

Design Methodology for Modifying an Existing Internal Combustion Engine to
Generate Power from a Stored Air System

by

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ABSTRACT

A low cost expander, combustor device that takes compressed air, adds thermal energy and then expands the gas to drive an electrical generator is to be designed by modifying an existing reciprocating spark ignition engine. The engine used is the 6.5 hp Briggs and Stratton series 122600 engine. Compressed air that is stored in a tank at a particular pressure will be introduced during the compression stage of the engine cycle to reduce pump work. In the modified design the intake and exhaust valve timings are modified to achieve this process. The time required to fill the combustion chamber with compressed air to the storage pressure immediately before spark and the state of the air with respect to crank angle is modeled numerically using a crank step energy and mass balance model. The results are used to complete the engine cycle analysis based on air standard assumptions and air to fuel ratio of 15 for gasoline. It is found that at the baseline storage conditions (280 psi, 70°F) the modified engine does not meet the imposed constraints of staying below the maximum pressure of the unmodified engine. A new storage pressure of 235 psi is recommended. This only provides a 7.7% increase in thermal efficiency for the same work output. The modification of this engine for this low efficiency gain is not recommended.

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LIST OF SYMBOLS

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1. Φ (phi).....	15
2. Υ (gamma)	16
3. θ (theta)	17
4. Δ (delta).....	20
5. ζ (zeta).....	22

Chapter 1

INTRODUCCION

PROBLEM STATEMENT

A low cost expander, combustor engine is to be developed by modifying an existing internal combustion engine to accept compressed air during its compression stage for the purpose of eliminating much of the parasitic pump work required for standard engine operation. The compressed air used will be stored in a storage tank at 280 psi.

Compressed air energy storage (CAES) is an energy storage system, similar to a dam or battery, in which excess base load energy is used to compress air and stored in a cavern or underground mine as potential energy (Swider, 2007). The principal motivation behind this storing is to take advantage of the significant difference between peak and off peak electricity prices. This stored compressed air is later combusted and expanded in a turbine during times of peak demand (Vasdz, 2001).

Interest in low cost expander. Southwest Solar Technologies (SST), an engineering company located in Phoenix, Arizona is developing a solar turbine that pairs with the CAES to increase their energy yield and to take advantage of the decoupling of compression. This allows them to run their compressors at night at a lower cost for storing and later produce power during peak hours (Technology, 2008).

For a proof of concept design, SST is researching the use of large air storage tanks for a small scale energy producing plant (power plant). The use of turbines for such small scale is not feasible. So SST has shown interest in

modifying an existing internal combustion (IC) engine to run on the stored compressed air.

Proposed Modifications.

Modified engine main components. The IC engine in this paper refers to a direct injection, spark ignition, reciprocating engine based on the Otto cycle. Figure 1 below is a functional representation of the modified engine. It shows the main functional components involved during the modified engine operation. Figure 1 is not to scale and only shows a single cylinder of an engine and excludes many components that might be part of the modification. The engine will be connected to the compressed air tank (1) through a connection (2) to the intake valve port controlled by the filling valve (3). As the filling valve opens, air from the compressed air tank flows in to the combustion chamber (11) through the filling connection. The exhaust valve (4) and port is connected to an exhaust/intake connection (5) which has a one-way valve (6) and in separate intake port (7) attached. The piston (8) moves up and down along the cylinder (9) to perform the required strokes for engine operation. A spark plug (10) provides the required ignition.

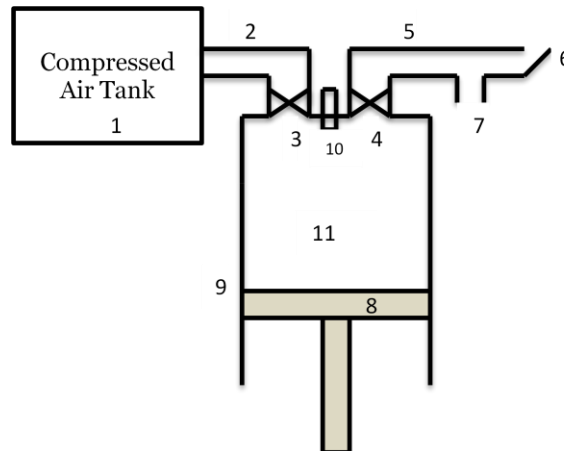


Figure 1. Modified IC engine functional components.

Process modifications to allow engine operation with compressed air and eliminate possible pump work. To understand how the modified engine works, knowledge of the standard engine cycle is required. An overview on the standard engine cycle is provided in the background information section in this chapter.

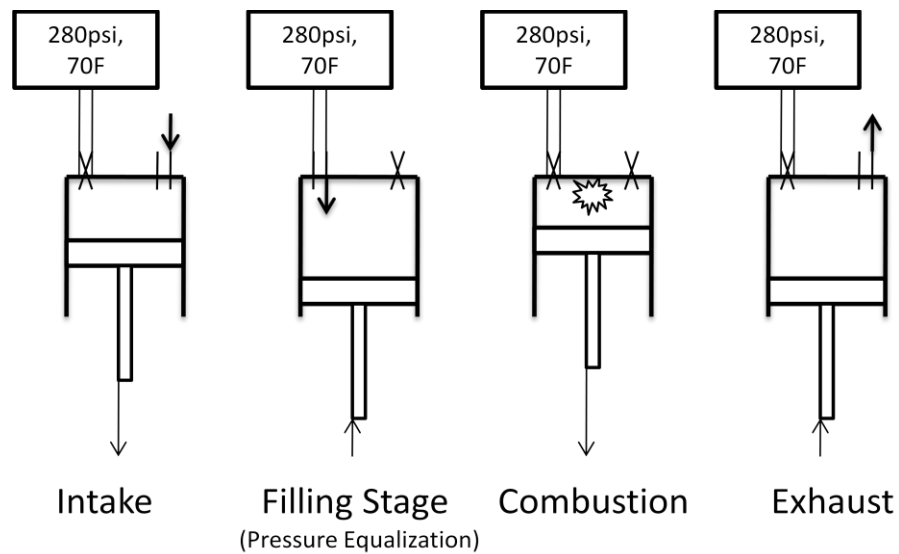


Figure 2. Modified IC engine cycle stages.

Combustion and Expansion Stroke. This stage is similar to the standard IC engine cycle.

Exhaust Blowdown and Exhaust Stroke. This stage is also similar to the standard IC engine cycle. At the expansion stroke, hot exhaust gases are pushed out as the piston continues to move from BDC to TDC with the exhaust valve (4) open.

The compressed air (CA) tank is connected to the standard intake port. Opening the filling valve (3) during the intake stroke will introduce the compressed air into the cylinder during intake. This is not desirable if the goal is

to eliminate pump work since work will be done against a higher pressure during the following compression stage. Due to this, the intake stroke will be modified as follows.

Intake Stroke. During this stage the piston travels from TDC to BDC with the exhaust valve still open from the exhaust stroke. The filling\intake valve remains closed while atmospheric air enters the combustion chamber through the exhaust valve. This stage equalizes the cylinder pressure to atmospheric pressure. The work done here is assumed to be negligible. Some modifications to the exhaust port might be required to avoid a vacuum that would cause the exhaust to re-enter. For example: a one-way valve.

Following the intake stroke, the piston starts moving from BDC to TDC. To avoid any compression, the exhaust valve needs to still remain open. Work is being done by the piston causing a change in volume in the cylinder against the atmospheric pressure.

Filling Stage. For the filling to take place, the exhaust valve is closed and the filling (intake) valve is opened at a particular crank angle before TDC (this angle needs to be determined from analysis). The filling valve is opened to allow the compressed air to fill the combustion chamber (CC) for just enough time so that the cylinder reaches the desired pressure before spark is initiated. The filling valve is closed around 10° before TDC or less depending on when fuel is injected or the spark plug is activated.

Compression does take place due to the upward movement of the piston during filling and after the closing of the valve before TDC. So strictly speaking, pump work is not entirely eliminated, but maybe significantly reduced.

Mechanical modifications to allow the modified engine operation.

Valve train modifications.

For the intake side:

- The intake valve needs to be open during the compression stroke so that filling can take place. Addition of material on the cam shaft is needed.
- Also the intake valve is closed during the intake stroke. Air is sucked in through the exhaust port. Subtraction of material is needed on the camshaft.

For the Exhaust Side:

- Exhaust valve is still open during the intake stroke. Addition of material to the camshaft is needed to keep the valve open after the exhaust stroke and into the compression stroke.

Other Mechanical Modifications that might be required.

- Intake and exhaust processes happen at the same port. Modification might be required not to cross-contaminate the intake and exhaust air. A one-way valve type modification could be used.

Challenges and Issues.

- For the valve modifications, valve timings for each of the stages of a particular engine is required.
- For the filling stage to take place with the least amount of pump work (compression) the filling valve needs to be opened for just enough time to fill to the desired pressure and closed right before combustion begins. If

this time period is small, mechanical valves will not be feasible. This filling time could vary with engine speed due to variations in piston speed.

- The exhaust valve is still open until the filling valve opens in the compression stage to eliminate pump work as much as possible.
- Pump work is not eliminated entirely.
- Lower engine speeds might be required to reduce fatigue on the valves or to stay within the original design.
- The modification costs might not be justified by the performance gain.

PREVIOUS WORK

Compressed air energy storage (CAES). According to Vasdaz, a CAES system consists of two main components 1) components that do the compression and 2) the storing technology.

The compression has to be highly efficient for the technology to be sensible. Multistage compression, inter-cooling, heat storage, recuperation etc are possibilities with this design (Vasdaz, 2001).

The storage technology typically used for storing compressed air on a large scale is currently the state of the art (SOA) for natural gas systems. These natural gas systems can be of the constant pressure or constant volume type.

An adiabatic storage system attempts to retain or store the heat produced during compression and re-use this heat for improving thermal efficiency of a power cycle.

For SST's small scale system, the use of such large storage systems is not reasonable. So, a compressed air storage tank is preferred for this design.

Utilizing compressed air in engine cycle.

Patents. The idea of introducing compressed air into an IC engine during its compression stroke is not a new idea. Below are two patents using the same concept to improve the efficiency of an IC engine.

- 1) USP #4 300 486 11/17/1981 Lowther, Frank E.

The goal of Lowther's (1982) idea is to develop an internal combustion engine in which no compression function is carried out. This is done by providing the compressed air to the engine from a compressed air storage tank which is outfitted with a pressure regulator controlled by a foot pedal to control

the air flow in to the combustion chamber. The engine can also be controlled using the foot pedal connected to the fuel injectors.

According to (Lowther, 1981) the compression that is carried out in the combustion chamber in IC engines is inefficient because the process takes place adiabatically with one stage in a hot cylinder. This compression process also uses a part of the power created by the engine.

There is a potential of substantial fuel saving if the compression is carried out separately and more efficiently.

Goals/Advantages:

- Reduce the fuel consumption of an IC engine with minimal modification, and improve mileage.
- The compression function is eliminated by feeding compressed air from a compressed air tank in to the combustion chambers.
- Reduce the size of the engine.
- When engine power is not required during operation the compression function is not carried out.

Description of invention:

In (Lowther, 1981) the invention the IC engine stages are such:

During the start of the power stroke both the intake and the exhaust valves are closed and a fuel air mixture is being ignited in the combustion chamber using spark or compression causing the expansion. At the beginning of the exhaust stroke the exhaust valve opens and exhaust takes place. This is followed by the intake stroke but, the intake valve remains closed and no intake takes place causing a partial vacuum in the combustion chamber. The intake valve and the exhaust valve remains closed until about 20° bTDC during the

compression stroke. At around 20° bTDC the intake valve opens and lets the pre-compressed air from the tank in to the combustion chamber. Fuel is injected and the fuel air mixture is ignited as the piston approaches TDC. The 20° bTDC is not a critical value and other timings can be used.

2) US 2010/0031934 02/11/2010 Tayyari, Seyyed Farhad.

In this design an external compressor is used to compress the air and provide it to the combustion chamber of the engine as the piston moves from BDC to TDC. Fuel is added, mixed and ignited to create the combusive force to push the piston from TDC to BDC.

Goal/Advantages:

- An external compressor is used to perform intake and compression, combustion and exhaust takes place in the combustion chamber. The cycle is designed so that there is a combustion stage in each revolution.
- The engine would have twice the power of a comparable 4 stroke engine with similar volume and cylinders.
- If the additional power is not required the engine RPM can be reduced to half to increase fuel efficiency and engine durability.

Description of invention:

Functional- The piston moves from BDC to TDC for the exhaust stroke with the exhaust valve opens pushing the exhaust out. At around 40° to 50° degrees bTDC the primary entry valve and the intake valve opens and the compressed air enters the combustion chamber pushing the remaining exhaust out of the exhaust valve. The exhaust valve is closed and fuel is injected as the intake valve closes. The fuel air mixture may or may not be further compressed,

but is then ignited when the piston reaches TDC causing the power stroke. The piston moves from TDC to BDC and then back to the exhaust stroke continuing the cycle. According to the above scheme there is a power stroke every cycle.

The approach taken in this paper is very similar to Lowther's design. In his approach the overall pump work is higher because the pump work during the exhaust stroke is not offset by the pump work during intake stroke due to the vacuum being created. He also does not give a justification of the 20° bTDC value.

In the proposed modification the exhaust stroke pump work is offset by the similar intake stroke pump work. It is also the goal of this paper to mathematically calculate the filling valve open angle.

In both the patents there is no attempt to model what happens during the filling or mathematically predict the engine performance gained.

A thorough search of technical journals available was done using the research databases available at Arizona State University. Engineering Village Compendex, Engineering Village Inspec, Knovel, Energy Citations Database and Google Scholar are the databases used. The keywords used during the search are compressed air energy storage, internal combustion engine, CAES IC engine, stored air system combustion engine, internal combustion external compression engine, gas spring model or analysis, air filling piston moving upward, reduce pump work in IC engine, filling air in cylinder.

Based on this through literature search, it is concluded that a thermodynamic analysis on the proposed modification has not been performed before. This provides a good opportunity to investigate the thermodynamics of

the proposed modification and gauge the performance and sensibility of the modification.

BACKGROUND INFORMATION

An internal combustion engine (ICE) can be classified in many different ways based on a variety of parameters such as type of ignition, type of cycle, valve location, etc (Pulkrabek, 2003). In a reciprocating engine a piston is connected to a crankshaft through a connecting rod, thus forming a relational configuration between the piston and crankshaft. As the crankshaft is rotating, it causes the piston to move up and down inside a cylinder which provides the space for the functions of an engine cycle. (Figure 3)

The position of the piston when it stops at the farthest point away from the crankshaft is called the top dead center (TDC). And the bottom dead center (BDC) is the position of the piston when it stops at the point closest to the crankshaft.

To facilitate the working of an engine, valves are used to allow for flow into (intake valve) and out of (exhaust valve) the cylinder. A spark plug is also used to initiate combustion by use of a spark.

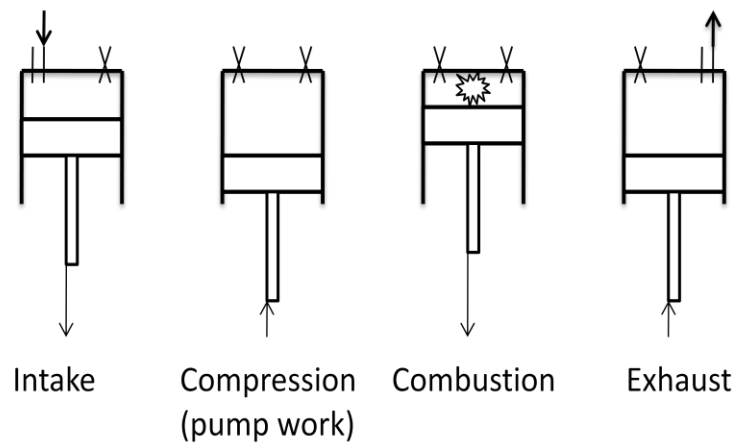


Figure 3. Standard engine cycle stages.

Standard four stroke engine cycle.

Intake Stroke: During this stage, the piston travels downward from top dead center (TDC) to bottom dead center (BDC) with the intake valve open and exhaust valve closed. This vacuum creating action draws air into the combustion chamber. Fuel may or may not be injected into the air flowing in depending on the type of fuel injection system (Pulkrabeck, 2003).

Compression Stroke: After the intake stroke, both the intake and exhaust valves are closed. During this stage the piston moves from BDC to TDC, compressing the air. Fuel could also be injected during the later part of this stage. At a very short period (spark advance) before compression stage ends, the spark plug is activated to start combustion (Pulkrabeck, 2003).

Combustion and Expansion Stroke: During this stage, the air fuel mixture is ignited which raises the temperature and pressure to high peak values. The high pressure created by the combustion pushes the piston from TDC to BDC creating work (Pulkrabeck, 2003).

Exhaust Blowdown and Exhaust Stroke: After the expansion, the exhaust valve is opened and the hot exhaust gases are pushed out of the cylinder due to the pressure differential caused by the hot gases in the cylinder. As the piston continues to move from BDC to TDC the remaining exhaust gases are pushed out through the exhaust valve (Pulkrabeck, 2003). The intake valve opens at TDC to allow for intake, and the cycle continues.

RESEARCH QUESTION

The research question and its secondary questions are listed below:

What is the decrease in fuel heat input (Q_{in}) to an existing engine if stored compressed is used air to reduce compression work done by the piston?

Other questions included in the scope of this work are:

- How is the operation of a standard SI engine modified?
- What modifications are required for the modified operation?
- How can the compression stage of the modified engine be thermodynamically and numerically modeled and compared to the standard compression stroke?
- What is the state of the air in the CC with respect to crank angle at the provided storage pressure and temperature of 280 psi and 70°F respectively?
- How does the above state compare to other storage pressures at the same temperature?
- Is the engine feasible at the provided storage pressure and other considered storage pressures?
- What is the thermal efficiency of the modified engine operation at the provided and considered storage pressures?

STATEMENT OF WORK

In this paper the engine considered is a reciprocating, single cylinder, four stroke, spark ignition gasoline engine. The specifications of the Briggs and Stratton 6.5 hp INTEK™ PRO Model Series 122600 are used for the numerical model.

Deliverables.

- Modifications necessary to achieve filling.
- Mathematical modeling of the compression (filling) stroke applicable to any SI engine.
- Thermodynamic behavior of the filling process.
- Filling schedule of valve movements vs. crank angle for the design engine.
- Predicted engine performance.

MODEL DEVELOPMENT

This chapter describes how the model was developed. The first section describes how the compression stage in a standard engine is modeled. The next section describes how this model is changed for the modified engine compression stroke. The equations for the remaining performance of the engine are then presented. Finally the model assumptions and constraints are presented in the final two sections of this chapter.

STANDARD ENGINE COMPRESSION STAGE

In a standard 4 stroke engine the compression stage begins a little after BDC when the intake valve closes ($\sim 10^\circ$ aBDC) (Pulkrabeck, 2003). The air contained within the combustion chamber (CC) or cylinder is compressed as the piston moves towards TDC. In most cases the spark may be initiated before TDC to start combustion.

Referring to Figure 4, the compression stage of the standard engine will be modeled from the crank angle at intake valve close Φ_i , to Φ_f (TDC).

At Φ_i the state of the air in the combustion chamber is assumed to be at atmospheric conditions (14.7 psi and 70°F). The valves are closed and air contained within the combustion chamber forms a closed system. If it is assumed that the system is well insulated (adiabatic) and irreversibilities due to mechanical friction are negligible (air-standard assumptions) the compression can be considered to be an isentropic process. This process is discussed in chapter 7.4 of Chengel (2006). Ideal gas and constant specific heats are also assumed.

