An Overview of Systematic Composition

with an Introduction to the Take-Away System

by

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ABSTRACT

A systematic approach to composition has been used by a variety of composers to control an assortment of musical elements in their pieces. This paper begins with a brief survey of some of the important systematic approaches that composers have employed in their compositions, devoting particular attention to Pierre Boulez's *Structures Ia*. The purpose of this survey is to examine several systematic approaches to composition by prominent composers and their philosophy in adopting this type of approach. The next section of the paper introduces my own systematic approach to composition: the Take-Away System. The third provides several musical applications of the system, citing my work, *Octulus* for two pianos, as an example. The appendix details theorems and observations within the system for further study.

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CHAPTER 1

A BRIEF OVERVIEW OF SYSTEMATIC COMPOSITION

Introduction

The purpose of this paper is to first examine several approaches to systematic composition from various composers in the past, then to introduce my own system, and finally, to explore various philosophies behind such an approach to composition. The first chapter will provide a brief historical overview of systematic composition. The intent is not to provide a complete or exhaustive history, but rather, to survey different systems that have been employed in the past, demonstrating the diverse interest in systematic composition. In particular, Boulez's *Structures Ia* will be addressed with comparisons between Boulez's systematic approach to composition and in my own work. The second chapter will then introduce the Take-Away System with the third chapter showing how this system can be applied to music citing musical examples from my composition, *Octulus*, for two pianos. In conclusion the philosophy behind systematic composition will be examined.

A Brief Overview of Systematic Composition

Throughout history, composers have used rules to assist them in composition. Within the tonal method, composers often adhered to conventions regarding melody, harmony, and form when writing their works. However, these composers ultimately had a large amount of control and freedom over each element of music and these works could rarely be seen as rigidly systematic.

After World War I, Schoenberg introduced the twelve-tone method. He saw this as a logical evolution from the musical trends of the preceding one hundred years.[1] Schoenberg remarks "these new sounds obey the laws of nature and of our manner of thinking—the conviction that order, logic, comprehensibility and form cannot be present without obedience to such laws."[2] Paul Lansky argues that ever since Schoenberg, and even more so today, "Analytic ideas are influenced by our compositional experiences, and compositional notations are inspired by analytic observations."[3] This might explain why an increased number of composers have sought to use systematic approaches to composition since the development of the twelve-tone method.

One systematic approach to composition was used by Steve Reich from 1965-1971 and was called the *gradual phase shifting process*.[4] Reich discovered this system by accident when he was playing two identical recordings on two different tape recorders at the same time.[5] According to Reich:

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[1] Arnold Schoenberg, Style and Idea - Selected Writings of Arnold Schoenberg, 1st ed. (New York: St. Martin's Press, 1975), 216.

[2] Schoenberg, 218.

[3] Paul Lansky, "Pitch Class Consciousness," Perspectives of New Music 13, no. 2 (04/1975): 30.

[4] Steve Reich, Writings On Music, 1965-2000, ed. Paul Hillier (New York: Oxford University Press, USA, 2002), 68.

> The two machines happened to be lined up in unison and one of them gradually started to get ahead of the other. The sensation I had in my head was that the sound moved over to my left ear, down to my left shoulder, down my left arm, down my leg, out across the floor to the left, and finally began to reverberate and shake and become the sound I was looking for . . . When I heard that, I realized it was more interesting than any one particular relationship, because it was the process (of gradually passing through all the canonic relationships) making an entire piece, and not just a moment in time.[6]

Reich first used his gradual phase shifting process in the piece *It's Gonna Rain* (1965) for tape and used it almost exclusively for seven years with the only exception being *Four Organs* (1970). Reich found that with some practice he could replicate the sound of the gradual phase shifting process live with human performers.[7] Other pieces using the gradual phase shifting process include *Piano Phase* (1967) for two pianos, *Violin Phase* (1967) for violin and three channels of tape (or for four violins), *Pendulum Music* (1968) for microphones, amplifiers, speakers and performers, and *Phase Patterns* (1970) for four electric organs.

Reich explains that the gradual phase process is an extension of the idea of an infinite canon or round.[8] In comparison to more traditional canons, however,

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[5] Ibid., 20-21.

[6] Ibid., 21.

[7] Ibid., 22-24.

[8] Ibid., 20.

Reich states that in his process the melodic material is shorter and the time interval between the melodic idea and its imitation is variable. Thus, Reich's gradual process music uses a small amount of material and the phasing of his system generates the piece. Reich explains that his process allows him to go "through a number of relationships between two identities without ever having any transitions. It was a seamless, uninterrupted musical process."[9]

Frederic Rzewski also developed an approach to composition in which a single idea could be systematically transformed into a large musical structure. Rzewski's system came to him "in a flash."[10] Some time earlier, Rzewski had tape-recorded a simple 65-note melody he had whistled while walking along the street. Later, on a train, he was pondering how to compose a work for a variable instrumentation and an indeterminate size of ensemble. Frans Bruggen had asked Rzewski to write a work for a group that Bruggen had formed with some of his students.

Rzewski applied an "additive procedure" on his 65-note melody. He describes the system by having the performers play:

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[9] Ibid., 20.

[10] Frederic Rzewski, Nonsequiturs: Writings & Lectures On Improvisation, Composition, and Interpretation = Unlogische Folgerungen: Schriften Und Vorträge Zu Improvisation, Komposition Und Interpretation, 1. Aufl. ed. (Köln: MusikTexte, 2007), 440.

> the first note, then the first two notes, then the first three, and so on until reaching note sixty-five, at which point I could begin to shrink it again towards the end by leaving out note 1, then notes 1 and 2, and so on, until ending with notes 63-64-65-64-65-65.[11]

Rzewski calls his approach "squaring" and he titled his 65-note melody for Bruggen *Les Moutons de Panurge* ("The Sheep of Panurge"). With squaring, Rzewski notes that the system creates a piece much longer (his twenty second melody becomes a piece around twenty minutes long) and with a different effect from the original inspiration. Rzewski uses his "squaring" idea on other pieces such as *Jefferson* (1970) for solo voice and piano and *Song and Dance* (1977): for flute, bassoon, clarinet, double bass, and vibraphone. Rzewski expands his squaring procedure to include rhythm and phrasing in his works *Coming Together* (1972) and *Attica* (1972)—both pieces for speaker, low instruments and ensemble.[12] With both Reich and Rzewski, a relatively simple system is utilized to control the overall structure and the detail of the piece.

Iannis Xenakis provided a unique outlook to composition reflective of his varied background in civil engineering, architecture, and music. Xenakis applied mathematical concepts of probability, set theory, group theory, and game theory, in his music. In some of Xenakis' works there is an explicit connection to a systematic approach to composition. In his work *Herma* (1960-1961) for solo piano, Xenakis states that the work is composed by combining sets of notes in logical ways.[13] Using the ideas of union and intersection from set theory, he states:

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[11] Ibid., 440-442.

[12] Ibid., 450.

> "I take all the notes of the two sets. Then I can take the sounds the two sets have in common. And finally I can take the sounds that the two sets don't have in common. There are, of course, other, more complex logical functions. In each case, I get a new set."[14]

Unlike Reich and Rzewski where the systematic process remains relatively audible to the listener, Xenakis takes liberties with his systems and his operations may not be easily heard. Even finding the sets and logical operations that Xenakis describes in his scores can be difficult, if not extremely frustrating as he admits to adjusting his systems to his own personal criteria of interest.[15] Evan Jones, in his analysis of *Herma* and *Tetora* shows how difficult this process can be and creates complex computer algorithms to determine which sets of notes Xenakis is utilizing.[16]

As opposed to Xenakis' mathematical perspective, John Cage used chance operations in his work *Music of Changes* (1951) for solo piano. Cage, influenced by the *I-Ching*, constructed charts for sounds and silences, durations, dynamics, number of layers, and tempi.[17] With his notes, Cage carefully arranged his charts so that all twelve chromatic pitches would occur in any row or column.[18] Once all twelve chromatic pitches were used, Cage took liberties and added other notes and extended techniques in his charts. Some of his extended techniques include: clusters, harmonics, string piano techniques, arm clusters, scraping the fingernails along a string inside the piano, slamming the keyboard lid closed, striking various parts of the piano with a wood sick, and dropping a cymbal beater vertically into the piano so that it hits the sound board.[19]

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[13] Balint Andras Varga, Conversations With Iannis Xenakis (London: Faber & Faber, 1996), 85.

[14] Ibid., 85.

[15] Evan Jones, "Residue-Class Sets in the Music of Iannis Xenakis: An Analytical Algorithm and a General Intervallic Expression," Perspectives of New Music 39, no. 2 (Summer, 2001): 234.

[16] Ibid., 240.

Cage then flipped coins to make his musical decisions relating to the succession and realization of each musical element.[20] Cage's system combines control and chance: "Charts were used for the *Music of Changes*, but in contrast to the method which involved chance operations, these charts were subjected to a rational control."[21] While Xenakis deviates from his system to exhibit greater control over his work, Cage's systematic use of chance operations relinquish some of his control over the work.

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[17] David Bernstein, "Cage and High Modernism," in The Cambridge Companion to John Cage, ed. David Nicholls (New York: Cambridge University Press, 2002), 203.

[18] John Cage, Silence: Lectures and Writings (Middletown, CT.: Wesleyan, 1961), 26.

[19] Bernstein, 207.

[20] Bernstein, 208.

[21] Cage, 25-26.

While John Cage was composing *Music of Changes,* Pierre Boulez was writing *Structures Ia* (1951-1952) for two pianos. Inspired by Messiaen's *Mode de valeurs et d'intensités* Boulez writes in a letter to John Cage in August of 1951, that his compositional attention for the past year had been "focused on the expansion and homogeneity of the field of serialism."[22] Boulez not only orders pitches, rhythms, articulations, and dynamics like Messiaen, but uses his organization of pitches to determine the order of rhythms, articulations and dynamics. In this sense, Boulez's systematic approach is much more exhaustive and rigidly applied than any of the previously mentioned composers.

To begin, Boulez used a series of twelve pitches taken from Messiaen's *Mode de valeurs et d'intensités* (Figure 1). Boulez numbers each pitch in the order it appears in the original series. When Boulez transposes the row down a half step (Figure 2), the numbers do not decrease by one. Instead, the numbers are still assigned to the order of the original row. Boulez also inverted the original series (Figure 3) similarly keeping the numbers assigned to the pitch order in the original series.

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[22] Pierre Boulez, The Boulez-Cage Correspondence, ed. Jean-Jacques Nattiez and Robert Samuels (Cambridge: Cambridge University Press, 1993), 99.

Figure 1. Boulez's original series of pitches in *Structures Ia*

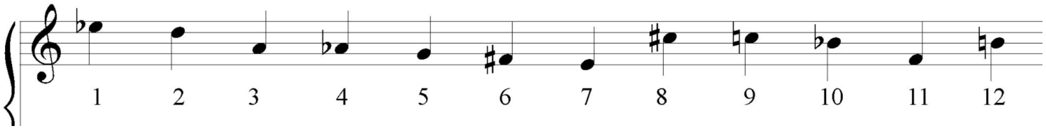


Figure 2. Transposition of Boulez's original series

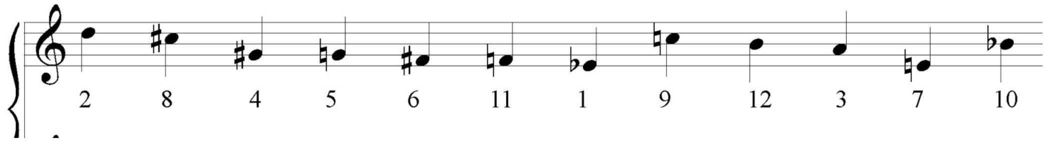
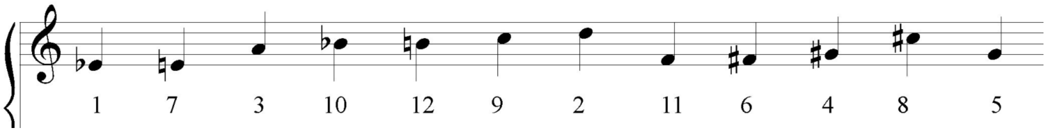


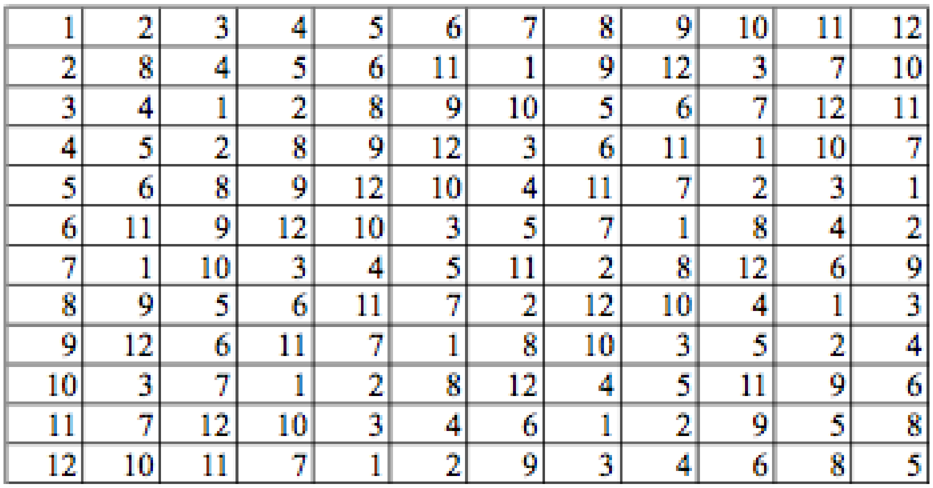
Figure 3. Inversion of Boulez's original series



Calculating all twelve transpositions of the row and its inversion, Boulez created two matricies (Figure 4) and used these charts to determine all of the pitches, rhythms, articulations and dynamics throughout the work. Matrix A contains the 12 transpositions of the original series and Matrix B contains the 12 transpositions of the *inversion* of the original series.

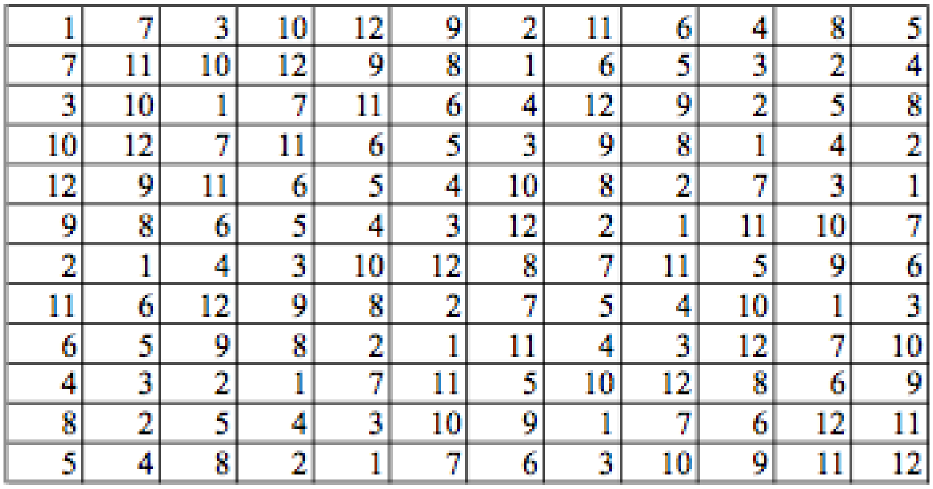
It is helpful to think of *Structures Ia* in two parts even though the work is divided into several sections with double bars and fermatas. For the pitch content Boulez uses each row in both Matrix A and B exactly once in the first part of *Structures Ia* and then uses the retrograde of each row in both matricies exactly once in the second part of the movement.

Figure 4: Boulez’s Matricies in *Structures I*



Matrix A

1	2	3	4	5	6	7	8	9	10	11	12
2	8	4	5	6	11	1	9	12	3	7	10
3	4	1	2	8	9	10	5	6	7	12	11
4	5	2	8	9	12	3	6	11	1	10	7
5	6	8	9	12	10	4	11	7	2	3	1
6	11	9	12	10	3	5	7	1	8	4	2
7	1	10	3	4	5	11	2	8	12	6	9
8	9	5	6	11	7	2	12	10	4	1	3
9	12	6	11	7	1	8	10	3	5	2	4
10	3	7	1	2	8	12	4	5	11	9	6
11	7	12	10	3	4	6	1	2	9	5	8
12	10	11	7	1	2	9	3	4	6	8	5



Matrix B

1	7	3	10	12	9	2	11	6	4	8	5
7	11	10	12	9	8	1	6	5	3	2	4
3	10	1	7	11	6	4	12	9	2	5	8
10	12	7	11	6	5	3	9	8	1	4	2
12	9	11	6	5	4	10	8	2	7	3	1
9	8	6	5	4	3	12	2	1	11	10	7
2	1	4	3	10	12	8	7	11	5	9	6
11	6	12	9	8	2	7	5	4	10	1	3
6	5	9	8	2	1	11	4	3	12	7	10
4	3	2	1	7	11	5	10	12	8	6	9
8	2	5	4	3	10	9	1	7	6	12	11
5	4	8	2	1	7	6	3	10	9	11	12

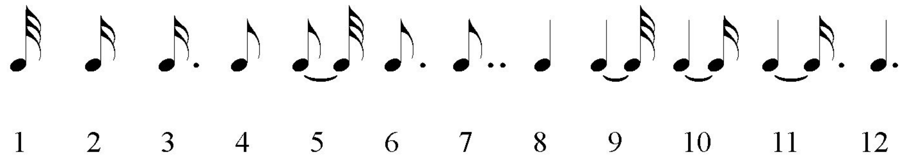
Piano I begins with the original series and plays all of the transpositions (rows) in Matrix A. The order of the transpositions is determined by the inversion of the original series [1, 7, 3, 10, 12, 9, 2, 11, 6, 4, 8, 5]. Thus, after Piano I plays the original series (the top row of Matrix A), it begins the seventh row in Matrix A, then the third row, followed by the tenth row, twelfth row, and so on, until it ends with the fifth row.

Piano II begins playing the inversion of the series (the top row of Matrix B) and plays all of the transpositions (rows) in Matrix B. The order Piano II plays these rows is dictated by the original series [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. After Piano II plays the inversion (the top row of Matrix B), it begins the row in Matrix B that starts with the number 2, followed by the row that starts with number 3, and so on until it ends by playing the row that begins with the number 12. It should be mentioned that in both piano parts Boulez sometimes has two or three rows occur concurrently throughout the movement. When this occurs, rows begin simultaneously and can be differentiated through articulations and dynamics.

After each piano has completed its matrix, Boulez has the pianos switch matricies for the second part of the movement. Piano I plays all of the rows in Matrix B in retrograde and Piano II plays all of the rows in Matrix A in retrograde. In the first part of the movement all of the transpositions of the original series and transpositions of the inversion of the original series are played, and in the second part of the movement all of the transpositions of the retrograde and retrograde inversion of the original series are played. Thus, the row and its representation in the matrices controls not only the note-to-note pitch decisions, but also the succession of rows for the entire piece. The succession of rows is executed in a completely systematic fashion.

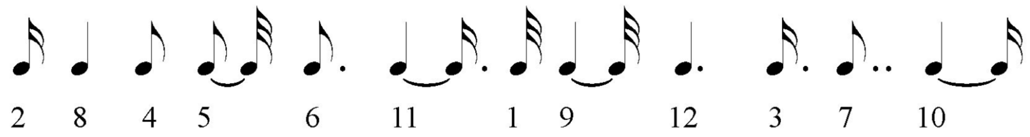
Rhythmically, Boulez follows a similar plan. He first writes all of the durations from one to twelve 32nd notes from smallest to largest. Then, Boulez assigns a number from one to twelve to each duration from shortest to longest (Figure 5). Like his pitch series, Boulez again takes his rhythmic ordering from Messiaen's *Mode de valeurs et d'intensités*.[23]

Figure 5. Boulez's durational serialization of rhythm (the row)



Boulez then applies the same matricies he used with pitch to transpose and invert his rhythms. Since the numbers in the matricies are linked with the pitches, the transposition of the durations of the series is not necessarily rhythmically intuitive. Transposing Boulez's original rhythm up one number, does not add one 32nd note to each note. Instead, the numbers are found in the second row of Matrix A and a very different rhythm appears (Figure 6). When the original series is inverted (Figure 7) another unique rhythm is produced.

Figure 6. Boulez's original rhythm transposed



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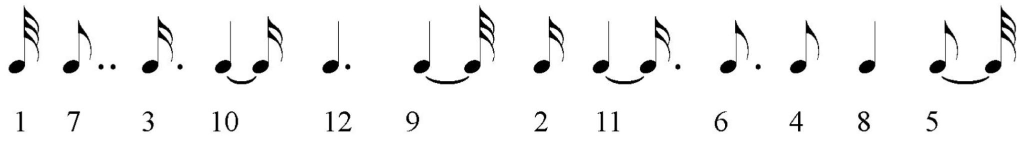
[23] Pierre Boulez, The Boulez-Cage Correspondence, ed. Jean-Jacques Nattiez and Robert Samuels (Cambridge: Cambridge University Press, 1993), 101.

Figure 7. Boulez's original rhythm inverted

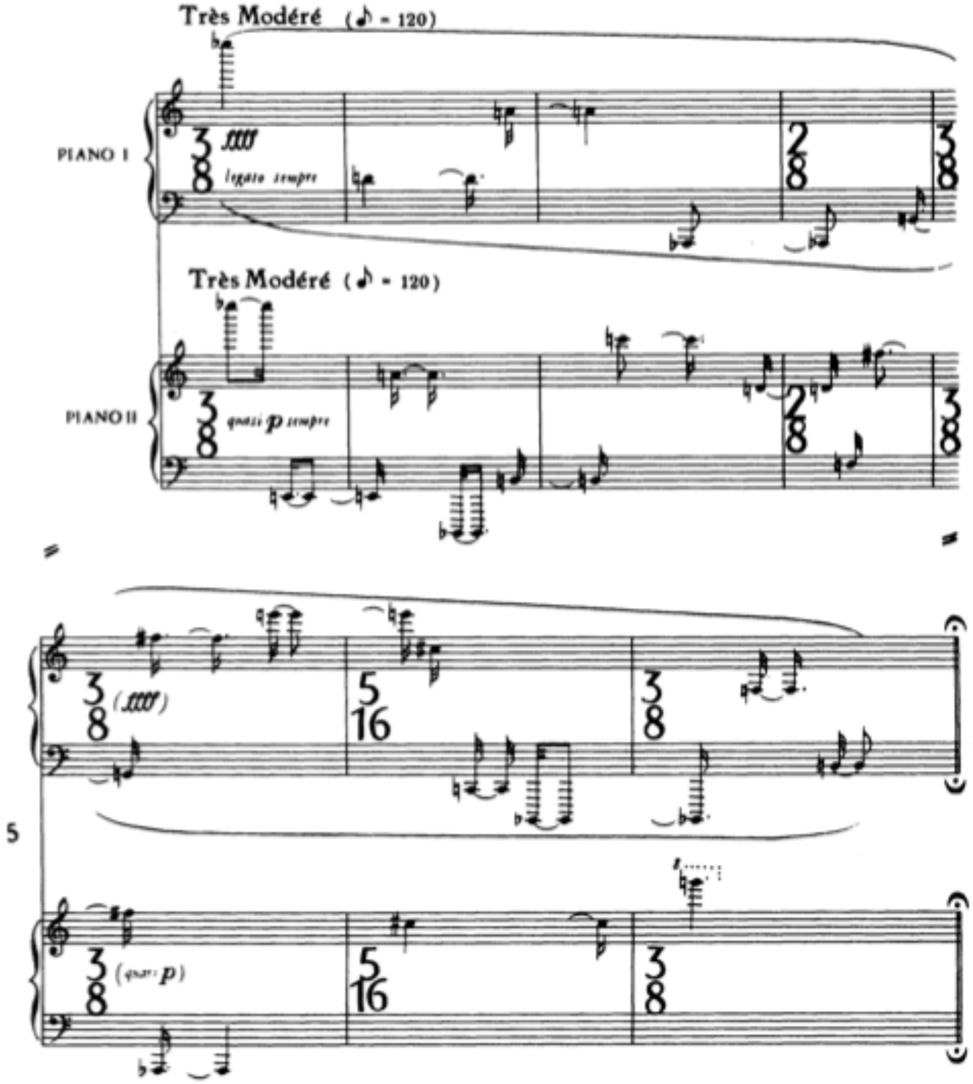


For the first section of the piece, the durations of the notes in Piano I follow the inversion matrix (Matrix B) in retrograde and the durations of the notes in Piano II follow the original matrix (Matrix A) in retrograde. Both piano parts (like with the ordering of the pitches in the second part) start with the bottom row of the matrix and end with the top row. In the opening measures of the work (Figure 8), Piano I's durations follow the bottom line of Matrix B in retrograde [12, 11, 9, 10, 3, 6, 7, 1, 2, 8, 4, 5] and Piano II's durations follow the bottom line of Matrix A in retrograde [5, 8, 6, 4, 3, 9, 2, 1, 7, 11, 10, 12].

In the second part of the movement, Piano I remains with the inversion matrix (Matrix B) and plays each row in Matrix B starting with row 12, then 11, then 9, then 10, 3, 6, 7, 1, 2, 8, 4, and ending with row 5 (the order of these rows is the bottom row in Matrix B in retrograde).

Piano II remains with the original matrix (Matrix A) and each duration follows the rows in Matrix A starting with row 5, then 8, 6, 4, 3, 9, 2, 1, 7, 11, 10, and ending with row 12 (the order of these rows is the bottom row of Matrix A in retrograde). With rhythm, the systematic exploitation of the matricies again determines the succession of durations (the row) and the succession of durational rows throughout the work.

Figure 8. Boulez's *Structures Ia:* mm. 1-7



Très Modéré (♪ = 120)
PIANO I
legato sempre
Très Modéré (♪ = 120)
PIANO II
quasi p sempre
5
(quasi p)

Pierre Boulez "Structures|Premier Livre|für 2 Klaviere"
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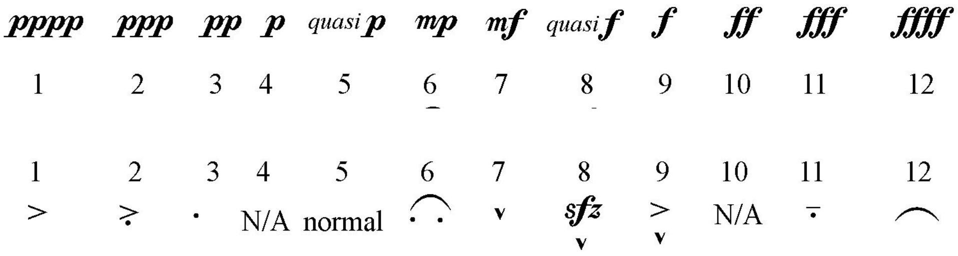
Boulez also serializes dynamics and articulations in *Structures Ia*. While he uses the same pitch and durational series as in Messiaen's *Mode de valeurs et d'intensités,* he deviates from his teacher's dynamic and articulation assignments. Boulez expands his dynamics to include twelve different dynamic markings (though he only uses ten of these in *Structures 1a*) and twelve different articulations (but again, only uses ten of these in the first movement). Figure 9 shows Boulez's arrangement of dynamics and articulations.

Figure 9. Boulez's numerical assignments of dynamics and articulations



Boulez selects his dynamics and articulations from the diagonal lines from the matricies he constructed for pitches and inversion of pitches (see Figure 10).[24] Since the diagonals only contain ten of the twelve integers, only ten dynamics and ten articulations are used throughout the piece. Piano I uses the original matrix and Piano II uses the inversion matrix. Boulez does not assign a different dynamic or articulation for each note. Instead, he employs one dynamic and one articulation per row.[25]

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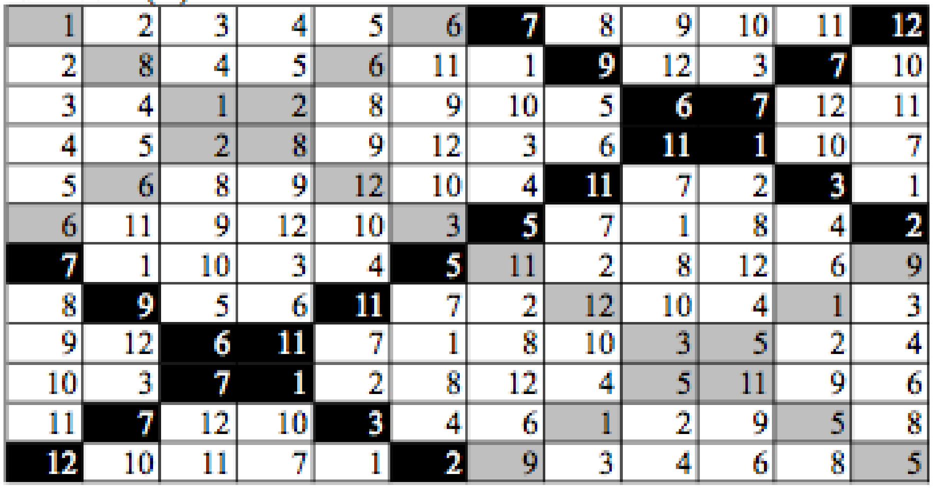
[24] Reginald Smith Brindle, The New Music: the Avant-Garde Since 1945, 2nd ed. (Oxford: Oxford University Press, USA, 1987), 29-30.

[25] Ibid., 30.

Figure 10. Diagonal lines in Boulez's matricies to determine dynamics and articulations

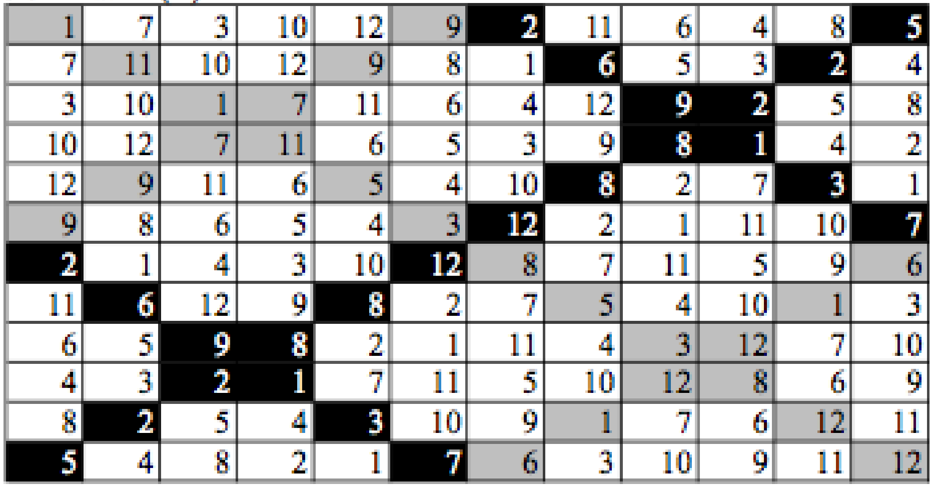
Original Matrix (A)



1	2	3	4	5	6	**7**	8	9	10	11	**12**
2	8	4	5	6	11	1	**9**	12	3	**7**	10
3	4	1	2	8	9	10	5	**6**	**7**	12	11
4	5	2	8	9	12	3	6	**11**	**1**	10	7
5	6	8	9	12	10	4	**11**	7	2	**3**	1
6	11	9	12	10	3	**5**	7	1	8	4	**2**
7	1	10	3	4	**5**	11	2	8	12	6	9
8	**9**	5	6	**11**	7	2	12	10	4	1	3
9	12	**6**	**11**	7	1	8	10	3	5	2	4
10	3	**7**	**1**	2	8	12	4	5	11	9	6
11	**7**	12	10	**3**	4	6	1	2	9	5	8
12	10	11	7	1	**2**	9	3	4	6	8	5

■ = dynamics in Piano I ▒ = articulations in Piano II

Inversion Matrix (B)



1	7	3	10	12	9	**2**	11	6	4	8	**5**
7	11	10	12	9	8	1	**6**	5	3	**2**	4
3	10	1	7	11	6	4	12	**9**	**2**	5	8
10	12	7	11	6	5	3	9	**8**	**1**	4	2
12	9	11	6	5	4	10	**8**	2	7	**3**	1
9	8	6	5	4	3	**12**	2	1	11	10	**7**
2	1	4	3	10	**12**	8	7	11	5	9	6
11	**6**	12	9	**8**	2	7	5	4	10	1	3
6	5	**9**	**8**	2	1	11	4	3	12	7	10
4	3	**2**	**1**	7	11	5	10	12	8	6	9
8	**2**	5	4	**3**	10	9	1	7	6	12	11
5	4	8	2	1	**7**	6	3	10	9	11	12

■ = dynamics in Piano II ▒ = articulations in Piano I

With dynamics, Piano I starts with the diagonal from the lower left corner of the original matrix (Matrix A) to the top right corner. Thus, the first twelve pitch rows of Piano I would have the dynamic designations of 12, 7, 7, 11, 11, 5, 5, 11, 11, 7, 7, 12. After this diagonal, the dynamic numbers start at the bottom center of the matrix and move up and to the left and when reaching the edge of the matrix, continue from the top center and move diagonally downward and to the right. Figure 11 shows the complete dynamic assignments for Piano I and II. Piano II follows a similar pattern using the Inversion Matrix (Matrix B). Boulez does deviate some from this system, and the discrepancies are notated with a * in Figure 11.

Figure 11. Dynamic scheme for Piano I and II in Boulez's *Structures Ia*.

a) Piano I:

ffff	***mf***	***mf***	***fff***	***fff***	*quasi* ***p***	*quasi* ***p***	***fff***	***fff***	***mf***	***mf***	***ffff***
12	7	7	11	11	5	5	11	11*	7	7	12
ppp	***pp***	***pppp***	***mp***	***f***	***mf***	***mf***	***f***	***mp***	***pppp***	***pp***	***ppp***
2	3	1	6	9	7	7	9	6	1	3	2

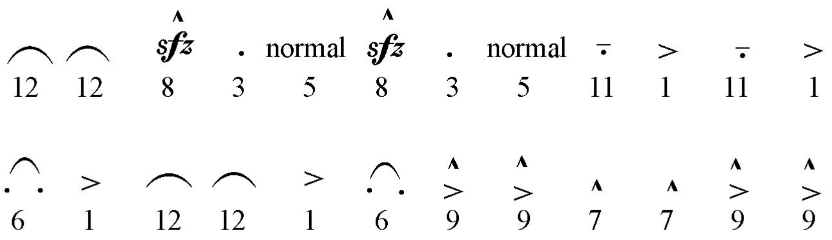
b) Piano II:

quasi ***p***	***ppp***	***ppp***	*quasi* ***f***	*quasi* ***f***	***ffff***	***ffff***	*quasi* ***f***	*quasi* ***f***	***ppp***	***ppp***	*quasi* ***p***
5	2	2	8	8	12*	12*	8	8	2	2	5
mf	***pp***	***pppp***	***f***	***mp***	***ppp***	***ppp***	***mp***	***f***	***pppp***	***pp***	***mf***
7	3	1	9	6	2	2	6	9	1	3	7

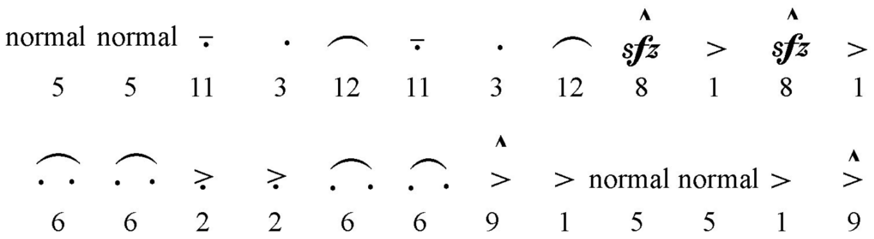
With articulations, Piano I starts in the lower right hand corner of the Inversion Matrix (Matrix B) and moves to the top left corner. The first twelve pitch rows of Piano I would have the articulation designations of 12, 12, 8, 3, 5, 8, 3, 5, 11, 1, 11, 1. After this diagonal, the articulation numbers start at the middle of the right side of the matrix and proceed diagonally downward and to the left. Figure 12 shows the complete articulation assignments for Piano I and II. Piano II follows a similar pattern using the Original Matrix (Matrix A).

Figure 12. Articulation scheme for Piano I and II in Boulez's *Structures Ia*

a) Piano 1:


sfz normal sfz normal
12 12 8 3 5 8 3 5 11 1 11 1
6 1 12 12 1 6 9 9 7 7 9 9

b) Piano II:


normal normal sfz sfz
5 5 11 3 12 11 3 12 8 1 8 1
normal normal
6 6 2 2 6 6 9 1 5 5 1 9

Structures Ia uses a total of 48 rows for both pitches and durations. Each matrix is used twice for each of these parameters. Using his durational system with rhythm, each instance of the row lasts exactly the same length (since it is always the sum of the same twelve durations). Other than Boulez's choice of how many rows to present concurrently, once Boulez has exhausted his system, the piece is over and so his system largely governs the form of the piece as well.

Connections Between Boulez's *Structures Ia* and *Octulus*

Like Reich and Rzewski, Boulez's *Structures Ia* is a work that is developed from very little material. However, unlike Reich and Rzewski, Boulez aspires to control more parameters within his system. After ordering and numbering a single series of pitches, Boulez employed transposition, inversion, retrograde, and retrograde inversion to determine all of the durations and pitches used throughout the movement. Further, Boulez used his two matricies to determine all of the articulations and dynamics.

There are elements that Boulez determines apart from his matricies. Notes seem to be placed in arbitrary registers throughout the entire range of the keyboard. In addition, the number of rows performed simultaneously and the various tempi used throughout the movement seem to be determined independently.

In *Octulus*, I similarly wanted to use a single source to generate pitch and rhythm. Using the Take-Away System, I was able to derive a 40-element cycle that generates almost all of the pitches and much of the rhythmic content throughout the work. Transposition, inversion, retrograde and retrograde inversion also all play a prominent role in the development of the piece.

Unlike Boulez's *Structures,* there are liberties with the durations and pitches in order to provide flexibility outside of the system. *Octulus A* is the most strict in using the Take-Away System on a variety of elements while in

Octulus B and *Octulus D* the Take-Away System is used primarily to control pitch. In *Octulus C,* the Take-Away System is used with both pitches and rhythms. Other musical elements that employ the Take-Away System in *Octulus* include register, form, harmony, and linear density. Unlike Boulez, I did not apply the Take-Away System to dynamics or articulations.

The Philosophy of Systematic Composition

Even though there have been a variety of systems employed by composers, the question arises of *why* some composers turned to a systematic approach to composition. Why would a composer surrender control of one or more aspects of their music to a system?

Steve Reich accepts his limited role as a composer in using a system. He writes that he discovers musical processes and only composes the musical material to run through that process. After the process is set up, he allows the system to run by itself.[26] In fact, he even considers his early gradual phase shifting pieces as processes rather than compositions.[27] He states:

> I saw that my methods [on the gradual phase shifting pieces] did not involve moving from one note to the next, in terms of each note in the piece representing the composer's taste working itself out bit by bit. My music was more of an impersonal process.[28]

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[26] Steve Reich, Writings On Music, 1965-2000, ed. Paul Hillier (New York: Oxford University Press, USA, 2002), 34.

[27] Ibid., 33.

[28] Ibid.

However, Reich would argue that the impersonality of his system actually gives performers more of an opportunity to bring their own emotional life to the piece.[29] In addition, Reich states that his system actually gives him *more* control over his works:

> Musical processes can give one a direct contact with the impersonal and also a kind of complete control and one doesn't always think of the impersonal and complete control as going together. By a "kind" of complete control I mean that by running this material through this process I completely control all that results, but also that I accept all that results without changes.[30]

For Frederic Rzewski, the rationale in using a system might appear more pragmatic with *Les Moutons de Panurge* in the fact that his squaring method allows for the construction of a much larger piece and also provided a solution to the requirements of the indeterminate ensemble for Frans Bruggen.

However, in *Jefferson* (1970) Rzewski states that the rigorous structuralism of the squaring technique counterbalances the otherwise restrained freedom of the vocal line and helps to reflect the sober and careful construction of the text (which is The Declaration of Independence).[31] In this work Rzewski uses his system with a vocal line that is not systematic at all. He can control certain aspects of the piece, but allows his system to control other elements of the work.

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[29] Ibid., 27.

[30] Ibid., 35.

[31] Frederic Rzewski, Nonsequiturs: Writings & Lectures On Improvisation, Composition, and Interpretation = Unlogische Folgerungen: Schriften Und Vorträge Zu Improvisation, Komposition Und Interpretation, 1. Aufl. ed. (Köln: MusikTexte, 2007), 446.

Likewise, John Cage exhibited control in the making of the charts for *Music of Changes* but left the execution of the charts to chance. He states, "One may conclude from this that in the *Music of Changes* the effect of the chance operations on the structure was balanced by a control of the materials."[32] With Rzewski, Reich, and Cage, systematic approaches were combined with elements of chance or intuition.

When Pierre Boulez was composing *Structures Ia* he states that he believes music has entered "a new stage of activity, the serial form."[33] In fact, Boulez's goal in *Structures Ia* was to "generalize the notion of the series itself." He generalizes his organization of the pitch row expanding it to durations, articulations, and dynamics.

Boulez did not continue to write works that exhibit control over so many parameters of music. In his article *Alea*, Boulez writes that every aspect of creative thought should not be automatically created. He states that it should

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[32] John Cage, Silence: Lectures and Writings (Middletown, CT.: Wesleyan, 1961), 24.

[33] Pierre Boulez, The Boulez-Cage Correspondence, ed. Jean-Jacques Nattiez and Robert Samuels (Cambridge: Cambridge University Press, 1993), 99.

"take its place as a particularly effective resource at a certain stage in a works' development,"[34] and additionally remarks:

> If you are only obsessed with organization, then practically you arrive at chaos, because an excess of organization in physics brings chaos. Chaos alone does not bring any order. Therefore, I have to accept the stream, which is an order, and within this stream I must accept the unforeseeable elements, which you cannot control. But I have to make the best use of them that is possible.[35]

Iannis Xenakis comments that the advantage of using probability, algorithms, statistics and logic for a starting point in a composition is that the principles helped to determine logical steps for moving a work forward. In addition, they "were also used to avoid the traditional ways of thinking about music."[36] Xenakis states quite frankly, "I think that my involvement in music, my privilege and also my duty, is to try and do something different. Otherwise, what the hell! The others have done much better than me in the past."[37]

None of these composers applied a system designed to control every aspect of music. Within each of these composer's works, there is always an element of control that is exercised by the composer outside of the system.

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[34] Pierre Boulez, Stocktakings from an Apprenticeship (Oxford: Oxford University Press, USA, 1991), 32.

[35] Rocco Di Pietro, Dialogues with Boulez (Lanham, Md.: Sacrecrow Press, 2001), 98.

[36] Richard Dufallo, Trackings: Composers Speak with Richard Dufallo. (New York: Oxford University Press, 1989), 178.

[37] Ibid., 183.

Boulez writes that composition is the result of constant choice.[38] In my own system, I likewise exert an amount of control over various elements of music. While I have used the system to dictate much of the pitch and rhythmic content of the work, certain decisions were made apart from the system. Like Xenakis, I have used my system to avoid traditional ways of thinking about music.

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[38] Pierre Boulez, Stocktakings from an Apprenticeship (Oxford: Oxford University Press, USA, 1991), 29.

CHAPTER 2

THE TAKE-AWAY SYSTEM

Introduction

My own systematic approach to composition stems from a fascination between order and chaos. In my own system, I have sought a way to allow a composer to organize a certain number of elements, and then employ a series of permutations that rearranges those elements in a way that is defined by the system. The name comes from the concept that each element will be selected one at a time and then "taken-away" from the set. The same operation will be placed upon the set as the elements are taken away one at a time.

The Take-Away System

The Take Away Modulus System is comprised of the following rules:

First, one needs a set with more than one element since a set with only one element cannot be rearranged. The number of elements in the set may be as large as desired. However, for the purposes of functionality, I will be focusing largely on sets with up to twelve elements. Elements in a set should be used once and not repeat.

