

Dispatch of Bulk Energy Storage in Power Systems with

Wind Generation

by

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ABSTRACT

This thesis concerns the impact of energy storage on the power system. The rapidly increasing integration of renewable energy source into the grid is driving greater attention towards electrical energy storage systems which can serve many applications like economically meeting peak loads, providing spinning reserve. Economic dispatch is performed with bulk energy storage with wind energy penetration in power systems allocating the generation levels to the units in the mix, so that the system load is served and most economically. The results obtained in previous research to solve for economic dispatch uses a linear cost function for a Direct Current Optimal Power Flow (DCOPF). This thesis uses quadratic cost function for a DCOPF implementing quadratic programming (QP) to minimize the function. A Matlab program was created to simulate different test systems including an equivalent section of the WECC system, namely for Arizona, summer peak 2009.

A mathematical formulation of a strategy of when to charge or discharge the storage is incorporated in the algorithm. In this thesis various test cases are shown in a small three bus test bed and also for the state of Arizona test bed. The main conclusions drawn from the two test beds is that the use of energy storage minimizes the generation dispatch cost of the system and benefits the power system by serving the peak partially from stored energy. It is also found that use of energy storage systems may alleviate the loading on transmission lines which can defer the upgrade and expansion of the transmission system.

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NOMENCLATURE

A	an $(m \times n)$ constraint matrix
b	an m -dimensional column vector of right hand side coefficients
C	the cost coefficient of the decision variables to be minimized
<i>CAES</i>	Compressed Air Energy Storage
C_B	Cost of battery in dollars per Wh
C_{BT}	Total cost of battery in dollars
C_E	Cost of electronics
C_{ET}	Total cost of electronics
c_i	The cost of the generator at i^{th} bus
C_i	Total initial investment
C_W	Cost of wind turbine in dollars per MW
C_{WT}	Total cost of wind turbines in dollars
<i>DCOPF</i>	Direct Current Optimal Power Flow
<i>DFIG</i>	Doubly Fed Induction Generator
<i>DP</i>	Dynamic programming
<i>EES</i>	Electrical energy storage systems
<i>EIA</i>	Energy Information Administration
$E_{sq \max}$	The maximal energy storage at storage i^{th} bus

$F_{\cos t}(k,n)$	The total cost from initial state to hour k state n
h	The number of interval of hours of a day.
$ITMAX$	Maximum allowed iterations
LB	Lower bound
LMP	Locational marginal price
LP	Linear programming
m	The set of states at hour $t - 1$
n_b	The number of buses in the system
N_D	Number of days for repay of the original investment
n_g	The number of generators
n_l	The number of transmission lines
$NREL$	National Renewable Energy Laboratory
n_s	The number of large scale storage system
P_A	Generation at bus A
P_B	Generation at bus B
$P_{\cos t}(k,n)$	The production cost for state (k,n)
P_{gi}	Generation in MW at bus i
P_{gi}	The real power output at generator bus i
$P_{gi \min}, P_{gi \max}$	The minimal and maximal real power output at generator i
P_{ij}	The power flow of transmission line $i-j$
$P_{ij \min}, P_{ij \max}$	The minimal and maximal power limits of transmission line $i-j$

P_L	Real power load in MW
<i>PSERC</i>	Power Systems Engineering Research Center
<i>PSO</i>	Particle swarm optimization
$P_{sq\ min}, P_{sq\ max}$	The minimal and maximal storage capacity at storage i
Q	$(n \times n)$ matrix describing the coefficients of quadratic terms
<i>QP</i>	Quadratic programming
$S_{\cos t}(k-1, m, k, n)$	The transition cost from state $(k-1, m)$ to state (k, n)
<i>SMES</i>	Superconducting magnetic energy storage
<i>TES</i>	Thermal energy storage
<i>UB</i>	Upper bound
<i>WECC</i>	Western Electricity Coordinating Council
X	The n -dimensional column vector of decision variables

Chapter 1. Introduction to Wind Energy and Large Scale Storage Systems

1.1 Introduction: wind energy integration

In the U.S. most electricity is generated from electric power stations that use coal and natural gas. These two despite being reliable and affordable also have drawbacks. These release greenhouse gases in the atmosphere and besides that are finite and unevenly distributed across the globe. There is an immediate need for some alternative fuels which can overcome the issues pertaining to conventional power stations such as solar power and wind power. These alternatives also have disadvantages as the wind energy resources are intermittent in nature. The same intermittency occurs with solar power. As a consequence of absorbing increasing amounts of wind and solar resources, the electrical power system will need more flexibility to respond to the combined instantaneous fluctuations in both load and renewable generation. Such response would come through providing regulation, load-following, and fast ramping services. Moreover, the system may also need to commit more dispatchable and flexible resources in the day-ahead time frame to meet load net of renewable generation due to inaccurate variable generation forecast. The capacity of generation should always be greater than or equal to the peak demand. This makes intermittent sustainable generation alternatives integration potentially difficult.

Energy storage technology has the capability to ease the inclusion of large-scale variable renewable electricity generation, such as wind and solar. Dur-

ing electricity generation wind and solar power emit no greenhouse gases. Compared to conventional generators, the electrical energy storage systems (EESS) have potentially faster ramping rate which can quickly respond to load fluctuations. This speed is the case for electronically controlled storage systems. Therefore, the EESS can be a spinning reserve source which provides a fast load following and reduces the need for spinning reserve sources from conventional generation.

1.2 The central objectives of this research

The wind generation industry is entering into the range of megawatt-scale production [1] and has been getting increasing attention on account of wind energy being available free of cost and also being a non-polluting source of electricity. But a barrier in wind energy integration to the grid is its intermittency and uncertainty. Upgrade of the transmission system is often necessary to mitigate congestion in the power system with increasing demand. However, transmission expansion solutions may not be effective because cost of building a transmission line is often high and obtaining approvals to install new lines will take time. The energy storage at the load could be a more flexible and economical solution to the planning of power system.

Renewable energy, due to its lower controllability, adds uncertainty in the operation of the power system which is a technical challenge for the existing power system. Uncertainty may require additional control action from the conven-

tional generation units and of renewables themselves thus increasing the cost of integration of the renewable resources [1].

This research focuses on the use of bulk energy storage in power systems for different energy storage capacities with wind energy penetration in the power system, thereby studying the operating cost of generation from conventional generators.

1.3 The contemporary literature of wind energy resources

Wind power in the world has seen a substantial growth in the past decade making it one of the fastest growing sources of electricity and one of the fastest growing markets in the world today. The analysis conducted by the NREL estimates that current wind technology could generate 37 trillion kilowatt-hours of electricity per year in U.S. [5]. With the increased wind power penetration and sizes of the wind farms such as over 1000 MW of offshore wind farms, their impact on the power system operation – stability, control, power flow will also increase. For large wind farms these sudden changes can lead to power system instability.

Wind farms produce enough electricity to power all of Virginia, Oklahoma or Tennessee [6]. To illustrate the contemporary importance of wind energy, note that:

- In 2010, 2.3 % of the electric energy generation came from the wind in the U.S.

- The state of Iowa is often cited as a high wind energy state, and existing wind projects could produce 20% of the state electricity [6].
- Minnesota, North Dakota, Oregon, Colorado and Kansas all receive more than 5% of their electricity from wind and other states are following close behind with ever-growing wind power fleets [6].
- According to the Annual Report by NREL [9], in 2007 in terms of nameplate capacity, wind power was the second largest new resource added to U.S. electricity grid behind 7,500 MW of new natural gas plants and ahead of 1,400 MW of new coal.

New wind plants contributed about 35% of the new nameplate capacity added to the U.S. electrical grid in 2007, compared to 19% in 2006, 12% in 2005, and less than 4% from 2000 through 2004 [7].

The U.S. Energy Information Administration (EIA) predicts that electric utilities plan on installing 72,157 MW of additional wind capacity between 2010 and 2014 [10]. Wind power has a number of benefits. Firstly, its primary energy source, the wind is globally abundant both on land (onshore) and at sea (offshore). Secondly, wind power is the most mature and cost effective renewable energy technology. Wind power also has some challenges. Good potential wind sites are often located far from the cities where electricity is required. This may require improving the contemporary transmission infrastructure to deliver the electricity to the load center.

1.4 Bulk energy storage

General remarks

Large scale energy storage uses forms of energy such as chemical, kinetic or potential to store energy later being converted to electricity:

- *Cut down reserve margin and reduce back-up power plants:* Energy storage technologies can provide an effective method of reducing the need for reserve margin and reserve power plants in order to respond to daily fluctuations in demand. Supplying peak electricity demand by using electricity stored during periods of lower demand, thereby reducing the need for expensive fossil-fired reserve generation plants.
- *Integrating renewable energy:* Electricity storage can smooth out this variability and allow unused electricity to be dispatched at a later time .Balancing electricity supply and demand fluctuations over a period of seconds and minutes and,
- *Cutting the cost:* As a result of aging electricity grid, electricity outages cost the U.S. approximately \$150 billion annually [8]. Electricity storage technologies can provide power to the grid to smooth out short-term fluctuations until backup generation is back to normal.
- *Deferral of transmission expansion:* The increasing demand of electricity requires additional transmission infrastructure. New transmission lines from power plants are a costly and time-consuming process. Storage can help to postpone the need to build new transmission lines [10].

As a possible remedy for volatility of the wind energy the major energy storage technology options are:

Pumped hydro

In pumped hydro storage, a body of water at a relatively high elevation represents potential or stored energy. During periods of high electricity demand and high prices, the electrical energy is produced by releasing the water to drop in elevation to flow back down through hydro turbines at a lower elevation and into the lower reservoir. During periods of low demand and low cost electricity water is pumped back from a lower-level reservoir. The potential use of this technology is limited by the availability of suitable geographic locations for pumped hydro facilities near demand centers or generation [4]. Pumped hydro storage is appropriate for load-leveling because it can be constructed at large capacities of hundreds to thousands of megawatts (MW) and discharged over long periods of time up to 4 to 10 hours [14]. The efficiency is about 70% - 80% which varies depending on the plant size [16].