As the piston moves upwards the pressure P, and volume V, are related by equation 1. Since the mass of air in the CC is constant, the pressure and temperature in the cylinder at any crank angle simplifies to equation 2.

$$\frac{P_2}{P_1} = \left(\frac{v_2}{v_1}\right)^\gamma \text{ also, } Tv^{\gamma-1} = \text{constant} \quad (1)$$

$$\frac{P_2}{P_1} = \left(\frac{V_2(\phi_2)}{V_1(\phi_1)}\right)^\gamma \text{ also, } TV^{\gamma-1} = \text{constant} \quad (2)$$

The volume of air in the CC at any crank angle is given by equation 3, where V_c is the clearance volume, B is the bore of the cylinder, r is the connecting rod length, a is crank offset, and s is the piston position with crank angle or the stroke length from BDC. This term is defined by equation 4. Φ is the crank angle taken to be 0° at BDC. (Pulkrabeck, 2003)

$$V(\phi) = V_c + \frac{\pi}{4} B^2 (r + a - s(\phi)) \quad (3)$$

$$s(\phi) = a * \cos(\pi + \theta) + \sqrt{r^2 - a^2 \sin^2(\pi + \theta)} \quad (4)$$

The mass of the air in the CC is constant and is calculated by the ideal gas equation (5). R is the gas constant for air, and M is the mass of the air.

$$P_1 V_1 = MRT_1, \quad (5)$$

Work done by the piston during this process (Chengel, 2006) is given by equation 6.

$$W_c = \int_{V(\phi_i)}^{V(\phi_f)} P dV = \frac{(P_f V_f - P_i V_i)}{1-\gamma} \text{ or } Cv(T_f - T_i) \quad (6)$$

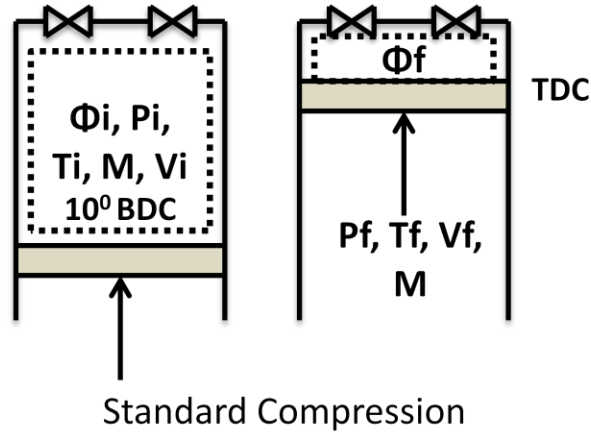


Figure 4. Standard engine compression thermodynamic model.

MODIFIED ENGINE COMPRESSION STROKE

Based on the process modifications discussed, the modified engine “compression stroke” Φ_i to Φ_f can be divided into 3 stages. Referring to Figure 5 they are: (a) no compression stage (Φ_i to θ_i), (b) filling stage (θ_i to θ_f), and (c) further compression stage (θ_f to Φ_f). Constant specific heats at 300°K are assumed for all stages.

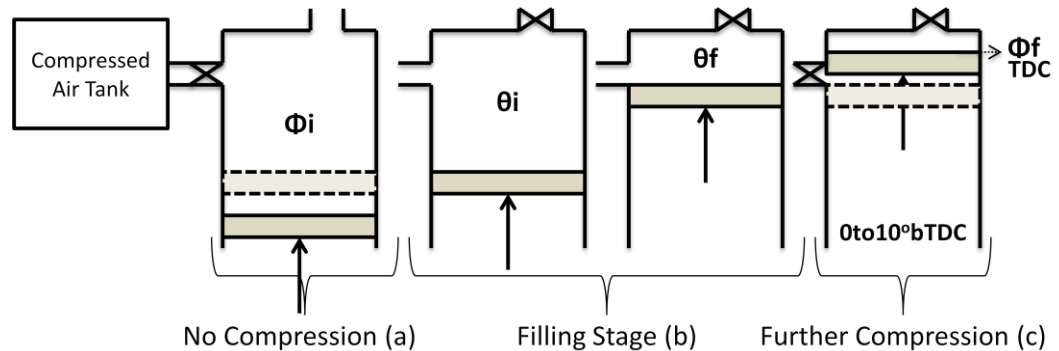


Figure 5. Modified engine detailed compression stroke.

- (a) No compression stage: During the no compression stage, the exhaust valve remains open (continuing from the intake stroke), while the filling valve remains closed. The pressure and temperature in the cylinder remains at atmospheric conditions while the volume changes as the piston moves from 10°aBDC (Φ_i) to filling valve opening at θ_i .

Pressure and temperature remain constant at 14.7 psi and 70°F. The volume changes according to equation 3 and equation 4. The mass of the air in the CC at any time during this stage is given by the ideal gas equation 5. θ_i will be determined after analyzing the filling stage.

Since the pressure in the CC is constant the work being done by the piston moving from ϕ_i to θ_i given by equation 6 simplifies to $W_b = P (V(\theta_i) - V(\phi_i))$.

(b) Filling stage: This stage begins when the filling valve opens at θ_i . The exhaust valve closes at this point. There is a pressure differential between the storage tank which is at constant pressure and the operating volume in the cylinder.

The CC is being filled up due to this pressure differential and will stop when the pressure in the CC equalizes with pressure P_{CT} in the compressed air storage tank.

The goal is to have the filling period as short and as late as possible, so that much of the compressive work that occurs after the valves are closed is eliminated.

At θ_i the state of the air in the CC is known from the previous stage. To find the state of the air in the CC with respect to crank angle a thermodynamic analysis is done. Referring to Figure 6, the control volume forms an unsteady flow process due to the introduction of air in to the CC. Both the size and mass of the control volume are changing with time.

In this case there is mass entering the system (\dot{m}_{in}) and not exiting, so the mass balance simplifies to equation 7. Here w is the angular speed of the engine and $\Delta\theta$ is the crank step

$$M_2 = M_1 + M_{in} = M_1 + \dot{m}_{in} * \Delta t = M_1 + \dot{m}_{in} * \Delta\theta / w \quad (7)$$

If the system in Figure 6 is adiabatic and the moving piston does boundary work on the system the energy balance simplifies to equation 8 and equation 9.

$$W_b = -M_{in}h_{in} + M_2u_2 - M_1u_1 \quad (8)$$

$$u_2 = \frac{W_b + M_{in}h_{in} + M_1u_1}{M_2} \quad (9)$$

Since the air is considered as an ideal gas, u and h are functions of temperature only ($u=C_v T$ & $h=C_p T$). C_v and C_p are constant volume and constant pressure specific heats respectively. For air the change in C_p and C_v over 300K to 600k is around 3% only, so it is considered to be constant at 300K. This assumption simplifies equation 9 to equation 10.

$$T_2 = \frac{W_b + \dot{m}_{in} * \Delta\theta / w C_p T_{in} + M_1 C_v T_1}{C_v * M_2} \quad (10)$$

Knowing the temperature T_2 , mass M_2 , volume V_2 (θ_2) of the next step the pressure is calculated by the ideal gas equation as in equation 5. Hence the state of the next step is determined.

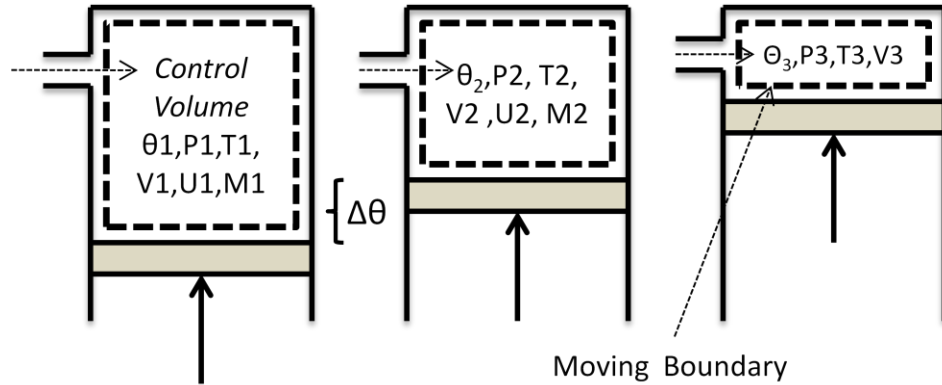


Figure 6. Filling stage thermodynamic model

The boundary work on the system for the step of $\Delta\theta$ and the mass flow rate of air in to the CC needs to be determined.

Boundary work or the work done by the piston is calculated for each step

of $\Delta\theta$ since the pressure varies with the work done by filling and boundary work. For each step, the previous pressure is assumed to be constant over the volume change. After the new pressure is calculated using equations 7 -10 the pressure is updated for the next step. The total boundary work is calculated by adding the step works for the swept volume. Refer to equation 11 where N is the number of steps.

$$Wb = \sum_{j=1}^{N-1} P_j * (V_{j+1} - V_j) \quad (11)$$

The rate at which the air is entering the cylinder depends on this pressure differential. As long as the pressure differential is greater than the critical pressure ratio the flow is choked and has a limited mass flow rate.

If the pressure differential is lesser than the critical pressure the flow rate depends on the new pressure in the cylinder and the pressure in the tank.

Choked and non choked flow. In the case in question, the mass of the air in the cylinder is not constant. It is increasing as the piston is moving up due to the filling. The mass flow rate into the cylinder depends on the resulting pressures in the two chambers.

As soon as the valve is opened, the air rushes into the CC to equalize the pressure. The pressure in the CC is increasing to equal the pressure in the compressed air tank (CT). This flow can be divided into two stages or regions based on the downstream pressure, choked and non-choked stages.

The critical pressure is defined as:

$$P_{crit} = P_{CT} * \lambda^{(\gamma/(\gamma-1))}, \lambda = 2/\gamma + 1 \quad (12)$$

If the pressure in the CC $P < P_{crit}$ the mass flow rate is only a function of the source pressure and temperature and is given by equation 13.

$$\dot{m}_{in} = P_{CT} A_v \sqrt{\frac{\gamma}{RT_{CT}}} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)} = F(P_{CT}, T_{CT}) \quad (13)$$

If the pressure in the CC $P > P_{crit}$ the mass flow rate is variable upon the source pressure, temperature and downstream condition. Mass flow rate is given by the equation 14

$$\dot{m}_{in} = P_{CT} A_v \sqrt{\frac{2}{RT_{CT}} \frac{\gamma}{\gamma-1} \left[\left(\frac{P}{P_{CT}}\right)^{2/\gamma} - \left(\frac{P}{P_{CT}}\right)^{\gamma+1/\gamma} \right]} = F(P_{CT}, T_{CT}, P) \quad (14)$$

This divides the filling into choked flow (P_i to P_{crit}) and non-choked flow (P_{crit} to P_{CT}) stages.

(c) Further compression stage. After the filling valve is closed, fuel is added and ignited to start the combustion process. The fuel filling is not included in this air standard model.

The mass of air inside is further compressed until the piston reaches TDC. Since this process now forms a closed system, isentropic relationships as in equation 1 and equation 2 can be invoked.

ENGINE PERFORMANCE

After the standard compression stage and the modified compression stage the cycle is considered to be the same for both cases (combustion and expansion). To compare the overall benefits and performance of the engine the states of the remaining stages are calculated as given below. These equations are from (Pulkrabeck, 2003).

Combustion in a spark ignition engine is constant volume. Heat (equation 15) is added to raise the temperature and pressure (equation 16) to peak levels. M_a is the mass of the air in the cylinder and M_f is the mass of the fuel. The air to fuel ratio (AF) is known $M_f = M_a / AF$. T_2 and T_3 is the temperature of the air in the cylinder after compression and combustion respectively. Q_{HHV} is the higher

heating value of the fuel and ζ_c is the efficiency of combustion. P_3 is the peak pressure in the CC.

$$Q_{in} = (Ma + Mf)Cv(T_3 - T_2) = MfQ_{HHV}\zeta_c \quad (15)$$

$$P_3 = P_2 \frac{T_3}{T_2} \quad (16)$$

After combustion, isentropic expansion follows. P_4 and T_4 are the pressure and temperatures in the CC after expansion.

$$P_4 = P_3 \left(\frac{1}{r_c}\right)^\gamma \quad (17)$$

$$T_4 = T_3 \left(\frac{1}{r_c}\right)^{\gamma-1} \quad (18)$$

Work done by expansion is $W_{EXP} = Cv(T_3 - T_4)$ and so the thermal efficiency of the engine is $\zeta_T = \frac{W_{exp}-Wc}{Q_{in}}$. Another way of basing engine performance is

through the mean effective pressure $mep = (W_{exp} - Wc)/Vd$, where Vd is the displacement volume.

ASSUMPTIONS

- Otto Cycle
- Spark Ignition Engine
- Air-standard model
- Simple compressible system and uniform unsteady flow during filling steps
- C_p, C_v constant $\gamma=1.4$
- Compression ratio=10
- Combustion efficiency=1
- Baseline air storage at 280 psi, 70° F
- Previous pressure assumed constant for $\Delta\theta$ to calculate work and mass flow rate.

CONSTRAINTS

- Stratton 6.5 hp INTEK™ PRO Model Series 122600
- Fill valve close 10° bTDC spark advance
- AF ratio stoichiometric =15 for gasoline
- (P, T) peak modified \leq (P,T) peak standard

NUMERICAL IMPLEMENTATION

This chapter describes how the equations presented in the previous chapter were implemented into a MATLAB model.

1) Primary Algorithms (unsteadyflow .m)

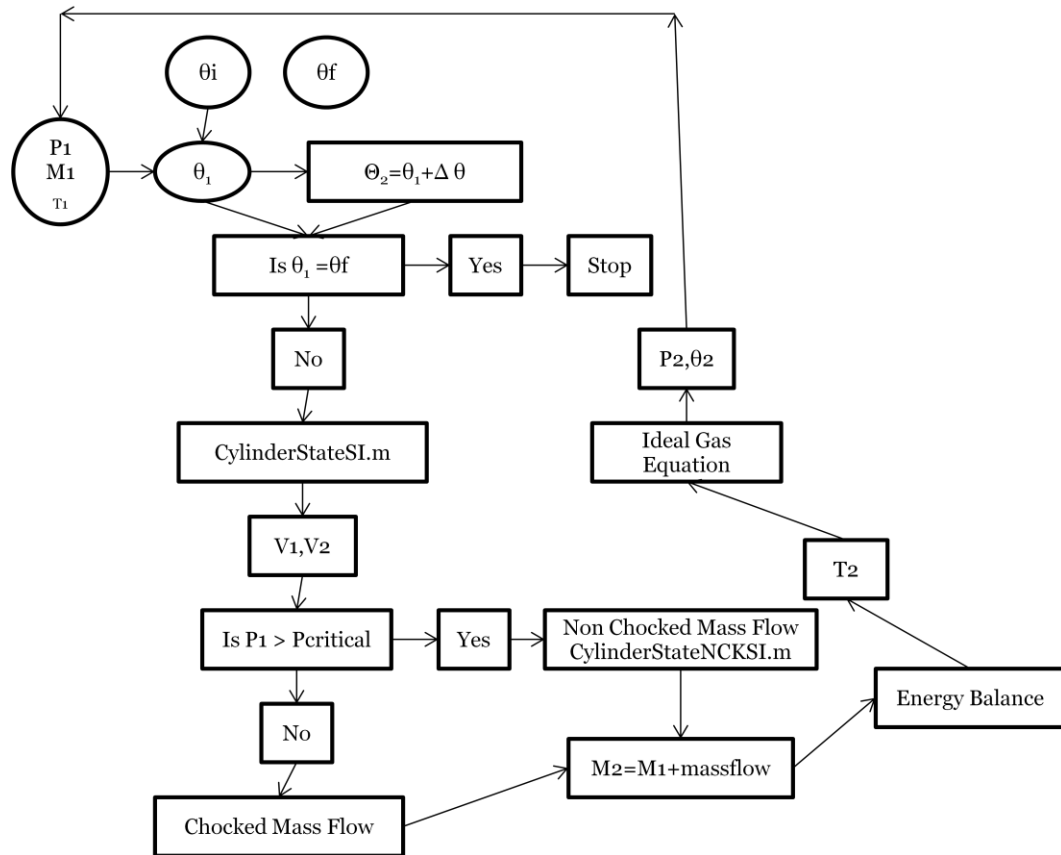


Figure 7. Block diagram for unsteadyflow.m, filling stage

The program “unsteadyflow.m” is a script file that runs through an algorithm to find the states of the air in the CC for the compression stage of both the standard and modified engines. It is divided into three parts (filling, no filling, further compression) for the modified stage and one part for the standard engine. The block diagrams for each of the parts of the program are shown in Figure 7, Figure 8, Figure 9, and Figure 10.

For all the parts the inputs are common and they are as follows:

Reservoir Condition (Compressed Air Tank Conditions)

- Pressure P_{in} (Pa)
- Temperature T_{in} (C)
- gamma γ , constant specific heat

Engine Operation Condition

- Engine Speed N (rev/min) which yields crank speed angular velocity ω (rad/sec)
- Compression stage beginning angle ϕ_i or intake valve close angle for standard engine
- Fill valve open angle θ_i for modified engine
- Fill valve close θ_F .
- Compression stage ending angle ϕ_f or TDC

Initial Conditions in the Combustion Chamber

- Pressure in the combustion chamber after intake stroke P_1 (Pa)
- Temperature in the combustion chamber after intake stroke T_1 (C)
- Gas constant R ($0.286E^3$ J/kg/K)

Inputs for Calculation

- $\Delta\theta$ for each section. It is smaller for the filling stage for more accuracy.
- Constant specific heats for air C_p and C_v (J/Kg/K)

The outputs of each part are the states of the air in the combustion chamber with respect to crank angle.

State of the Air in the Combustion Chamber

- Pressure in the combustion chamber P_2 (psi)
- Temperature in the combustion chamber T_2 (C)
- Volume at each step V_1, V_2 (m^3)
- Mass of the air in the combustion chamber M_1, M_2 (kg)
- Mass flow rate of the air in to the combustion chamber from the fill port \dot{m}_{in} (kg/sec) at every step.

Filling Time Period

- Crank Angle required to reach the compressed tank pressure in the combustion chamber.
- Time for intake derived from the crank angle.

Engine Cycle States and Performance

- Pressure and temperature after compression stage P_c (psi) and T_c
- Pressure and temperature after combustion stage P_3 (psi), T_3
- Pressure and temperature after expansion stage P_4 (psi), T_4
- Pump Work for both standard and modified case (J)
- Work Expansion W_{exp} (J)
- Mass of fuel M_f (kg) based on constant air fuel ratio AF
- Heat added in to the engine during combustion Q_{in} (J)
- Thermal Efficiency
- Mean effective pressure (psi)

For each part the algorithm is followed to complete the stroke from 10°aBDC to TDC.

A θ_i is guessed; if pressure at fill valve close, θ_f equals the pressure in the compressed air tank, then the fill valve opening angle is found. If not a new θ_i is guessed. For each of the part the equations as presented in the model development section are followed.