Second, the modulus of the set will equal the number of elements in the set. Thus, a set with seven elements will be using modulus seven. Mathematically, a modulus can be defined as the number of classes we have broken the integers up into. Very interesting results could occur by using an operation of a different modulus on a set (using modulus seven on a set of five elements). However, these observations are outside the scope of this study.

Third, elements in the set will be arbitrarily notated as [*a*, *b*, *c*, *d*…etc.] and the number of the element as it is arranged in the set will be the primary concern with the operation (*a* is the first element, *b* is the second element, etc.).

Fourth, the operation on the set will be arithmetic and is concerned with the arrangement of the elements. Meaning if I take the set [*a, b, c, d, e, f, g, h*] and add two to the placement of element *c*, I arrive at element *e*. In the same set, if I add two to the seventh element, *g*, I would arrive at the first element since the ninth element = first element mod8 ($g + 2 = a$). Note that the operation has no implication on the value of the element within the set (the value of $a \neq$ the value of *e*, nor does the value of $a + 2$ necessarily equal the value of *c*) since the operation is not concerned with the quantitative differences between elements (*a* is not necessarily greater than or less than *b*).

Fifth, an element is selected by choosing every *n*-th element of a set (where n is the modulus of the set). However, once an element is selected, it is removed from that set and placed in a new set (it is "taken away"). Whatever element is selected first from the original set is now the first element in the new set. Whatever the third element is that is selected from the original set is now the third element of the new set. The operation is then repeated resuming from the exact spot of the element that had just been removed.

Finally, once all elements have been selected and removed from the original set, the new set, the new arrangement of elements can be used as a set to repeat the process all over again.

An Example of the System

Take a set of eight elements. The modulus for this set will be set to correspond directly to the eight elements in the set. Let the eight elements of the set be denoted as: [a, b, c, d, e, f, g, h]. Thus, in the arrangement of our original set *a* is the first element, *b* is the second element, *c* is the third element…and so forth. Following our rule to select every *n*-th element (where *n* is the modulus), we will be selecting every eighth element. Therefore in our original set *h* is the eighth element of the set so it is removed from the original set and now placed as the first element in our new set. Our original set now only has seven elements [a, b, c, d, e, f, g] and we resume exactly where we left off (where the element *h* was). Counting eight elements form this spot in our now original set of seven elements, we end on the first element *a* (wrapping around the set twice). We now remove *a* from our original set and place it as the second element in our new set. Our original set now only has six elements [b, c, d, e, f, g] and our new set contains two elements [h, a].

Repeating the process and counting another eight elements from where we left off with the six remaining elements, we next end up on element *c*. Thus our new set now contains three elements [h, a, c] and our original set is down to five elements [b, d, e, f, g]. If we carry-on in a like manner, our next elements will be *f,* then *e*, then *b,* then *g,* and finally *d*. Our new set is now completely rearranged [h, a, c, f, e, b, g, d] and our original set is now empty.

We may then repeat the Take-Away System on our new set to get another ordering of our set. The second permutation would result [d, h, c, b, e, a, g, h]. If we apply the system again, we then have [f, d, c, a, e, h, g, b]. With two more permutations, the results are [b, f, c, h, e, d, g, a] and then [a, b, c, d, e, f, g, h], which takes back to our original ordering of the elements. The Take-Away System takes an original ordering of elements in a set and rearranges those elements through a number of permutations and eventually returns back to the original ordering.

CHAPTER 3

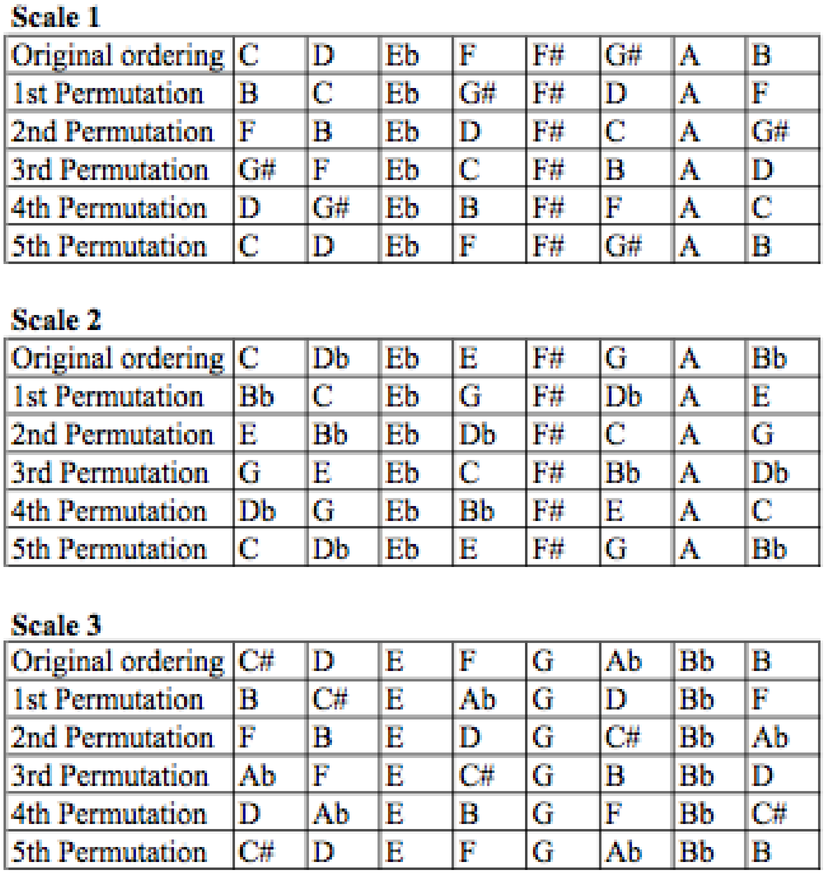
APPLYING THE TAKE-AWAY SYSTEM

Applying the Take-Away System to Pitch

In my own work *Octulus*, I have almost exclusively used a modulus of 8 on an eight-element set with the Take-Away System. I will therefore focus most of the application of the system on modulus 8. Implementing the Take-Away System to notes is perhaps the most intuitive application. Limited to a modulus of 8, I chose a symmetrical octatonic scale with a one-to-one correspondence of each element in my set to a note. Given the cyclic construction of the octatonic scale (altering between whole-steps and half-steps), the octatonic scale afforded the avoidance of favoring any one note (designating a tonic) as well as the opportunity to "modulate" to and from other symmetrical octatonic scales by applying basic transformations on the set.

Beginning with the two octatonic scales starting on *C* (pitch-classes will always be designated with capital letters, while lower-case letters will refer to elements in the set), I could order the sets in ascending order as [C, D, Eb, F, F#, G#, A, B] and [C, Db, Eb, E, F#, G, A, Bb]. I will arbitrarily refer to the preceding scales as Scales 1 and 2. By applying the Take-Away System on each set the first permutation would be [B, C, Eb, G#, F#, D, A, F] and [Bb, C, Eb, G, F#, Db, A, E] respectively. The third octatonic scale (that does not have a '*C*') [C#, D, E, F, G, Ab, Bb, B] would result in the first permutation of [B, C#, E, Ab, G, D, Bb, F] (referred to as Scale 3). Figure 13 shows the complete permutation cycle of the three symmetrical octatonic scales.

Figure 13. Complete permutation cycle of three octatonic scales.



Scale 1

Original ordering	C	D	Eb	F	F#	G#	A	B
1st Permutation	B	C	Eb	G#	F#	D	A	F
2nd Permutation	F	B	Eb	D	F#	C	A	G#
3rd Permutation	G#	F	Eb	C	F#	B	A	D
4th Permutation	D	G#	Eb	B	F#	F	A	C
5th Permutation	C	D	Eb	F	F#	G#	A	B

Scale 2

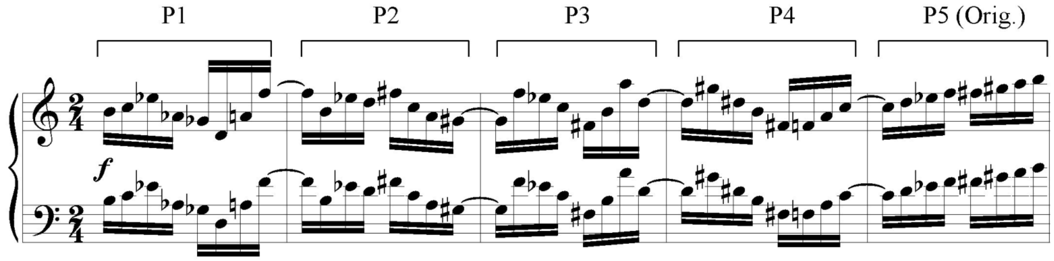
Original ordering	C	Db	Eb	E	F#	G	A	Bb
1st Permutation	Bb	C	Eb	G	F#	Db	A	E
2nd Permutation	E	Bb	Eb	Db	F#	C	A	G
3rd Permutation	G	E	Eb	C	F#	Bb	A	Db
4th Permutation	Db	G	Eb	Bb	F#	E	A	C
5th Permutation	C	Db	Eb	E	F#	G	A	Bb

Scale 3

Original ordering	C#	D	E	F	G	Ab	Bb	B
1st Permutation	B	C#	E	Ab	G	D	Bb	F
2nd Permutation	F	B	E	D	G	C#	Bb	Ab
3rd Permutation	Ab	F	E	C#	G	B	Bb	D
4th Permutation	D	Ab	E	B	G	F	Bb	C#
5th Permutation	C#	D	E	F	G	Ab	Bb	B

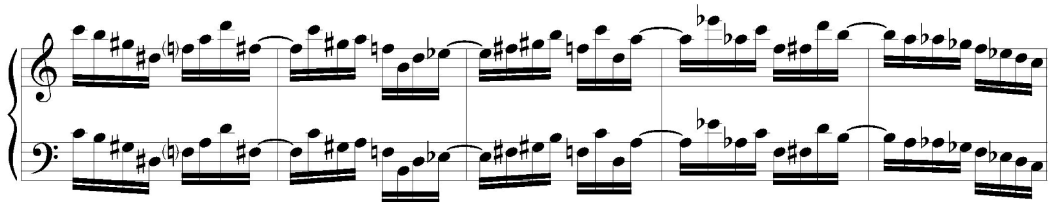
Figure 14 shows a musical rendering of Scale 1 beginning with the first permutation (from *Octulus B* mm. 1-5 in Piano 1). Each note in the cycle receives a sixteenth note with ties over repeated notes to distinguish each permutation. With a 40-note cycle, the duration lasts a total of five measures (in 2/4 time).

Figure 14. *Octulus B* mm. 1-5 (Piano I) showing the five permutations of Scale 1



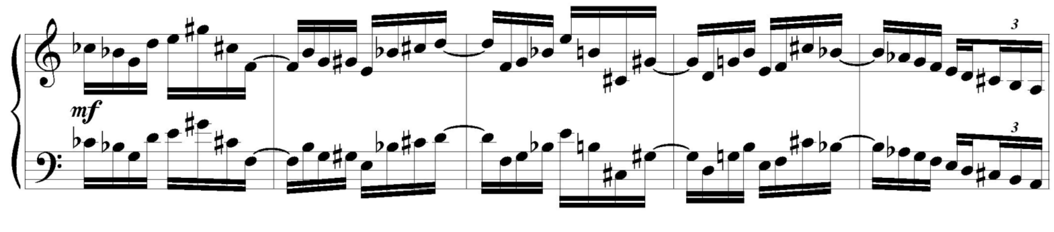
It is then possible to apply transformations upon the entire 40-element cycle such as transposition, retrograde, inversion, and retrograde-inversion. Figure 15 shows a realization of the cycle using Scale 1 inverted and transposed up a half step.

Figure 15. *Octulus B* mm. 21-25 (Piano I)



When an octatonic row is inverted, a different octatonic scale is produced. In the preceding inversion (Figure 15), the original scale was preserved through transposition. However Figure 16 shows how the direct inversion of Scale 1, results in a “modulation” to Scale 3 [C#, D, E, F, G, Ab, Bb, B].

Figure 16. *Octulus B* mm. 114-118 (Piano 2)



One interesting property of the Take-Away System is that the original arrangement of the row will eventually return after a finite number of permutations (see Theorem 4 in Appendix A). So when the original arrangement of the set is assigned to an ascending scale, the presence of ascending and descending scales will exist (see mm. 5, 25, or 118 in Figures 14, 15, or 16).

In all four movements of *Octulus,* almost all of the pitch content is comprised of one of the three scales either using all or part of the permutation cycles directly or using one of the aforementioned transformations on a permutation cycle. In *Octulus C* the permutation cycle is divided into three or four note chords instead of the linear movement seen in *Octulus B* (see Figure 14). Figure 17 shows the permutation cycle of Scale 1 in a simplification of both piano parts. Likewise, the permutation of Scale 2 can be seen in *Octulus D* in the bass line of the chordal progression shown in Figure 18.

Figure 17. *Octulus C* compilation and simplification of mm. 1-13

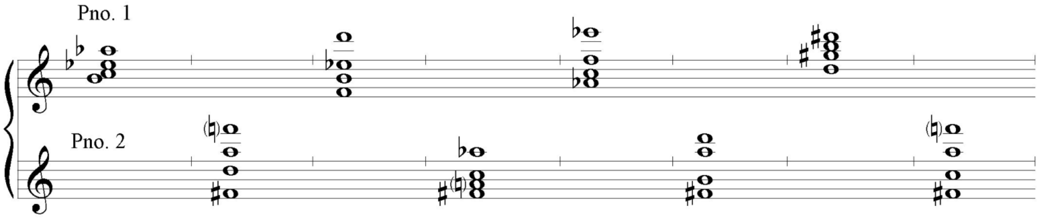
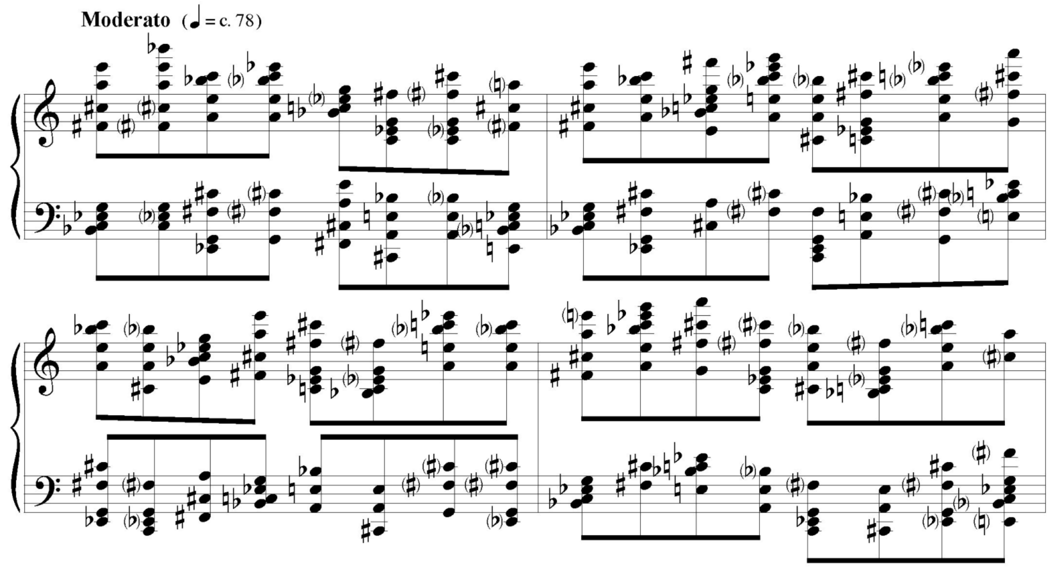


Figure 18. *Octulus D* compilation of mm. 2-5

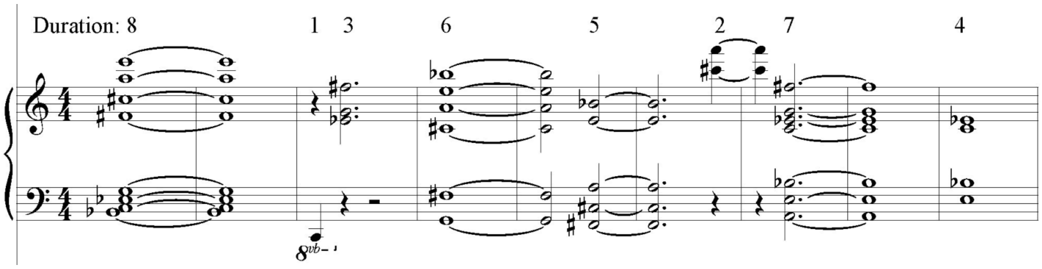


Applying the Take-Away System to Rhythm

In *Octulus* I have mainly applied the Take-Away System to durations of rhythms like Boulez's *Structures Ia*. In using the Take-Away System with rhythm, I have substituted the elements variable names [a, b, c, d, e, f, g, h] with each element's original arrangement number [1, 2, 3, 4, 5, 6, 7, 8]. In *Octulus* I have mainly applied the first permutation of this set [8, 1, 3, 6, 5, 2, 7, 4] to the rhythm.

The opening bars of *Octulus A* reveal how each harmony lasts according to these durations (with the quarter note equaling the beat unit). Figure 19 is a simplification of mm. 1-9 of *Octulus A* illustrating the rhythmic ordering in one grand staff.

Figure 19. *Octulus A* mm. 1-9 (compilation and simplification of both piano parts)



Durations can be altered by using the retrograde of the first permutation (Figure 20), multiplying a constant to each duration to get a similar ratio (Figure 21), or by using the inverted durations (Figure 22). In the latter example, each element in the first permutation of the set is subtracted from the modulus plus one so our first permutation [8, 1, 3, 6, 5, 2, 7, 4] becomes [1, 8, 6, 3, 4, 7, 2, 5]. Figure 23 shows the numerical representation of the permutation cycle as well as the numerical representation of the transformations on the cycle.

Figure 20. *Octulus A* mm. 10-18 (compilation and simplification of both piano parts)

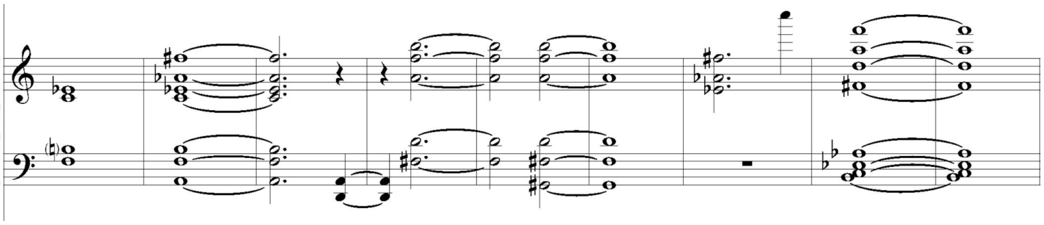


Figure 21. *Octulus A* mm. 37-42 (Piano 2 Left Hand)

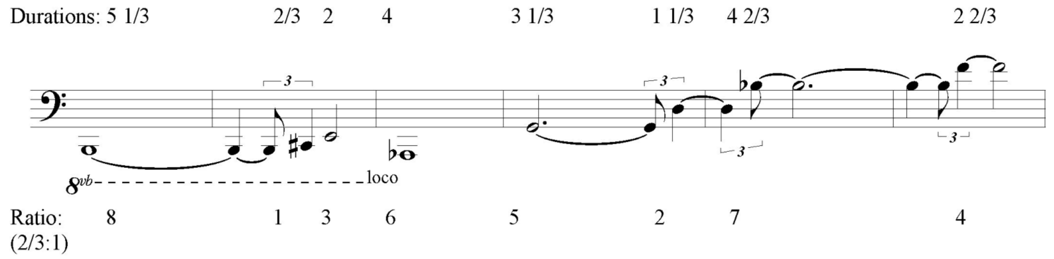

Durations: 5 1/3
2/3
2
4
3 1/3
1 1/3
4 2/3
2 2/3
3
3
3
3
8vb
loco
Ratio:
(2/3:1)
8
1
3
6
5
2
7
4

Figure 22. *Octulus A* mm. 93-97 (Piano 1 Right Hand)

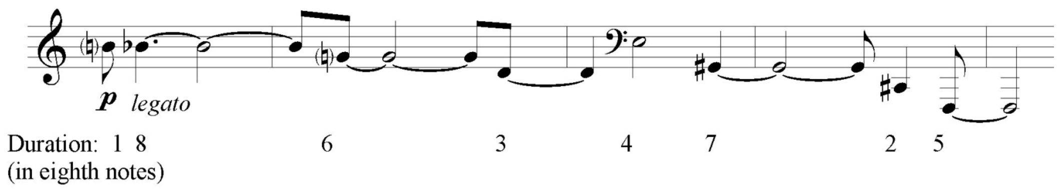
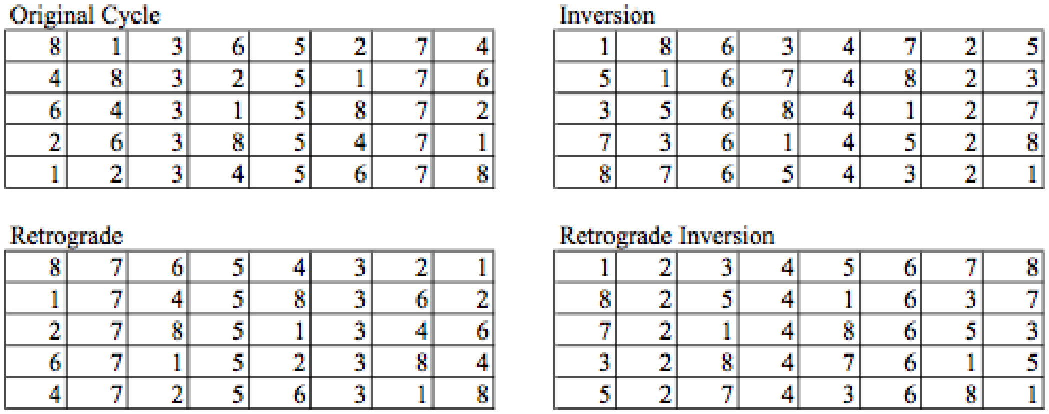

p
legato
Duration:
(in eighth notes)
1
8
6
3
4
7
2
5

Figure 23. Numerical representation of the system (mod 8) with basic transformations



Original Cycle

8	1	3	6	5	2	7	4
4	8	3	2	5	1	7	6
6	4	3	1	5	8	7	2
2	6	3	8	5	4	7	1
1	2	3	4	5	6	7	8

Inversion

1	8	6	3	4	7	2	5
5	1	6	7	4	8	2	3
3	5	6	8	4	1	2	7
7	3	6	1	4	5	2	8
8	7	6	5	4	3	2	1

Retrograde

8	7	6	5	4	3	2	1
1	7	4	5	8	3	6	2
2	7	8	5	1	3	4	6
6	7	1	5	2	3	8	4
4	7	2	5	6	3	1	8

Retrograde Inversion

1	2	3	4	5	6	7	8
8	2	5	4	1	6	3	7
7	2	1	4	8	6	5	3
3	2	8	4	7	6	1	5
5	2	7	4	3	6	8	1

Durational rhythms can be seen in other places throughout *Octulus*. Some other notable examples include *Octulus A* mm. 129-144 (Piano II), *Octulus B* mm. 59-82 and mm. 233-256 (Pianos I & II), and *Octulus C* mm. 14-18, 21-25, 27-32, 34-38 (Pianos I & II).

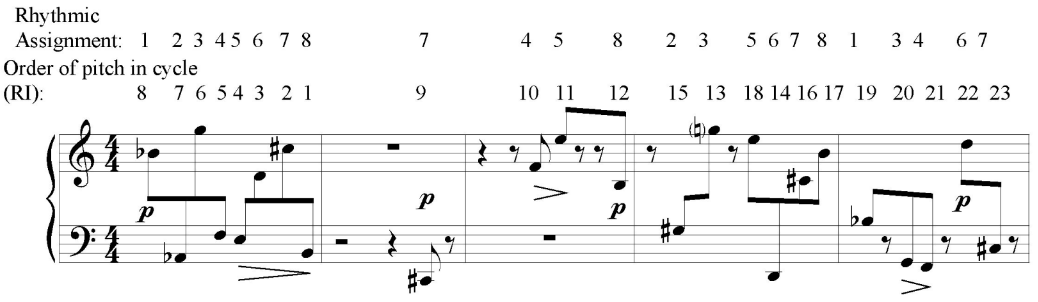
A different systematic approach to rhythm is employed in *Octulus A* mm. 49-72. I have divided each measure into eight parts. In this case, each eighth-note/rest in a 4/4 measure would be numbered one through eight and will be referred to as time points (1-8). A note is assigned to a specific time point in the measure using transformations of the cycle (see Figure 24a). The linear density (number of notes per measure) and register are also based on a different transformation of the cycle and will be discussed later.

Figure 24. *Octulus A* mm. 49-52 (Piano 2)

a) Analytical Chart of *Octulus A* mm. 49-52 (Piano 2)

Density (number of pitches per measure Follows original cycle:	8								1	3			6					
Pitches used Follows Retrograde Inversion of Scale 1	B	C#	D	E	F	G	Ab	Bb	C#	F	E	B	G	D	Ab	C#	B	E
Point in the measure (1-8) Follows the Retrograde	8	7	6	5	4	3	2	1	7	4	5	8	3	6	2	7	8	5
Register Follows the Inversion	1	8	6	3	4	7	2	5	1	6	7	4	8	2	3	5	6	8

b) Annotated rhythms of *Octulus A* mm. 49-53 (Piano 2)


Rhythmic Assignment: 1 2 3 4 5 6 7 8 7 4 5 8 2 3 5 6 7 8 1 3 4 6 7
Order of pitch in cycle (RI): 8 7 6 5 4 3 2 1 9 10 11 12 15 13 18 14 16 17 19 20 21 22 23
p
p
p
p

In Figure 24 (*Octulus A* mm. 49-53) the pitches of Piano 2 are found by the retrograde inversion of the cycle using Scale 1 omitting repeated notes: [B, C#, D, E, F, G, Ab, Bb, C#, F, E, B…]. However, the notes are not quite in this order because rhythmically, they are assigned a time point within the measure using the retrograde of the cycle (again omitting repeated numbers) [8, 7, 6, 5, 4, 3, 2, 1, 7, 4, 5, 8…]. So the first pitch in the set, *B,* occurs on time point eight in measure 49. The second pitch, *C#,* occurs on time point 7, the third note, *D*, on time point 6, etc.

Applying the Take-Away System to Register

Applying the Take-Away System to register for a piece with piano poses the problem of how to divide the eighty-eight keys on the keyboard. In *Octulus A*, subsets or the entire range of the keyboard were divided in equal sections. In mm. 49-56, for example, a four-octave range was used (see Figure 25a). Notes in this section were appointed to a register using a process similar to that used for the rhythmic durations (see Figure 23). In mm. 49-56, if a note had the register designation of a 1 or a 2 it would indicate the lowest octave, 3-4 would be in the second octave, 5-6 the third octave, and 7-8 would place the note in the fourth octave.

Figure 25 Applying the Take-Away System to Register

Figure 25a. Register chart of *Octulus A* mm. 49-56 and realization (mm. 49-53)

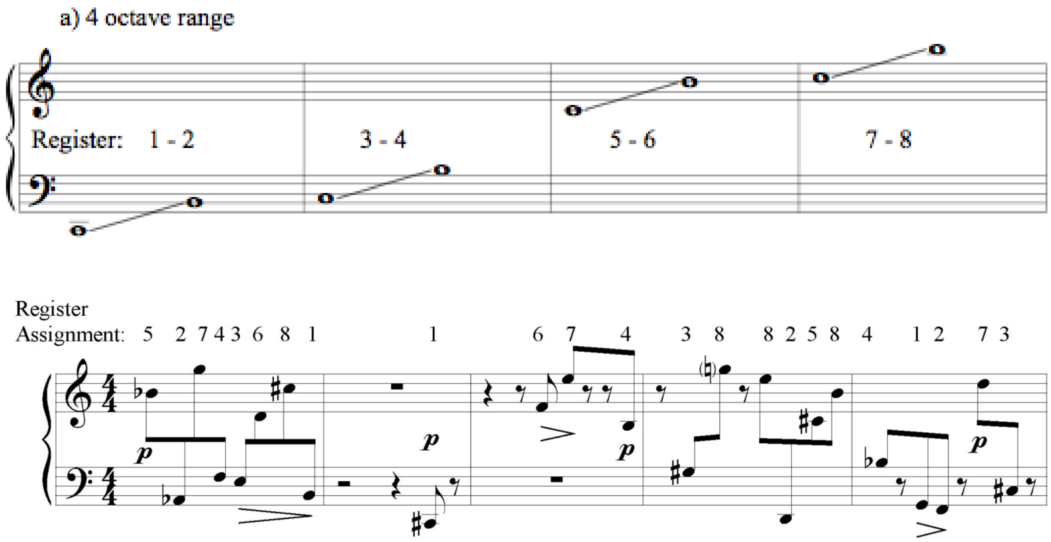

a) 4 octave range
Register: 1 - 2
3 - 4
5 - 6
7 - 8
Register
Assignment: 5 2 7 4 3 6 8 1 1 6 7 4 3 8 8 2 5 8 4 1 2 7 3
p
p
p
p

Figure 25b. Register chart *Octulus A* mm. 57-64 and realization (mm. 57-61)

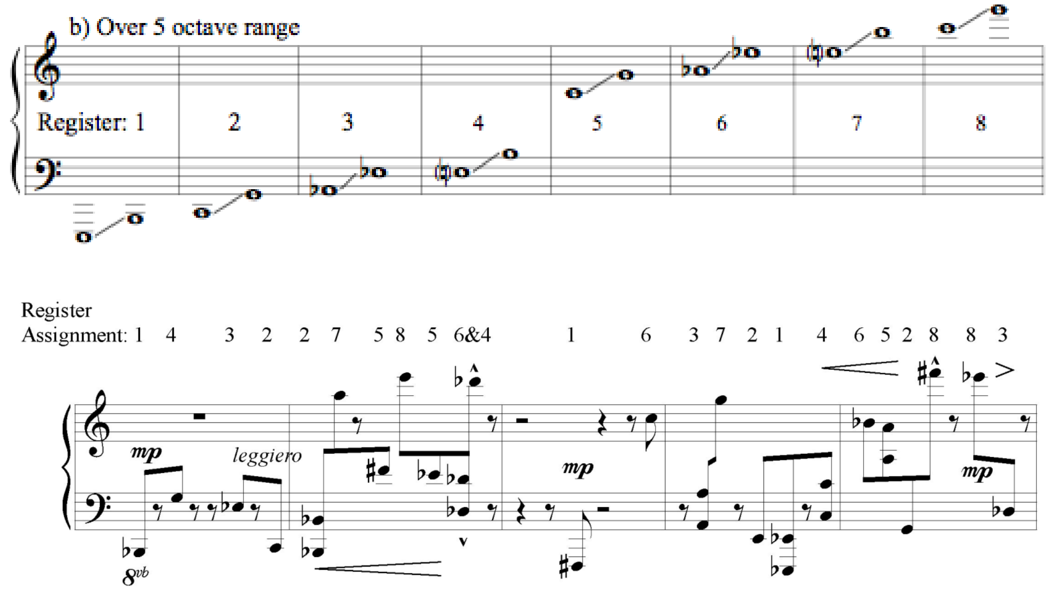
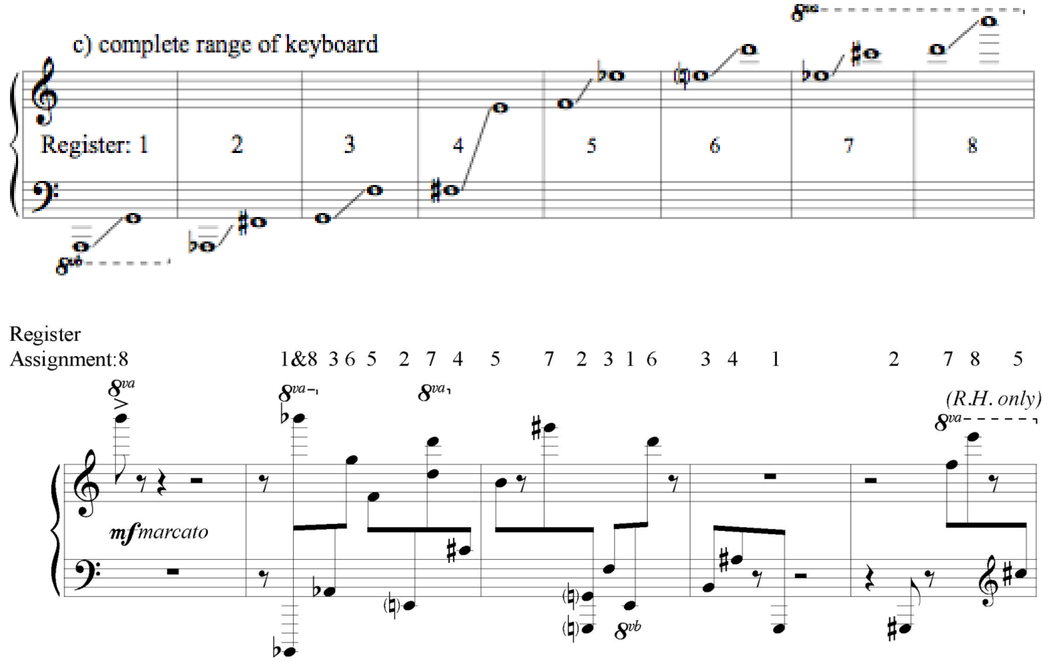

b) Over 5 octave range
Register: 1
2
3
4
5
6
7
8
Register
Assignment: 1 4 3 2 2 7 5 8 5 6&4 1 6 3 7 2 1 4 6 5 2 8 8 3
mp
leggiero
mp
mp
8vb

Figure 25c. Register chart *Octulus A* mm. 65-72 and realization (mm. 65-69)


c) complete range of keyboard
Register: 1
2
3
4
5
6
7
8
8va
8vb
Register Assignment:8
1&8 3 6 5 2 7 4 5 7 2 3 1 6 3 4 1 2 7 8 5
8va
8va
8va
(R.H. only)
8va
mf marcato
8vb

In mm. 57-64, this range was expanded to over 5 octaves (Figure 25b). In these measures, each register had the span of a perfect fifth and the register number corresponded to each fifth (1-8). If a note were assigned to a register in which it did not exist (for example if the range of the register was the perfect fifth between *C*-*G* and the note assigned to that register was a *B*), both appropriate notes above and below that register would be used (both *B*'s above and below the fifth would be played simultaneously).

In mm. 65-72, the entire range of the keyboard was utilized. With an expanded range of 88 notes, the keyboard could be divided into 8 ten-note sections (Figure 25c). Like the preceding eight measures, if a note was designated to a register in which it did not exist, the appropriate notes directly above and below that register would be used.

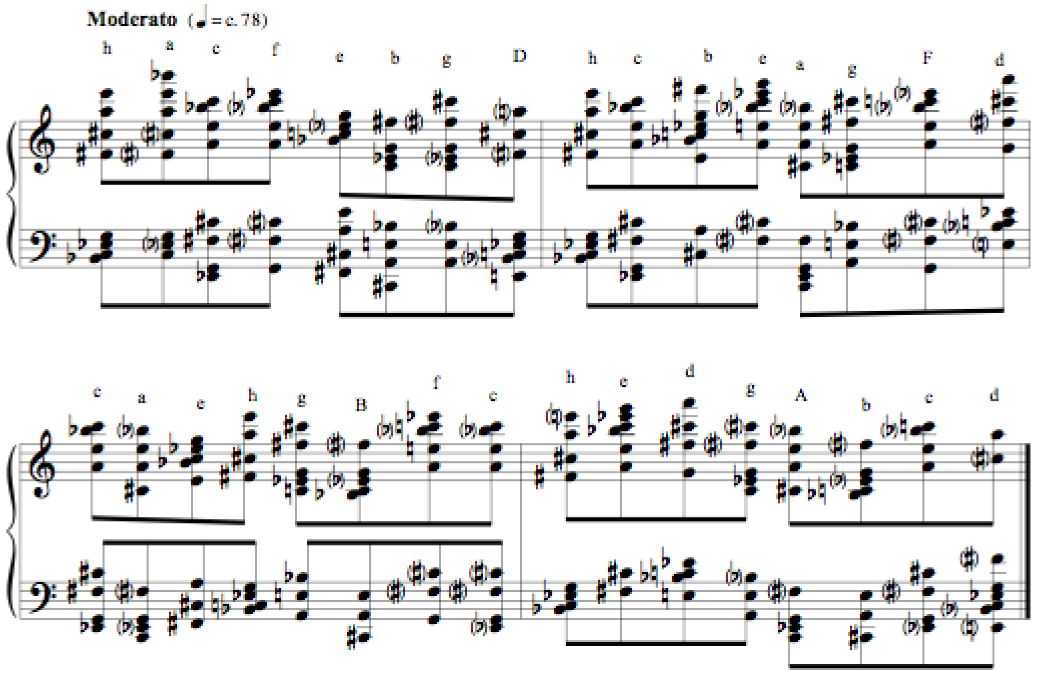
Applying the Take-Away System to Form

The Take-Away System could be applied to form by using the first permutation of the set as a series of ratios. In *Octulus A*, the eight formal structures are 32 measures, 4 measures, 12 measures, 24 measures, 20 measures, 8 measures, 28 measures, and 20 measures (multiplying each element of the set [8, 1, 3, 6, 5, 2, 7, 4] by four). An arch form is employed with the style in these sections in *Octulus A*, as retrograde plays a large role in the pitch construction.

Applying the Take-Away System to Harmonies and Density

Using the eight notes of the first permutation with any of the three octatonic scales as a bass line, an 8-note, 8-chord harmonic progression was created by using a rotational system within the first permutation. Notes within each chord were ordered in ascending order. This technique is seen with Scale 2 in mm. 2-10 of *Octulus D* (Figure 26 shows mm. 2-5 condensed onto one piano). In Figure 26, the letters above each chord refer to the element in the cycle that corresponds to the bass note. Capital letters refer to the beginning of a new permutation.

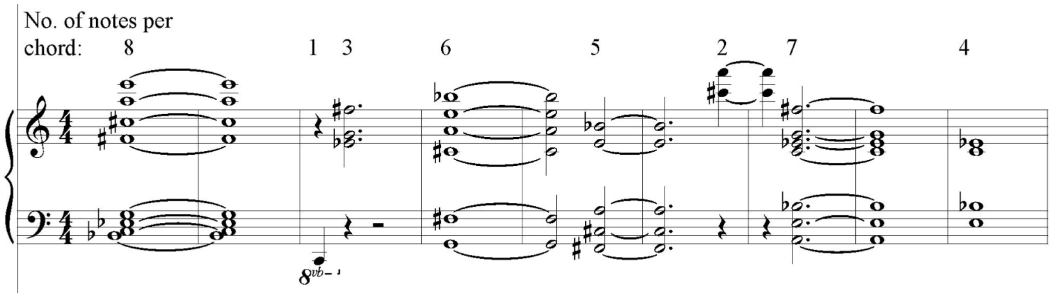
Figure 26. *Octulus D* mm. 2-5 (compilation of Pianos I & 2


Moderato (♩ = c. 78)
h a c f e b g D h c b e a g F d
c a e h g B f e h e d g A b c d

The first permutation of Scale 2 is [Bb, C, Eb, G, F#, C#, A, E] and so the first chord would have *Bb* in the bass and then the chord would stack the nearest *C* on top of the *Bb*, then closest ascending *Eb* above the *C*, then the closest ascending *G* above the *Eb*…and so on until the last and highest note of the set, *E*. The next chord would start with a *C* (the second element in the cycle) and repeat the process. However, upon reaching the last note in the eight-note set, *E*, the set again arrives on a *Bb* for the highest note of the chord. This process is continued with the bass line transcending the first permutation and playing all 40 notes of the cycle of the system. To avoid duplicating any pitches in any of the eight-note chords, only the first eight chords are used. Thus, only eight harmonies are employed but the order of these chords changes with the different permutations of the bass line. I have omitted replicating chords where note repetition occurs in-between permutations. The bulk of the chordal progressions in *Octulus D*, are built on eight chords that have a 35-note cycle (40 minus the five duplicates).

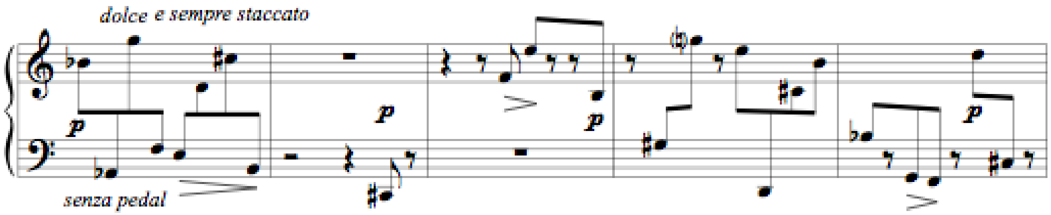
In *Octulus A* mm. 1-9 (Figure 27) the number of notes in each chord is also dependent on the system. The chords still follow the rotational idea already mentioned over the first permutation, but the number of notes in each chord is dictated by the first permutation set [8, 1, 3, 6, 5, 2, 7, 4]. The chords are again stacked in order (from the bass note up) but only the specified number of notes is used per chord.

Figure 27. *Octulus A* mm. 1-9



The Take-Away System can also be used to organize linear density. In *Octulus A* mm. 49-72 all or part of the permutation cycle and its transformations (inversion, retrograde, and retrograde inversion) are used not only with pitches, rhythms and register for each note as previously discussed, (see Figure 24) but also in the number of notes of the cycle in each measure. For example, Figure 28 illustrates that Piano 2 uses eight notes in m. 49, one note in m. 50, three notes in m. 51, six notes in m. 52…(following the first permutation [8, 1, 3, 6, 5, 2, 7, 4]).

Figure 28. *Octulus A* mm. 49-53 (Piano 2)



Conclusions

The Take-Away System can control any element of music. With a finite number of permutations, a set can be ordered and then rearranged in ways that are systematically derived, thus combining facets of both order and chaos. Since the system changes with each modulus, two pieces using the Take-Away System with different moduli would have a unique harmony and sound. Even using the same modulus, different applications of the system could be applied giving two pieces using the same system a different sound.

In *Octulus* I sought to explore four different applications of the Take-Away System in an effort to show the versatility within the system and to explore its many facets. While a symmetrical scale was used in *Octulus,* any type of scale with any number of notes, or in any intonational system could be used. The Take-Away System lends itself to using the time-honored ideas of inversion, retrograde, retrograde-inversion, and transposition. Further, symmetrical scales can invert to a reordered and transposed scale thus producing the idea of a modulation.

Iannis Xenakis states, “After a while, when you have thought a lot and gotten used to the theoretical means you have developed, it becomes second nature to you. You don’t even need to calculate.”[39] It is my goal to work more with the Take-Away System to become even more familiar with the many opportunities the system affords. For this reason, Appendix A outlines some observations and theorems regarding the Take-Away System that I discovered as I composed *Octulus* and worked with the system. By exploring some of its more interesting mathematical traits, I hope to become better versed in the system and to gain the familiarity that Xenakis describes.

Some composers who have used a systematic approach to their compositions change or adapt their systems over time. Steve Reich comments on his departure from his system, “*Clapping Music* (1972) marks the end of my use of the gradual phase shifting process . . . I felt the need to find new techniques.”[40]

Boulez’s attitude also changed. He accepted more of the chaotic and unforeseen elements in life. He states, “For me the fundamental fact of life is deterministic with a lot of aleatoric events that one sorts through. So life is not black or white; it’s gray, and if gray is condensed, all the small dots turn to black;

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[39] Richard Dufallo, Trackings: Composers Speak with Richard Dufallo. (New York: Oxford University Press, 1989), 183.

