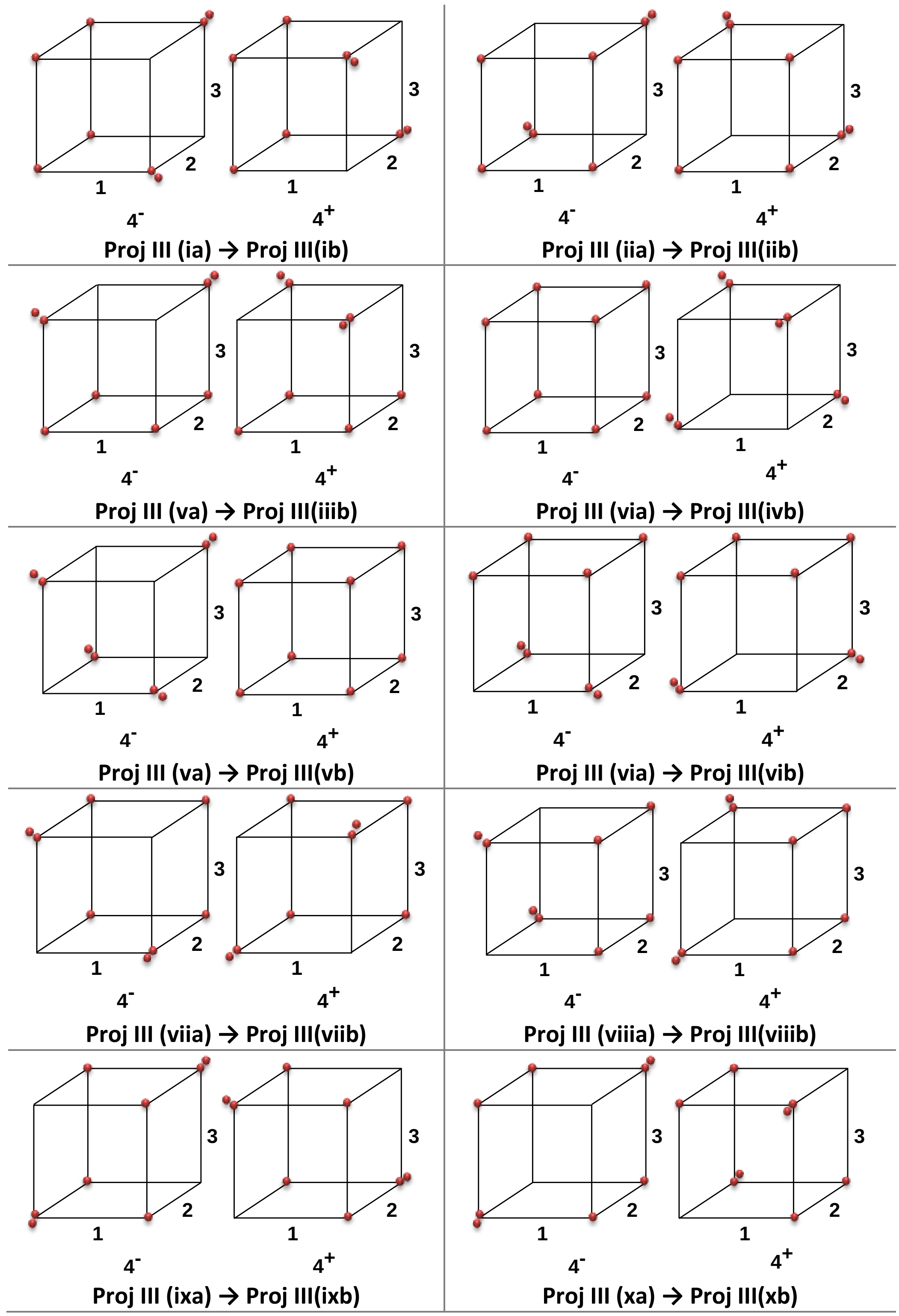


Figure 4.8 (contd.) Four factor projections for 9 - 14 NC designs

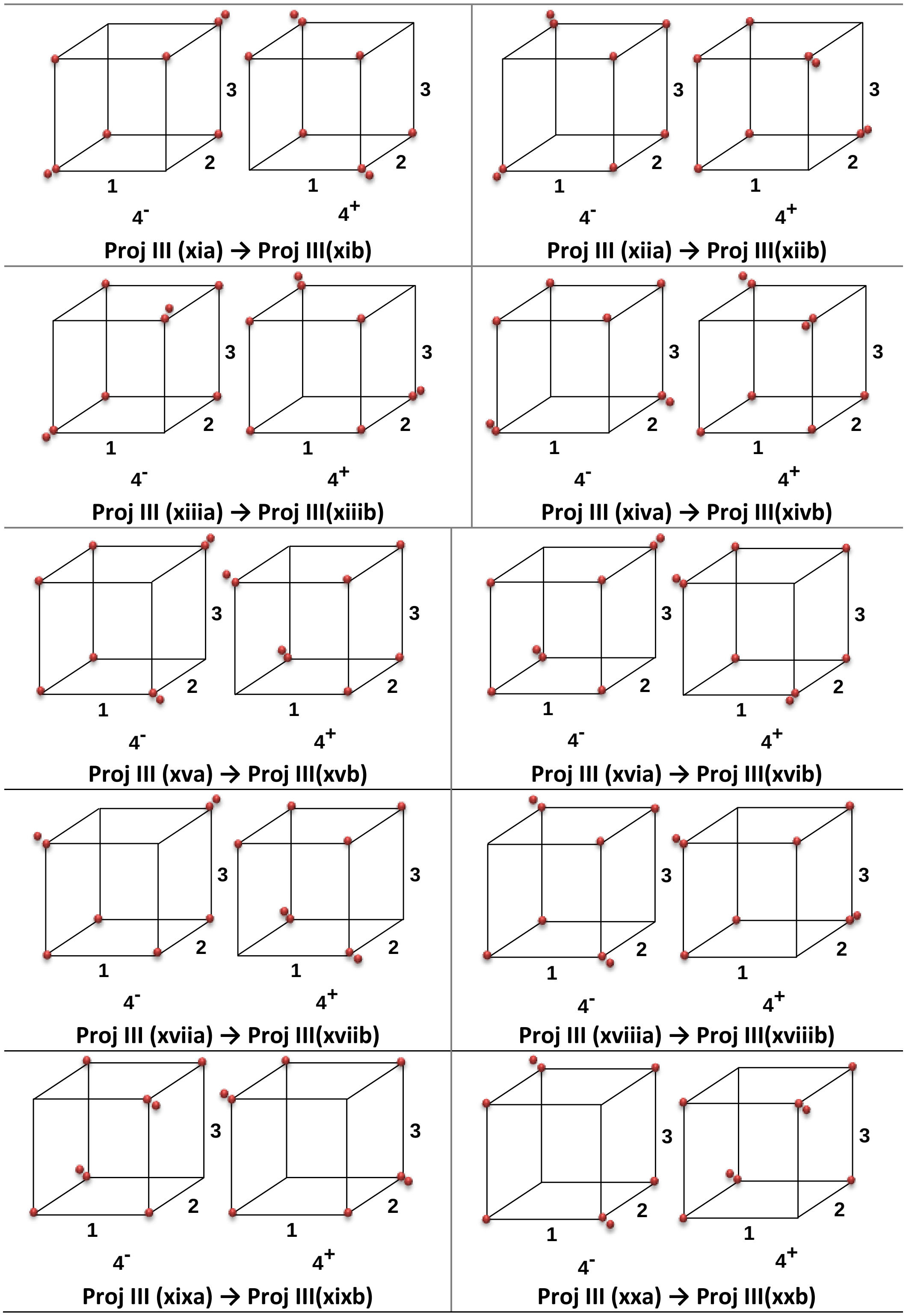


Figure 4.8 (contd.) Four factor projections for 9 - 14 NC designs

4.5.1. Three factor projections

For 9 – 14 factor NC designs, the three factor projections are of two types; projection type I is a full factorial projection with eight distinct design points each replicated twice. The second type of three factor projection (projection type III) also results in eight distinct designs points but as can be seen in Figure 4.7, four points are not replicated and the other four points are replicated thrice. Projection type three actually results in two different projections that are isomorphic in nature. When rotated on any axis projection type III (a) results in projection type III (b) and vice versa. Table 4-8, Table 4-11, Table 4-14, Table 4-17, Table 4-20 and Table 4-23 gives the details of how many of the three factor projections result in the two projections types for the 9 – 14 factor NC designs. Table 4-9, Table 4-12, Table 4-15, Table 4-18, Table 4-21 and Table 4.24 lists out the exact columns that result in projection type I, III (a) or III (b).