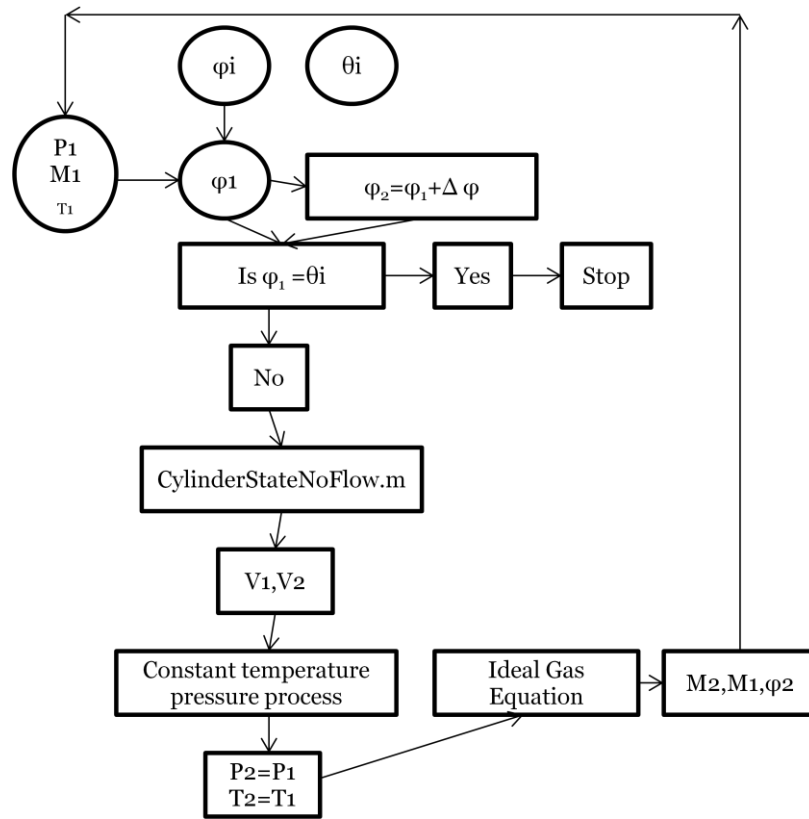


Figure 8. Block diagram for unsteadyflow.m, no filling stage

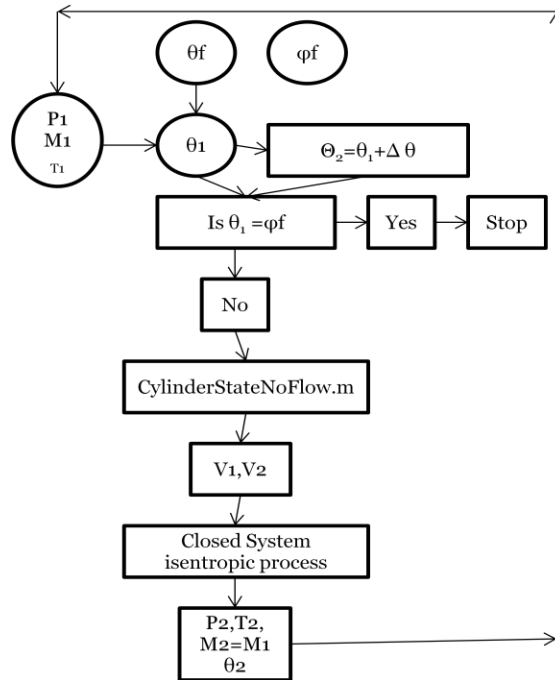


Figure 9. Block diagram for unsteadyflow.m further compression stage

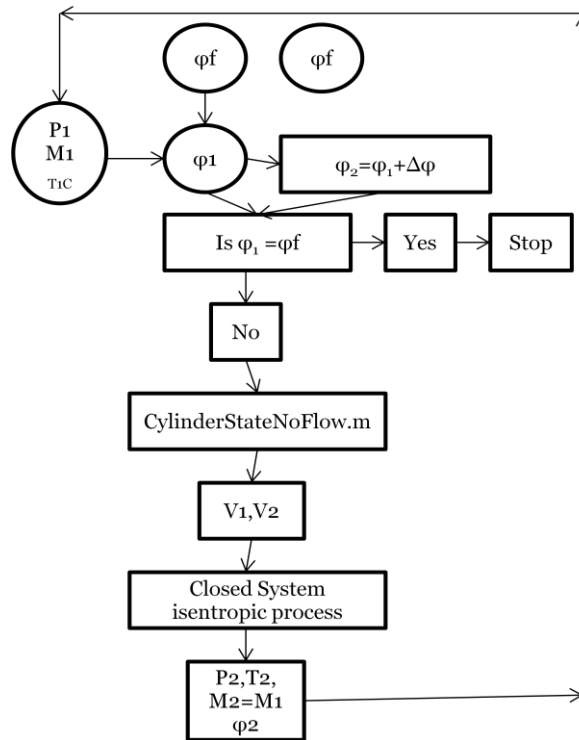


Figure 10. Block diagram for standard compression stage

2) Secondary Algorithms (CylinderStateSI.m , CylinderStateNCKSI.m,
CylinderStateNoFlow.m)

This is a function that calculates the operating volume of a combustion chamber with respect to piston position as specified by the crank angle and mass flow rate of the air from the compressed air tank into the combustion chamber.

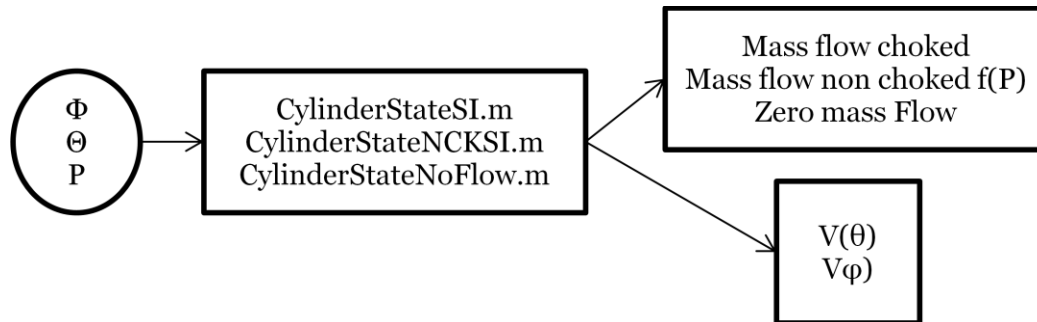


Figure 11. Block diagram for inline functions in unsteady flow.m

All three programs have the same inputs and they are as follows:

Variables

- Crank Angle θ_i
- Current Pressure in the cylinder P_1 (Pa)

Required Parameters

Engine Characteristics.

- Bore (m) b , Connecting rod length (m) r , Stroke (m) S ,
Crank offset length (m) $a=S/2$,
- Compression Ratio (rc)
- Displacement Volume V_d , Clearance V_{ol} (m^3)
 $V_c=V_d/(rc-1)$
- Valve diameter d_v , Effective area of intake valve
 $A_v=\pi*0.25*d_v^2$ ((Pulkrabeck, 2003)),

- Engine Speed N (rev/min) which yields crank speed angular velocity ω (rad/sec)

Compressed Tank Conditions Air

- Pressure P_{in} (Pa), Temperature $T_{in}(C)$, gas constant R , Ratio of specific heats γ

All three of the inline functions have the same outputs namely:

Operational Outputs

- Operational volume of the combustion chamber , V (m^3)
- Mass flow rate of the air in to the combustion chamber, \dot{m}_{in} (kg/sec)

Other Outputs

- Critical Pressure P_{crit} (psi) based on the compressed air tank conditions

The difference in the functions is whether there is flow or not, and if there is mass flow, whether the flow is choked or not. This is decided by the script `unsteadyflow.m` by checking if the calculated pressure is above or below the critical pressure and directed to the appropriate inline function. If the fill valve is closed `CylinderStateNoFlow.m` is used, if fill valve is open and the pressure in the combustion chamber is below critical pressure `CylinderStateSI.m` is used and if the fill valve is open and the pressure in the combustion chamber is above critical pressure `CylinderStateNCKSI.m` is used. The equations used to calculate the mass flow rates are explained in the model development section under ‘choked and no choked flow’.

RESULTS

BASELINE CASE

Baseline Engine. The baseline engine closely follows the Briggs and Stratton 6.5 hp INTEK™ PRO Model Series 122600 specifications. Table 1 presents the specifications used for the mathematical model. This data was obtained from Briggs & Stratton Engine Specifications, (2003) and Crankshaft Drawings, (2003). The valve size is determined using the optimal valve size derivation given in (Pulkrabeck, 2003)

In Figure 12 and Figure 13 general states of the air in the CC during the compression stage of a standard engine are plotted with respect to the crank angle. Compression is taken to begin at 10° aBDC and end at TDC.

Table 1

Baseline Engine Specifications

Specification	English Units	Metric Units
Bore (B)	2.69 in	68.3 mm
Stroke (S)	2.04 in	51.8 mm
Displacement Volume (Vd)	11.58 in ³	189.9 cc
Compression Ratio r_c	10	10
Clearance Volume $V_c = V_d / (r_c - 1)$	1.28 in ³	21.1 cc
Crank Offset ($a = S/2$)	1.02 in	25.9 mm
Connecting Rod ($r = 4a$)	4.08 in	103.6 mm
Valve Diameter (dv) (2 valves)	0.272 in	6.9 mm
Effective Valve Area ($\pi * dv^2 / 4$)	0.058 in ²	37.4 mm ²
Engine RPM	3600rpm	3600rpm

Standard Operation. Figure 12 shows that the pressure in the CC rises from 14.7 psi (101.35 kPa) at 10° BDC to 366.59 psi (2527.56 kPa) at TDC.

Figure 13 shows that the temperature rises from 70 °F (21.11° C) to 868.02° F (464.46°C).

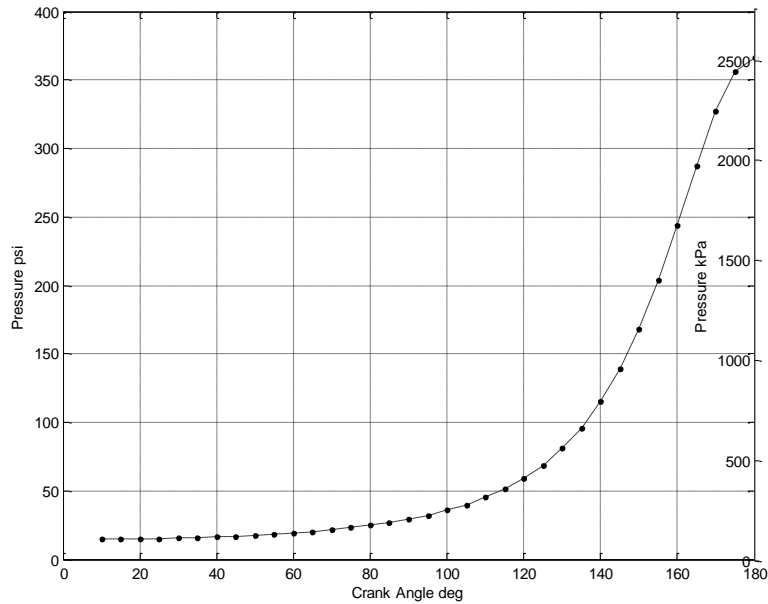


Figure 12. Pressure vs. crank angle standard engine

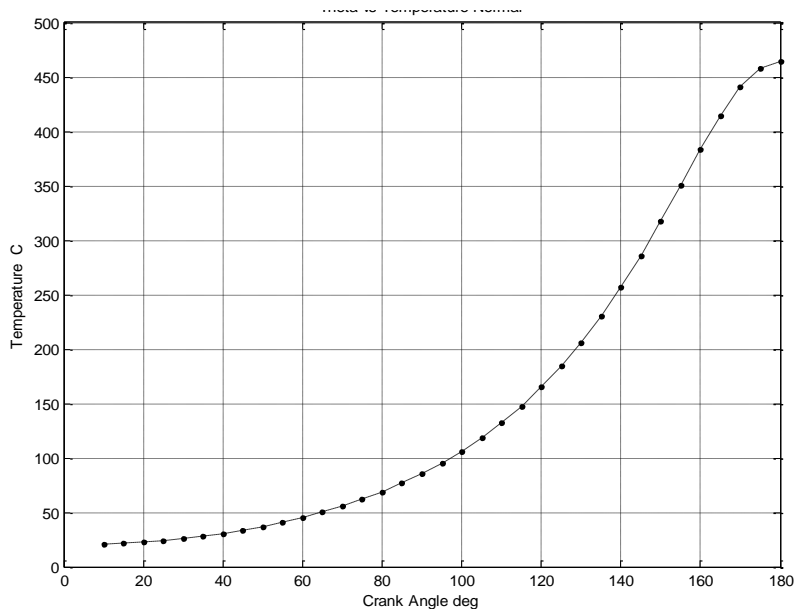


Figure 13. Temperature vs. crank angle standard engine

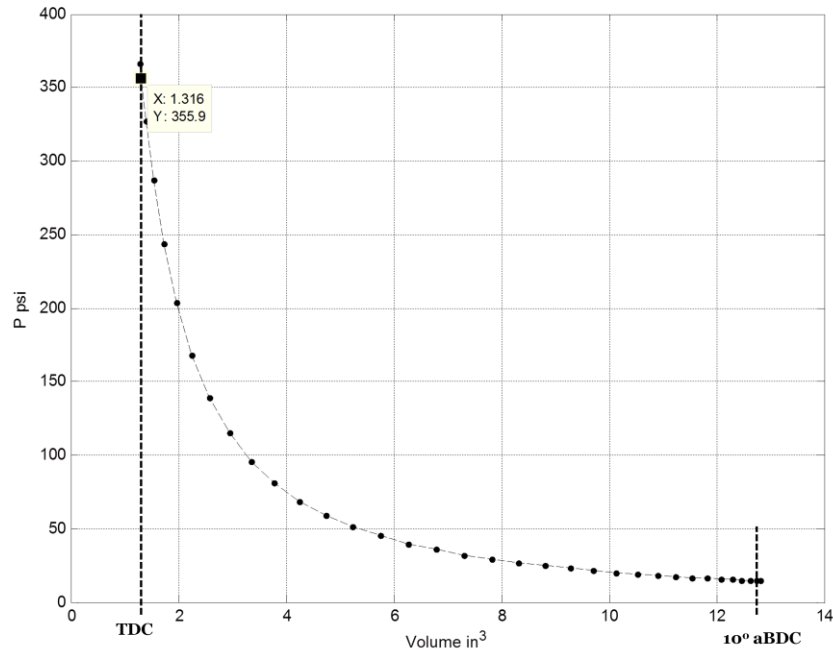


Figure 14. Pressure vs. volume standard engine

The mass of the air in the CC is constant at 0.253 g and the volume changes based on equation 3. Figure 14 is the pressure vs. volume diagram for the baseline engine. The pump work performed by the piston is calculated to be 80.18 J.

Performing a thermodynamic cycle analysis provides the states for each stage of the engine cycle. The air to fuel ratio is assumed to be close to stoichiometric at 15 for gasoline (47300 kJ/kg)(Fuel-Higher Calorific Values). Table 2 gives a list of the pressure and temperature values for each stage, the work of expansion (W_{EXP}), heat added to the system (Q_{in}) and thermal efficiency (ζ_T).

Table 2

Standard Engine States at Cycle Stages

	Pressure	Temperature
1	101.3 kPa (14.7 psi)	21.1° C (70° F)
2	2527.5 kPa (366.59 psi)	464.46 °C (868.03° F)
3	16638.4 kPa (2412.9 psi)	4581.80 °C (8279.24° F)
4	66.4 kPa (96 psi)	1665.32° C (3029.58° F)
W_{Comp}	80.18 J	
Q_{in}	797.8 J	
W_{EXP}	566.15	
ζ_T	60.9%	

Base Line Compressed Air Tank Conditions. For the baseline case the compressed air is stored at conditions described in Table 3. These are also the conditions provided by SST.

Table 3

Storage Tank Thermodynamic Specifications

Parameter	English Units	Metric Units
Storage Pressure (P _{CT})	280 psi	19.5 atm (193KPa)
Storage Temperature (T _{CT})	70 F	21.1 C

Baseline modified engine valve timing. It is assumed that for the baseline engine the spark is initiated at 170° aBDC so the fill valve is closed at this crank angle. For the CC to reach the pressure of the baseline compressed air before sparking a $\Delta\theta$ of 35.4° is required as shown in Table 4. At these conditions the work done by the piston during the compression stroke until the end of filling part is 15.6 J and 3.7 J for further compression part. This gives a 76.3% gain in pump work.

Table 4

Baseline Engine Filling Valve Timing

Valve Specs	Angles
θ_i	134.6°
θ_f	170°
Crank Angle	35.4°

State of air in baseline modified engine. For the baseline conditions the states of the air at the engine stages are given in Table 5.

The mass of the air at the end of compression is 0.322 g; this is 21.7% more air compared to the normal cycle. At the end of the compression stroke (state 2) the temperature is substantially lower than in the standard engine. The AF ratio is assumed to be stoichiometric, the same as in the standard engine. The W_{EXP} , Q_{in} and ζ_T are also calculated in table 5. There is a 24% increase in heat added to the cycle for a 76% decrease in pump work. This gives a 7.6% increase in thermal efficiency.

The top pressure of the modified engine cycle has exceeded the standard cycle top pressure by 21%. This is not an acceptable design.

Table 5

Baseline Engine States at Cycle Stages

	Pressure	Temperature
1	101.3 kPa (14.7 psi)	21.1°C (70°F)
2	2275.3 kPa (330.01 psi)	222.4 °C (432.37 °F)
3	21178.6 kPa (3071.7 psi)	4339.8 °C (7843.66°F)
4	846.9 kPa (122.83 psi)	1563.3°C (2845.94 °F)
W_{Comp}	19.3 J (76% saved)	
Q_{in}	1051.4 J	
W_{EXP}	684.7 J	
ζ_T	65.5%	

DYNAMICS OF FILLING BASELINE CASE

Figure 15 to Figure 20 show the variations of mass flow rate, pressure, volume and temperatures with respect to the crank angle for the baseline case.

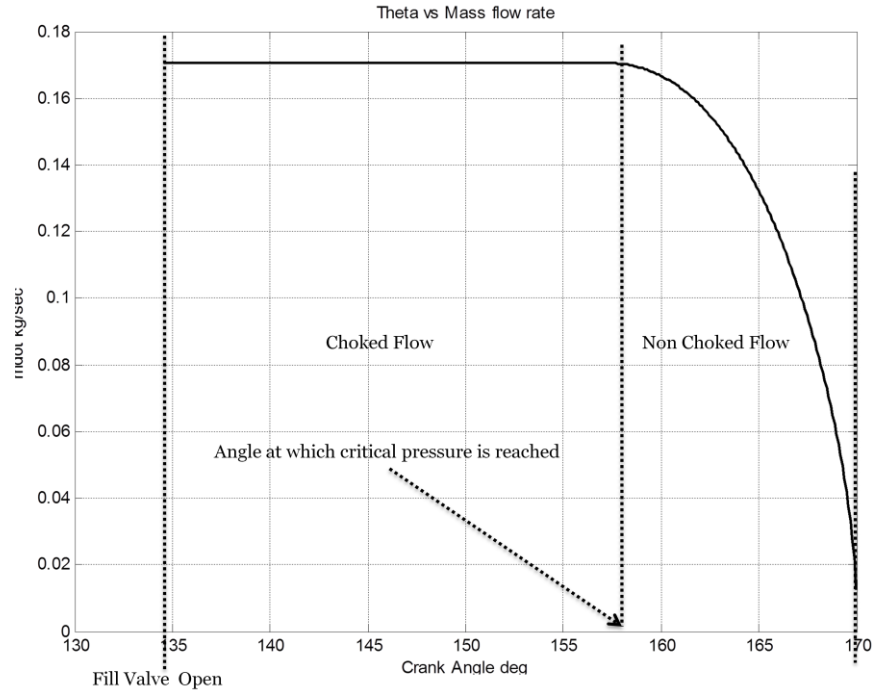


Figure 15. Mass flow rate of air into the combustion chamber after fill valve opens, (280psi, 170°).

Figure 15 shows the variation of mass flow rate of the air from the compressed air tank in to the CC from the fill valve open to fill valve close. Note that the mass flow rate is constant within the choked flow region i.e. when the pressure in the CC is below the critical pressure (147.9 psi) and decreases to zero rather quickly (12°) as the pressure in the CC equalizes with the storage tank pressure at fill valve close.

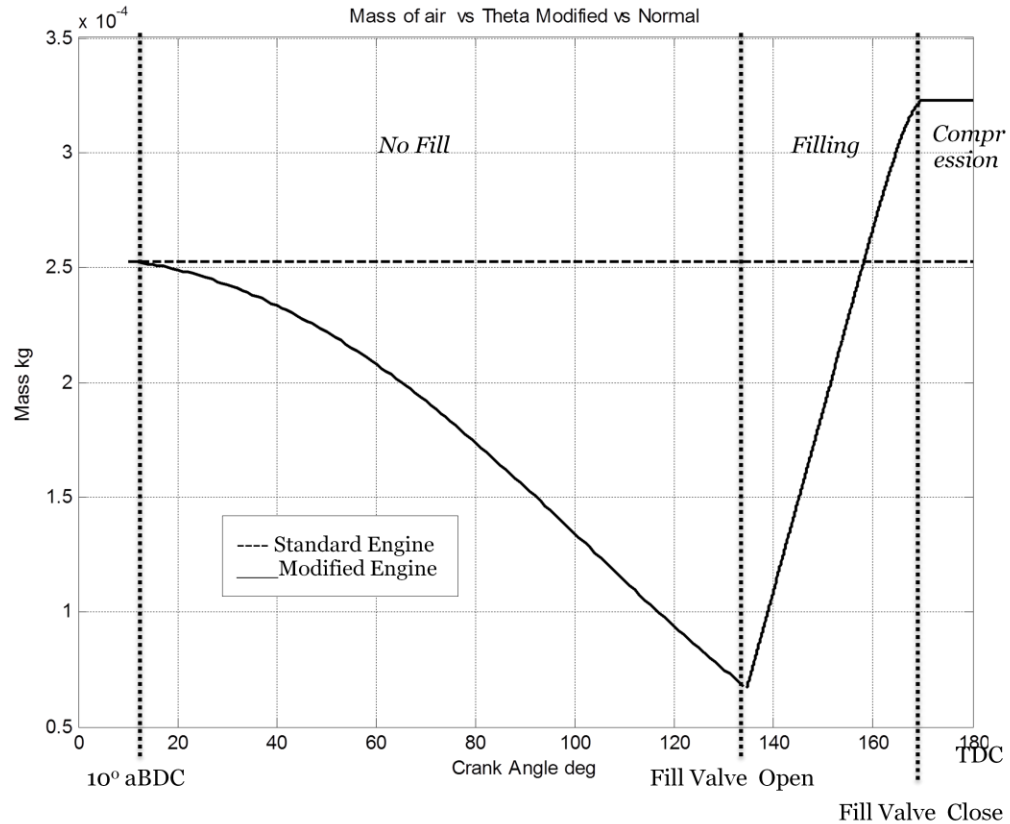


Figure 16. Mass of the air in the CC from start of compression to fill valve close compared to the standard engine (28opsi, 170°).

