# Estimating the Soil–Water Characteristic Curve

# Using Grain Size Analysis and Plasticity Index

by

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# ABSTRACT


The infrastructure is built in Unsaturated Soils. However, the geotechnical practitioners insist in designing the structures based on Saturated Soil Mechanics. The design of structures based on unsaturated soil mechanics is desirable because it reduces cost and it is by far a more sustainable approach.

The research community has identified the Soil–Water Characteristic Curve as the most important soil property when dealing with unsaturated conditions. This soil property is unpopular among practitioners because the laboratory testing takes an appreciable amount of time. Several authors have attempted predicting the Soil–Water Characteristic Curve; however, most of the published predictions are based on a very limited soil database.

The National Resources Conservation Service has a vast database of engineering soil properties with more than 36,000 soils, which includes water content measurements at different levels of suctions. This database was used in this study to validate two existing models that based the Soil–Water Characteristic Curve prediction on statistical analysis. It was found that although the predictions are acceptable for some ranges of suctions; they did not performed that well for others. It was found that the first model validated was accurate for fine-grained soils, while the second model was best for granular soils.

For these reasons, two models to estimate the Soil–Water Characteristic Curve are proposed. The first model estimates the fitting parameters of the Fredlund and Xing (1994) function separately and then, the predicted parameters are fitted to the Fredlund and Xing function for an overall estimate of the degree

of saturation. Results show an overall improvement on the predicted values when compared to existing models. The second model is based on the relationship between the Soil–Water Characteristic Curve and the Pore-Size Distribution of the soils. The process allows for the prediction of the entire Soil–Water Characteristic Curve function and proved to be a better approximation than that used in the first attempt. Both models constitute important tools in the implementation of unsaturated soil mechanics into engineering practice due to the link of the prediction with simple and well known engineering soil properties.

DEDICATION

To my mother and my family

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This work was like building a bridge, where after thousands of difficulties many cars can pass safely on it. Now, I am a little more prepared to teach my daughters about life and to my students about engineering.

Thank you for doing something great: ‘To teach’.

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# CHAPTER 1

# INTRODUCTION

## 1.1 Overview

The Soil-Water Characteristic Curve (SWCC) is the relationship between matric suction and water content. This property is vital when solving engineering problems or designing in unsaturated soils. For example, this function allows for the determination of the hydraulic conductivity at different degrees of water content or saturation, which is very important when estimating fluid flow underneath covered areas such as foundations and pavement systems.

Most of the infrastructure is founded in unsaturated soils. Even though constitutive relationships that utilize the concepts of unsaturated soils have been proposed for the classic areas of interest to geotechnical engineers, the application or implementation into engineering practice has been rather slow. One of the reasons for the delay in the application of unsaturated soil mechanics in practice is with no doubt the time required for the determination of the SWCC in the laboratory, and also the specialized equipment and training needed.

In 2008, AASHTO approved the Mechanistic Empirical Pavement Design Guide (MEPDG). This new pavement design guide incorporates the effects of environmental conditions such as precipitation and temperature in the determination of changes of unbound material properties during the life of the pavement structure. This model makes use of unsaturated soil principles which in turn requires the input of the SWCC. To aid in the implementation of the MEPDG, an alternative way to determine the SWCC via laboratory testing, is a method that estimates or derives the SWCC based on well-known soil index properties.

Several attempts have been made to estimate the SWCC based on grain-size distribution (GSD) and well-known index properties such as Plasticity Index. Also, several approaches have been used to solve the problem including three major approaches (Zapata, 1999):

1) Statistical estimation of water contents/degree of saturation at selected matric suction values.

2) Correlation of soil properties with the fitting parameters of the SWCC function by means of nonlinear regression analysis.

3) Estimation of the SWCC using a physics-based conceptual model.

A comprehensive comparison of the different approaches and models can be found in Zapata, 1999; where yet another model was proposed based on the second approach. This approach was also taken by Perera, 2003 and further refined by Witczak et al., 2006.

In this study, the models published by Zapata, 1999 and Witczak et al., 2006 (MEPDG model) were validated with a large database of matric suction and other index soil properties collected as part of the National Cooperative Highway Research Program (NCHRP) 9-23A project (Zapata, 2010). Furthermore, two different procedures to estimate the SWCC are proposed. The first procedure is based on the correlation of soil properties with the fitting parameters of the SWCC analytical function proposed by Fredlund and Xing, 1994. The second procedure is based on the estimation of the SWCC based on a physics-based conceptual model which relates the grain-size distribution of the soil and index properties with the pore-size distribution. The first procedure will greatly aid in the implementation of the new MEPDG pavement design guide, while the second procedure presents an alternative approach that is both conceptually sound and easy to implement by engineering practitioners. Both procedures are based on the database collected during the NCHRP 9-23A project, which consisted of soil properties, including matric suction measurements, for more than 36,000 soils.

## 1.2 Historical Background

The NCHRP 9-23A project entitled "A National Catalog of Subgrade Soil-Water Characteristic Curve (SWCC) Default Inputs and Selected Soil Properties for Use with the ME-PDG" was carried out at Arizona State University in 2010 (Zapata, 2010). The objective of this project was the creation of a national database of pedologic soil families that reflected the input soil properties for subgrade materials needed in the implementation of the approved AASHTO ME-PDG (Darter et al., 2006). The database focuses upon the Soil-Water Characteristic Curve (SWCC) parameters, which are key parameters in the implementation of Level 1 environmental analysis as well as measured soil index properties needed in all hierarchical levels of the climatic/environmental engine of the guide, the “Enhanced Integrated Climatic Model (EICM)”. These parameters are primarily used to estimate the equilibrium moisture content in unbound materials which directly affect the pavement performance due to changes in the resilient modulus of the soil.

The NCHRP 9-23A project allowed for the creation of a database of more than 31,000 soils throughout the continental US, Puerto Rico, Hawaii and Alaska, which included soil index properties and moisture retention measurements for at least two or three levels of suction. This extensive database, perhaps the largest available, allowed for the determination of the fitting parameters to define the SWCC Function. Other properties available included the grain-size distribution, hydrometer analysis, liquid limit, plastic limit, and saturated hydraulic conductivity.

As part of this work, the database collected allowed for the validation of two models available for the determination of the fitting parameters for the SWCC function. The first model, developed by Zapata in 1999 consisted in a useful family of curves of SWCCs for both granular soils and fine-grained soils based on GSD parameters such as the percent passing No. 200 sieve (P200), the diameter corresponding to 60% passing (D60) and the PI. The equations developed in this study were initially adopted in the NCHRP 1-37A project entitled Design Guide for New and Rehabilitated Pavement Design and later replaced by a set of equations initially developed by Perera, 2003 and then refined as part of the NCHRP 1-40D project entitled Models Incorporated into the

Current Enhanced Integrated Climatic Model in 2006 (Witczak et al., 2006). The refined equations gave rise to the second model that was validated as part of this work.

This research work proposes two different procedures to estimate the SWCC. The first procedure makes use of a statistical analysis to estimate the SWCC fitting parameters needed in the Fredlund and Xing equation (Fredlund and Xing, 1994), which could be easily incorporated into the EICM. The second procedure makes use of a physics-based conceptual model and uses the entire GSD to estimate the SWCC function, by relating the suction values with the particle diameter.

## 1.3 Research Objectives

The main objectives of this research are to:

- Validate the SWCC prediction models previously proposed by Zapata, 1999 (Zapata Model) and by Witczak et al., 2006 (MEPDG Model) by using the database collected as part of the NCHRP 9-23A project from the National Resources Conservation Service.

- Propose a new set of Soil–Water Characteristic Curve parameters for the Fredlund and Xing equation based on correlations with soil index properties.

- Propose a new approach to estimate the Soil–Water Characteristic Curve based on a physics-based conceptual model whereas the entire grain–size distribution and soil index properties are related to the pore-size distribution of the soil.

## 1.4 Methodology

The methodology used to achieve the objectives of this research has been divided into the following three main stages:

1. *Creation of the database*. This involves the acquisition of the data, the conversion of the database into formats that makes it easier to manipulate, the selection of variables of interest to this research work, the recognition and elimination of inconsistent data, the statistical analysis of variables, and the analysis of variability of the data. Under this task, the suction measurements obtained will be used in the generation of the SWCC fitting parameters needed for the possible correlations with several soil properties such as gradation and consistency limits.

2. *Validation of two existing models* using the collected database. This task allows for the evaluation of the models previously developed by Zapata,1999; and the models developed by Perera, 2003, and enhanced as part of the project NCHRP 1-40D in 2006 (MEPDG model). The validation study will serve as benchmark to the effort pursue as part of this thesis work.

3. *Generation of new models to predict the SWCC*. As explained above, two different approaches were followed. The first procedure consists in finding relationships between the SWCC fitting parameters for the Fredlund and Xing function and well-known index properties. This procedure is of interest because the fitting parameters are needed as input values in the new pavement design guide (MEPDG). The second procedure makes use of the entire grain-size distribution curve and relates the particle diameter to their corresponding suction value. Once again, some index properties such as passing #200 and Plasticity index are used to calibrate the model found.

## 1.5 Organization of the Thesis

This research work is organized according to the outlined methodology as follows:

Chapter 1 presents the *Introduction* of the thesis. This introduction includes an overview of the importance of the work to be pursued, a brief historical background, the objectives of the thesis and an outline of the methodology followed in the development of the project.

Chapter 2 contains the *Literature Review* where the main geotechnical concepts about unsaturated soil mechanics and the soil–water characteristic curve are defined. Previous work on the prediction of the SWCC is summarized.

Chapter 3, *Database Collection*, presents the acquisition of the database. This chapter includes the process followed to acquire the data, the conversion of the database into formats easier to manipulate, the selection of appropriate variables needed for this research work, the removal of data that presented inconsistencies, the statistical analysis of variables, and the analysis of variability of the data to determine the restrictions into the final results.

Chapter 4 presents the effort done as part of the *Validation* of two existing SWCC predicting models by using the database collected and presented in Chapter 3.

A new set of models to estimate the Soil-Water Characteristic Curve fitting parameters of the model proposed by Fredlund and Xing (1994) is presented in Chapter 5. These new set of equations are based on the soils index properties and SWCC parameters obtained directly from testing results.

Chapter 6 proposes a different approach to estimate the SWCC, which is based on the similarity between the SWCC and the GSD curves.