[40] Steve Reich, Writings On Music, 1965-2000, ed. Paul Hillier (New York: Oxford University Press, USA, 2002), 68.

and if it's more dispersed, then it turns to white. And that's my philosophy of life."[41]

Historically, composers have used systematic approaches for a variety of reasons: for pragmatic reasons, balance, control, or a conviction to explore new musical options. The Take-Away System provides an appealing method for composing in its ability to combine order and chaos, its finite number of permutations, its ability to regulate any musical parameter and its flexibility to be applied to sets containing any number of elements. I believe that there are many exciting musical possibilities with the Take-Away System and that it is worthy of further exploration and analysis.

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[41] Rocco Di Pietro, Dialogues with Boulez (Lanham, Md.: Sacrecrow Press, 2001), 98.

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APPENDIX A

THEOREMS WITH THE TAKE-AWAY SYSTEM

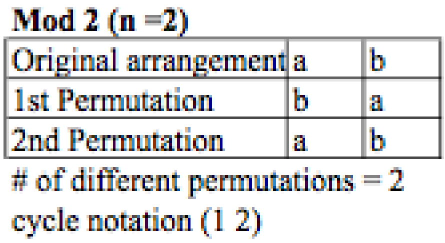
Theorem 1: Given a set with *n* elements with the arrangement [a, b, . . . n]. The first element selected for the new set will always be *n* (1 maps to *n*). Regardless of the modulus, since we are always selecting the *n*-th element and starting with the first element *(a)*, the *n*-th element will always equal *n*. The term mapping refers to how the element's arrangement will change in each permutation using cycle notation. Permutation can be thought of as the change from the original arrangement of the set to a completely new arrangement (the order of each set remaining constant). An iteration would refer to one instance of applying the modulus (moving *n* elements in a set).

Theorem 2. The second element chosen will always be the first element in the original set (2 maps to 1). Take a set with *n* elements with the arrangement [a, b, . . . n]. Since the first element selected will be *n*, after the first iteration, our original set will be [a, b, . . . n-1]. Since we resume from where we have left off, our second iteration will begin on element *a*. Moving *n* spots in a set with *n*-1 elements will take us back to the first element, or *a*.

Theorem 3. The third element chosen in a set with three or more elements will always be the third element in the set (3 maps to 3). Take a set with *n* elements, with $n \geq 3$ and arranged [a, b, . . . n]. If we take our prior two observations, we can note that after two iterations, our original set will be [b . . . n – 1] and our new set will be [n, a]. Since we are resuming the third iteration from where we left off in the second iteration, we will begin with element *b*. Moving *n* elements from *b* in a set with ($n - 2$) elements will move us one element forward to ($b + 1$) elements, which equals element *c*.

Theorem 4. The number of permutations on a set (or the number of different arrangements of a set) with the Take-Away method is always finite. The Take Away system always yields a cyclical permutation. In Figure 30, I have worked out the system for modulus two to twelve. With each different modulus, I have provided the cycle notation (where each element maps) as well as the number of permutations that are needed to reorder the set.

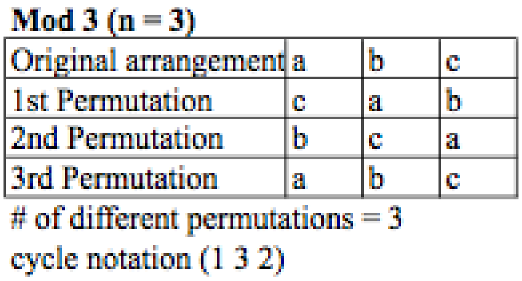
Figure 29. Take-Away System using modulus two to twelve



Mod 2 (n =2)

Original arrangement	a	b
1st Permutation	b	a
2nd Permutation	a	b

# of different permutations = 2
cycle notation (1 2)

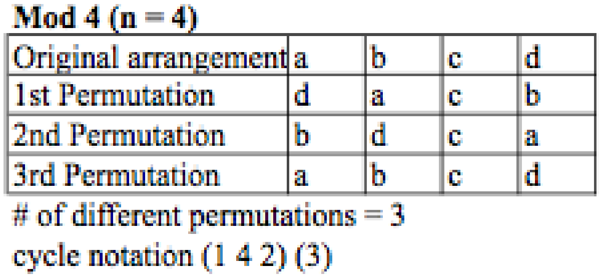


Mod 3 (n = 3)

Original arrangement	a	b	c
1st Permutation	c	a	b
2nd Permutation	b	c	a
3rd Permutation	a	b	c

# of different permutations = 3
cycle notation (1 3 2)

Figure 29. (cont.)

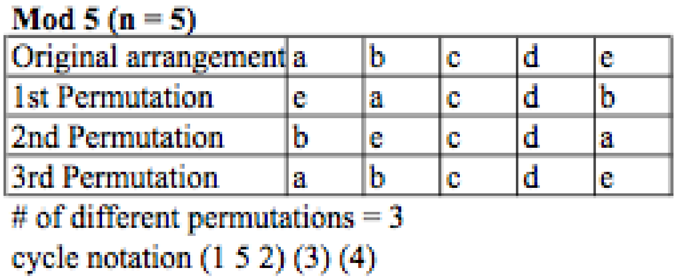


Mod 4 (n = 4)

Original arrangement	a	b	c	d
1st Permutation	d	a	c	b
2nd Permutation	b	d	c	a
3rd Permutation	a	b	c	d

# of different permutations = 3

cycle notation (1 4 2) (3)

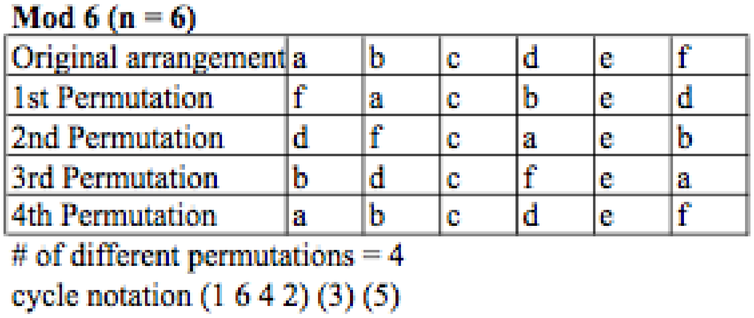


Mod 5 (n = 5)

Original arrangement	a	b	c	d	e
1st Permutation	e	a	c	d	b
2nd Permutation	b	e	c	d	a
3rd Permutation	a	b	c	d	e

# of different permutations = 3

cycle notation (1 5 2) (3) (4)

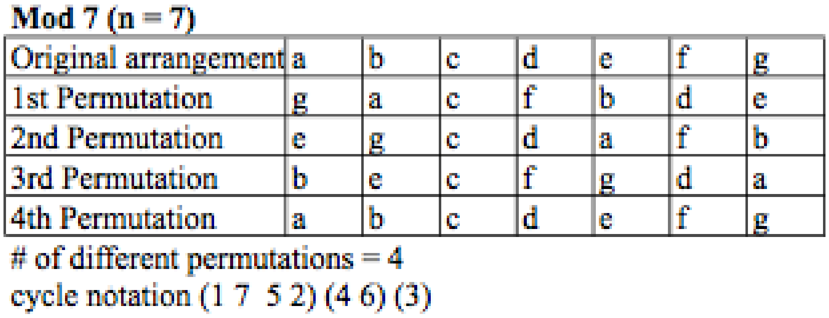


Mod 6 (n = 6)

Original arrangement	a	b	c	d	e	f
1st Permutation	f	a	c	b	e	d
2nd Permutation	d	f	c	a	e	b
3rd Permutation	b	d	c	f	e	a
4th Permutation	a	b	c	d	e	f

# of different permutations = 4

cycle notation (1 6 4 2) (3) (5)



Mod 7 (n = 7)

Original arrangement	a	b	c	d	e	f	g
1st Permutation	g	a	c	f	b	d	e
2nd Permutation	e	g	c	d	a	f	b
3rd Permutation	b	e	c	f	g	d	a
4th Permutation	a	b	c	d	e	f	g

# of different permutations = 4

cycle notation (1 7 5 2) (4 6) (3)

Figure 29. (cont.)

Mod 8 (n = 8)

Original arrangement	a	b	c	d	e	f	g	h
1st Permutation	h	a	c	f	e	b	g	d
2nd Permutation	d	h	c	b	e	a	g	f
3rd Permutation	f	d	c	a	e	h	g	b
4th Permutation	b	f	c	h	e	d	g	a
5th Permutation	a	b	c	d	e	f	g	h

# of different permutations = 5
cycle notation (1 8 4 6 2) (3) (5) (7)

Mod 9 (n = 9)

Original arrangement	a	b	c	d	e	f	g	h	i
1st Permutation	i	a	c	f	d	e	b	g	h
2nd Permutation	h	i	c	e	f	d	a	b	g
3rd Permutation	g	h	c	d	e	f	i	a	b
4th Permutation	b	g	c	f	d	e	h	i	a
5th Permutation	a	b	c	e	f	d	g	h	i
6th Permutation	i	a	c	d	e	f	b	g	h
7th Permutation	h	i	c	f	d	e	a	b	g
8th Permutation	g	h	c	e	f	d	i	a	b
9th Permutation	b	g	c	d	e	f	h	i	a
10th Permutation	a	b	c	f	d	e	g	h	i
11th Permutation	i	a	c	e	f	d	b	g	h
12th Permutation	h	i	c	d	e	f	a	b	g
13th Permutation	g	h	c	f	d	e	i	a	b
14th Permutation	b	g	c	e	f	d	h	i	a
15th Permutation	a	b	c	d	e	f	g	h	i

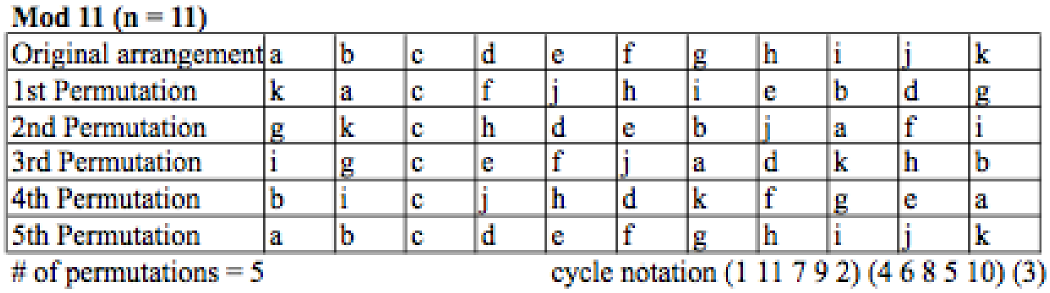
# of different permutations = 15
cycle notation (1 9 8 7 2) (4 6 5) (3)

Mod 10 (n = 10)

Original arrangement	a	b	c	d	e	f	g	h	i	j
1st Permutation	j	a	c	f	b	i	e	g	d	h
2nd Permutation	h	j	c	i	a	d	b	e	f	g
3rd Permutation	g	h	c	d	j	f	a	b	i	e
4th Permutation	e	g	c	f	h	i	j	a	b	d
5th Permutation	b	e	c	i	g	d	h	j	f	a
6th Permutation	a	b	c	d	e	f	g	h	i	j

# of permutations = 6 cycle notation (1 10 8 7 5 2) (4 6 9) (3)

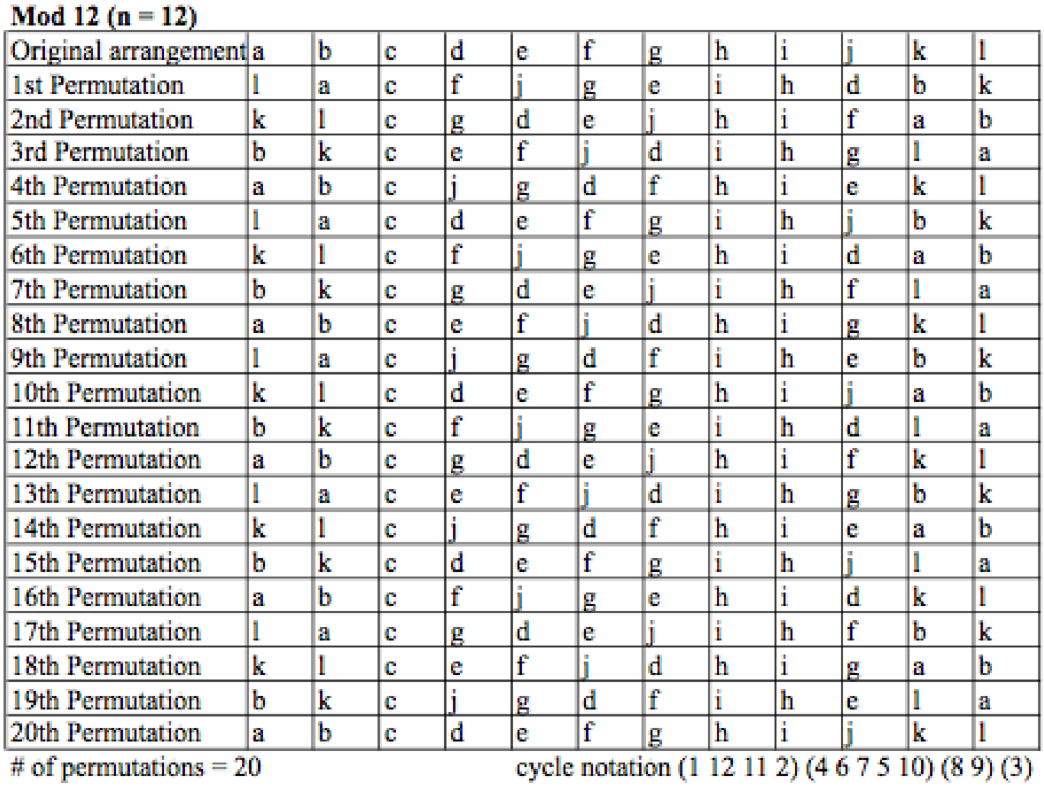
Figure 29. (cont.)



Mod 11 (n = 11)

Original arrangement	a	b	c	d	e	f	g	h	i	j	k
1st Permutation	k	a	c	f	j	h	i	e	b	d	g
2nd Permutation	g	k	c	h	d	e	b	j	a	f	i
3rd Permutation	i	g	c	e	f	j	a	d	k	h	b
4th Permutation	b	i	c	j	h	d	k	f	g	e	a
5th Permutation	a	b	c	d	e	f	g	h	i	j	k

# of permutations = 5 cycle notation (1 11 7 9 2) (4 6 8 5 10) (3)



Mod 12 (n = 12)

Original arrangement	a	b	c	d	e	f	g	h	i	j	k	l
1st Permutation	l	a	c	f	j	g	e	i	h	d	b	k
2nd Permutation	k	l	c	g	d	e	j	h	i	f	a	b
3rd Permutation	b	k	c	e	f	j	d	i	h	g	l	a
4th Permutation	a	b	c	j	g	d	f	h	i	e	k	l
5th Permutation	l	a	c	d	e	f	g	i	h	j	b	k
6th Permutation	k	l	c	f	j	g	e	h	i	d	a	b
7th Permutation	b	k	c	g	d	e	j	i	h	f	l	a
8th Permutation	a	b	c	e	f	j	d	h	i	g	k	l
9th Permutation	l	a	c	j	g	d	f	i	h	e	b	k
10th Permutation	k	l	c	d	e	f	g	h	i	j	a	b
11th Permutation	b	k	c	f	j	g	e	i	h	d	l	a
12th Permutation	a	b	c	g	d	e	j	h	i	f	k	l
13th Permutation	l	a	c	e	f	j	d	i	h	g	b	k
14th Permutation	k	l	c	j	g	d	f	h	i	e	a	b
15th Permutation	b	k	c	d	e	f	g	i	h	j	l	a
16th Permutation	a	b	c	f	j	g	e	h	i	d	k	l
17th Permutation	l	a	c	g	d	e	j	i	h	f	b	k
18th Permutation	k	l	c	e	f	j	d	h	i	g	a	b
19th Permutation	b	k	c	j	g	d	f	i	h	e	l	a
20th Permutation	a	b	c	d	e	f	g	h	i	j	k	l

# of permutations = 20 cycle notation (1 12 11 2) (4 6 7 5 10) (8 9) (3)

Theorem 5. When n > 6, the sixth element in the original set will become the fourth element in the new set (6 maps to 4 when n > 6). Given the set [a, b, c, d, e . . . n] we know that the first three elements selected in the Take-Away System will be *n, a,* and *c* respectively. Thus, after three iterations our original set will be [b, d, e, . . . n-1]. Resuming from where element *c* was in the original set and adding *n* places, we will end on element *f* (or three after our third element in the original system), since the order of our original set is now (*n* – 3). Further, carrying on with the same logic, we can state **(Theorem 6) when n > 10, the tenth element in the original set will become the fifth element in the new set (10 maps to 5 when n > 10); and when n > 15, the fifteenth element will map to the sixth element (15 maps to 6 when n > 15).** One can begin to see a pattern of the first few elements in the order they are chosen when *n* reaches a large enough number: [*n, a, c, f, j, o*....]. If we replace the letters with the number in the arrangement: [*n,* 1, 3, 6, 10, 15….]. The first elements would follow the pattern [n, 1, 3 (1+2), 6 (3+3), 10 (6+4), 15 (10+5), 21 (15+6)…]. These numbers after the second element are found by taking the previous element and adding to it the number two less than its arrangement order. Note that the value of *n* must increase for each added known element.

APPENDIX B

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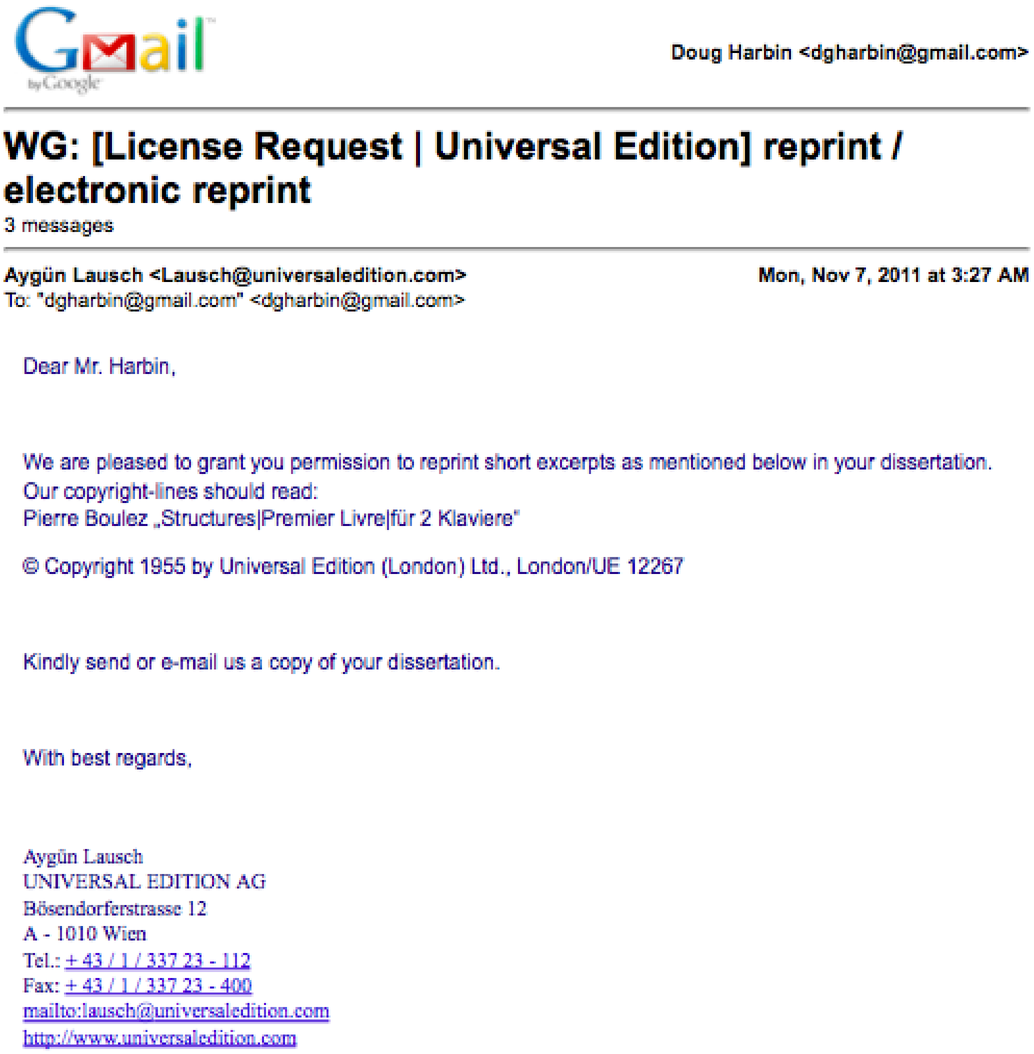

Doug Harbin <dgharbin@gmail.com>

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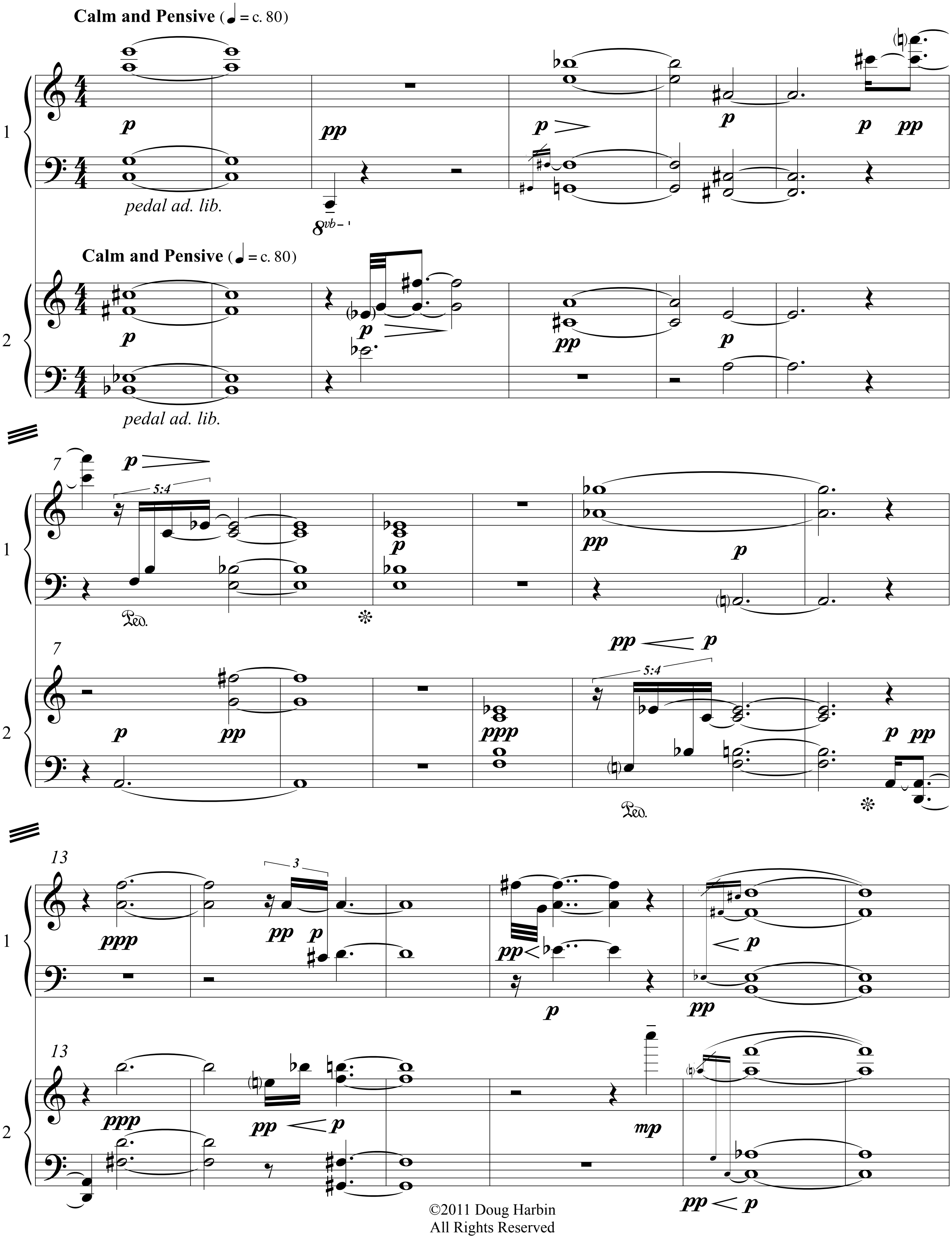
APPENDIX C

OCTULUS

Octulus A

Doug Harbin


Calm and Pensive (♩ = c. 80)
1
2
pedal ad. lib.
p
pp
mp
ppp
8vb
5:4
3
7
13
Ped.
*

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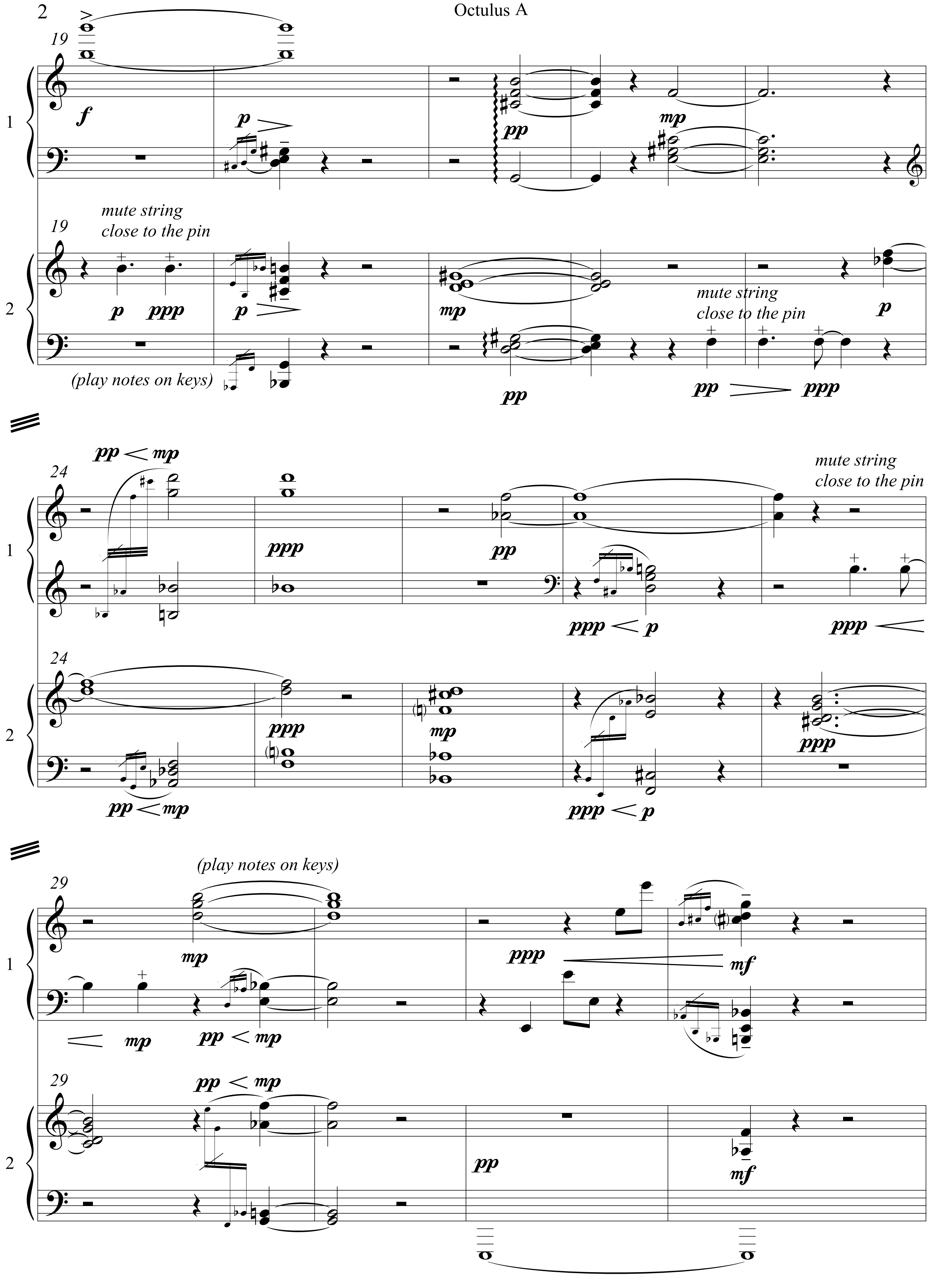
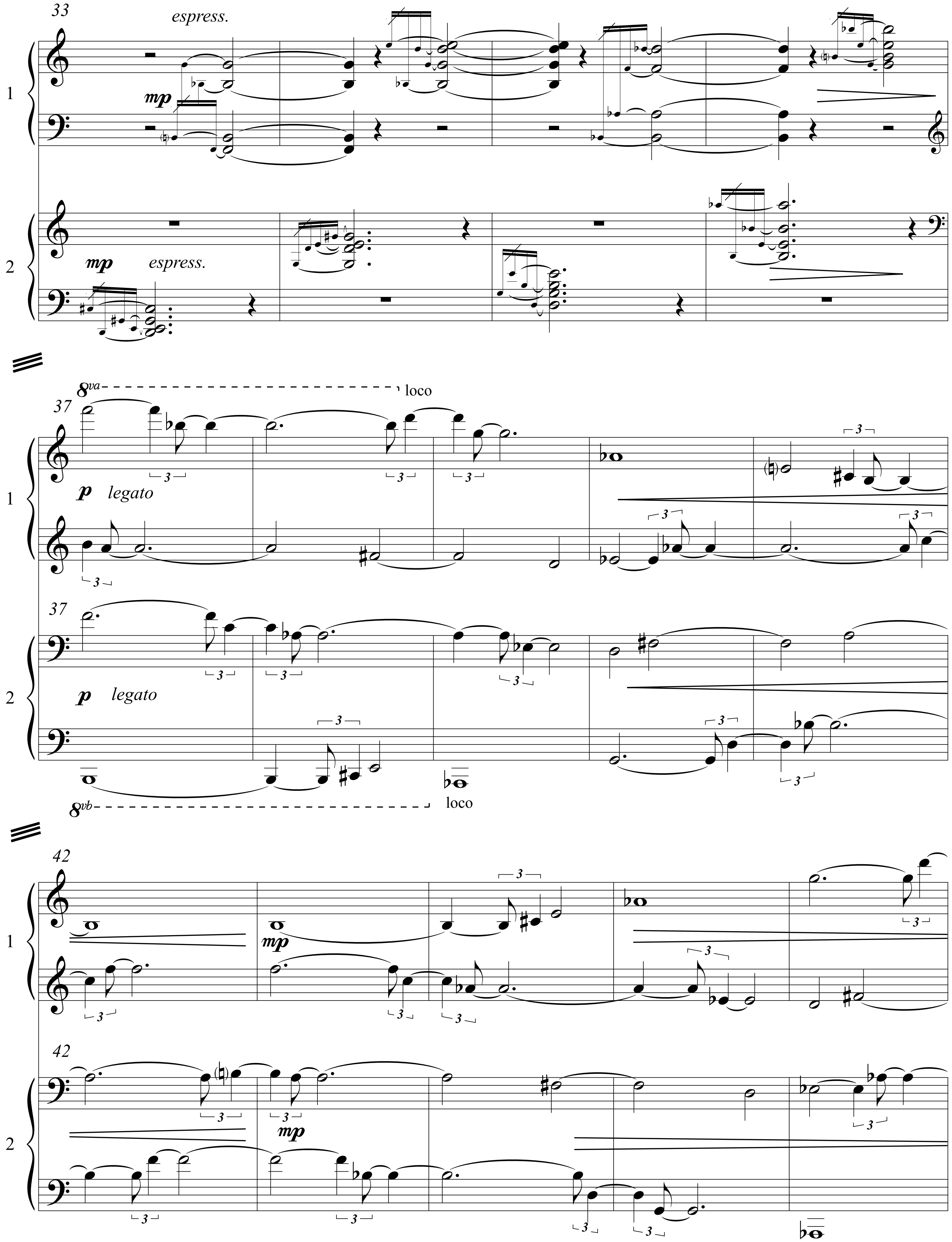
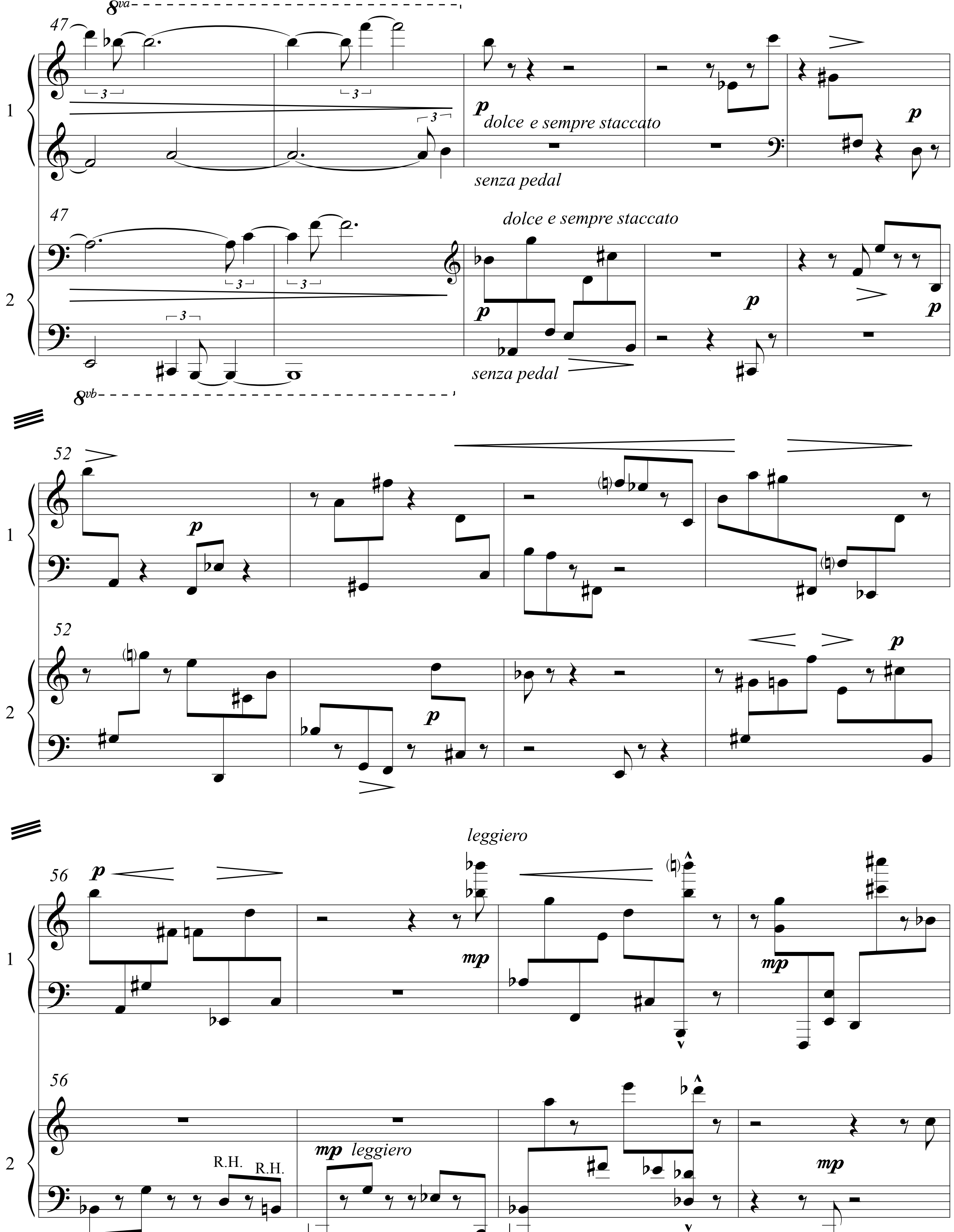
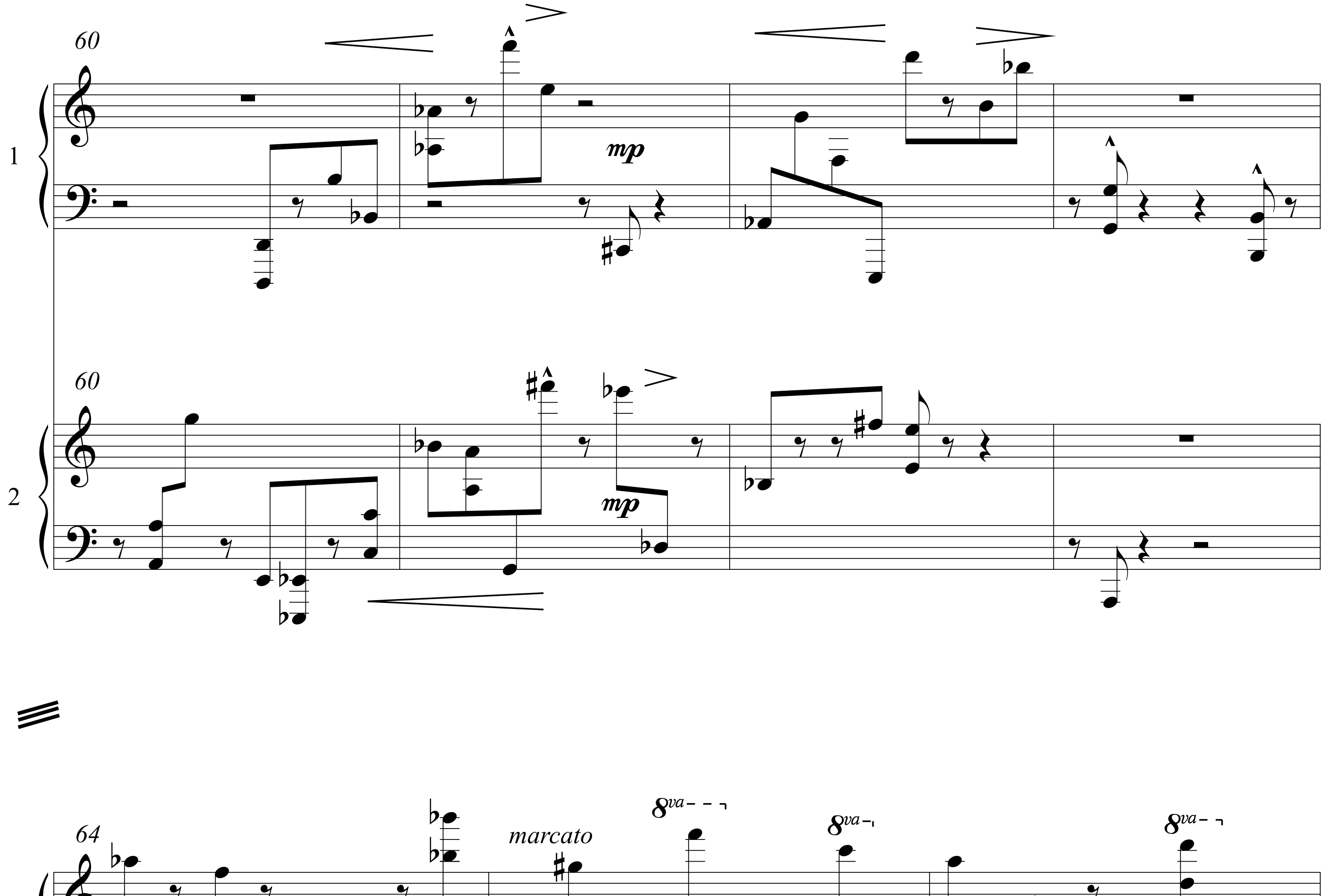
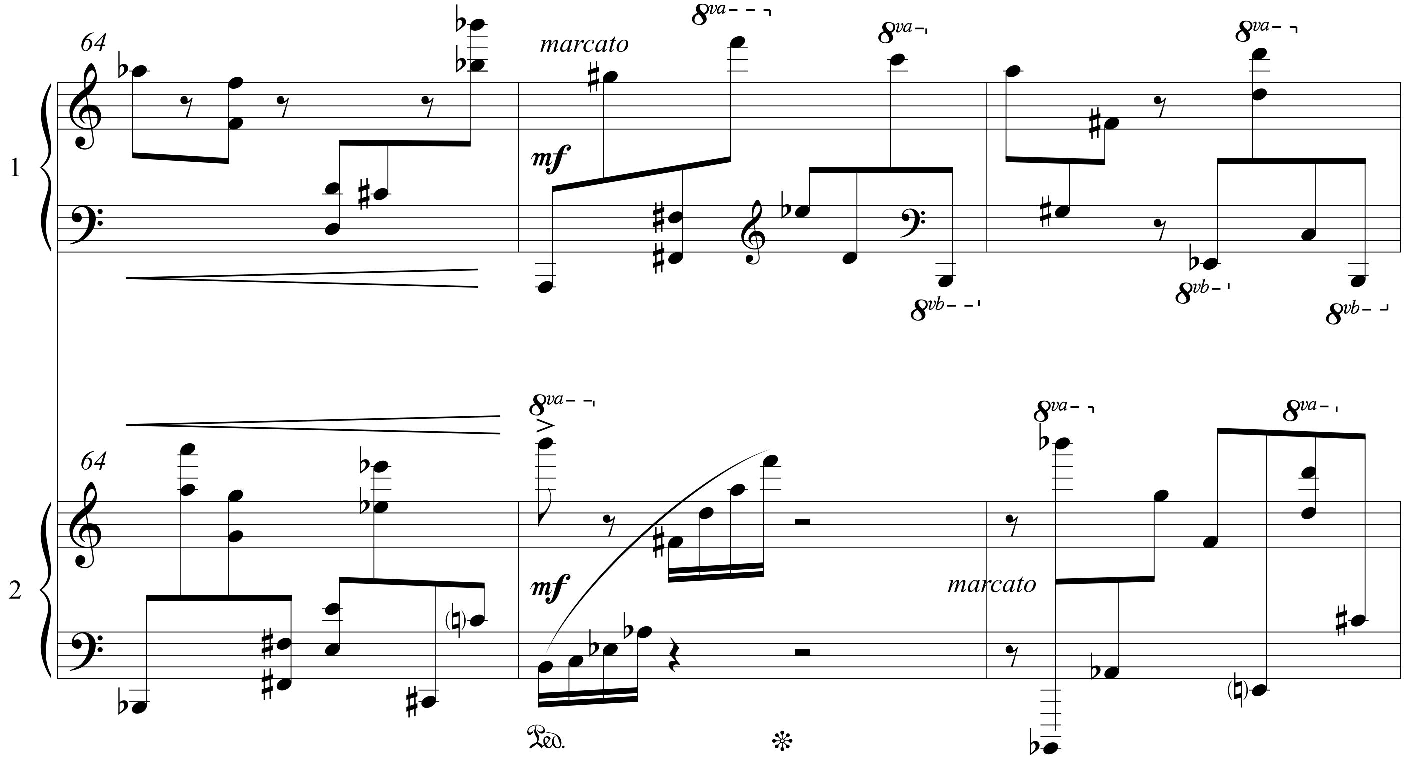
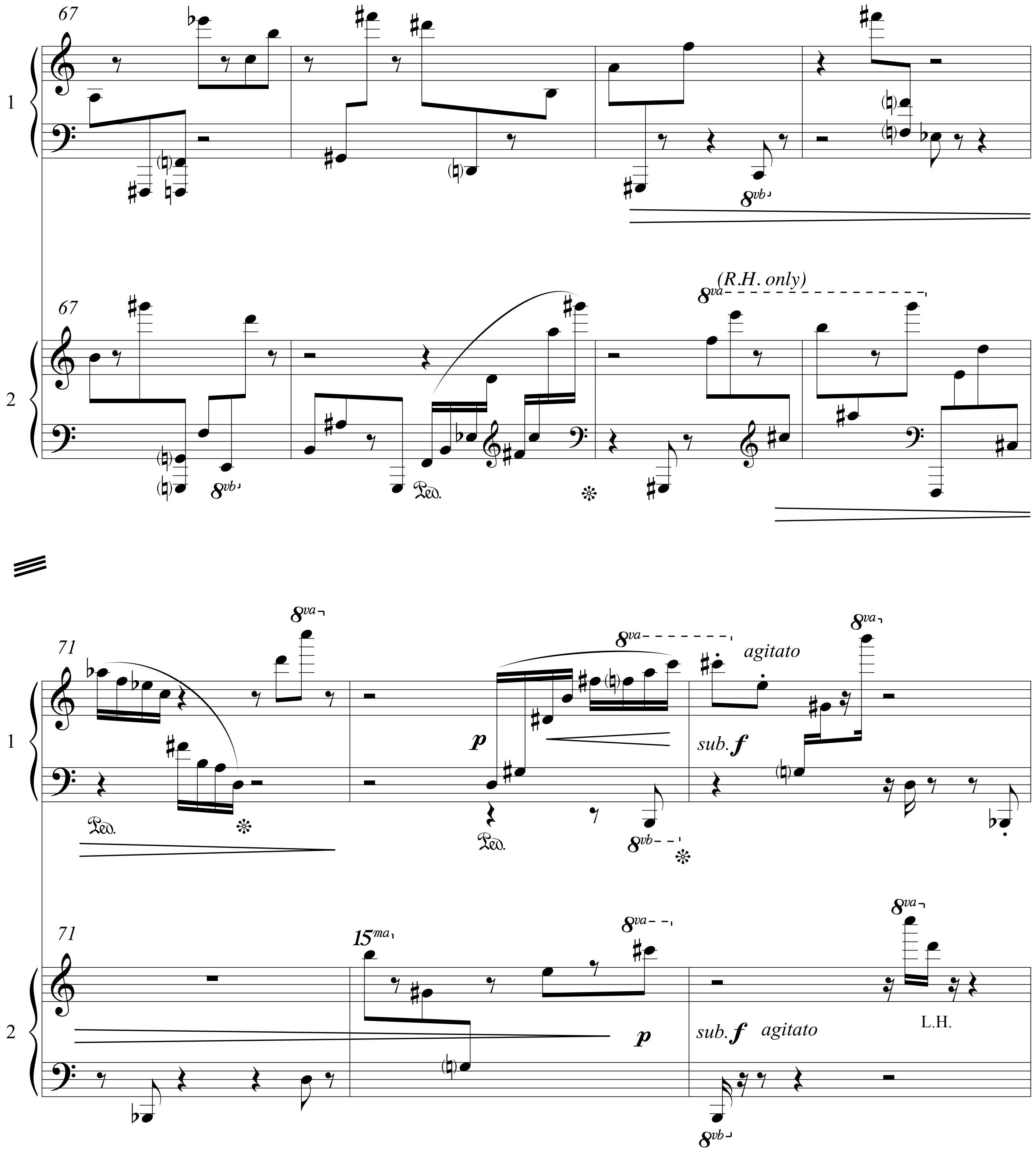

