

The Woodcock Johnson Three and Math Learning Disabilities

by

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A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Arts

ARIZONA STATE UNIVERSITY

August 2010

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by

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has been approved

June 2010

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ABSTRACT

This study investigated the link between the cognitive clusters from the Woodcock-Johnson III Tests of Cognitive Ability (WJ III COG) and Broad Math, Math Calculation Skills, and Math Reasoning clusters of the Woodcock-Johnson III Tests of Achievement (WJ III ACH) using data collected over seven years by a large elementary school district in the Southwest. The students in this study were all diagnosed with math learning disabilities. Multiple regression analyses were used to predict performance on the Broad Math, Math Calculation Skills, and Math Reasoning clusters from the WJ III ACH. Fluid Reasoning (Gf), Comprehension-Knowledge (Gc), Short-Term Memory (Gsm), and Long-term Retrieval (Glr) demonstrated strong relations with Broad Math and moderate relations with Math Calculation Skills. Auditory Processing (Ga) and Processing Speed (Gs) demonstrated moderate relations with Broad Math and Math Calculation Skills. Visual-Spatial Thinking (Gv) and Processing Speed (Gs) demonstrated moderate to strong relations with the mathematics clusters. The results indicate that the specific cognitive abilities of students with math learning disabilities may differ from their peers.

ACKNOWLEDGMENTS

This study would not have been possible without the help and unceasing support of the committee members, Jill Stamm and Amanda Sullivan and most especially the chair, Linda Caterino, whose guidance has been invaluable, and to whom I am eternally grateful. The Woodcock-Munoz Foundation funded part of the data collection for this study.

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Chapter 1

INTRODUCTION

Overview

Educators and researchers, as well as parents and administrators have debated the diagnosis of learning disabilities since the passage of the Education for the Handicapped Act (EHA P.L. 92-142) in 1975, [later amended and re-titled the Individuals with Disabilities Education Improvement Act (IDEIA) in 2004 (P.L. 108-446)]. Much of the controversy lies in disagreement about the definition of the terms “intelligence” and “learning disabilities.” The dispute about intelligence revolves around two central questions: 1) What is intelligence, and 2) Do the currently available assessment measures test it correctly? Theorists differ in their perception of intelligence as a general, overarching quality, “g” (or general factor) or one made up of multiple talents (Wasserman & Tulsy, 2005). They have also had difficulty translating these theories into empirically validated testing techniques.

In 1999, after completing factor analysis studies, Kevin McGrew, together with Richard Woodcock, John Horn, and John Carroll, integrated over fifty years of theory and research into the *Cattell-Horn-Carroll* theory (CHC), a hierarchical model of intelligence that separates intelligence into fluid and crystallized reasoning. The CHC model contains three “strata.” Stratum III describes the overall ability, referred to as “g”. Stratum II includes the broad abilities. According to Carroll (1993, as quoted in Flanagan, Ortiz, & Alfonso, 2007), these

represent “basic constitutional and long standing characteristics of individuals that can govern or influence a great variety of behaviors in a given domain.” (p. 271). Stratum I contains narrow abilities, which Carroll says, “represent greater specializations of abilities, often in quite specific ways that reflect the effects of experience and learning or the adoption of particular strategies of performance.” (1993, as cited in Flanagan, Ortiz, & Alfonso, 2007, p. 271). The next chapter describes the broad and narrow abilities in greater detail.

Some of the disagreement about the definition of learning disabilities is reflected in the broad wording of the federal statute that prescribes its use in the school system. The Individuals With Disabilities Education Act (IDEA, 2004), defines a learning disability as:

A disorder in one or more of the basic psychological processes involved in understanding or using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations.

Because the language in IDEA does not specify how to determine this disorder, or the exact nature of its manifestations, it has not been implemented uniformly in school districts (Sattler, 2001).

Use of the CHC theory to identify learning disabilities has yielded promising results. Some researchers consider it the best-validated means of testing intelligence (Esters & Ittenbach, 1999). Unfortunately, even supporters of the CHC model have not used uniform testing batteries, and the composite scores

reached by different batteries have not been found to be interchangeable (Floyd, Bergeron, McCormack, Anderson, & Hargrove-Owens, 2005).

One means of measurement has received significant support as a method for identifying learning disabilities. The Woodcock-Johnson Psycho-Educational Battery-III, Tests of Cognitive Ability (WJ III COG; Woodcock, McGrew, & Mather, 2001), which was developed in accordance with the CHC theoretical model has been substantially researched, and has shown considerable success in measuring current levels of cognitive and academic performance in students (Fiorello & Primerano, 2005; Flanagan & Harrison, 2005).

The most widely used means of identifying learning disabilities has involved the “discrepancy model”. The basis for this technique is the belief that a learning disability involves a difference between a person’s ability and her academic performance. According to this model, someone with a learning disability in a specific area should score significantly lower on a measure of academic achievement (performance) than on one of intelligence (ability). Unfortunately, the application of this technique has not been uniformly applied.

While many achievement tests, such as the Woodcock-Johnson Test of Academics III (Woodcock, McGrew, & Mather, 2001) and the Wechsler Individual Achievement Test, Second Edition (WIAT-II; Wechsler, 2001), have research supporting their validity, the scores they generate have not always been used as the developers of the tests intended. Some of the criticism of the discrepancy model for identifying learning disabilities has centered on the use of

subtests, rather than composite scores, to identify specific learning disabilities, such as math disability. These subtests have shown less reliability and more variance than composite scores (Watkins, 2003). However, according to McGrew (1997), as cited in Fiorello and Primerano (2005):

Most of the anti-specific ability research in school psychology has been conducted with measures that are based on an outdated conceptualization of intelligence (viz. Wechsler batteries), and have used research methods that have placed primary emphasis on *prediction* with little attention to *explanation* and *theoretical understanding* of the relations between general and specific cognitive abilities and school achievement (p. 191; italics in original).

While some research has supported the use of overall intelligence, or “g” as the best predictor of school performance, according to Flanagan and McGrew (1997), these results often stem from the choice of particular statistical analyses that attempt to “partition variance into that accounted for by a general factor score or scores versus that accounted for separately by the variance in subtest scores” (p.191). Flanagan and McGrew (1997) assert that this technique does not adequately measure the importance of the effects of the different variables, and cite several articles in support of their premise. Additionally, they state that studies attempting to show how abilities predict performance are less meaningful than those that explain how those abilities affect performance (Flanagan & McGrew, 1997).