Compressed air

Compressed air energy storage (CAES) is a hybrid generation technology in which energy is stored by compressing air within an air reservoir and in some cases injecting air at high pressure into underground geologic formations, using a compressor at off-peak and low-cost electric energy. When demand for electricity is high, the compressed air is released and burnt with fuel to drive the generator such as gas-fired turbines. Thereby, allows the turbines to generate electricity us-

ing less natural gas [4]. This is also an appropriate load-leveling because it can be constructed in capacities for few hundred MW and can be discharged over long periods of time (4-24 hours) [14].

Batteries

Energy storage batteries store the electrical energy in the form of a chemical reaction by creating electrically charged ions inside the battery. The reversal of this reaction will result in the discharge of the battery producing electrical energy from the chemical reaction [14]. There are a number of battery technologies under consideration for large-scale energy storage like lead-acid, lithium-ion, and sodium sulfur. Among these lead-acid batteries are mostly used because of their relatively low cost. Batteries can provide power quality, load-leveling and is easy to install [18]. Table 1.1 shows the comparison of lead-acid, nickel-cadmium and lithium-ion batteries.

Batteries store dc charge, and power conversion is required to interface a battery with an AC system. Small, modular batteries with power electronic converters can provide four-quadrant operation (bidirectional current flow and bidirectional voltage polarity) with rapid response. But there are some technical problems with use of batteries:

- The cell will discharge itself so they are only suitable for short-term electricity storage.
- They have a tendency to age resulting in a decreasing storage capacity.

Table 1.1. Specification of batteries

Battery type	Lead acid	Nickel cadmium	Lithium-ion
Specification			
Energy density (Wh/kg)	30-50	45-80	150-190
Cell voltage (V)	2	1.2	3.6
Overcharge tolerance	High	Moderate	Low
Cycle life (80% discharge)	200-300	1000	500-1000
Charge time (h)	8-16	1	2-4
Toxicity	Very high	Very high	Low
Cost (\$/Wh)	0.125-0.2	0.4-0.8	0.2-0.36

*Sources of data: [16]-[18]

Thermal energy storage

Thermal energy storage (TES) can be divided in two different types. Firstly, TES applicable to solar thermal power plants and secondly its end-use [20]. TES for a solar thermal power plant consists of a synthetic oil or molten salt that stores solar energy in the form of heat collected by solar thermal power plants to enable smooth power output during daytime cloudy periods and to extend power production for 1-10 hours past sunset [21]. End-use TES stores electricity from off-peak periods through the use of hot or cold storage in underground aquifers, water or ice tanks, or other storage materials and uses this stored energy to reduce the electricity consumption of building heating or air conditioning systems during times of peak demand [22]. During off-peak periods ice can be made from water using electricity, and the ice can be stored until next day when it is used to cool

either the air in a large building, thereby shifting the demand off-peak. Using thermal storage can reduce the size and initial cost of cooling systems, lower energy costs and maintenance costs.

Hydrogen

Hydrogen storage involves using electricity to split water into hydrogen and oxygen through a process called electrolysis. Compressed hydrogen is the simplest system to conceive. When electricity is needed the hydrogen can be used to generate electricity through a hydrogen powered combustion engine or a fuel cell. Hydrogen fuel cells can be used in power quality applications where 15 seconds or more of ride-through are required. On a life-cycle cost basis for long duration applications, fuel cell technology competes with battery systems at discharge times greater than about 2 hours, depending on cost assumptions, and with hydrogen-fueled engines at discharge times greater than about 4 hours. Typical energy efficiency of a fuel cell is between 40-60%, or up to 85% efficient if waste heat is captured for use [23]-[24].

Flywheels

A flywheel is an electromechanical storage system in which energy is stored in the form of kinetic energy of rotating mass. The charging or discharging of the flywheel storage system takes place by changing the amount of kinetic energy present in the accelerating or decelerating rotor, respectively [4]. The flywheel is coupled with an electrical machine which acts as a motor to drive the

flywheel while charging and acts as a generator to discharge the stored energy by decelerating the rotor to stationary position. During charging, an electric current flows through the motor increasing the speed of the flywheel. During discharge, the generator produces current flow out of the system slowing the wheel down [25].

Ultra capacitors / super capacitor

Capacitors store their energy in an electrostatic field rather than in chemical form. These consist of two parallel electrode plates which are separated by a dielectric. When the voltage is applied across the terminals the positive and negative charges get accumulated over the electrodes of opposite polarity. The capacitor stores energy by increasing the electric charge accumulation on the metal plates and discharges energy when the electric charges are released by the metal plates. Ultra-capacitors are now available in the range of up to 100 kW with very a short discharge time of up to ten seconds [26]. Ultra-capacitors have temperature independent response, low maintenance and long lifetimes, but they have relatively high cost. These devices also have high loss and they are intended to be operated only for a few seconds.

Super Conducting Magnetic Energy Storage (SMES)

Superconducting magnetic energy storage is an energy storage device that stores electrical energy in magnetic field without conversion to chemical or me-

chanical form. In SMES, a coil of superconducting material allows DC current to flow through it with virtually no loss at very low temperatures. This current creates the magnetic field that stores the energy. On discharge, switches tap the circulating current and release to serve the load with high power output in short interval of time [25]. Although the SMES device itself is highly efficient and has no moving parts, it must be refrigerated to maintain superconducting properties of the wire materials. Therefore, SMES devices require cryogenic refrigerators and related subsystems, thus increasing maintenance costs [14].

Table 1.2 summarizes some of these storage technologies and their characteristics.

1.5 Organization of this thesis

This thesis is organized into five chapters. Chapter 2 presents basic concepts of optimal dispatch including different economic dispatch methodologies. These concepts are used in the formation and solution of the algorithm for optimal energy storage.

Chapter 3 demonstrates the idea of optimal scheduling of energy storage using a small illustrative example. Chapter 4 illustrates application of this algorithm in the state of Arizona as a test bed. The test bed is a subset (equivalent) of the Western Electricity Coordinating Council system.

Chapter 5 presents conclusions, contributions from the test beds studied in Chapter 4 and lines of future work regarding the use of large scale energy storage in power systems.

There are two appendices provided. Appendix A shows the corresponding Matlab algorithm for the DC optimal power flow developed during this research. Appendix B describes the quadratic programming algorithm.

Table 1.2 A comparison of bulk energy storage technologies

Storage method	Capacity	Capital cost (\$/MWh)	Weight (kg/MWh)	Efficiency	Maintenance cost (\$/MWh)	Maturity	Discharge time	Power level (MW)	Response time (ms)	Lifetime (years)
Pumped Hydro	22,000 MWh	7,000	3,000	0.8	4	Commercial	12 hours	<2000	30	40
CAES	2,400 MWh	2,000	2.5	0.85	3	Commercial	4-24 hours	100-300	3000-15000	30
Batteries	200 MWh			0.7-0.85		Commercial	1-8 hours	<30	30	2-10
Thermal energy	400 MWh	550	300,000	0.8	15	Commercial	6 hours	260		40
Hydro-gen	0.3-2000 kWh	15,000	30	0.45-0.8	10	Commercial				10
Flywheel (low speed)	50 kWh	300,000	7,500	0.9	3	Commercial	min to 1 h	<0.100 (each)	5	20
Flywheel (high speed)	750 kWh	25,000,000	3,000	0.93	4	Recent commercial				20
Ultra capacitor	0.5 kWh	28,000,000	10,000	0.95	5	Commercial	10s	0.200	5	40
SMES	0.8 kWh	10,000	10	0.97	1	Commercial	10s	0.100	5	40

*Sources of data [12]-[26]

Chapter 2. Optimal Dispatch of Energy Storage Systems

2.1. Power system operation

The operation of power systems involves the best utilization of the available energy resources. The operation generally subjected to various constraints to transfer electrical energy from generating stations to the consumers with maximum safety without interruption of supply.

Prior to restructuring of the power system in the U.S., unit commitment (identifying the generators which when dispatched, will give the available least-cost operation of available generation resources to meet the electrical load) [32] and economic dispatch were performed by vertically integrated utilities. This operating strategy is done to minimize the production cost of generation. Occasionally, there are power exchanges or interchanges between utilities to take economical advantage of power interchanges. Power pools were formed by several interconnected utilities to effectuate this exchange. Traditionally, coordinating unit commitment and economic dispatch were performed by a central dispatch office [31].

There are three stages in system control, namely unit commitment, security analysis and economic dispatch [36]:

- Unit commitment involves the hour-by-hour ordering of generator units start-up/shut-down in the system to match the anticipated load.

- With a given power system topology and a given number of generators, security analysis assesses the system response to a set of contingencies and provides a set of constraints that should not be violated if the system is to remain in secure state.
- Economic dispatch orders the minute-to-minute loading of the connected generating plants so that the cost of generation is minimum subject to constraints. Figure 2.1 illustrates the operation and data flow in a modern power system.

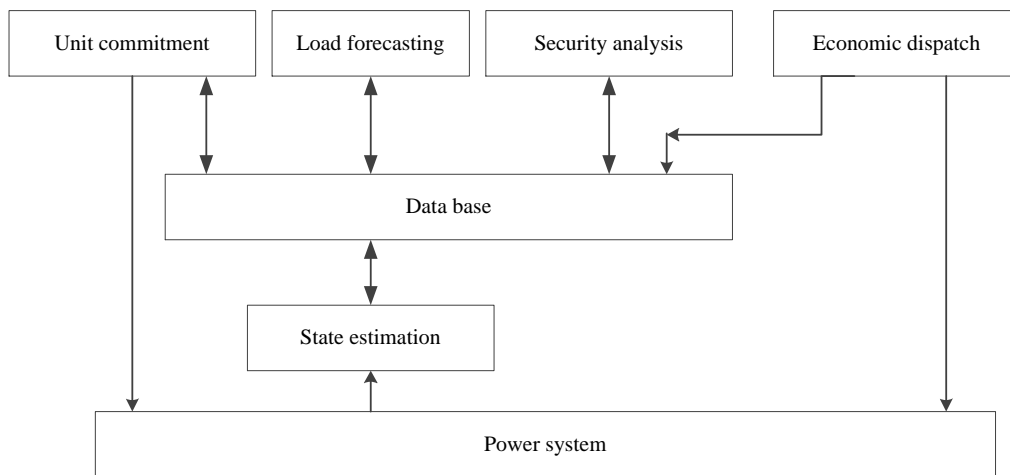


Figure 2.1 Power system control activities

2.2. The theory of optimal dispatch

The definition of optimal or economic dispatch provided in EPCRA section 1234 is “The operation of generation facilities to produce energy at the lowest cost to reliably serve consumers, recognizing any operational limits of generation and transmission facilities” [27].