Finally, Chapter 7 includes the conclusions per chapter and recommendations for future studies.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Introduction

Considering the main objective of this work the prediction of the Soil–Water Characteristic curve, it is convenient to review fundamental principles related to the Unsaturated Soil Mechanics.

Four main concepts are reviewed in detail: the stress state variables and the moisture flow in unsaturated soils; the matric suction and the soil–water characteristic curve. Finally, the approaches that have been attempted to predict the soil–water characteristic curve are presented.

In unsaturated soils, four phases in equilibrium are defining the system: the soil particle, the contractile skin, and the phases of air and water. In each phase, the measurable stresses ($\sigma$, $u_a$ and $u_w$) at equilibrium are formulated in equilibrium under the context of continuum mechanics. Fredlund & Morgenstern, 1977, defined the stress state in an unsaturated soil by using two independent stress tensors. The formulations are presented.

The second topic considered is the flow generated on the air and water phases under the applied pressure gradients. That gives rise to the matric suction, which has been considered the driving potential responsible of fluid flow, Fredlund & Rahardjo, 1993. This concept is briefly analyzed due to the importance in unsaturated soil mechanics. The clear knowledge about the causes of the driving potential on the air and water phases is fundamental in understanding the concepts of air and water flow in an unsaturated soil.

The concept of Matric Suction is explained under section 2-4. This concept is fundamental to understand the Soil–Water Characteristic Curve (SWCC) and its importance in unsaturated soil mechanics. The last reviewed topic relates to the different approaches to obtain the SWCC Function. The different approaches presented are based on the concepts outlined by Zapata, 1999 and Fredlund et al., 2003.

## 2.2 Stress State Variables

Terzaghi, 1943, introduced terms to understand the unsaturated soil behavior. His works was focused on saturated soil for which he defined the concept of “effective stress variable” as the most important variable or “effective” variable to define the state of stress in such soil. The effective stress is defined as:

Effective Stress: $\sigma' = \sigma - u_w$ ..................................................................(2-1)

Where:

$\sigma'$ = Effective stress

$\sigma$ = Total stress

$u_a$ = Pore water pressure

After Terzaghi, several researchers attempted to express the stress state of unsaturated soils. In the 1950's, Bishop, introduced the pore air pressure as an independent and measurable variable in order to define the effective stress in unsaturated soils (Bishop, 1959). Bishop proposed the following expression to estimate the effective stress:

$$\sigma' = (\sigma - u_a) + \chi( u_a - u_w) \quad \text{(2-2)}$$

Where:

$u_a$ = Pore air pressure

$\chi$ = Parameter related to the degree of saturation

In the 1960's, most of the research was focused in trying to define the stresses driving the behavior of unsaturated soils or trying to vary Bishop's equation. In this decade, many experiments ere performed and theoretical equations were presented by Donald, Blight, Aitchinson, Bishop, Coleman, Jennings, Burland, Richards, Matyas and others (Fredlund, 1979). Most of the models were based on measurable parameters such as total stress, pore–water pressure and pore–air pressure.

In the 1970's, Fredlund and Morgenstern, 1977, presented a new theoretical stress analysis for unsaturated soils based on two independent stress state variables: $(\sigma - u_a)$ and $( u_a - u_w)$, and considering the soil as a multiphase element. Assuming soil particles incompressible and chemically inert, the analysis inferred that any two of three possible normal stress variables could describe the stress state in an unsaturated soil:

$(\sigma - u_a)$ and $(u_a - u_w)$..............................................................................(2-3)

$(\sigma - u_w)$ and $(u_a - u_w)$..............................................................................(2-4)

$(\sigma - u_a)$ and $(\sigma - u_w)$..............................................................................(2-5)

Based on the stress equilibrium condition for an unsaturated soil, Fredlund presented an equation of forces in equilibrium considering the phases: air, water, and contractile skin. With these phases in equilibrium, he was able to establish three stress state variables: $u_a$ (which can be eliminated assuming soil particles and water are incompressible), $(\sigma - u_a)$, and $(u_a - u_w)$.

The stress state for an unsaturated soil can be expressed with the following stress tensors:

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yw} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \end{bmatrix} \quad (2\text{-}5)$$

and

$$\begin{bmatrix} (u_a - u_w) & 0 & 0 \\ 0 & (u_a - u_w) & 0 \\ 0 & 0 & (u_a - u_w) \end{bmatrix} \quad (2\text{-}5)$$

Where:

$\tau_{ij}$ = Shear stress in the i–plane and the j–direction

$\sigma_x - u_a$ = Net normal stress in x–direction

$\sigma_y - u_a$ = Net normal stress in y–direction

$\sigma_y - u_a$ = Net normal stress in z–direction

## 2.3 Moisture Flow in Unsaturated Soils

The moisture flow can be analyzed in terms of energy or "head" when water–air flows from a point of high energy to a point of low energy. This energy gradient is known as "hydraulic head gradient". The behavior of the moisture flow is described under the principles of Bernoulli and Darcy. These principles apply equally for both saturated and unsaturated soil.

Bernoulli's law consider the total energy or head as the sum of three heads: velocity head, pressure head and the position head. In geotechnical practice, the velocity head is very low when comparing with pressure head and position head (Fredlund & Rahardjo, 1993). The pressure head ($p/\gamma_w$) or total suction is made of two components: matric suction and osmotic suction. Therefore, the pressure head and position head, combined, define the hydraulic head gradient in an unsaturated or saturated soil. Equation 2-8, expresses the hydraulic head gradient, *h*, at any point in the soil mass.

$$h = (p/\gamma_w) + H \qquad (2\text{-}8)$$

Where:

$p$ = Total suction (matric suction + osmotic suction)

$H$ = Position head (elevation)

On the other hand, Darcy's law considers the flow of water–air proportional to the hydraulic gradient. This law can be written as:

$v = -k\ (dh/dx)$ ………………………...………………..…………. (2.9)

Where:

$v$ = Velocity of water flow through an unsaturated soil

$k$ = Coefficient of permeability or hydraulic conductivity

This hydraulic conductivity as a function of several factors: fluid viscosity, pore–size distribution, grain–size distribution, voids ratio, roughness of mineral particles and the degree of soil saturation. In unsaturated soil, the hydraulic conductivity varies depending on the stress state of the soil (Fredlund, 2006) and particularly, on the matric suction which greatly affects the amount of water into and out of the soil. It is important to recognize that the permeability of the soil can be represented by two functions depending whether the process is drying or wetting. Therefore, hysteresis in the soil–water characteristic curve drives hysteresis in the permeability function and hence, a close connection between the Soil–Water Characteristic Curve and the hydraulic conductivity function (expressed as hydraulic conductivity versus soil suction) should be expected.

One important concept related to the water in unsaturated soil was developed by Lorenzo A. Richards in 1931 (Richards, 1931). The equation described by Richards can be written as follows:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\theta)\left(\frac{\partial \psi}{\partial z} + 1\right)\right] \qquad (2.10)$$

Where:

$K(\theta)$ = Hydraulic conductivity as a function of volumetric water content

$\psi$ = Pressure head

$z$ = Elevation above a vertical datum

$\theta$ = Volumetric water content

$t$ = Time

Richard's concept, which describes the flow of water in unsaturated soils, is based on the laws of hydrodynamics where the movement of water is due to gravity and the pressure gradient acting on a soil element.

## 2.4 Matric Suction and the Soil–Water Characteristic Curve

Considering one of the three possible combinations of stress state variables given by equations (2-3), (2-4), and (2-5), Fredlund (2006) defined the net normal stress ($\sigma - u_a$) and the matric suction ($u_a - u_w$) as the most applicable combination in engineering practice. Net normal stress is the stress state caused by external loads removed/applied to the soil, and matric suction is the stress state caused by environmental conditions due to eliminate variations at the ground surface or by the groundwater table fluctuations.

Matric suction is defined in the standard ASTM D 5298 – 03 *Measurement of Soil Potential–Suction Using Filter Paper* as:

> "The negative pressure relative to ambient atmospheric pressure on the soil water, to which a solution identical in composition with the soil water must be subjected in order to be in equilibrium through a porous permeable wall with the soil–water".

In other words, matric suction is the measure of negative pore–water pressure due to changes in the relative humidity (water vapor pressure caused by the difference in air and water pressures across the water surface).

Values of matric suction in the field (vadose zone) can range from high pressures (1,000,000 kPa, Fredlund 2006) under dry conditions (zero water degree of saturation on the ground surface, in some cases) to zero at the ground water table. Soils close to the surface are frequently affected by environmental conditions causing a negative effect on the soil. These soils are called expansive or swelling, collapsible, and residual soils.

A plot between the matric suction and the volumetric water content or degree of saturation is called Soil–Water Characteristic Curve (SWCC) (see Figure 2-1). In engineering purposes, the degree of saturation (percentage of voids filled with water) is commonly used.

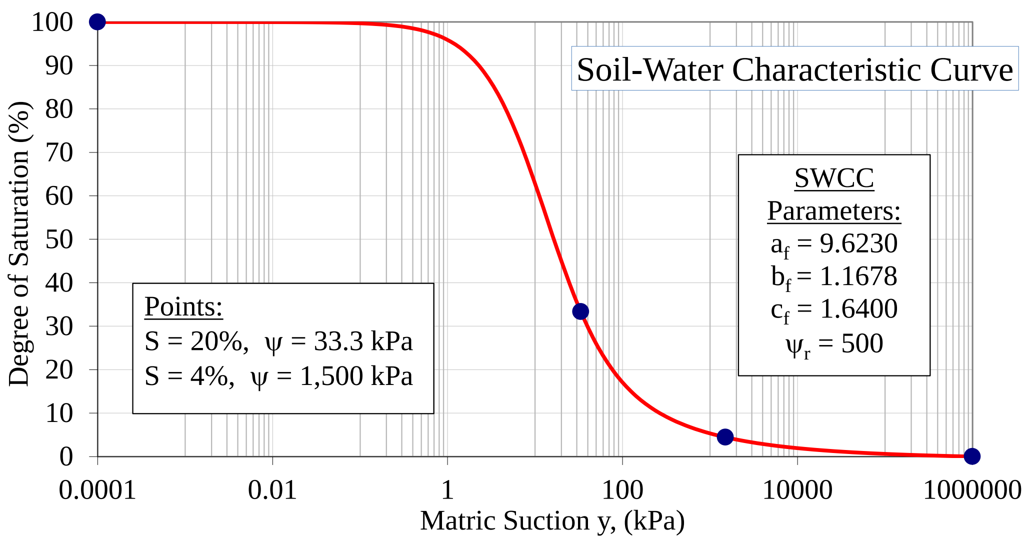



Figure 2-1. Soil–Water Characteristic Curve in Terms of Degree of Saturation

## 2.5 Approaches to Obtain Unsaturated Soil Properties

In order to obtain the SWCC, several levels of sophistication have been identified. For example, Fredlund et al., 2003, defined four hierarchical levels. The following sub–sections will brief explain each level of analysis.