The use of testing techniques that have empirical validation has come to the forefront in the field of education, especially as the No Child Left Behind Act (2001) has increased pressure on educators to identify students with learning difficulties and to intervene before they begin to fail. According to Mazzoco (2005), many researchers have studied proper identification and intervention strategies for students with reading disabilities (RD). Neurologists have even identified specific areas of the brain that function differently in students with reading disabilities than typically functioning students (Mazzoco, 2005). This research has substantiated early intervention as important in mediating the neurobiological differences inherent in children with reading disabilities. However, research into other learning disabilities, such as math disabilities has lagged behind. While techniques have been developed for screening children with math disabilities (MD), no consensus has been reached about the nature of what is being measured (Mazzoco, 2005).

Statement of the Problem

Because of the paucity of research into math learning disabilities, the specific cognitive abilities that affect math achievement are less well established than those that affect reading achievement. According to Flanagan and McGrew (1997), the best way to establish empirically validated interventions, is to better explain which cognitive abilities affect math performance. By understanding the cognitive deficits that underlie disabilities, psychologists can design interventions that compensate for or ameliorate these deficits.

Purpose of the Study

This study attempts to replicate studies conducted by McGrew and Hessler (1995), using the WJ-R, and those by Floyd, Evans, and McGrew (2003) using the WJ III. The purpose of this study is to determine whether the areas of cognitive ability that predict math achievement in the general population also predict math achievement in a clinical sample of children with math learning disabilities. Unlike previous studies that looked at the standardization sample from the WJ-R and WJ III, the subjects in this study represent a clinical sample of students who have all been diagnosed with math learning disabilities. This information can aid in the design of interventions that target students' intra-individual strengths and weaknesses.

This study aims to answer the following questions:

1. Does the CHC factor Gf, as measured by the WJ III cognitive subtests Concept Formation and Analysis-Synthesis demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 1: Gf will demonstrate a significant relation with total math skills, since previous research has shown that this ability is very important for math achievement at all ages.

2. Does the CHC factor Gc, as measured by the WJ III cognitive subtests Verbal Comprehension and General Information demonstrate a significant

relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 2: Gc will demonstrate a significant relation with total math skills, since previous research has shown that this ability is important for math achievement at all ages.

3. Does the CHC factor Gsm, as measured by the WJ III cognitive subtests Memory for Words and Numbers Reversed demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 3: Gsm will demonstrate a significant relation with total math skills, as previous research has shown that this ability is important for math achievement.

4. Does the CHC factor Gv, as measured by the WJ III cognitive subtests Spatial Relations and Picture Recognition demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 4: Gv will not demonstrate a significant relation with total math skills, as previous research has shown that this ability is

important mainly for higher-level math skills, and this is an elementary school population.

5. Does the CHC factor Ga, as measured by the WJ III cognitive subtests Sound Blending and Auditory Attention demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 5: Ga will not demonstrate a significant relation with total math skills, as previous research has not demonstrated a consistent relationship.

6. Does the CHC factor Glr, as measured by the WJ III cognitive subtests Visual Auditory Attention and Retrieval Fluency demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 6: Glr will not demonstrate a significant relation with total math skills, as previous research has not demonstrated a consistent relationship.

7. Does the CHC factor Gs, as measured by the WJ III cognitive subtests Visual Matching and Decision Speed demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Hypothesis 7: Gs will demonstrate a significant relation with total math skills, as previous research has shown that this skill is especially important for math at the elementary school level.

Chapter 2

LITERATURE REVIEW

Importance of Diagnosing Math Learning Disabilities

In recent years, an upsurge of research has taken place in the diagnosis of reading learning disabilities. Researchers such as Torgeson (1998), Lyon (2002), and Shaywitz (2001, all as cited in Wolfe & Nevills, 2004) have provided new understanding into the structure of the underlying processing deficits associated with reading learning disabilities. However, research into the etiology, course, and diagnosis of math learning disabilities has not kept pace.

Mazzoco lists various disciplines that have studied math learning disabilities, including cognitive psychology, child development, education, clinical neuropsychology, and behavioral neurogenetics (2005). She defines the main issues of interest as the sources, course, and individual differences in those individuals with math learning disabilities. Mazzoco states that one of the greatest difficulties is differentiating normal variation from those with abnormalities. While she reports that research has empirically validated techniques for later diagnosis of math learning disabilities, earlier diagnosis is more difficult. Mazzoco reports that her studies have demonstrated that low scores on some assessments conducted on kindergarteners can be predictive of later math difficulties, but researchers have not yet determined what broad skills underlie the specific items that these tests measure. These measures have also not been validated on preschoolers, first or second graders (Mazzoco, 2005). Adding to the

confusion, is that, according to Mazzoco, “no core deficit has been identified for MD” (2003, p. 219). Unlike reading learning disabilities, researchers have not yet identified the specific aspects of math learning disabilities, or its neurobiological basis.

Prevalence

Math learning disabilities have been shown to have a lower prevalence rate than reading learning disabilities. Researchers estimate the rate of reading learning disabilities at 10-15% of the overall population (Lyon, Fletcher, Fuchs, & Chhabra, 2006). The rate of math learning disabilities has been estimated to fall somewhere between 5-8% of the overall population (Geary, 2004; Lyon, Fletcher, & Barnes, 2005; Mazzoco, 2005). It has also shown considerable heritability. Oliver, Harlaar, Hayiou, Kovas, Walker, and Petrill (2004) studied 2,178 same-sex twin pairs in the United Kingdom. They reported a heritability estimate for low math ability of .65. However, these figures are difficult to validate, because like other learning disabilities defined by the IDEA, no universal definition for math learning disabilities exists.

Definitions of Math Learning Disabilities

When diagnosing math learning disabilities, not every practitioner uses the same definition. According to Mazzoco (2005), this presents difficulties, because “amidst the consensus and controversy that exist among researchers and practitioners, confusion naturally arises when many individuals address a phenomenon that is not yet fully understood” (p. 318) According to Mazzoco

(2005), researchers have studied math learning disabilities using different terminology including mathematics difficulties, mathematics disabilities, dyscalculia, and poor math achievement; however, no clear definitions of these have been offered, which makes comparisons between these studies difficult. While research may purport to measure the same construct, enough differences exist between studies to suggest that there are differences in their operational definitions, as well as their terminology (Landerl, Bevan, & Butterworth, 2004). Researchers who employ a discrepancy-based definition often differ in the amount of discrepancy used between cognitive ability and achievement, as well as the standardized tests used to measure these differences. The skills needed to succeed on various tests may vary based on the choice of the test, and may not be limited to math knowledge alone (Landerl et al., 2004). Thus, no consensus has yet been reached on a widely accepted research definition of math learning disabilities (Mazzoco, 2005).