4.5.2. Four factor projections

For 9 – 14 factor NC designs, the four factor projections can be categorized into three basic types of projections; projection type I (full factorial projection with 16 distinct points and no replicates), projection type II (eight distinct design points replicated twice) and projection type III (12 distinct design points, four of which are replicated twice and eight are not replicated). Projection type II results in two projections that are isomorphic in nature. Projection type III results in a maximum of 40 different projections; 20 of which are isomorphic in nature resulting in a total of 40 different projections. Table 4-10, Table 4-13, Table 4-16, Table 4-19, Table 4-22 and Table 4-25 lists out the 20 projections and their isomorphic projections.

Table 4-8 Projections for 9 factor NC design

Proj Type Number	Type of Projections	3-factor projections	4-factor projections
I	Full factorial projections	68	64
II	2 FI Completely Confounded with other 2 FIs	0	14 (2 types)
III	Main effects Partially Confounded with Two Factor Interactions	16 (2 types)	48 (16 types)

Table 4-9 Three factor projections for 9 factor NC design

Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
III	ADH, AGH, AHJ, BHJ, CGH, EFH, EGH, EHJ	AFH, BDH, BFH, BGH, CDH, CFH, CHJ, DEH

Table 4-10 Four factor projections for 9 factor NC design

Projection Type	**Projections**	
I	ABCG, ABCH, ABCJ, ABDE, ABDF, ABDJ, ABEF, ABEG, ABEH, ABEJ, ABFG, ABGJ, ACDE, ACDF, ACDG, ACEF, ACEG, ACEH, ACEJ, ACFJ, ACGJ, ADEG, ADEJ, ADFG, ADFJ, ADGJ, AEFG, AEFJ, AFGJ, BCDE, BCDG, BCDJ, BCEF, BCEG, BCEH, BCEJ, BCFG, BCFJ, BDEF, BDEG, BDFG, BDFJ, BDGJ, BEFJ, BEGJ, BFGJ, CDEF, CDEJ, CDFG, CDFJ ,CDGJ, CEFG, CEGJ, CFGJ, DEFG, DEFJ, DEGJ, DFGH, DFHJ, DGHJ, EGHJ, FGHJ	
Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
II (i)	ABDG, ABFJ, ACDJ, ACFG, BDEJ, BEFG, CDEG, CEFJ	ADEF, AEGJ, BCDF, BCGJ, DFGJ, ABCE
III (ix)	ABFH,ACFH,BCDH,BCFH	ABHJ, ACGH, AEGH, AEHJ, BEHJ, CEGH
III (x)	BDEH,CDEH	
III (xi)	BDFH,BDGH,BFGH,CDFH	ADGH, EFGH
III (xii)		DEHJ
III (xiii)	CDHJ,CFHJ	AFHJ, BDHJ, BFHJ, BGHJ
III (xvi)	CGHJ	ADHJ, AGHJ, EFGJ, EFHJ
III (xviii)	ADEH	DEFH, DEGH
III (xix)	ADFH	AFGH, CDGH, CFGH
III (xx)	ABDH, BGH, ACDH, ACHJ, BCHJ	AEFH, BCGH, BEFH, BEGH, CEFH, CEHJ

Table 4-11 Projections for 10 factor NC design

Proj Type Number	Type of Projections	3-factor projections	4-factor projections
I	Full factorial projections	88	35
II	2 FI Completely Confounded with other 2 FIs	0	5 (2 types)
III	Main effects Partially Confounded with Two Factor Interactions	32 (2 types)	170 (32 types)

Table 4-12 Three factor projections for 10 factor NC design

Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
III	ABE, ABJ, ABK, ACD, ACE, ACK, ADH, AEF, AEH, AHJ, BDG, BEG, BGH, CGJ, CGK, DGK, EGJ, FGH, GHJ, GHK	ABD, ACJ, ADF, AFJ, AFK, AHK, BCG, CFG, DFG, DGJ, EFG, EGK

Table 4-13 Four factor projections for 10 factor NC design

Projection Type	**Projections**	
I	ABCF, ABCH, ABFG, ABFH, ACFH, ACGH, ADEG, ADEJ, ADEK, ADJK, AEJK, AGJK, BCDE, BCDF, BCDH, BCDK, BCEF, BCEH, BCEJ, BCFJ, BCFK, BCHJ, BCHK, BCJK, BDEH, BDEJ, BDEK, BDFH, BDFJ, BDFK, BDHJ, BDJK, BEFH, BEFJ, BEFK	
Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
II	BCFH, BDEF	BCDJ, BCEK, BDHK
II		
III (i)	ACFG	ABGH, ADGK, AEGJ
III (iii)	AFGK	ABGJ, AEGH
III (v)	ACEG	ACDG
III (vi)	ABGK, ADGH	AFGJ
III (viii)	AFGH	ABCG, ADGJ, AEGK
III (ix)		AGHJ
III (x)	BCFG	ACGK
III (xi)	ACFJ, AFHK	ABCE, ABCK, ABHJ, ADEH, BDEG
III (xii)	ADFG	ABDG, BCGJ, BCGK
III (xiii)	ABDF, ADFJ, ADFK	ABDH, BCGH
III (xiv)	AFJK	ABDE, ABDJ, ABDK, ACJK, ADFH
III (xv)	AEFG, BDGJ	ABEG, BDGK, BEGJ

Table 4.13 (contd.) Four factor projections for 10 factor NC design

Projection Type	Projection (a)	Isomorphic Projection (b)
III (xvi)	ACDF, ADHK, AEFJ, AEFK, AEHK	ABEF, ABEH, ACDH, ACEF, ACEH, ADHJ, AEHJ, BDGH, BEGH
III (xvii)	ACDJ, ACEJ, AHJK	ABEJ, ABEK, ABJK, ACDE, ACDK, ACEK, AEFH
III (xviii)	BDFG, BEFG	ACGJ
III (xix)	ABCJ, ABFJ, ABFK, ABHK, ACFK, ACHK	ABCD, ACHJ, ADEF, AFHJ, BCDG, BCEG, BEGK, BEHJ, BEHK, BEJK, BFGH, BFGJ, BFGK, BFHJ, BFHK, BFJK, BGHJ, BGHK, BGJK, BHJK, CDEF, CDEG, CDEH, CDEJ, CDEK, CDFG, CDFH, CDFJ, CDFK, CDGH, CDGJ, CDGK, CDHJ, CDHK, CDJK, CEFG, CEFH, CEFJ, CEFK, CEGH, CEGJ, CEGK, CEHJ, CEHK, CEJK, CFGH, CFGJ, CFGK, CFHJ, CFHK, CFJK, CGHJ, CGHK, CGJK, CHJK, DEFG, DEFH, DEFJ, DEFK, DEGH, DEGJ, DEGK, DEHJ, DEHK, DEJK, DFGH, DFGJ, DFGK, DFHJ, DFHK, DFJK, DGHJ, DGHK, DGJK, DHJK, EFGH, EFGJ, EFGK, EFHJ, EFHK, EFJK, EGHJ, EGHK, EGJK, EHJK, FGHJ, FGHK, FGJK, FHJK, GHJK
III (xx)		AGHK

Table 4-14 Projections for 11 factor NC design

Projection Type Number	Type of Projections	3-factor projections	4-factor projections
I	Full factorial projections	117	94
II	2 FI Completely Confounded with other 2 FIs	0	8 (2 types)
III	Main effects Partially Confounded with Two Factor Interactions	48 (2 types)	228 (43 types)

Table 4-15 Three factor projections for 11 factor NC design

Projection Type	Projection (a)	Isomorphic Projection (b)
III	ABC, ACD, AEG, AFJ, AFL, BCJ, BCL, BDF, BEG, BEH, BEJ, BFJ, BJK, CDL, CGJ, CGL, CHL, DEJ, DFG, DHK, EGL, EJL, FGL, GHK, GKL, HJK	ABK, ACG, ACH, ADF, AEH, AEJ, AFH, AHK, AKL, BDK, BFH, BFL, BGK, CDJ, CHJ, DEG, DEH, DKL, EHL, FGH, FGJ, JKL