### 2.5.1 Level 1.

Level 1 implies direct measurement of suction values and then the use of one of the universal models to fit the data to the whole range of suction. This level properly applies for large projects with high risk and high consequences due to failures. This level is usually followed by researchers that use equipment and techniques with advance level of investigation in unsaturated soil mechanics.

Universal models are basically a series of empirical equations developed by several researchers in order to best–fit the suction/water content values with a soil–water characteristic model. The best fit parameters of these models can be obtained by using a regression analysis that minimizes the least squared errors. These models can be categorized as two–fitting–parameter and three–fitting–parameter models. These parameters are related to the soil–water characteristic curve in this manner: the first parameter depends on the air entry value of the soil; the second parameter depends on the rate of water extraction of the soil after exceeding the air entry value, and the third parameter, when it is used, is basically a function of the residual water content at high values of suction.

Models with two parameters include those proposed by Garner (1958), Brooks & Corey (1964), Brutsaert (1967), Laliberte (1969), Farrel & Larson (1972), Campbell (1974), and McKee & Bumb (1987) (in Fredlund, 2006).

Some of the most commonly used models with three parameters were proposed by Van Genuchten, 1980, Fredlund and Xing, 1994, Leong and Rahardjo, 1997, and Sillers, 1997.

*Van Genuchten, 1980*, proposed the next equation with three fitting parameters:

$$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{\left[1+\left(\frac{h}{a}\right)^b\right]^c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2.11)}$$

Where:

$\theta_w$ = Volumetric water content

$\theta_r$ = Residual volumetric water content

*a* = Soil parameter which is a function of the air entry value of the soil

*b* = Soil parameter which is a function of the rate of water extraction of the soil after exceeding the air entry value

*c* = Soil parameter which is a function of the residual water content of the soil

*Fredlund & Xing, 1994,* proposed the following equation with three fitting parameters:

$$\theta_w = C(\psi) \times \left[ \frac{\theta_{sat}}{\left\{ ln\left[ e + \left( \frac{\psi}{a} \right)^n \right] \right\}^m} \right] \text{.......................................... (2.12)}$$

$$C(\psi) = 1 - \frac{ln\left(1 + \frac{\psi}{\psi_r}\right)}{ln\left[1 + \left(\frac{1{,}000{,}000}{\psi_r}\right)\right]} \text{............................................. (2.13)}$$

Where:

$\theta_w$ = Volumetric water content

$a$ = Soil parameter which is a function of the air entry value of the soil (kPa)

$n$ = Soil parameter which is a function of the rate of water extraction of the soil after exceeding the air entry value

$m$ = Soil parameter which is a function of the residual water content of the soil

$h_r$ = Soil parameter which is basically function of the suction at which residual water content occurs (kPa)

In summary, the implementation of unsaturated soil mechanics under level 1 requires testing to find directly the unsaturated soil property functions. Being these tests highly expensive, this level should be considered mainly for projects of great importance.

### 2.5.2 Level 2.

To find the SWCC under Level 2, there is no need for direct measurements. In this case, conceptual model to infer the SWCC (unsaturated soil property) from direct measurements of grain size distribution can be used. This approach was postulated by Fredlund et al., 1997, where a model was presented to estimate the SWCC from directly measured soil properties such as gradation, dry density, void ratio, and specific gravity. In this case, Fredlund et al. used a computational program (SoilVision®) to obtain the SWCC based on the least–square errors curve fitting algorithm. Their model requires a conceptual model as well as statistical computations of the SWCC parameters.

This approach has limitations. First, it requires a specific shape curve (sigmoid). It also requires a minimum number of particle sizes, which have a strong influence in the equation, and also requires three soil properties: Specific gravity, void ratio, and dry density. The prediction of the SWCC following this approach seems to be reasonable for non–plastic soils (Zapata, 1999).

In summary, at Level 2 analysis, unsaturated soil property functions can be inferred from other function measured in the laboratory, such as the grain–size distribution. This method has shown to be reliable for non–plastic soils. However, it makes use of a physics–based conceptual model which makes it a good candidate for reliable predictions provided a large database is used in the analysis.

### 2.5.3 Level 3.

At Level 3 analysis, a basic soil index property is correlated to estimate the SWCC. This level is frequently used for the preliminary studies of most projects. Statistical analyses are used at this level based on databases of previous test results.

There are two ways to estimate the soil–water characteristic curve at this level of analysis. First, the SWCC can be estimated from a database, by relating the SWCC with the gradation, with the classification, or with other index soil properties. Secondly, the SWCC can be estimated by relating a particular soil with a similar soil for which a SWCC exists or has been measured. This level implies a good criterion of estimation, and therefore the estimation at this level is less accurate. It is appropriate for small projects or projects with low risk of failure.

### 2.5.4 Level 4.

At this level, correlations are used to estimate the SWCC. This level has the lowest level of complexity, and could be applied to projects with low risk. This level implies the use of the soil classification to estimate the SWCC, and then to use this function to empirically estimate other unsaturated soil property functions.

It is obvious that Level 1 has the highest level of accuracy while Level 4 involves correlations and hence, it has a great level of variability.

In the progress of unsaturated soil mechanics techniques, the soil–water characteristic curve is a specialized test which involves laboratory equipment which it is quite complex to operate. This situation has created the need to estimate procedures, approaches, or use correlations to characterize unsaturated soils. This methodology is fundamental in the implementation of unsaturated soil mechanics into geotechnical engineering practice.

Zapata, 1999, presented a comprehensive review of approaches to predict the soil–water characteristic curve. In her work, the predictions were organized in three categories. The first category is based upon statistical estimation of water contents at selected matric suction values. The second category encompasses those models that, by regression analysis, correlate soil properties with the fitting parameters of analytical equations that represent the SWCC. The models in the third category estimate the SWCC using a physical conceptual model approach. One particular approach followed under the last category converts the Grain–Size Distribution into the Pore–Size Distribution which in turn can be developed into the SWCC by means of a packing parameter to relate or calibrate the relationship between the GSD and the SWCC.

## 2.6 Approaches to Predict the Soil–Water Characteristic Curve

Zapata, 1999, recognizes different approaches for the prediction of the SWCC at Level 3 analysis:

***Approach 1A***

In this approach, the water content at a particular suction values is estimated by using statistical correlation with grain–size distribution parameters and volume mass soil properties.

Researchers such as Van Genuchten, 1980, Mckee & Bumb, 1987, Van Genuchten & Mualem, 1980, Gardner, 1958, Williams et al., cited by Zapata, 2010, have predicted volumetric water content from equations calculated by regressions which have fitting parameters that are function of soil properties such as percentage of clay content, organic matter content, dry density, etc. These models were found with a very limited database.

***Approach 1B***

Besides the grain–size distribution and volume mass soil properties, several researchers have used one or more suction values to statistically estimate the water content in the SWCC. They have found that by adding one or two measurements of suction greatly improve the precision of the models. However, this concept requires determining one point in the SWCC from one specific value of matric suction, being this a limitation of this approach. As with the approach 1A, limited databases were used to develop these models.

### *Approach 1C*

Similarly to the approaches mentioned above, this approach is base on statistical estimation considering the grain size distribution and volume mass soil properties. Particularly, this approach use models based on a small database from a particular location. This approach has to be carefully considered understanding clearly the assumptions, soil tested, the precision, and the soil properties employed in this thesis.

### *Approach 2*

In this approach, correlations are based on regressions analysis. The water content can be computed by statistical correlations of soil properties with the fitting parameters of the SWCC.

The uncertainty associated with predicting models depends on the database used in the computations and the tests applied to validate the model. The models proposed and based on Level 2 analysis have proofed reliable for granular soils which operate in a low matric suction range of values. On the other hand, predictions of the SWCC for fine–grained soils can be considered unreliable. Some of features affecting the prediction of the SWCC on fine–grained soils include the shape of soil particle, organics coating, the entrapped air, and some adsorptive forces acting on the surface of soil particles. Nevertheless, this method is desirable if a large database is available for proper calibration.

## 2.7 Final Remarks

The literature review shown in this chapter allows us visualize that the estimation of the Soil–Water Characteristic Curve at any level of difficulty has been approached by several researches and most of them use very limited database and therefore, it is concluded that there is room for improvement of the estimation.

# CHAPTER 3

# DATABASE COLLECTION

## 3.1 Source of the Database

The United States Department of Agriculture (USDA) has an organization dedicated to conserving all natural resources particularly on private land. The private land is more than 70% of the land in this country, and for this reason, it is an objective of the Federal Government to ensure that the natural resources on these lands are protected and conserved. This federal organization is called the Natural Resources Conservation Service (NRCS) and has the objective of collecting, storing, maintaining, and distributing the soil survey information for private land owner in the United States (Soil Survey Staff, 1993).

Using the latest in science and technology, NRCS has been working with a multidisciplinary group of professionals on projects and research to get productive land in a healthy environment. These projects and research are developed in the field directly, and are complemented with analysis from tests developed in the laboratory (Manual, Soil Survey Division Staff, 1993).

One of the fundamental goals of the NRCS is soil conservation. Understanding that soil is a foundation for agricultural and sustainable development, it must be conserved between the highest standards of quality. In this regard, the USDA is working consistently to have soils well characterized in all private and public areas of the United States and its Trust Territories. The characterization involves investigation, inventory, classification, mapping, and interpretation of the quality of soils. This soil survey information is one of the most important tools for the planning and management of the majority of urban and rural projects where soil is involved. While the database was obviously intended for agricultural purposes; the USDA entered into a joint agreement with the then Bureau of Public Roads (BPR) to also measure key soil properties useful into the field of highway/pavement engineering. The engineering properties from this database will be used as the primary source of information in order to pursue the main objectives of this research work (Soil Survey Staff, 1975 & 1994).