Early Definitions of Math Learning Disabilities

Prior to the passage of PL 94-142 in 1975, children were often identified as needing help with math when they were low achievers, regardless of their intellectual ability. PL 94 142 and its later iterations (i.e., Individuals With Disabilities Education Act, 1997; & IDEIA, 2004, as cited in Sattler, 2001) identified thirteen qualifying areas for Special Education services, including Specific Learning Disabilities. IDEA defined a Specific Learning Disability as “a disorder in one or more of the basic psychological processes involved in using

language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations” (IDEA, 1997). Children could qualify for special education if they showed a significant discrepancy between ability and achievement in math calculation or math reasoning. Unfortunately, the definition of “significant discrepancy” varies from state to state, and often from school district to school district. Additionally, while most psychologists agree that math learning disabilities involves disabilities in underlying processes, they have not yet reached a consensus about the nature of these processes.

Criticism of the discrepancy model has come not only from educators concerned with the lack of help for low achieving children who did not show discrepancies, but also from researchers studying LD. According to Mazzocco (2005), “in the RD and general LD literature, there exists little, if any empirical support for the effectiveness of a discrepancy-based model; moreover, there is strong evidence of the inappropriateness and ineffectiveness of such IQ-achievement discrepancy definitions” (p.222).

Contemporary Definitions

With the update of IDEA in 2004 came significant changes in mandated evaluation techniques. Local districts can opt to qualify children based on a lack of improvement after the use of empirically based intervention as well as the traditional ability-achievement discrepancy. However, the lack of a discrepancy requirement does not necessarily eliminate the use of standardized assessments.

Gregg, Coleman, and Knight (2003) report several eligibility techniques used to assess possible learning disabilities. These include the discrepancy model (previously described), a cutoff model in which children qualify because of low performance, and Response to Intervention (RTI). Gregg et al. describe the cut-off model as one that uses the norm-referenced scores provided by the Woodcock Johnson III, including percentile scores, relative proficiency scores, and standard scores that can inform professionals about the relative achievement level of a student (2003). The latter technique, widely known as Response to Intervention (RTI) entails identifying disabilities purely on behavior, without regard to intellectual ability. Children whose academic skills lag behind their peers undergo several tiers of interventions (Fletcher-Janzen & Reynolds, 2008). The first tier involves school-wide screening, similar to the annual curriculum based measures currently used in districts nationwide. Those that do poorly undergo classroom based (i.e., second tier) interventions. The children who still do not respond are identified as learning disabled and placed in special education to undergo specialized (i.e., third tier) interventions. Expanding on the three techniques listed above is the clinical model, which incorporates information from a variety of sources, including RTI and intelligence and achievement tests. The WJ III fits into this model as well (Wodrich & Schmitt, 2006).

Cognitive Profiles of Math Learning Disabilities

Unlike children with reading disabilities who have been shown to demonstrate difficulties with specific processing skills such as phonological

processing and rapid automatic naming (McGrew & Wendling, in press), children with math disabilities show significant heterogeneity in their cognitive profiles (Mazzocco, 2005). They may have deficits in short-term memory, difficulties with acquiring the automatic procedures necessary to solve mathematics problems quickly, and visuospatial difficulties (2005). For instance, Bull and Scerif (2001) found that scores on tasks that measure executive functioning were predictive of math learning disabilities. Areas that correlated the most with math learning disabilities included, difficulty switching cognitive strategies, poor working memory span, and difficulty inhibiting irrelevant information.

Definition of the Cattell-Horn-Carroll Theory

Raymond B. Cattell developed a theory of intelligence based on factor analysis in 1941 in response to the work of Charles Spearman, published originally in 1904 (Wasserman & Tulsky, 2005). While Spearman asserted that a broad factor - "g" - underlies intelligence, Cattell believed it to have two main factors, crystallized intelligence (Gc), defined as access to acquired knowledge, and the ability to store new knowledge, and fluid intelligences (Gf), the ability to adapt to novel situations through reasoning (Wasserman & Tulsky, 2005).

In the sixties, Cattell and his student, John L. Horn broadened the number of abilities to five, adding visualization, retrieval capacity, and cognitive speed (as

cited in McGrew, 2005). By 1981, Horn reported that evidence supported the existence of nine factors: fluid intelligence (Gf), crystallized intelligence (Gc), short-term acquisition and retrieval (Gsm), visual intelligence (Gv), auditory intelligence (Ga), long-term storage and retrieval (Glr), cognitive processing speed (Gs), quantitative knowledge (Gq), and reading and writing skills (Grw) (cited in McGrew, 2005).

In 1993, John B. Carroll proposed a model of intelligence that was hierarchical and had three levels or strata. He based this on a meta-analysis of 461 test-based datasets (Wasserman & Tulskey, 2005). He believed that intelligence contained an overarching general ability, akin to Spearman's "g", but had eight or more broad-ability factors and up to 65 narrow abilities.

After McGrew conducted extensive factor analyses that validated areas of both Cattell-Horn's Gf-Gc model and Carroll's three-stratum model, the theories were combined into the Cattell-Horn-Carroll (CHC) theory of cognitive abilities (McGrew & Woodcock, 2001). The broad CHC abilities which were essentially the same as Horn's were redefined and included Comprehension-Knowledge (Gc), Long-Term retrieval (Glr), Visual-Spatial Thinking (Gv), Auditory Processing (Ga), Fluid Reasoning (Gf), Processing Speed (Gs), Short-Term Memory (Gsm), Reading-Writing (Grw), and Mathematics (Gq) (McGrew & Woodcock, 2001). Table 1 lists the CHC broad abilities and gives a brief description of each.

Cross Battery Approach

Prior to the publication of the WJ III in 2001, no single battery measured all broad areas of the CHC model of intelligence. As research supporting this model grew, so did the popularity of a different kind of assessment of intellectual ability. Rather than using single measures of intelligence, psychologists began selecting subtests from different measures that purported to represent different CHC clusters. Woodcock first suggested this type of approach in a 1990 article in which he described a theory-driven “battery-free” approach to assessment of fluid and crystallized ability (Gf-Gc; McGrew, 2005). Woodcock conducted confirmatory analyses of the major intelligence batteries of the day, including the Woodcock Johnson Psycho-Educational Battery (Woodcock & Johnson, 1977), Woodcock Johnson Psycho-Educational Battery –Revised (Woodcock & Johnson, 1989), Wechsler Scales (Wechsler, 1981), the Kaufman Assessment Battery for Children (Kaufman & Kaufman, 1983), and the Stanford-Binet Intelligence Scales, Fourth Edition (Thorndike, Hagen, & Sattler, 1986). He then described how individual subtests of these batteries corresponded to the Cattell-Horn Gf-Gc model (McGrew, 2005). Flanagan, Genshaft, and Harrison then expanded upon this technique in their 1997 book, *Contemporary Intellectual Assessment: Theories, Tests, and Issues*. Flanagan and Harrison (2005) have since published the second edition. In 1998, McGrew and Flanagan published specific guidelines to the CHC cross battery approach. They reported which subtests of major intelligence batteries corresponded to individual CHC clusters as affirmed by extensive factor analyses (Alfonso, Flanagan, & Radwan, 2005).