Table 4-16 Four factor projections for 11 factor NC design

Projection Type	Projections	
I	ABDE, ABDG, ABDH, ABDJ, ABEF, ABEL, ABFG, ABGH, ABGJ, ABGL, ABHL, ABJL, ACEF, ACEK, ACEL, ACFK, ACJK, ACJL, ADEK, ADEL, ADGH, ADGK, ADGL, ADHJ, ADHL, ADJK, ADJL, AEFK, AFGK, AGHJ, AGJK, AGJL, AHJL, BCDE, BCDG, BCDH, BCEF, BCEK, BCFG, BCFK, BCGH, BCHK, BDEL, BDGJ, BDGL, BDHJ, BDHL, BDJL, BEFK, BEKL, BGHJ, BGHL, BHJL, BHKL, CDEF, CDEK, CDFH, CDFK, CDGH, CDGK, CEFG, CEFH, CEFJ, CEFL, CEGH, CEGK, CEHK, CEJK, CEKL, CFGK, CFHK, CFJK, CFJL, CFKL, DEFK, DEFL, DFHJ, DFHL, DFJK, DFJL, DGHJ, DGHL, DGJK, DGJL, EFGK, EFHJ, EFHK, EFJK, EFKL, EGHJ, EGJK, FHJL, FHKL, GHJL	
Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
II (i)	ABHJ, AGHL, BDGH, CEFK, DHJL	ABDL, ADGJ, BGJL
III (i)	CDEH, EFGH, FJKL	CDFG, CEJL, CGHK, EHJK
III (ii)	ACKL, BCDK, BCFH, BFGK, CFHJ, DEKL, EFHL	ACFJ, AEFL, BCEG, CEGJ, DFHK
III (iii)	ABEK, BFKL	AEFG, BCEL, BDEF, CDFL, CHKL, DEFJ
III (iv)	ACGK, ADFK, AEJK, CDJK, DEGK, FGJK	BFHK
III (v)	ABCE, ACDK, AEGK, AFJK, CGJK, DEJK, DFGK	ABCF, ACDE, BEHK
III (vi)	BCKL, BEFG, CDEL	ABFK, ACFG, ADEF, BDEK, CDFJ, DEFH, EHKL
III (vii)	ACFL, BCDF, BCEH, CFHL, EFJL	ACEJ, AEKL, BCGK, BEFL, CEHJ, DFKL
III (viii)	CDHK, EGHK, FHJK	CDEG, CFGH, CJKL, EFGJ
III (ix)	ABFH, ADEH, ADKL, AJKL, DJKL	ABEG, ABFJ, BDEJ, BHJK, CEGL, CFGL, DGHK, EFGL
III (x)	ABDK, ABGK, AFGH, BFGH	AFGL, BCDL, BCGJ, BCGL, BCHL, CGKL, DHJK, EGKL, FGKL, GHJK
III (xi)	ABHK, ACEH, ACFH, AEFH, BDGK, CDHJ	BCEJ, BCFJ, BEFJ, CDGL, CDHL, CGHL, EGJL
III (xii)	ACHJ, AEHL, BDKL, DEHL	ACGJ, ACGL, ACHL, ADFG, AEJL, BGKL, CHJK, DEGL, FGHK
III (xiii)	ABKL, ACHK, ADFH, AEHK, AFHK, AHKL	ADFJ, ADFL, DEHK
III (xiv)	ACGH, AEHJ, BFHL, DEGH, FGHJ	AFHJ, AFHL, BFHJ, CDJL, CHJL, DEGJ, DEHJ, FGHL, FGJL
III (xv)	ACDJ, BEHL, BJKL, DFGH, DFGJ, HJKL	ABCJ, ABCL, ACDL, AEGL, BDFG, BEGL, BEJL, DEJL, DFGL
III (xvi)	ABCG, ABCH, ACDF, BDFH, BDFL, BEGK, DHKL	ABCD, BCJK, BDFJ, BEJK, BFJK, GHKL

Table 4.16 (contd.) Four factor projections for 11 factor NC design

Projection Type	Projection (a)	Isomorphic Projection (b)
III (xvii)	ABCK, ACDG, ACDH, AEGH, AEGJ, BDFK, BFJL	AFJL, BCJL, BEGH, BEGJ, BEHJ, CGJL
III (xviii)	AFGJ, BCDJ, BCHJ, BFGJ, CDKL, EJKL	ABJK, AHJK, BDHK, BFGL, BGHK, CDEJ
III (xix)	AFKL, BCFL, BEFH, CGHJ, EGHL	ACEG, AEFJ, BDJK, BGJK, CDGJ, DEFG, EHJL
III (xx)	ABFL, ADEG, BDEG, BDEH, CEHL, CFGJ, GJKL	ABDF, ABEH, ABEJ, ADEJ, ADHK, AGHK, AGKL, DGKL

Table 4-17 Projections for 12 factor NC design

Projection Type Number	Type of Projections	3-factor projection s	4-factor projections
I	Full factorial projections	156	144
II	2 FI Completely Confounded with other 2 FIs	0	15 (2 types)
III	Main effects Partially Confounded with Two Factor Interactions	64 (2 types)	336 (40 types)

Table 4-18 Three factor projections for 12 factor NC design

Projection Type	Projection (a)	Isomorphic Projection (b)
III	ACD, ACL, ACM, ADH, ADJ, AFM, AHK, AHM, BCL, BDJ, BEK, BEL, BFM, BGK, BGL, BHK, CEF, CGJ, DEF, DEJ, DEK, DFG, DGJ, DGL, EFM, EHJ, EJM, ELM, FGH, FGM, GJM, GKM	ACK, ADF, AEK, AEL, AGK, AGL, AHL, AJM, BCD, BCK, BCM, BDF, BDH, BHL, BHM, BJM, CEJ, CFG, CFK, CFL, CJK, CJL, DEL, DGK, EFH, EKM, FHK, FHL, GHJ, GLM, HJK, HJL

Table 4-19 Four factor projections for 12 factor NC design

Projection Type	Projections	
I	ABCE, ABCF, ABCG, ABCJ, ABDE, ABDG, ABDK, ABDL, ABEF, ABEH, ABEJ, ABEM, ABFG, ABFH, ABFK, ABFL, ABGH, ABGJ, ABGM, ABHJ, ABJK, ABJL, ABKM, ABLM, ACEG, ACEH, ACFH, ACFJ, ACGH, ACHJ, ADEG, ADEM, ADGM, ADKL, ADKM, ADLM, AEFG, AEFJ, AEGH, AEGJ, AEGM, AFGJ, AFHJ, AFJK, AFJL, AFKL, AJKL, AKLM, BCEG, BCEH, BCFH, BCFJ, BCGH, BCHJ, BDEG, BDEM, BDGM, BDKL, BDKM, BDLM, BEFG, BEFJ, BEGH, BEGJ, BEGM, BFGJ, BFHJ, BFJK, BFJL, BFKL, BJKL, BKLM, CDEG, CDEH, CDEM, CDFH, CDFJ, CDFM, CDGH, CDGM, CDHJ, CDHK, CDHL, CDJM, CDKL, CDKM, CDLM, CEGK, CEGL, CEGM, CEHK, CEHL, CEHM, CEKL, CFHM, CFJM, CGHK, CGHL, CGHM, CGKL, CHJM, CHKM, CHLM, CKLM, DEGH, DEHM, DFHJ, DFHM, DFJK, DFJL, DFKL, DFKM, DFLM, DGHM, DHJM, DHKL, DHLM, DHLM, DJKL, DJKM, DJLM, EFGK, EFGL, EFJK, EFJL, EFKL, EGHK, EGHL, EGHM, EGJK, EGJL, EHKL, EJKL, FGJK, FGJL, FGKL, FHJM, FJKM, FJLM, FKLM, GHKL, GJKL, HKLM, JKLM	
Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
II (i)	ABCH, ABDM, ABEG, ABFJ, ABKL	CDHM, CEGH, CFHJ, CHKL, DEGM, DFJM, DKLM, EFGJ, EGKL, FJKL
III (i)	BCEJ, BCFG, BEFH, BGHJ, CDEL, CDGK, CEKM, CGLM, DFHK, DHJL	ACEF, ACGJ, AEHJ, AFGH
III (ii)	AFGL, BEHM, CDFL, CDJK, EHKM, GHLM	AGHM, BFGK
III (iii)	AEFK, AEJL, AGJK, AJKM, AJLM, BCGM, BDGH, DEHL, DGHK	ACEM, ADEH, BEFL, BEJK, BFKM, BFLM, BGJL
III (iv)	ADFK, ADFL, BCDE, CFKM, CJLM, FHLM, HJKM	BCDG, BDFK, BDFL, CFLM, CJKM, FHKM, HJLM
III (v)	ACDE, ADJK, ADJL	ACDG, BDJK, BDJL
III (vi)	ACGM, ADGH, AFKM, AFLM, BEFK, BEJL, BGJK, DEHK, DGHL	AEFL, AEJK, AGJL, BCEM, BDEH, BJKM, BJLM
III (vii)	AEHM, BFGL, EHLM, GHKM	AFGK, BGHM, CDFK, CDJL
III (viii)	BCEF, BCGJ, BEHJ, BFGH, CDEK, CDGL, CELM, CGKM	ACEJ, ACFG, AEFH, AGHJ, DFHL, DHJK
III (ix)	ABCK, ABDF, ABHL, ABJM, ADEL, ADGK, AFHL, BFHL, CHJK, CHJL	ABCL, ABDJ, ABFM, ABHK, AEFM, BDEK, BDGL, BEFM, CDEF, CDGJ, EGJM
III (x)	ACFK, ACJK, AHJL, BCFK, BCJK, BHJL, CFHK, CFHL, FHJK, FHJL	ADEJ, ADGJ, AFGM, BDEJ, BDGJ, BFGM, DEHJ, EFGM, FGJM, FGKM
III (xi)	ACEK, ACGK, AEGK, AEGL, AEHL, AGHL, BCHM, BCJM, BHJM, CFJK, CFJL	ACFM, ACHM, AFHM, BCEL, BCGL, BEGK, BEGL, BEHK, BGHK, DEGJ, EFJM, EFLM, EJLM, GJKM