The fuel cost (\$/h) of a thermal unit is often expressed as an approximately quadratic function of the power output (MW) of the unit. Therefore the incremental cost (\$/MWh) is almost linear with respect to the unit power output. Without considering other parameters (e.g., transmission losses, reactive losses, line constraints, unit output power constraints), the most economical generation levels occur when the incremental costs of all available units are equal. This simple rule is known as the ‘equal incremental cost rule’ and this is a result of elementary analysis and formulation of the problem as a Lagrange multiplier optimization [28]. If a unit has a higher incremental cost at an output level than other units, it would be cheaper to generate the MW from another unit with a lower incremental cost. The ‘equal incremental cost rule’ needs to be modified when the generator output limits and the transmission losses are taken into consideration. When the MW output level of a unit reaches its upper limit, the unit output is fixed at the upper limit even if the system load increases. Other units which have not reached their maximum limits would share the load increase based on the ‘equal incremental cost’ rule. To account for the transmission losses, the incremental costs are modified with a ‘penalty factor’. The penalty factor is a measure of additional transmission losses due to an incremental increase in the unit output [31].

There are many conventional methods that are used to solve the economic dispatch problem such as the Lagrange multiplier method, lambda iteration. These methods need to compute the economic dispatch each time load changes. As a result, long computation times may result.

2.3. Economic dispatch methodologies

There are various techniques including traditional and modern optimization methods developed for the economic dispatch without security-constrained (i.e. operation of the power system under credible contingencies). These methods can be classified as conventional optimization methods and intelligent search methods [33]. The conventional optimization methods include lambda-iteration, linear programming (LP), quadratic programming (QP), dynamic programming, and mixed integer programming. Among these methods lambda-iteration method is simple, more favorable, and used in many commercial economic dispatch programs. Some of the intelligence search methods are neural network and particle swarm optimization (PSO).

The system incremental fuel cost rate, called system lambda, is the key to find the most economical generation output of all on-line units. However, when the cost function is more complex than a piecewise linear function or a quadratic function, other methods are more suitable than the lambda-iteration method [31]. The conventional optimization methods are discussed below in brief:

The lambda-iteration method:

In lambda iteration method, lambda is the variable introduced in solving constraint optimization problem and is called Lagrange multiplier. All the inequality constraints to be satisfied in each trial, the equations are solved by the iterative method [31]:

Step 1. Assume a suitable value of $\lambda^{(0)}$ this value should be more than the largest intercept of the incremental cost characteristic of the various generators.

Step 2. Compute the individual generations i.e. calculate P_{gi} for $i = 1, 2, \dots, N$.

Step 3. First iteration, check the equality constraint i.e. tolerance, $\epsilon = P_L - \sum P_{gi}$ for $i = 1, 2, \dots, N$. If not satisfied set a new value of λ and repeat the above steps.

Step 4. Check the convergence. If ΔP_{gi} in step 3 are below the user-defined tolerance, the solution converges. Otherwise, go to step 2.

Linear programming (LP) method:

Linear programming maximizes or minimizes the objective, which is dependent on a finite number of variables. These variables may or may not be independent of each other, and in most cases are subject to certain conditions referred to as constraints. LP method finds a point in the optimization surface where this function has the smallest (or largest) value. Linear programs are problems that can be expressed in canonical form:

$$\text{Minimize} \quad C^T X \quad (2.1)$$

$$\text{subject to} \quad A_{\text{eq}} X = b_{\text{eq}} \quad (2.2)$$

$$AX \leq b \quad (2.3)$$

Where

X the vector of variables to be determined

C the cost coefficient of the decision variables to be minimized

A, A_{eq} an $(m \times n)$ constraint matrix

B, b_{eq} an m -dimensional column vector of right hand side constraints

The method for solving economic dispatch by LP uses an iterative technique to obtain the optimal solution [33]:

Step 1. Select the set of initial control variables.

Step 2. Solve the power flow problem to obtain a feasible solution that satisfies the power balance equality constraint.

Step 3. Linearize the objective function and inequality constraints around the power flow solution and formulate the LP problem.

Step 4. Solve the LP problem and obtain optimal incremental control variables ΔP_{gi} .

Step 5. Update and form the new control variables $P_{gi\ new} = P_{gi\ old} + \Delta P_{gi}$.

Step 6. Obtain the power flow solution with updated control variables.

Step 7. Check the convergence. If ΔP_{gi} in step 4 are below the user-defined tolerance, the solution converges. Otherwise, go to step 3.

Quadratic programming method:

Quadratic programming is a special form of nonlinear programming whose objective function is quadratic and constraints are linear. The most often used objective function in power system optimization is the generator cost func-

tion, which generally is a quadratic. The linear programming method can also be used in the quadratic programming model of economic dispatch (see Appendix B).

Dynamic programming (DP) method:

The basic idea of the theory of DP is that of viewing an optimal policy as one determining the decision required at each time in terms of the current state of the system. This absolute problem is normally solved by discretization of the entire dispatch period into a number of small time intervals over which the load is assumed to be constant and the system is considered to be in steady-state [37]. There are two DP algorithms. They are forward and backward dynamic programming. The start-up cost of a unit is a function of the time. The forward approach is often adopted since the initial condition is known. The backward DP algorithm is appropriate when the terminal condition is known. Suppose a system has n units. There is $2^n - 1$ combination.

The recursive algorithm is used to compute the minimum cost in hour k with state n is [31],

$$F_{\text{cost}}(k,n) = \min[P_{\text{cost}}(k,n) + S_{\text{cost}}(k-1, m:k,n) + F_{\text{cost}}(k-1,m)] \quad (2.4)$$

where

$F_{\text{cost}}(k,n)$ The total cost from initial state to hour k state n

$S_{\text{cost}}(k-1, m:k,n)$ The transition cost from state $(k-1, m)$ to state (k,n)

m The set of states at hour $t-1$

$P_{\text{cost}}(k,n)$ The production cost for state (k,n)

This thesis uses the process of applying the quadratic programming method to a minimization problem. The QP method is a very powerful solution algorithm because of their rapid convergence near the solution. This property is especially useful for the power system application because an initial guess near the solution is easily attained.

2.4. Formulation of the optimal bulk storage problem

A general minimization problem can be written in the following form:

$$\text{Minimize} \quad f(X) \quad (\text{the objective function}) \quad (2.5)$$

$$\text{subject to:} \quad h_i(X) = 0 \quad i = 1, 2, \dots, m \quad (\text{equality constraints}) \quad (2.6)$$

$$g_j(X) \leq 0 \quad j = 1, 2, \dots, n \quad (\text{inequality constraints}) \quad (2.7)$$

There are m equality constraints and n inequality constraints and the number of variables is equal to the dimension of the vector X . The system described has constraints that capture line ratings, generator ratings, bus power conservation and the Kirchhoff laws.

The mathematical model of real power economic dispatch with security constraints can be written as follows:

$$\text{Minimize} \quad f(X) = c_i P_{gi} + P_{gi}^T Q P_{gi} \quad i \in n_g \quad (2.8)$$

$$\text{subject to} \quad A_{eq}X = b_{eq} \quad (2.9)$$

$$AX \leq B \quad (2.10)$$

such that

$$\begin{aligned} \sum P_{gi} &= \sum P_{Lk} & i \in n_g; k \in n_l \\ P_{ij \min} &\leq P_{ij} \leq P_{ij \max} & ij \in n_t \\ 0 &\leq P_{gi} \leq P_{gi \max} & i \in n_g \\ P_{sq \min} &\leq P_s \leq P_{sq \max} & q \in n_s \\ 0 &\leq E_s \leq E_{sq \max} & q \in n_s \end{aligned}$$

Where

P_L	The real power load in MW
P_{ij}	The power flow of transmission line ij in MW
$P_{ij \min}, P_{ij \max}$	The minimal and maximal power limits of transmission line ij in MW
P_{gi}	The real power output at generator bus i in MW
$P_{gi \min}, P_{gi \max}$	The minimal and maximal real power output at generator i in MW
$P_{sq \min}, P_{sq \max}$	The minimal and maximal storage capacity at storage i in MW
$E_{sq \max}$	The maximal energy storage at storage i MWh
c_i	The cost of the generator i
n_l	The number of transmission lines
n_g	The number of generators
n_s	The number of large scale storage system
Q	$(n \times n)$ symmetric matrix describing the coefficients of quadratic terms
X	The n -dimensional column vector of decision variables (note: X contains: (1) control variables such as generation and storage power levels as well as (2) problem unknowns such as line flows and bus voltage phase angles)

Problem of dimensionality

Generally, the number of unknowns X increases like $(n_b + n_s + n_l + n_b - 1)h$.

The number of equality constraints increases like $(n_b + n_b)h + 1$.

The number of inequality constraints increases like $(2n_l + n_s)h + n_s(2h - 2)$.

where n_b The number of buses in the system.

h The number of interval of hours of a day.

Equality constraints

The equality constraints (A_{eq}) of the optimal power flow (OPF) reflect the physics of the power system. The following equality constraints are enforced during QP.

- Conservation of power at each bus: The physics of the power system are enforced through the power flow equations which require that the net injection of real power at each bus sum to zero. The corresponding generation limits of individual generator are accommodated in the upper bound (UB) and lower bound (LB) of the programming.
- Line load versus phase angle at each bus: Assumption is the voltage at the nodes is 1 p.u.

$$P_{ij} = (\delta_i - \delta_j) / (x_{ij}).$$

- Charge /discharge schedule for all the storage elements should sum up to zero.

$$\sum P_{si} = 0 \quad i \in n_s.$$

Inequality constraints

In addition to the equality constraints, there are inequality constraints (A) in the model. The inequality constraints in the OPF reflect the limits on physical devices in the power system as well as the limits created to ensure system security. Physical devices that require enforcement of limits are:

- Line loads
- Conservation of energy (storage)
- Power to storage element.

Chapter 3. A Small Illustrative Example

3.1 Objectives of a small illustrative example

In this section, a simple three bus power system test bed is used to demonstrate the idea of optimal scheduling of energy storage. The basic formulation of the problem is given in this section. It is assumed that the given data are:

- Loads
- Wind power
- LMPs at generation buses.