The Cross Battery approach consists of three pillars: the CHC theory itself, CHC broad clusters, and CHC narrow clusters. According to Alfonso, Flanagan, and Radwan (2005) this approach is “a systematic means of supplementing single intelligence batteries to ensure that the abilities considered most important vis-à-vis the referral are well represented in the assessment” (Alfonso, Flanagan, & Radwan, 2005, p. 198). Alfonso, Flanagan, and Radwan reported which subtests of major intelligence batteries corresponded to individual CHC clusters as affirmed by extensive factor analyses (2005).

The suggestion that the cross battery approach be supplemental implies that individual subtests be used after interpretation of single comprehensive intelligence tests. However, rather than supplement tests with additional subtests, many practitioners simply chose individual subtests from different intelligence batteries without regard to the standardization process that guides interpretations. Intelligence tests are normed with specific samples and are intended to be reported as a single test. In addition, subtests from different assessments that purport to measure identical CHC broad abilities are not interchangeable. Floyd, Bergeron, McCormack, Anderson, and Hargrove-Owens (2005) found that combining CHC scores from different assessments significantly lowered the reliability of the results. Thus, there was a movement to ensure that all (or most) of the major factors be included in one battery.

Woodcock-Johnson III

In 2001, Richard Woodcock, Kevin McGrew, and Nancy Mather published the Woodcock-Johnson III, a norm-referenced, individually administered assessment of intelligence and academic achievement. They based their revised test on the CHC theory of intelligence. The WJ III COG consists of two batteries, the Standard Battery (tests 1-10), and Extended Battery (tests 11-20). The subtests can be grouped together into a broad measure of intellectual ability, seven CHC factors, as well as three cognitive categories, and seven clinical clusters. Each CHC factor consists of subtests that claim to measure a narrow CHC ability, such as working memory and perceptual speed. The WJ III COG also organizes the CHC model into the WJ III Cognitive Performance Model. This posits the cognitive performance results from four main influences: Stores of Acquired Knowledge (Gc, Gq, Grw), Thinking Ability (Glr, Gv, Ga, Gf,) Cognitive Efficiency (Gsm, Gs), and Facilitator-Inhibitors (Internal, External). Table 2 lists the WJ III subtests that comprise the CHC factors.

The Achievement Tests are divided into the Standard Battery (i.e., tests 1-12) and Extended Battery (i.e., tests 13-22). These subtests combine into six clusters: Reading, Oral Language, Mathematics, Written Language, Academic Knowledge, and Supplemental Clusters. Each of the clusters, except Academic Knowledge consists of two to six subtests, depending on the cluster and the examiner's choice. Academic Knowledge consists of only one subtest in the Extended Battery.

The Mathematics Achievement subtests consist of Calculation, Math Fluency, and Applied Problems in the Standard Battery with the addition of Quantitative Concepts in the Extended Battery (Mather, Wendling, & Woodcock, 2001). The three standard math subtests comprise the cluster of Broad Math. Calculation and Math Fluency comprise the cluster Math Calculations Skills, and Applied Problems and Quantitative Concepts comprise the cluster Math Reasoning (Mather, Wendling, & Woodcock, 2001).

The WJ-R had been devised according to the CHC model, and purported to measure seven of the broad cognitive abilities described within the model. McGrew (1995), together with various researchers (e.g., Evans, et al.; 2002, Floyd & Evans; 2003, Hessler, 1995), used the standardization data from the WJ-R for a number of studies that attempted to determine which cognitive clusters best predict performance in different academic areas. They analyzed the relationship between the cognitive clusters and reading achievement (1993), written language (McGrew & Knopik, 1993), and mathematics achievement (McGrew & Hessler, 1995). They found that the clusters measuring crystallized intelligence (Gc), auditory processing (Ga), long-term retrieval (Glr), and processing speed were most predictive of reading achievement, while Gc, Ga, and Gs were the best predictors of writing achievement (McGrew & Knopik, 1993).

McGrew and Hessler (1995) investigated the correlation between the WJ-R cognitive cluster and mathematics achievement. They looked at the seven CHC cognitive clusters of the WJ-R (Long-Term Retrieval, Short-Term Memory,

Visual Processing, Auditory Processing, Processing Speed, Comprehension Knowledge, and Fluid Reasoning) and two mathematics clusters, Basic Mathematics (which consists of Calculation and Quantitative Concepts), and Math Reasoning (which comprises Applied Problems) in 5,386 participants (aged 2-95 yrs) from the standardization sample. Their analysis yielded significant correlations across both math criteria and across all age groups. In their study, most multiple regression coefficients were in the .70-.80 range. The Processing Speed, Comprehension-Knowledge, and Fluid Reasoning cluster showed the most consistent relationships with math achievement across all age groups, especially with Basic Math. They also found a relationship between Long-Term Retrieval and Basic Math, as well as between Short-Term Memory and Basic Math. However, they found little relationship between either Auditory Processing or Visual Processing and Basic Math. They also found similar relationships between the cognitive clusters and Math Reasoning, with Processing Speed, Comprehension-Knowledge, and Fluid Reasoning showing the strongest relationship, and Short-Term Memory and Visual Processing also showing a relationship, primarily at earlier ages. Multiple regression analysis with the WJ-R cognitive clusters as predictors and the achievement measures as criteria at 21 different age groups revealed significant relationships among cognitive clusters and achievement (McGrew & Hessler, 1995, p. 21). McGrew and Hessler concluded that the combined clusters accounted for 50%-70% of math achievement variance across the life span. Table 3 outlines those broad and

narrow abilities that Flanagan, Ortiz, and Alfonso report have consistently shown to affect math performance (2007).