Table 4.19 (contd.) Four factor projections for 12 factor NC design

Projection Type	**Projection (a)**	**Isomorphic Projection (b)**
III (xii)	AEKM, AGLM, EFHK, EFHL, GHJK, GHJL	ADFG, AELM, AGKM, BDFG, CEJM, CFGH, CFGM, DELM, DGKM
III (xiii)	BCDF, BCDH, BDHL, BDHM, CEJK, CEJL	ADFM, BCDJ, BDFM, BDHK, CFGJ, EFHJ, GHJM
III (xiv)	AEKL, AGKL, BCDK, BCDM, BCKM, BDFH, BHLM, CFGK, CFGL, CFKL, CJKL, FHKL, HJKL	ACKL, ACKM, ADFH, ADFJ, AHLM, BCDL, BCKL, BDFJ, BDHJ, DGKL, EFHM
III (xv)	BEKM, BGLM, CEFH, DEFH, DEKM, DGLM, EHJK, EHJL, FGHJ	BELM, BGKM, CEFM, CGJM, DEFM, DEJM, DFGH, DFGM, DGJM
III (xvi)	ACDF, ADHL, ADJM, BDJM, CEFG, CEFK, CEFL, CGJK, CGJL, DFGK, FGHK, FGHL	ACDH, ACDJ, ADHK, ADHM, DEFG, DFGJ, DFGL, EHJM
III (xvii)	ACDK, AHKL, BCLM, BHKL, BHKM, CEFJ, DEFL, DEJL, DEKL, DGJK	ACDL, ACDM, ACLM, ADHJ, AHKM, BEKL, BGKL, DEFJ, DEFK, DEJK, DGJL, FGHM
III (xviii)	ACFL, ACJL, AHJK, BCFL, BCJL, BHJK, CGHJ, DGHJ, FGLM	ADEF, BDEF, CEHJ, EFGH
III (xix)	ACEL, ACGL, ACHL, ACJM, AFJM, AHJM, BCHL, BEHL, BFHM, BFJM, BGHL, DEGK, EFKM, EJKM, GJLM, GKLM	ACHK, AEHK, AGHK, BCEK, BCFM, BCGK, BCHK, CEGJ, DEGL, EKLM
III (xx)	ABCD, ABCM, ABDH, ABHM, AFHK, BDEL, BDGK, BFHK, EGHJ, EGLM	ABEK, ABEL, ABGK, ABGL, ADEK, ADGL, AEJM, AGJM, BEJM, BGJM, CDEJ, CDFG, EGKM

Table 4-20 Projections for 13 factor NC design

Proj Type Number	Type of Projections	3-factor projections	4-factor projections
I	Full factorial projections	198	180
II	2 FI Completely Confounded with other 2 FIs	0	15 (1 type)
III	Main effects Partially Confounded with Two Factor Interactions	88 (2 types)	520 (40 types)

Table 4-21 Three factor projections for 13 factor NC design

Proj Type	Projection (a)	Isomorphic Projection (b)
III	ABC, ABH, ABK, ACL, ACN, ADF, AFJ, AFN, AJK, AKL, ALM, BCE, BDJ, BEK, BEM, BFK, BGN, BHJ, BHN, BJM, BMN, CDM, CEJ, CEL, CFH, CFL, CFM, CGH, DHK, DLN, EFG, EFJ, EFN, EHL, EKL, EKN, FHJ, FHK, FKM, FMN, GJL, GKM, HJL, HLN, JLM, LMN	ABM, ACJ, ADH, ADM, AFG, AGH, AGM, AHL, AKN, BCF, BDF, BDN, BEH, BFG, BGJ, CDH, CDJ, CDN, CEN, CGJ, CGM, CGN, DEF, DEH, DEM, DFL, DJK, DJL, DKM, DKN, EGH, EGM, EJK, ELM, FGL, FHN, FJM, FKL, GHK, GJK, GKN, GLN

Table 4-22 Four factor projections for 13 factor NC design

Proj Type	Projections	
I	ABDE, ABDG, ABDL, ABEF, ABEG, ABEJ, ABEN, ABFL, ABGL, ABJL, ABJN, ABLN, ACDE, ACDG, ACDK, ACEF, ACEG, ACEH, ACEM, ACFK, ACGK, ACHK, ACHM, ACKM, ADEJ, ADEK, ADEL, ADEN, ADGJ, ADGK, ADGL, ADGN, ADJN, AEFH, AEFK, AEFL, AEFM, AEGJ, AEGK, AEGL, AEGN, AEHJ, AEHK, AEHN, AEJL, AEJM, AEKM, AELN, AEMN, AFHM, AGJN, AHJM, AHJN, AHKM, AHMN, AJLN, AJMN, BCDG, BCDK, BCDL, BCGK, BCGL, BCHK, BCHL, BCHM, BCJK, BCJL, BCJN, BCKM, BCKN, BCLM, BCLN, BDEG, BDEL, BDGH, BDGK, BDGM, BDHL, BDHM, BDKL, BDLM, BEFL, BEGL, BEJL, BEJN, BELN, BFHL, BFHM, BFJL, BFJN, BFLM, BFLN, BGHL, BGHM, BGKL, BGLM, BHKL, BHKM, BJKL, BJKN, BKLM, BKLN, CDEG, CDEK, CDFG, CDFK, CDGL, CDKL, CEFK, CEGK, CEHK, CEHM, CEKM, CFGK, CFJK, CFJN, CFKN, CGKL, CHJK, CHJM, CHJN, CHKL, CHKN, CHLM, CHMN, CJKL, CJKM, CJLN, CJMN, CKLM, CKLN, CKMN, DEGJ, DEGK, DEGL, DEGN, DEJN, DFGH, DFGJ, DFGK, DFGM, DFGN, DFHM, DFJN, DGHJ, DGHL, DGHN, DGJM, DGKL, DGLM, DGMN, DHJM, DHJN, DHLM, DHMN, DJMN, EFHM, EGJN, EHJM, EHJN, EHKM, EHMN, EJLN, EJMN, FGHM, FGJN, FHLM, FJKN, FJLN, GHJM, GHJN, GHLM, GHMN, GJMN, HJKM, HJKN, HKLM, HKMN, JKLN, JKMN	
Proj Type	**Projection (a)**	**Isomorphic Projection (b)**
II (i)	ABEL, ACEK, ADEG, AEHM, AEJN, BCKL, BDGL, BHLM, BJLN, CDGK, CHKM, CJKN, DGHM, DGJN, HJMN	
III (i)	ABDN, ABGJ, ADJL, AGLN, BCDH, BCGM, BDKM, BGHK, CDEF, DFHN, DFJM, EGLN, FGJK	ACFM, AFHK, BEFN, CHJL, CLMN, DEKL, EFKM
III (ii)	ACFG, BEGJ, CFGN, DEJL, DFKN, FGHN, FGJM	ABFJ, AFLM, AGKL, AHJK, BCHN, BCJM, BKMN, CEFH, EFHL, EHKN, HKLN, JKLM
III (iii)	ADJM, BDEN, CDFJ, CGLM, DFHL, DKLM, EGJM	ABDK, ABFH, ACDL, ACHN, ACMN, AFLN, BCDE, BEFM, BEGK, BFJK, CEGL, CEHJ, CFJL, EKMN
III (iv)	ADHN, AFGK, AGHJ, AGMN, BDFM, BFGH, CDHL, DEFK, DEHJ, DEMN, EGHN, FGLM, GHKL	ACJM, ADHJ, ADMN, AGHN, BCFJ, BCFN, BDFH, BFGM, DEHN, DFLM, EGHJ, EGMN, EJKM, FKLN
III (v)	ABCD, ADFK, AFJL, BCEG, CGHL, DHKL, EFGK	ABCG, AJKM, BFKN, BHJK, CEJM, CFLN, EFJL
III (vi)	ABGK, ACGL, BDEJ, BHKN, BJKM, CDLM, EFLN, GKLM, HJKL	ABFM, ACHJ, AGJM, AKMN, BEFH, CDFN, CEHN, CEMN, DEJM, FGHL
III (vii)	ABFN, ACDF, ADKL, BCHJ, BCMN, BDEK, BEGN, CDEL, CEFM, DELN, EGKL, FGHJ, FGMN	AFHL, AHKN, CFGJ, DFJK, EFLM, EHJK, FJKL