Figure 16 shows the mass of the air in the cylinder as a function of crank angle. For the standard engine the mass in the CC is constant. For the modified engine the mass in the CC is not constant since the air is being pushed out through the exhaust valve as the piston moves from BDC to fill valve open (~134°). The exhaust valve closes and the filling valve opens to let air fill the CC until the filling valve closes. The rate at which the mass increases is linear until the critical pressure is met. The mass of the air in the CC is higher at the end of filling than a standard engine, 0.253 g vs. 0.325g respectively. This is due to a much higher rate of filling for the modified engine than during the intake stroke of a standard engine.

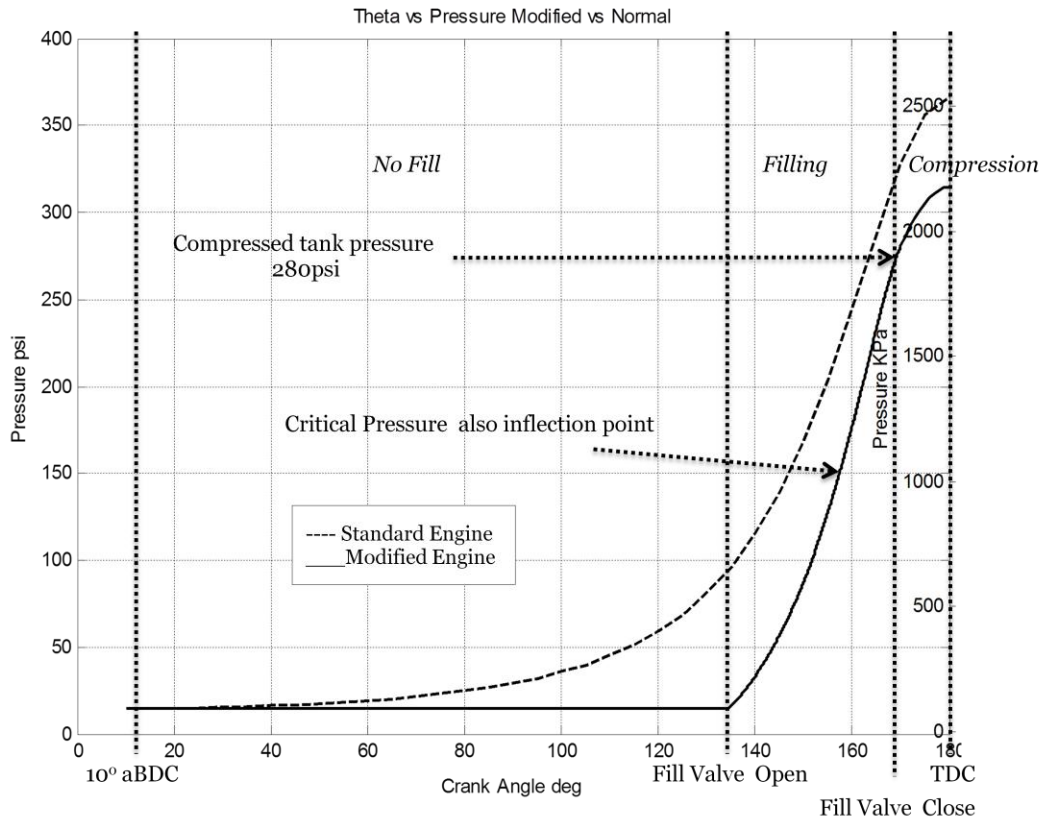


Figure 17. Pressure of air in the combustion chamber from start of compression stage to TDC, (280psi, 170°).

Figure 17 shows the pressure variation with crank angle for both the standard and modified engine. The pressure in the CC for the modified engine remains at 14.7 psi until the fill valve opens since the exhaust valve is open. As the exhaust valve is closed and the fill valve is opened, the filling and reducing boundary increases the pressure in the CC. After the fill valve is closed the increase in pressure follows the same trend as in the standard engine since now the pressure only depends on the volume change as given in equation 1. It is interesting to note that the final pressure in the modified engine at TDC is lower than that for the standard engine. This is because at the end of filling (170°) the pressure (280 psi) ends below that of a standard engine (~327 psi) and since after filling the curve is parallel a lower pressure is reached. To attain the same pressure at the end of compression for the modified engine as the standard

engine, the storage pressure should equal to the pressure in the CC for standard engine at the fill valve close angles. For example for the fill valve closed at 160° , the storage pressure should be ~ 250 psi to the attain same states as the standard engine. This is also shown in Figure 30.

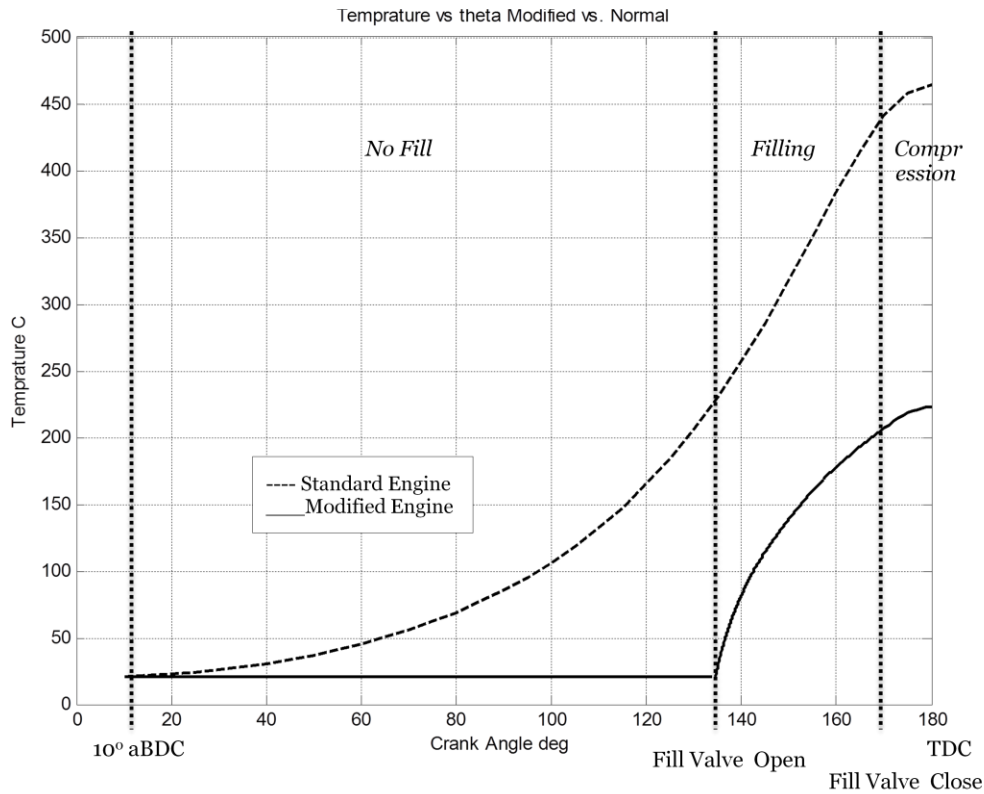


Figure 18. Temperature of air in the CC from the start of compression stage to fill valve close (280psi, 170°).

The temperature change with respect to the crank angle is shown in Figure 18. The temperature of the air in the CC is constant until the fill valve opens because there is not compressive heating and the exhaust valve is open. As soon as the fill valve is opened and the exhaust valve is closed, the temperature starts to increase due to primarily the enthalpy imparted by the air entering the CC (94% of total energy) and boundary work by the piston (refer to equation 6).

After the fill valve is closed, the temperature increase is due to compressive heating.

The temperature at the end of the compression stage is much lower than that of the standard engine, this could be due to the short time involved in the filling and compression process ($\sim 45^\circ$) compared to 170° in a standard engine compression stage.

The lower temperature after the compression stage would require an increase in the amount of heat input required to obtain the same peak temperature ($\sim 26\%$ more) as the standard engine. This increases the work of expansion by $\sim 21\%$ due to the increased mass of air and fuel. It would be wise to make sure the peak pressure is not surpassed so that the engine is not damaged.

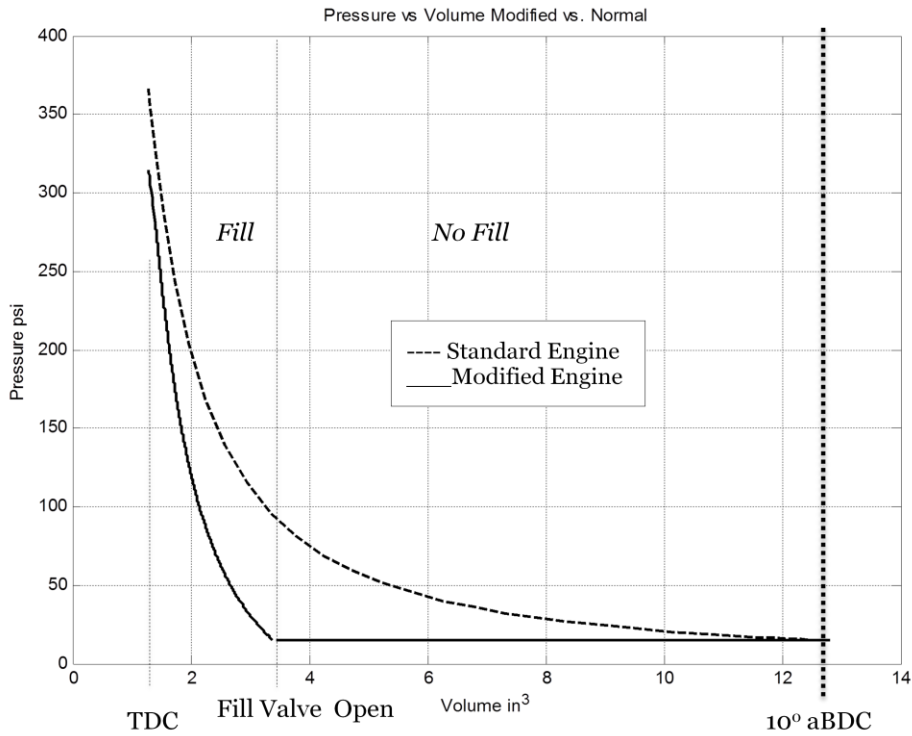


Figure 19. Pressure vs. volume of standard engine vs. the modified engine compression stage (280psi, 170°).

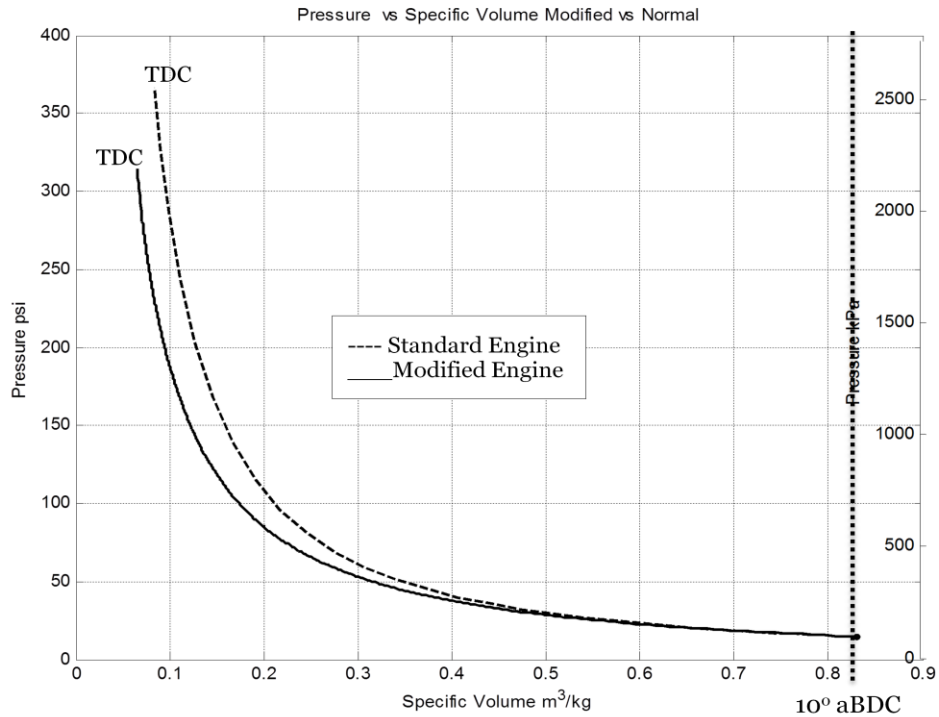


Figure 20. Pressure vs. specific volume of standard engine vs. the modified engine compression stage (280psi, 170°).

Figure 19 and Figure 20 are the pressure vs. volume and pressure vs. specific volume diagrams of the standard engine and modified engine. It is clear from Figure 19 that the pump work done by the modified engine during the compression stroke is much lower than that of the standard engine compression. This is due to 1) the pump work during the no fill stage does not increase the pressure but only decreases the volume and 2) the short volume change for the pressure to increase from initial conditions to desired pressure during the fill stage. The pump work saved is the difference between the two pump works (~76% saved).

In Figure 20 the volume occupied by one kg of air is given with respect to the pressure in the CC. As can be seen, at the end of the compression stage, there is more mass of air in the CC for the modified engine.

STEP SIZE ANALYSIS

As mentioned in the modeling section, the filling stage is analyzed for each step of $\Delta\theta$ because the pressure and mass flow rate is both unknown at any instant of time. The state one is known (M_1 , T_1 , P_1 , and V_1). The next state is after a single step of $\Delta\theta$ and is at V_2 . Two main processes take place in-between the two states, filling and compression. In the analysis these processes are assumed to happen at state one. P_1 is assumed to be constant for the step of $\Delta\theta$ to find the boundary work, and mass flow in to the CC. The new mass in the CC and work yields the conditions at state point two (M_2 , T_2 , P_2). This process is continued until the end of the filling stage.

It is critical to gauge how sensitive the parameters calculated are with values of $\Delta\theta$. This will provide a convergence measure and justify the use of smaller $\Delta\theta$ for relative accuracy.

Figure 21 thru Figure 24 and Table 6 shows the sensitivity of the mass flow rate, pressure, temperature and work for the baseline engine with decreasing step length ($\Delta\theta$). The storage pressure is increased to 300psi.

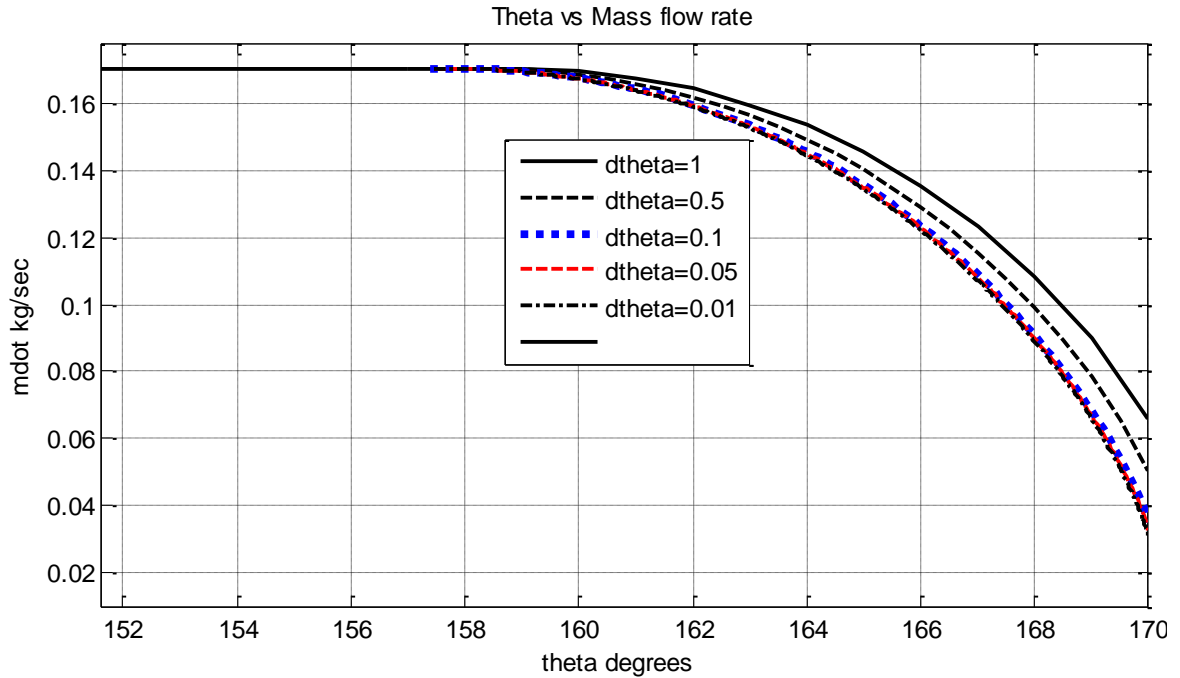


Figure 21. Sensitivity of mass flow with $\Delta\theta$

In Figure 21, the mass flow rate of sensitivity is shown. The mass flow rate is not sensitive in the choked flow region since the mass flow rate is only a function of the upstream pressure which is kept constant.

The point at which P critical begins varies from $\Delta\theta=1$ to 0.1 by 2% but, then remains constant at lower $\Delta\theta$ values.

The mass flow rates vary at a maximum (46% difference) between $\Delta\theta = 1$ to 0.1. From 0.1 to 0.01 the mass flow rate converges with less than 1% difference. Since non-choked mass flow rate is a function of cylinder pressure it could be assumed that the pressure values behave similarly.