And the constraints are:

- Line loads
- The energy and power ratings of the storage.

And the Kirchhoff's laws:

- Conservation of power at each bus
- Line load versus phase angle at each bus.

3.2 Description of the test bed

The test bed proposed as a small example is denominated as test bed #1. A 3-bus system was considered of how storage can improve integration of renewable resources was developed and used for preliminary test of calculation technique and proof of concept. The 3-bus system is shown in Figure 3.1. The system data and line data are shown in Table 3.1. The LMP (locational marginal price,

incremental cost of energy delivered at a bus) of the day is shown in Figure 3.2. Load and renewable energy generation (wind) at the bus B and C are shown in Figure 3.3 and 3.4 respectively. A 100 MVA base is chosen for calculations.

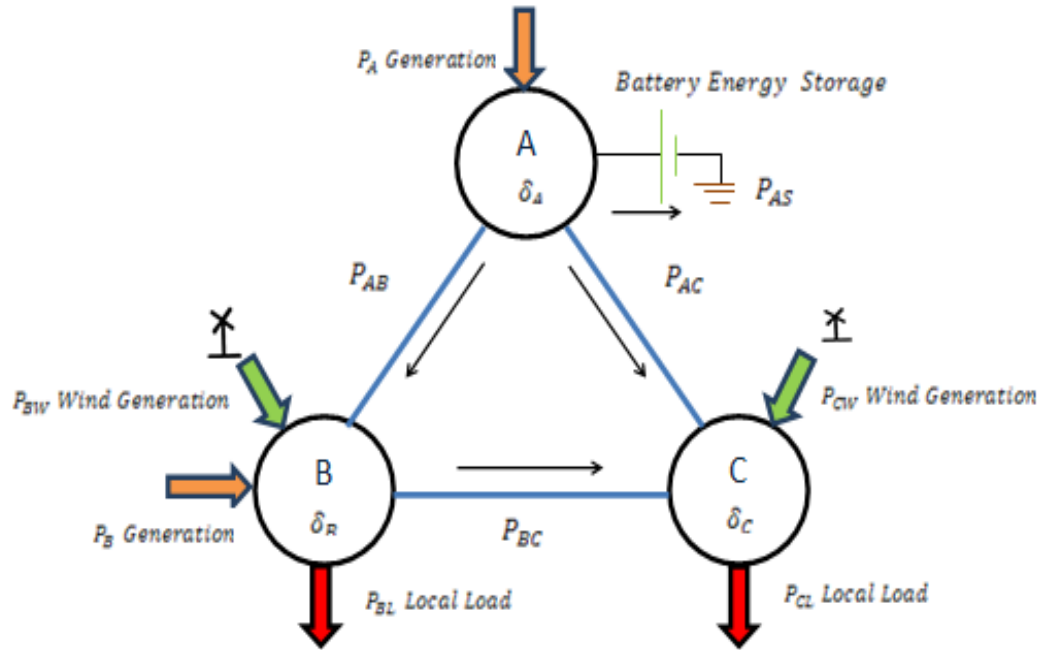


Figure 3.1 Three bus test bed: test bed # 1

Table 3.1 Transmission line ratings

Transmission line		Reactance (Ω)	Thermal rating (MW)
From	To		
A	B	0.01	190
A	C	0.02	100
B	C	0.03	200

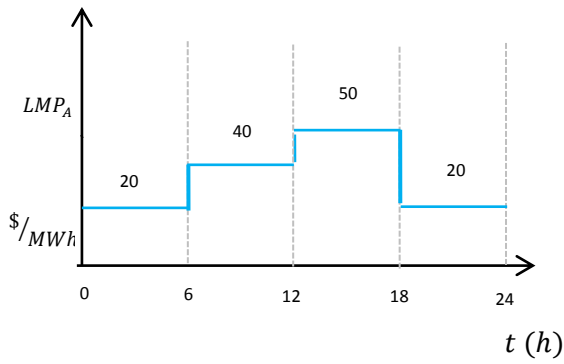


Figure 3.2 LMP at bus A for test bed #1 (\$/MWh)

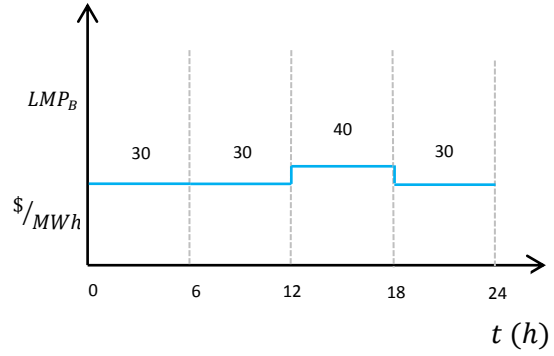


Figure 3.3 LMP at bus B for test bed #1 (\$/MWh)

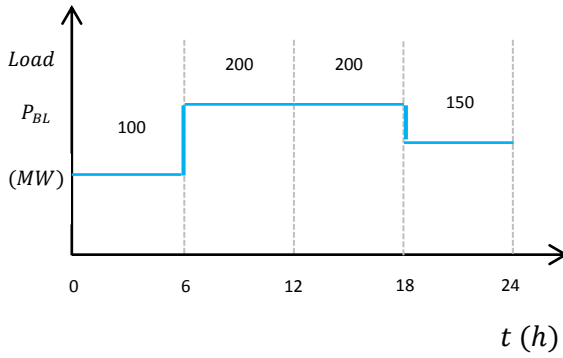


Figure 3.4 Load at bus B for test bed #1 (MW)

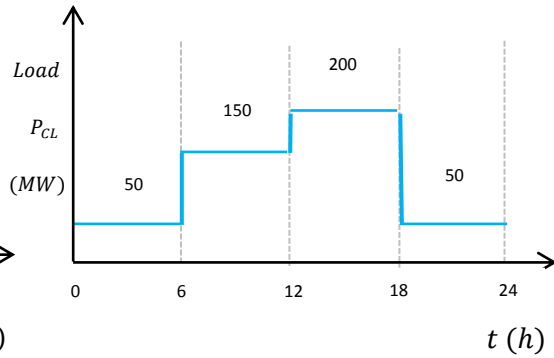


Figure 3.5 Load at bus C for test bed #1 (MW)

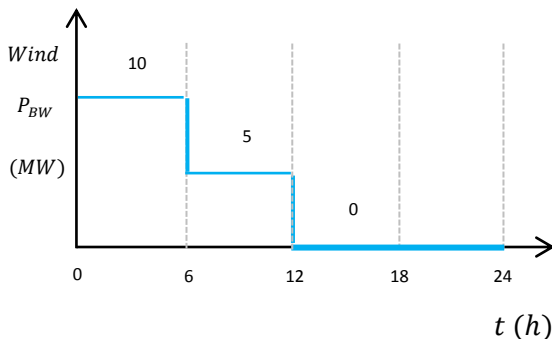


Figure 3.6 Wind generation at bus B for test bed #1 (MW)

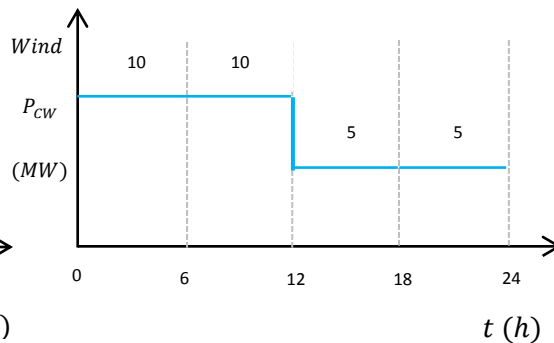


Figure 3.7 Wind generation at bus C for test bed #1 (MW)

3.3 Formulation of the problem

The main objective is to maximize beneficial impacts of storage, mainly reflected as minimizing generation dispatch cost. A storage facility is considered to be present at Bus A in Figure 3.1. A QP based algorithm is carried out to optimize generation and storage scheduling with maximal use of renewable generation. For the tests reported, all the wind generation is used. The unknowns, optimum generation schedule and storage (store/discharge) schedule are calculated, minimizing the purchase price of energy for one day. The information used and constraints considered are mentioned below:

Given Information

- Loads
- Wind power
- LMPs at generation buses

Constraints

- Line loads
- Energy and power of storage
- Conservation of power at each bus
- Voltage phase angle at each bus

Bus A is assumed to be the reference bus. The voltage phase angle at each bus constrained to lie between $-30^\circ \leq \delta \leq 30^\circ$. Three cases are studied calculating the economic dispatch of generation at bus A (P_A) and bus B (P_B) for minimum cost with:

- No storage and no constraints line ratings.
- Constraint on line ratings and one storage unit at bus A.
- Constraint on line ratings and two storage units at bus A and B.

All the three cases are studied for a one day time horizon broken into 4 intervals each have a span of 6 hours.

3.4 Study of case 1 (base case)

In this case study, the system is initially assumed without energy storage and without constraints on line ratings are considered. After executing the economic dispatch considering the limits on generation, the output of generating units P_A and P_B computed is listed in Table 3.2. The minimum generation cost of the system without energy storage using QP in Matlab is 181,520 dollars per day. At interval 1 and 4, the load is being supplied by the cheap unit A. At intervals 2 and 3, the cheap unit B has to supply power.

Table 3.2 Case 1 study results, test bed #1

Operational data		Interval (each 6 hours)			
		1	2	3	4
Generation (MW)	Bus A	130	0	0	195
	Bus B	0	335	395	0
Line flows (MW)	Bus A to Bus B	88.33	-70	-97.5	140
	Bus A to Bus C	41.67	70	97.5	55
	Bus B to Bus C	-1.67	70	97.5	-100
Voltage angle (radians)	Bus B	-0.0088	0.007	0.0097	-0.014
	Bus C	-0.0083	-0.014	-0.0195	-0.011

3.5 Study of case 2

In this case study, with the energy storage at bus A having rating of 20 MW and energy capacity of 120 MWh. Economic dispatch of generations P_A and P_B is calculated for minimum cost using QP in Matlab with storage and considering the limits on generations and thermal ratings of the transmission lines. The dispatch results are shown in Table 3.3. At the first low-load hour, the storage is charged by 20 MW which is from the cheap unit A. At intervals 2 and 3, the output from the storage is discharged mitigating the congestion on the line from bus A to bus C. In addition the output from the battery is replacing generation from expensive unit A. The total generation dispatch cost is 179,380 dollars per day which is less than that in case 1. This saving could be much higher in the large system.