## 3.2 Characteristics of the Database

The information obtained from the NRCS is divided into three main databases, which depend primarily on the scale used for mapping the different soil units.

1. The Soil Survey Geographic (SSURGO) database, at the farm to rural area scale ranging from 1:12,000 to 1:63,360

2. The State Soil Geographic (STATSGO) database, at the regional scale of 1:250,000, and

3. The National Soil Geographic (NATSGO) database, at the national scale of 1:5,000,000

The components of map units in each database are different and correspond to different levels of detail.  SSURGO database, for example, provides the most detailed level of information. Its information is focused on local levels, where the data is used for specific planning and management of farms, ranches, and rural areas. STATSGO was designed for regional levels, river basins, states, or regional governments with the purpose of planning, management and monitoring natural resources, lands and aquifers. Its information cannot be used for interpretation or planning at the county level. NATSGO has a lower level of detail and is basically used for national and regional resource appraisal, planning, and monitoring. Its information (maps and databases) was processed from states' general soil maps, and its attributes were generated from generalization of detailed maps.

It is important to note that each database contains the same amount if detail in tabular form. However, the maps (spatial data) are a function of the area scale. For example, a map unit at the SSURGO level represents a single soil component; while at the NATSGO level, a map unit can contain up to 21 components. Each component represents a soil profile with information of up to ten layers.

The source of information used in this study is based on the State Soil Geographic (STATSGO) database. Due to the level of detail and scale, the STATSGO database is specifically used by the NRCS in agricultural matters to evaluate regional soils and water quality, soil erodibility, soil wind erosion, crop growth, soil productivity, hydrology and ecology; and in general, to generate environmental quality models. Although STATSGO was primarily oriented for agricultural purposes, its information, has been used cautiously in studies of other fields of science such as biology, chemical engineering, geology, geotechnical engineering, environmental science, etc.

### 3.3 Data Collection

The US general soil map and data needed for this analysis was downloaded from the website: http://soildatamart.nrcs.usda.gov/ (reference). It contains two types of data: Spatial and Tabular files. The spatial files have information necessary to process the graphical expressions of the different soil units. They provide shapefiles, which allow the user to analyze spatial information, edit data and create maps in a GIS® (Geographic Information System) based format. The tabular files provide engineering and agricultural soil properties in Microsoft® Access format. This format allows the user to handle an immense volume of data. The tables are organized by group of attributes according to the technological field; and therefore, it is possible to classify and query the database. For the purpose of this study only the tabular information was downloaded and processed. However, there are sufficient capabilities for mapping the soil information, should further research requires visual representation of the soil properties obtained.

The US general soil map downloaded from the USDA website was obtained from a generalized analysis of detailed soil survey maps. In areas where soil survey maps were not available, information about geology, topography, vegetation, and climate, together with images was obtained from the Land Remote Sensing Satellite (LANDSAT) that allowed for the definition of the most probable attributes and areas for the STATSGO dataset (general soil map). Most of the soil areas are defined cartographically by using the satellite images together with the soil survey map. With regards to the characteristics, properties or attributes of a particular soil unit area; these were obtained as estimates of properties from sampling areas based on statistical extrapolation from areas where its properties were well-known.

The database developed by the USDA-NRCS is being constantly updated by organizing the layers according to new studies, validating soil properties from new information received and including new soil properties according to correlations provided by new research and studies. An example of the correlations used by NRCS is mentioned in Feng et al., 2009 for the Saturated Hydraulic Conductivity, Ksat. Ksat is expressed in mm/h and it is estimated by the equation (3-1),

$$Ksat = 1930\left(SAT - \theta_{33}\right)^{(3-\lambda)} \quad \text{..................................................................(3-1)}$$

Where:

$SAT$ = Saturated Moisture at 0 kPa (%v)

$\theta_{33}$ = Moisture at 33 kPa (%v)

$$\lambda = \frac{1}{A} \qquad (3\text{-}2)$$

$$A = \frac{[\ln(1500) - \ln(33)]}{[\ln(\theta_{33}) - \ln(\theta_{1500})]} \qquad (3\text{-}3)$$

This equation and other correlations used in the database are also presented by Saxton and Rawls, 2006.

The accuracy or errors encountered in the database are extremely important for the objectives pursued in this research work. For example, a study performed for the Western states (Feng et al., 2009) indicated a Root Mean Squared Error (RMSE) for sand and clay content between 4% and 7%. This validation was accomplished through comparisons with tests and studies which were carried out directly in the field by the USDA-NRCS Soil Survey Laboratory.

In GIS format, general soil map units are linked to attributes in order to indicate the location of each soil map unit and its soil properties. Although most of the continental US areas are defined, some areas do not have available information. The area where digital soil data is available is shown in Figure 3-1. The tabular data contained in the database represent a mean range of properties for the soil comprised in each soil map unit. The representative value is used in this study to define the soil property for each type of soil. The tabular data contain soil information that serves as an attribute of the soil map unit in GIS format.

The database main downloaded from the NCRS website contained information for 1,227,117 soils throughout the continental US, Alaska, Hawaii and Puerto Rico, with more than 150 geotechnical, chemical, and physical properties for all the layers and soil unit maps considered at the SSURGO level. The information is grouped in "map units". The “map units” are areas that represent a group of soil profiles with generally the same or similar characteristics. These map units contain information organized according to the schematic diagram shown in Figure 3-2. Each map unit is identified with a code called Mukey. Each Mukey or map unit is made of several "components", which are soil profiles with slightly different soil properties. The percentage of area, within the map unit, covered by each component is available. For the purpose of this project, it was assumed that the component with the largest percentage of coverage was representative of the entire map unit. Each profile is typically comprised of 3 to 5 layers or "soil units", with some profiles containing information of up to 11 layers. The depth covered by the typical profile averages about 60 inches, with some profiles approaching 100 inches.

### 3.3.1 Master File Properties and Characteristics.

Soil properties that are known to impact the moisture retention properties of the soil were extracted from the NRCS main database. From the main database, 52 soil properties were extracted in order to pursue the objectives of this research work. These geotechnical properties that constitute the Master database are summarized in Tables 3-1 and 3-2.

### 3.3.2 Preliminary Reduction of Soil Unit Data.

Each soil type found in the database had information from several boring logs. In most of the cases, the information was similar or very similar and therefore, it allowed for the initial reduction of the Master database. This process was carefully performed by choosing the boring log with the most complete information. In some cases, the information collected from two boring logs was complimentary and therefore, the information was combined to produce a complete description of the soil.

### 3.3.3 Selecting the Proper Component to Represent the Map Unit.

As previously noted, the Master database consisted of information for different components within each map unit. It was necessary to further reduce the database to reflect only one set of soil properties per map unit. For the purpose of this project, it was assumed that the component with the largest percentage of coverage was representative of the entire map unit. After this criterion was applied, the number of soils was reduced to 36,462 items.

### 3.3.4 Properties Included in the SWCC Predicting Analysis.

Table 3-3 presents the final selection of soil properties included in the SWCC predicting analysis and summarizes the percentage of data available, for each soil engineering variable considered in the final database. The soil properties needed to estimate the SWCC parameters include the volumetric water content at 10, 33, and 1,500 kPa; and the saturated volumetric water content (i.e., satiated water content or porosity). In addition, parameters such as grain-size distribution values, consistency limits, saturated hydraulic conductivity, groundwater table depth and bedrock information were included.

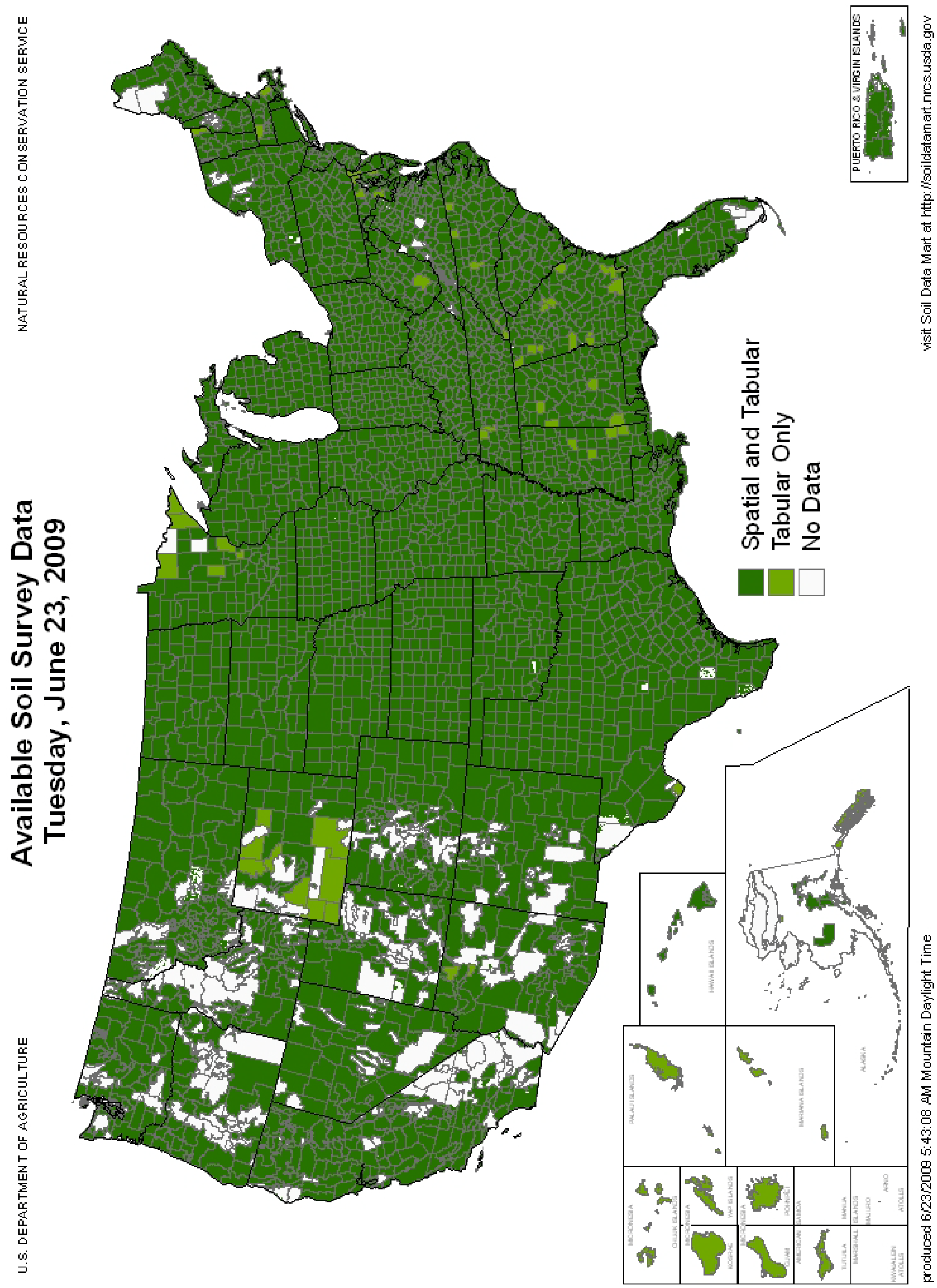



Figure 3-1. Available Soil Survey Data

(http://soildatamart.nrcs.usda.gov, June 2009)