Osmon, Smerz, Braun, and Plambeck (2006) evaluated 138 college age students for math learning disabilities using the WJ-R Cognitive and several measures of executive function. The students had been referred for testing based on suspicion of a math learning disability. Osmon et al. (2006) hypothesized that math learning disability can be predicted by deficits in spatial ability and executive functioning. In addition to the WJ-R, they used the Benton Judgment of Line Orientation (JLO) (Benton, Hamsher, Varney, & Spreen, 1983), and the Category Test (CT) (Reitan, & Wolfson, 1993), two neuropsychological measures of spatial ability and executive function. They measured math ability using the Calculation and Applied Problems subtests of the WJ-R Achievement Test, and then conducted a MANOVA, then one-way ANOVAs (Math impaired vs. Math unimpaired), and found significant main effects for Long Term Retrieval, Auditory Processing, Visual-Spatial Thinking, Comprehension-Knowledge, and Fluid Reasoning, as well as for the two tests of spatial ability and executive functioning. This contradicts McGrew and Hessler's findings that Visual Processing accounts for math disabilities only at the primary grades. The different results may be due to the additional visual tests that Osmon et al. used, but they may also be due to the age of the sample and the different statistical analyses used. Osmon et al. (2006) conducted cluster analysis of their results, and concluded that those with math learning disabilities can be split into three separate

groups: those with spatial deficits only, those with executive functioning deficits only, and those with deficits in both spatial ability and executive functioning.

Floyd, Evans, and McGrew (2003) used the standardization sample to examine the relationship between the WJ III COG and mathematics achievement. Floyd, Evans, and McGrew used the cognitive clusters of the WJ III (Comprehension-Knowledge, Long-Term Retrieval, Visual Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, and Short Term Memory) and achievement measures of Math Reasoning with 4,498 participants and Math Calculation with 3,064 participants for ages 6-19 in a national sample. Using multiple regression analysis, they found that all of the cognitive clusters from the WJ III COG that measured CHC broad and narrow abilities were significantly related to math achievement across age groups (2003). For elementary school children, Comprehension-Knowledge (Gc) showed the strongest relation with both Math Reasoning and Math Calculation Skills (2003). Fluid Reasoning (Gf) also showed moderate relations with Math Calculation and Math Reasoning. The correlations with Math Reasoning increased with the age of the students. Short term Memory (Gsm) also showed moderate relations with Math Reasoning and Math Calculation for elementary school children, as did Processing Speed (Gs). Long-Term Retrieval (Glr) showed moderate relations with Math Reasoning and Calculation up to age eight, and Auditory Processing (Ga) showed moderate relations with Math Calculation only to age six (2003).

Visual-Spatial Thinking (Gv) showed no significant relations with Math Calculation skills and Math Reasoning (2003).

Flanagan, Ortiz, and Alfonso (2007) reviewed the research and reported that the strongest and most consistent associations were between Gf, Gc, and Gs and Math Achievement. Specifically, they found that within the broad area of Gf, the inductive and general sequential reasoning abilities show the strongest relation with math achievement. The Concept Formation subtest of the WJ III COG measures inductive reasoning, while the Analysis Synthesis subtest measures general sequential reasoning (Flanagan, Ortiz, & Alfonso, 2007). Within the broad area of Gc, the narrow abilities of language development and lexical knowledge (as measured by Verbal Comprehension on WJ III COG), and listening abilities (not measured on WJ III COG) have a strong relationship with math learning disabilities. In Gs, Perceptual Speed (measured by Visual Matching on WJ III COG) is especially important in the elementary school grades. They also found a moderate relationship between the memory span and the working memory areas of Gsm measured by Numbers Reversed on WJ III COG (Flanagan, et al. 2007).

While the previous studies investigated the link between the CHC clusters and math achievement, the 2003 study did not investigate the relation between the clusters and Broad Math. In addition, neither study looked at correlations between individual subtests. Unlike previous studies that looked at representative samples, the population of the current study is a clinical sample. All of the participants

have been diagnosed with a learning disability in math reasoning, math calculations, or both.

Chapter 3

METHOD

Participants

The sample for this study came from a larger database, consisting of archival data collected from the special education files of a large Southwestern school district with a student population of more than 25,300 students across 32 schools. The district includes students in kindergarten through eighth grade. The files were from the years 2001-2007. The reported grade of the sample ranged from K.8 (eighth month of kindergarten) to 8.1 (first month of eighth grade). Table 4 outlines the sample breakdown by grade.

The larger database from which this sample was drawn is a clinical sample of students who were evaluated for special education by certified school psychologists in the district using the WJ III COG/ACH. Graduate students gathered the data, with permission, from the special education files of the district and created a list of 4000 students all of whom had received either a WJ III COG or ACH and a clinical diagnosis. The students within the database were assigned random numbers to avoid possible identification. The database also included demographic and background information for each student.

The ethnicity of this sample was 42.6% White, 33.6% Hispanic, 10.9% Black, 4.7% Native American, 0.8% Asian/Pacific Islander, 1.6% Other, 3.1% Multiethnic, and 1.6% not reported. The gender breakdown of the sample was

55.8% Male and 44.2% Female. The mean grade of the sample was 4.4 with a standard deviation of 1.9.

To be included in the present study, the individuals needed to have received at least the standard WJ III COG battery, which includes the first seven subtests (Verbal Comprehension, Visual-Auditory Learning, Spatial Relations, Sound Blending, Concept Formation, Visual Matching, and Numbers Reversed) and at least one of the three subtests from the WJ III ACH battery that measures mathematical ability (Calculation, Math Fluency, and Applied Problems). Additionally, subjects needed to receive a diagnosis of Specific Learning Disability in either Math Calculation or Math Reasoning. Of the original population, 146 were determined to fit the parameters of this study. Of those, 17 were eliminated because the scores in their IQ profiles were more than two standard deviations below the mean, which falls below the criteria for average intelligence set by the American Psychiatric Association's *Diagnostic and Statistical Manual of Mental Disorders, fourth edition text revision (DSM-IV-TR, 2000)*. Because the definition of learning disabilities rules out cognitive impairment, subjects with scores this low may not qualify as having learning disabilities. This left 129 participants.

Procedure

Because this study used archival data with no threat to the anonymity of the subjects, Arizona State University's IRB approved it and the district granted permission to investigate the files. The Woodcock-Munoz foundation, an

organization that funds research using the Woodcock-Johnson test batteries, provided funding for the data collection. In return, they received the data to include in their own national clinical database. The researcher, along with several other graduate students, perused all of the special education files in the district office and selected those students who had received a full standard WJ III cognitive battery, which includes the first seven subtests (Verbal Comprehension, Visual-Auditory Learning, Spatial Relations, Sound Blending, Concept Formation, Visual Matching, and Numbers Reversed) or a full standard achievement WJ III battery, which includes three subtests of each academic area (math, reading and writing). The data from these students were entered into SPSS.

Materials

The Woodcock-Johnson III Test of Cognitive Abilities (WJ III COG) is an individually administered intelligence test designed for the assessment of children and adults from age two to ninety. Index and IQ scores have a mean of 100 and a standard deviation of 15, and scores between 90 and 110 are considered average. This study used the standard and extended battery.