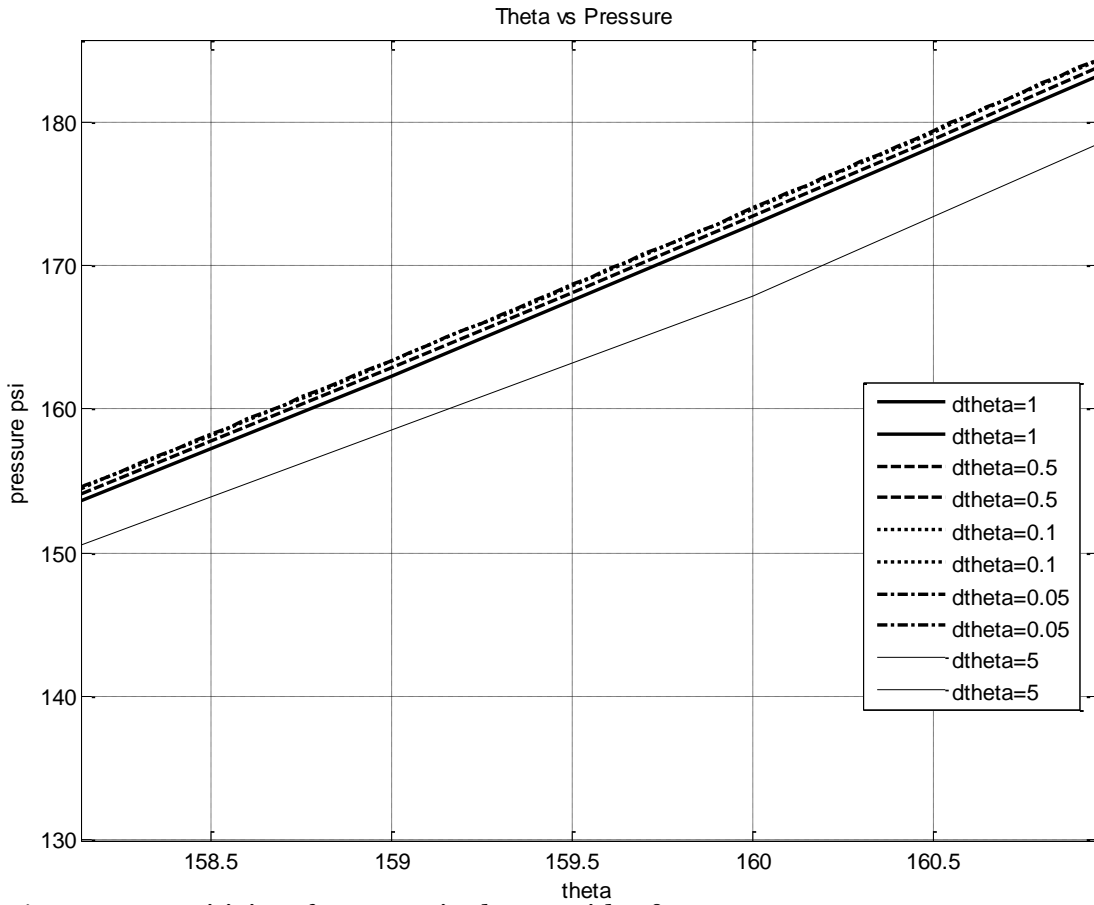


Figure 22. Sensitivity of pressure in the CC with $\Delta\theta$

Figure 22 shows the variation in pressure in the CC with smaller $\Delta\theta$. As it can be seen the sensitivity of the pressure is surprisingly small (about 3%) between $\Delta\theta=5^\circ$ to 0.05° . The pressure values remain constant at $\Delta\theta$ below 0.1° .

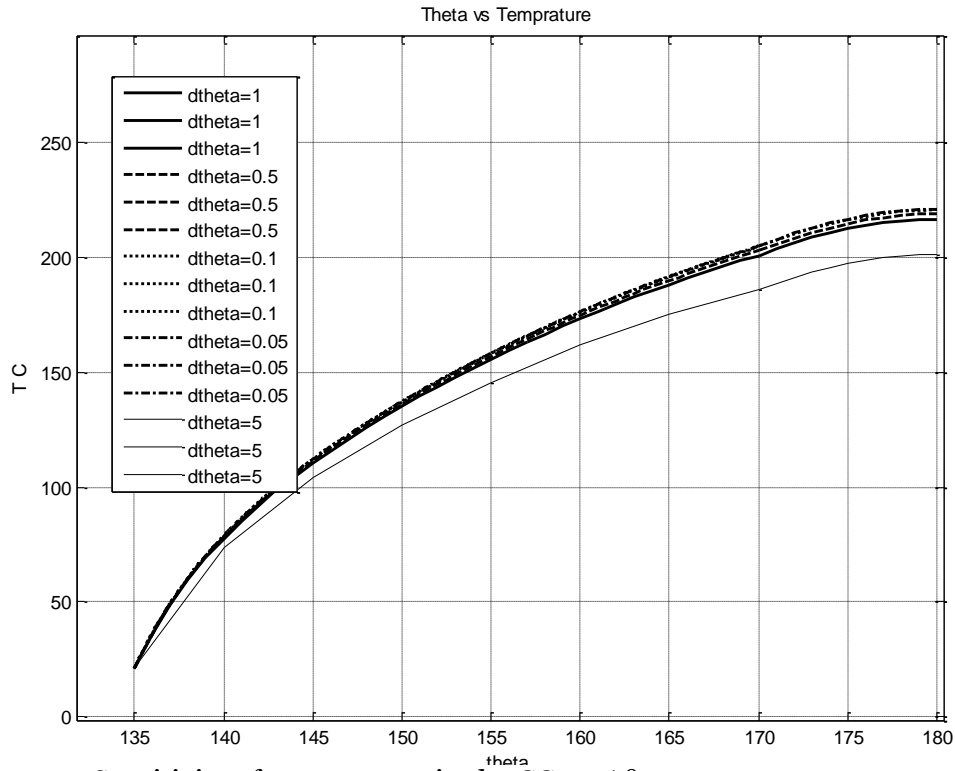


Figure 23. Sensitivity of temperature in the CC vs. $\Delta\theta$

Figure 23 shows the sensitivity of temperature in the CC with decreasing $\Delta\theta$. The temperature values seem more sensitive to changes in $\Delta\theta$. The temperature varies at a maximum of ~11% going from $\Delta\theta = 5^\circ$ to 0.05° , whereas between 1 and 0.05 the difference is only 2%.

Figure 24 shows the variation of the pressure volume diagram with changing $\Delta\theta$ values. Since the pressure was not very sensitive to $\Delta\theta$ the same is expected here since volume (V) is a function of θ .

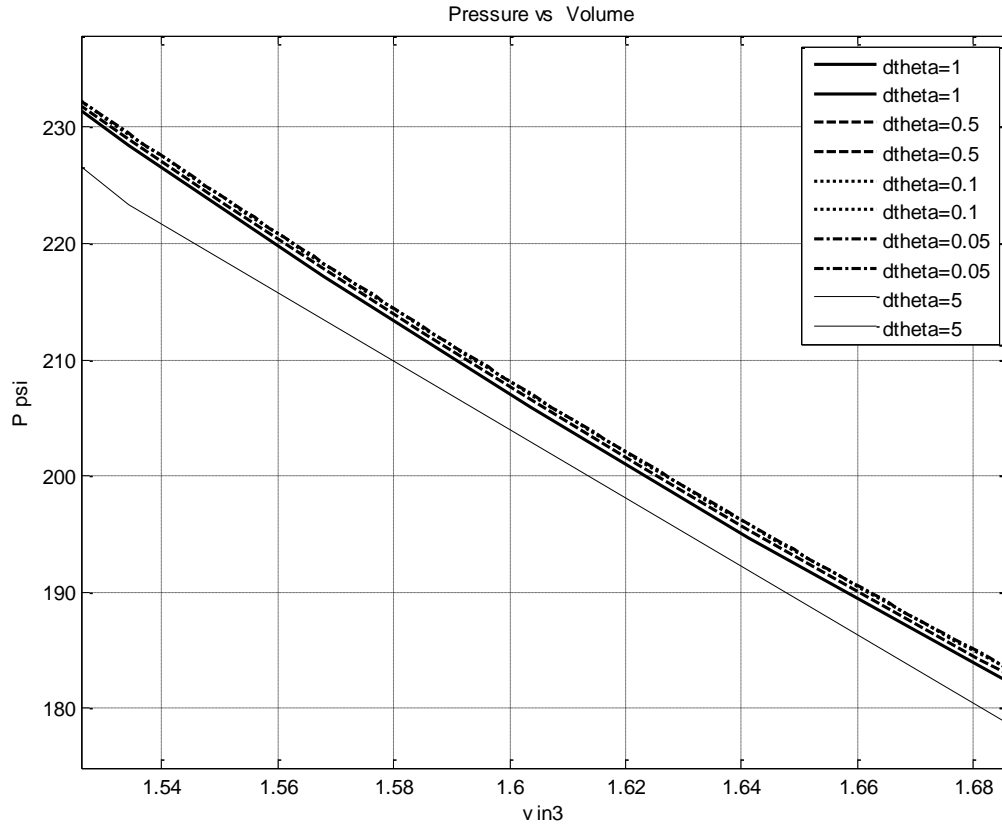


Figure 24. Sensitivity of pressure with volume vs. $\Delta\theta$

Table 6

Sensitivity of Work at Filling Stage with change in $\Delta\theta$

$\Delta\theta$	Work Filling Stage (J)
5	15.715
1	15.719
0.5	15.719
0.1	15.719
0.05	15.719

Table 6 shows the sensitivity of the work calculated with change in $\Delta\theta$. Since the pressure volume values did not vary substantially the same is expected for the calculated work. A maximum of 3% variation is observed from $\Delta\theta$ 5° to 1° , but at lower values the work calculated is constant.

The mass flow is the most sensitive, but from the above analysis it is clear that a $\Delta\theta$ equal to or below 0.1° is a very reasonable estimate. $\Delta\theta = 0.01^\circ$ is chosen to get a good resolution for the ending pressure at fill valve close which is the tank pressure.

FILLING DYNAMICS

Figure 25 to Figure 29 show the trends of mass flow rate, pressure, temperature, and pressure volume at higher and lower tank storage pressures from the baseline case. The fill valve close angle is held constant at 170° aBDC.

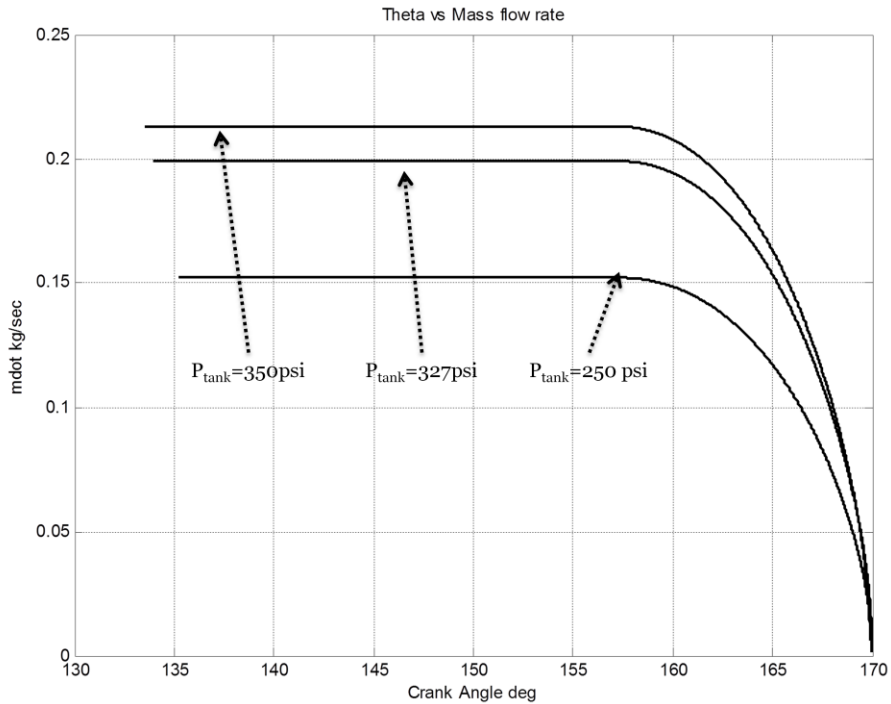


Figure 25. Mass flow rate trend for fill valve close at 170° aBDC

In Figure 25 the mass flow rate in to the CC with higher and lower tank pressures are given. The tank storage pressure is also the target pressure at fill

valve close. At higher tank pressure the mass flow rate is generally higher. This is expected since the mass flow rate in the choked flow region is only a function of upstream pressure. In the non-choked flow area the mass flow rate is a function of both tank pressure and pressure in the CC. As shown in the figure, note that the rate of change of mass flow rate for the higher pressure is faster than for the lower pressure. At the same engine speed, the faster initial rate of filling increases the pressure in the CC and thus decreases the rate of filling later.

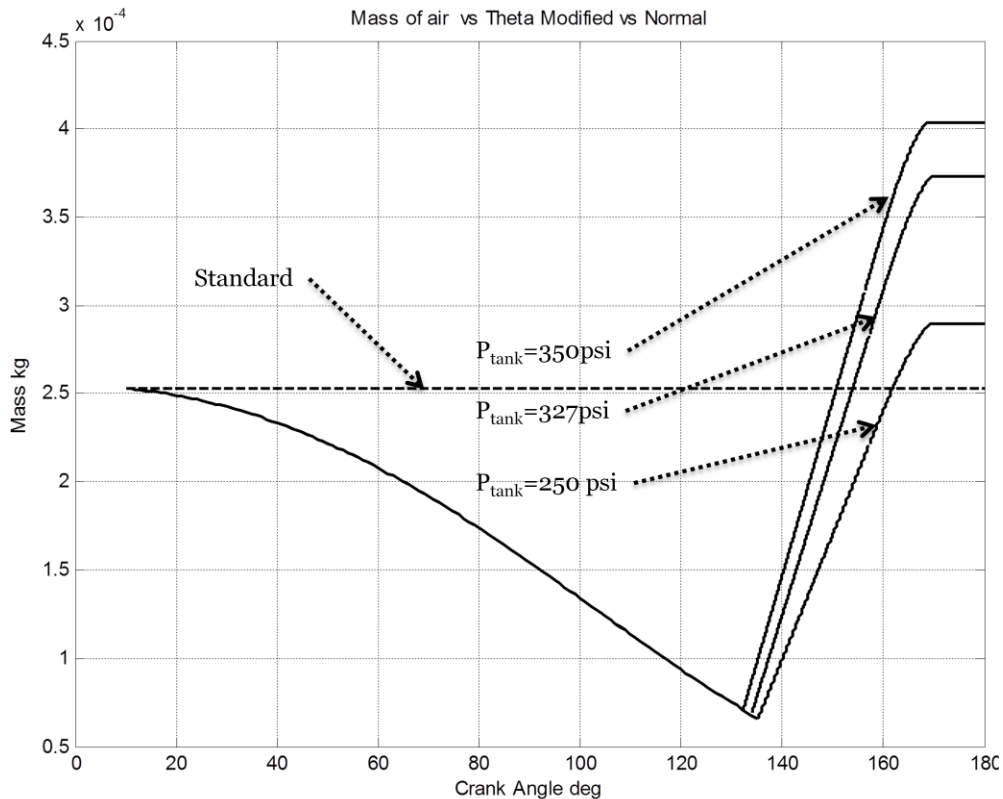


Figure 26. Mass of air in the CC vs. crank angle trend for fill valve close at 170° aBDC.

Figure 26 shows the mass of the air inside the CC with respect to the crank angle for higher and lower pressures from the baseline case. The higher the pressure the more mass ends up in the CC. For the higher pressure it can be seen

that that the filling starts a little earlier, this reduces the amount of air lost. The higher mass flow rates for the higher pressures also explain this trend well.

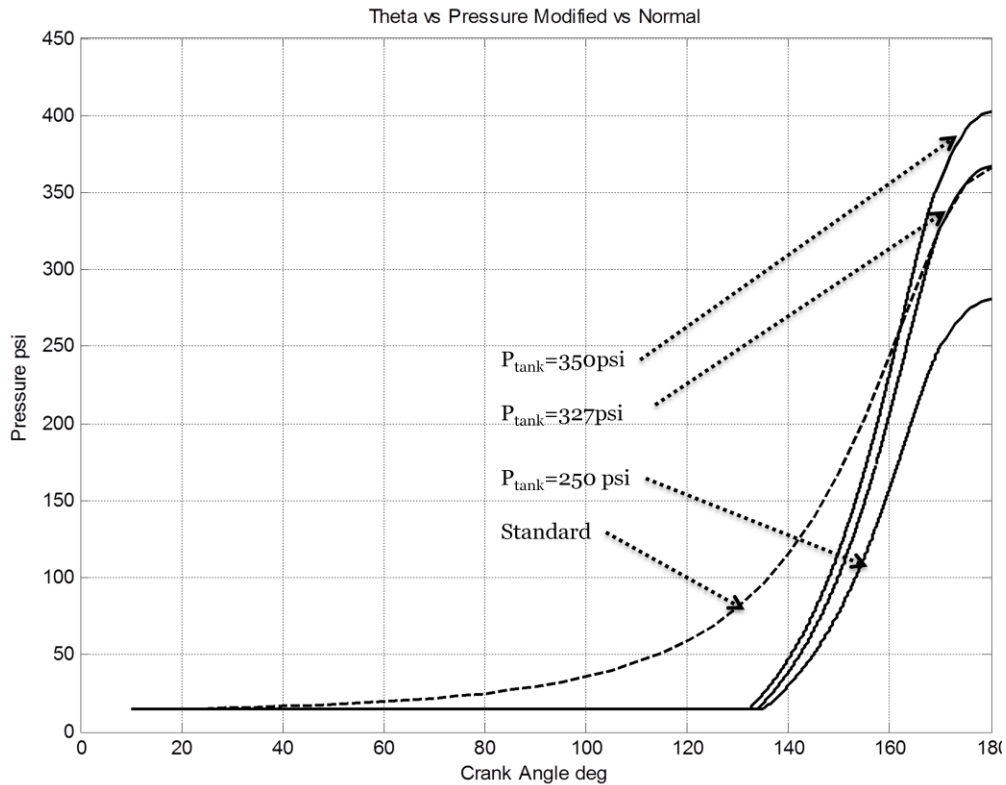


Figure 27. Pressure vs. crank angle trend for fill valve close at 170° aBDC

Figure 27 shows the pressure in the CC vs. the crank angle for higher and lower pressures from the baseline case. At 170°, the pressure in the standard engine is ~327psi. It can be seen that if this is the target pressure for the modified engine, the ending pressure at TDC will be the same as that of the standard engine. This was predicted earlier in the paper.

If the tank pressure is above the standard pressure at fill valve close the ending pressure is higher and vice versa for the lower target pressure.

The rate at which the pressure rises is higher for the higher pressure. This is due to the increased mass of air in the CC.

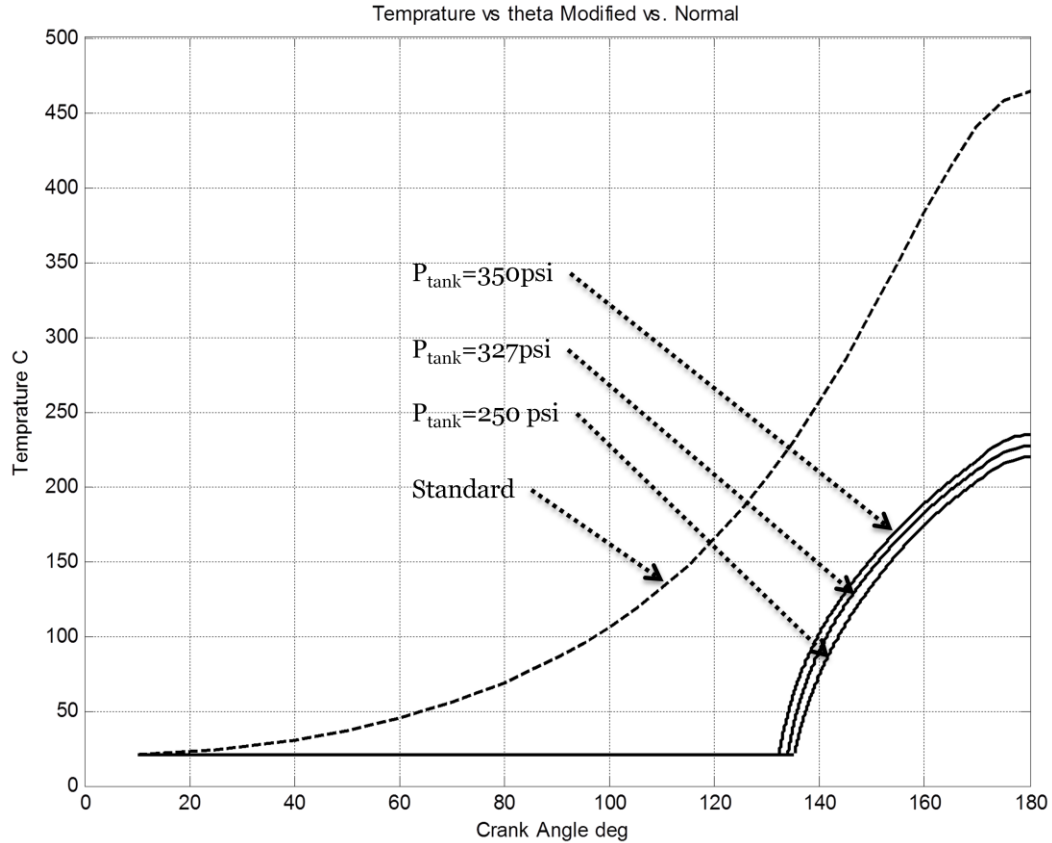


Figure 28. Temperature vs. crank angle trend for fill valve close at 170° aBDC

Figure 28 shows the variation of temperature in the CC for higher and lower pressures from the baseline case. It can be seen that the higher pressure has a very small effect on the temperature of the air in the CC. This trend can be attributed to most of the pump work being done by filling rather than the change in boundary.