19
1
2
f
p
pp
mp
mute string
close to the pin
p
ppp
p
mp
mute string
close to the pin
p
(play notes on keys)
pp
pp
ppp
24
pp < mp
ppp
pp
mute string
close to the pin
ppp < p
ppp
ppp
mp
ppp < p
ppp
pp < mp
mp
29
(play notes on keys)
mp
ppp
mf
mp
pp < mp
pp < mp
pp
mf

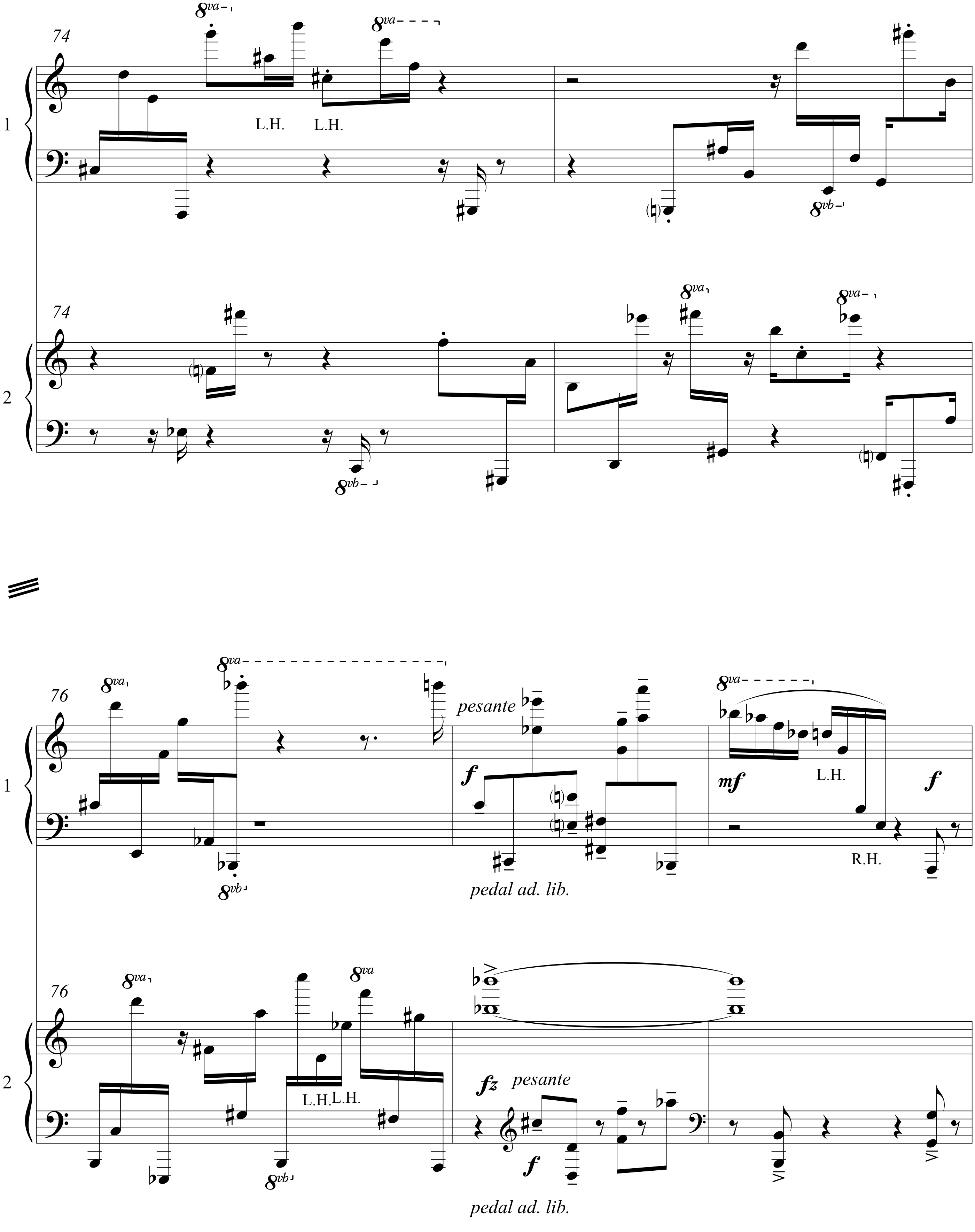

33
espress.
mp
1
2
mp
espress.
37
8va
loco
p
legato
37
p
legato
8vb
loco
42
mp
42
mp

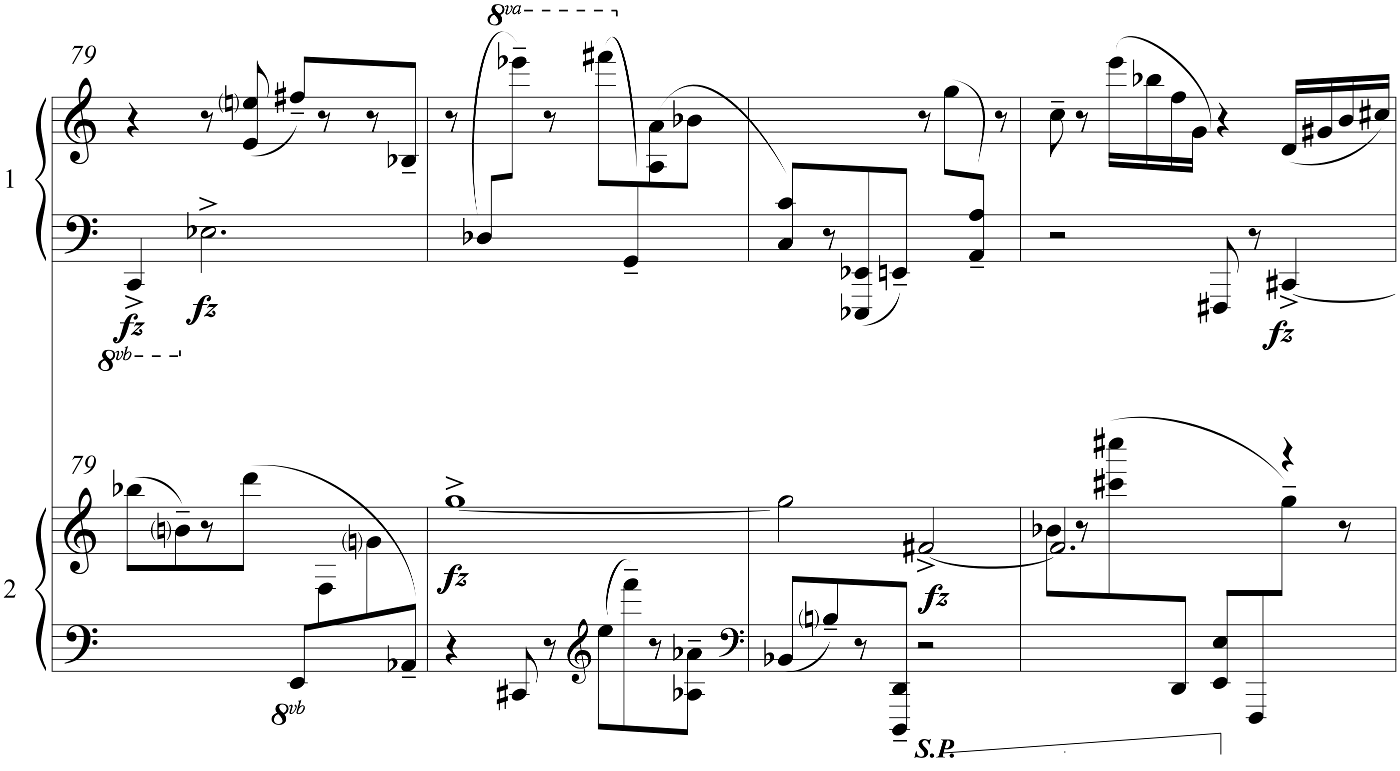

8va
47
1
dolce e sempre staccato
p
senza pedal
2
8vb
52
56
leggiero
mp
R.H.
8vb

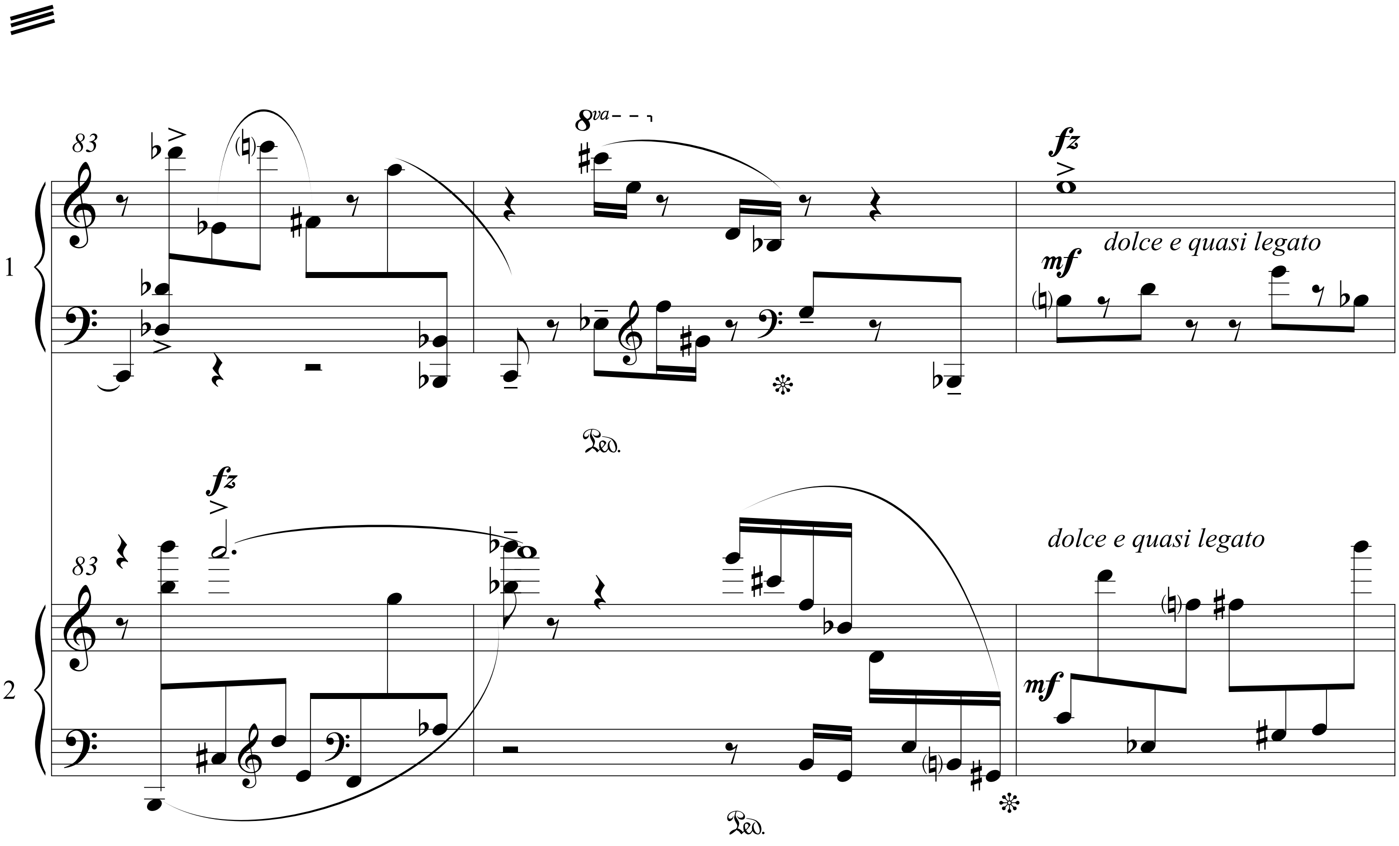

60
1
2
mp
mp
64
marcato
8va
8vb
mf
Ped.
*

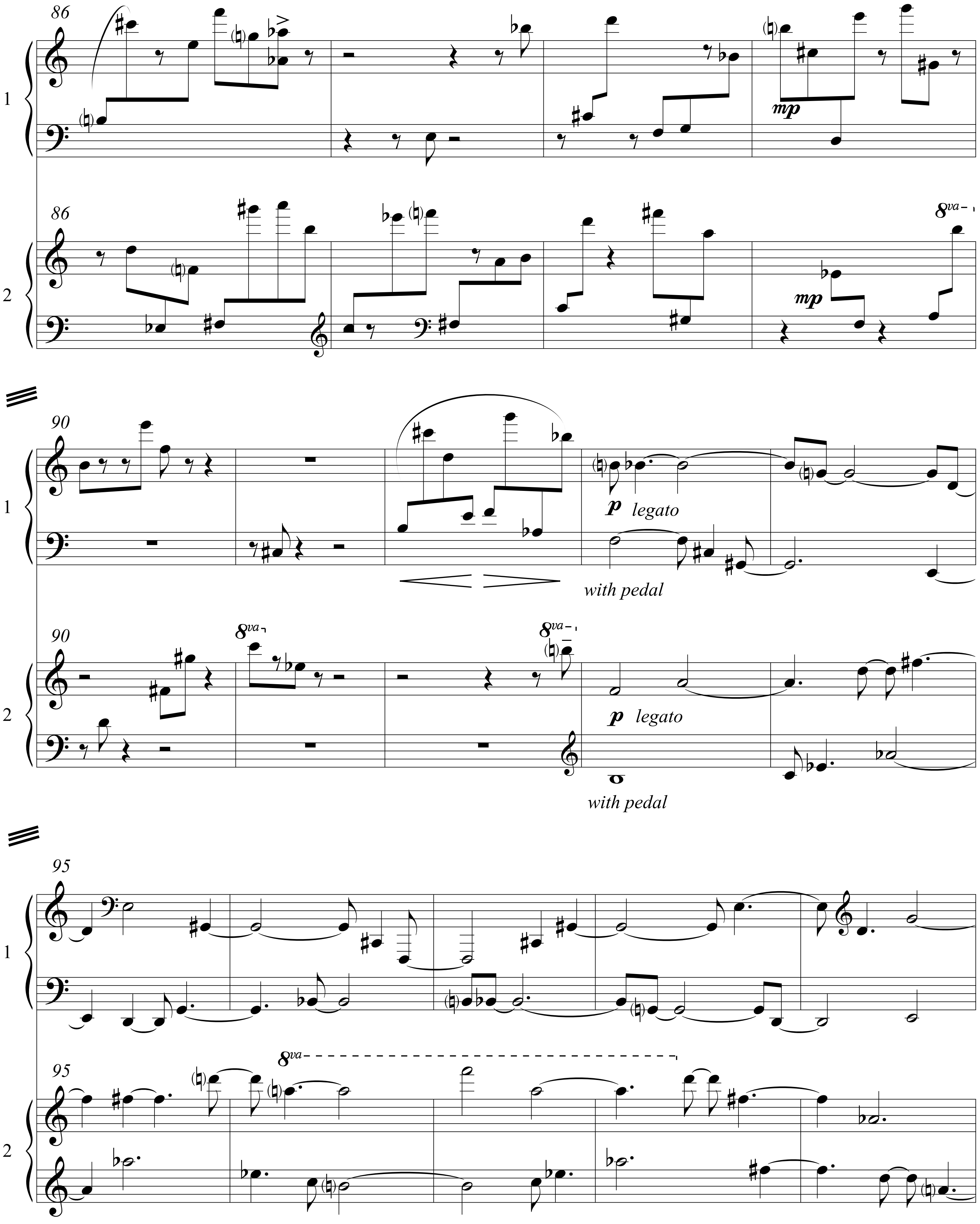


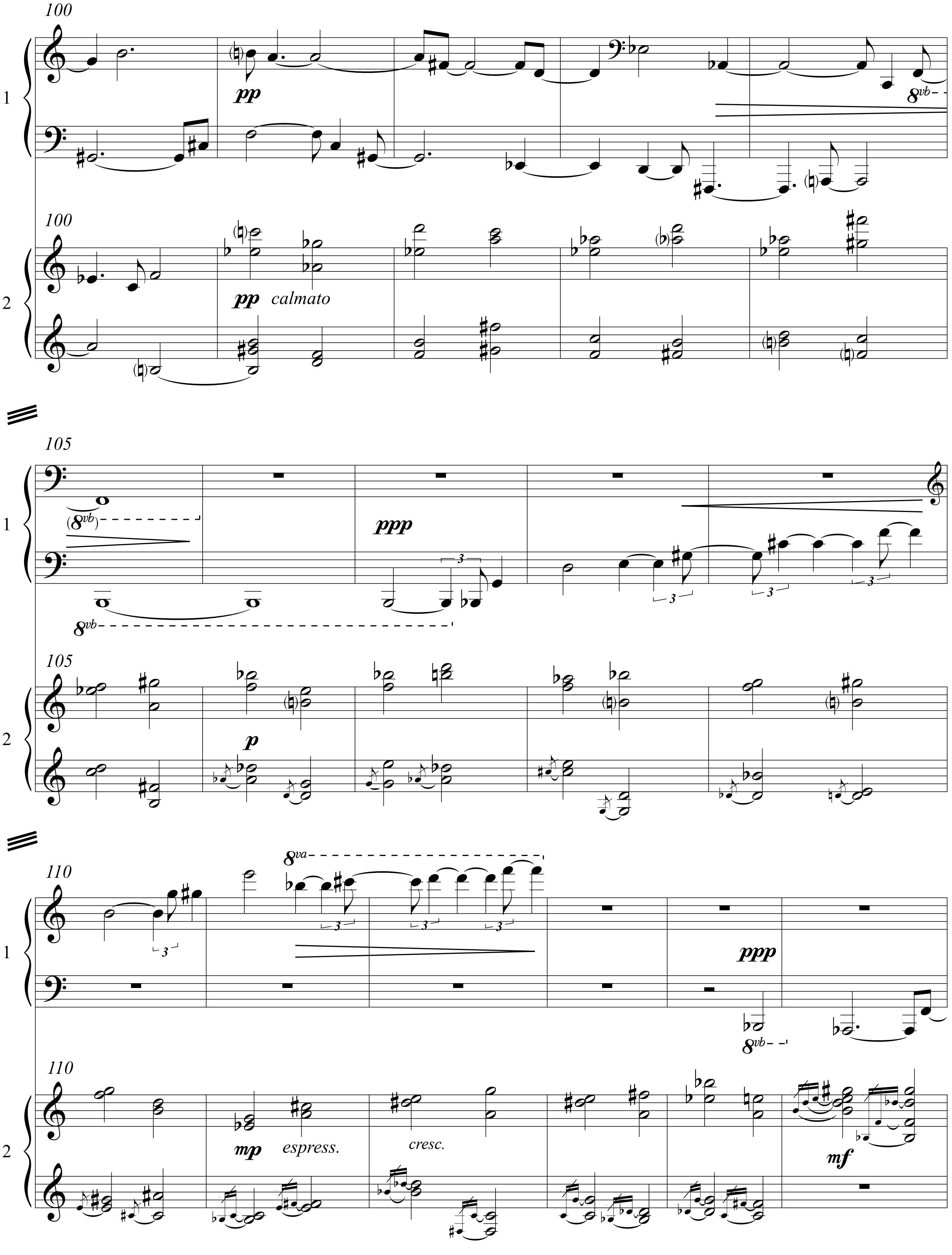

67
1
2
8va
8vb
(R.H. only)
Ped.
71
15ma
p
sub. f
agitato
L.H.

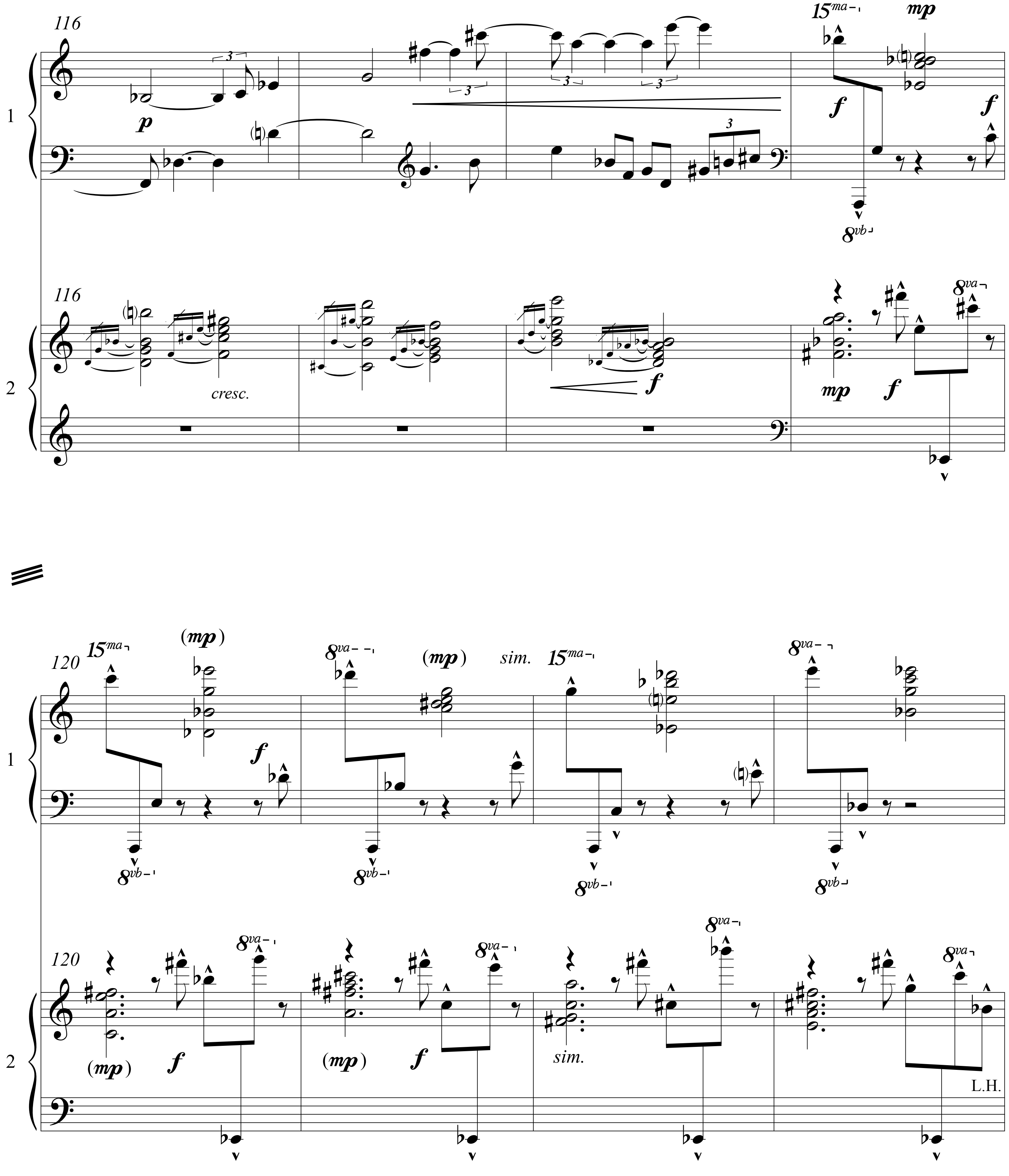

74
8va
L.H.
L.H.
8va
8vb
1
74
8va
8va
8vb
2
76
8va
8va
pesante
f
8va
mf
L.H.
f
R.H.
8vb
pedal ad. lib.
1
76
8va
8va
L.H.
L.H.
8vb
fz
pesante
f
pedal ad. lib.
2

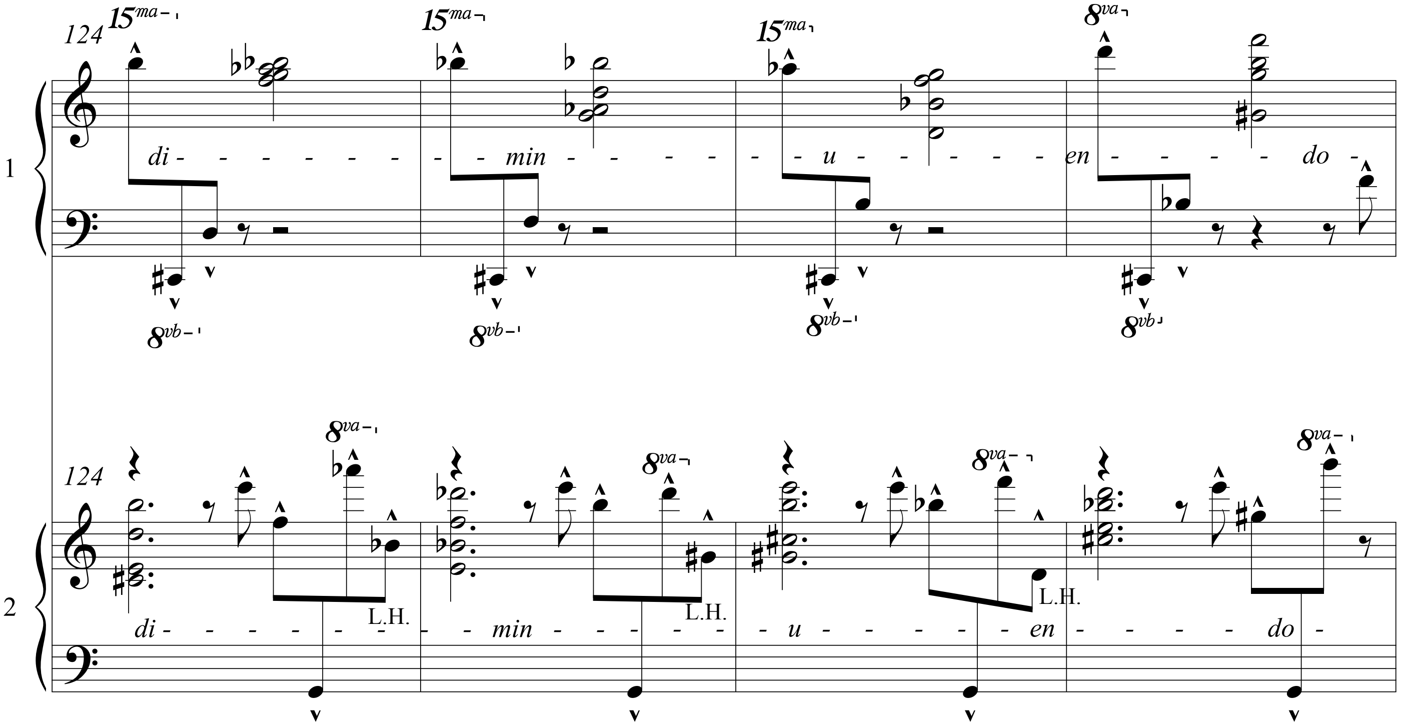

79
1
2
8va
8vb
fz
S.P.

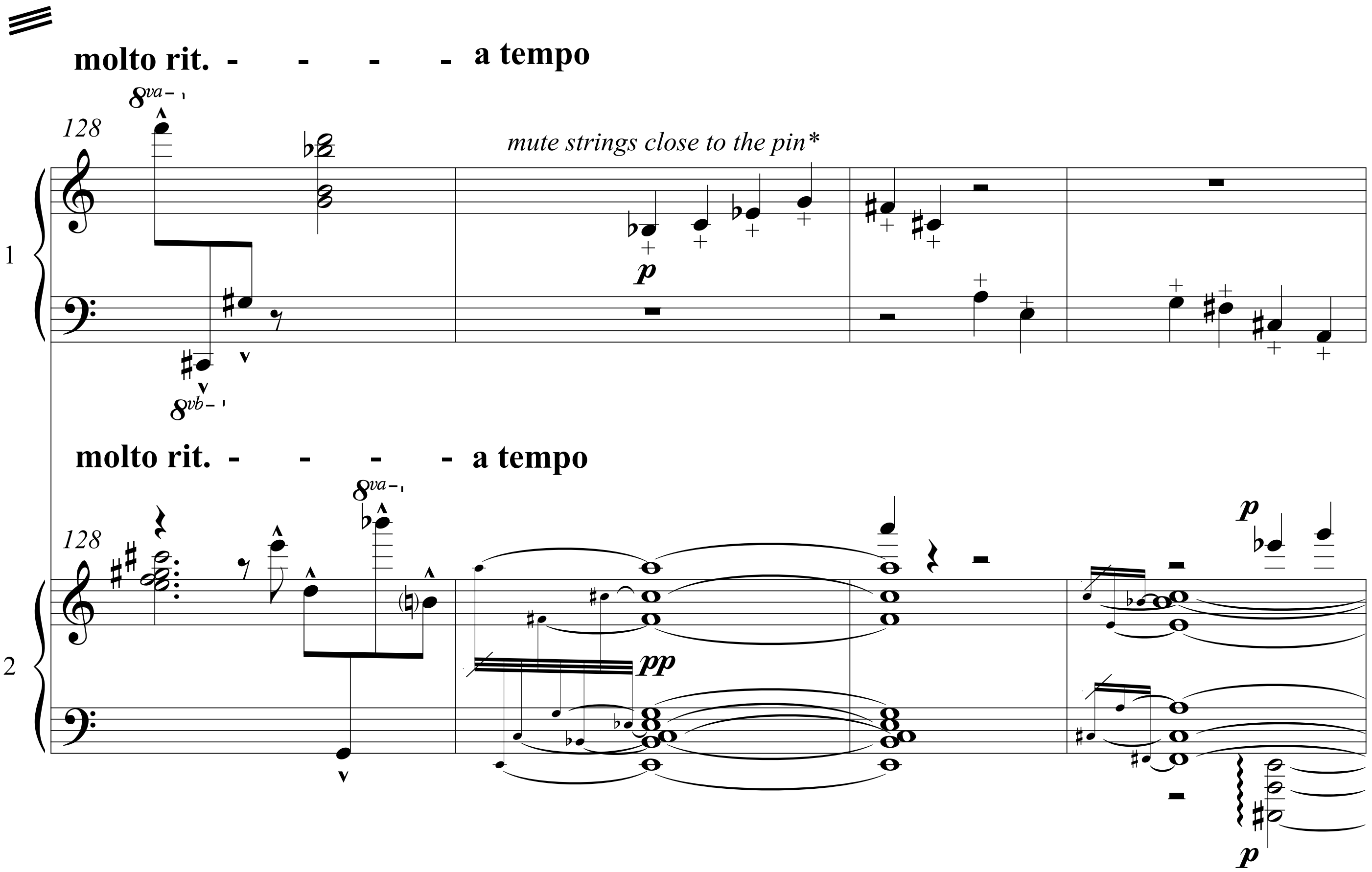

83
1
2
8va
fz
mf
dolce e quasi legato
Ped.


86
1
mp
86
2
8va
mp
90
1
p legato
with pedal
90
2
8va
8va
p legato
with pedal
95
1
95
2
8va


100
1
pp
8vb
100
2
pp
calmato
105
(8vb)
ppp
8vb
105
p
110
8va
ppp
8vb
110
mp
espress.
cresc.
mf

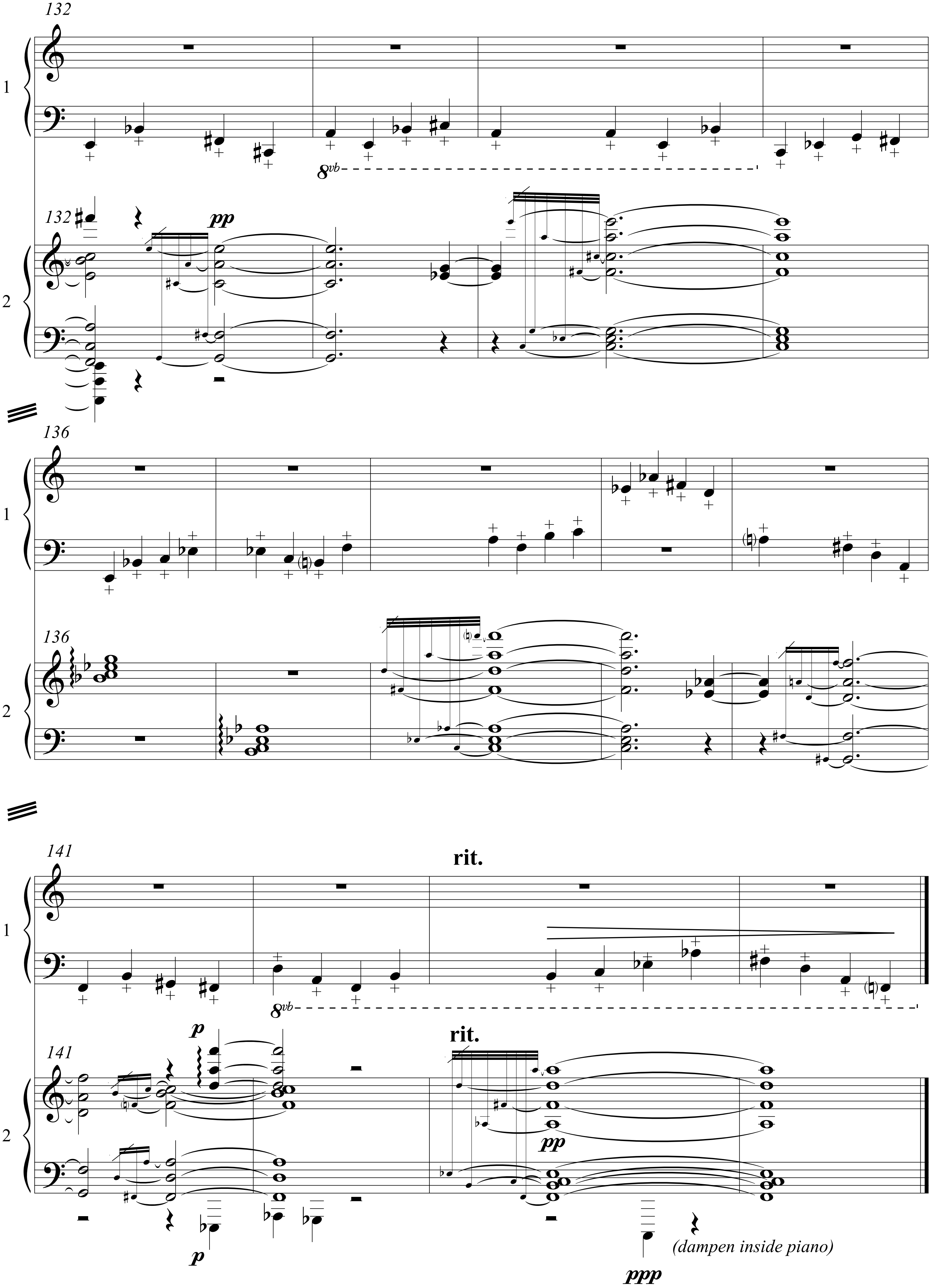

116
1
2
p
3
cresc.
f
15ma
mp
8vb
8va
120
(mp)
sim.
L.H.


124
15ma
8va
8vb
di - - - - - - - - min - - - - - - - u - - - - - - en - - - - - do -
1
2
L.H.

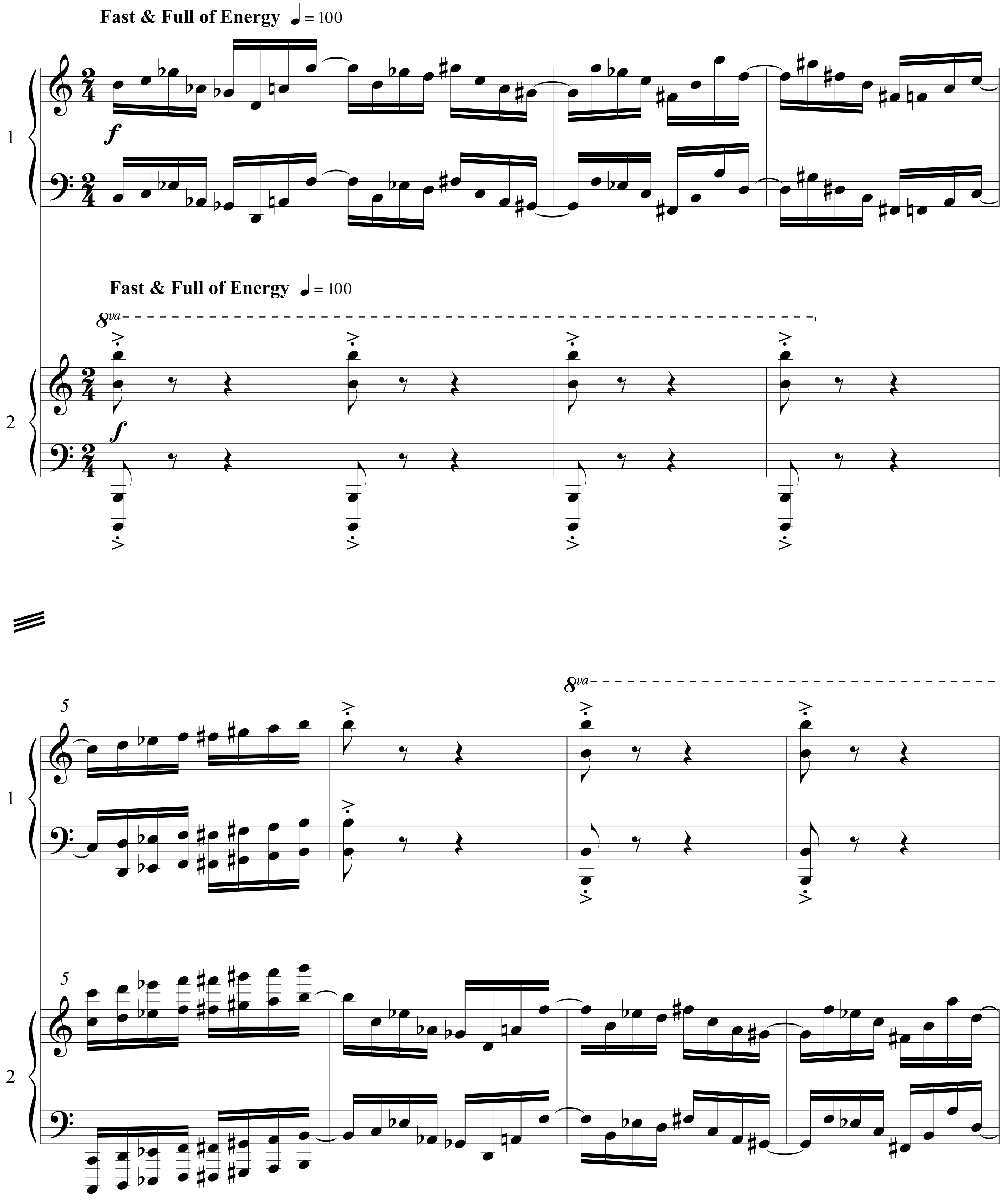

molto rit. - - - - a tempo
128
8va
8vb
mute strings close to the pin*
p
pp
1
2

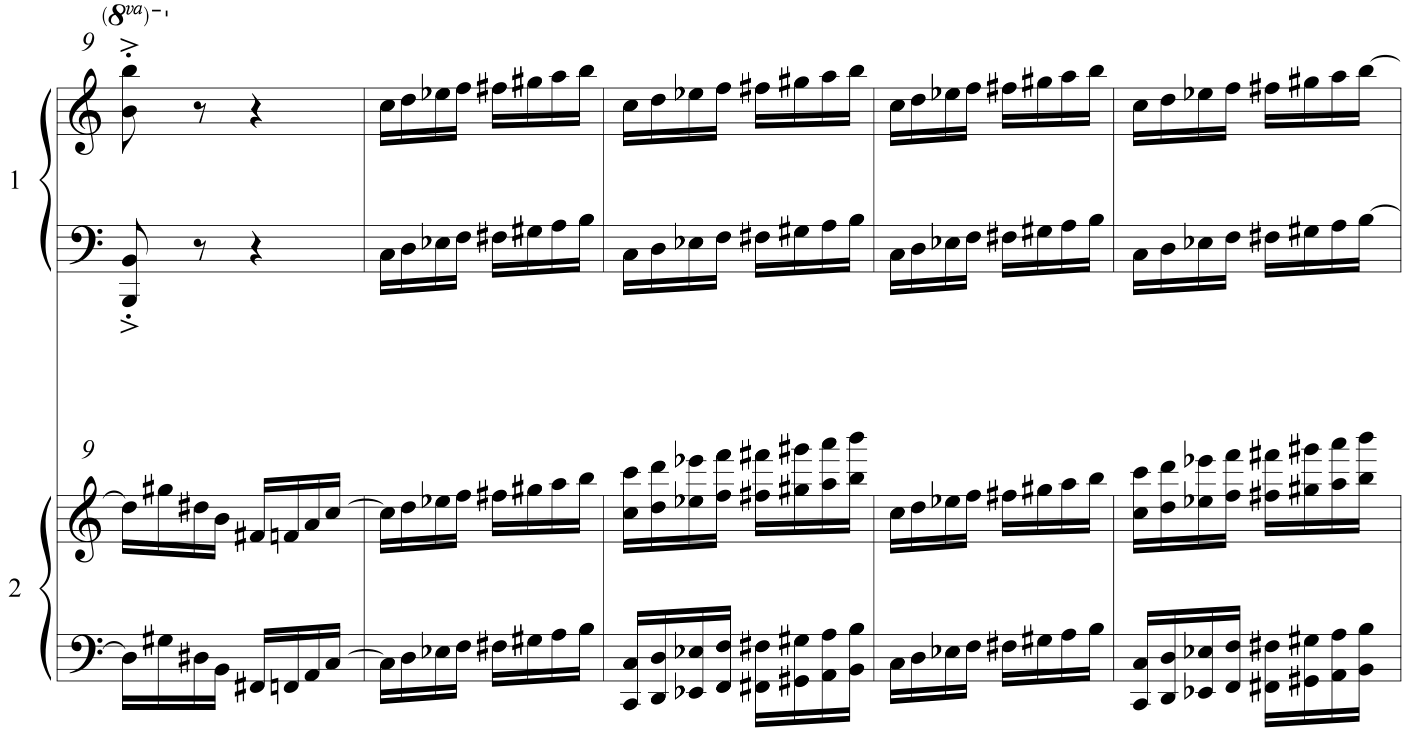
---

* Performer might attach felt to the bottom of a thin board to simplify the muting process