From case 2 it is observed, the energy is stored during the minimum cost of generation and discharged when the cost of generation is high. At the end of the day the storage element is completely discharged. In other words the battery (storage element) charges during the first interval and discharges during the second and third interval to minimize the cost of generation

Table 3.3 Case 2 study results, test bed #1

Operational data		Interval (each 6 hours)	1	2	3	4
Generation (MW)	Bus A		150	0	0	195
	Bus B		0	330	380	0
Line flows (MW)	Bus A to Bus B		88.33	-65.83	-85	140
	Bus A to Bus C		41.67	70.83	100	55
	Bus B to Bus C		-1.67	69.17	95	-100
Voltage angle (radians)	Bus B		-0.0088	0.0066	0.0085	-0.014
	Bus C		-0.0083	-0.0142	-0.02	-0.011
Storage (MW)	Bus A		20	-5	-15	0

3.6 Study of case 3

In case 3 the economic dispatch is solved with limits on line ratings and two storage units at bus A and bus B having combined rating of 20 MW and energy capacity of 120 MWh (10 MW and 60 MWh each). The results are shown in Table 3.4. The cost of economic dispatch of generation per day calculated is \$ 179,080.

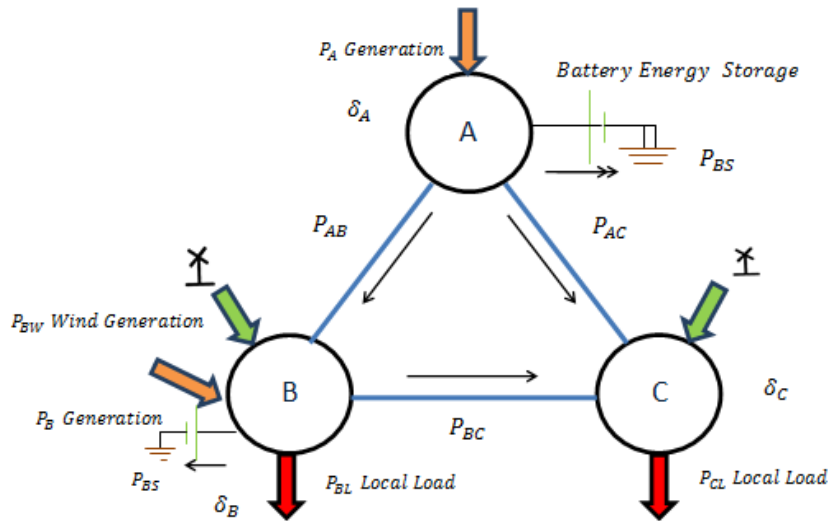


Figure 3.8 Three bus test bed: test bed # 1 with two storage units

Inference drawn from this case is that the spreading the storage unit reduces the generation production cost. This is because of line rating constraints limiting concentrated energy storage. At the first low-load hour, the storage at bus A and bus B is charged by 10 MW each which is from the cheap unit A. At interval 4, the output from the storage is discharged mitigating the congestion on the line. In addition the output from the battery is replacing generation from expensive unit A and B. The total generation dispatch cost is 179,080 dollars per day which is less than that in case 2.

Table 3.4 Case 3 study results, test bed #1

Operational data		Interval (each 6 hours)			
		1	2	3	4
Generation (MW)	Bus A	150	0	0	195
	Bus B	0	335	375	0
Line flows (MW)	Bus A to Bus B	96.67	-70	-89.7	140
	Bus A to Bus C	43.33	70	99.17	55
	Bus B to Bus C	-3.33	70	95.83	-100
Voltage angle (radians)	Bus B	-0.0097	0.0070	0.0089	-0.014
	Bus C	-0.0087	-0.014	-0.0198	-0.011
Storage (MW)	Bus A	10	0	-10	0
	Bus B	10	0	-10	0

3.7 Impact of storage: observations from test bed # 1

The implementation and use of renewable energy may not always be possible due to constraints of transmission and component ratings, when storage is

added; these constraints are partially relaxed, this can be observed from cases mentioned above.

High prices are one of the largest barriers facing renewables. During peak demand on the electric grid, electric companies pay more for electricity. Often additional power needs at this time are supplied by natural gas or oil, which has higher fuel costs. The opposite is true during times of low demand, when electricity costs are lower, during this time the energy can be stored and discharged when the demand and fuel cost is high thereby reducing the overall cost of generation per day; this can be observed from the above discussed cases 1, 2 and 3.

The grid needs a consistent, stable supply of energy that can be adjusted during times of peak demand. Black out occurs when supply does not keep up with demand. High demand on the power grid often requires power plants to be fired up to cover short-term electricity demand at a higher price. Large-scale of use of renewable energy will require that it can adapt to variable levels of demand on the power grid. Energy storage combined with these renewable energy resources may firm up the power output.

The cost of delivery and generation (fuel) can be minimized by increasing the capacity of storage elements and fuel cost can be further optimized by disbursing the storage unit across the power system. This concept is illustrated in Figure 3.9. Here, corresponding to the total power (MW, across two storage units), six hours of energy storage (MWh) is considered.

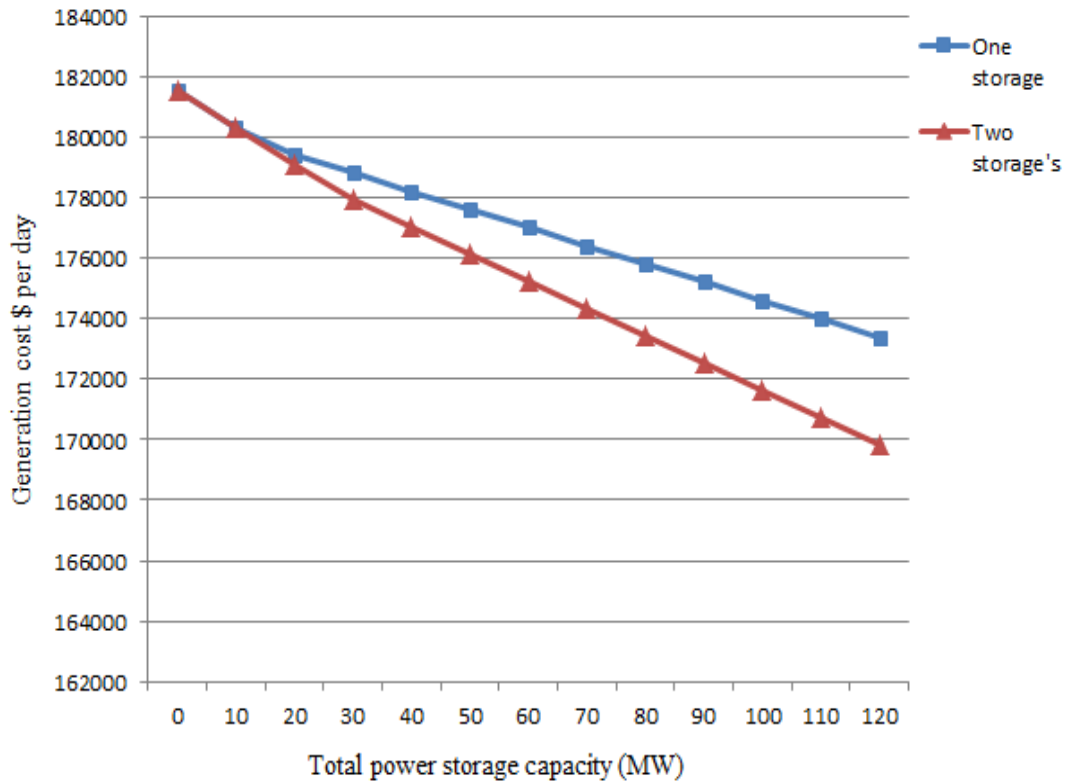


Figure 3.9 Fuel cost comparison with one and two storage unit

Energy storage gives additional degrees of freedom in the optimal dispatch problem, thereby potentially allowing the additional use of renewable energy. The simple example of test bed #1 shown has wind penetration in the range of 5 % (Wind peak power / Peak demand power). Much more significant improvements in operating strategies occur at higher storage capacities. This is shown in the Table 3.5.

Table 3.5 Cost comparison with one and two storage units.

Total Storage Capacity (MW)	Fuel cost per day to serve the load (\$/day)	
	One storage unit	Two storage unit
0	181,520	181,520
10	180,320	180,320
20	179,380	179,080
30	178,810	177,910
40	178,210	177,010
50	177,610	176,110
60	177,010	175,210
70	176,410	174,310
80	175,810	173,410
90	175,210	172,510
100	174,610	171,610
110	174,010	170,710
120	173,400	169,810

Chapter 4. Illustrative Example using the State of Arizona as a Test Bed

4.1 Description of the test bed: State of Arizona

The previous chapter provided an introduction test system to demonstrate the idea of optimal scheduling of energy storage. This chapter looks at more realistic and well-studied example. In this section, the effect of energy storage on the minimization of the objective function using the State of Arizona as a test bed with different storage capacities and wind generation is studied. This benchmark system which represents a portion of the Western Electricity Coordinating Council (WECC) as of April 2009 does not include storage. Therefore, while the use of its network topology, generation bounds as well as transmission line ratings bounds, appropriate values for the storage parameters are added in the profile. The load, wind power and LMPs (assumed, at generation bus) at each bus are also given. The heavy summer case of 2009 is considered (actual load and generation data).

In the test case, an objective function is minimized. Again, this corresponds to minimum operating cost. The constraints and formulation of the problem is the same as provided in the previous chapter. A QP based algorithm is carried out to optimize generation and storage scheduling with maximal use of renewable generation. The optimum generation schedule, storage (store / discharge) schedule, and line flows are control variables, and these quantities are calculated.

A one day time horizon broken into 3 intervals each having a span of 8 hours is studied. The objective of the constrained economic dispatch is to schedule the generation outputs economically including storage over one day. The simplifications made are:

- Reactive power flows are not modeled or considered
- A simple linear relationship is assumed between bus voltage phase angle and line active power flows
- Transmission line losses are neglected.