This trend also means that approximately the same amount of heat (depending on the mass of air) will be required to raise the temperature of the air during combustion to get the same peak temperature as in the standard engine.

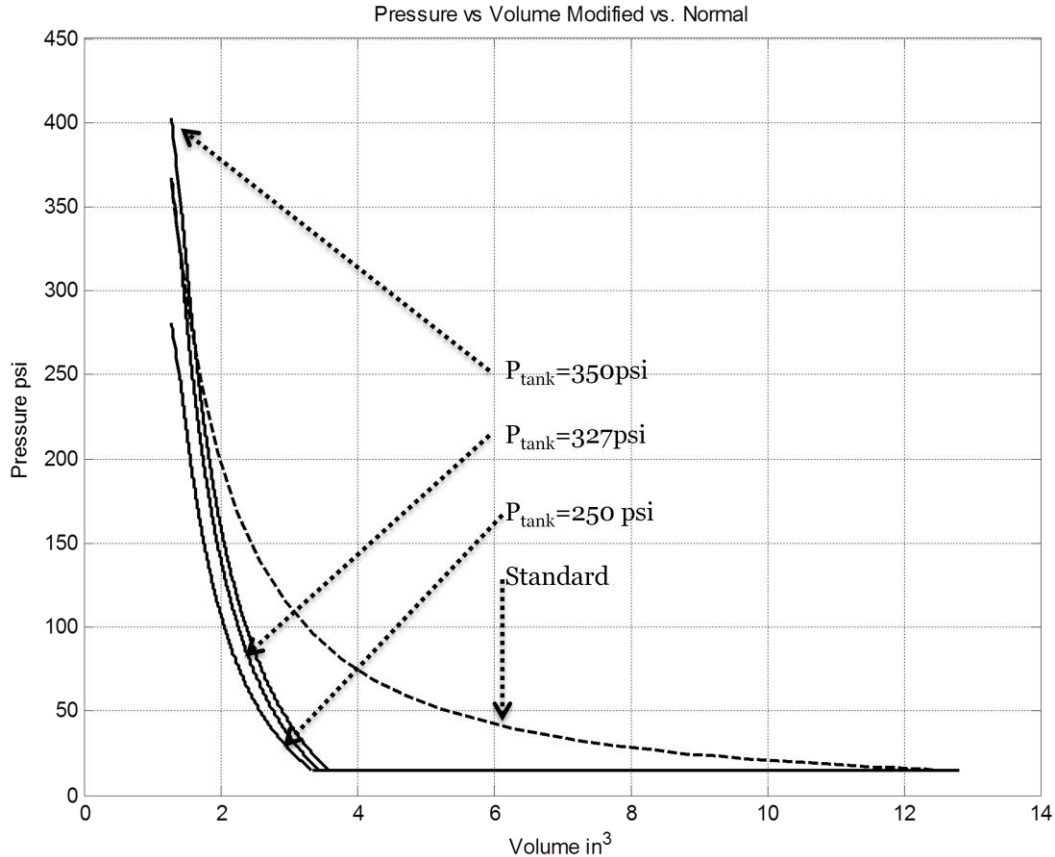


Figure 29. Pressure vs. volume trend for fill valve close at 170° aBDC

Figure 29 shows the pressure volume diagram for the higher and lower pressures from the baseline case. It is interesting to note that the area under the 250 psi curve is lower than that for the 350 psi curve. This means less piston work is done within the stroke for the lower pressure case. This is quite contrary to what was expected because a higher tank pressure was thought to do the least amount work.

If the ending pressure (TDC) is required to be constant then this trend probably will not hold because the lower pressure will need more change in volume to achieve it.

With the lower pressure more heat input might be required to get the same pressure after combustion as in the standard engine. But with the higher

pressures care needs to be taken to make sure the pressure after combustion does not exceed the design pressure for the standard engine.

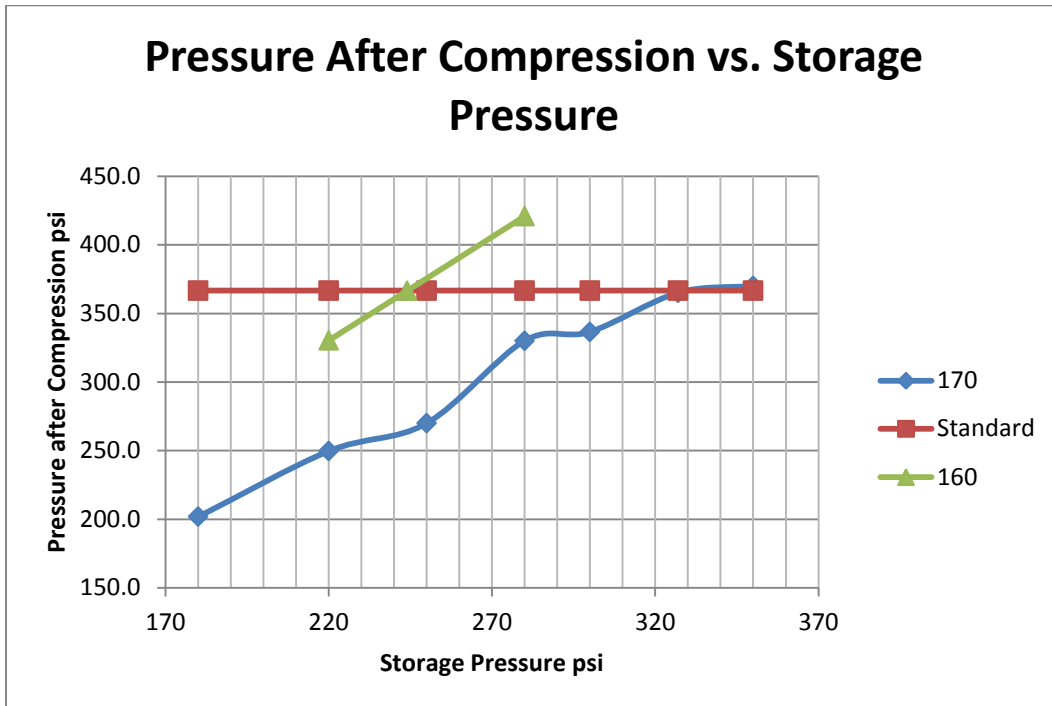


Figure 30. The final pressure after compression stage in the CC vs. storage pressure for different fill valve closing angles.

Figure 30 shows the final pressures in the CC after the compression stage for the modified engine for different storage tank pressures. This is done for the 170° fill valve close angle and also for 160° as a comparison. As shown in Figure 27, if the pressure at the valve close (the storage pressure) is equal to the standard pressures at that angle then the ending pressure follows the standard engine. This is ~ 245 psi for the 160° close angle. Therefore lowering the storage pressure lowers the ending pressure in the CC.

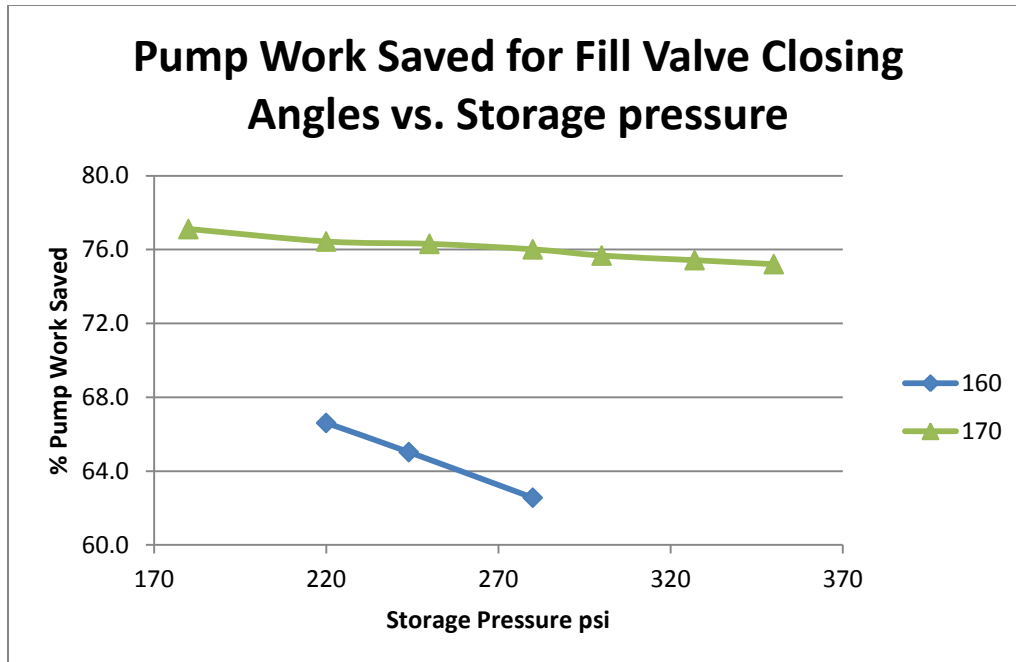


Figure 31. Percentage pump work saved from filling modification for different fill valve closing angles.

Figure 31 shows the percentage of pump work saved from the standard engine by filling at 170° aBDC and 160° aBDC fill valve closing angles.

It is seen that more pump work is saved when the filling target pressure is lower for a particular fill valve closing angle. This is also seen from the area under the pressure volume diagram in Figure 29 that, if the storage pressure is lower than the standard pressure at the fill valve close angle, the total work done by the piston is lower since the piston is working against a lower average pressure in the CC pressure.

But for a reasonable comparison between standard and modified engines, the final pressure on the modified engine needs to be equal to the compression pressure for the standard engine pressure. The storage pressures to attain this are in Figure 30 (324psi for 170° aBDC, and 244psi for 160° aBDC) and the pump work saved is 75% and 65% respectively.

Less pump work is saved when filling valve closing is earlier in the compression stroke because of the extended further compression. Also referring to Table 7, column 8 and 10 it can be seen that the pump work during filling decreases as the storage pressure increases, but the pump work during the further compression stage increases.

From Figure 32 and Figure 33 it can be seen that the time required for the filling to reach the storage pressure is longer at higher pressures.

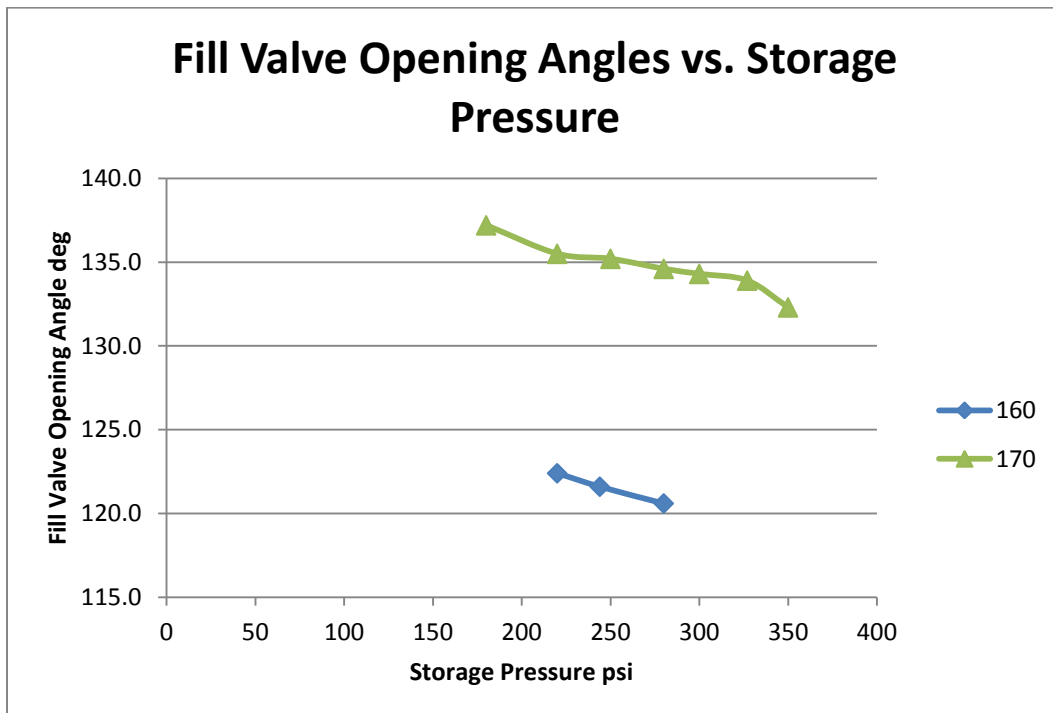


Figure 32. Fill valve opening angles to achieve desired pressure for different fill valve closing angles

Figure 32 and Figure 33 shows that it takes more time to reach the target pressure if the storage pressure is higher for any particular closing angle.

But it is interesting to note that if the closing angle is earlier in the compression stroke (160° vs. 170°) the filling takes place in a longer period of time. In general it takes longer to reach the desired pressure or storage pressure if the filling is started earlier in the compression stroke. If the target pressure is

the same the mass flow rate in to the cylinder is the same and so is the critical pressure. Earlier in the compression stroke the piston is faster and slows down as it reaches TDC. This causes the pressure rise to be to be higher earlier in the stroke thus reaching the critical pressure faster after which the mass flow rate decreases exponentially. This explains why it takes longer to reach a target pressure if filling starts earlier.

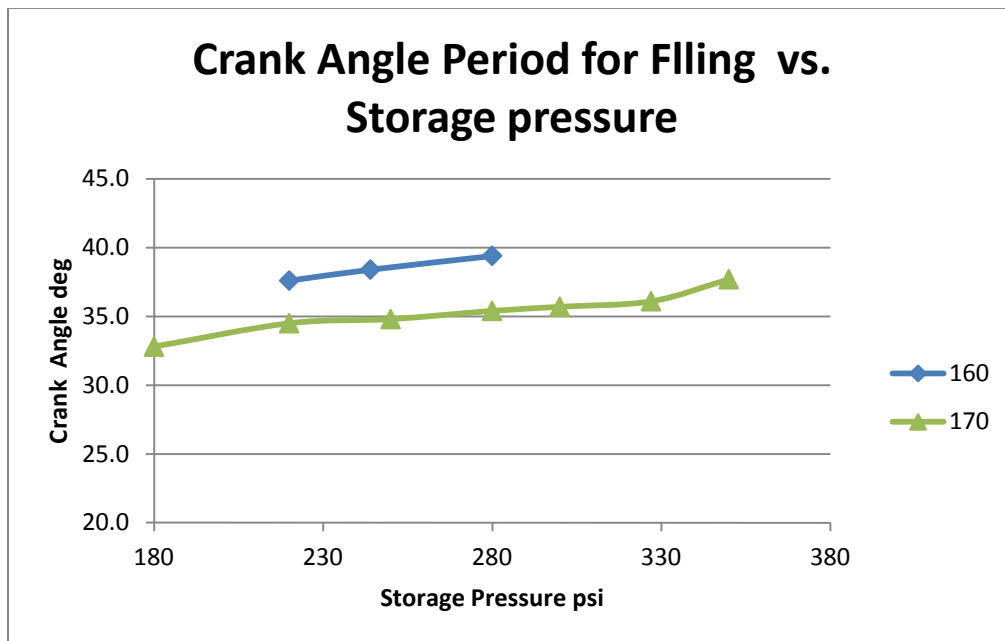


Figure 33. Crank period required to achieve desired filling pressure for different fill valve close angles.

In the following section, the engine cycle for the modified engine is completed with a constant AF ratio of 15 (stoichiometric for gasoline engine) using the air standard Otto cycle analysis as explained in the engine performance section in modeling. The fill valve closes at 170° aBDC.

MODIFIED ENGINE PERFORMANCE

Figure 34 and Figure 35 show the pressure and temperature of the air in the CC of the modified engine after the constant volume heat addition (combustion) stage respectively.

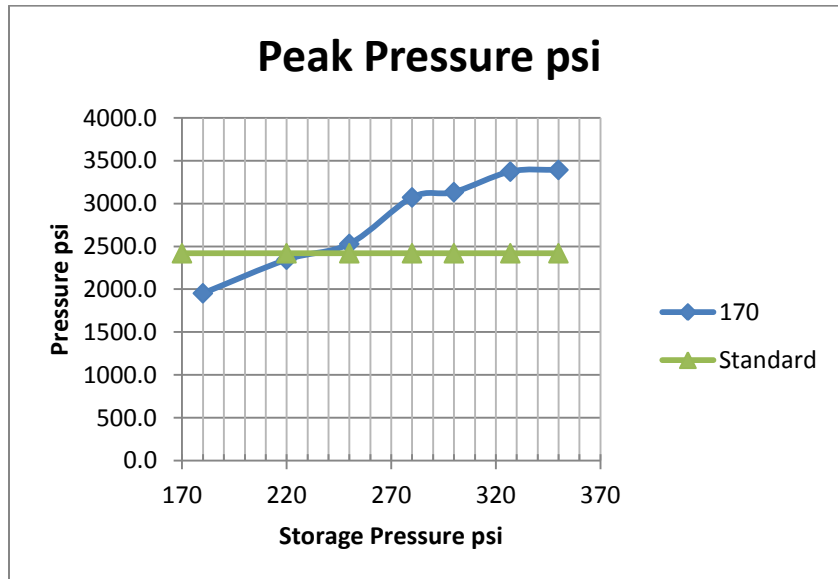


Figure 34. Pressure after combustion in the CC vs. storage pressures, constant AF.

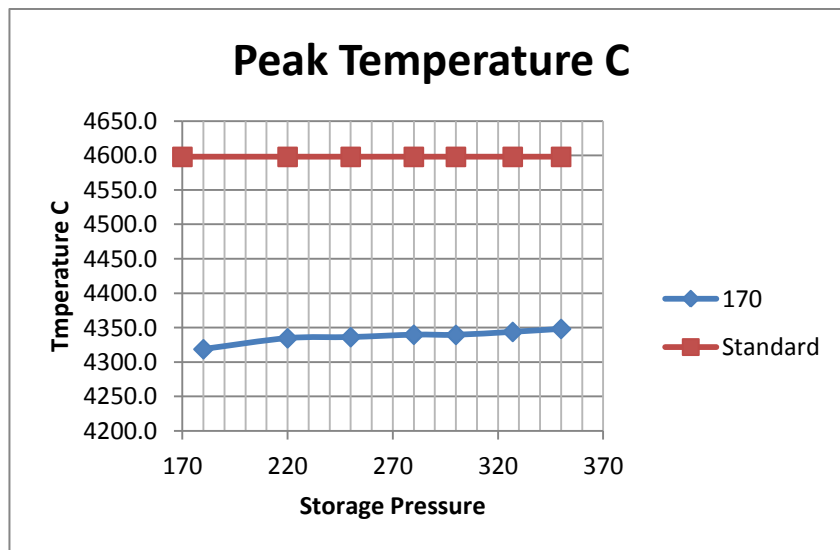


Figure 35. Temperature after combustion in the CC vs. storage pressures, constant AF.

As it can be seen in Figure 34, at the baseline storage pressure condition (280 psi), the peak pressure in the modified engine is 20% higher than that of the standard engine. This storage pressure modification is not possible for the chosen engine. In Table 7 column 1, the mass of air in the CC is ~ 21% higher than in the standard engine. Even though the same specific heat will be added to the engine, the increase in mass will drive the pressure beyond desirable for the engine. This shows that being able to control the mass flow rate will be beneficial in this sort of an application.

The peak temperature on the other hand, stays under the peak temperature of the standard engine because of the low starting temperatures. The T2 's (Table 7) of the modified cases are ~50% lower than the standard while the P2 for the baseline is only 10% lower than the standard.