Table 4.22 (contd.) Four factor projections for 13 factor NC design

Proj Type	Projection (a)	Isomorphic Projection (b)
III (viii)	ABDJ, ABGN, ACFH, ADLN, AFKM, AGJL, BCDM, BCGH, BDHK, BEFJ, BGKM, CEFG, CHLN, CJLM, DFHJ, DFMN, EFHK, EGJL, KLMN	FGKN
III (ix)	ABFG, ACDH, ACGM, ADKN, AEGH, AEGM, AGKN, BCDN, BCGJ, BDEH, CEGM, DEJK, DGJK, DGKN, EGJK	AEFJ, AEFN, AEKL, AJLM, BFHJ, BFMN, BLMN, DFHK, DHLN, FGKM, FLMN, GHJL
III (x)	ACDJ, ACGJ, ADEH, ADEM, ADKM, BDEF, BDKN, BEGH, CDEH, CDKN, CGKN, CGLN, DEGH, DEGM, DELM, DFGL, DFKL	ABEK, ABFK, ACEL, ACFL, AFHJ, AFMN, BFHK, BHLN, BJLM, CEFJ, CEHL, CEKL, CFKM, DLMN, EFHJ, EFMN, EHJL, HLMN
III (xi)	ABDM, ABGM, ADGH, ADGM, BCDF, CDEN, CDGJ, CDGN, CEGN, DEKM, DFJL, EGLM, FGKL, GHJK, GKLN	ABJK, ACFN, ACKL, BDHJ, BEFK, BEJM, BGHN, BGMN, BHMN, CDFM, CEFL, CFGH, EFKN, EHKL
III (xii)	AFGL, AGHK, BDFL, BFGL, BGJK, CDJK, CDJL, CGJK, EGHK, FGLN	ABMN, ADHK, AHLN, BCFH, BCFL, BCFM, BEHL, BGJL, CDHK, CGJL, DEFG, DEFJ, DEFN, DEHL, DJLM, ELMN
III (xiii)	ADHL, AFGH, AFGM, AGHL, BCFG, BDFG, BFGJ, DEFL, DJKM, DJKN, GHKN, GJKN	ACJK, AHLM, BCFK, BDFK, BEHJ, BEHN, BFGN, BGJM, DEHK, DFLN, DJLN, EGHL, EJKL, EJKN, FJMN, GHKM, GJKM
III (xiv)	ADHM, AGHM, BDFN, CDHJ, CDHN, CDJN, CGJM, CGJN, CGMN, DEFH, DEFM, DEHM, DJKL, DKMN, EGHM	ACJL, ACJN, AFGJ, AFGN, BDFJ, BEHK, BEHM, BFGK, BGJN, CDHM, CDJM, FKLM, GJKL
III (xv)	ABCF, ADFL, AFJM, BCEN, BDJK, BDJL, BFKL, CEJK, CELM, CFHN, CGHK, EFGL, EFJM	ABCE, ABHJ, ABHN, ALMN, BCEJ, BCEL, BEKL, BEKN, BFKM, BHJL, CFHJ, CFHK, CFMN, EHLN, FHJL, GJLM, HJLM, JLMN
III (xvi)	ABCJ, ABHL, ABKN, ADFG, AJKN, BCEH, DHKM, DHKN, EFGH, EFGM, EFJK, EHLM, EKLM, FHJM, FHKL, GJLN	ABCL, ABCN, ABKL, ACLM, ADFJ, ADFN, AFJK, AJKL, AKLM, BCEK, BCEM, BDJM, BEMN, BHJM, BJMN, FHKM, FKMN, HJLN
III (xvii)	ABCM, ABHM, ABKM, ADFH, ADFM, AKLN, BCEF, BDJN, CDMN, CEJN, CELN, CGHJ, CGHM, CGHN, FHJN, FHKN, GKMN	ABCH, ABCK, ABHK, ACLN, AFJN, BEKM, BHJN, CEJL, CFHL, CFHM, CFLM, EFGJ, EFGN, EFJN, EKLN, FHJK
III (xviii)	ABEH, ACDN, ACEN, ACGN, ADEF, AFHN, BEGM, BEJK, BELM, BGKN, BGLN, CDEM, CDKM, CFGL, CFJM, CFKL, EFHN	ABEM, ABJM, ACEJ, AGKM, AHJL, BDLN, CDLN, CEFN, CEKN, CGKM, EGKM, FGJL, FHLN, FJLM, GLMN

Table 4.22 (contd.) Four factor projections for 13 factor NC design

Proj Type	**Projection (a)**	**Isomorphic Projection (b)**
III (xix)	ABDH, ABGH, ACHL, ACKN, AFKN, BDGJ, CDGM, CEGJ, CFGM, DHJK	ABLM, ACFJ, ADLM, AGLM, AHKL, BDGN, BDHN, BDMN, BGHJ, CDEJ, CDFH, CDGH, DKLN, FHMN, FJKM
III (xx)	ABDF, ADJK, AEJK, AELM, AFKL, AGJK, BCDJ, BCGN, BDEM, BFHN, BFJM, CDFL, CEGH, DGHK, DGLN, EFKL, EGKN, FGHK	ACDM, ACGH, AEFG, AEHL, AEKN, BEFG, DEKN, DFKM, DGJL, DGKM, DHJL, EJLM, GHLN

Table 4-23 Projections for 14 factor NC design

Proj Type Number	Type of Projections	3-factor projections	4-factor projections
I	Full factorial projections	252	252
II	2 FI Completely Confounded with other 2 FIs	0	21 (1 type)
III	Main effects Partially Confounded with Two Factor Interactions	112 (2 types)	728 (30 types)

Table 4-24 Three factor projections for 14 factor NC design

Proj Type	**Projection (a)**	**Isomorphic Projection (b)**
III (a)	ABG, ABK, ABN, ACG, ACK, ACM, ADH, ADL, ADO, AEJ, AEM, AEN, AGH, AHJ, AJL, AKL, AMO, ANO, BDE, BDK, BDO, BEH, BEJ, BFG, BFK, BFM, BGJ, BHM, BHN, BJO, BLM, BLN, BLO, CDE, CDG, CDO, CEJ, CEL, CFG, CFK, CFN, CHM, CHN, CHO, CJK, CJO, CLM, CLN, DEF, DFH, DFL, DGM, DGN, DHM, DKM, DKN, DLN, EFM, EFN, EGH, EGL, EGN, EHK, EKL, EKM, FGL, FHJ, FHK, FJL, FJO, FMO, FNO, GHO, GJM, GJN, GLO, GMO, HJN, HKO, JKM, JKN, JLM, KLO, KNO	ABM, ACN, ADE, AGL, AHK, AJO, BDG, BEL, BFN, BHO, BJK, CDK, CEH, CFM, CGJ, CLO, DFO, DHN, DLM, EFJ, EGM, EKN, FGH, FKL, GNO, HJM, JLN, KMO