The Woodcock-Johnson III Tests of Achievement (WJ III ACH) is an individually administered test of academic performance designed for the assessment of children and adults from age 2 to 95, and grades K.0 through 18.0. The WJ III ACH was co-normed with the WJ III COG, allowing for increased reliability in comparing scores. The WJ III ACH has 22 subtests that measure five areas of academic achievement: reading, math, written language, knowledge, and

oral language (Mather et al., 2001). The standard battery is comprised of seven subtests, and the extended battery has 14. Additional subtests can provide supplemental scores. This study uses the math battery.

Data Analysis Plan

Descriptive statistics were calculated including means and standard deviations. Frequencies were reported for ethnicity, gender, and area of disability.

Multiple regression analysis was used to study the relationships between the predictor variables and the criterion variables of Broad Math (subtests Calculation, Math Fluency and Applied Problems), Math Calculation Skills (Calculation and Math Fluency), and Math Reasoning (Applied Problems and Quantitative Concepts) clusters of the WJ III Achievement from Cognitive Performance model as well as the broad CHC Clusters (Comprehension-Knowledge, Long-Term Retrieval, Visual Spatial Thinking, Auditory Processing, Fluid Reasoning, Processing Speed, and Short Term Memory). Differences between the results in the clinical population and those of the nonclinical population were determined.

Research Questions

Question 1. Does the CHC factor Gf, as measured by the WJ III cognitive subtests Concept Formation and Analysis-Synthesis show a strong relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Gf measure (comprised of the subtests Concept Formation and Analysis-Synthesis), and the criterion variables, the WJ III achievement clusters of Broad Math (comprised of Calculation, Math Fluency and Applied Problems), Math Calculation Skills (Calculation and Math Fluency), and Mathematics Reasoning (Applied Problems and Quantitative Concepts).

Question 2. Does the CHC factor Gc, as measured by the WJ III cognitive subtests Verbal Comprehension and General Information show a positive relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Gc measure (and the criterion variables, the WJ III achievement clusters of Broad Math (comprised of Calculation, Math Fluency and Applied Problems), Math Calculation Skills (Calculation and Math Fluency), and Mathematics Reasoning (Applied Problems and Quantitative Concepts).

Question 3. Does the CHC factor Gsm, as measured by the WJ III cognitive subtests Memory for Words and Numbers Reversed show a positive relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Gsm measure (comprised of the

subtests Numbers Reversed and Auditory Working Memory), and the criterion variables, the WJ III achievement clusters of Broad Math, Math Calculation Skills and Mathematics Reasoning.

Question 4. Does the CHC factor Gv, as measured by the WJ III cognitive subtests Spatial Relations and Picture Recognition demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Gv measure, and the criterion variables, the WJ III achievement clusters of Broad Math, Math Calculation Skills, and Mathematics Reasoning.

Question 5. Does the CHC factor Ga, as measured by the WJ III cognitive subtests Sound Blending and Auditory Attention demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Ga measure, and the criterion variables, the WJ III achievement clusters of Broad Math (comprised of Calculation, Math Fluency and Applied Problems), Math Calculation Skills (Calculation and Math Fluency), and Mathematics Reasoning (Applied Problems and Quantitative Concepts).

Question 6. Does the CHC factor Glr, as measured by the WJ III cognitive subtests Visual Auditory Attention and Retrieval Fluency demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Glr measure, and the criterion variables, the WJ III achievement clusters of Broad Math (comprised of Calculation, Math Fluency and Applied Problems), Math Calculation Skills (Calculation and Math Fluency), and Mathematics Reasoning (Applied Problems and Quantitative Concepts).

Question 7. Does the CHC factor Gs, as measured by the WJ III cognitive subtests Visual Matching and Decision Speed demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Multiple regression analysis was used to scrutinize the relation between the predictor variable, the WJ III cognitive Gs measure, and the criterion variables, the WJ III achievement clusters of Broad Math (comprised of Calculation, Math Fluency and Applied Problems), Math Calculation Skills (Calculation and Math Fluency), and Mathematics Reasoning (Applied Problems and Quantitative Concepts).

Chapter 4

RESULTS

This section includes the statistical findings of the study. Each research question is addressed. Table 5 shows the sample sizes and descriptive statistics for clusters included in the regression models. Tables 6-12 show the results of the analyses. Following McGrew's (1993) guidelines, coefficients of .10 to .29 are defined as moderate relations, and coefficients of .30 and above are strong relations. Relations are only defined as significant when $p < .05$.

Research Questions and Analyses

Question 1. Does the CHC factor Gf, as measured by the WJ III cognitive subtests Concept Formation and Analysis-Synthesis demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 1. Multiple regression analyses were conducted to evaluate how well Gf predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. There was a strong relation between Gf and Broad Math. Gf also showed a strong relation with Math Reasoning and a moderate relation with Math Calculation. Table 6 summarizes these findings.

Question 2. Does the CHC factor Gc, as measured by the WJ III cognitive subtests Verbal Comprehension and General Information show a positive relation

with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 2. Multiple regression analyses were conducted to evaluate how well Gc predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. Gc demonstrated the strongest relation with Broad Math, and a moderate relation with Math Calculation. However, Gc did not demonstrate a significant relation with Math Reasoning or with Math Calculation. Table 7 summarizes these findings.

Question 3. Does the CHC factor Gsm, as measured by the WJ III cognitive subtests Memory for Words and Numbers Reversed show a positive relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 3. Multiple regression analyses were conducted to evaluate how well Gsm predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. Gsm showed a moderate relation with Broad Math and Math Calculation. It demonstrated the strongest relation with Math Reasoning. Table 8 summarizes these findings.

Question 4. Does the CHC factor Gv, as measured by the WJ III cognitive subtests Spatial Relations and Picture Recognition demonstrate a significant

relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 4. Multiple regression analyses were conducted to evaluate how well Gv predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. Gv showed a moderate relation with Broad Math and Math Calculation and a strong relation with Math Reasoning. Table 9 summarizes these findings.

Question 5. Does the CHC factor Ga, as measured by the WJ III cognitive subtests Sound Blending and Auditory Attention demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 5. Multiple regression analyses were conducted to evaluate how well Ga predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. Ga showed a moderate relation with Broad Math and Math Calculation, but did not demonstrate a significant relation with Math Reasoning. Table 10 summarizes these findings.

Question 6. Does the CHC factor Glr, as measured by the WJ III cognitive subtests Visual-Auditory Learning and Retrieval Fluency demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 6. Multiple regression analyses were conducted to evaluate how well Glr predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. Glr showed a moderate relation with Broad Math and Math Calculation, but did not demonstrate a significant relation with Math Reasoning. Table 11 summarizes these findings.