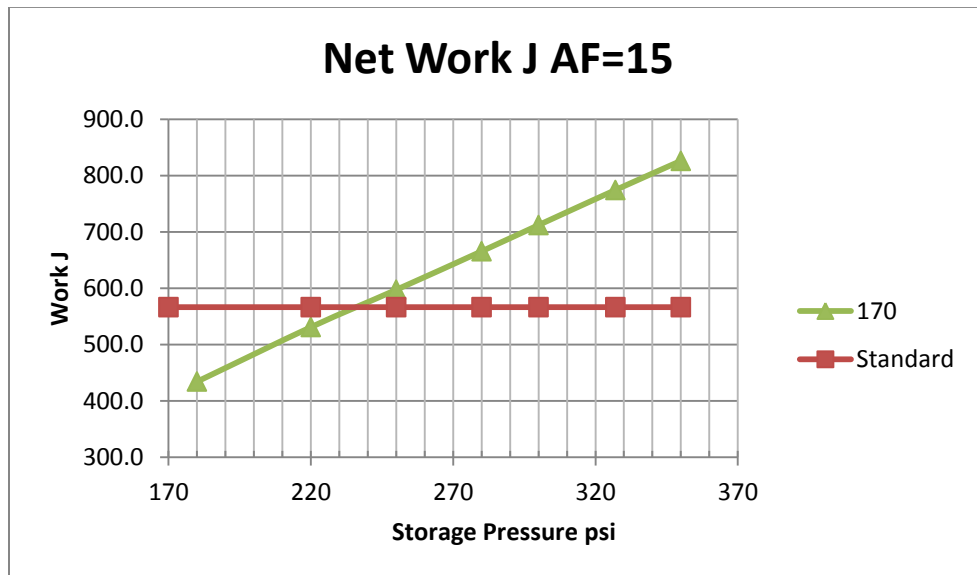


Figure 36. Net work done by the modified engine at the various storage pressures with the net work of the standard engine.

Storage pressures below ~235 psi are the only possibilities for the selected engine under the specified modification. (Continued on page 60)

Table 7

Engine Cycle States and Performance for Selected Engine in Standard and Modified Configuration with Various Storage Pressures.

Mass Air E-4 kg	Mf kg E-5 kg	Pcr psi	θ_i	d θ	Wfill J	WS J	Wcomp J	WS _j	% Pump	T ₂ C	P ₂ psi	T ₃ C	P ₃ psi	T ₄ C	Q _{in} J	W _{exp} J	W _{net} J	ζ_r	FuelUse E-8 kg/J
2.53	1.69				62.1		18.1			464.5	366.6	4581.8	2412.9	1659.6	797.8	566.2	486.0	60.9	3.47
2.14	1.43	180	137.2	32.8	16.0	55.7	2.4	24.5	77.1	201.5	201.9	4318.8	1953.6	1554.9	674.2	452.6	434.2	64.4	3.28
2.59	1.73	220	135.5	34.5	15.7	62.1	3.2	18.1	76.4	217.3	249.7	4334.7	2345.8	1561.3	816.2	549.8	530.9	65.0	3.25
2.90	1.93	250	135.2	34.8	15.7	66.4	3.3	13.8	76.3	218.9	270.0	4336.3	2529.2	1561.9	914.5	616.2	597.2	65.3	3.24
3.22	2.15	280	134.6	35.4	15.6	70.3	3.7	9.9	76.0	222.4	330.0	4339.8	3071.7	1563.3	1015.4	684.7	665.5	65.5	3.23
3.44	2.29	300	134.3	35.7	15.6	72.8	3.9	7.4	75.7	222.2	336.5	4339.6	3133.3	1563.2	1084.7	731.5	712.0	65.6	3.22
3.73	2.49	327	133.9	36.1	15.4	75.9	4.3	4.3	75.4	226.6	365.0	4343.9	3372.4	1564.9	1176.2	793.9	774.2	65.8	3.21
3.97	2.65	350	132.3	37.7	15.3	78.4	4.6	1.8	75.2	230.9	370.0	4348.2	3392.4	1566.7	1251.9	845.7	825.9	66.0	3.20

The standard engine states and performance are given in bold. The modified engine is analyzed for 180, 220 250 280 300 324, 350 storage pressures for a fill valve close at 170° aBDC.

Figure 36 shows the net work calculated for the modified engine for various storage pressures. As it is seen there is more work out of the engine at the baseline condition but the specified engine is restricted to a 235 psi storage pressure. Below this pressure the net work of the engine drops below standard engine work out. At the maximum possible storage pressure (235 psi) the net work out of the engine is the same as the standard engine, hence no work output is gained.

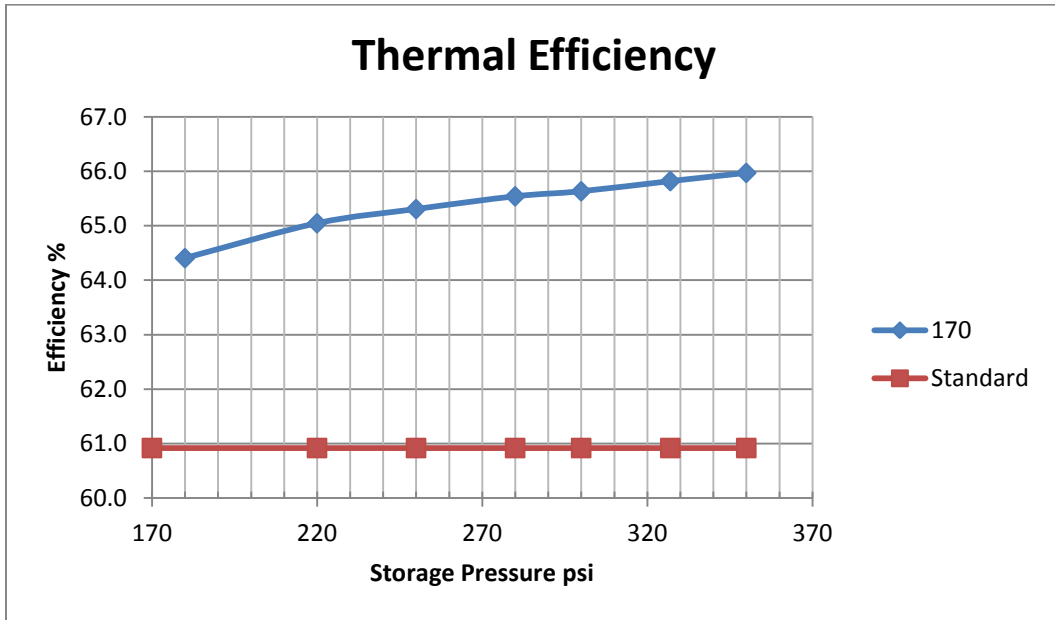


Figure 37. Thermal efficiency of the modified engine and standard engine vs. storage pressures.

Figure 37 shows the thermal efficiency of the modified engine operation for the various pressures with the standard engine. The efficiency of the modified engine operation increases with increase in storage pressure. As seen, the efficiency gain is not substantial for any of the storage pressures. For the chosen engine and storage pressure limit, the thermal efficiency for the same net work out is ~65.1%. That is only a 5 percentage point increase in efficiency, which is about 7.7% less fuel usage for the same net work out.

CONCLUSIONS

For the chosen Briggs and Stratton 6.5 hp engine, at baseline conditions with compressed air stored at 280 psi and 170° aBDC fill valve close angle, the engine fails the design constraint of staying below the standard engine maximum pressure. From the analysis a new storage pressure and design parameters are recommended as given in Table 8.

Table 8

Possible Design for Chosen Engine in Modified Operation.

Parameter	Design Point
Fill Valve Opening	~135.2 aBDC
Fill Valve Closing	170.0 aBDC
Storage Pressure Temperature	235 psi , 70°F
Gain in Thermal Efficiency	~5 % points (7.6% fuel saving)

Compressed air stored at a desired pressure in a storage tank is introduced into the CC of a reciprocating, 4 stroke, IC, SI engine via a connection to the intake port during its compression stroke at ~45° bTDC. This is done to reduce the pump work done by the piston. The valve train of the engine is modified to operate in this mode.

During the intake stroke the intake valve is closed and exhaust valve remains open and continues to be open during the compression stroke until the fill valve (intake valve) is opened to start the filling process. The fill valve is closed at 10° bTDC for spark advance. At this point the pressure in the CC should equal the storage pressure. All valves are closed at combustion and exhaust takes place through the exhaust valve during the exhaust stroke.

Only the compression process of the modified engine cycle is modified. The other processes of the engine cycle continue as a standard engine cycle. The compression process has 3 parts namely: no fill, filling, and further compression.

The filling process of the modified engine is thermodynamically modeled using a uniform unsteady flow, energy and mass balance model. Numerically the model is solved for the Briggs and Stratton 6.5 hp engine using a crank step approach in MatLab. The standard engine compression stage is modeled using isentropic relationships.

At the baseline storage conditions, the pressure of the air in the CC remains constant until the fill valve opens ($\sim 134.6^\circ$ aBDC). The filling begins and pressure in the CC rises to 280 psi. At the end of the compression stroke the final pressure in the CC at TDC is ~ 310 psi, which is $\sim 14\%$ lower than that in a standard engine (Figure 17). The temperature in the CC also rises only after filling begins (Figure 18). The temperature rises to $\sim 225^\circ$ C at the end of the compression stroke. The temperature rise is only half as much as the standard case. The mass of the air in the CC is $\sim 22\%$ higher than that of a standard engine.

For other storage pressures, the general trend is as follows: As the storage pressure increases, the mass flow rate of air in to the CC increases (Figure 25, Figure 26). This also results in a higher mass of air in the CC at TDC. At higher storage pressures the filling valve needs to be opened slightly earlier. The pressure rise is faster at higher pressures and the final pressure at TDC is also higher as the storage pressure increases (Figure 27, Figure 30). The temperature in the CC is slightly higher as the storage pressure increases (Figure 28).

The lower temperatures results in substantial performance loss for the modified operation. The higher mass in the CC allows the engine to attain the same net work at the lower temperatures.

As mentioned in the beginning of this chapter the modified engine operation fails to meet the imposed constraints.

RECOMMENDATIONS

The modification of this engine is not recommended for such low efficiency gains in thermal efficiency.

FURTHER RESEARCH

The research done can be improved by doing the following:

Broadening the research question

- Generalize results for other IC engines
 - Investigate other compressions ratios.
 - Investigate dependency of results with engine size.

Improve mathematical model

- Implement C_p and C_v dependency on air temperature in the numerical model.
- Implement effects of increasing storage air temperature through recuperation.

Engine Modifications

- Possibilities of controlling mass flow rate in to the CC
- Electronic valve

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APPENDIX A
MATLAB CODE

```

clear
clc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
%An existing IC engine is being modified to accept compressed air
at the compression
%stage. This is done to eliminate the parasitic pump work during
engine operation
% The air is compressed separately and efficiently during off
peak hours
%and provided to the engine during peak hours to produce
electricity

% The engine is connected to a compressed air tank via a
connection to the
% intake valve port, the intake valve serves as the filling
valve. To eliminate
% as much pump work as possible 1) the filling valve needs to be
opened for
% just the time required to fill the cylinder with the required
pressure before spark.
% The pump work after spark(~10deg bTDC) is unavoidable because
of
% combustion design.
% 2) The exhaust valve remains open as the piston moves up
during the
% compression stage, and only closed when the filling valve
opens.This
% ensures no specific pump work due to the piston moving up is
being done.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
% Piston moving from 10aBDC (ref 0=BDC) to TDC (ref 180=TDC) is
analysed
% for both modified and nomodified engine.
% phi_i=10, phi_f=180
theta_i=136.3*pi/180; % Fixing theta initial 'filling valve
opening' %rad

theta_f=170*pi/180; % Fixing theta final 'filling Valve closing'
%rad
% vary initial angle to complete intake by 5 to 10 deg bTDC
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%(1) We need to find the crank angle at which the pressure in the
cylinder equals the critical pressure

%(2) Also the initial angle that leads to reservoir
pressure (280psi)

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Reservoir Tank Conditions%%(inlet conditions) SI Units
Pin=230*6894.76; %psia to Pa
Tin=(70-32)*5/9; %F to C
gamma=1.4; %assumed to be ideal gas constant Cp Cv for air
gamma=Cp/Cv for 300K
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Engine Operating Characteristic
%-----
N=3600; %Engine speed (Rev/min)
w=N*pi/30;% Crank speed angular velocity rad/sec
%-----
%%To change the engine characteristics modify CylinderStateSI
%%CylinderStateNCKSI and CylinderStateNoFlow

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Initially %% exhaust valve is open so pressure and temp is
atmospheric
P1=14.7*6894.76; %Pressure in cylinder after intake (psia)to pa
T1=(70-32)*5/9; %Temperature in cylinder after intake (F) to (C)
R=0.286e3;%gas constant J/kg/K

% As the filling valve opens and exhaust valve closes there is
pressure
% differential between the storage tank and the operation volume
isn the cylinder.
% The cylinder is being filled up due to this pressure
differential.
% The rate at which the air is entering the cylinder depends on
this
% pressure differential. As long as the pressure differential is
greater
% than the critical pressure ratio the flow is choked and has a
limited mass flow rate.

% If the pressure differential is lesser than the critical
pressure the
% flow rate depends on the new pressure in the cylinder and the
pressure
% in the tank

% CylinderStateSI program calculates the operating volume of the
cylinder, the
% mass added to the cylinder and massflow rate due to the choked
flow,
% it is valid within the critical pressure range Pi to Pstar
only.

```

```

% Initial Properties
X=CylinderStateSI(theta_i,theta_i); % [V, M_added, massflow rate
]=Cylinder state
V1=X(1); %m^3
M1=P1*V1/R/(T1+273.15); %kg
mdot_choked=X(3); %kg/sec
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% At critical pressure P_str=Pin*(2/(gamma+1))^(gamma/(gamma-1));
For air
% with constant gamma Pin/P ~1.9 is the critical ratio

P_str=Pin*(2/(gamma+1))^(gamma/(gamma-1)); %(critical pressure
Pa)

%Solution procedure and methodology is provided in more detail in
paper

%The system is an unsteady flow system, we know initial
properties
% theta1 P1 V1 T1 M1 mdot (1)if we fix the theta2 V2 is fixed
%(2) If we assume that the mass flow is constant for the period
between
% theta1 and theta2 the M2 is fixed. Knowing V2 and M2 we can
find the T2
%and V2 using mass balance and energy balance for the system.
% -Wb=Min*hin+M1U1-M2U2
% If we assume that for the period between theta1 and theta2 the
the
% piston pressure is P1, the boundary work Wb is P1(V2-V1)
% U2 is solved
% And for ideal gas constant specific heats assumption U=CvT
% h=CpT (T=0K ref temperature) T2 is solved

%For and ideal gas P2V2=M2RT2

dtheta=.01*pi/180; % Very small increment of theta
dsf=1
S=theta_i:dtheta:theta_f; % Filling period

D=[theta_i*180/pi , P1, V1, T1, M1, mdot_choked]; % initializing
matrix

for T=1:length(S)

theta_1=S(T);
theta_2=S(T+1);
X1=CylinderStateSI(theta_1,theta_1);
V1=X1(1);
P1=D(T,2);
T1=D(T,4);
M1=P1*V1/R/(T1+273.14);

X2=CylinderStateSI(theta_2,theta_2);

```

```

V2=X2(1);
M2=M1+mdot_choked*dtheta/w;

B_Work=P1*(V2-V1); % (N/m2)*m3--N m--J

Cv=0.718e3; %J/kg/K
Cp=Cv*gamma; %J/kg/K
h_in=Cp*(Tin+273.14); % J/kg
U1=Cv*(T1+273.14); %J/kg

U2=(mdot_choked*dtheta/w*h_in-B_Work+M1*U1)/M2;
T2=(U2/Cv)-273.14;
P2=M2*(T2+273.14)*R/V2;
if P2>=P_str
    break;
end
D((T+1),:)= [theta_2*180/pi ,P2, V2, T2, M2, mdot_choked];

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Iterations are stopped when the pressure in cylinder exceed
critical press
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% After the pressure in the cylinder reaches critical pressure
the flow is
% not choked ie the mass flow varys with time(crank angle)as the
pressure
% increases.
% The mass flow rate is now a function of the cylinder pressure
also.
% When the presr in the cylinder reaches 280psi the mass flow
rate is zero.

% CylinderStateNCKSI program calculates the mass flow rate of the
air
% entering the cylinder based on cylinder pressure and tank
pressure
% and the operating volume of the cylinder,
%
% it is valid within the critical pressure range Pstar to Pfinal
.

P1=D(T,2);
V1=D(T,3);
T1=D(T,4);
M1=D(T,5);
theta_int=D(T,1)*pi/180; %angle at which critical pressure is
reaced approx

P_str/6894.76

S2=theta_int:dtheta:theta_f;

```



```

DD=[D(T,1), P1, V1 T1 M1 D(T,6)]; % transition point

for TT=1:length(S2)-1

theta_1=S2(TT);
theta_2=S2(TT+1);
P1=DD(TT,2);

X1=CylinderStateNCKSI(theta_1,P1); % [massflow
rate,V]=CylinderStateNCKSI
V1=X1(2);
T1=DD(TT,4);
M1=P1*V1/R/(T1+273.14);
mdot=X1(1);

X2=CylinderStateNCKSI(theta_2,P1);
V2=X2(2);
M2=M1+mdot*dtheta/w;

B_Work=P1*(V2-V1); % (N/m2)*m3--N m--J

Cv=0.718e3; %J/kg/K
Cp=Cv*gamma; %J/kg/K
h_in=Cp*(Tin+273.14); % J/kg
U1=Cv*(T1+273.14); %J/kg

U2=(mdot*dtheta/w*h_in-B_Work+M1*U1)/M2;
T2=(U2/Cv)-273.14;
P2=M2*(T2+273.14)*R/V2;

DD((TT+1),:)= [theta_2*180/pi ,P2, V2, T2, M2, mdot];
if P2>=Pin
break
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Iterations are stopped when the pressure in cylinder exceeds
tank pressure
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

D(:,2)=D(:,2)/6894.76; %changing pressure to psi

DD(:,2)=DD(:,2)/6894.76; %changing pressure to psi

Crank_Angle=(theta_f-theta_i)*180/pi
Time_for_intake=(theta_f-theta_i)/w
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%Engine Compression for unmodified engine
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
phi_i=10*pi/180; % If in an unmodified engine all valves close
at 10deg aBDC
phi_f=180*pi/180; % Air in being compressed till TDC even though
spark may
                % be initiated earlier

% CylinderStateNoFlow program is used to find the operating
volume of the cylinder only.

X=CylinderStateNoFlow(phi_i,0); %[V, M_added,
mdotin]=CylinderStateNoFlow
V1=X(1);
P1=14.7*6894.76; % Pressure in cylinder after intake (psia)to pa
T1=(70-32)*5/9; % Temperature in cylinder after intake (F)
R=0.286e3;      % Gas constant J/kg/K

M1=P1*V1/R/(T1+273.14); %kg

A=phi_i:5*pi/180:phi_f; % array of crank angles

DDD=[phi_i*180/pi,P1,T1,V1,M1];

for Ts=1:length(A)-1

    X1=CylinderStateNoFlow(A(Ts),0); %[V,
M_added]=CylinderStateNoFlow
    theta_1=A(Ts);
    P1=DDD(Ts,2);
    V1=X1(1);
    T1=DDD(Ts,3);
    M1=P1*V1/R/(T1+273.14); % Ideal Gas

    X2=CylinderStateNoFlow(A(Ts+1),0); %[V,
M_added]=CylinderStateNoFlow
    theta_2=A(Ts+1);
    V2=X2(1);
    M2=M1+X2(2); % M1=M2 no mass added
    P2=P1*(V1/V2)^gamma; %isentropic relationship
    T2=(T1+273.14)*(V1/M1*M2/V2)^(gamma-1)-273.14;% isentropic
relationship

    DDD((Ts+1),:)= [theta_2*180/pi,P2,T2,V2,M2];
end

DDD(:,2)=DDD(:,2)/6894.76;
DDD;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

% figure(6)
% plot(DDD(:,1),DDD(:,2),'.')
% title('Theta vs Pressure Normal')
% xlabel('theta')
% ylabel('pressure psi')
% grid on
%
% figure(7)
% plot(DDD(:,1),DDD(:,4)./DDD(:,5),'.')
% title('Theta vs Specific Volume Normal')
% xlabel('theta')
% ylabel('v m3/kg')
% grid on
%
% figure(8)
% plot(DDD(:,1),DDD(:,3),'.')
% title('Theta vs Temprature Normal')
% xlabel('theta')
% ylabel('T C')
% grid on
%
% figure(9)
% plot(DDD(:,4)./DDD(:,5),DDD(:,2),'.')
% title('Pressure vs Specific Volume Normal')
% ylabel('P psi')
% xlabel('v in3/#')
% grid on
%
% figure (10)
% plot(DDD(:,4)./(.0254)^3,DDD(:,2),'.')
% title('Pressure vs Volume Normal')
% ylabel('P psi')
% xlabel('V in3')
% grid on
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
% State in the modified engine cylinder before filling begins 10
to theta_i
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%At this stage the exhaust valve is open as the piston is moving
up and
%the filling valve is closed. So there is no air flow in to the
cylinder

%The only changing parameter is the volume of the cylinder and
thus mass of
%air but the specific voulme remains constant

S=phi_i:dsf*pi/180:theta_i;
P=14.7*6894.76;