Table 4-25 Four factor projections for 14 factor NC design

<table>
<tr><th>Projection Type</th><th colspan="2">Projections</th></tr>
<tr><td>I</td><td colspan="2">ABCD, ABCE, ABCH, ABCJ, ABCL, ABCO, ABDF, ABDJ, ABEF, ABEO, ABFH, ABFJ, ABFL, ABFO, ABHL, ACDF, ACDJ, ACEF, ACEO, ACFH, ACFJ, ACFL, ACFO, ACHL, ADFG, ADFK, ADFM, ADFN, ADGJ, ADGK, ADJK, ADJM, ADJN, ADMN, AEFG, AEFH, AEFK, AEFL, AEGK, AEGO, AEHL, AEHO, AEKO, AELO, AFGJ, AFGM, AFGN, AFGO, AFHM, AFHN, AFHO, AFJK, AFJM, AFJN, AFKM, AFKN, AFKO, AFLM, AFLN, AFLO, AGJK, AGKM, AGKN, AGKO, AGMN, AHLM, AHLN, AHLO, AHMN, AJMN, AKMN, ALMN, BCDF, BCDH, BCDL, BCDM, BCDN, BCEF, BCEG, BCEK, BCEM, BCEN, BCFH, BCFJ, BCFL, BCFO, BCGH, BCGL, BCGM, BCGN, BCGO, BCHJ, BCHK, BCJL, BCJM, BCJN, BCKL, BCKM, BCKN, BCKO, BCMO, BCNO, BDFJ, BDHJ, BDHL, BDJL, BDJM, BDJN, BDMN, BEFO, BEGK, BEGO, BEKO, BEMN, BEMO, BENO, BFHL, BGHK, BGHL, BGKL, BGKM, BGKN, BGKO, BGMN, BHJL, BHKL, BJMN, BKMN, BMNO, CDFJ, CDHJ, CDHL, CDJL, CDJM, CDJN, CDMN, CEFO, CEGK, CEGO, CEKO, CEMN, CEMO, CENO, CFHL, CGHK, CGHL, CGKL, CGKM, CGKN, CGKO, CGMN, CHJL, CHKL, CJMN, CKMN, CMNO, DEGJ, DEGK, DEGO, DEHJ, DEHL, DEHO, DEJK, DEJL, DEJM, DEJN, DEKO, DELO, DEMN, DEMO, DENO, DFGJ, DFGK, DFJK, DFJM, DFJN, DFMN, DGHJ, DGHK, DGHL, DGJL, DGJO, DGKL, DGKO, DHJK, DHJO, DHKL, DHLO, DJKL, DJKO, DJLO, DJMO, DJNO, DMNO, EFGK, EFGO, EFHL, EFHO, EFKO, EFLO, EGJK, EGJO, EHJL, EHJO, EHLM, EHLN, EHMN, EHMO, EHNO, EJKO, EJLO, EJMN, EJMO, EJNO, ELMN, ELMO, ELNO, FGJK, FGKM, FGKN, FGKO, FGMN, FHLM, FHLN, FHLO, FHMN, FJMN, FKMN, FLMN, GHJK, GHJL, GHKM, GHKN, GHLM, GHLN, GHMN, GJKL, GJKO, GKLM, GKLN, GLMN, HJKL, HJLO, HKLM, HKLN, HKMN, HLMO, HLNO, HMNO, JMNO, KLMN, LMNO</td></tr>
<tr><th>Proj Type</th><th>Projection (a)</th><th>Isomorphic Projection (b)</th></tr>
<tr><td>II (i)</td><td>ABCF, ADFJ, AEFO, AFGK, AFHL, AFMN, BCDJ, BCEO, BCGK, BCHL, BCMN, DEJO, DGJK, DHJL, DJMN, EGKO, EHLO, EMNO, GHKL, GKMN, HLMN</td><td></td></tr>
<tr><td>III (i)</td><td></td><td>ABEH, ABLO, ACEL, ACHO, ADGM, ADKN, AGJN, AJKM, BDFH, BEGN, BEKM, BFJL, BGMO, BKNO, CDFL, CEGN, CEKM, CFHJ, CGMO, CKNO, DEGL, DEHK, DGHO, DKLO, EHJN, EJLM, FGJM, FJKN</td></tr>
</table>

Table 4.25 (contd.) Four factor projections for 14 factor NC design

Proj Type	Projection (a)	Isomorphic Projection (b)
III (ii)		ABDL, ABHJ, ACDH, ACJL, AHMO, ALNO, BDFM, BEFK, BFLO, BGHN, BGLM, BKLN, CDFN, CEFG, CFHO, CGHM, CGLN, CHJK, CKLM, DEHM, DELN, DFGN, DFKM, EJKL, FGJO, FHNO, FLMO, GJLO, HJKO
III (iii)		ABDN, ABEG, ABJN, ACDM, ACEK, ACJM, ADGO, ADKO, AEGJ, AEHN, AELM, BDHK, BDNO, BEFH, BFJM, BGHJ, BHKM, BJMO, CDMO, CEFL, CFJN, CHKN, CJNO, EFHM, EFLN, FJKO
III (iv)		ADEG, ADEK, AGLM, AGLN, AHKM, AHKN, BDGH, BDGL, BJKL, CDKL, CGJL, DHNO, DLMO, EFJK, FGHM, FGHN, FKLM, FKLN, HJMO, JLNO
III (v)	ABGO, ACKO, BDEN, BEJM, BFKO, CDEM, CEJN, CFGO	ABKO, ACGO, AEJK, AGHM, AGHN, AKLM, AKLN, BDEM, BDKL, BEJN, BFGO, BGJL, CDEN, CDGH, CDGL, CEJM, CFKO, CJKL, DEFG, DEFK, DHMO, DLNO, EGHJ, FGLM, FGLN, FHKM, FHKN, HJNO, JLMO
III (vi)	ABEK, ACEG, AEHM, AELN, BDMO, BHKN, BJNO, CDNO, CHKM, CJMO, EFHN, EFLM, EGJL, EHJK, GHJO	ABDM, ABJM, ACDN, ACJN, AJKO, BEFL, BFJN, CDHK, CEFH, CFJM, CGHJ, DFGO, DFKO, EFGJ
III (vii)	ABDH, ABJL, ACDL, ACHJ, AHNO, ALMO, BEFG, BGHM, BGLN, BKLM, CEFK, CGHN, CGLM, CKLN, DFGM, DFKN, FHMO, FLNO	AGJO, BDFN, BFHO, BHJK, CDFM, CFLO, DEHN, DELM
III (viii)	ADGN, ADKM, AGJM, AJKN, BDFL, BFHJ, CDFH, CFJL, DEGH, DEKL, DGLO, DHKO, FGJN, FJKM, JKLO	ABEL, ABHO, ACEH, ACLO, BEGM, BEKN, BGNO, BKMO, CEGM, CEKN, CGNO, CKMO, EHJM, EJLN