Question 7. Does the CHC factor Gs, as measured by the WJ III cognitive subtests Visual Matching and Decision Speed demonstrate a significant relation with math achievement, as measured by the WJ III achievement math cluster in a clinical population of students with math learning disabilities?

Results and Analyses for Question 7. Multiple regression analyses were conducted to evaluate how well Gs predicted math achievement as measured by the WJ III clusters of Broad Math, Math Calculation, and Math Reasoning. Gs demonstrated a moderate relation with Broad Math. Gs also showed a moderate relationship with Math Calculation. However, it did not demonstrate a significant relation with Math Reasoning. Table 12 summarizes these findings.

Chapter 5

DISCUSSION

The results of this study replicated several studies from research involving CHC factors and the Woodcock-Johnson III Tests of Cognitive and Academic Ability. However, unlike previous studies, this study involved a clinical sample in which all of the subjects had been diagnosed with math learning disabilities. This may provide insight into some ways in which children with math learning disabilities differ from their peers.

Gf and Mathematics

As expected, Gf, which measures reasoning and problem solving ability, demonstrated a consistently moderate to strong relation with mathematics achievement. Previous research has established a consistent relationship between Gf and math achievement (Flanagan, Ortiz, & Alfonso, 2007). Floyd, Evans, and McGrew report, “Gf appears to represent some of the prominent constructs in studies of mathematics skill development, such as problem-solving schemata, strategy use, and strategic change” (2003).

Gc and Mathematics

Previous research has demonstrated a consistent relationship between Gc and mathematics achievement. According to Schrank, Flanagan, Woodcock, and Mascolo (2002), meta-analyses have shown that the Gc narrow abilities of “Language Development (LD), Lexical Knowledge (VL), and Listening Ability (LS) are important at all ages. These abilities become increasingly more important

with age” (pp. 132). According to Floyd, Evans, and McGrew, “general cultural knowledge and knowledge of mathematics concepts, facts and the procedures to conduct arithmetic stem largely from the acquisition and modification of declarative and procedural knowledge structures” (2003, p. 163). Gc measures crystallized intelligence, or the knowledge acquired through experience and education. Because it depends so strongly on exposure to academics, Floyd, Evans, and McGrew report that it may also be considered a form of academic achievement. Thus, the strong relationship between Gc and Broad Math demonstrated in the current study is not surprising, nor is the moderate relation between Gc and Math Calculation. What is surprising, however, is that Gc demonstrated no significant relationship with Math Reasoning. This may be due to the young age of the sample. Floyd, Evans, and McGrew (2003) found that the relation between Gc and Math Reasoning steadily increased with age.

Gsm and Mathematics

Previous research has established the role that short-term memory plays in the acquisition of math skills (Flanagan, Ortiz, & Alfonso, 2007). Floyd, Evans, and McGrew (2003) found that working memory had a particularly strong relationship with math calculation and reasoning. In this study, Gsm showed a consistent moderate to strong relationship with all areas of math achievement.

Gv and Math Achievement

Previous research has shown a link between Visual Spatial Thinking (Gv) and higher-level math, such as geometry and calculus (Flanagan, Ortiz, &

Alfonso, 2007). However, it has not shown any consistent link with the acquisition of early math skills. In their meta-analysis, McGrew and Wendling (in press) commented on the lack of consistent findings regarding Gv and math learning disabilities. According to McGrew, many studies have noted that aspects of visual-spatial ability are a core deficit in those math learning disabilities, and yet the overall Gv ability has not shown a consistent significant relationship with math abilities. In this study, however, Gv showed a moderate to strong relation with all areas of math achievement.

Ga and Mathematics

Previous research has not demonstrated a consistent link between Auditory Processing (Ga) and math achievement (Flanagan, Ortiz, & Alfonso, 2007). However, Floyd, Evans, and McGrew (2003) found significant relations between Ga and Math Calculations in early childhood: however, these effects dissipated with age. Because the sample for this study contained only elementary school students, this may explain the moderate relation between Ga and Math Calculation, as well as Broad Math. The non-significant relationship between Ga and Math Reasoning is consistent with previous findings.

Glr and Math achievement

Glr showed a strong relation with Broad Math, and a moderate relation with Math Calculation, but no significant relation with Math Reasoning. Research has not consistently shown a link between Glr and math achievement. However, Floyd, Evans, and McGrew (2003), found that long-term retrieval was important

to early development of math calculation skills. They concluded, “Rote recall of mathematical facts from declarative memory (and not more complex cognitive operations representing procedural memory) is required to complete simple math problems (p.165). This may explain the lack of connection between Glr and Math Reasoning, as Math Reasoning measures higher-level abilities. Thus, for this sample declarative memory affected lower level math achievement, but not higher-level mathematics achievement.

Gs and Mathematics

Previous research has established a connection between Gs and the acquisition of primary math skills (Floyd, Evans, & McGrew, 2003). Gs showed a moderate relationship with Broad Math and Math Calculation, but did not demonstrate a significant relation with Math Reasoning. Previous research has established the relationship between speed of processing and math skills (Flanagan, Ortiz, & Alfonso, 2007). Both Broad Math and Math Calculation contain a measure of math fluency, which measures the student’s ability to quickly complete simple arithmetic problems. Math Reasoning however contains no timed tests, so the lack of connection between it and Gs aligns with previous research.

Major Implications

Research into reading learning disabilities has uncovered differences in the cognitive processes of those with reading disabilities (Wolfe & Nevills, 2004). The results of this study point to the possibility that those with math learning

disabilities also process information differently than their peers. Previous research, mostly involving the standardization sample for iterations of the Woodcock-Johnson (WJ-R, WJ III) has not consistently shown a significant relationship between Auditory Processing (Ga) and math achievement, and Visual-Spatial Intelligence (Gv) has only demonstrated consistent relations with higher level math (Flanagan, Ortiz & Alfonso, 2007). However, in this study, they both demonstrated moderate to strong relationships with Broad Math and Math Calculation, and Gv showed a strong relationship with Math Reasoning. This may point to processing deficits in Gv and Ga in those with math learning disabilities. If so, these students may have greater difficulty than their peers processing the information presented in standard lectures, as well as the visual cues often given to aid students who have difficulties processing information auditorily. Difficulties with auditory and visual processing may also inhibit their ability to conquer tasks basic to mathematic skills, such as learning numbers and being able to identify patterns (Flanagan, Ortiz, & Alfonso, 2007). Brain scans of those with reading learning disabilities have demonstrated that they continue to process text as novice readers, rather than developing efficient cognitive strategies (Wolfe & Nevills, 2004). Perhaps those with math learning disabilities also fail to develop the efficient strategies used by their peers. Further research using brain scans may reveal more information in this area.