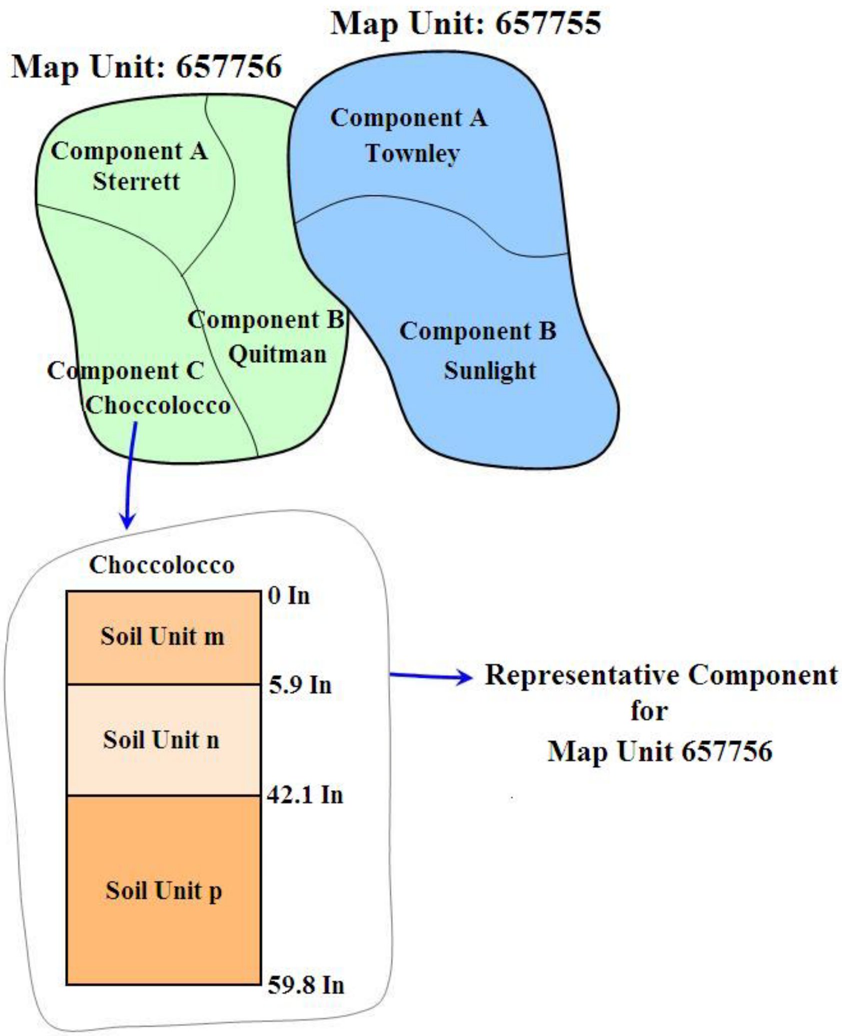



Figure 3-2. Schematic Representation of Map Unit, Component and Soil Unit

Table 3-1. Initial Soil Properties Selected for the Master Database

| Column Label | Column Name |
|---|---|
| Component Name | compname |
| AASHTO Classification | aashtocl |
| AASHTO Group Index | aashind_r |
| Unified Classification | unifiedcl |
| Top Depth - Representative Value | hzdept_r |
| Bottom Depth - Representative Value | hzdepb_r |
| Thickness - Representative Value | hzthk_r |
| #4 - Representative Value | sieveno4_r |
| #10 - Representative Value | sieveno10_r |
| #40 - Representative Value | sieveno40_r |
| #200 - Representative Value | sieveno200_r |
| Total Clay - Representative Value | claytotal_r |
| LL - Representative Value | ll_r |
| PI - Representative Value | pi_r |
| Db 0.1 bar H2O - Representative Value | dbtenthbar_r |
| Db 0.33 bar H2O - Representative Value | dbthirdbar_r |
| Db 15 bar H2O - Representative Value | dbfifteenbar_r |
| Ksat - Representative Value | ksat_r |
| 0.1 bar H2O - Representative Value | wtenthbar_r |
| 0.33 bar H2O - Representative Value | wthirdbar_r |

Table 3-1. Initial Soil Properties Selected for the Master Database (Cont'd)

| Column Label | Column Name |
|---|---|
| 15 bar H2O - Representative Value | wfifteenbar_r |
| Satiated H2O - Representative Value | wsatiated_r |
| LEP - Representative Value | lep_r |
| CaCO3 - Representative Value | caco3_r |
| Gypsum - Representative Value | gypsum_r |
| CEC-7 - Representative Value | cec7_r |
| Water Table Depth - Annual - Minimum | wtdepannmin |
| Water Table Depth - April - June - Minimum | wtdepaprjunmin |
| Bedrock Depth - Minimum | brockdepmin |
| Corrosion Concrete | corcon |
| Corrosion Steel | corsteel |
| EC - Representative Value | ec_r |
| Available Water Storage 0-150 cm | aws0150wta |
| SAR - Representative Value | sar_r |
| pH H2O - Representative Value | ph1to1h2o_r |
| Kw | kwfact |
| Kf | kffact |
| AWC - Representative Value | awc_r |
| Db oven dry - Representative Value | dbovendry_r |
| Comp % - Representative Value | comppct_r |
| Hydrologic Group | hydgrp |
| MAAT - Representative Value | airtempa_r |
| Elevation - Representative Value | elev_r |
| ENG - Local Roads and Streets | englrsdcd |
| Mapunit Key | mukey |
| Component Key | cokey |
| Chorizon Key | chkey |
| Chorizon AASHTO Key | chaashtokey |
| Chorizon Unified Key | chunifiedkey |

Table 3-2. Description of the Soil Properties Initially Selected from the Main Database

| Column Label | Description |
|---|---|
| Mapunit Symbol | The symbol used to uniquely identify the soil mapunit in the soil survey. |
| Map Unit Name | Correlated name of the mapunit (recommended name or field name for surveys in progress). |
| Component Name | Name assigned to a component based on its range of properties. |
| AASHTO Classification | A rating based on a system that classifies soils according to those properties that affect roadway construction and maintenance. Soils are classified into seven basic groups plus eight subgroups, for a total of fifteen for mineral soils. Another class for organic soils is used. The groups are based on determinations of particle-size distribution, liquid limit, and plasticity index. The group classification, including group index, is useful in determining the relative quality of the soil material for use in earthwork structures, particularly embankments, subgrades, subbases, and bases. (AASHTO) |
| AASHTO Group Index - Representative Value | The empirical group index formula devised for approximately within-group evaluation of the "clayey granular materials" and the "silty-clay materials". |
| Unified | Unified Soil Classification System - A system for classifying mineral and organo-mineral soils for engineering purposes based on particle size characteristics, liquid limit, and plasticity index. |
| Top Depth - Representative Value | The distance from the top of the soil to the upper boundary of the soil horizon. |
| Bottom Depth - Representative Value | The distance from the top of the soil to the base of the soil horizon. |
| Thickness - Representative Value | A measurement from the top to bottom of a soil horizon throughout its areal extent. |
| #4 - Representative Value | Soil fraction passing a number 4 sieve (4.70mm square opening) as a weight percentage of the less than 3 inch (76.4mm) fraction. |
| #10 - Representative Value | Soil fraction passing a number 10 sieve (2.00mm square opening) as a weight percentage of the less than 3 inch (76.4mm) fraction. |

Table 3-2. Description of the Soil Properties Initially Selected from the main Database (Cont'd)

| **Column Label** | **Description** |
|---|---|
| #40 - Representative Value | Soil fraction passing a number 40 sieve (0.42mm square opening) as a weight percentage of the less than 3 inch (76.4mm) fraction. |
| #200 - Representative Value | Soil fraction passing a number 200 sieve (0.074mm square opening) as a weight percentage of the less than 3 inch (76.4mm) fraction. |
| Total Clay - Representative Value | Mineral particles less than 0.002mm in equivalent diameter as a weight percentage of the less than 2.0mm fraction. |
| LL - Representative Value | The water content of the soil at the change between the liquid and plastic states. |
| PI - Representative Value | The numerical difference between the liquid limit and plastic limit. |
| Db 0.1 bar H2O - Representative Value | The oven dried weight of the less than 2 mm soil material per unit volume of soil at a water tension of 1/10 bar. |
| Db 0.33 bar H2O - Representative Value | The oven dry weight of the less than 2 mm soil material per unit volume of soil at a water tension of 1/3 bar. |
| Db 15 bar H2O - Representative Value | The oven dry weight of the less than 2 mm soil material per unit volume of soil at a water tension of 15 bars. |
| Dp | Mass per unit of volume (not including pore space) of the solid soil particle either mineral or organic. Also known as specific gravity. |
| Ksat - Representative Value | The amount of water that would move vertically through a unit area of saturated soil in unit time under unit hydraulic gradient. |
| 0.1 bar H2O - Representative Value | The volumetric content of soil water retained at a tension of 1/10 bar (10 kPa), expressed as a percentage of the whole soil. |
| 0.33 bar H2O - Representative Value | The volumetric content of soil water retained at a tension of 1/3 bar (33 kPa), expressed as a percentage of the whole soil. |
| 15 bar H2O - Representative Value | The volumetric content of soil water retained at a tension of 15 bars (1500 kPa), expressed as a percentage of the whole soil. |