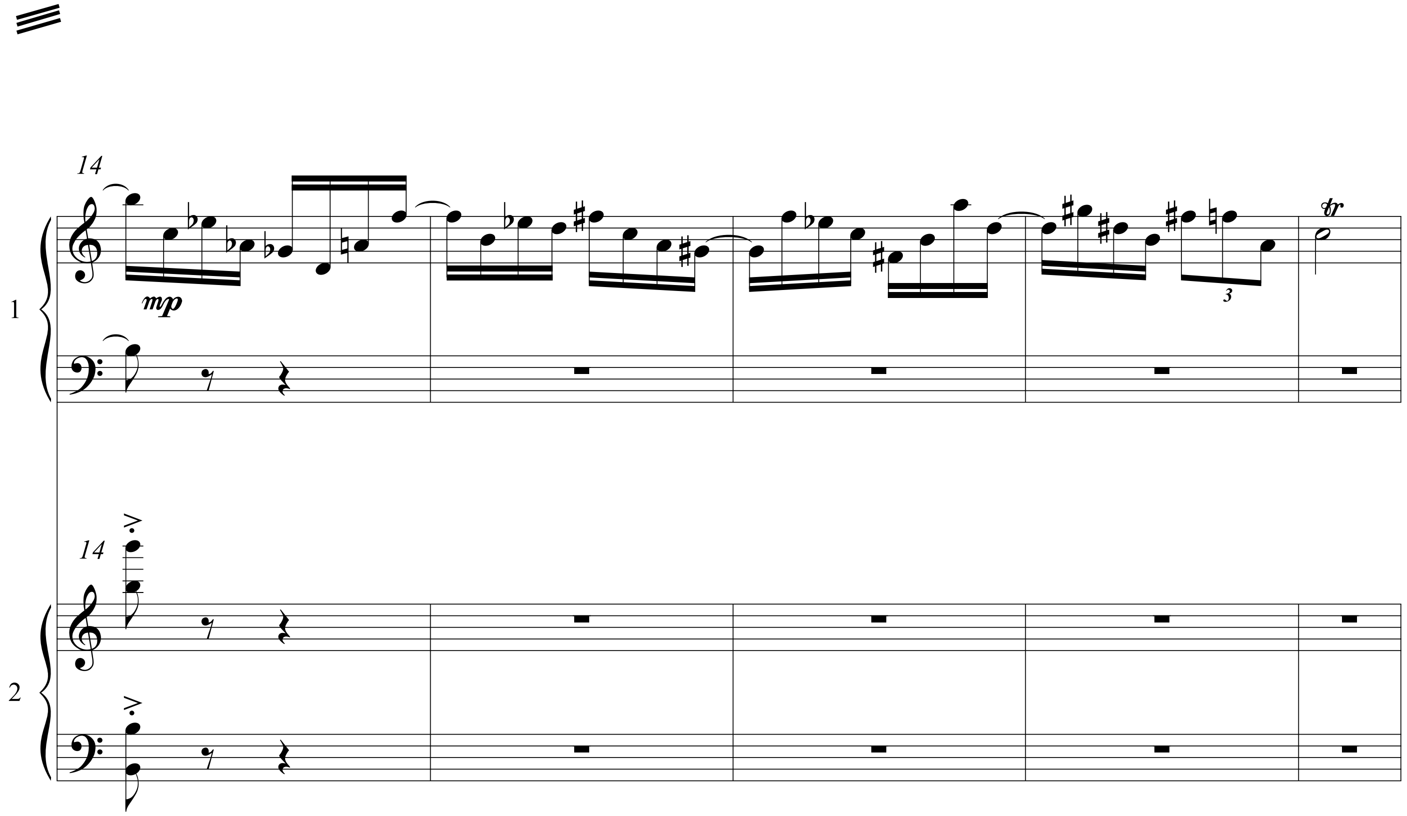

132
1
2
8vb
pp
136
141
rit.
p
rit.
pp
p
ppp
(dampen inside piano)

Octulus B

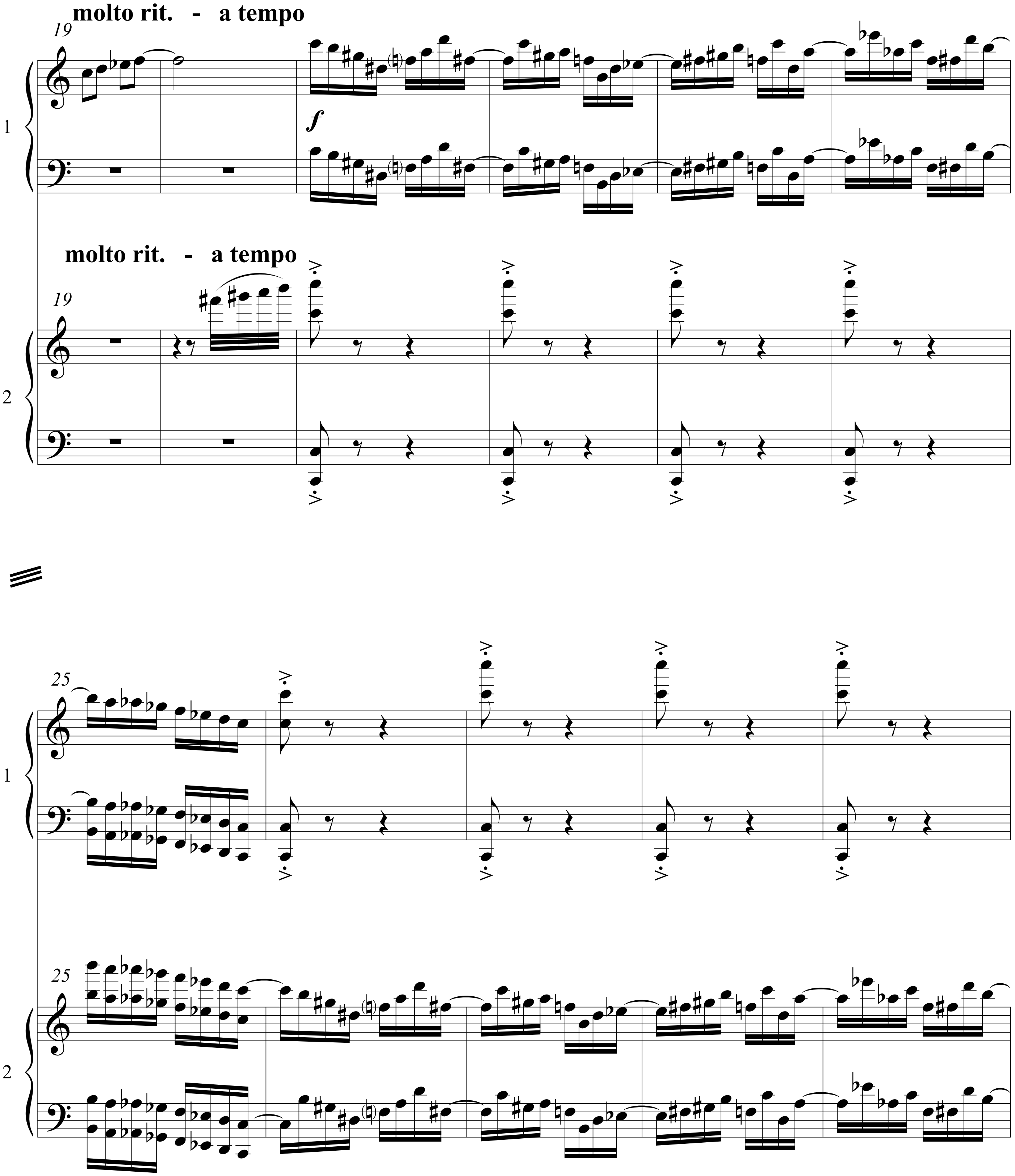

Fast & Full of Energy ♩= 100
1
f
Fast & Full of Energy ♩= 100
8va
2
f
5
8va
1
5
2

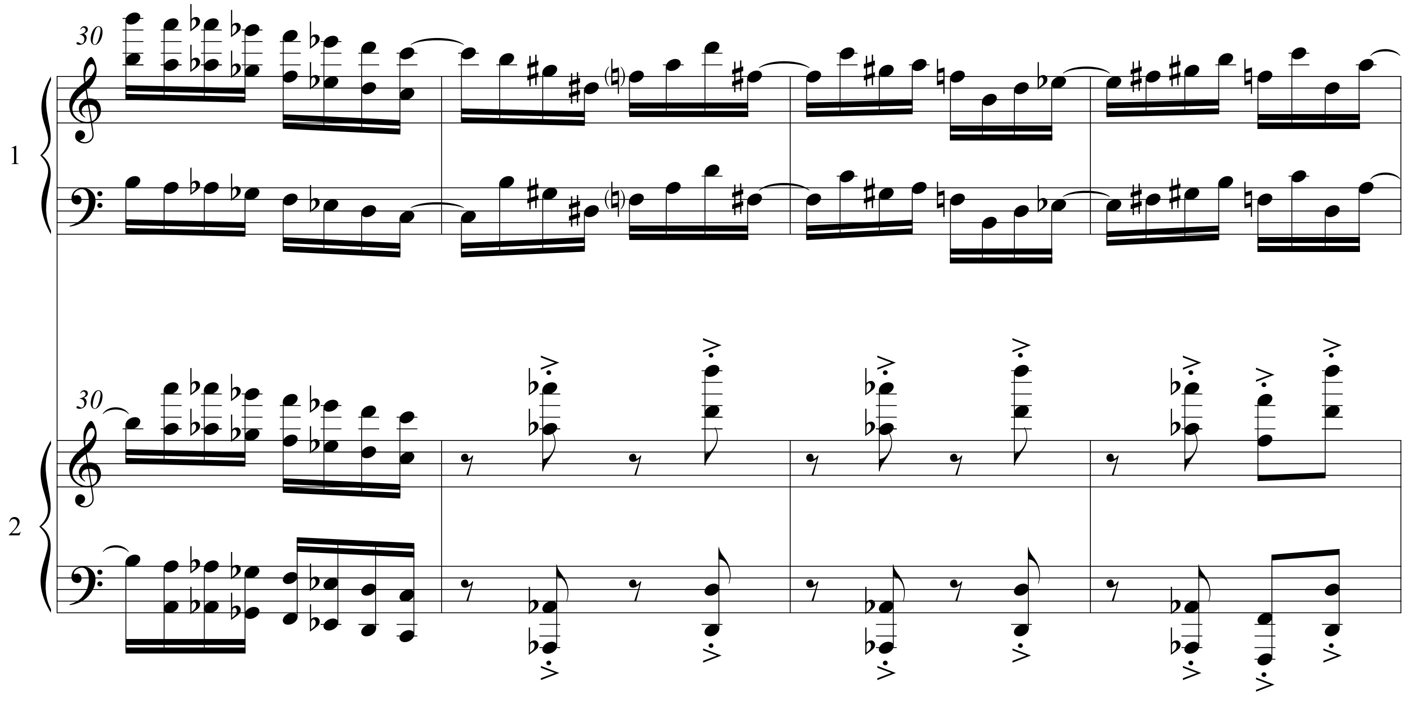


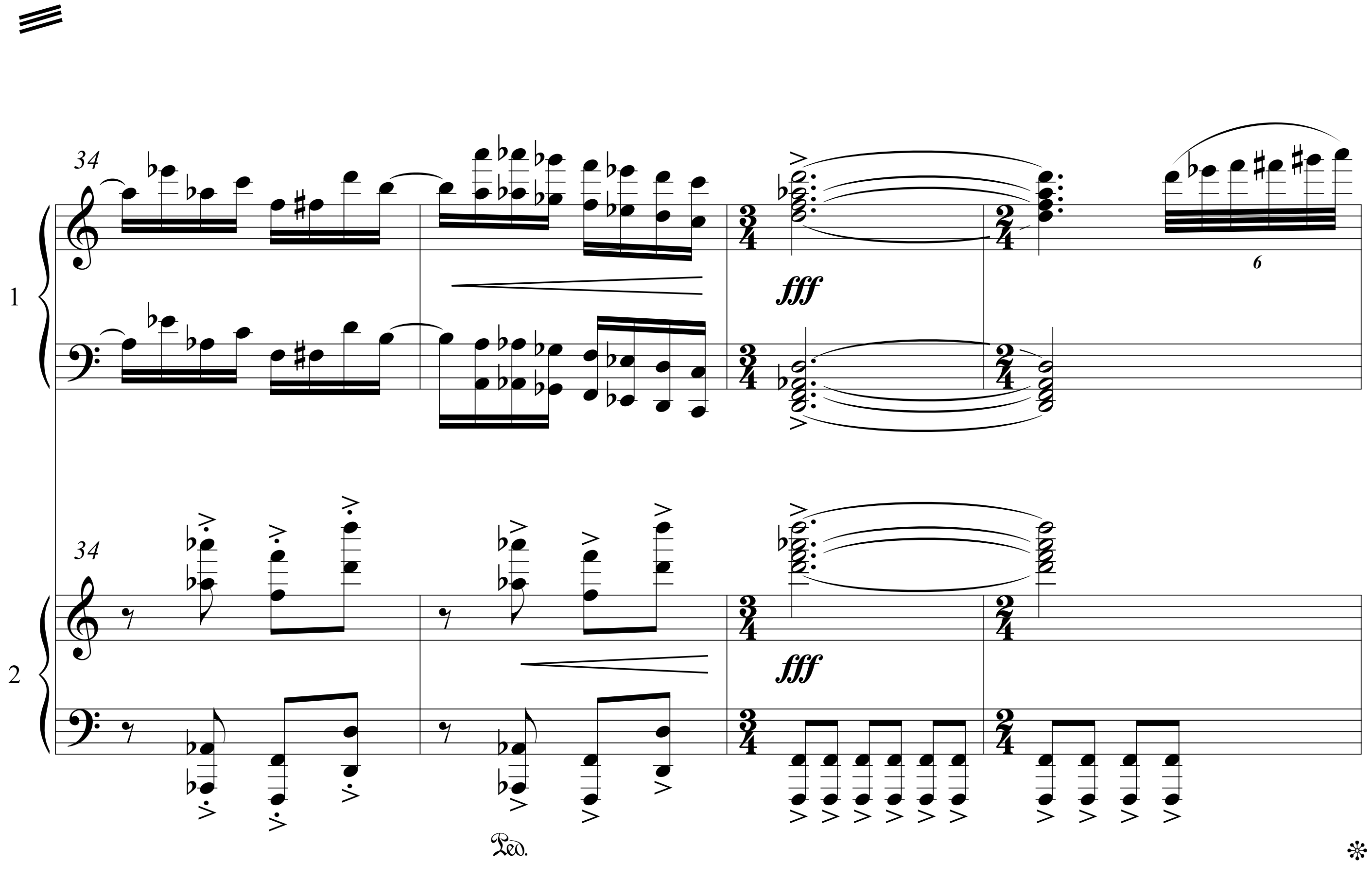
(8va)