This study also showed no significant relationship between Crystallized Intelligence and Math Reasoning, while previous research has shown that this

ability generally becomes more important with age. This too may point to differences in the way students with math learning disabilities process information. If the students with math learning disabilities have not been able to learn mathematical skills to automaticity, they may not be referring to their fund of knowledge to complete mathematical tasks. Instead, they react to each task as a novice would, which decreases their efficiency and speed (Flanagan, Ortiz, & Alfonso, 2007).

Implications for Future Research

Because of the small sample size of this study, generalization to the population of children with math learning disabilities is limited in scope. Future research into children with math learning disabilities with a larger sample size can allow for broader analyses and generalization of findings. Research that attempts to replicate these findings may confirm the differences found here. Research into age differences may be helpful in understanding these findings and future researchers may investigate whether these differences continue as the students age. Research centering on those in high school and beyond may be helpful. Future studies with larger sample sizes should also look at gender differences. Investigations into the underlying neurological processes, such as those done by Shaywitz and Shaywitz on children with reading learning disabilities (Shaywitz, 2003) may also yield more information on differences between those with math disabilities and those without.

Implications for Practice

Research has supported the efficacy of evidence-based interventions to mediate or bypass specific cognitive deficits (Wodrich & Schmitt, 2006). For instance, children with deficits in processing speed may benefit from additional time to complete academic tasks and children who have difficulty with memorization may require number strips to remind them of basic calculation facts (McCarney & Wunderlich, 2006). If children with math learning disabilities display commonalities in their cognitive profiles, teachers could better devise strategies to aid them based on their processing strengths and weaknesses.

Limitations

Several limitations of this study should be considered. The biggest limitation is the size of the sample. Because the study was based on archival data and was limited to data found in one district, the sample size was smaller than ideal. The size of the sample limited the number of analyses possible. For instance, since Floyd, Evans, and McGrew (2003) used the standardization sample, they had 4,498 subjects and were able to analyze their sample by age group and gender; however, the size of this sample would have rendered that analysis meaningless. The small size of the sample also prevented any meaningful analysis of differences by gender or ethnicity. In addition, because this sample consisted only of elementary school children, conclusions could not be reached about the influence of cognition on older children. This study used only one

measure of CHC factors (WJ III COG) and math achievement (WJ III ACH), so generalization to other measures of CHC factors should be made with caution.

Summary

While many of the findings of this study confirm previous findings, some vary from the established research base. These may relate to the age of the sample. However, they may also indicate ways in which children with math learning disabilities differ from their peers. Several areas of cognition that showed moderate relations with Broad Math and Math Calculation in this study have previously only shown to affect math skills in the first few years of school. Brain scans of children with reading learning disabilities have indicated that their brains continue to process information as novice readers (Wolfe and Neville, 2004). Perhaps students with math learning disabilities have similar cognitive deficits. Future research should explore this possibility.

Table 1

CHC Factors

CHC factor	Description
Fluid Intelligence (Gf)	Ability to process novel tasks.
Crystallized Intelligence (Gc)	Acquired knowledge, and ability to apply this knowledge
Short-Term Memory (Gsm)	Ability to hold information in awareness and quickly use it
Visual Processing (Gv)	Ability to perceive process and analyze visual patterns and stimuli
Auditory Processing (Ga)	Ability to perceive, process, and analyze auditory patterns and stimuli
Long-Term Storage and Retrieval (Glr)	Ability to store and easily retrieve new or previously learned items from long-term memory
Processing Speed (Gs)	Ability to quickly perform cognitive tasks

Table 2

WJ III Subtests That Comprise CHC Clusters

CHC factor	WJ III subtests
Gf	Concept Formation, Analysis Synthesis
Gc	Verbal Comprehension, General Information
Gv	Spatial Relations, Picture Recognition
Gsm	Memory for Words, Numbers Reversed
Glr	Visual-Auditory Learning, Retrieval Fluency
Ga	Sound Blending, Auditory Attention
Gs	Visual Matching, Decision Speed

Table 3

CHC Broad and Narrow Abilities That Affect Math

CHC factor	Description	Narrow abilities shown to influence math performance
Gf	Fluid reasoning	Inductive (I) and General Sequential Reasoning (RG)
Gc	Comprehension-	Language Development (LD), Lexical Knowledge (VL) and Listening Abilities (LS),
Gsm	Short term memory and retrieval	Memory Span (MS), Working Memory (WM)
Gs	Speed of processing	Perceptual Speed (P)

Table 4

Frequencies by Grade

Grade	N
K	1
1	10
2	20
3	21
4	23
5	25
6	16
7	3
8	10
Total	129

Table 5

Sample Size and Descriptive Statistics for Clusters included in the Regression

Models

	N	M	SD
Broad Math	124	80.56	10.29
Math Calculation	88	78.30	12.04
Math Reasoning	23	81.78	7.38

Table 6

Standardized Regression Coefficients for WJ III Gf Cluster with Math

Achievement

Variable	Regression coefficient
Broad Math	.32**
Math Calculation	.23*

* $p < .05$. ** $p < .01$.

Table 7

Standardized Regression Coefficients for WJ III Gc Cluster with Math

Achievement

Variable	Regression coefficient
Broad Math	.33**
Math Calculation	.22*
Math Reasoning	-.14

* $p < .05$. ** $p < .01$.

Table 8

Standardized Regression Coefficients for WJ III Gsm Cluster with Math

Achievement

Variable	Regression coefficient
Broad Math	.30**
Math Calculation	.29**
Math Reasoning	.40*

* $p < .05$. ** $p < .01$.

Table 9

Standardized Regression Coefficients for WJ III Gv Cluster with Math

Achievement

Variable	Regression coefficient
Broad Math	.22**
Math Calculation	.21*
Math Reasoning	.39*

* $p < .05$. ** $p < .01$.

Table 10

Standardized Regression Coefficients for WJ III Ga Cluster with Math

Achievement

Variable	Regression coefficient
Broad Math	.22**
Math Calculation	.22*
Math Reasoning	.29

* $p < .05$. ** $p < .01$.

Table 11

Standardized Regression Coefficients for WJ III Glr Cluster with Math

Achievement

Variable	Regression coefficient
Broad Math	.30**
Math Calculation	.24*
Math Reasoning	.20

* $p < .05$. ** $p < .01$.

Table 12

*Standardized Regression Coefficients for WJ III Gs Cluster with Math
Achievement*

Variable	Regression coefficient
Broad Math	.21**
Math Calculation	.29**
Math Reasoning	.02

* $p < .05$. ** $p < .01$.

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