The portion of the WECC under study is mainly the state of Arizona having the description profile indicated in Table 4.1.

Table 4.1 Description profile: state of Arizona power system

Number of buses n_b	792	Number of generators	182
Number of lines n_l	1079	Number of wind farms *	2

*Assumed

4.2 Case 4

Case 4 is a ‘base case’ study for this test bed. In case 4, no storage units are scheduled and two wind farms are located at Flagstaff and Springerville. Economic dispatch of generations is calculated for the minimum cost using QP in the Matlab optimization toolbox. According to the constraints considered in this work, only active power constraints are considered. Therefore, the respective

maximum and minimum operating long term thermal ratings of the transmission lines, generation limits and voltage phase angle limits at each bus is accommodated in the upper (UB) and lower bound (LB) of the program.

Table 4.2 shows the operational data for case 4. The cost of economic dispatch of generation per day calculated is 12.049 million dollars per day (M\$/day).

Table 4.2 Case 4 study results, Arizona test bed

<i>Wind</i>		<i>Storage</i>			
P_1 (MW)	P_2 (MW)	P_1 (MW)	W_1 (MWh)	P_2 (MW)	W_2 (MWh)
400	300	0	0	0	0

4.3 Case 5 – storage added

In case 5, the wind power capacity and storage capacity is increased. The case is divided under low, medium and high depending on wind power penetration and storage capacity of the power system. Two wind farms are considered located at Flagstaff and Springerville along with two storage units both at Navajo. The storage units have two ratings, one relating to the power electronic converters (this is the power rating of the unit), and the other as the ultimate energy storage capability (this is the energy rating, e.g., in MWh). For case 5, it is assumed that the power rating (MW) times 6 hours is the energy (MWh) rating.

Table 4.3 tabulates the description of the wind power and storage as well as the solution cost. It is observed that with increasing energy storage, the operating

cost reduces. Note that electrical energy is stored during times when generation cost is low and when production exceeds consumption. The stored energy is discharged during the period when the production cost from conventional generating plants is high.

Table 4.3 Case 5 study results, Arizona test bed

Operational data		Scenario		
		Low 1	Medium 2	High 3
Wind	P_1 (MW)	400	600	800
	P_2 (MW)	300	500	600
Storage	P_1 (MW)	50	100	300
	P_2 (MW)	50	150	250
	W_1 (MWh)	300	600	1800
	W_2 (MWh)	300	900	1500
Cost	QP (Million dollars / day)	11.772	11.453	11.066

4.4 Case 6 – increase in the number of storage units

In case 6, three scenarios are studied. The number of energy storage units is increased from 2 to 4 to 6. In each scenario, the total power and energy stored is

the same, i.e. total power capacity = $P_{ST} = 700$ MW and total energy $W_{ST} = 4200$ MWh. These levels are shared among the storage units. The wind power is the same as assumed in case 4. Table 4.4 tabulates the number of storage units and economic dispatch cost in millions of dollars per day obtained using QP for the respective scenarios.

The results indicate that the cost of delivery and generation (fuel) can be minimized by increasing the capacity of storage elements. The fuel cost can be further optimized by selecting optimum locations for the two storage units.

Table 4.4 Case 6 study results, Arizona test bed

Operational Data		Scenario	1	2	3
		Storage	Number of Units		2
P (MW) each unit			350	175	116.67
W (MW) each unit			2100	1050	700
Cost	QP (M\$/day)		11.616	11.563	11.484

4.5 Case 7 – large scale implementation

This case resembles more of a practical scenario. In other words, a large number of wind machines are accommodated. Note that the 2025 renewable port-

folio standard for Arizona is 15%; a higher percentage of wind generation is accommodated in case 7. In case 7, 15% of the total load is derived from wind generation. Also, ten energy storage units are represented having a total capacity of 700 MW with 6 hours of energy storage (i.e., the total energy rating is 6 times 700 or 4200 MWh). Table 4.5 shows the system description.

Table 4.5 Case 7 system description

Number of buses n_b	792	Number of lines n_l	1079
-----------------------	-----	-----------------------	------

The wind availability considered throughout the day for case 7, and this is shown in Fig. 4.1. The wind turbines are assumed to generate power at name plate rated capacity. The cost of economic dispatch of generation computed is 10.863 million dollars per day (M\$/day).

The inference made from this case is the optimal location of energy storage units is at the generation buses. This observation is made for storage units such as batteries; however, obviously, the location of pumped-hydro storage is dictated by geography and topography. Storage units can be placed next to wind farms to produce a consistent flow of power. Locations like Bullhead City have a high potential of wind production [5]. Siting wind generation at such locations may be dependent on ratings of the adjacent transmission facilities. Storage unit placed at these locations can store excess wind energy and discharge during later

periods. Therefore, use of storage can reduce the cost of upgrade of the electricity link and defer the expansion of the transmission network.

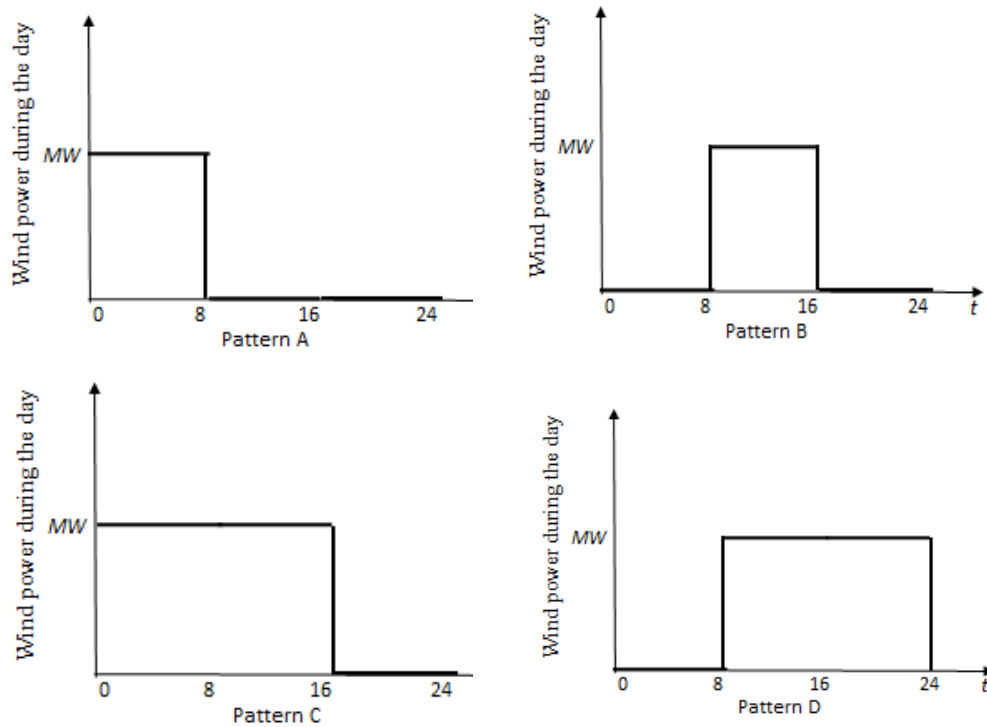


Figure 4.1 Wind generation patterns for case 7, t is in hours

4.6 Calculation of payback period

The payback period in capital budgeting refers to the period of time required to return an investment, to repay the sum of the original investment. An approximate payback period is calculated for the above discussed cases 5, 6 and 7. Mathematically, the length of time required to recover the cost of an investment is calculated as:

$$= \text{Cost of Project} / \text{Annual Cash Inflows}$$

There are two main problems, with the payback period method:

- It ignores any benefits that occur after the payback period and, therefore, does not measure profitability.
- It ignores the time value of money.

Annual cash inflows is the savings obtained from cases 5, 6, and 7 when compared with case 4. Following assumption is made:

- The energy storage system is a lead-acid battery and wind turbine is a doubly fed induction generator (DFIG), type 3 is assumed.
- Cost of lead-acid battery, $C_B = 0.17$ \$/Wh.
- Cost of wind turbine, $C_W = 1.2$ to 2.6 million \$/MW of name plate capacity.
- Cost of electronics (converter), $C_E = \$ 250$ per kW.

Let, N_D = Number of days for repay of the original investment.

Case 5: The wind power capacity and storage capacity is increased. The case is divided under low, medium and high depending on wind power penetration and storage capacity of the power system. The case 5 test bed has two energy storage systems and two wind turbines with the electronic converters.

Low case scenario: Total cost of battery storage is,

$$\begin{aligned} C_{BT} &= \text{Number of units} \times \text{Storage capacity (Wh)} \times \text{Cost of lead acid battery} \\ &\quad (\$/\text{Wh}) \\ &= 2 \times 300 \times 10^6 \times 0.17 = \$ 102 \text{ million.} \end{aligned}$$

Cost of electronics for the two storage units,

$$C_{ET} = 2 \times 250 \times 50000 = \$ 25 \text{ million.}$$

Cost of wind turbines,

$$C_{WT} = (400+300) \times 1.2 = \$ 840 \text{ million.}$$

Total initial investment is,

$$C_i = C_{BT} + C_{ET} + C_{WT} = \$ 967 \text{ million.}$$

Saving's with respect to case 4,

$$S = \$ 0.277 \text{ million /day.}$$

Thus, an approximate payback period is,

$$S \times N_D = C_i$$

$$\Rightarrow N_D = C_i / S = 3490.97 \text{ days} = 9.56 \text{ years}$$

Note: The above calculation does not take account of maintenance and battery replacement with inflation rate for the total system.

By the same token, the approximate payback period in years for medium case and high case scenario is 7.52 years and 6.63 years respectively.

Case 6: The number of energy storage units is increased from 2 to 4 to 6. In each scenario, the total power and energy stored is kept the same, i.e. total power capacity = $P_{ST} = 700$ MW and total energy $W_{ST} = 4200$ MWh. The payback period for 2, 4 and 6 storage units is found to be 10.93 years, 9.74 years and 8.38 years.

Case 7: This case resembles more of a practical scenario. In other words, a large number of wind machines are accommodated. Here, 15% (4400 MW) of the total

load is derived from wind generation. Also, ten energy storage units are represented having a total capacity of 700 MW with 6 hours of energy storage (i.e., the total energy rating is 6 times 700 or 4200 MWh). The payback period is 14.25 years. Figure 4.2 represents the payback period of each case.