Table 3-2. Description of the Soil Properties Initially Selected from the main Database (Cont'd)

| Column Label | Description |
|---|---|
| Satiated H2O - Representative Value | The estimated volumetric soil water content at or near zero bar tension, expressed as a percentage of the whole soil. |
| LEP - Representative Value | The linear expression of the volume difference of natural soil fabric at 1/3 or 1/10 bar water content and oven dryness. The volume change is reported as percent change for the whole soil. |
| CaCO3 - Representative Value | The quantity of Carbonate (CO3) in the soil expressed as CaCO3 and as a weight percentage of the less than 2 mm size fraction. |
| Gypsum - Representative Value | The percent by weight of hydrated calcium sulfate in the less than 20 mm fraction of soil. |
| CEC-7 - Representative Value | The amount of readily exchangeable cations that can be electrically adsorbed to negative charges in the soil, soil constituent, or other material, at pH 7.0, as estimated by the ammonium acetate method. |
| Water Table Depth - Annual - Minimum | The shallowest depth to a wet soil layer (water table) at any time during the year expressed as centimeters from the soil surface, for components whose composition in the map unit is equal to or exceeds 15%. |
| Water Table Depth - April - June - Minimum | The shallowest depth to a wet soil layer (water table) during the months of April through June expressed in centimeters from the soil surface for components whose composition in the map unit is equal to or exceeds 15%. |
| Bedrock Depth - Minimum | The distance from the soil surface to the top of a bedrock layer, expressed as a shallowest depth of components whose composition in the map unit is equal to or exceeds 15%. |
| Corrosion Concrete | Susceptibility of concrete to corrosion when in contact with the soil. |
| Corrosion Steel | Susceptibility of uncoated steel to corrosion when in contact with the soil. |
| EC - Representative Value | The electrical conductivity of an extract from saturated soil paste. |

Table 3-2. Description of the Soil Properties Initially Selected from the main Database (Cont'd)

| **Column Label** | **Description** |
|---|---|
| Available Water Storage 0-150 cm | Available water storage (AWS). The volume of water that the soil, to a depth of 150 centimeters, can store that is available to plants. It is reported as the weighted average of all components in the map unit, and is expressed as centimeters of water. AWS is calculated from AWC (available water capacity) which is commonly estimated as the difference between the water contents at 1/10 or 1/3 bar (field capacity) and 15 bars (permanent wilting point) tension, and adjusted for salinity and fragments. |
| pH H2O - Representative Value | The negative logarithm to the base 10, of the hydrogen ion activity in the soil using the 1:1 soil-water ratio method. A numerical expression of the relative acidity or alkalinity of a soil sample. (SSM) |
| Kw | An erodibility factor which quantifies the susceptibility of soil particles to detachment and movement by water. This factor is adjusted for the effect of rock fragments. |
| Kf | An erodibility factor which quantifies the susceptibility of soil particles to detachment by water. |
| AWC - Representative Value | The amount of water that an increment of soil depth, inclusive of fragments, can store that is available to plants. AWC is expressed as a volume fraction, and is commonly estimated as the difference between the water contents at 1/10 or 1/3 bar (field capacity) and 15 bars (permanent wilting point) tension and adjusted for salinity, and fragments. |
| Db oven dry - Representative Value | The oven dry weight of the less than 2 mm soil material per unit volume of soil exclusive of the desiccation cracks, measured on a coated clod. |
| Comp % - Representative Value | The percentage of the component of the mapunit. |
| Hydrologic Group | A group of soils having similar runoff potential under similar storm and cover conditions. |

Table 3-2. Description of the Soil Properties Initially Selected from the main Database (Cont'd)

| **Column Label** | **Description** |
|---|---|
| MAAT - Representative Value | The arithmetic average of the daily maximum and minimum temperatures for a calendar year taken over the standard "normal" period, 1961 to 1990. |
| Elevation - Representative Value | The vertical distance from mean sea level to a point on the earth's surface. |
| ENG - Local Roads and Streets | The rating of the map unit as a site for local roads and streets, expressed as the dominant rating class for the map unit, based on composition percentage of each map unit component. |
| Chorizon Key | A non-connotative string of characters used to uniquely identify a record in the Horizon table. |

Table 3-3. Summary of Final Database Statistics

| Soil-Property | Unit | n | % Data | Max | Min | Average | Median | Mode | StDev |
|---|---|---|---|---|---|---|---|---|---|
| Top Depth of Layer | cm | 36,462 | 100 | 241 | 0 | 28 | 8 | 0 | 39 |
| Bottom Depth of Layer | cm | 36,462 | 100 | 254 | 2 | 67 | 41 | 152 | 58 |
| Thickness of the Layer | cm | 36,462 | 100 | 218 | 2 | 39 | 28 | 20 | 32 |
| Passing Sieve # 4 | % | 36,462 | 100 | 100 | 19 | 85 | 93 | 100 | 18 |
| Passing Sieve # 10 | % | 36,462 | 100 | 100 | 13 | 80 | 88 | 100 | 21 |
| Passing Sieve # 40 | % | 36,458 | 100 | 100 | 6 | 69 | 73 | 95 | 22 |
| Passing Sieve # 200 | % | 36,455 | 100 | 100 | 0 | 51 | 50 | 43 | 25 |
| Passing Sieve 0.002 mm | % | 36,461 | 100 | 90 | 0 | 21 | 20 | 15 | 13 |
| Saturated Hydraulic Conductivity | µm/s | 36,460 | 100 | 423 | 0 | 21 | 9 | 9 | 30 |
| Volumetric Water Content at Suction 10 kPa | % | 4,357 | 12 | 37 | 0 | 15 | 15 | 14 | 5 |
| Volumetric Water Content at Suction 33 kPa | % | 36,462 | 100 | 55 | 0 | 21 | 21 | 14 | 9 |
| Volumetric Water Content at Suction 1500 kPa | % | 36,462 | 100 | 42 | 0 | 12 | 10 | 7 | 7 |
| Saturated Volumetric Water Content | % | 36,462 | 100 | 70 | 10 | 37 | 38 | 41 | 10 |
| Liquid Limit | % | 32,494 | 89 | 125 | 0 | 32 | 30 | 25 | 12 |
| Plasticity Index | % | 36,400 | 100 | 66 | 0 | 10 | 8 | 3 | 10 |
| Elevation | m | 31,708 | 87 | 3,963 | -5 | 973 | 823 | 305 | 753 |
| Bedrock Depth - Minimum | cm | 10,218 | 28 | 202 | 0 | 54 | 41 | 77 | 36 |
| Water Table Depth - Annual - Minimum | cm | 10,065 | 28 | 168 | 0 | 51 | 46 | 0 | 42 |
| Water Table Depth - April - June - Minimum | cm | 9,363 | 26 | 168 | 0 | 51 | 46 | 0 | 42 |

## 3.4 Working with the Database

In addition to the initial set of properties extracted from the database, further reduction of data was necessary in order to find the SWCC fitting parameters and the GSD fitting parameters. The computed parameters were also incorporated into the Final database generated from this study:

### 3.4.1 Calculating Soil-Water Characteristic Soil Parameters.

The Soil-Water Characteristic Curve (SWCC) is defined as the relationship between soil water content or degree of saturation and soil matric suction (Fredlund, 2006). Several researchers have proposed universal models to define the SWCC, as previously discussed in Chapter 2. However, the model implemented in the MEPDG is the model given by Fredlund & Xing, 1994 and shown in the equations 2-12 and 2-13 in Chapter 2; and therefore, it was desirable to find the fitting parameters of this model due to the practical application in pavement design and analysis.

This model represents a sigmoid with four fitting parameters, named $a_f$, $b_f$, $c_f$ and $h_{rf}$ in the MEPDG. The best set of these fitting parameters can be found by fitting the measured volumetric water content retained at tensions of 1/10 bars (10 kPa), 1/3 bars (33 kPa) and 15 bars (1,500 kPa) which are obtained from the primary database (explained in section 3-2 of this chapter).

The first step in order to find the best four SWCC fitting parameter was to define the available points. Two points were obtained from the complete database:

the volumetric water content at 1/3 bar (33 kPa) and the volumetric water content at 15 bars (1,500 kPa) of suction. With these data and the saturated volumetric water content it was possible to calculate the Degree of Saturations at the same suctions. Degree of Saturation is the ratio between the volumetric water content and the saturated volumetric water content for a specific suction. The third point is at zero suction when soil has 100% of saturation (Drying process was used for these tests). The fourth point is assumed at 1,000,000 kPa when the lowest values of saturation are reached, this assumption was defined by Dr. Fredlund (Fredlund & Xing, 1994, and Sillers & Fredlund, 2001).