Table 4.25 (contd.) Four factor projections for 14 factor NC design

Proj Type	Projection (a)	Isomorphic Projection (b)
III (ix)		ABDO, ABEJ, ACDO, ACEJ, AEGH, AEKL, AFHJ, AFJL, AFMO, AFNO, AGMO, AKNO, BCDE, BCDO, BCEJ, BCFG, BCFK, BCHM, BCHN, BCJO, BCLM, BCLN, BDHM, BDLN, BEFM, BFJO, BGLO, BJLM, BKLO, CDHM, CDLN, CEFN, CFJO, CGHO, CJLM, DEGN, DEKM, DJKM, DJKN, EFGL, EFHK, EJKM, FGMO, FKNO, GHJN, GKLO
III (x)		ABDK, ABFG, ABFK, ABHN, ABLN, ACDG, ACFG, ACFK, ACHM, ACJK, ACLM, ADFH, ADFL, AEFM, AEFN, AEGN, AEKM, BEGH, BFHK, BHJN, CEGL, CEKL, CFHK, CHJN, CHKO, DFGL, DFJL, DGJM, DGJN, EGJN, GHKO, GJKM, GJKN, GJLM, HJKN, HKLO, HKNO
III (xi)		ABCG, ABCK, ABEN, ACEM, ADGH, ADJL, ADKL, ADMO, ADNO, AEHJ, AJKL, AMNO, BDFK, BDJO, BDLO, BEGJ, BFHM, BFLM, BHLM, BHLN, BJLO, CDFG, CDHO, CDJO, CFHN, CFJK, CFLN, CHJO, CHLM, CHLN, DGHM, DGKM, DGKN, DGLN, DHKM, DKLN, EFGN, EFKM, EGKL, FGJL, FJMO, FJNO, FMNO, GHLO, GHMO, GLMO, JKLM, KLNO
III (xii)		ADEF, AGLO, AHKO, BDGM, BDGN, BFNO, BJKM, BJKN, CDKM, CDKN, CEHK, CFMO, CGJM, CGJN, EFJL, EFJO, EGMO, EKNO, FGHO, FKLO
III (xiii)		ABMO, ACNO, ADEJ, ADEM, ADEN, AHKL, BDGJ, BELM, BELN, BELO, CEHM, CEHN, CEHO, CGJK, CGJO, FGHJ, FGHK
III (xiv)		ABMN, ADEH, ADEL, ADEO, BDGK, BDGO, BJKO, CDKO, CEHJ, CEHL, CFMN, DLMN, EFJM, EFJN, EGMN, FGHL, HJMN

Table 4.25 (contd.) Four factor projections for 14 factor NC design

Proj Type	Projection (a)	Isomorphic Projection (b)
III (xv)	ACGJ, ADHN, ADLM, AHJM, AJLN, BFGH, BFKL, CFGH, CFKL, DEFJ, DGNO, DKMO, EGNO, EKMO, FHJM, FJLN, JKMO	ABGJ, ADHM, ADLN, AGHO, AHJN, AJLM, AKLO, BDEF, BDKM, BDKN, BEHK, BFGL, BFMO, BGJM, BGJN, CDEF, CDGM, CDGN, CFGL, CFNO, CJKM, CJKN, DEFM, DEFN, DFHJ, DFHK, DGMO, DKNO, EFMO, EFNO, EGHO, EGLO, EHKO, EKLO, FGLO, FHJN, FHKO, FJLM, JKNO
III (xvi)	ABGL, ACGL, ADHK, AEJO, AGHK, AHJO, BDEL, BEHO, BEJK, BGJK, CDEH, CDGJ, CELO, CFGJ, DEFO, DFHN, DFLM, EHKN, FHKL, GJNO	ABGH, ABKL, ABNO, ACGH, ACKL, ACMO, ADHJ, AEJL, AEMO, AENO, AGHJ, AHJL, BDEH, BDEJ, BEHM, BEHN, BEJO, BFGJ, BGJO, CDEJ, CDEL, CEJK, CEJO, CELM, CELN, DEFH, DEFL, DFHM, DFLN, EGHK, EHKL, EHKM, FHJL, FHJO, GJMO
III (xvii)	ABGM, ABKM, ACGN, ACKN, ACMN, AGHL, AHJK, AJLO, BDEG, BEHL, BEJL, BFGN, BFKN, BFMN, BHMO, BHNO, CDEK, CDGK, CFGM, CFKM, CLMO, CLNO, DFHO, DFLO, DHMN, EGHM, EGLM, EKLN, EKMN, JLMN	ABGK, ABGN, ABKN, ACGK, ACGM, ACKM, ADHL, ADHO, ADLO, AEJM, AEJN, AEMN, BDEK, BDEO, BDKO, BEHJ, BFGK, BFGM, BFKM, BHMN, BLMN, BLMO, BLNO, CDEG, CDEO, CDGO, CEJL, CFGK, CFGN, CFKN, CHMN, CHMO, CHNO, CJKO, CLMN, DFHL, DGMN, DKMN, EFMN, EGHL, EGHN, EGLN, EKLM, FHJK, FJLO, GJMN, JKMN
III (xviii)	ABDG, ABFN, ABJK, ACDK, ACFM, ADFO, AEFJ, AEGM, AEKN, BDFO, BEFJ, BHJM, CDFO, CEFJ, CHJM, DFGH, DFKL, DHJM, GJLN, HJLN, HKMO	ABFM, ABHM, ABLM, ACFN, ACHN, ACLN, BEGL, BEKL, BHKO, CEGH, DFJO, DFMO, DFNO, DHJN, EFHJ, EGJM, HJKM, HJLM
III (xix)	ABCN, ABHK, ACHK, ADGL, ADJO, BDHO, BDJK, BFJK, CDLO, CEGJ, CHLO, CJLO, DGHN, DGLM, DHLM, DKLM, EFGM, EFKN, EGKN, FGKL, FJKL, GHNO, GLNO, GMNO, JKLN, KLMO	ABCM, ABEM, ACEN, AGJL, AGKL, AJMO, AJNO, BDFG, BFHN, BFLN, BHJO, BHLO, CDFK, CDJK, CFHM, CFLM, DHKN, DHLN, EGKM, KMNO
III (xx)	AFGH, AFKL, AGNO, AKMO, BCDK, BCEH, BCFM, BCGJ, BCLO, BDHN, BDLM, BJLN, CDHN, CDLM, CJLN, DEGM, DEKN, DJLN, EFGH, EFKL, FGNO, FKMO, GHJM, GKMO	ABDE, ABJO, ACDE, ACJO, AEGL, AEHK, AFGL, AFHK, AFJO, BCDG, BCEL, BCFN, BCHO, BCJK, BEFN, BGHO, CEFM, CGLO, CKLO, DJLM, EJKN, GKNO

4.6. Analysis Method

Based on the projection properties of the 9 – 14 factor NC designs, it can be clearly seen that the full three and four factor models with main effects and their interactions can be fit. The projections show that there are eight distinct design points for all three factor projections and atleast 12 distinct design points for the four factor projections. This indicates that the full factorial model can be fit for all the three and four factor projections for the 9 – 14 factor NC designs. Therefore using all possible subsets regression method for analyzing these designs is a logical method to analyze these designs. I tested a variation of the all possible subsets regression along with stepwise regression on two examples. The method used is listed below.

Step 1: Fit all possible subsets from one to ten terms with only main effects.

Step 2: Pick the best main effects only model and add all the two factor interactions for the selected main effects.

Step 3: Fit all possible subsets from one to ten terms for this modified list of factors

Step 4: Pick the best 2 or 3 models and fit the ordinary least squares model to it and select the terms from the model which is the best fit amongst these models.

4.6.1. Example I

This example is from Junqua, Duran, Gancet and Goulas (1997), where they study microbial transglutaminase production using a designed experiment approach. In the example they study five factors casein (X_1), glycerol (X_2), peptones (X_3), yeast extract (X_4) and oligoelements (X_5). I added two dummy variables to extend the design to a nine factor design. The original experiment was run as a 32 run full factorial experiment with five center runs. I used the results from the original experiment to simulate data for the NC nine factor design in 16 runs with the same coefficients and RMSE as the original experiment. The analysis of the original experiment showed that X_1, X_2, X_4 and X_1X_2 are the significant effects. The analysis method described in the previous section is used to analyze this simulated experiment. The simulated dataset is shown in Table 4-26.