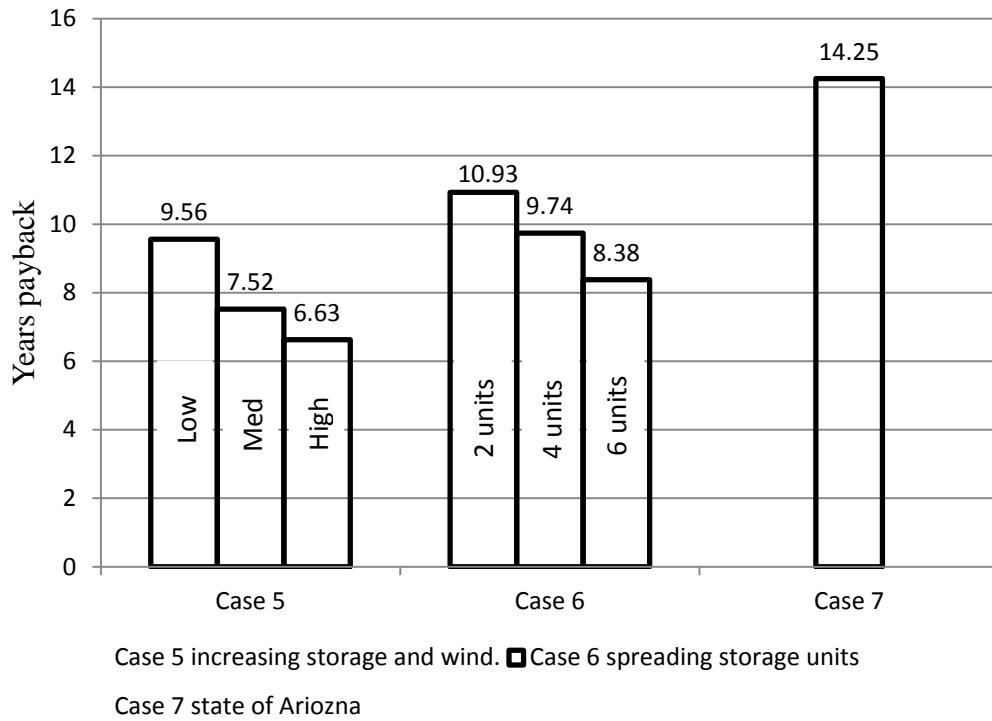


Figure 4.2 Payback period

Chapter 5. Conclusions and Future Work

5.1 Conclusions and main contributions

It has been shown in this thesis that energy storage devices not only facilitate the large scale integration of renewable energy resources into the grid, but also assist in the economic dispatch of generation. In this research, an equivalent section of the WECC system, namely for Arizona, summer peak 2009 was considered. The following main conclusions can be made from the results presented in the previous chapters:

- In the base case (Case 4) without energy storage, the minimum generation dispatch cost of the system is 12.049 million dollars per day and the economic dispatch with the energy storage in system for a comparable case (Case 5) the total generation dispatch cost is 11.772 million dollars per day. The savings increases with an increase in storage capacity. Quantitatively, for the cited case an increase in savings of 0.277 to 0.596 million dollars per day is attained for addition of 900 MWh.
- The test bed state of Arizona with accommodation of 15% (4400 MW) of the total load being served from wind production and 4200 MWh of energy storage the total generation dispatch cost is 10.863 million dollars per day. This figure is observed for the summer 2009 peak period.
- Large scale energy storage can be used to mitigate the overloading of the transmission lines at places where the wind energy potential is high and

connection to the grid is expected. For example, in Case 7, wind generation sited at Bullhead City AZ was studied and energy storage at this site is allowed. The addition of 250 MW of wind despite the 140 MW adjacent existing transmission. In this example energy storage is rated at 720 MWh, with a converter rating of 120 MW.

- Defer the upgrade of the transmission systems when renewable resources are added. This results as the usage of energy storage can reduce the power transfer through adjacent lines during peak load periods. Also, the use of storage can decrease the congestion cost as energy storage systems can shift the load from the peak to off-peak load periods. This advantage was illustrated in Case 7 in which up to 116 MW in the period 0800 to 1600 hours is shifted to the period 1600 to 0000 hours. This 116 MW shift was in a line of rating of 398 MVA.
- Disbursing the storage units across the state of Arizona reduces the generation production cost. This is the case because of line rating constraints limiting concentrated energy storage. Case 6 illustrates this point through the comparison of the utilization of 2, 4, and 6 storage units.

Secondary contributions are:

- Quadratic programming has been illustrated as an optimization method for scheduling energy storage.
- Examples have been shown with an actual power system for the state of Arizona.

- Sample code in Matlab has been developed (see Appendix A).

5.2 Future work

In the present work, the objective cost function is based on the power generated from power plants. The thesis mainly focuses on economic dispatch using the state of Arizona as a test bed. This work can be extended by modeling the system external to Arizona and implementing the following:

- Address the dimensionality problem.
- Include the impact of energy storage on the reduction of spinning reserve.
- Model the transmission and storage devices losses occurring in the power system.
- Model the storage technologies characteristics to better represent each of them.
- Include a dynamic response study to study the system stability. Also, check voltage stability in the steady state and in the dynamic case.
- Study the power quality issues in the grid due to the appearance of high levels of DC/AC and AC/DC conversion.
- Perform reactive power studies.

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APPENDIX A
MATLAB CODE

A.1 Matlab code used in this project

```
clear;
clc;
tic;
%% Main system data
nb=792; % Number of bus
nl=1079;% Number of lines
h=3;    % Intervals of each 24/h hours
ng=nb;  % Storage and generation assumed at all buses. Zero stor-
age and
nst=nb; % generation is accommodated in lower and upper bounds
Base=100;

%% Read data
B = xlsread('AZ.xlsx','Line Records','b1:b10000'); % From bus
D = xlsread('AZ.xlsx','Line Records','d1:d10000'); % To bus
J = xlsread('AZ.xlsx','Line Records','k1:k10000'); % Reactance
U=xlsread('AZ.xlsx','Bus Records','n2:n10000');    % Load
V=xlsread('AZ.xlsx','Bus Records','o2:o10000');    % Wind
Y=xlsread('AZ.xlsx','Line Records','m1:m10000');    % Line rat-
ings
S=xlsread('AZ.xlsx','Storage','q1:q10000');        % Energy
stored
P=xlsread('AZ.xlsx','Storage','p1:p10000');        % Power
Stored
G=xlsread('AZ.xlsx','Storage','r2:r10000');        % Generation
ratings

%% Cost function to be minimized
c=zeros(1,ng*h+nl*h+nst*h+(nb-1)*h);
H=zeros(ng*h+nl*h+nst*h+(nb-1)*h,ng*h+nl*h+nst*h+(nb-1)*h);
Q=xlsread('AZ.xlsx','Storage','s2:s10000');
C=xlsread('AZ.xlsx','Bus Records','t2:t10000');
n=0;k=0;
for ro=1:h:ng*h;
    q=Q(ro+k-n)*Base;
    for added=1:h;
        H(ro+added-1,ro+added-1)=2*q;
    end
    k=k+1;
    n=n+h;
end
f=C*Base;
for i=ng*h+1:ng*h+nl*h+nst*h+(nb-1)*h;
    f(i,1)=0;
end

%% Formation of bus dictionary
busdict=zeros(nb,1);
busdict(1)=10435; %
running=1;
```

```

for iline=1:nl;
    ifrom = B(iline);
    ito = D(iline);
    xline=J(iline);
    ifound=0;
    for i=1:running;
        if busdict(i)==ifrom;
            ifound=i;
            ifromn=i;
        end;
    end;
    if ifound == 0;
        running=running+1;
        busdict(running)=ifrom;
        ifromn=running;
    end;
    ifound=0;
    for i=1:running;
        if busdict(i)==ito;
            ifound=i;
            iton=i;
        end;
    end;
    if ifound == 0
        running = running+1;
        busdict(running)=ito;
        iton=running;
    end;
end;
busdict1=sort(busdict);
%% Formation of equality constraints
% Formation of Aeq
aeq=sparse(nb*h+nl*h+1, (3*nb-1+nl)*h);
inb=sparse(eye(nb*h, nb*h));
aeq(1:nb*h, 1:nb*h)=inb; % Generation at buses
aeq(1:nb*h, (nb)*h+1:(nb+nst)*h)=-inb; % Storage at buses
% Stored energy at end of day should be zero
aeq(nb*h+nl*h+1, (nb)*h+1:(nb+nst)*h)=24/h;
% Line injection power
inh=sparse(eye(h, h));
for k=1:nl;
    stb=B(k);
    stbb=0;
    for look=1:nb;
        if busdict1(look)==stb;
            stbb=look;
        end;
    end;
    endb=D(k);
    endbb=0;
    for look=1:nb;
        if busdict1(look)==endb;
            endbb=look;
        end;
    end;
end;

```

```

        end;
    end;
    aeq(h*(stbb-1)+1:h*(stbb-1)+h,nb*h+nst*h+1+h*(k-1):...
        nb*h+nst*h+h*(k-1)+h)=-inh;
    aeq(h*(endbb-1)+1:h*(endbb-1)+h,nb*h+nst*h+1+h*(k-1):...
        nb*h+nst*h+h*(k-1)+h)=inh;
end;
for k=1:nl;
    x=J(k);
    stb=B(k);
    stbb=0;
    for look=1:nb;
        if busdict1(look)==stb;
            stbb=look;
        end;
    end;
    endb=D(k);
    endbb=0;
    for look=1:nb;
        if busdict1(look)==endb;
            endbb=look;
        end
    end
    if stbb~=1;
        aeq(nb*h+1+(k-1)*h:nb*h+(k-1)*h+h,(2*nb*h+nl*h)+1+(stbb-
2)*h:...
            (2*nb*h+nl*h)+(stbb-2)*h+h)=-1/x*inh;
        end
        if endbb~=1;
            aeq(nb*h+1+(k-1)*h:nb*h+(k-1)*h+h,(2*nb*h+nl*h)+1+(endbb-
2)*h:...
            (2*nb*h+nl*h)+(endbb-2)*h+h)=1/x*inh;
        end
    end
end
inl=sparse(eye(nl*h,nl*h));
aeq(nb*h+1:(nb+nl)*h,nb*h+nst*h+1:(nb+nst+nl)*h)=inl;% Power Flow
vs. delta
%Formation of beq
beq=zeros(nb*h+nl*h+1,1);
for k=1:nb*h
    load=U(k)/Base;
    wind=V(k)/Base;
    beq(k,1)=load-wind;
end
%% Formation of inequality constraints
% Formation of A
a=sparse(nl*h*2+nst*h,ng*h+nl*h+nst*h+(nb-1)*h);
a(1:nl*h,(ng+nst)*h+1:(ng+nst+nl)*h)=inl; % Upper line
rating
a(nl*h+1:(nl+nl)*h,(ng+nst)*h+1:(ng+nst+nl)*h)=-inl;% Lower line
rating
% Maximum energy stored
n=0;m=0;