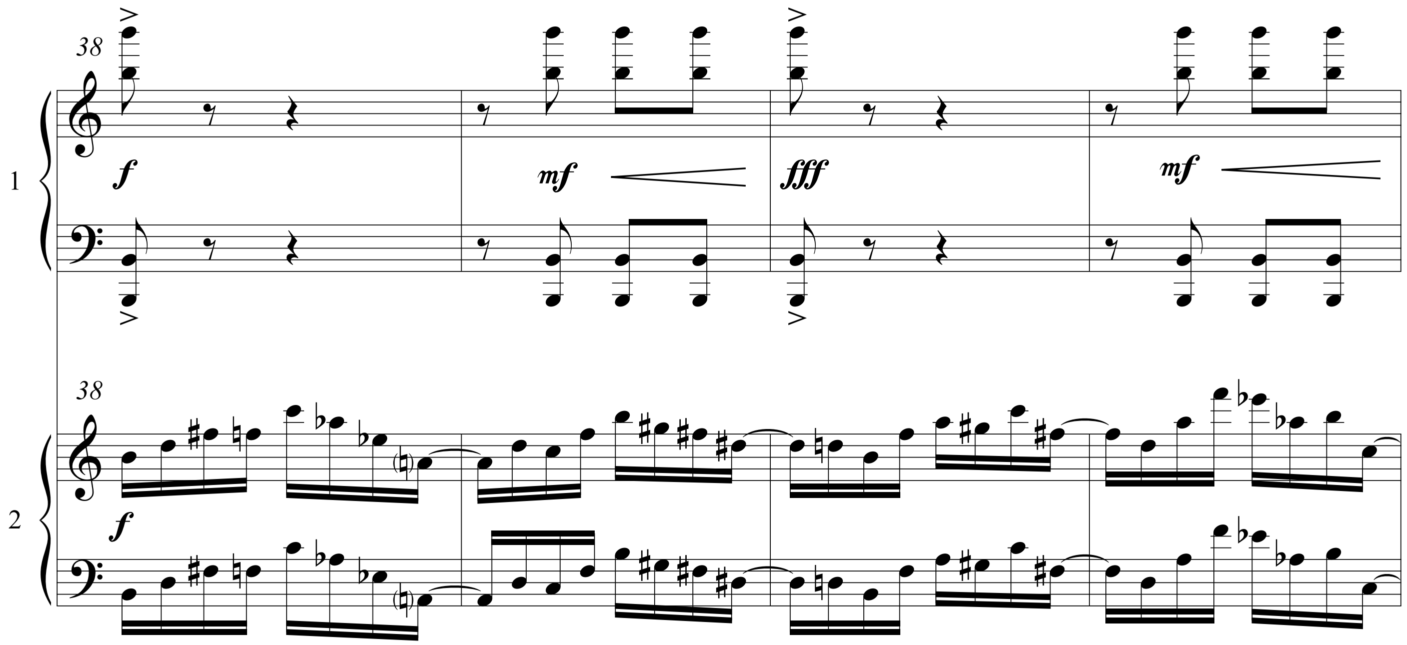


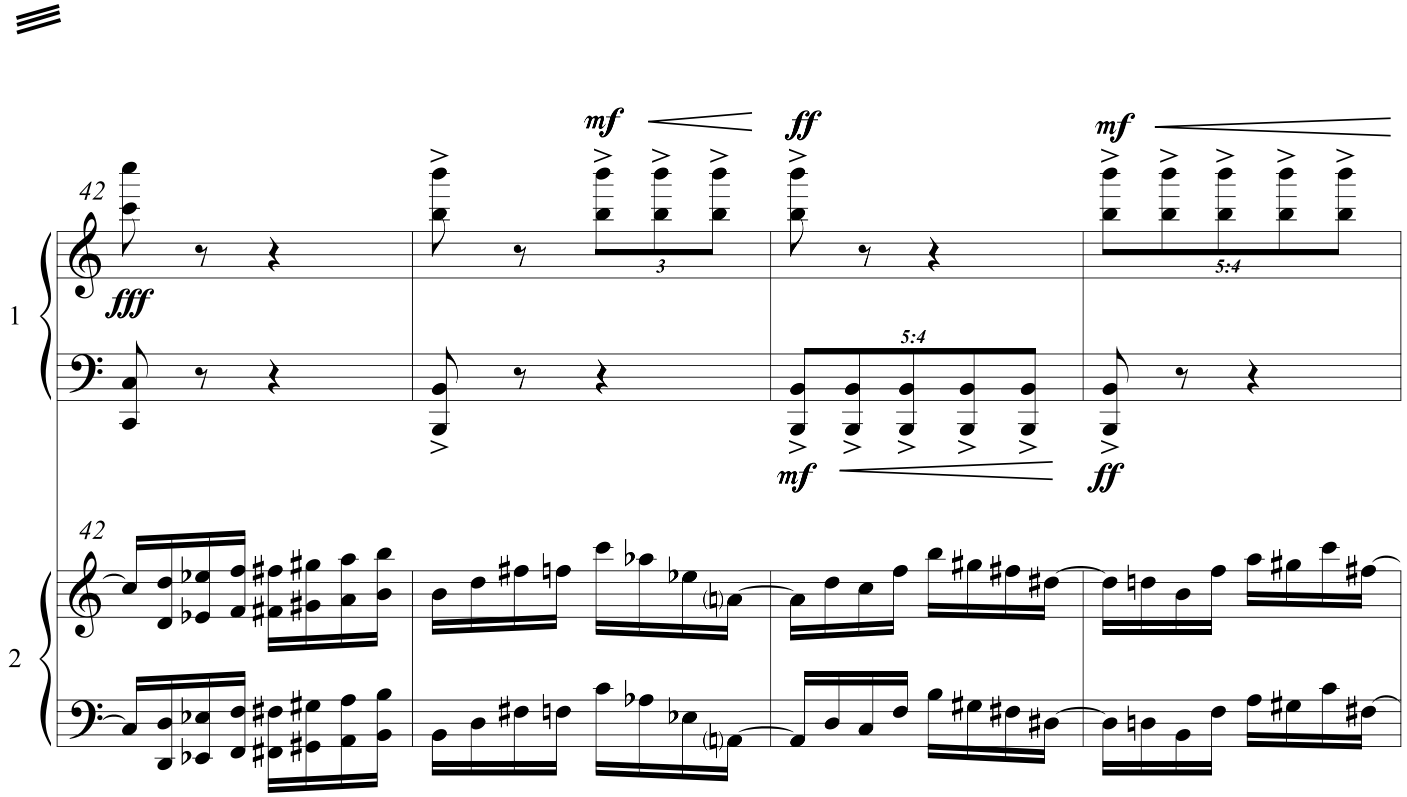
mp
tr

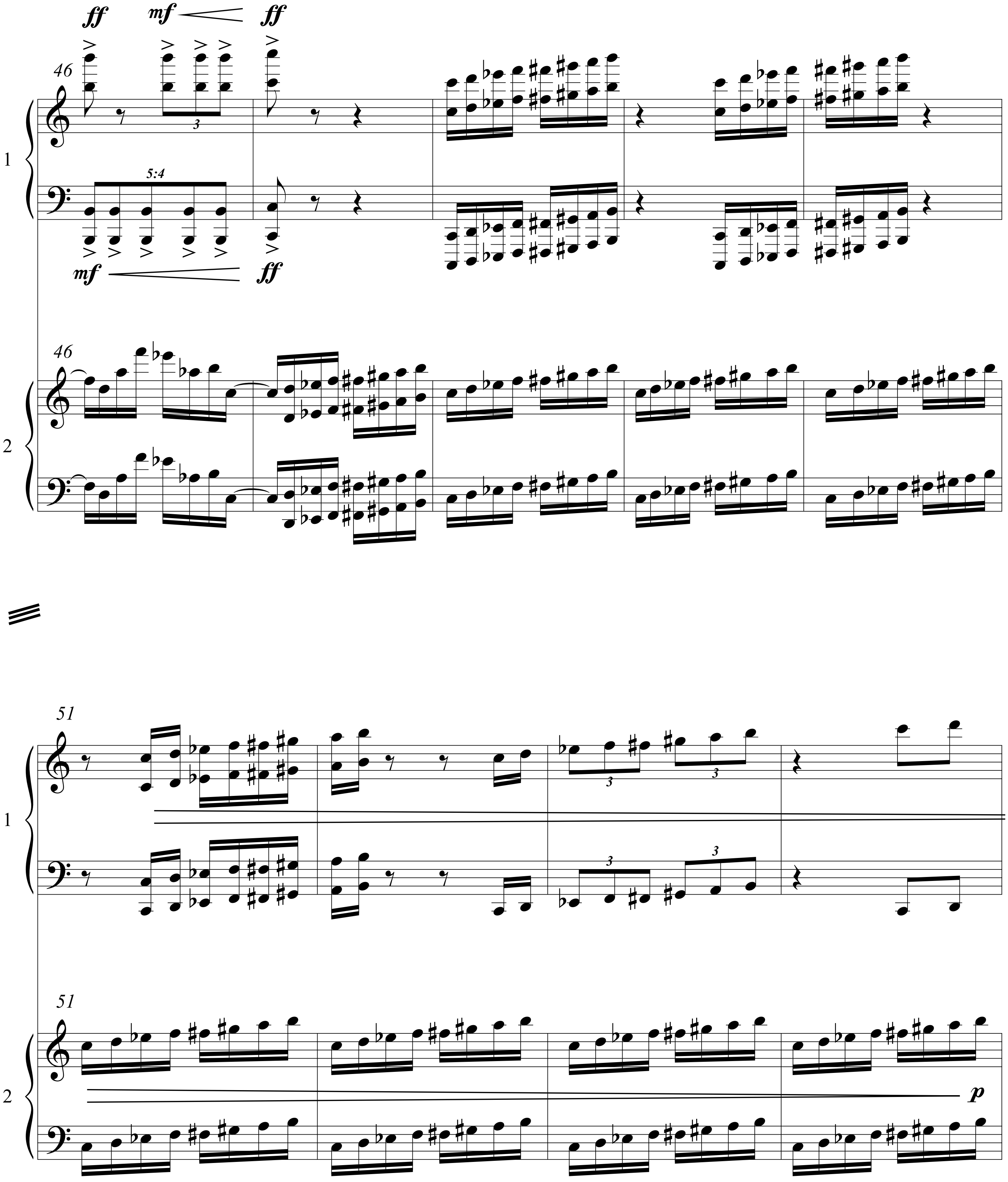

molto rit. - a tempo
f
molto rit. - a tempo

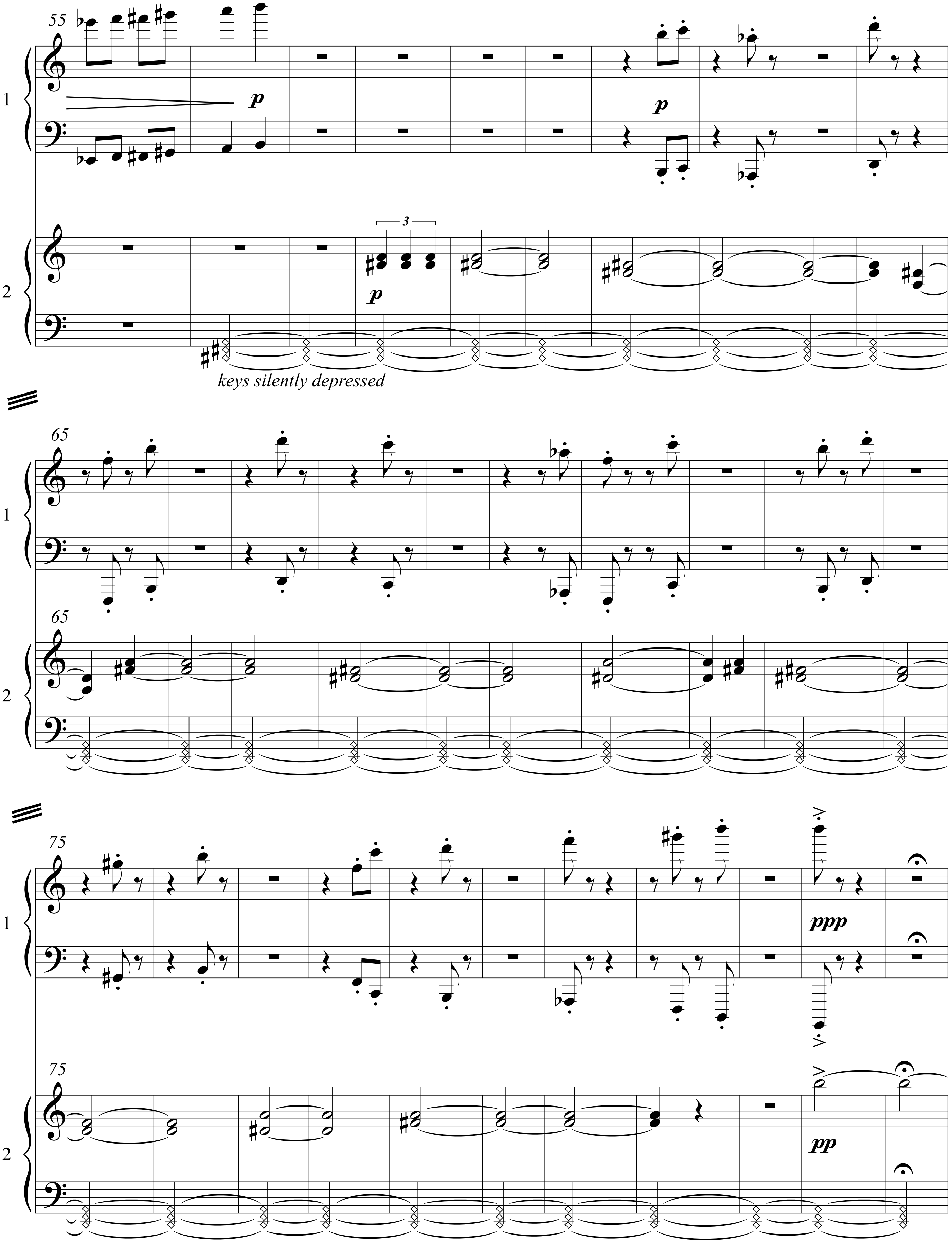


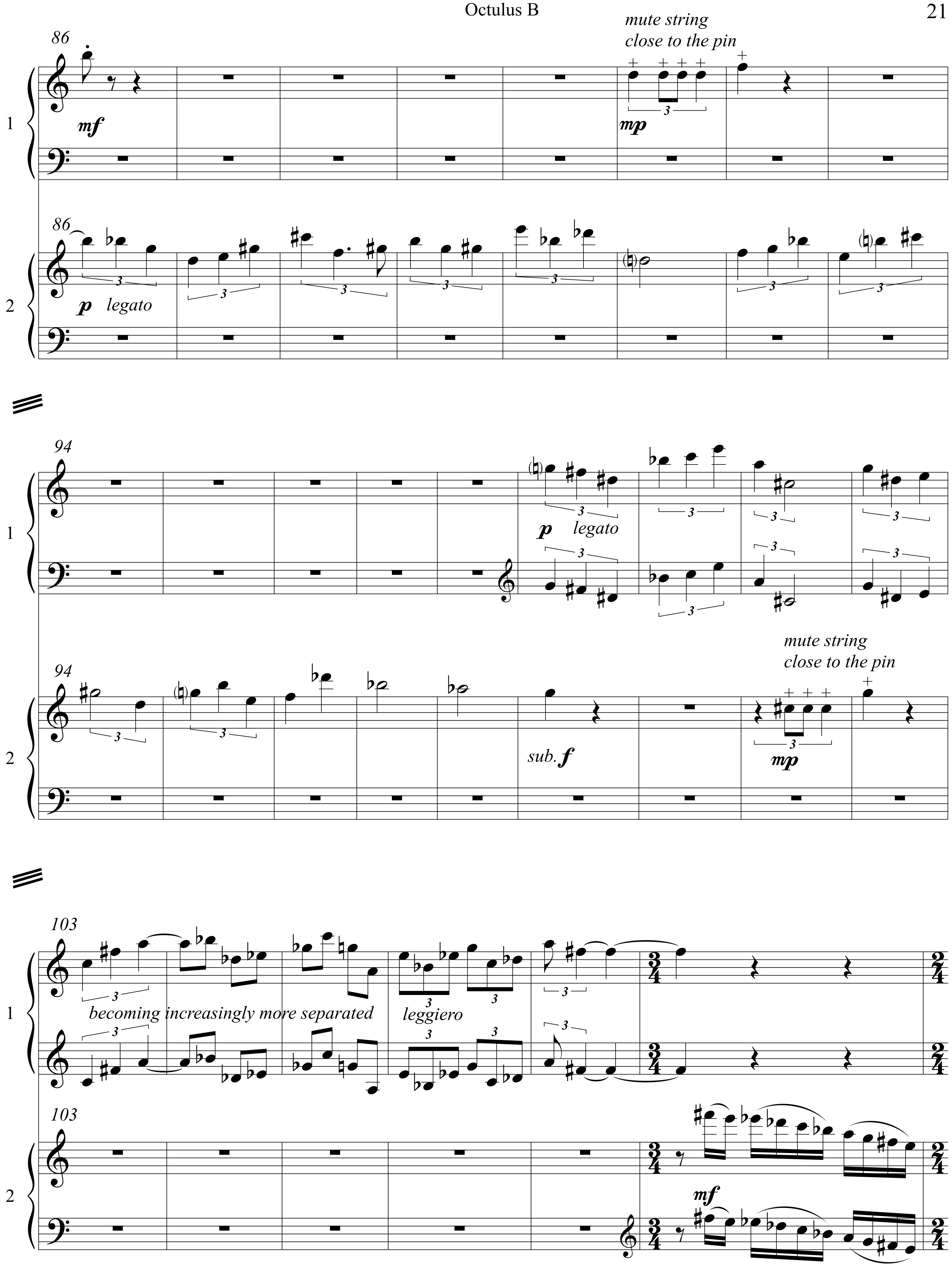

34
1
6
fff
34
2
fff
Ped.
*

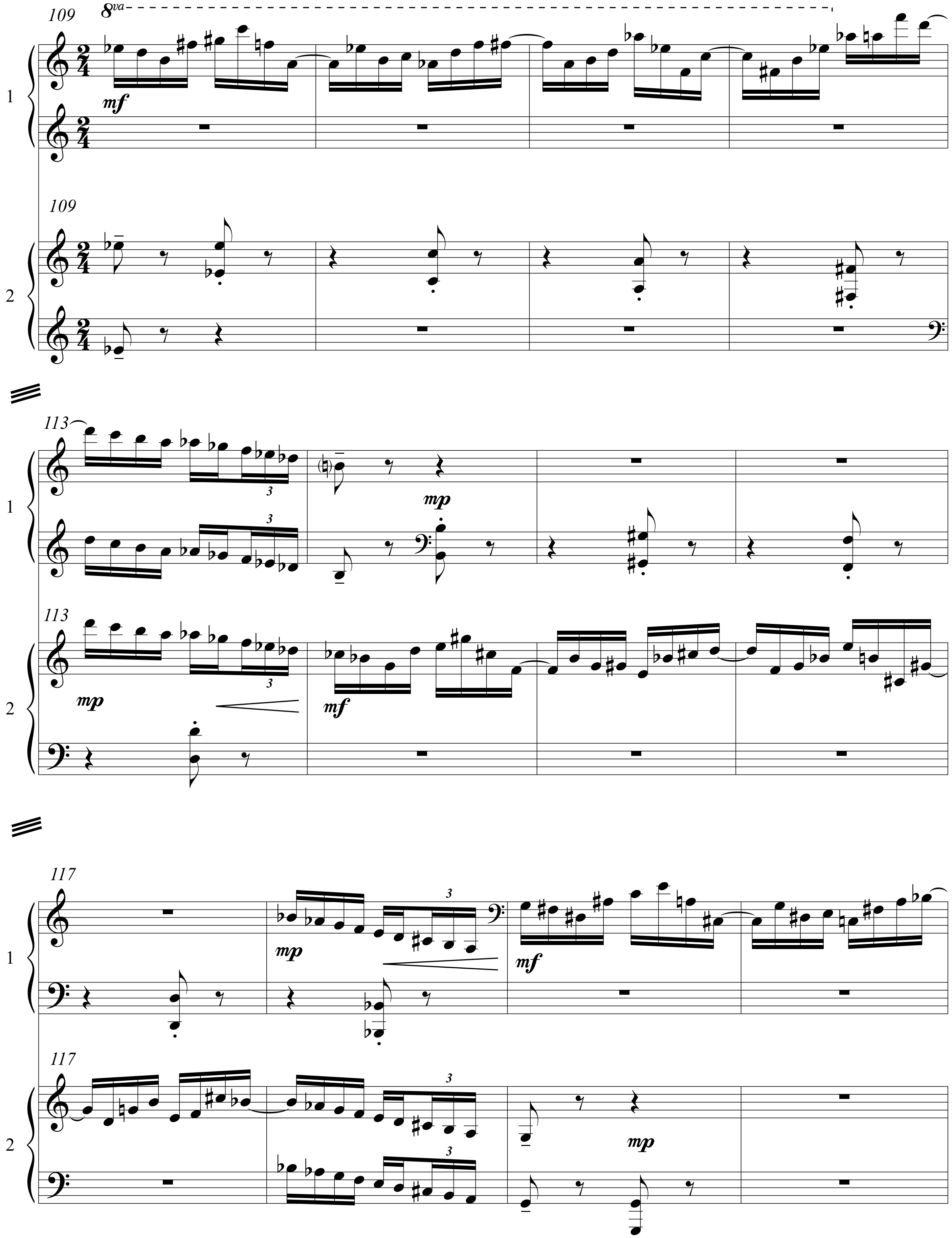

38
1
2
f
mf
fff

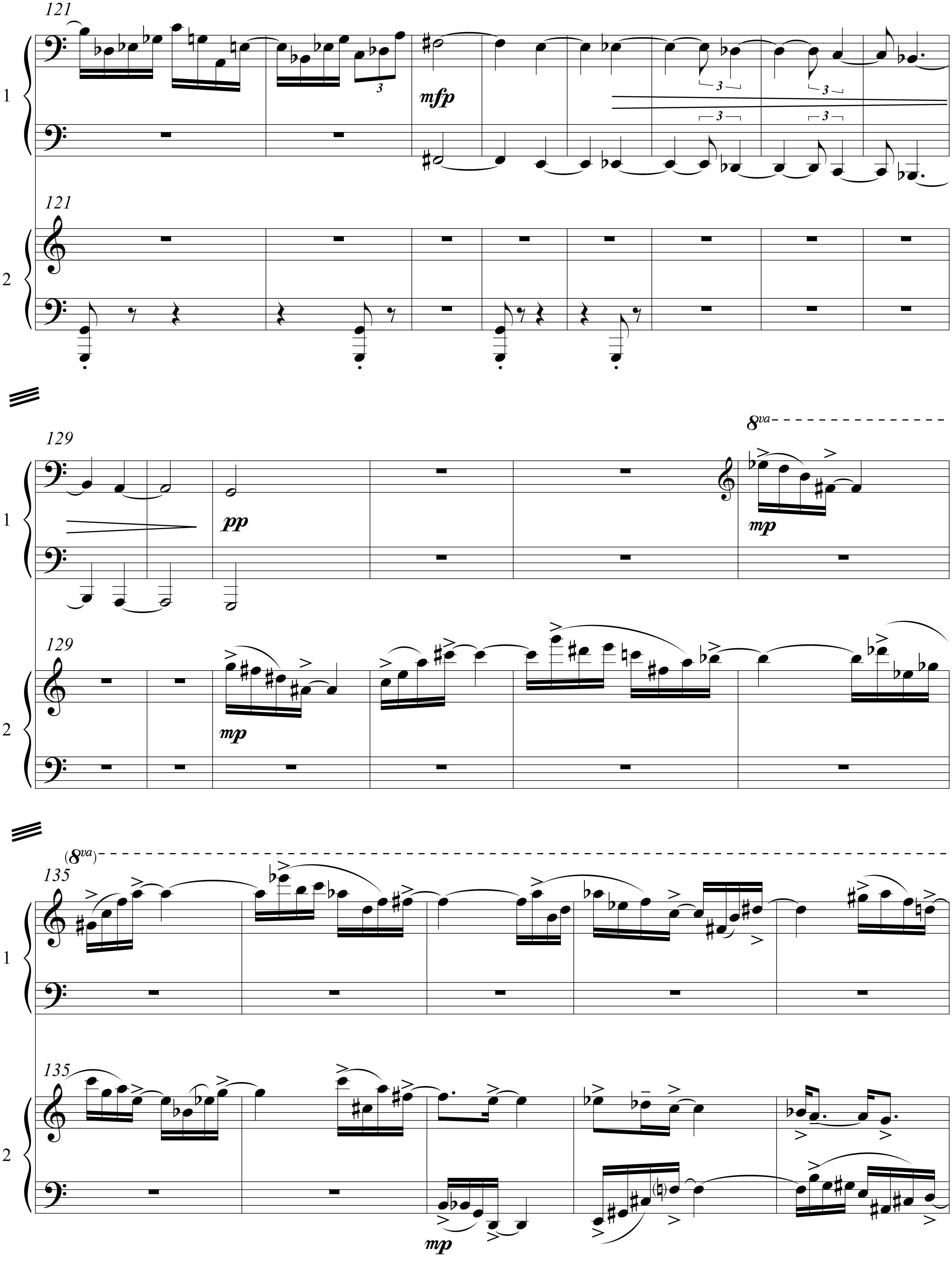

42
1
2
fff
mf
ff
3
5:4

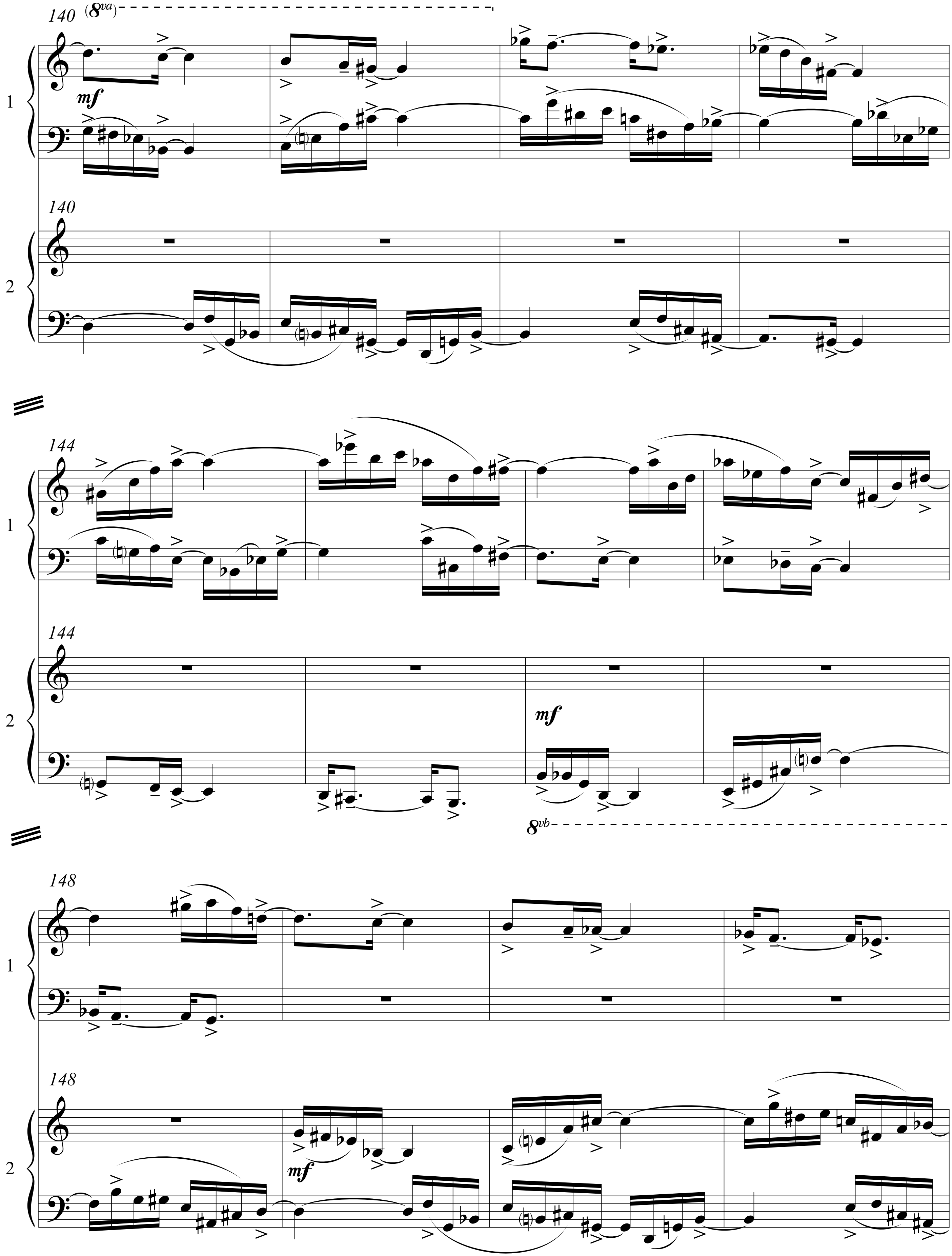

ff
mf
ff
5:4
mf
ff
p

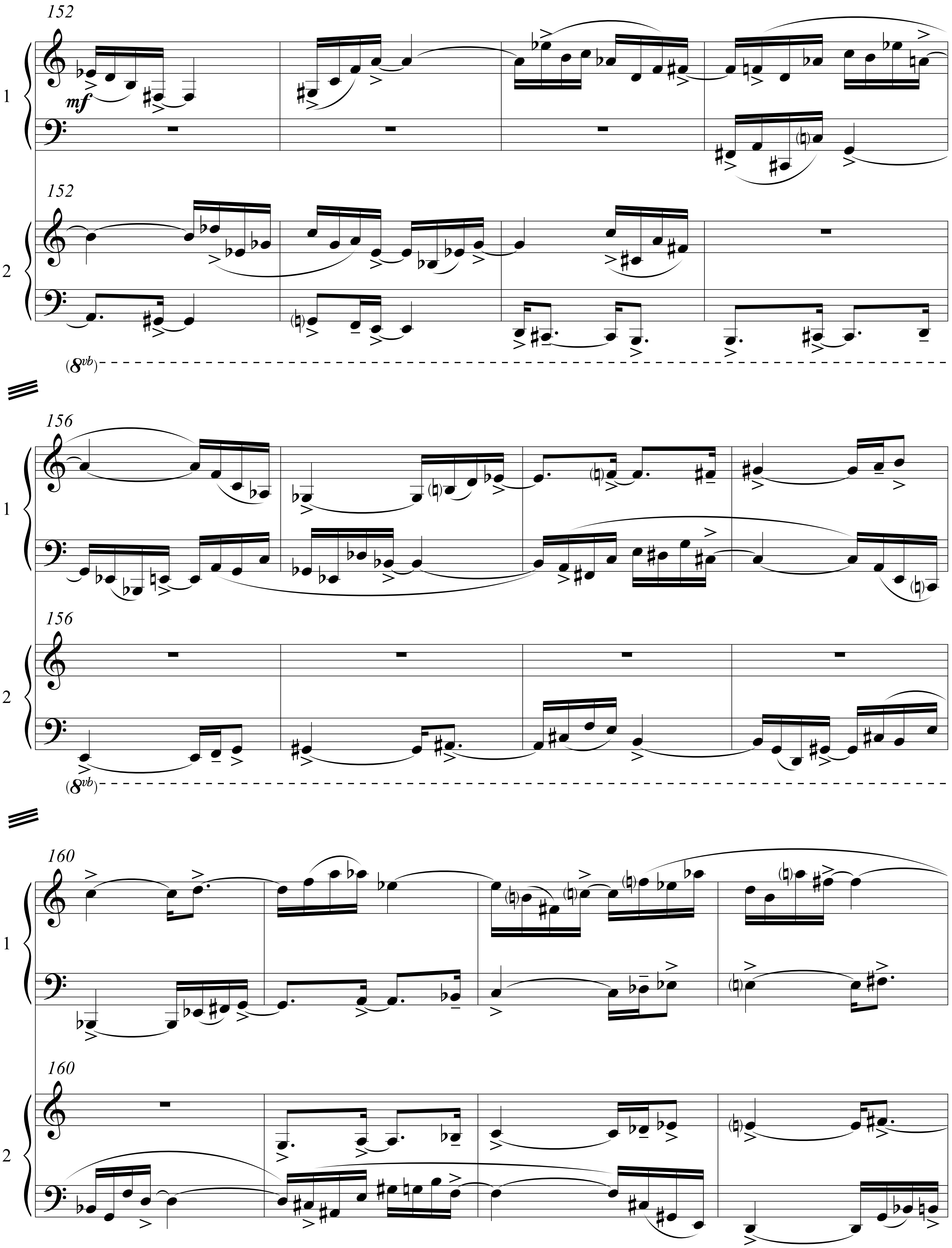

55
1
2
p
p
p
3
keys silently depressed
65
65
75
75
ppp
pp

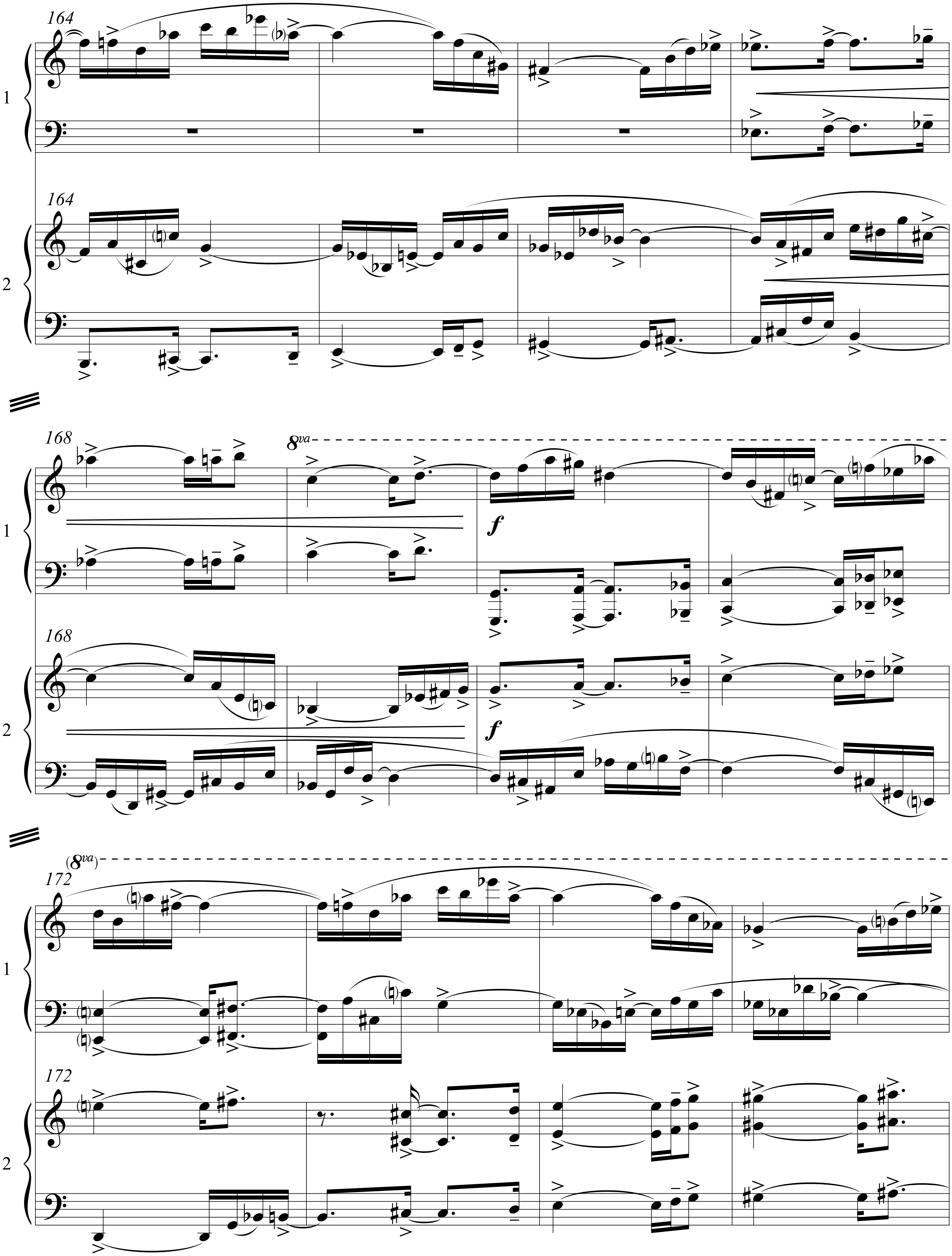

86
mute string
close to the pin
mf
mp
1
86
p legato
2
94
p legato
1
94
mute string
close to the pin
sub. f
mp
2
103
becoming increasingly more separated
leggiero
1
103
mf
2

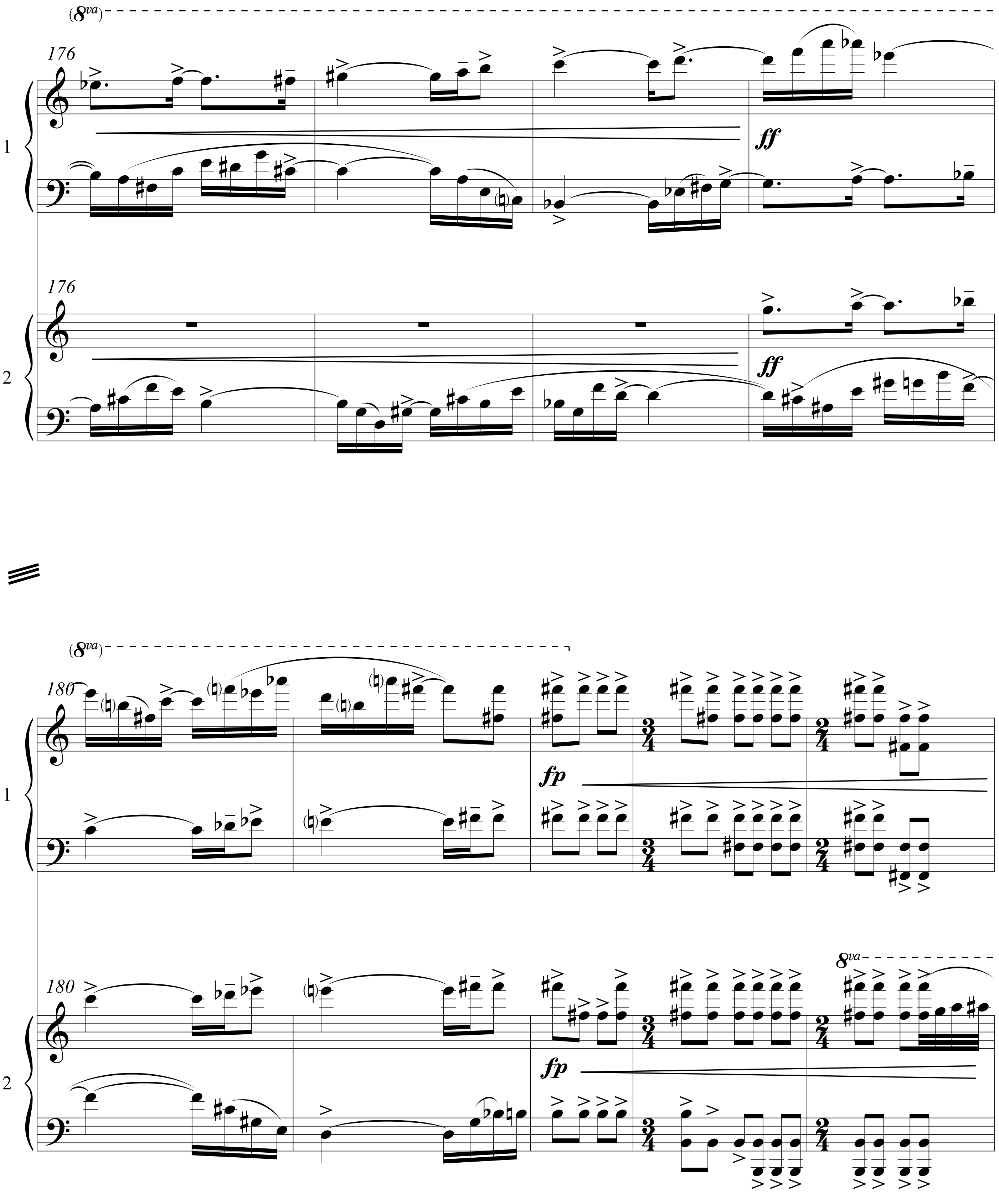

109
8va
mf
1
2
113
mp
mf
3
117
mp
mf

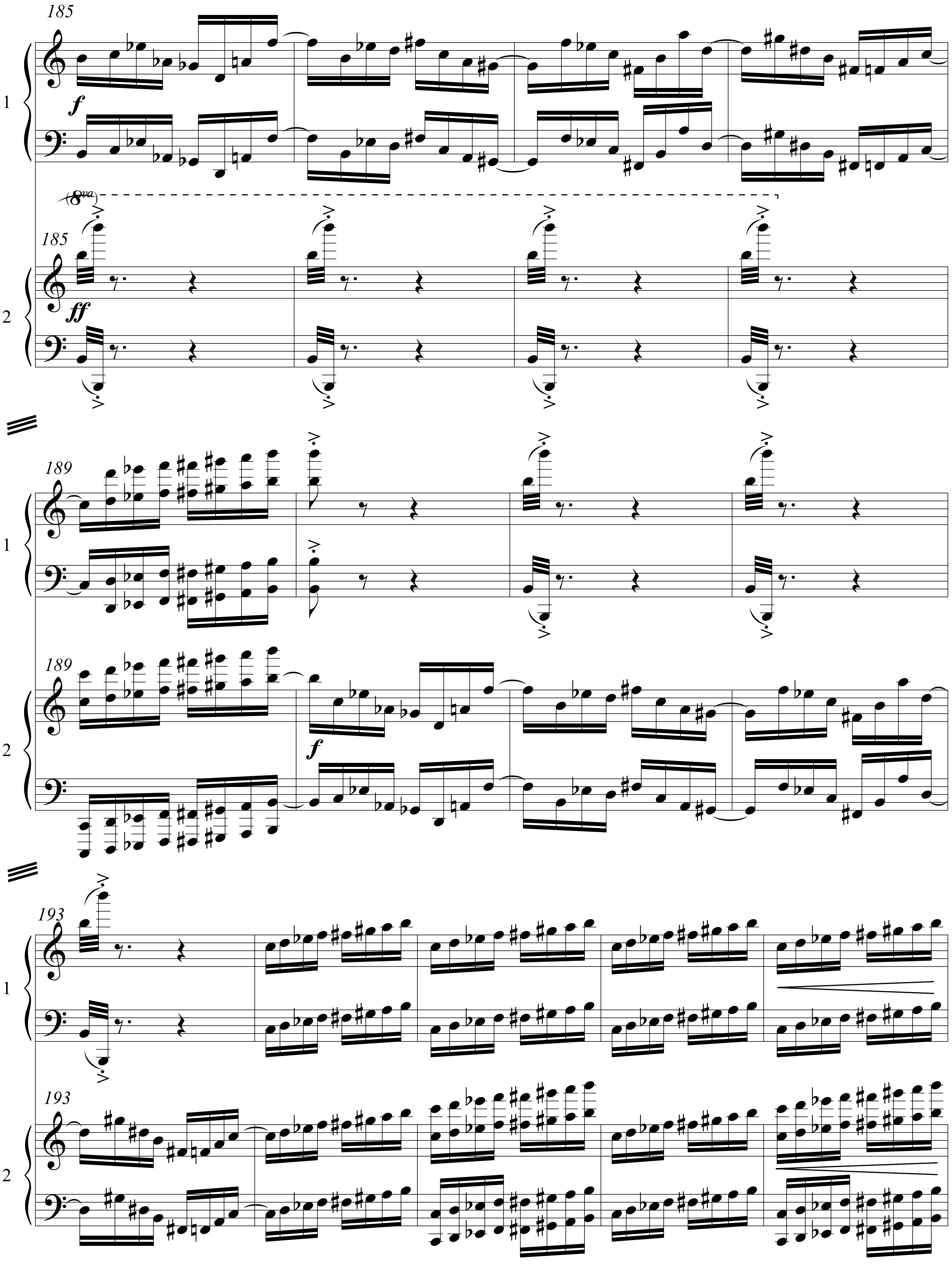

121
1
2
3
mfp
129
8va
pp
mp
135
(8va)
mp

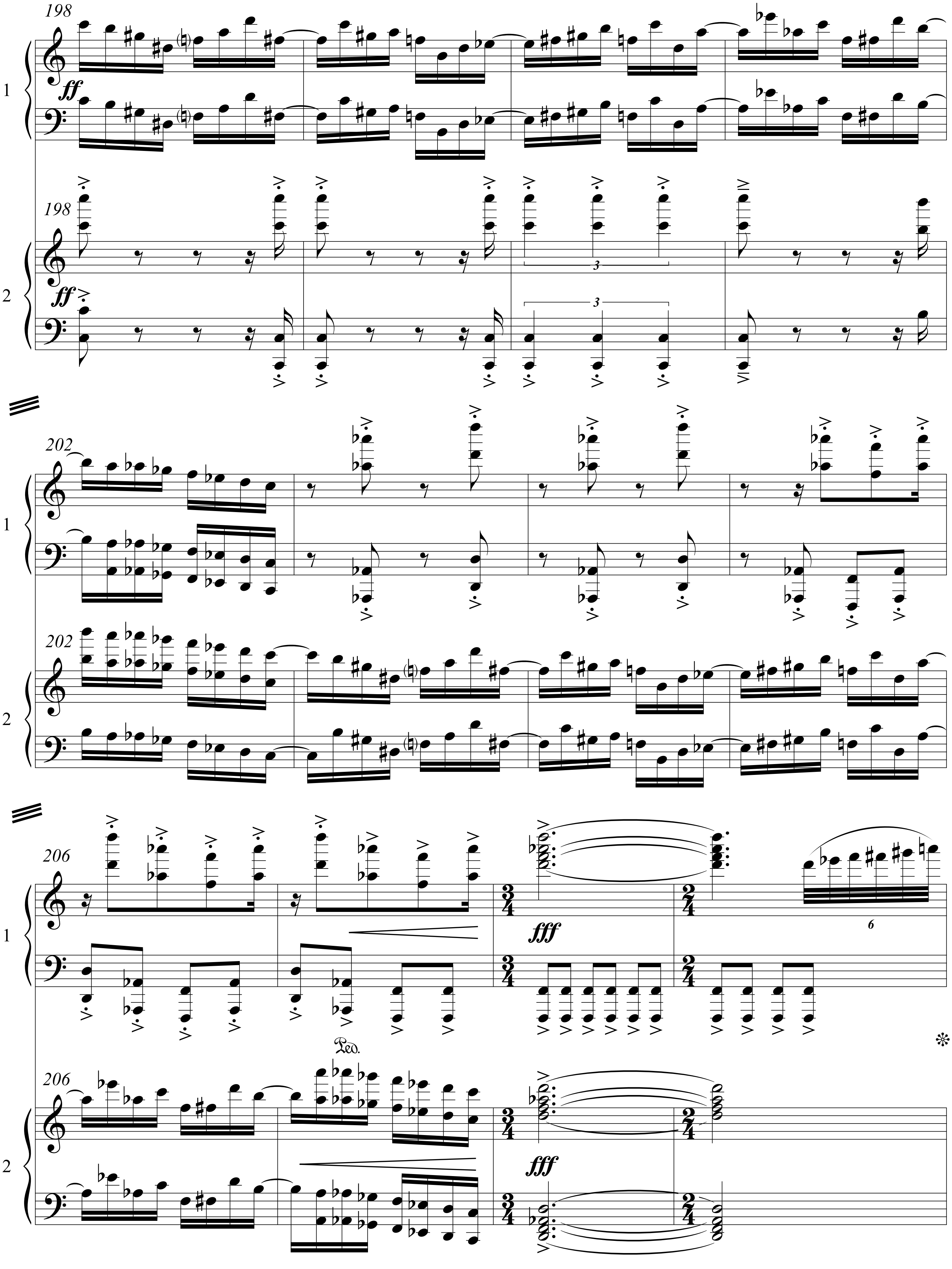

(8va)
mf
mf
8vb
mf
(8vb)

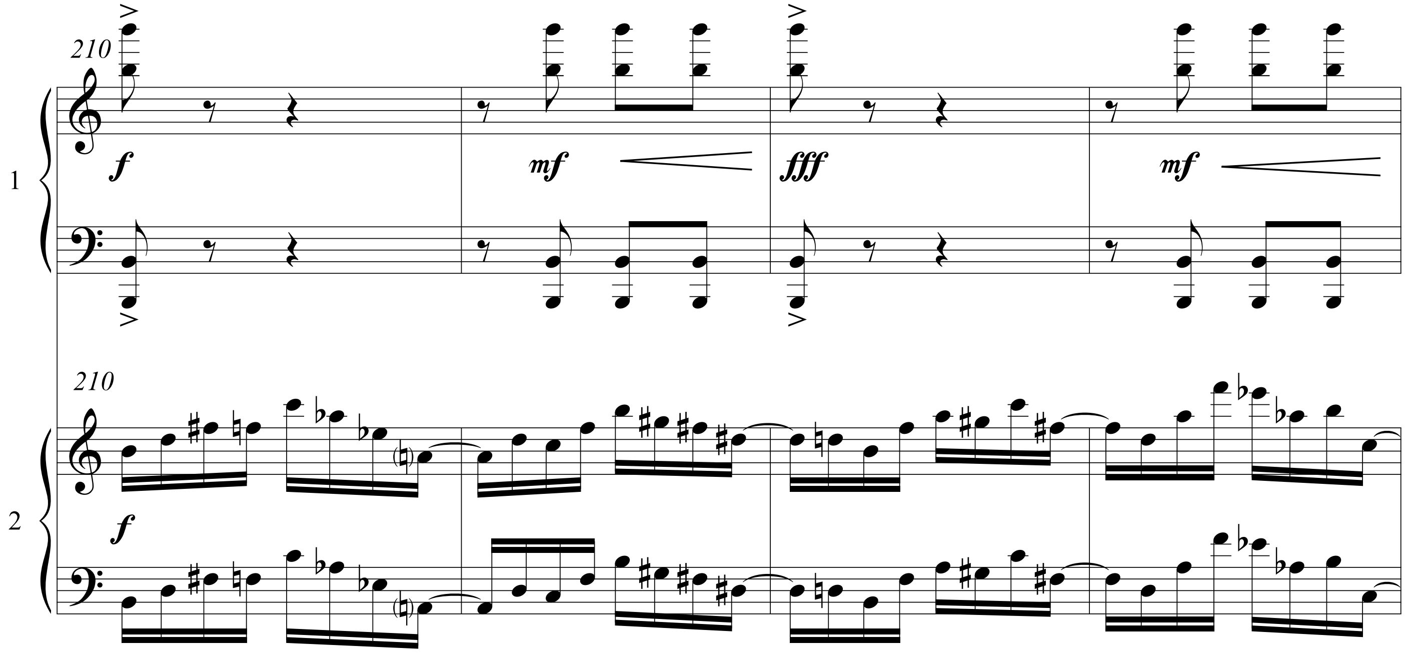

mf
(8vb)
(8vb)
(8vb)

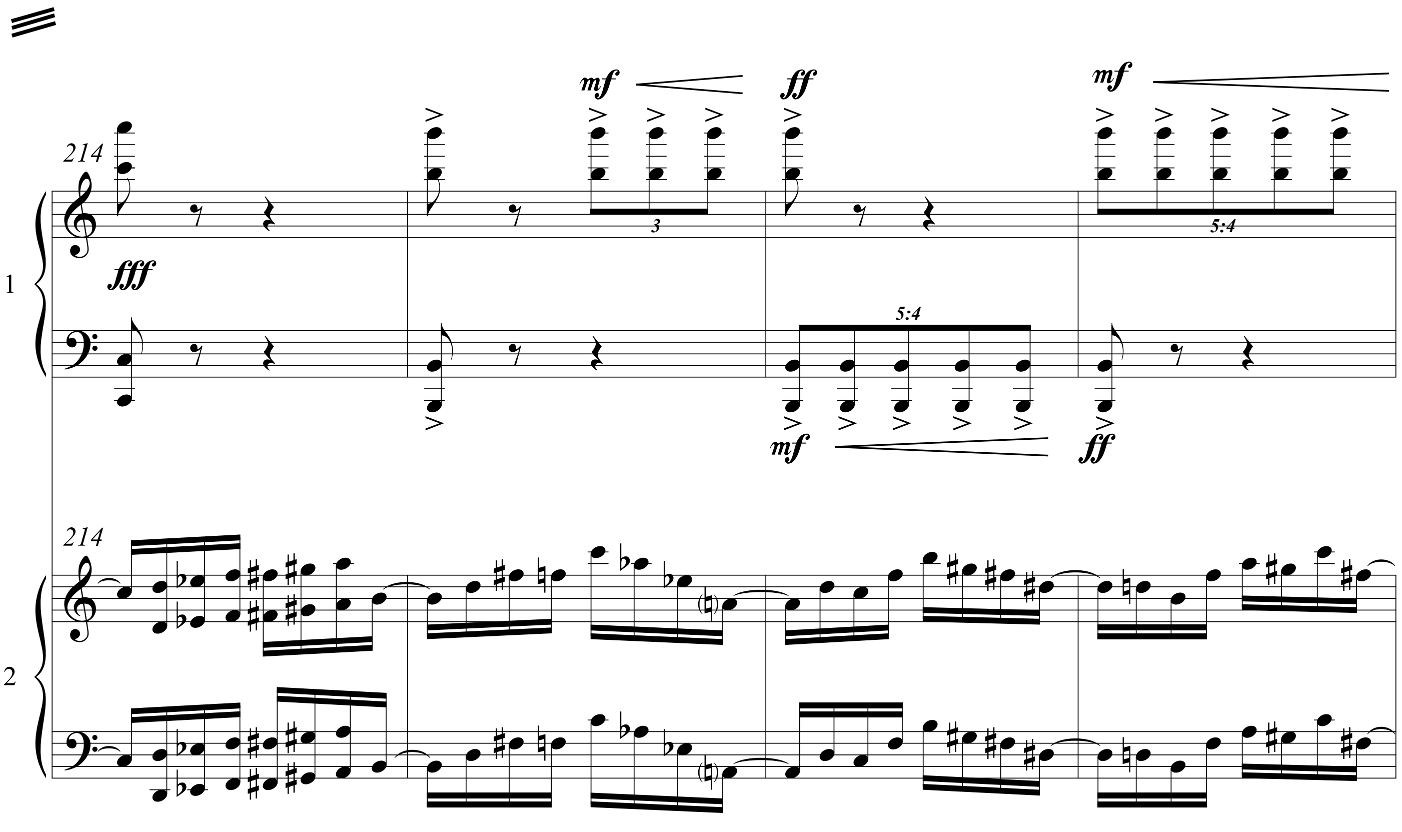

8va
f
(8va)

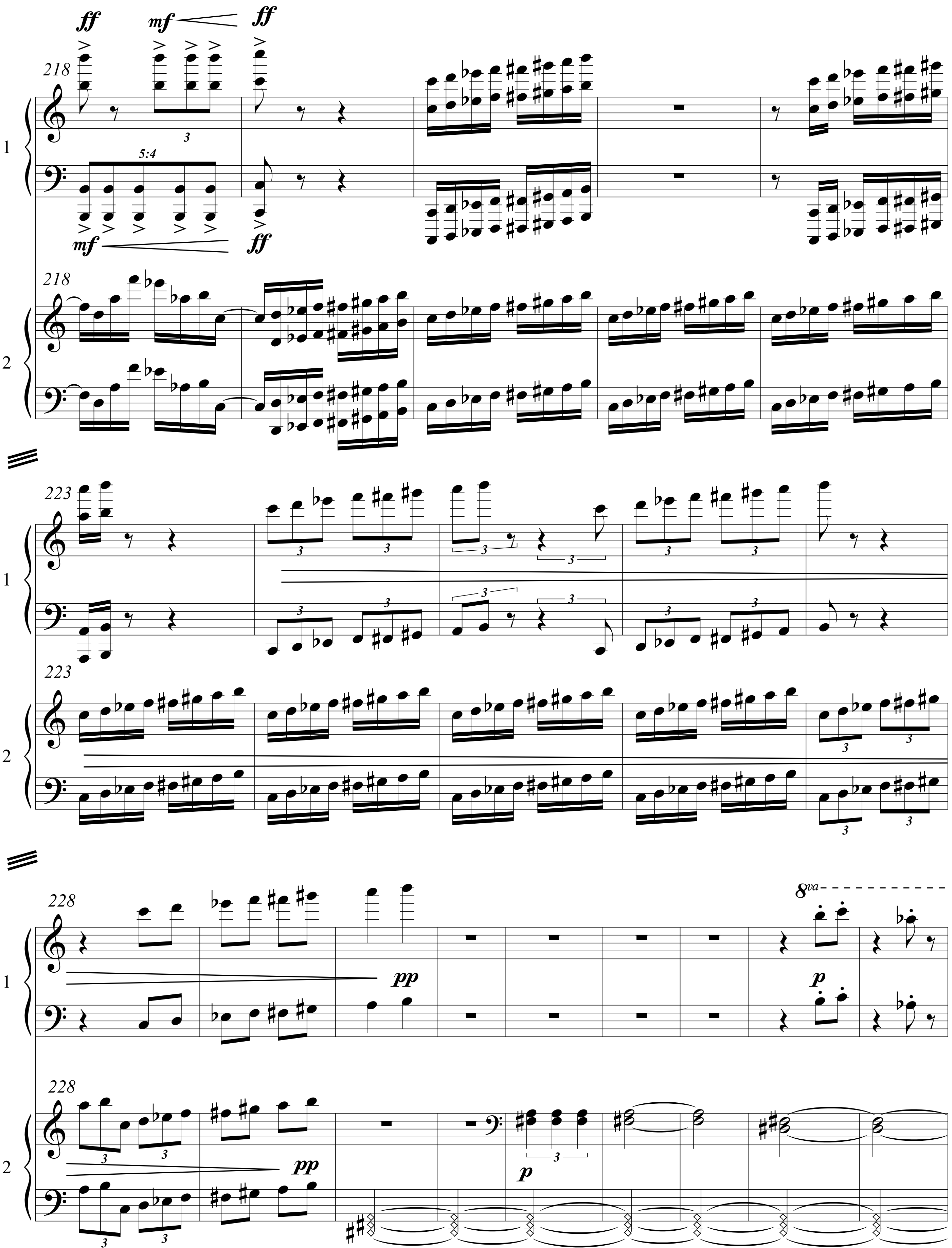

(8va)
176
1
ff
176
2
ff
(8va)
180
1
fp
3/4
2/4
180
2
fp
8va

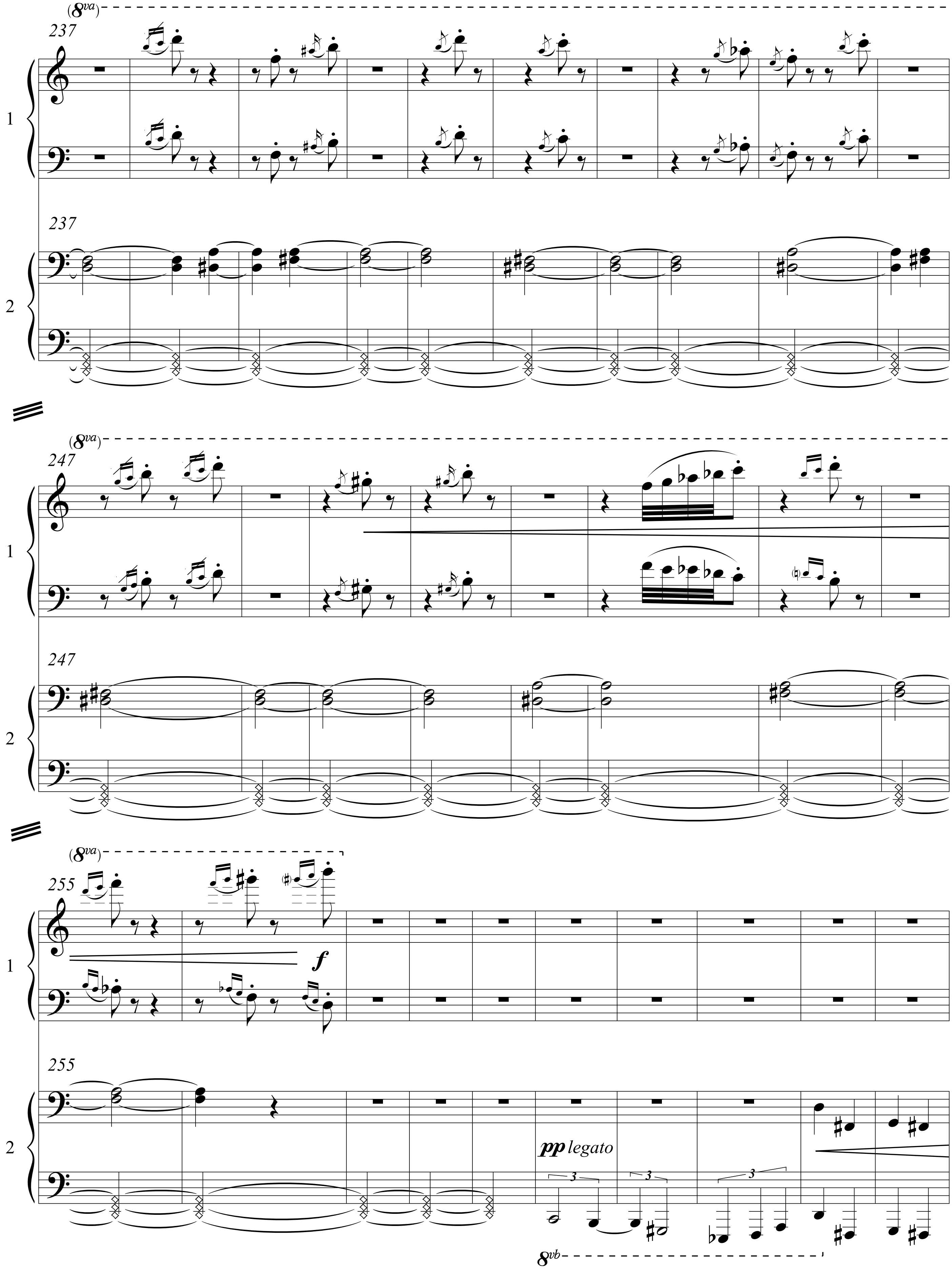

(8va)
ff
f

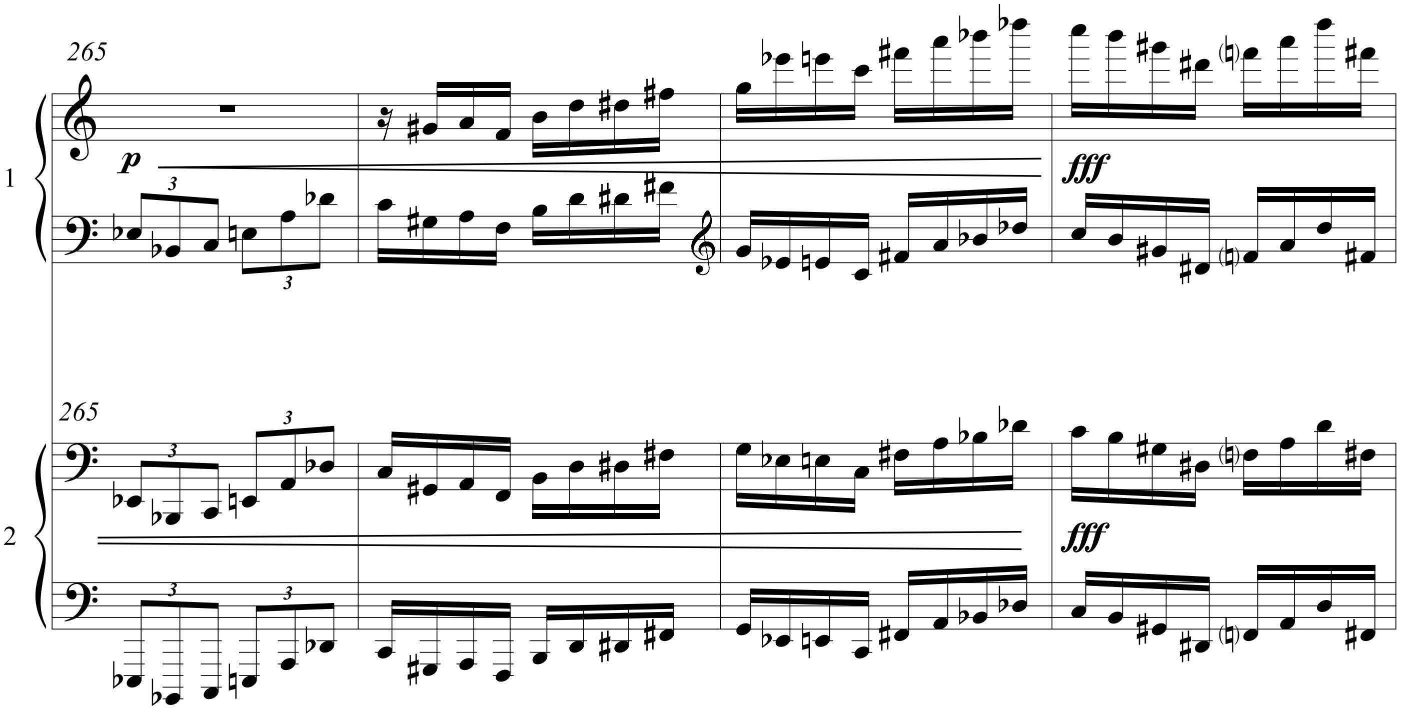

ff
ff
fff
❊
Ped.
fff

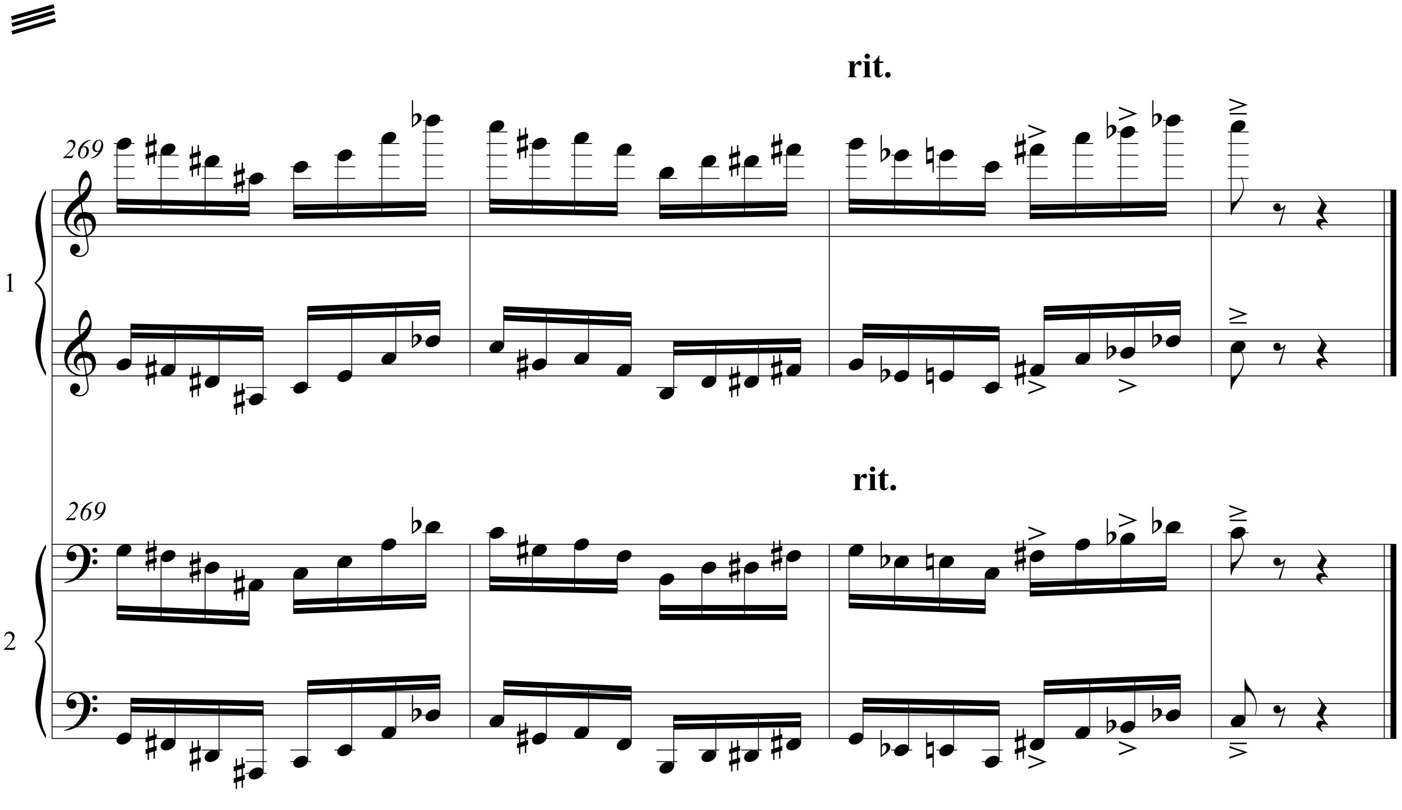

210
1
2
f
mf
fff
mf
210
f


214
1
2
fff
mf
ff
mf
3
5:4
5:4
mf
ff
214

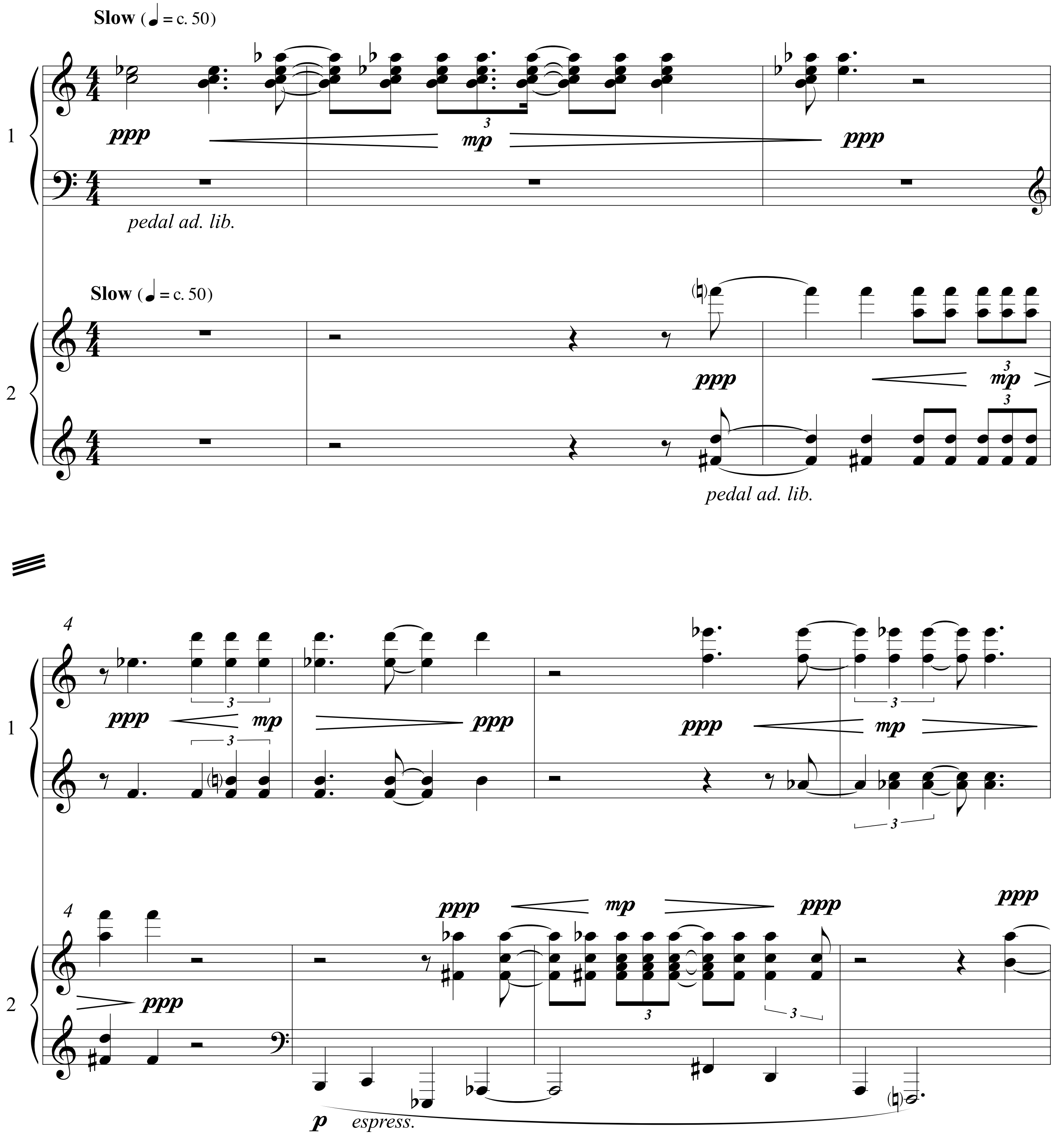

ff
mf
ff
218
5:4
3
mf
ff
1
218
2
223
3
1
223
2
228
8va
pp
p
1
228
pp
p
3
2
keys silently depressed