Table 4-26 The 9 factor no-confounding design for the microbial transglutaminase production experiment

Run	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	Growth
1	-1	-1	-1	-1	-1	-1	1	-1	1	0.0188893887
2	-1	-1	-1	1	-1	1	-1	1	-1	0.0289614421
3	-1	-1	1	-1	1	1	1	1	-1	-0.001691386
4	-1	-1	1	1	1	-1	-1	-1	1	0.0365263064
5	-1	1	-1	-1	1	1	-1	1	1	0.0724725282
6	-1	1	-1	1	1	-1	1	-1	-1	0.0872040587
7	-1	1	1	-1	-1	-1	-1	1	-1	0.0586051129
8	-1	1	1	1	-1	1	1	-1	1	0.1055086723
9	1	-1	-1	-1	1	-1	-1	-1	-1	0.0185123407
10	1	-1	-1	1	1	1	1	1	1	0.0528304058
11	1	-1	1	-1	-1	1	-1	-1	1	0.0482017164
12	1	-1	1	1	-1	-1	1	1	-1	0.0572336741
13	1	1	-1	-1	-1	1	1	-1	-1	0.1481654593
14	1	1	-1	1	-1	-1	-1	1	1	0.1619445557
15	1	1	1	-1	1	-1	1	1	1	0.1560683984
16	1	1	1	1	1	1	-1	-1	-1	0.1900055485

Table 4-27 All Possible Factor Models up to nine terms (main effects only) comparison

No.	Model	No of terms	R - Square	RMSE	AICc	BIC
1	X2	1	0.63	0.04	-54.48	-54.16
2	X1	1	0.22	0.05	-42.48	-42.16
3	X4	1	0.05	0.06	-39.28	-38.96
4	X1,X2	2	0.85	0.02	-65.60	-66.15
5	X2,X4	2	0.68	0.04	-53.14	-53.69
6	X2,X8	2	0.64	0.04	-51.08	-51.63
7	**X1,X2,X4**	**3**	**0.90**	**0.02**	**-67.81**	**-69.94**
8	X1,X2,X8	3	0.86	0.02	-61.84	-63.98
9	X1,X2,X9	3	0.86	0.02	-61.82	-63.96
10	**X1,X2,X4,X8**	**4**	**0.91**	**0.02**	**-63.39**	**-68.09**
11	**X1,X2,X4,X9**	**4**	**0.91**	**0.02**	**-63.36**	**-68.06**
12	**X1,X2,X3,X4**	**4**	**0.91**	**0.02**	**-63.25**	**-67.95**
13	X1,X2,X4,X8,X9	5	0.91	0.02	-57.66	-66.25
14	X1,X2,X3,X4,X8	5	0.91	0.02	-57.55	-66.14
15	X1,X2,X3,X4,X9	5	0.91	0.02	-57.52	-66.11
16	X1,X2,X3,X4,X8,X9	6	0.92	0.02	-49.97	-64.36
17	X1,X2,X4,X6,X8,X9	6	0.92	0.02	-49.65	-64.04
18	X1,X2,X3,X4,X6,X8	6	0.92	0.02	-49.54	-63.93
19	X1,X2,X3,X4,X6, X8,X9	7	0.92	0.02	-39.13	-62.18
20	X1,X2,X3,X4,X5, X8,X9	7	0.92	0.02	-38.60	-61.64
21	X1,X2,X3,X4,X7, X8,X9	7	0.92	0.02	-38.56	-61.61
22	X1,X2,X3,X4,X5, X6,X8,X9	8	0.92	0.02	-23.19	-59.47
23	X1,X2,X3,X4,X6, X7,X8,X9	8	0.92	0.02	-23.15	-59.43
24	X1,X2,X3,X4,X5, X7,X8,X9	8	0.92	0.02	-22.62	-58.89
25	X1,X2,X3,X4,X5, X6,X7,X8,X9	9	0.92	0.03	0.79	-56.72

The top three main effects models for one to nine terms are listed in Table 4-27. The best model is the main effects model with X_1, X_2 and X_4. Next I add all the two factor interactions; X_1X_2, X_1X_4 and X_2X_4 and then fit all possible subsets to these main effects and interactions.The top three models for one to six terms are listed in Table 4-28. The second, third and fourth best models each with four terms in the first step were also tested to check if these including any of these terms gave better model fits. But the best model fit is the one with X_1, X_2, X_4 and X_1X_2. Including any other term does not improve the model fit. This model is fit is shown in Figure 4.9. The RSquare Adj value for this model is 0.962. The terms identified using this analysis method is identical to the true model.

Table 4-28 All Possible subsets Models up to nine terms (main effects and two factor interactions) comparison

No.	Model	No of terms	R - Square	RMSE	AICc	BIC
1	X2	1	0.63	0.04	-54.48	-54.16
2	X1	1	0.22	0.05	-42.48	-42.16
3	X1*X2	1	0.07	0.06	-39.62	-39.30
4	X1,X2	2	0.85	0.02	-65.60	-66.15
5	X2,X1*X2	2	0.70	0.03	-54.18	-54.72
6	X2,X4	2	0.68	0.04	-53.14	-53.69
7	X1,X2,X1*X2	3	0.92	0.02	-71.50	-73.63
8	X1,X2,X4	3	0.90	0.02	-67.81	-69.94
9	X1,X2,X1*X4	3	0.85	0.03	-61.29	-63.42
10	**X1,X2,X4,X1*X2**	**4**	**0.97**	**0.01**	**-82.46**	**-87.16**
11	X1,X2,X1*X2,X1*X4	4	0.92	0.02	-66.25	-70.95
12	X1,X2,X1*X2,X2*X4	4	0.92	0.02	-66.24	-70.94
13	X1,X2,X4,X1*X2,X1*X4	5	0.97	0.01	-76.05	-84.64
14	X1,X2,X4,X1*X2,X2*X4	5	0.97	0.01	-76.02	-84.61
15	X1,X2,X1*X2,X1*X4,X2*X4	5	0.92	0.02	-59.67	-68.26
16	X1,X2,X4,X1*X2,X1*X4,X2*X4	6	0.97	0.01	-67.70	-82.09

Fit Group

Response Y

Summary of Fit

RSquare	0.972204
RSquare Adj	0.962097
Root Mean Square Error	0.011387
Mean of Response	0.077465
Observations (or Sum Wgts)	16

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	0.04988815	0.012472	96.1866
Error	11	0.00142631	0.000130	Prob > F
C. Total	15	0.05131447		<.0001*

Lack Of Fit

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0774649	0.002847	27.21	<.0001*
X1	0.0266554	0.002847	9.36	<.0001*
X2	0.0450319	0.002847	15.82	<.0001*
X4	0.0125619	0.002847	4.41	0.0010*
X1*X2	0.0148938	0.002847	5.23	0.0003*

Figure 4.9 Model fit for X_1, X_2, X_4, X_1X_2

Chapter 5

CONCLUSIONS AND FUTURE WORK

The regular fractional factorial designs with six, seven or eight factors in 16 runs are widely used. However due to the complete confounding of the two-factor interactions with one another, these designs often require the experimenter to perform runs to resolve ambiguities whenever any of the two-factor interactions are identified as being active. The NC designs allow for the estimation of all main effects along with some of the two-factor interactions since there is no complete confounding in these designs.

The simulation study confirmed that stepwise regression does not work well once the total number of active terms exceeds four. However the study also showed that NC designs allow for estimation of two factor interactions without the need to run additional runs. Furthermore, once the true model contains interactions, regular fractional factorial designs are unable to compete with the nonregular designs due to the complete confounding of the two-factor interactions.

The simulation study shows that although stepwise regression may not be the best method to use for the analysis of nonregular designs, it is reasonably effective if the number of active terms (main effects and interactions included) is not more than four. There is no statistically significant difference between using a 2-stage stepwise regression method and a stepwise regression method. Both model selection methods used the AICc criterion.

I believe that the NC designs are good alternatives to the FF designs specially when running another set of experiments is not an alternative. With the NC designs, the experimenter would be able to study both the main effects and the interactions from the initial 16 runs of the experiment when the effect sparsity principle holds true.

The projection properties of the NC designs show that these designs allow for the estimation of all main effects along with some of the two-factor interactions since there is no complete confounding in these designs. I presented two intuitive approaches to analyzing these designs based on the projection properties. Systems with four active factors are likely to have a significant interaction. Therefore being able to estimate the two-factor interactions without the need for design augmentation is a desirable characteristic. Based on the projection properties of the NC designs all the main effects and their interactions can be estimated for up to four active factors or in other words models with up to 11 terms (including the intercept) can be fit as there are 12 distinct designs points for the four factor projections of these designs.

As part of this dissertation I looked at a few examples of NC designs and analyzed them using all subsets regression and two stage stepwise regression using all subsets. The methods are intuitive approaches to analyze these designs. Running a simulation study to evaluate the effectiveness of the analysis method for NC designs would be an ideal extension to this dissertation.

The Dantzig Selector (2007) has been used to identify active terms in nonregular designs. Candes and Tao explain how the β's can be estimated when p is much larger than n. Since in the case of NC designs where both main effects and interactions are being estimated, the p is much larger than n specially as the number of terms in the design matrix increase, this could be another analysis method worth exploring.

Box et al (2005), Montgomery and Runger (1996), Li and Mee (2002) and Li and Lin (2003) study the foldover plans for regular orthogonal designs. Another extension to this work would be identification of additional runs in cases where additional runs are to be run. Either foldover plans or addition of individual runs to the designs would allow the experimenter to run experiments using the NC designs and have a plan on how to run further experiments, if additional experiments are to be run.

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