```

```

for i=1:nst;
a(2*nl*h+1+m:2*nl*h+(h-1)+m,ng*h+1+n:ng*h+h-1+n)=...
    sparse(tril(ones(h-1,h-1))*24/h;
k=a(2*nl*h+1+m:2*nl*h+(h-1)+m,ng*h+1+n:ng*h+h-1+n);
    if h==3;
        a(2*nl*h+h+m:2*nl*h+h+m,ng*h+1+n:ng*h+h-1+n)=-k(2:h-
1,1:h-1);
        j=m;
    else
        a(2*nl*h+h+m:2*nl*h+h+h-3+m,ng*h+1+n:ng*h+h-1+n)=-k(2:h-
1,1:h-1);
        j=m;
    end
n=n+h;m=2*h+m-3;
end
% Maximum stored power
inb=sparse(eye(nb*h,nb*h));
a(2*nl*h+2*h-2+j:2*nl*h+2*h-3+j+nst*h,(nb)*h+1:(nb+nst)*h)=inb;
% Formation of b
b=zeros(nl*h*2+nst*h,1);
k=0;
for j=1:nl;
    kline=Y(j)/Base;
    b(1+k:h+k,1)=kline;           % Line ratings
    k=k+h;
end
k=0;
for j=1:nl;
    kline=Y(j)/Base;
    b(nl*h+1+k:nl*h+h+k,1)=kline; % Line ratings
    k=k+h;
end
m=0;j=0;n=0;
for i=1:nst;
    s=S(i)/Base;                 % Energy stored
    b(2*nl*h+1+m:2*nl*h+h-1+m,1)=ones(h-1,1)*s;
    if h==3;
        b(2*nl*h+h+m:2*nl*h+h+m,1)=ones(h-2,1)*0;
        j=m;
    else
        b(2*nl*h+h+m:2*nl*h+h+h-3+m,1)=ones(h-2,1)*0;
        j=m;
    end
m=2*h+m-3;
end
k=0;
for i=1:nst;
    p=P(i)/Base;                 % Maximum Power stored
    b(2*nl*h+2*h-2+j+k:2*nl*h+3*h-3+j+k,1)=ones(h,1)*p;
    k=k+h;
end
%% Construct lb and ub vectors
lb=zeros((3*nb-1+nl)*h,1);

```

```

k=0;n=0;
for ro=nb*h+1:h:2*nb*h;
    lb(ro)=0;
    es=ro-nb*h-h*k+n;
    rate=P(es)/Base;
    for added=1:h-1;
        lb(ro+added)=-rate;
    end;
    k=k+1;n=n+1;
end;
n=0;k=0;
for ro = 2*nb*h+1:h:2*nb*h+nl*h;
    y=-Y(ro-2*nb*h-h*k+n)/Base;
    for added=1:h;
        lb(ro+added-1)=y;
    end
    k=k+1;n=n+1;
end
lb(2*nb*h+nl*h+1:(3*nb-1+nl)*h,1)=-pi/6;    % voltage angle within
30 degree
ub=-lb;
n=0;k=0;
for ro=1:h:nb*h;
    g=G(ro+k-n)/Base;
    for added=1:h;
        ub(ro+added-1)=g;
    end
    k=k+1;n=n+h;
end
k=0;n=0;
for ro=nb*h+1:h:2*nb*h;
    ub(ro+h-1)=0;
    es=ro-nb*h-h*k+n;
    %es=((ro-h*(n+2)-1)/h+k);
    rate=P(es)/Base;
    for added=1:h-1;
        ub(ro+added-1)=rate;
    end;
    k=k+1;n=n+1;
end;
options=optimset('Algorithm','interior-point-convex');
[X,fval]=quadprog(H,f,a,b,aeq,beq,lb,ub,[],options);
%Aeq=full(aeq);
toc;

```

APPENDIX B

QUADRATIC PROGRAMMING ALGORITHM

B.1 Quadratic programming

QP model of economic dispatch

Let the initial operating point of generator i be P_{geni}^0 . Expanding the non-linear objective function using Taylor series [33],

$$\begin{aligned} f_i(P_{geni}) &= f_i(P_{geni}^0) + \left. \frac{df_i(P_{geni})}{dP_{geni}} \right|_{P_{geni}^0} \Delta P_{geni} + \frac{1}{2} \left. \frac{df_i^2(P_{geni})}{dP_{geni}^2} \right|_{P_{geni}^0} \Delta P_{geni}^2 + \dots \\ &= a \Delta P_{geni}^2 + b \Delta P_{geni} + c \\ f_i(\Delta P_{geni}) &= a \Delta P_{geni}^2 + b \Delta P_{geni} \end{aligned}$$

where $a = \frac{1}{2} \left. \frac{df_i^2(P_{geni})}{dP_{geni}^2} \right|_{P_{geni}^0}$ $b = \left. \frac{df_i(P_{geni})}{dP_{geni}} \right|_{P_{geni}^0}$ $c = f_i(P_{geni}^0)$ are constant

and $\Delta P_{geni} = P_{geni} - P_{geni}^0$

Power balance equation

Since loads are constant for the given time and using Kirchoff's law, the following expression of power balance equation obtained:

$$\sum P_{geni} = \sum P_{Lk}$$

Linearization of branch flow constraints

The real power flow equation of a branch is:

$$P_{ij} = V_i^2 g_{ij} - V_i V_j (-g_{ij} \cos \theta_{ij} + b_{ij} \sin \theta_{ij})$$

Where

P_{ij} The sending end real power on transmission branch ij

V_i The node voltage magnitude of bus i

θ_{ij} The difference of bus voltage angles between the sending and receiving

end of the line ij

g_{ij} The conductance of transmission branch ij

b_{ij} The susceptance of transmission branch ij

Linearizing the power flow equation and considering a high voltage network, the value of θ_{ij} is very small. In addition, assuming the magnitudes of all the bus voltages equal to 1.0 p.u. and the reactance of the line is much bigger than resistance of the line:

$$\Delta P_{ij} = -b_{ij}\Delta\theta_{ij} = \frac{\Delta\theta_i - \Delta\theta_j}{X_{ij}}$$

Generator and storage power constraint

$$0 \leq P_{geni} \leq P_{geni\ max} \quad i \in NG$$

$$P_{sq\ min} \leq P_s \leq P_{sq\ max} \quad q \in n_s$$

QP Algorithm

Quadratic programming is the problem of finding a vector X that minimizes a quadratic function, subject to linear constraints:

$$\text{Minimize} \quad X^T Q X + C^T X \quad (1)$$

$$\text{Such that} \quad A_{eq} X \leq b_{eq} \quad (2)$$

$$AX \leq b \quad (3)$$

$$lb \leq X \leq ub \quad (4)$$

where C is an n -dimensional row vector of cost of generation, Q is an $(n \times n)$ symmetric matrix describing the coefficients of the quadratic terms, the decision

variables are denoted by the n -dimensional column vector X , and the constraints are defined by an $(m \times n)$ A , A_{eq} matrix and an m -dimensional column vector b , b_{eq} of right-hand-side coefficients.

When the objective function $f(X)$ is convex for all feasible points, the problem has a unique local minimum, which is also the global minimum.

The equation (3) can be expressed as [33]

$$g(X) = (AX-b) \leq 0 \quad (5)$$

The Lagrange function for the equation (1) and (5),

$$L(X, \mu) = CX + X^T QX + \mu g(X)$$

where μ is an m -dimensional row vector.

According to the optimization theory, the Kuhn-Tucker (KT) conditions for a local minimum are given as follows:

$$\left. \begin{array}{l} \frac{dL}{dX_j} \geq 0, j = 1, \dots, n \\ C + 2X^T Q + \mu A \geq 0 \end{array} \right\} \quad (6)$$

$$\left. \begin{array}{l} \frac{dL}{d\mu_i} \leq 0, i = 1, \dots, m \\ AX - B \leq 0 \end{array} \right\} \quad (7)$$

$$\left. \begin{array}{l} X_j \frac{dL}{dX_j} = 0, j = 1, \dots, n \\ X^T (C^T + 2QX + A^T \mu) \geq 0 \end{array} \right\} \quad (8)$$

$$\left. \begin{array}{l} \mu_j g_i(X) = 0, i = 1, \dots, m \\ \mu(AX - B) \geq 0 \end{array} \right\} \quad (9)$$

$$\left. \begin{array}{l} X \geq 0 \\ \mu \geq 0 \end{array} \right\} \quad (10)$$

Introduction of nonnegative variables y to the inequalities in equation (6) and nonnegative variables v to the inequalities in equation (7), to obtain the equations:

$$C^T + 2QX + A^T \mu^T - y = 0 \quad (11)$$

$$AX - B + v = 0 \quad (12)$$

Then, the KT conditions are written as:

$$2QX + A^T \mu^T - y = -C^T \quad (13)$$

$$AX + v = B \quad (14)$$

$$X \geq 0, \mu \geq 0, y \geq 0, v \geq 0 \quad (15)$$

$$y^T X = 0, \mu v = 0 \quad (16)$$

The KT conditions in equations (13) to (16) have a linear form with the variables X , μ , y , and v . An interior point convex algorithm can be used to solve the equations (13) to (16). The interior point convex algorithm performs the following steps [35]:

1. Presolve/Postsolve
2. Generate initial point
3. Predictor-corrector
4. Multiple corrections