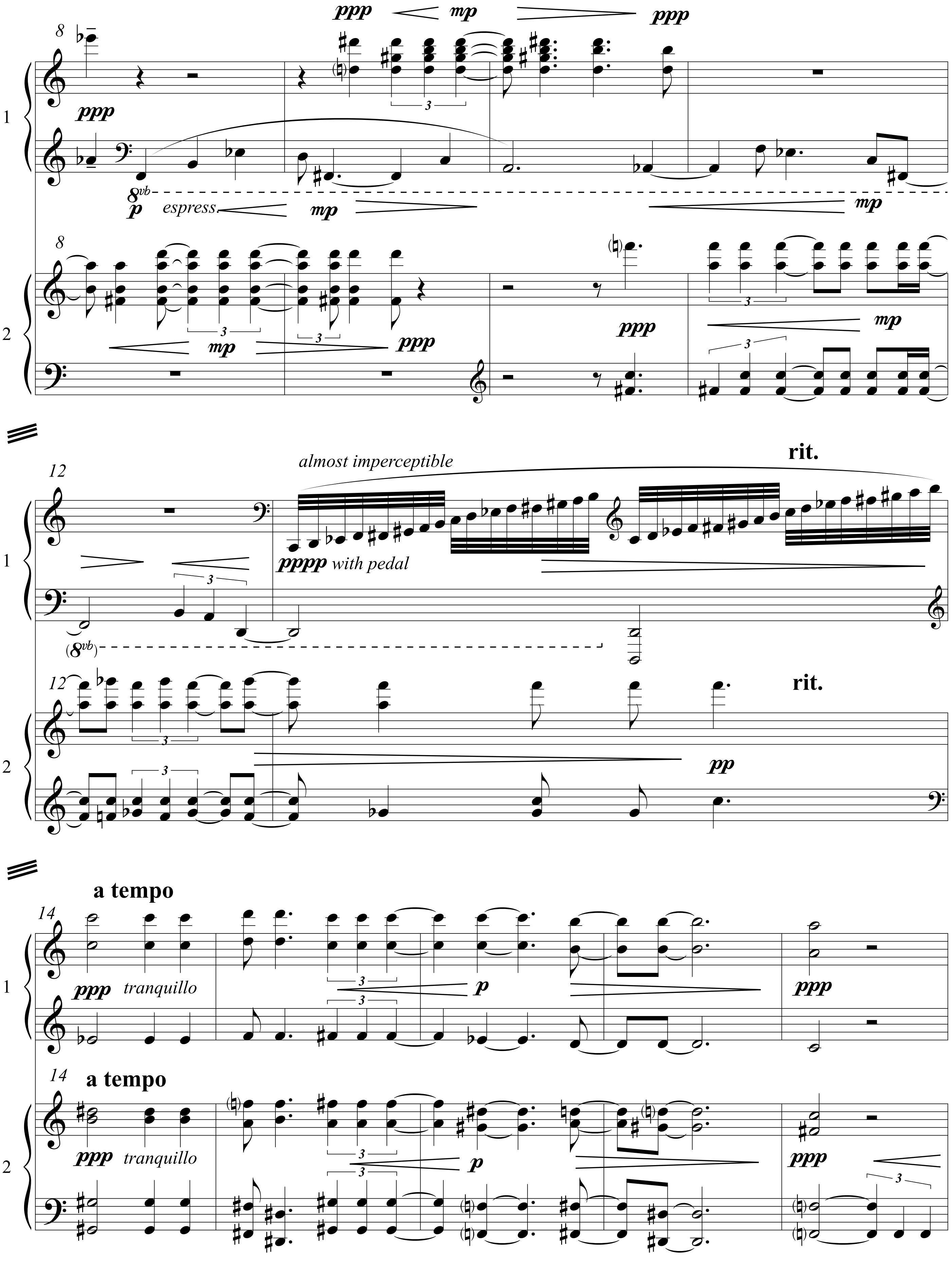

(8va)
(8va)
(8va)
f
pp legato
8vb

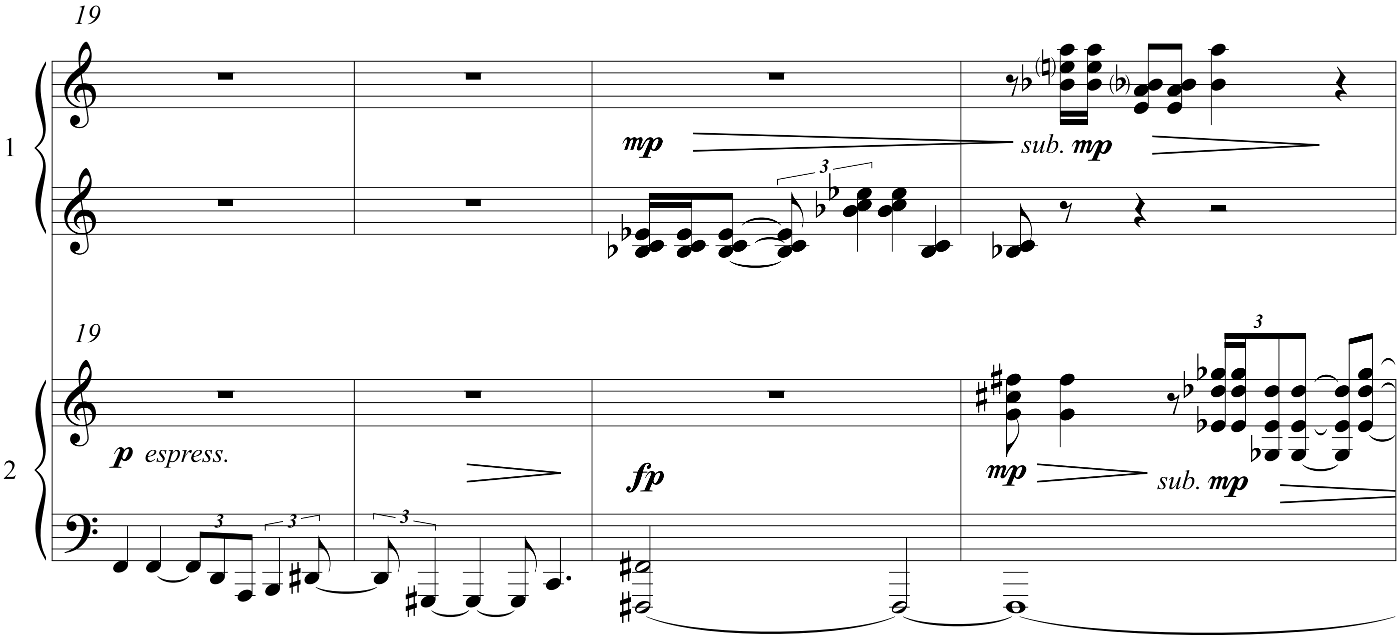

p
fff

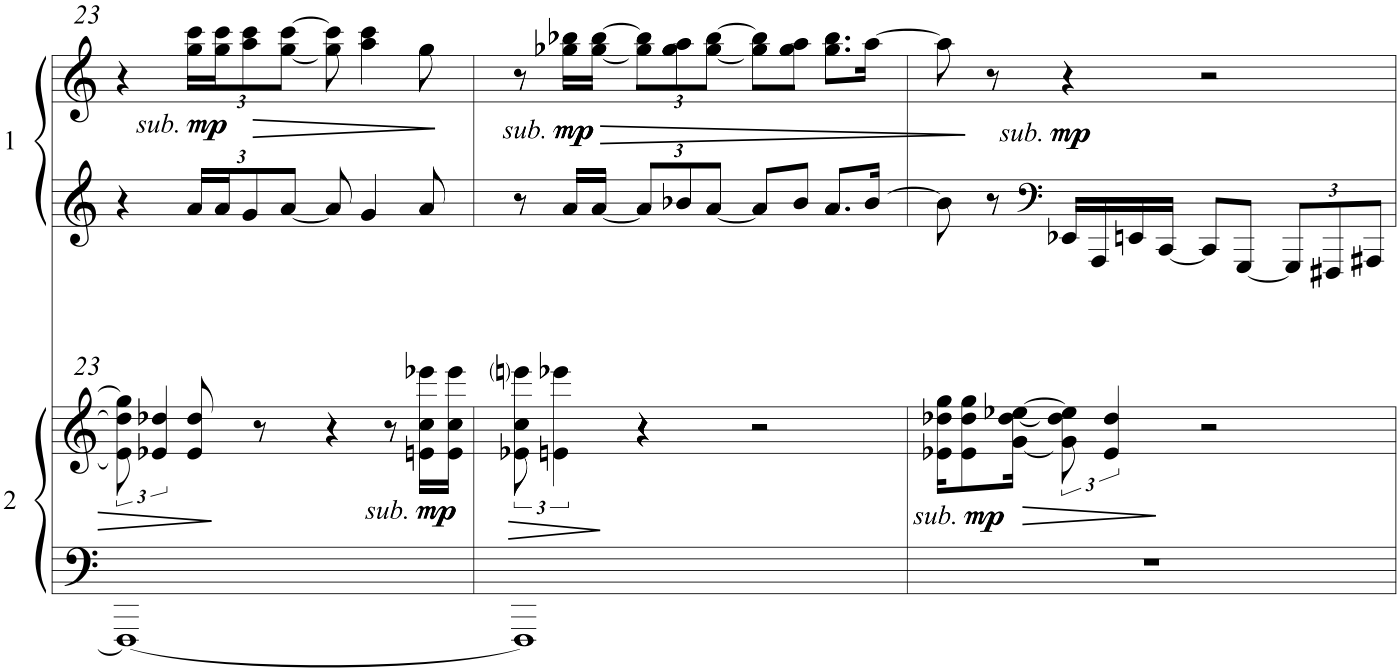

rit.

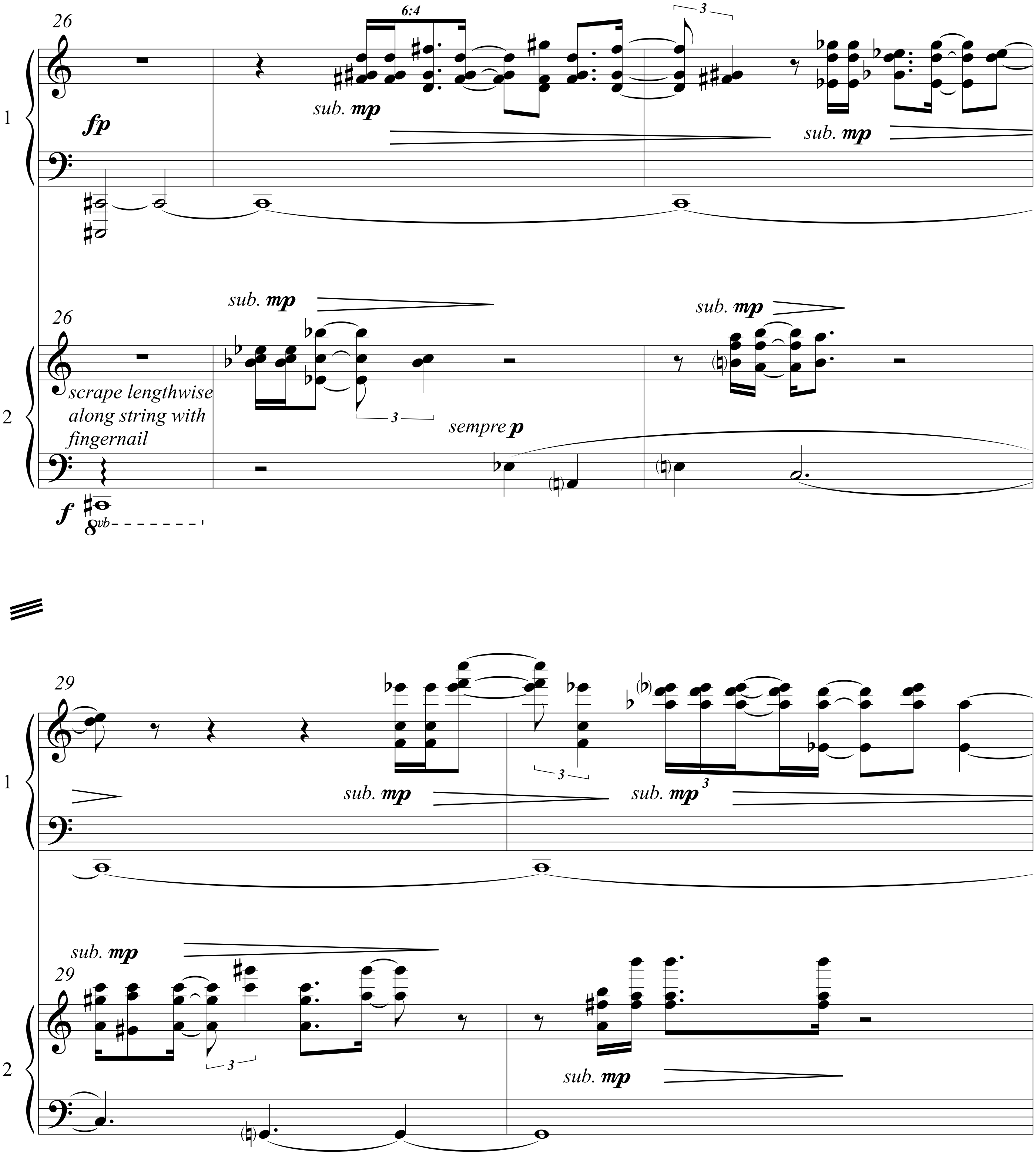
Octulus C

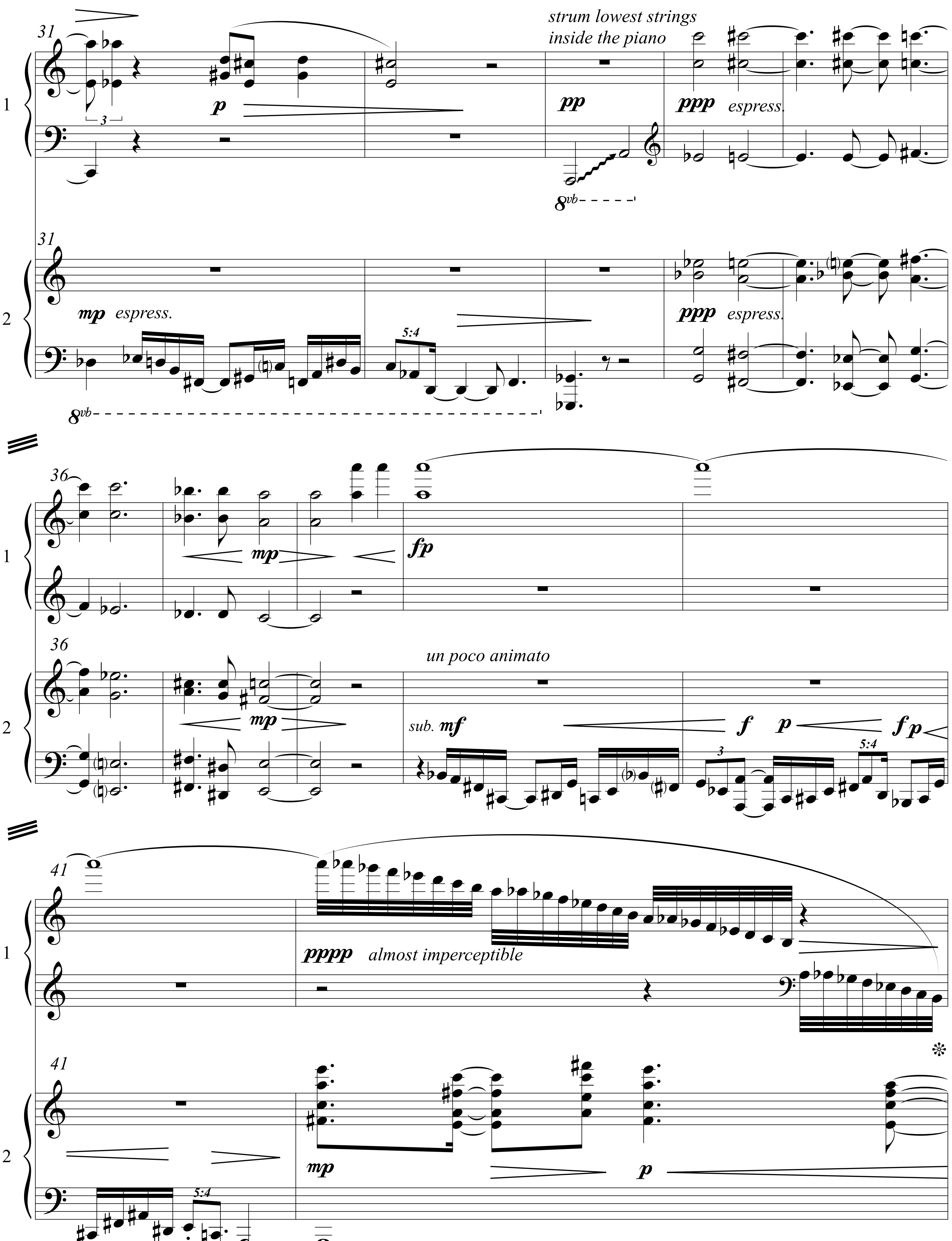

Slow (♩ = c. 50)
1
ppp
mp
ppp
3
pedal ad. lib.
Slow (♩ = c. 50)
2
ppp
mp
3
pedal ad. lib.
4
ppp
mp
ppp
3
4
ppp
mp
ppp
3
p
espress.

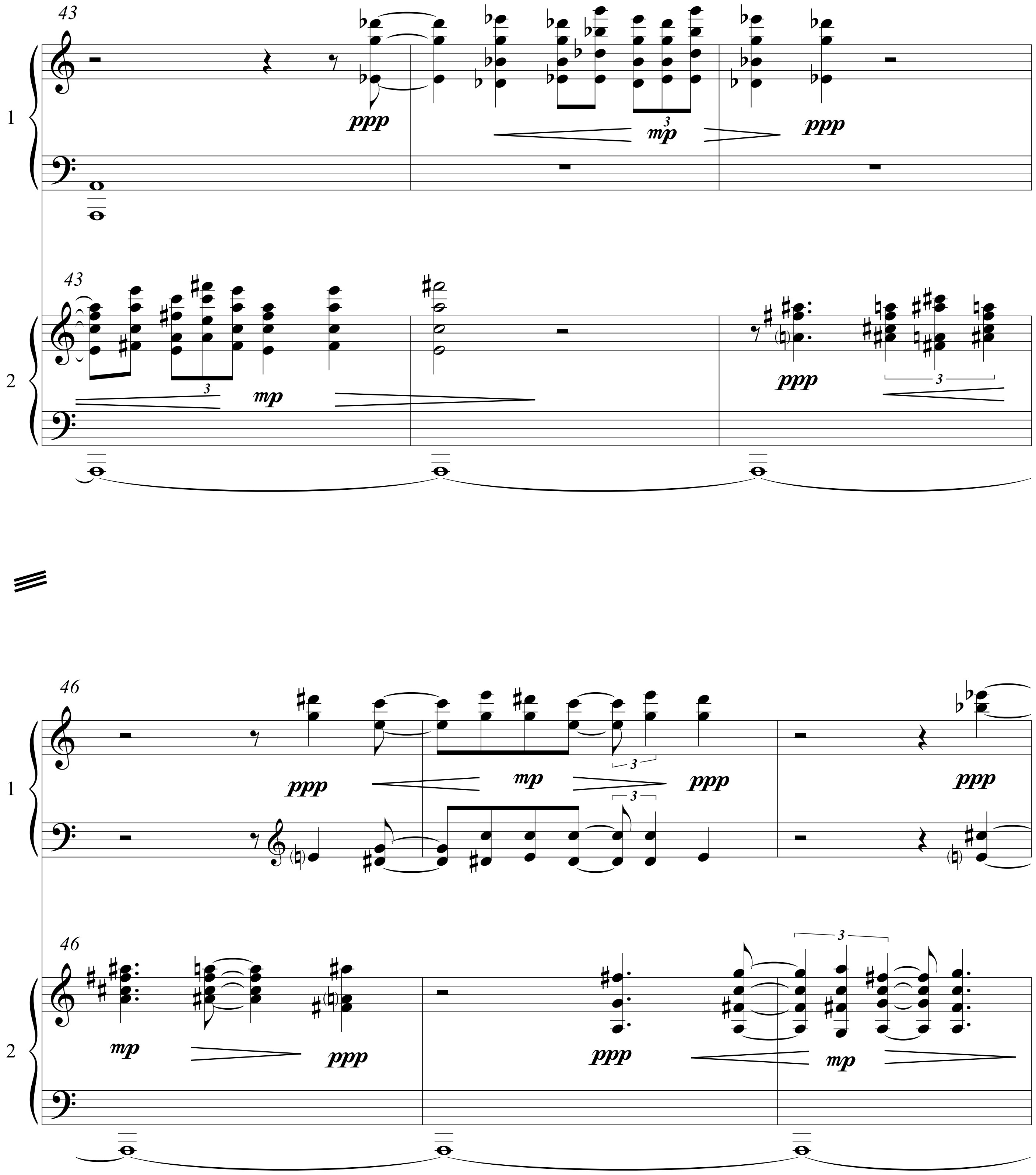

8
ppp
mp
ppp
1
8vb
p espress.
mp
mp
8
2
mp
ppp
ppp
mp
12
almost imperceptible
rit.
pppp with pedal
(8vb)
12
rit.
pp
a tempo
14
ppp tranquillo
p
ppp
14
a tempo
ppp tranquillo
p
ppp
3

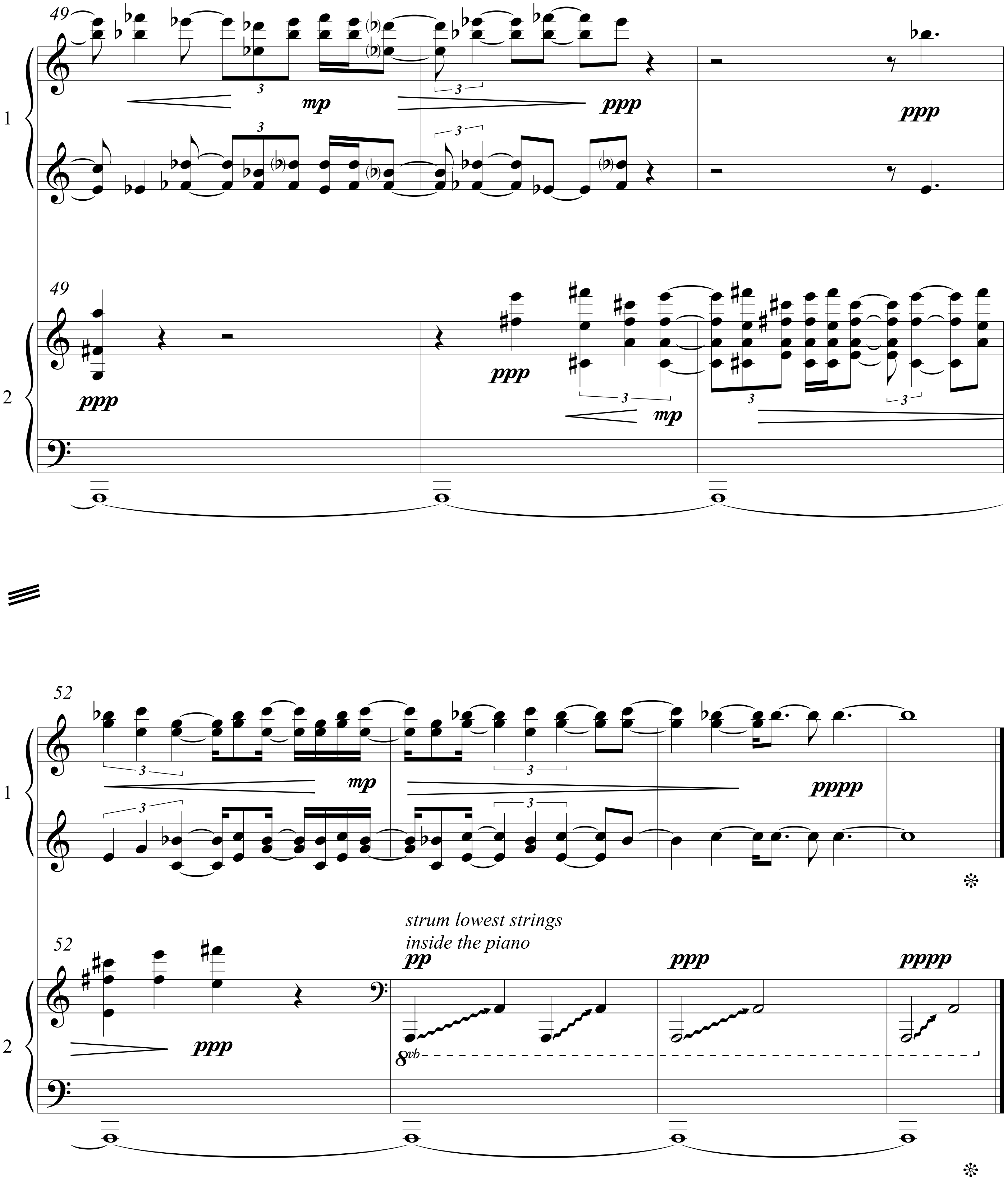

19
1
2
19
p espress.
fp
mp
sub. mp
mp
sub. mp
3


23
1
2
23
sub. mp
sub. mp
sub. mp
sub. mp
sub. mp
3

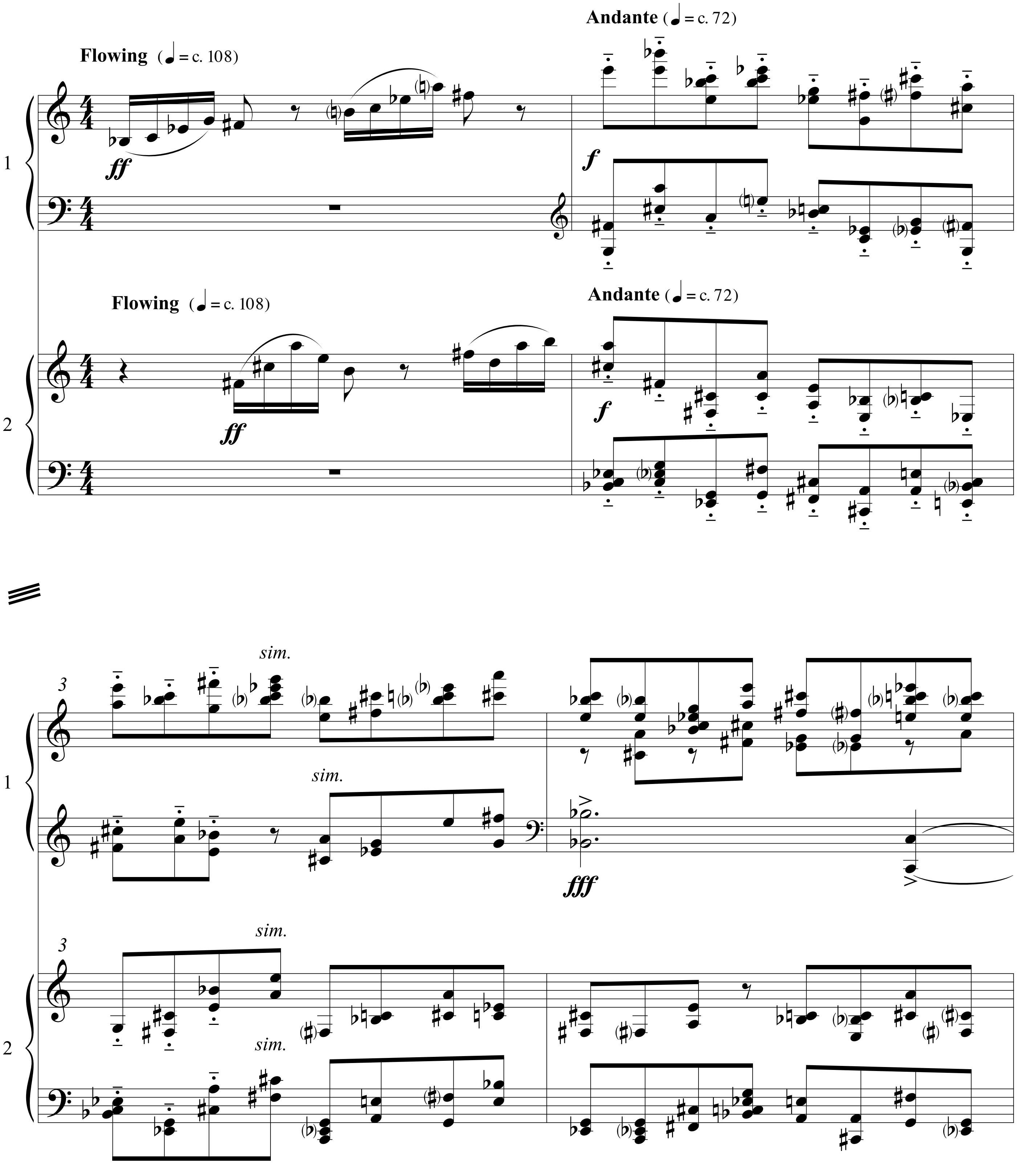

26
6:4
3
1
fp
sub. mp
sub. mp
26
2
sub. mp
sub. mp
3
scrape lengthwise along string with fingernail
sempre p
f
8vb
29
3
sub. mp
sub. mp
3
29
sub. mp
3
sub. mp

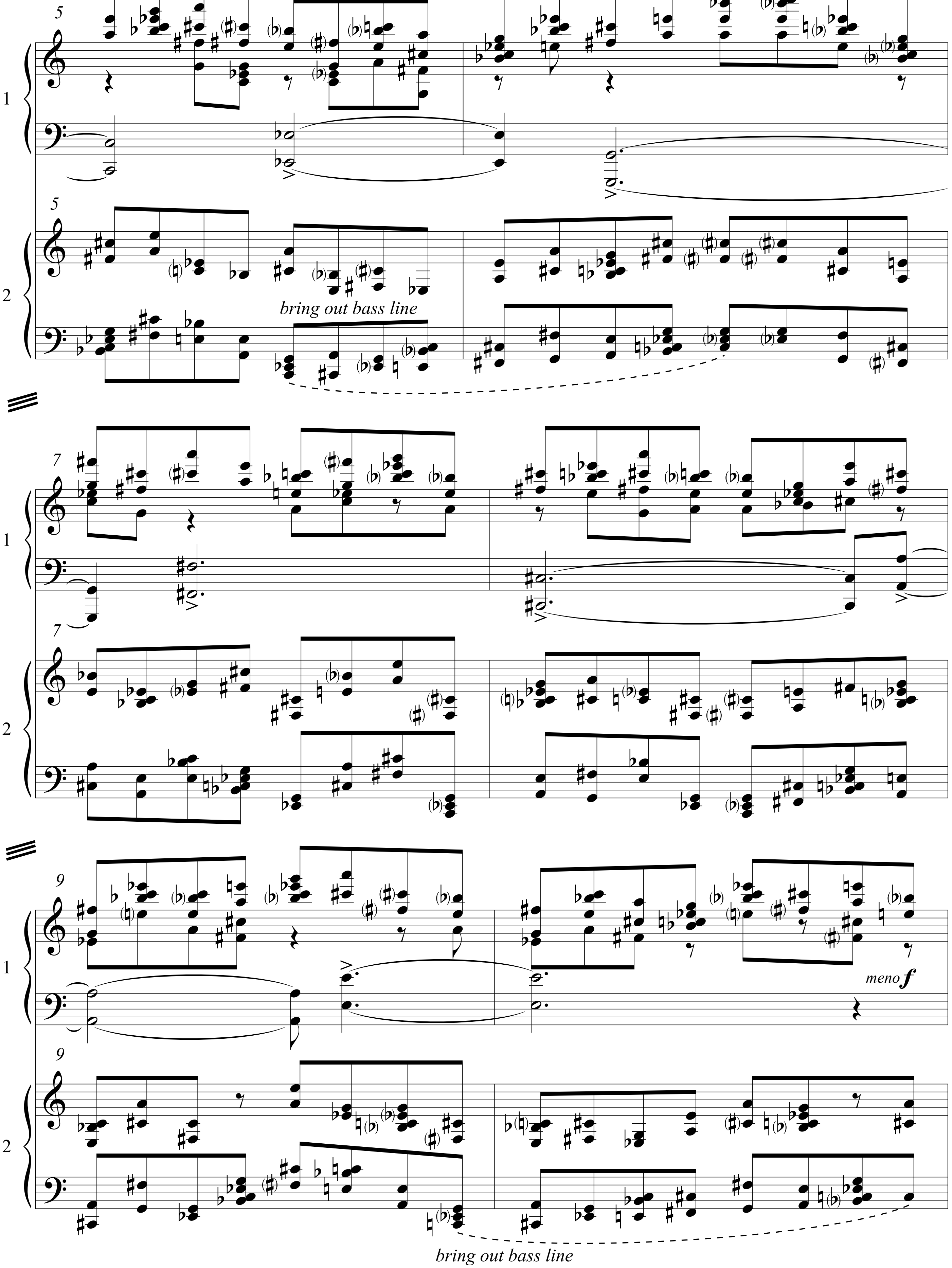

31
1
2
strum lowest strings
inside the piano
p
pp
ppp
espress.
8vb
3
mp espress.
5:4
36
mp
fp
un poco animato
sub. mf
f
p
fp
41
pppp almost imperceptible
❊
mp
p
S.P.

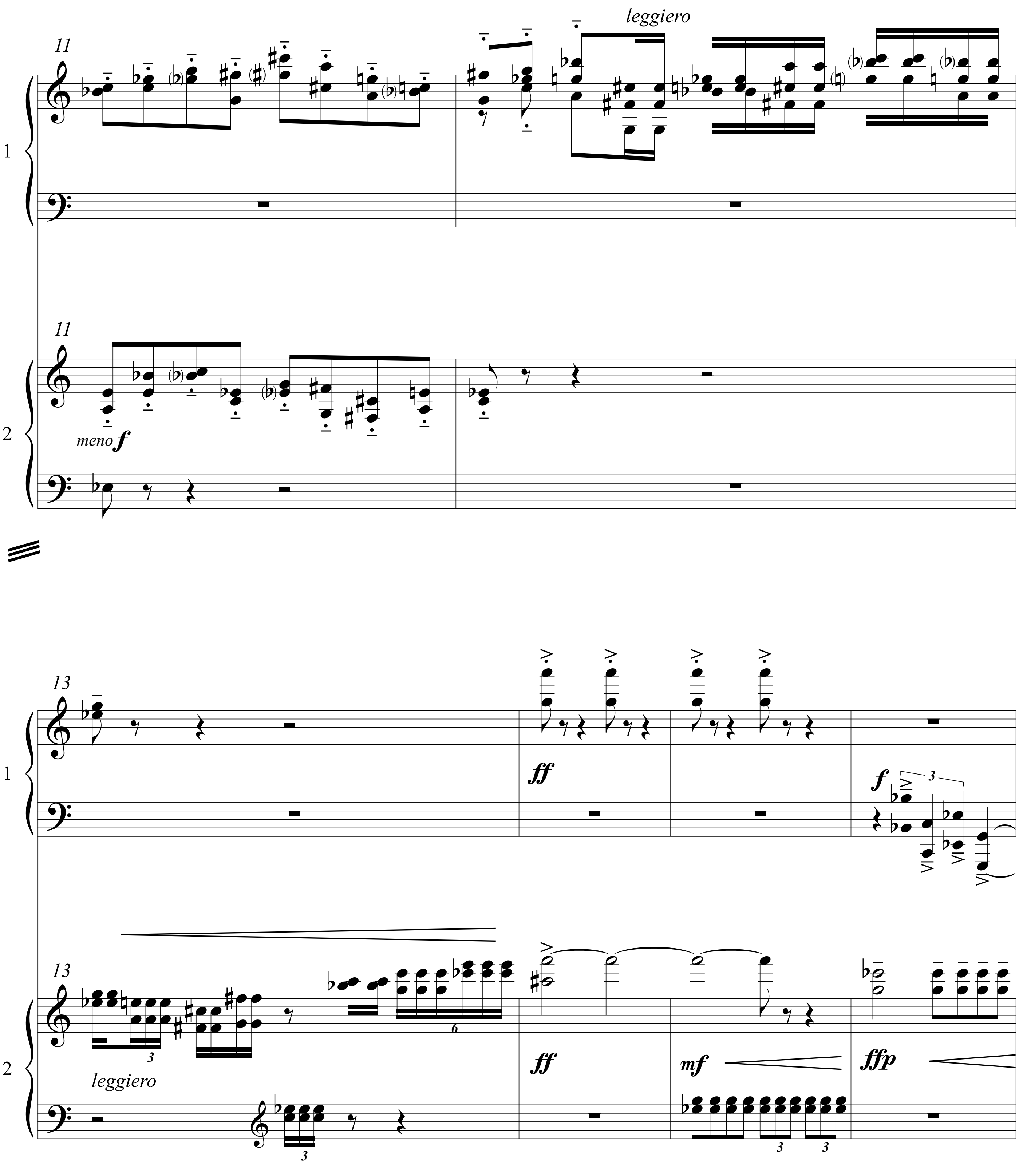

43
1
ppp
mp
3
ppp
43
2
3
mp
ppp
3
46
1
ppp
mp
3
ppp
3
ppp
46
2
mp
ppp
ppp
3
mp

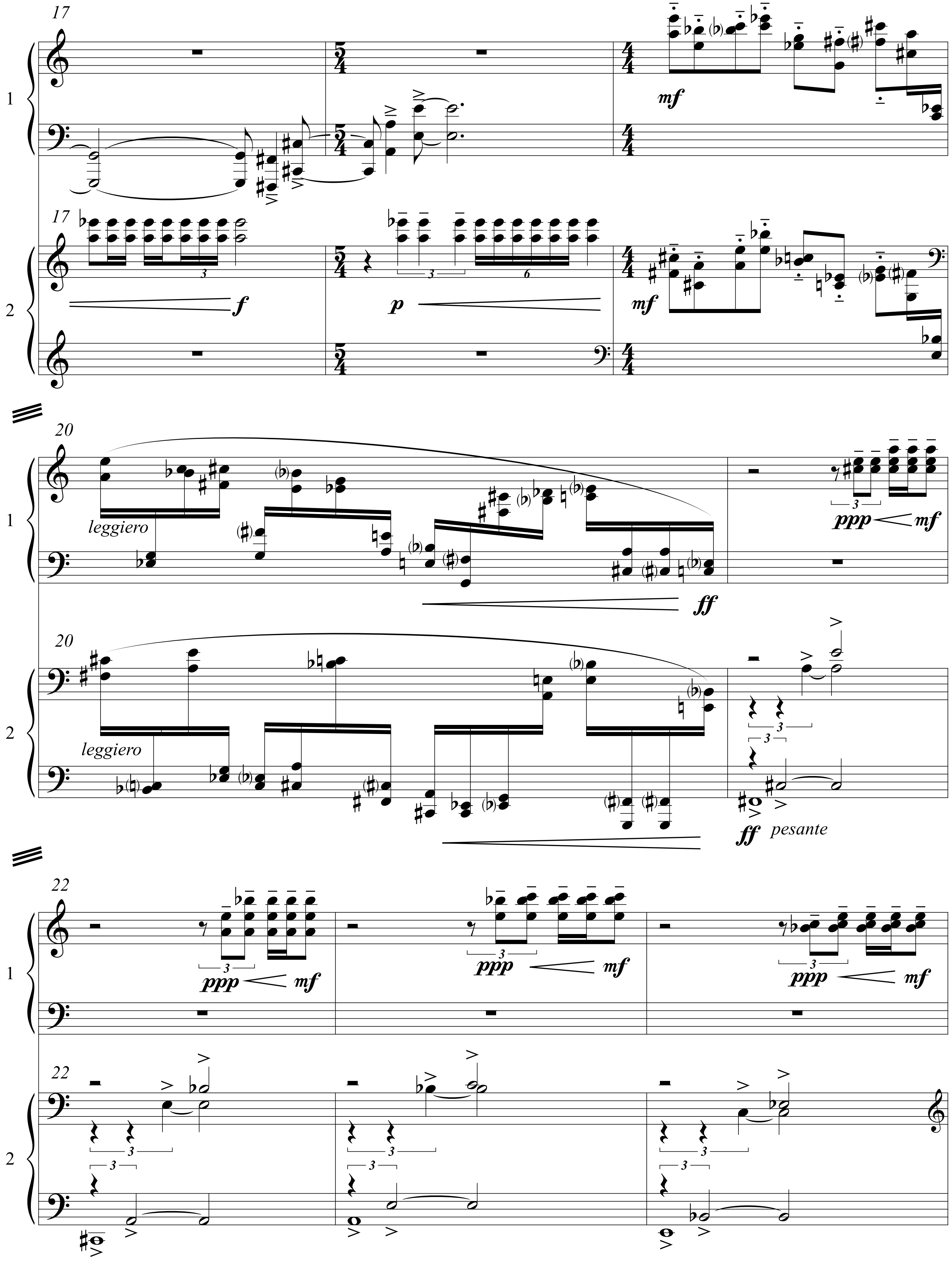

49
1
mp
ppp
ppp
3
49
2
ppp
ppp
mp
3
52
1
mp
pppp
3
52
2
ppp
strum lowest strings
inside the piano
pp
ppp
pppp
8vb
❊
❊