```

```

Tm=(70-32)*5/9;
Z=CylinderStateNoFlow(phi_i,0);
V=Z(1);
M=P*V/R/(Tm+273.14);

SS=[phi_i*180/pi P V Tm M 0]; %initializing
for s=2:length(S)
    P=14.7*6894.76;
    Tm=(70-32)*5/9;
    Z=CylinderStateNoFlow(S(s),0);
    V=Z(1);
    M=P*V/R/(Tm+273.14); %ideal gas assumption
    SS(s,:)=[S(s)*180/pi P V Tm M 0];
end
SS(:,2)=SS(:,2)/6894.76;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% State in the modified engine cylinder after filling over
% theta_f to TDC
%-----
%-----
%At this stage both the exhaust valve and filling valve is closed
% as the piston is moving up. Spark is usually initiated at
%10bTDC
%but this will not be modeled here.

%The filling valve is closed. So there is no air flow in to the
cylinder

% The air is being compressed and assumed to be isentropic

F=theta_f:dsf*pi/180:phi_f;

FT=DD(TT+1,:);
FT(2)=FT(2)*6894.76; %Pa

for f=1:length(F)-1
    theta_1=F(f);
    P1=FT(f,2);
    V1=FT(f,3);
    T1=FT(f,4);
    M1=FT(f,5);

    X2=CylinderStateNoFlow(F(f+1),0);
    theta_2=F(f+1);
    V2=X2(1);
    M2=M1+X2(2);
    P2=P1*(V1/V2)^gamma; %isentropic
    T2=(T1+273.14)*(V1/M1*M2/V2)^(gamma-1)-273.14; %isentropic

    FT((f+1),:)= [theta_2*180/pi,P2,V2,T2,M2,0];
end
FT(:,2)=FT(:,2)/6894.76;

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Variables=['ThetaDeg' 'P (psi)' 'V(m^3)' 'Tem(F)' 'M(kg)'
'mdot(kg/sec)']
NoFill=SS;
Filling=[D;DD];
Compr=FT;

[SS(1,:);
D(1,:);
D(length(D),:);
DD(2,:);
DD(length(DD),:);
FT(length(FT),:)]

[DDD(1,:);
DDD(length(DDD),:)]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%

figure(1)
plot(D(:,1),D(:,6),'r')
hold on
plot(DD(:,1),DD(:,6),'')
title('Theta vs Mass flow rate')
xlabel('Crank Angle deg')
ylabel('mdot kg/sec')
grid on
%
figure(2)
plot(SS(:,1),SS(:,2),'g')
hold on
plot(D(:,1),D(:,2),'')
plot(DD(:,1),DD(:,2),'r')
plot(FT(:,1),FT(:,2),'k')
title('Theta vs Pressure ')
xlabel('Crank Angle deg')
ylabel('pressure psi')
grid on

figure(3)
plot(SS(:,1),SS(:,3)./SS(:,5),'g.')
hold on
plot(D(:,1),D(:,3)./D(:,5),'.')
plot(DD(:,1),DD(:,3)./DD(:,5),'r.')
plot(FT(:,1),FT(:,3)./FT(:,5),'k.')
title('Theta vs Specific Volume ')
xlabel('Crank Angle deg')
ylabel('v m3/kg')
grid on
%
figure(4)
plot(SS(:,1),SS(:,4),'g')
hold on
plot(D(:,1),D(:,4),'')

```

```

plot(DD(:,1),DD(:,4),'r')
plot(FT(:,1),FT(:,4),'k')
title('Theta vs Temperature ')
xlabel('Crank Angle deg')
ylabel('T C')
grid on

figure(5)
plot(SS(:,3)./0.0254^3,SS(:,2),'g')
hold on
plot(D(:,3)./0.0254^3,D(:,2),'r')
plot(DD(:,3)./0.0254^3,DD(:,2),'r')
plot(FT(:,3)./0.0254^3,FT(:,2),'k')
title('Pressure vs Volume ')
ylabel('P psi')
xlabel('v in^3')
grid on

figure(11)
plot(SS(:,1),SS(:,2),'k','LineWidth',2)
hold on
plot(D(:,1),D(:,2),'k','LineWidth',2)
plot(DD(:,1),DD(:,2),'k','LineWidth',2)
plot(DDD(:,1),DDD(:,2),'--k','LineWidth',2)
plot(FT(:,1),FT(:,2),'k','LineWidth',2)
title('Theta vs Pressure Modified vs Normal')
xlabel('Crank Angle deg')
ylabel('Pressure psi')
grid on
%
figure(12)
plot(SS(:,3)./SS(:,5),SS(:,2),'k','LineWidth',2)
hold on
plot(D(:,3)./D(:,5),D(:,2),'k','LineWidth',2)
plot(DD(:,3)./DD(:,5),DD(:,2),'k','LineWidth',2)
plot(DDD(:,4)./DDD(:,5),DDD(:,2),'--k','LineWidth',2)
plot(FT(:,3)./FT(:,5),FT(:,2),'k','LineWidth',2)
title('Pressure vs Specific Volume Modified vs Normal')
xlabel('Specific Volume m^3/kg')
ylabel('pressure psi')
grid on
%
figure(13)
plot(SS(:,3)./0.0254^3,SS(:,2),'k','LineWidth',2)
hold on
plot(D(:,3)./0.0254^3,D(:,2),'k','LineWidth',2)
plot(DD(:,3)./0.0254^3,DD(:,2),'k','LineWidth',2)
plot(DDD(:,4)./0.0254^3,DDD(:,2),'--k','LineWidth',2)
plot(FT(:,3)./0.0254^3,FT(:,2),'k','LineWidth',2)
title('Pressure vs Volume Modified vs. Normal')
xlabel('Volume in^3')
ylabel('Pressure psi')
grid on

figure(14)

```

```

plot(SS(:,1),SS(:,4),'k','LineWidth',2)
hold on
plot(D(:,1),D(:,4),'k','LineWidth',2)
plot(DD(:,1),DD(:,4),'k','LineWidth',2)
plot(DDD(:,1),DDD(:,3),'--k','LineWidth',2)
plot(FT(:,1),FT(:,4),'k','LineWidth',2)
title('Temperature vs theta Modified vs. Normal')
xlabel('Crank Angle deg ')
ylabel('Temperature C')
grid on

figure(15)
plot(SS(:,1),SS(:,5),'k','LineWidth',2)
hold on
plot(D(:,1),D(:,5),'k','LineWidth',2)
plot(DD(:,1),DD(:,5),'k','LineWidth',2)
plot(DDD(:,1),DDD(:,5),'--k','LineWidth',2)
plot(FT(:,1),FT(:,5),'k','LineWidth',2)
title('Mass of air vs Theta Modified vs Normal')
xlabel('Crank Angle deg')
ylabel('Mass kg')
grid on

SS(:,2)=SS(:,2).*6894.76;
D(:,2)=D(:,2).*6894.76;
DD(:,2)=DD(:,2).*6894.76;
FT(:,2)=FT(:,2).*6894.76;
DDD(:,2)=DDD(:,2).*6894.76;

b=2.69*0.025;      %Bore (m)
r=4*1.020*0.025;  %Connecting rod length (m)
St=2.04*0.025;    %Stroke (m)
a=St/2;           %Crank offset length (m) a=S/2

Wb_Eng1=((Pin*DDD(1,4)*(DDD(1,2)/Pin)^(1/gamma)-
DDD(1,2)*DDD(1,4))/(1-gamma));
Wb_Eng=(DDD(length(A),2)*DDD(length(A),4)-DDD(1,2)*DDD(1,4))/(1-
gamma);
Wb_Eng2=(Wb_Eng-Wb_Eng1);

for v2=1:s-1
dVSS(v2)=SS(v2,3)-SS(v2+1,3);
end
for v3=1:T-1
dVD(v3)=D(v3,3)-D(v3+1,3);
end
for v4=1:TT
dVDD(v4)=DD(v4,3)-DD(v4+1,3);
end
for v5=1:f

```

```

dVFT(v5)=FT(v5,3)-FT(v5+1,3);
end

G1=SS(1:v2,2).*dVSS'; D(1:v3,2).*dVD'; DD(1:v4,2).*dVDD';
G2=FT(1:v5,2).*dVFT';

Wb_ModEng1=-sum(G1);
Wb_ModEng2=-sum(G2);

LostPerfm=Wb_Eng2-Wb_ModEng2;
Wsaved=Wb_Eng1-Wb_ModEng1;
QextraKJperKg=Cv*(DDD(length(DDD),3)-FT(length(FT),4))/1e3;
Saved_PumpWork=(Wsaved)/(Wb_Eng);
fprintf('%s\n', 'WSaved      Percent      ExtraWork      StdEngWork
QExtra')
fprintf('%-10.3f %-10.3f %-10.3f %-10.3f %-10.3f\n', Wsaved,
Saved_PumpWork, LostPerfm, Wb_Eng, QextraKJperKg)

fileName=sprintf('UFlow_%d_%d.txt', Pin/6894.76, theta_f*180/pi);
dlmwrite(fileName, [Wb_Eng1, Wb_Eng, Wb_ModEng1, LostPerfm, Wsaved, Sa
ved_PumpWork]; [NoFill; Filling; Compr])

%(pi*b^2/4)*(a*sin(pi+A)+a^2*sin(pi+A).*cos(pi+A))./(sqrt(r^2-
a^2*(sin(pi+A)).^2)).*(1*pi/180);
AF=15;
Ma_m=FT(length(FT),5);
Mf_m=Ma_m/AF;
M_m=Ma_m+Mf_m;
%for std eng
T3=4869.19;
Qin_m=M_m*Cv*(T3-(FT(length(FT),4)+273.14)) %Joules
T4=1938.46;
Wexp_m=M_m*Cv*(T3-T4);

Wc_m=Wb_ModEng1+Wb_ModEng2;
Thermeff=(Wexp_m+Wc_m)/Qin_m
[Wb_ModEng1, Wb_ModEng2, Wb_Eng1, Wb_Eng2, Wexp_m]

function A=CylinderStateNoFlow(theta, theta_i)
%Engine Specs%
%-----
%Clearance Volume
b=2.69*0.0254;      %Bore (m)
r=4*1.020*0.0254;  %Connecting rod length (m)
S=2.04*0.0254;     %Stroke (m)
a=S/2;             %Crank offset length (m) a=S/2
rc=10;            %Compression Ratio rc=Vd+Vc/Vc
Vd=pi*b^2/4*S;    %Displacement Volume m^3
Vc=Vd/(rc-1);     % Clearance Vol (m^3)
dv=0.272*0.0254;  %m
Av=pi*0.25*dv^2;% effective area of intake valve %eq 5-4 IC Book
m^3

```



```

%Engine Operating Characteristic
%-----
N=3500; %Engine speed (Rev/min)
w=N*pi/30;% Crank speed  angular velocity rad/sec
%-----

Tin=(70-32)*5/9; %C
R=0.287e3; %gas constant  J/kg/K
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
gamma=1.4; % ratio of specific heats assumed to be constant refer
to IC book

%We can use the isentropic equation to calculate pressure in
terms of time

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%Choked Flow Analysis%
gamma=1.4; % ratio of specific heats assumed to be constant refer
to IC book
%Critical Pressure
P_star=Po*(2/(gamma+1))^(gamma/(gamma-1)); % Pa  for air with
gamma = 1.4  refer Thermo Book Online
mdot_inCK=Po*Av/sqrt(R*(To+273.14))*sqrt(gamma)*(2/(gamma+1))^(gamma/(gamma-1)); % kg/s
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%

%Cylinder Volume and mass added wrt crank angle

s=a*cos(pi+theta)+sqrt(r^2-a^2*(sin(pi+theta))^2); % m stroke
length from BDC
V=Vc+(pi*b^2/4)*(r+a-s); %m^3 volume of the cylinder wrt crank
angle
M_added=mdot_inCK*(theta-theta_i)/w; % extra mass added due to
choked flow of air in to the cylinder
A=[V,M_added,mdot_inCK];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%
%Non Choked Flow Analysis%
% Defining the mass flow rate equation
K1=Pin*Av*sqrt((2/R/(Tin+273.14))*(gamma/(gamma-1)));
s=a*cos(pi+theta)+sqrt(r^2-a^2*(sin(pi+theta))^2); %stroke length
from BDC
V=Vc+(pi*b^2/4)*(r+a-s); %volume of the cylinder wrt crank angle
% the top part is actually X(1)
mdot_inNCK=K1*sqrt((P/Pin)^(2/gamma)-(P/Pin)^((gamma+1)/gamma));

```

```
%JJ=mdot_inNCK(length(mdot_inNCK))-K1*sqrt((P/Po)^(2/gamma)-  
(P/Po)^((gamma+1)/gamma));  
J=[mdot_inNCK,V];
```

APPENDIX B
HAND CALCULATIONS

INPUTS:

Engine Characteristics:

$$b = 2.69 * 0.0254 \text{ (m)}$$

$$S = 2.04 * 0.0254 \text{ (m)}$$

$$R = 4 * 1.020 * 0.0254 \text{ (m)}$$

$$rc = 10$$

$$V_d = 1.899 \text{ E}^{-4} \text{ (m}^3\text{)},$$

$$V_c = 2.11 \text{ E}^{-5} \text{ (m}^3\text{)},$$

$$d_v = 0.272 * 0.0254, \text{ (m) (valve size)}$$

$$A_v = \pi * 0.25 * d_v^2 = 3.748 \text{ E}^{-5} \text{ (m}^2\text{)}$$

Engine Operation:

$$N = 3600 \text{ rpm}$$

$$\omega = N * \pi / 30 = 376.99 \text{ (rad/sec)}$$

Compressed Tank Air Properties:

$$P_{CT} = 280 * 6894.76 \text{ (psi to Pa)}$$

$$T_{CT} = (70 - 32) * 5/9 \text{ (}^\circ\text{F to }^\circ\text{C)}$$

$$R = 0.286 \text{ E}^3 \text{ (J/kg/K)}$$

$$\gamma = 1.4$$

$$C_v = 718 \text{ (J/kg/K)}$$

Spark Advance (Fill Valve close angle):

$$\theta_F = 170^\circ \text{ from BDC (10}^\circ \text{ bTDC)}$$

Filling takes place in between the unknown θ_i to θ_F

PROCEDURE:

If we guess:

$$\theta_i = 138^\circ \text{ (filling valve opens)}$$

Filling:

After the no fill stage the CC is in atmospheric condition so:

$$\theta_1 = 138^\circ$$

$$P_1 = 14.7 * 6894.76 \text{ (psi to Pa)}$$

$$T_1 = (70 - 32) * 5/9 \text{ (}^\circ\text{F to }^\circ\text{C)}$$

$$R = 0.286 \text{ E}^3 \text{ (J/kg/K)}$$

$$\gamma = 1.4$$

$$s = 0.1214 \text{ (m)}$$

$$V_1 = 5.0864 \text{ E}^{-5} \text{ (m}^3\text{)}$$

$$M_1 = 6.1255 \text{ E}^{-5} \text{ (kg)}$$

Since $P_1 < P_{critical}$, the flow in to the CC is choked flow:

$$P_{critical} = 1.0199 \text{ E}^6 \text{ (Pa)}$$

$\dot{m}_{in}=0.1705 \text{ (kg/sec)}$
 $\Delta\theta=1^\circ$

$\Theta_2=139^\circ$
 $V_2=4.9557E^{-5} \text{ (m}^3\text{)}$
 $M_2=6.9152E^{-5} \text{ (kg)}$
 $\text{Boundary Work } W_b=P_1(V_2-V_1)=-0.1325 \text{ (J)}$
 $U_1=2.1127E^5 \text{ (J/kg)}$
 $U_2=2.2284E^5 \text{ (J/kg)}$

$T_2=37.21 \text{ (C)}$
 $P_2=1.2386 \text{ e}5 \text{ (Pa)}$

P2 < Pcritical so we are still in choked flow

Next Step all 2s are now 1s

$\Theta_1=139^\circ$
 $T_1=37.21 \text{ (C)}$
 $P_1=1.2386E^5 \text{ (Pa)}$
 $V_1=4.9557E^{-5} \text{ (m}^3\text{)}$
 $M_1=6.9152E^{-5} \text{ (kg)}$
 $U_1=21127E^5 \text{ (J/kg)}$

$\Theta_2=140^\circ$
 $V_2=4.8272e^{-5} \text{ (m}^3\text{)}$
 $M_2=7.7047e^{-5} \text{ (kg)}$
 $\text{Boundary Work } W_b=P_1(V_2-V_1)=-0.1591 \text{ (J)}$
 $U_2=2.3237E^5 \text{ (J/kg)}$

$T_2=50.5 \text{ (C)}$
 $P_2=1.4774 \text{ E}5 \text{ (Pa)}$

θ	P	V	T	M	\dot{m}
138	1.0135e5	5.0864e-5	21.111	6.1256e-5	0.17053
139	1.2386e5	4.9557e-5	37.217	6.9152e-5	0.17053
140	1.4774e5	4.8272e-5	50.502	7.7047e-5	0.17053

.....]

Continued till P2=> Pcritical

$\Theta=160$ $P_2 1.0404E5 \text{ Pa}$ is $> P_{critical}$ –Switch to no choked flow equations.

Now the flow is non choked:

$\Theta_1=159^\circ$
 $V_1=2.8947e^{-5} \text{ (m}^3\text{)}$
 $P_1 = 9.7289 \text{ e}5 \text{ (Pa)}$
 $T_1 = 160.5 \text{ (}^\circ\text{C)}$
 $M_1= 2.2705E^{-4} \text{ (kg)}$
 $U_1=3.1140E^5 \text{ (J/kg)}$

$\theta_2=160^\circ$
 $V_2=2.823e-5 \text{ (m}^3\text{)}$
 $\dot{m}=0.1703 \text{ (kg/sec)}$
 $M_2=2.3493 e-4 \text{ (kg)}$
 $\text{Boundary Work } W_b=P_1(V_2-V_1)=-0.6977 \text{ (J)}$
 $U_2=3.1384 E_5 \text{ (J/kg)}$

 $T_2=163.96 \text{ (C)}$
 $P_2=1.0404 E_6 \text{ (Pa)}$

Since $P_2 < P_{cr}$, the filling still continues:

$\theta_1=162^\circ$
 $V_1= 2.627e-5 \text{ (m}^3\text{)}$
 $P_1 = 1.1823e6 \text{ (Pa)}$
 $T_1 = 170.36 \text{ (C)}$
 $M_1= 2.506 e-4\text{(kg)}$
 $U_1= 3.184 e_5\text{(J/kg)}$

 $\theta_2=163^\circ$
 $V_2= 2.6277e-5\text{(m}^3\text{)}$
 $\dot{m}=0.16784\text{kg/sec}$
 $M_2= 2.58e-4\text{(kg)}$
 $\text{Boundary Work } W_b=P_1(V_2-V_1)=- 0.7303 \text{ (J)}$
 $U_2= 3.2058e_5 \text{ (J/kg)}$

 $T_2= 173.34 \text{ (C)}$
 $P_2= 1.256 e_6 \text{ (Pa)}$

θ	P	V	T	M	\dot{m}
159	9.7289e5	2.8947e-5	160.56	0.00022705	0.17053
160	1.0404e6	2.823e-5	163.97	0.00023493	0.17031
161	1.1103e6	2.7546e-5	167.23	0.00024282	0.17048
162	1.1823e6	2.6895e-5	170.36	0.00025068	0.1697
163	1.256e6	2.6277e-5	173.34	0.00025845	0.16784

.....]
Continues till θ_f

At θ_f , $P_f=1.7653 e_6 \text{ Pa (256.43 psi)}$

P_f should be the tank pressure—so θ_i needs to be a little smaller.

*For $\theta_i=134.7^\circ$ $P_f=279.97$ at $\theta_f=170^\circ$
 Crank period required = 35°*

$W_{\text{pump}} = P_1 \Delta V$ at every step or sum of W