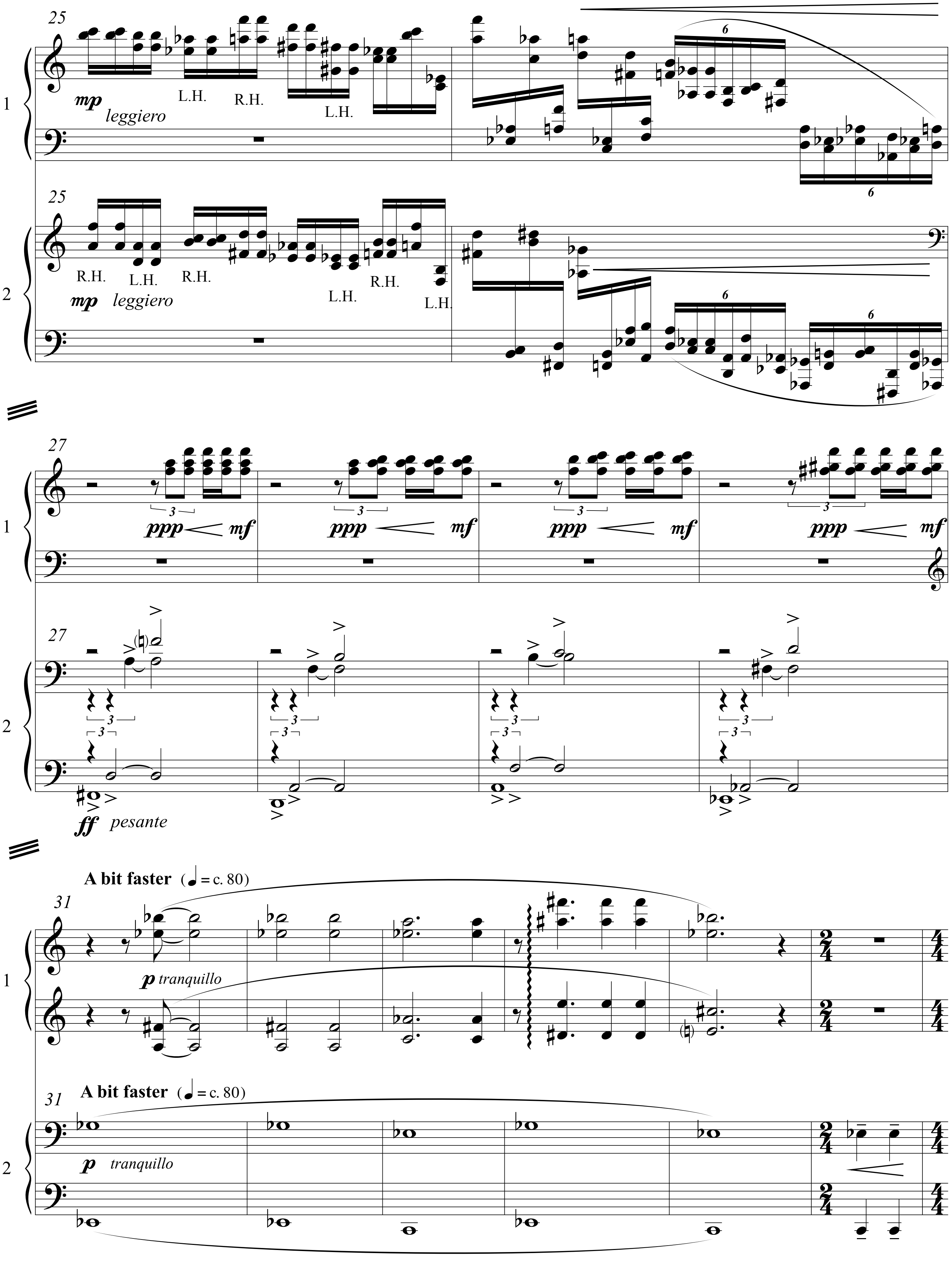
Octulus D

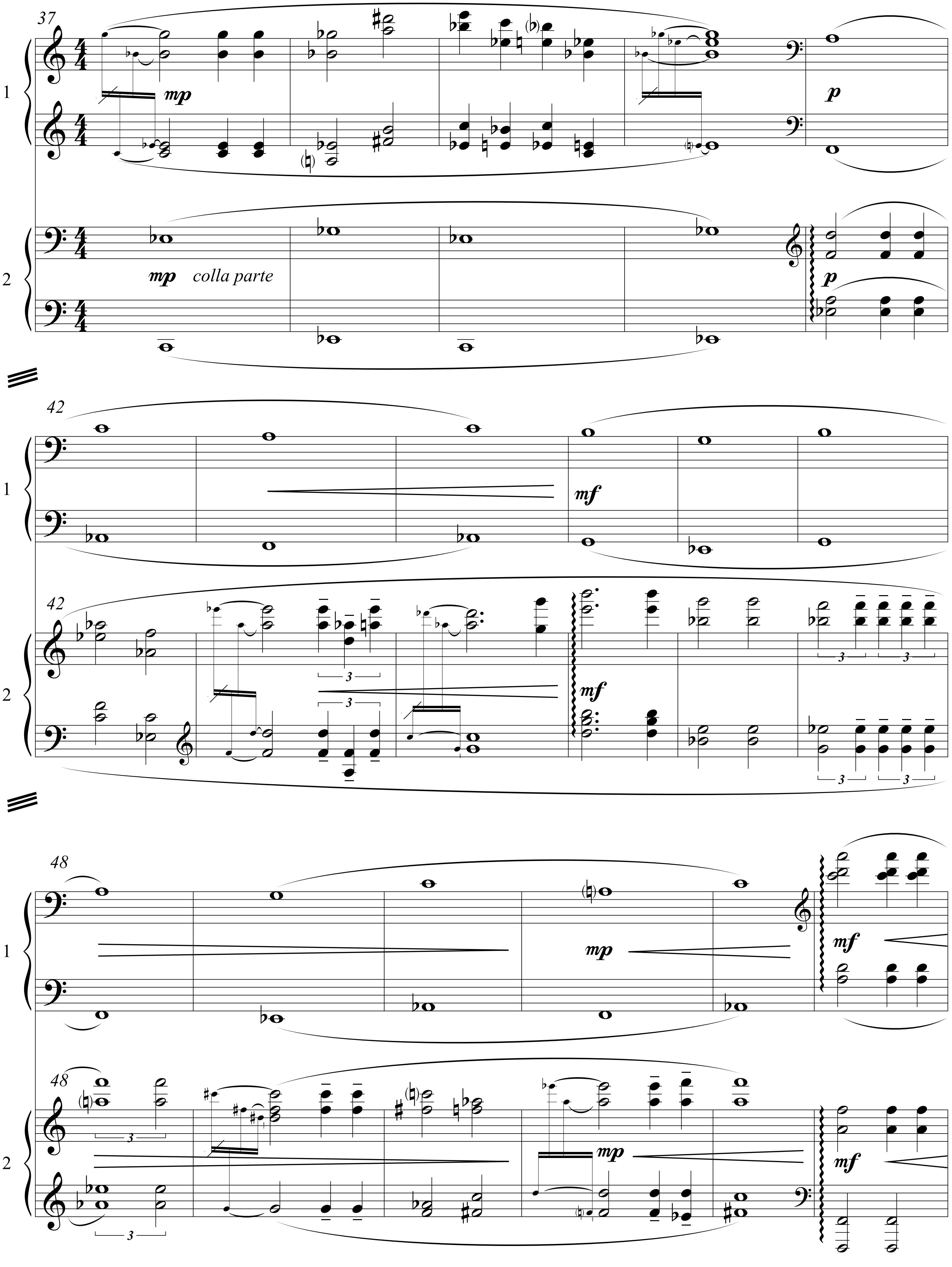

Flowing (♩ = c. 108)
Andante (♩ = c. 72)
ff
f
1
Flowing (♩ = c. 108)
Andante (♩ = c. 72)
ff
f
2
3
sim.
sim.
fff
1
3
sim.
sim.
2

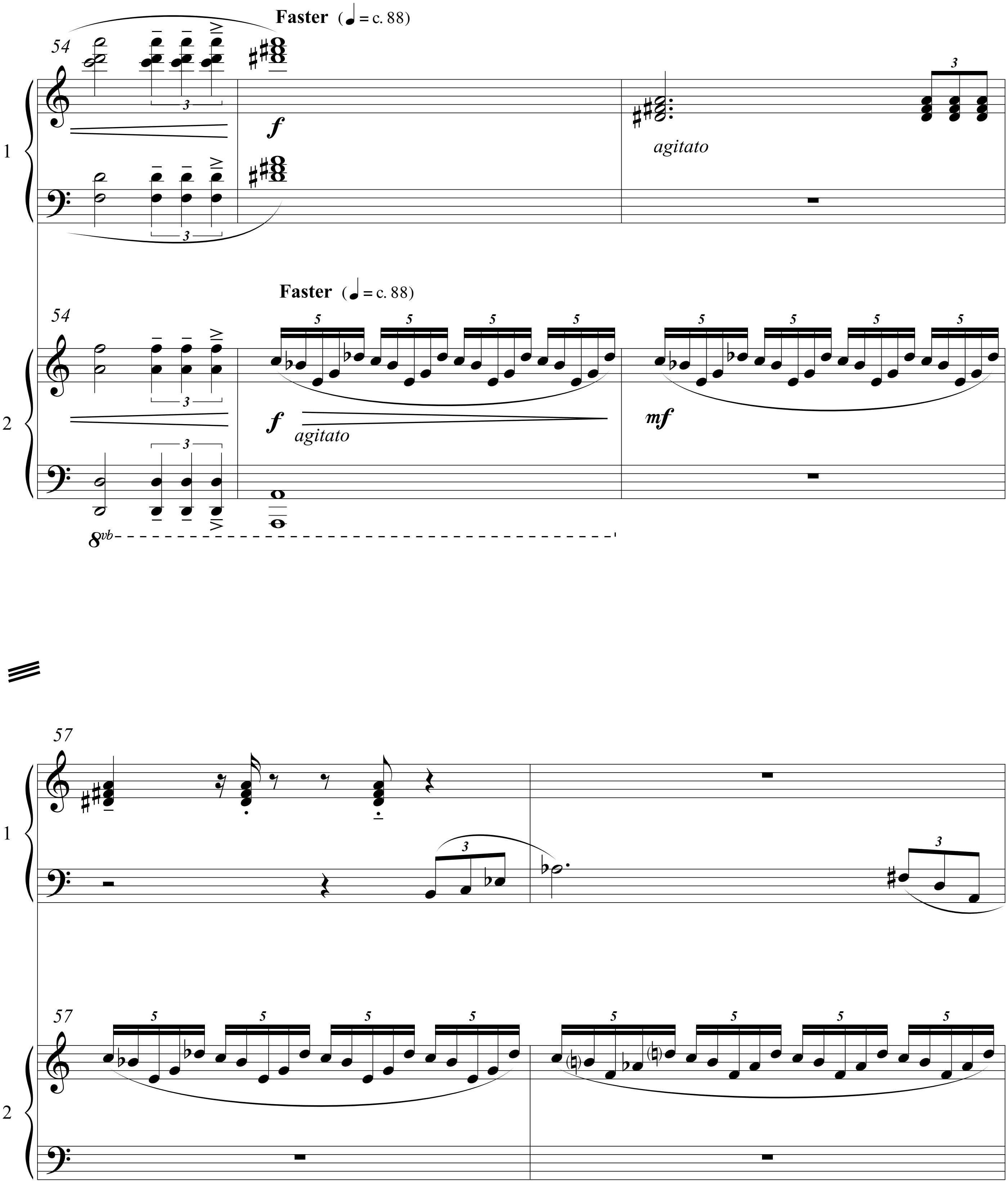

bring out bass line
meno f
bring out bass line

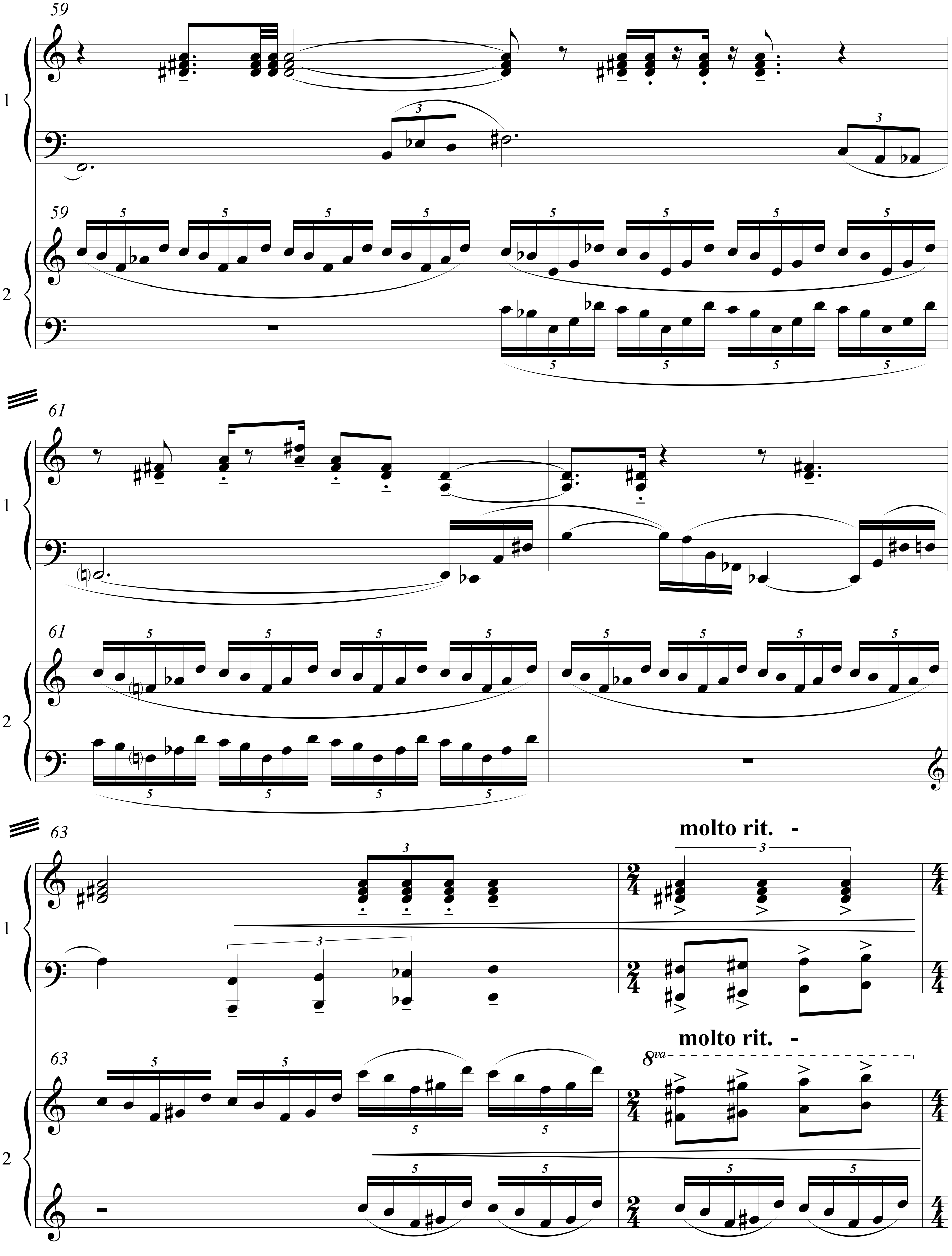

leggiero
meno f
ff
f
leggiero
ff
mf
ffp

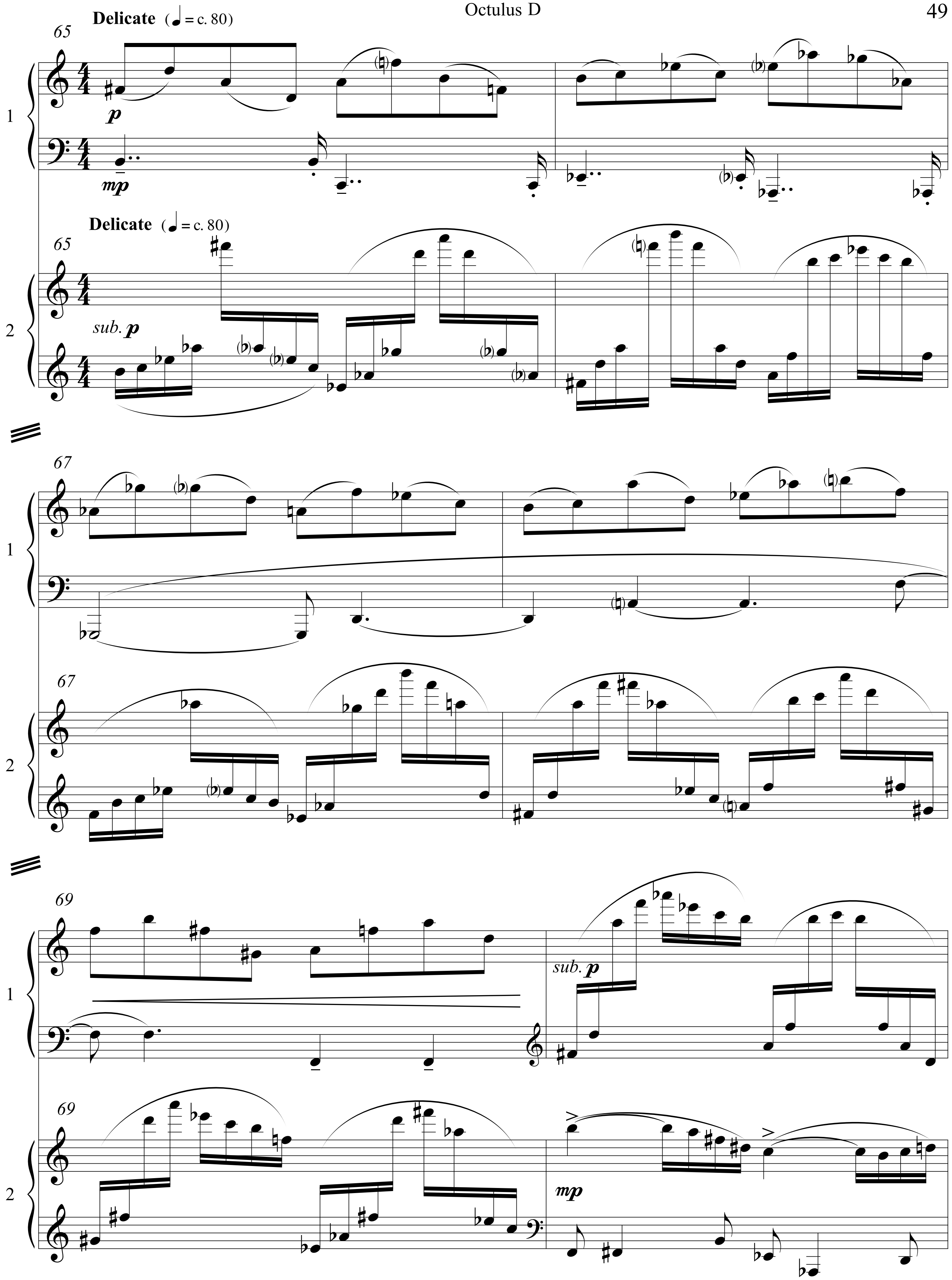

17
1
2
mf
f
p
3
6
20
leggiero
ppp
ff
pesante
22

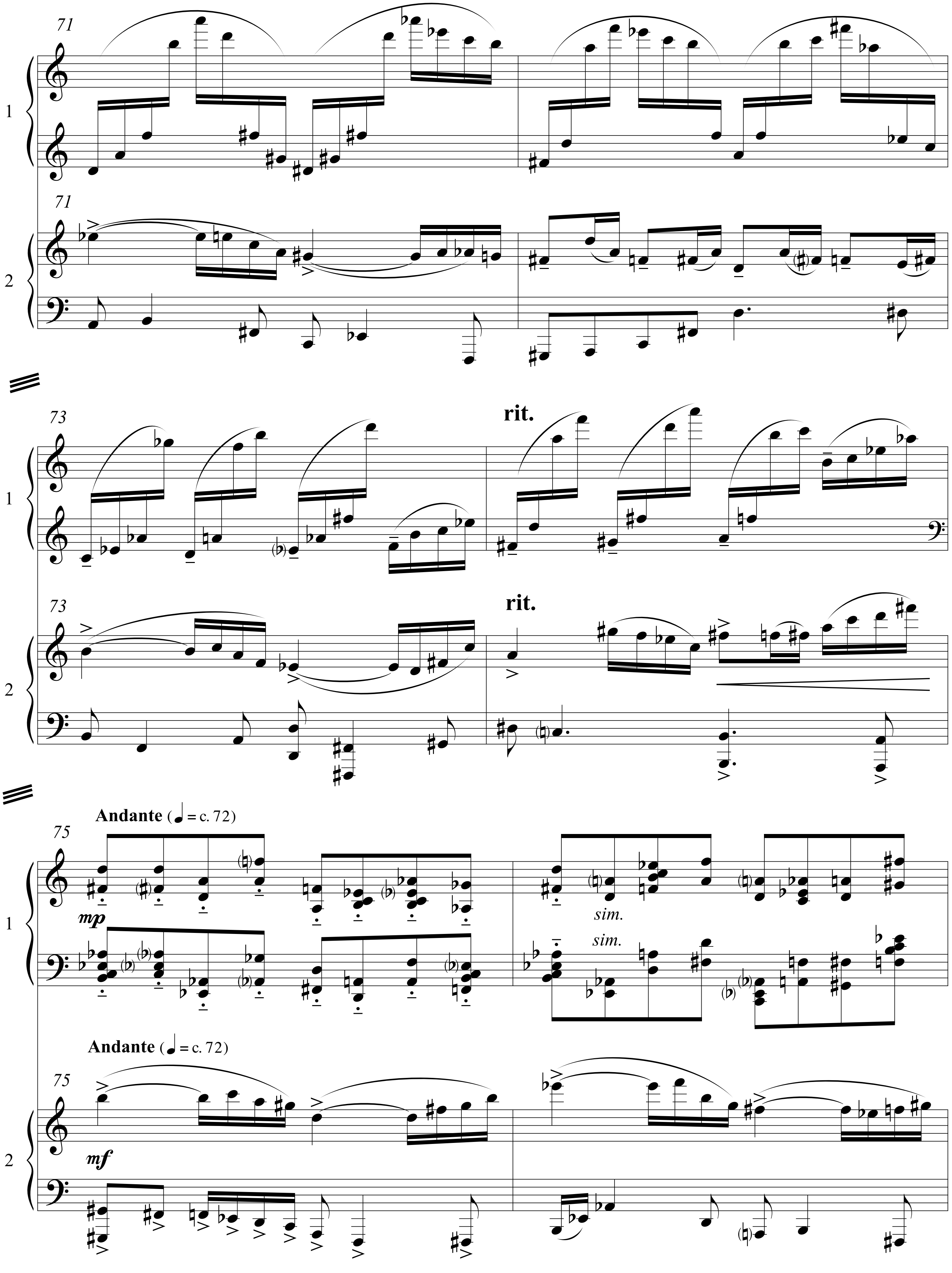

25
1
2
mp
leggiero
L.H.
R.H.
6
27
ppp
mf
ff
pesante
31
A bit faster (♩ = c. 80)
p tranquillo
p
tranquillo
2/4
4/4

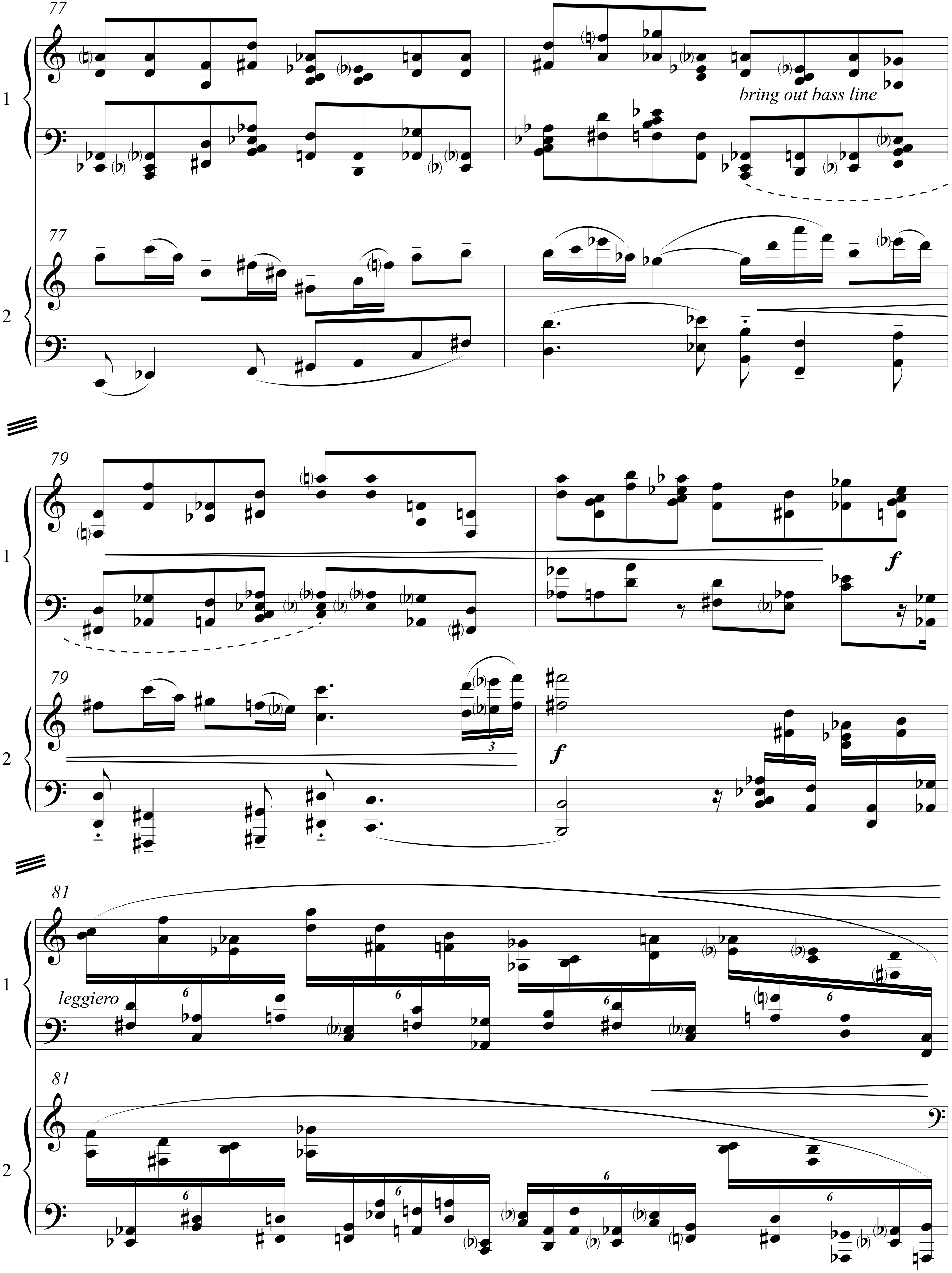

37
1
2
mp
mp
colla parte
p
p
42
42
mf
mf
3
48
48
mp
mp
mf
mf

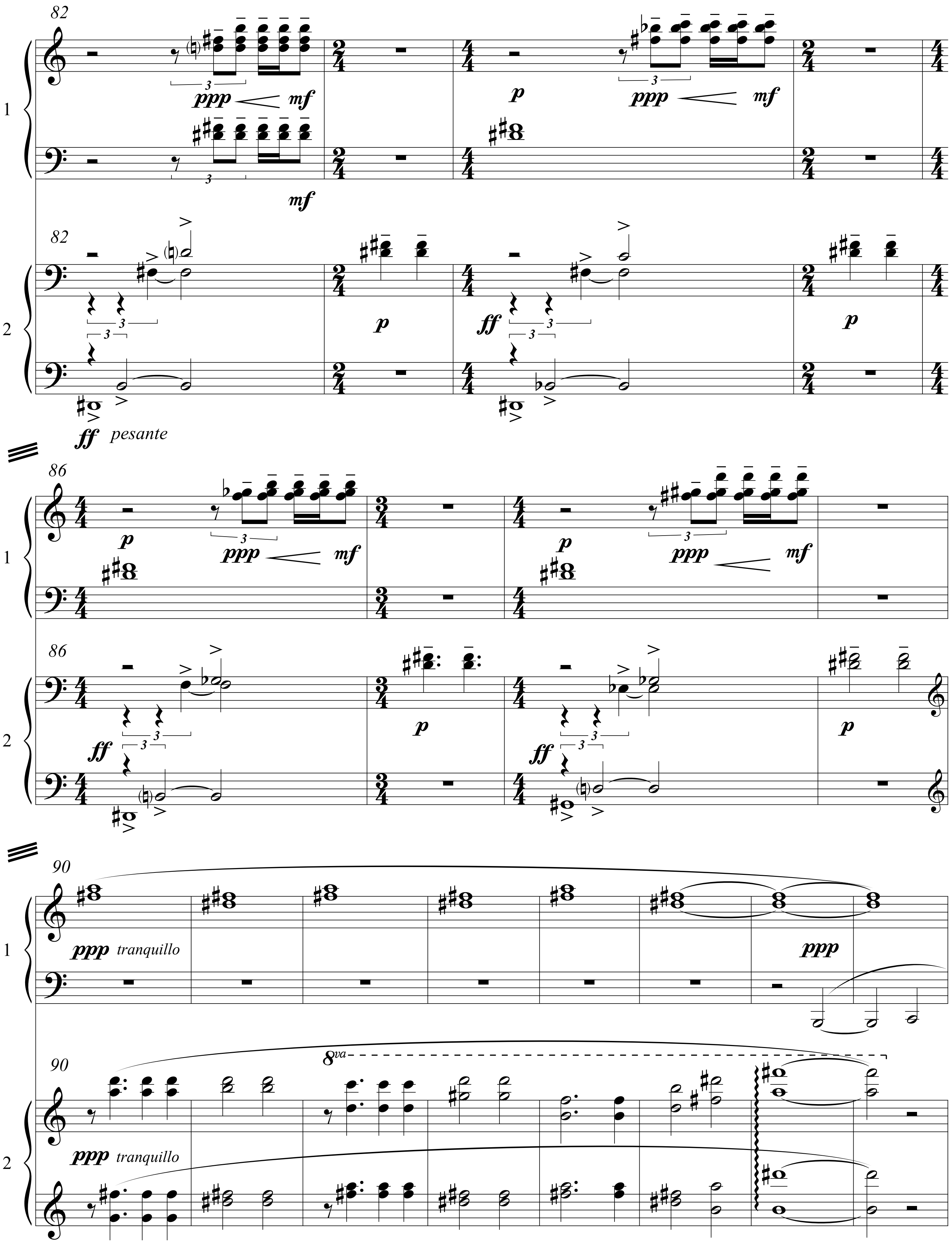

54
Faster (♩ = c. 88)
f
agitato
1
54
Faster (♩ = c. 88)
f
agitato
mf
2
8vb
57
1
57
2

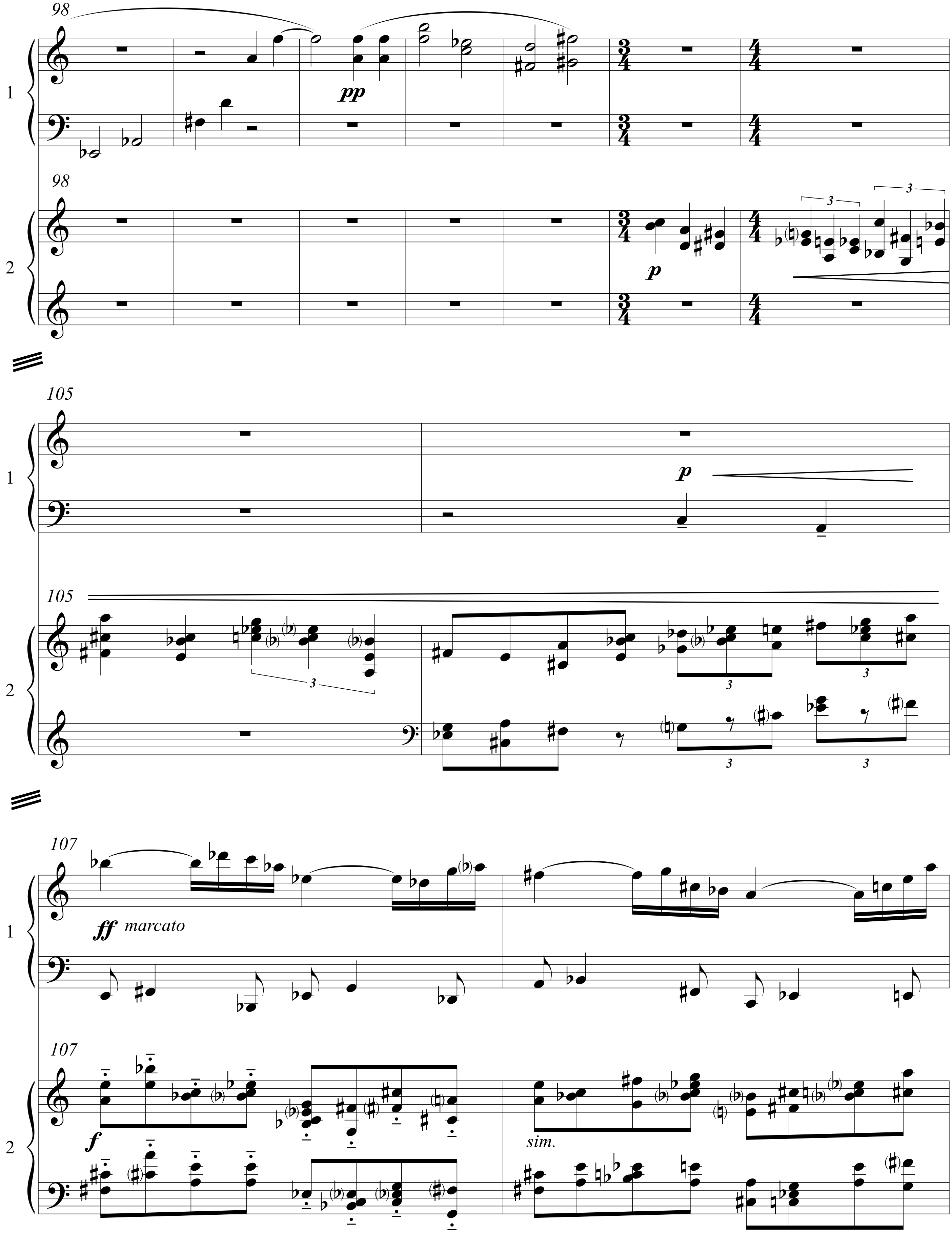

molto rit.
8va

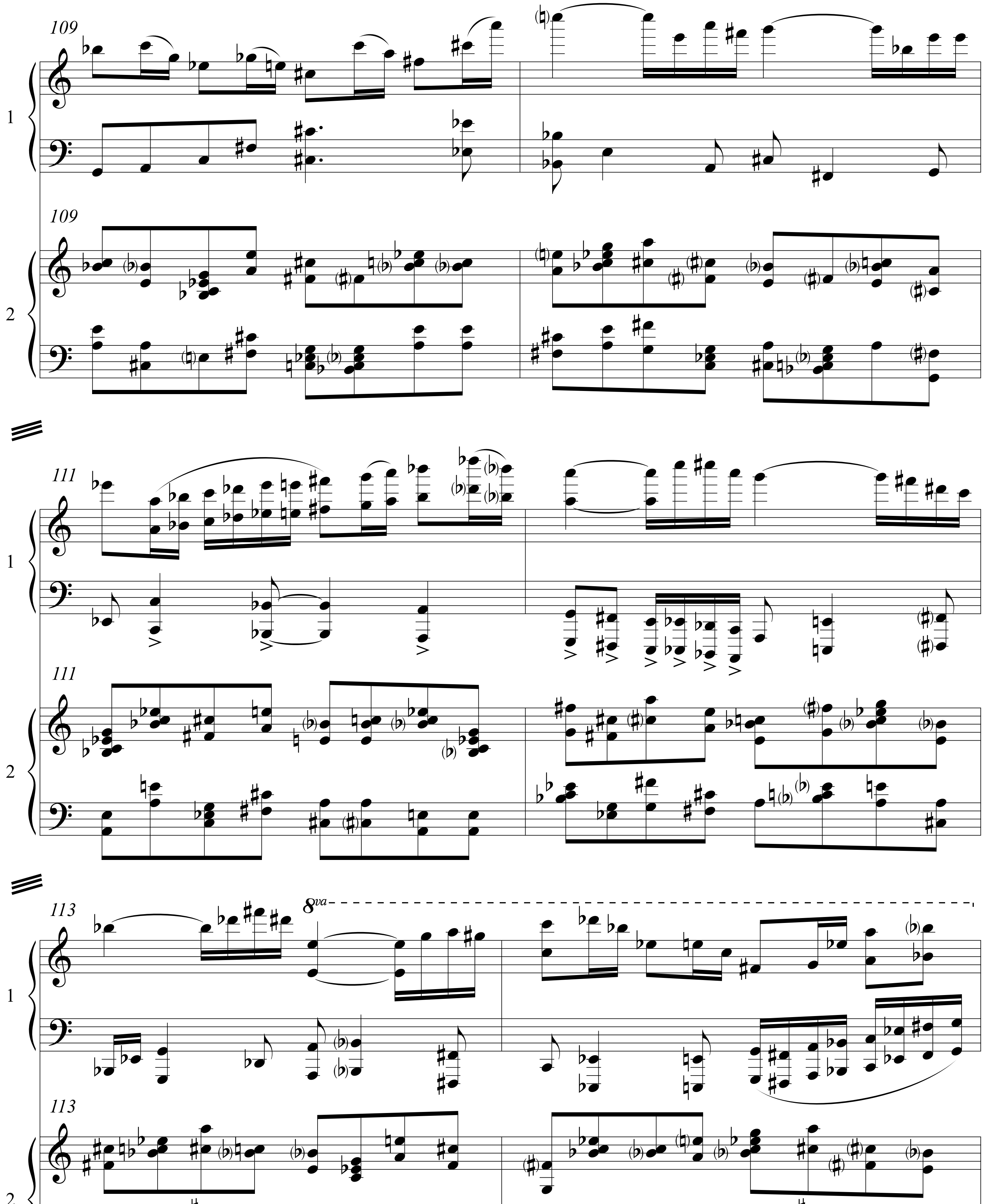

Delicate (♩ = c. 80)
p
mp
Delicate (♩ = c. 80)
sub. p
sub. p
mp

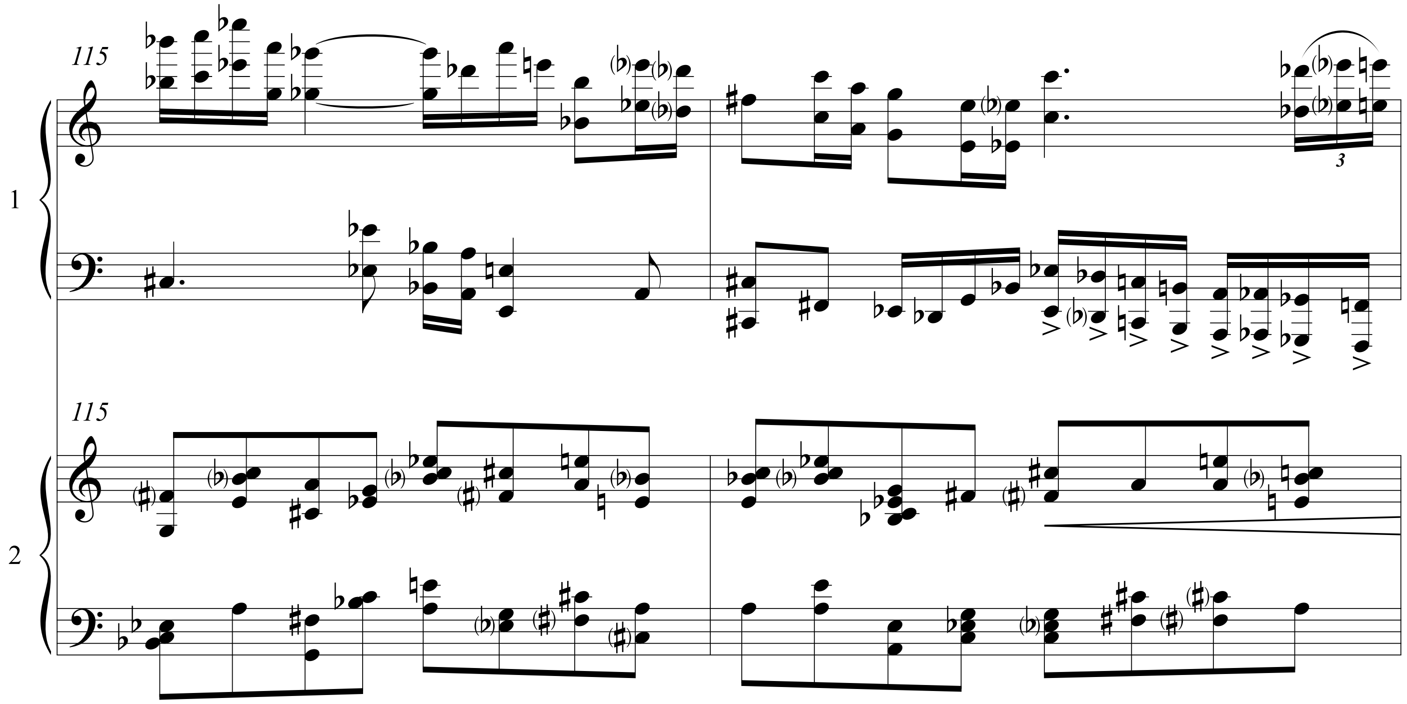

rit.
rit.
Andante (♩ = c. 72)
mp
sim.
sim.
Andante (♩ = c. 72)
mf

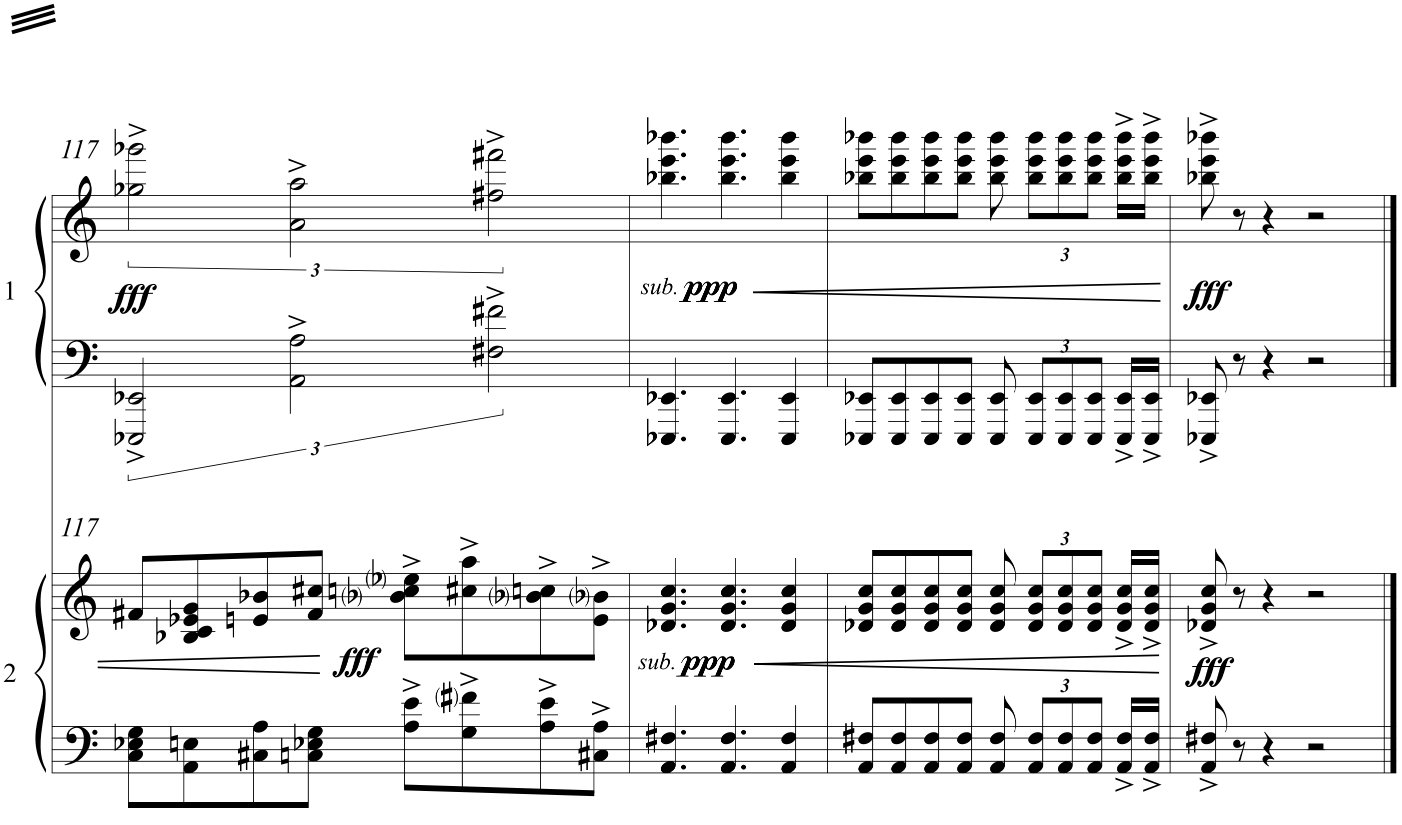

bring out bass line
f
f
leggiero


82
1
ppp
mf
p
ppp
mf
mf
82
2
p
ff
p
ff
pesante
86
1
p
ppp
mf
p
ppp
mf
86
2
ff
p
ff
p
90
1
ppp
tranquillo
ppp
90
2
8va
ppp
tranquillo


98
1
2
pp
p
3
105
p
107
ff
marcato
f
sim.


8va




117
fff
sub. ppp
fff
1
117
fff
sub. ppp
fff
2
3