The second step was to calculate the best fitting parameters by a non-linear least squared regression. This regression was developed by using the Tool Solver of Excel® from Microsoft (Microsoft Corp., Redmond, WA). Figure 3-3 shows the spreadsheet used for this calculation.

| Suction (Bar) | Suction (kPa) | Dry Density (gm/cc) | Note | Vol. w/c (%) | Sat vol w/c (%) | Sat (%) |
|---|---|---|---|---|---|---|
| 0.1 | 10 | | N/A | | 39.0 | |
| 0.33 | 33.33 | | N/A | 25.8 | | 66.2 |
| 15 | 1500 | | N/A | 14.2 | | 36.4 |

| **SWCC Parameters** | | |
|---|---|---|
| Parameter | final | initial |
| af | 1.1972 | 10 |
| bf | 1.4156 | 1 |
| cf | 0.4969 | 2 |
| hr | 500.0 | 500 |

| Objective Function |
|---|
| 1.05642E-09 |

| xe Suction (psi) | Vol. Water Content: % | ye Sat | yp | Constraints | | |
|---|---|---|---|---|---|---|
| 0.0001 | 39.0 | 100.0 | 100.0 | 0.000 | | |
| 4.8309 | 25.8 | 66.2 | 66.2 | 0.000 | | |
| 217.3913 | 14.2 | 36.4 | 36.4 | 0.000 | | |

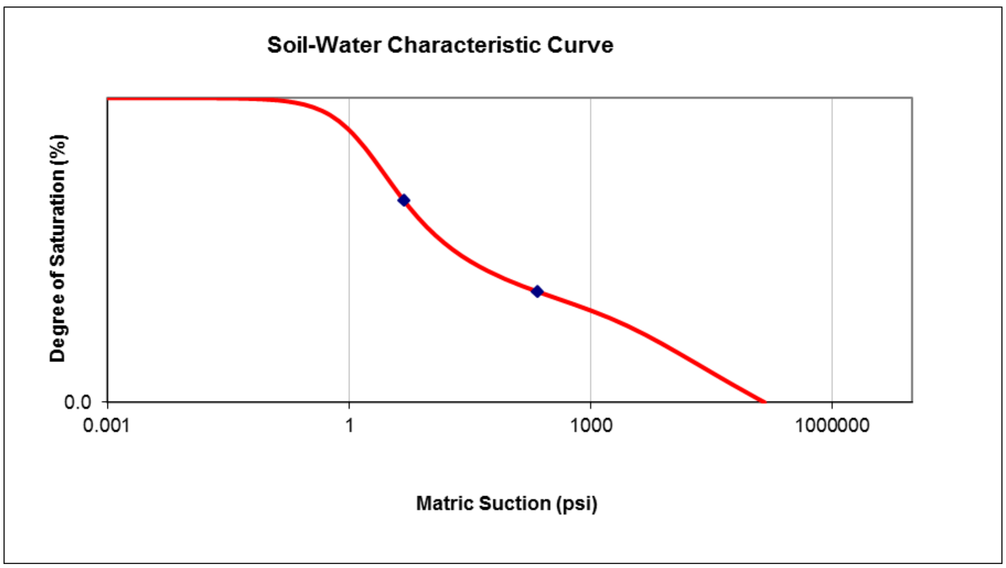



Figure 3-3. Spreadsheet with the SWCC Parameter Calculations

For this calculation, it is important to consider initial values for the Solver. These initial values should be assumed to be very similar to the final parameters in order to reach the lowest value of the objective function; which in this case is the difference between the squared of degree of saturations measured and predicted. In this project it was assumed:

| For plastic soils: | For non-plastic soils: |
|---|---|
| $a_f = 10$ | $a_f = 10$ |
| $b_f = 1$ | $b_f = 1$ |
| $c_f = 2$ | $c_f = 2$ |
| $h_{rf} = 500$ | $h_{rf} = 100$ |

In order to simplify, especially because a database of more than 31,000 items was employed, these initial parameters were used. Developing more detailed work would be ideal by using different initial parameters according to the wPI value; however, that work would require too much time. Finishing this calculation was necessary to re-process the Solver in many cases, especially with the non-plastic soil where the calculations showed many errors or ‘not found’ values.

Once the calculation for the SWCC was defined, the third step was to develop a VBA project which is usually called ‘Macro’ in Excel®. This program allows the execution of one process several times. The code for this macro for calculating the SWCC Parameters is as following:

```
Sub Macro1()

' Macro1 Macro

' Macro recorded 9/24/2010 by user

On Error GoTo Desc

    Sheets(1).Select

    Application.Goto Reference:="R1C1"

    Range("A1:AR1").Select

    Range(Selection, Selection.End(xlDown)).Select

    Range(Selection, Selection.End(xlDown)).Select

    Set RawData = Selection

    Sheets(2).Select

        For j = 2 To RawData.Rows.Count

            If RawData(j, 27) <> "" Then

            If RawData(j, 24) <> "" Or RawData(j, 25) <> "" Or RawData(j,
26) <> "" Then

                Range("E3").FormulaR1C1 = RawData(j, 24)

                Range("E4").FormulaR1C1 = RawData(j, 25)
```

```
Range("E5").FormulaR1C1 = RawData(j, 26)

Range("G3").FormulaR1C1 = RawData(j, 27)

Range("B8").FormulaR1C1 = 10

Range("B9").FormulaR1C1 = 1

Range("B10").FormulaR1C1 = 2

Range("B11").FormulaR1C1 = 100

Range("D15").FormulaR1C1 = Range("C15").Text

Range("D16").FormulaR1C1 = Range("C16").Text

Range("D17").FormulaR1C1 = Range("C17").Text

SolverReset

SolverOk SetCell:="$D$9", MaxMinVal:=2, ValueOf:="0",
ByChange:= _

    "$B$8:$B$11,$D$15:$D$17"

SolverAdd CellRef:="$E$15:$E$17", Relation:=2,
FormulaText:="0"

SolverAdd CellRef:="$B$8:$B$11", Relation:=3,
FormulaText:="0.0001"
```

```
SolverOptions MaxTime:=100, Iterations:=10000,
Precision:=0.000001, AssumeLinear _
    :=False, StepThru:=False, Estimates:=1, Derivatives:=1,
SearchOption:=1, _
    IntTolerance:=5, Scaling:=False, Convergence:=0.0001,
AssumeNonNeg:=True
 SolverSolve True, 1
 Range("B8").Select
Range(Selection, Selection.End(xlDown)).Select
Set Solution_ = Selection
RawData(j, "AO") = Solution_(1)
RawData(j, "AP") = Solution_(2)
RawData(j, "AQ") = Solution_(3)
RawData(j, "AR") = Solution_(4)
cHANGEsCALE
    End If
End If
```

```
    Next
Exit Sub
Desc:
MsgBox "Error"
End Sub
```

Finishing this step was necessary to re-process the data with errors in the results. This part of the work showed errors for several reasons. Non-plastic soils usually present problems because the rate of decrease of the degree of saturation is high with small changes of suction; these cases cause problems when finding the optimum in Solver. The Soils with high plasticity present problems as well, because the sigmoidal shape is lost and the approximation to the objective function takes a long time. As such, many times are shown as errors. It is important to emphasize the fact that a regression with only two points is large and not very precise. Many times the regression in the Solver cannot stop at an appropriate point. In these cases, having good initial parameters is the best way to reach the optimum. The complete database includes these parameters.

### 3.4.2 Calculating Grain-Size Distribution Parameters.

The Grain- Size Distribution Curve was represented by Wagner, 1994 as a sigmoidal shape with a lognormal distribution, presenting a high similarity to the soil-water characteristic curve given by Fredlund & Xing, 1994. The model equation for the grain-size distribution given by Wagner, 1994 is:

$$P_p(D) = \frac{1}{\ln\left[\exp(1) + \left(\frac{g_a}{D}\right)^{g_n}\right]^{g_m}} \left[1 - \left[\frac{\ln\left(1 + \frac{D_r}{D}\right)}{\ln\left(1 + \frac{D_r}{D_m}\right)}\right]^7\right] \text{..........................(3-4)}$$

Where:

$P_p(D)$ = percent of Passing a particular grain-size, $d$

$g_a$ = fitting parameter associated to the initial break in the GSD,

$g_n$ = fitting parameter associated to the maximum slope of GSD,

$g_m$ = fitting parameter associated to the curvature of the GSD,

$D$ = particle diameter in mm.

$D_r$ = residual particle diameter in mm.

$D_m$ = minimum particle diameter in mm.

Similarly as the SWCC parameter were calculated, the GSD fitting parameters were calculated finding the best set of GSD parameters. In this case the points are given in the database and correspond to the Gradation of the soil, Passing #4, #10, #40, and #200 US sieves. With these points, the regression was developed using Solver of Excel®. A spread sheet similar to the spread sheet used to calculate the SWCC parameters is showed in Figure 3-4.

| Particle Size | | % Pass |
|---|---|---|
| # | (mm) | (%) |
| 4 | 4.750 | 95.0 |
| 10 | 2.000 | 91.5 |
| 40 | 0.425 | 82.5 |
| 200 | 0.075 | 57.5 |

| Grain-Size Distribution Parameters | | |
|---|---|---|
| | Final | Initial |
| ga | 0.0001 | 1 |
| gn | 0.5837 | 0.5 |
| gm | 71.8474 | 0.5 |

| Objective Function |
|---|
| 1.06438 |

| De Diameter (mm) | % Passing Measured | % Passing Predicted | Constraints |
|---|---|---|---|
| 1,000 | 100.0 | 99.8 | 0.000 |
| 4.750 | 95.0 | 95.2 | 0.000 |
| 2.000 | 91.5 | 92.2 | 0.000 |
| 0.425 | 82.5 | 81.8 | 0.000 |
| 0.075 | 57.5 | 57.7 | 0.000 |

| Graph | |
|---|---|
| Particle Size (mm) | Passing (%) |
| 0.0001 | 0.00 |
| 0.001 | 0.18 |
| 0.01 | 17.31 |
| 0.075 | 57.68 |
| 0.1 | 62.76 |
| 0.425 | 81.80 |
| 1 | 88.51 |
| 2 | 92.17 |
| 4.75 | 95.20 |
| 10 | 96.86 |
| 100 | 99.17 |
| 1000 | 99.78 |

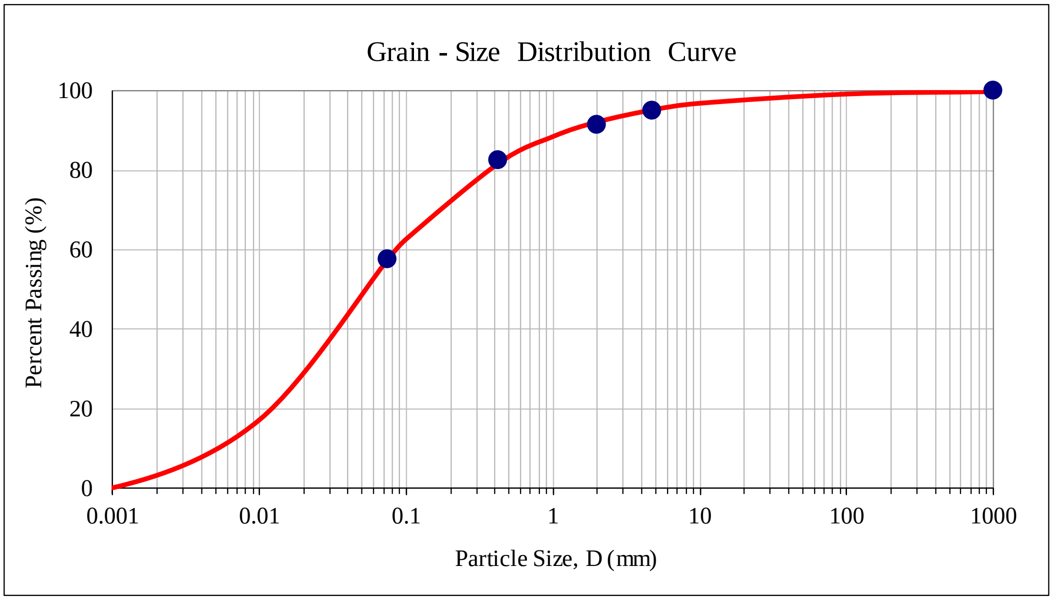



Figure 3-4. Spreadsheet with the GSD Parameter Calculations

For this calculation, is important to consider the initial values to use in the Solver as well. For this point, each soil or item of the database was run three times in the Solver. In this way, it was possible to define the best set of GSD parameters. The idea was to change the $a_g$ parameter because this parameter defines the break point of the curve. These three options allow for the choosing of the minimum value for the objective function based on the least squared error:

Objective Function = (square measured value – squared estimated value)

Once the calculation was correctly defined, the next step was developed using Macro in Visual Basic of Excel® in order to execute this process *n* times, *n* being the number of items of the database. The code for this macro is:

```
Sub Macro2()

' Macro2 Macro

' Macro recorded 9/24/2010 by user

'On Error GoTo Desc

    Sheets(1).Select

    Application.Goto Reference:="R1C1"

    Range("A1:AR1").Select

    Range(Selection, Selection.End(xlDown)).Select

    Range(Selection, Selection.End(xlDown)).Select
```

```
    Set RawData = Selection

    Sheets(2).Select

        For j = 2 To RawData.Rows.Count

            If RawData(j, 27) <> "" Then

            If RawData(j, 24) <> "" Or RawData(j, 25) <> "" Or RawData(j, 26) <> ""
Then

                Range("E3").FormulaR1C1 = RawData(j, 24)

                Range("E4").FormulaR1C1 = RawData(j, 25)

                Range("E5").FormulaR1C1 = RawData(j, 26)

                Range("E6").FormulaR1C1 = RawData(j, 27)

                Range("B9").FormulaR1C1 = 100

                Range("B10").FormulaR1C1 = 0.5

                Range("B11").FormulaR1C1 = 0.5

                Range("D16").FormulaR1C1 = Range("C16").Text

                Range("D17").FormulaR1C1 = Range("C17").Text

                Range("D18").FormulaR1C1 = Range("C18").Text

                Range("D19").FormulaR1C1 = Range("C19").Text
```

```
        Range("D20").FormulaR1C1 = Range("C20").Text



        SolverReset

        SolverOk SetCell:="$D$10", MaxMinVal:=2, ValueOf:="0",
ByChange:= _
            "$B$9:$B$11,$D$16:$D$20"

        SolverAdd CellRef:="$E$16:$E$20", Relation:=2, FormulaText:="0"

        SolverAdd CellRef:="$B$9:$B$11", Relation:=3,
FormulaText:="0.0001"

        SolverOptions MaxTime:=100, Iterations:=30000, Precision:=0.000001,
AssumeLinear _
            :=False, StepThru:=False, Estimates:=2, Derivatives:=2,
SearchOption:=1, _
            IntTolerance:=5, Scaling:=False, Convergence:=0.00001,
AssumeNonNeg:=True

         SolverSolve True, 1

        Range("B9").Select

        Range(Selection, Selection.End(xlDown)).Select
```

```
            Set Solution_ = Selection

            RawData(j, "AO") = Solution_(1)

            RawData(j, "AP") = Solution_(2)

            RawData(j, "AQ") = Solution_(3)

            RawData(j, "AR") = Range("D10").Text

        End If

    End If

  Next

Exit Sub

Desc:

MsgBox "Error"

End Sub
```

The data obtained after this process was added to the complete database. These parameters are indispensable calculating the Particle-Size values.

### 3.4.3 Calculating Particle Size Values.

Particle Size (called Effective Size as well) is the grain size or grain diameter of the soil through which a defined percentage of the total material is passing. For example, $D_{60}$ is the diameter in the grain-size distribution curve corresponding to the 60% finer. Figure 3-5 graphically shows the concept of Particle Size. This particle size is a good geotechnical property to estimate the hydraulic conductivity; many researches have shown similarities in the models between Grain-Size Distribution, Soil-Water Characteristic Curve and the Saturated Hydraulic Conductivity. The database developed for this work is an excellent platform for finding more correlations between them.

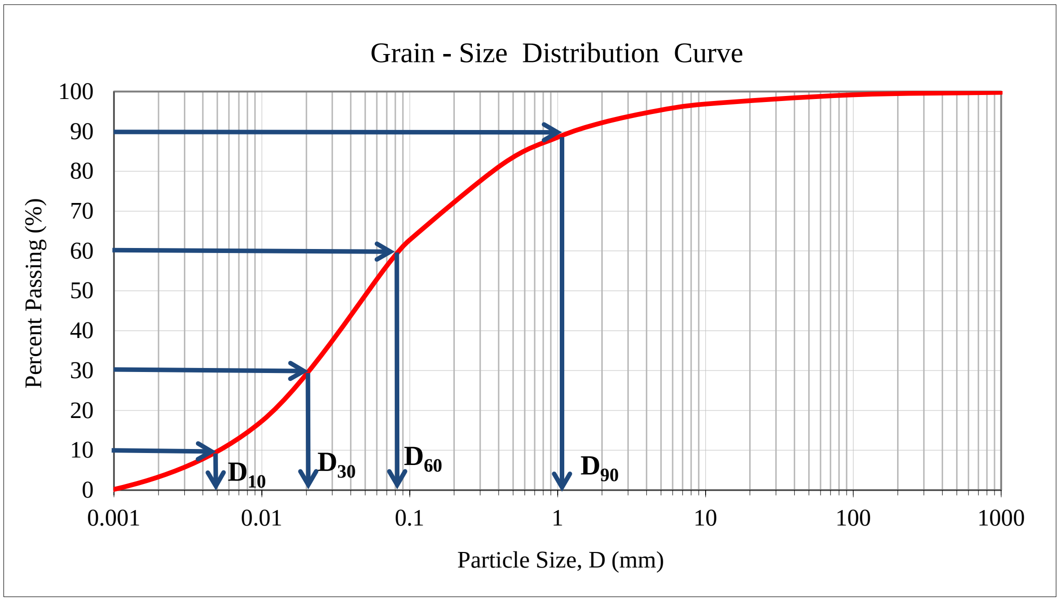



Figure 3-5. Finding Particle Size Values

Having the model equation for the grain-size distribution (see equation 3-4), which was implemented by Fredlund (Fredlund at al., 1997) into SoilVision® as a model to predict the SWCC, the next step was to calculate through this equation the Particle Size for 10, 20, 30, 60, and 90% of Passing. This step required calculating into the equation, the Dvalues from a defined percentage of passing.

This work was developed using the Tool Goal Seek in Microsoft Excel®, and using a Macro in Microsoft VBA® to repeat the process *n* times (*n* is the number of items considered in the database). After this, which required a long time to process, the database was considered complete.

Particle Size values are really important in the prediction of the SWCC parameters for non-plastic soils. Many researches have studied the geotechnical behavior for granular soils, and the effective size is an excellent measure for this type of soil. Ayra & Paris, 1981, Gupta & Larson, 1979, Wagner & Ding, 1994, Fredlund at al., 2000, are some researchers who have worked by using this concept.

Two factors added using the particle size values are the Coefficient of Uniformity, $C_u$ and the Coefficient of Curvature, $C_c$. These expressions are defined as:

$$C_u = \frac{D_{60}}{D_{10}} \qquad \text{(3-5)}$$

$$C_c = \frac{(D_{30})^2}{(D_{10})(D_{60})} \qquad \text{(3-6)}$$

Where:

$D_{10}$ = grain diameter in mm corresponding to 10% passing, by weight

$D_{30}$ = grain diameter in mm corresponding to 30% passing, by weight

$D_{60}$ = grain diameter in mm corresponding to 60% passing, by weight

Coefficient of uniformity, $C_u$ is an expression to define the shape of the grain-size distribution. When a granular soil is well graded, $C_u$ is higher than 15. The sand of a beach poorly graded has a $C_u$ between 2 and 3. A granular soil with a $C_u$ = 1, is a soil with particles of the same size. Coefficient of Curvature, $C_c$ is an expression related to the shape of the particle size distribution, values of $C_c$ between 1 and 3 are considered well graded soils. These geotechnical expressions, which refer to the grain-size distribution are related to the SWCC, and are also related to the saturated hydraulic conductivity.

### 3.4.4 Other Calculations.

The group index is an engineering concept developed by AASHTO that categorizes the probable "service performance" of the soil, particularly when it is used as a highway pavement subgrade. The group index can be calculated by the empirical equation given in the standard AASHTO M 145-91 (Standard Specification for Classification of Soils and Soil-Aggregate Mixtures for Highway Construction Purposes):

$$GI=(P_{200}-35)[0.2+0.005(LL-40)]+0.01(P_{200}-15)(PI-10) \quad \text{(3-7)}$$

Where:

$P_{200}$ = Passing the No. 200 sieve

$LL$ = Liquid Limit

$PI$ = Plasticity Index

Note that the first term is related to the liquid limit and the latter to the plasticity index. The final GI value is based on the following qualifications:

- If the GI calculated is negative, it is taken to be zero
- The GI calculated is rounded to the nearest whole number
- There is no upper limit
- The GI for the following soils must be taken as zero: A-1-a, A-1-b, A-2-4, A-2-5, and A-3
- For soils A-2-6 and A-2-7, the GI must be calculated by the equation:

$$GI=0.01(P_{200}-15)(PI-10) \quad (3\text{-}8)$$

Salient grain size distribution parameters such as the $D_{60}$, Passing 200 and Plasticity Index are required to estimate the weighted plasticity index (wPI). This property is estimated as follows:

$$wPI = \frac{P_{200} \times PI}{100} \quad (3\text{-}10)$$

The wPI will depend on the type of soil being considered. For coarse soils the $wPI = 0$, and for soils with more than 12% of fines, the $wPI > 0$.

The relationship between the Group Index (equation 3-7) and the wPI (equation 3-10) is shown in Figure 3-6. For this analysis were used the entire database with wPI greater than zero.

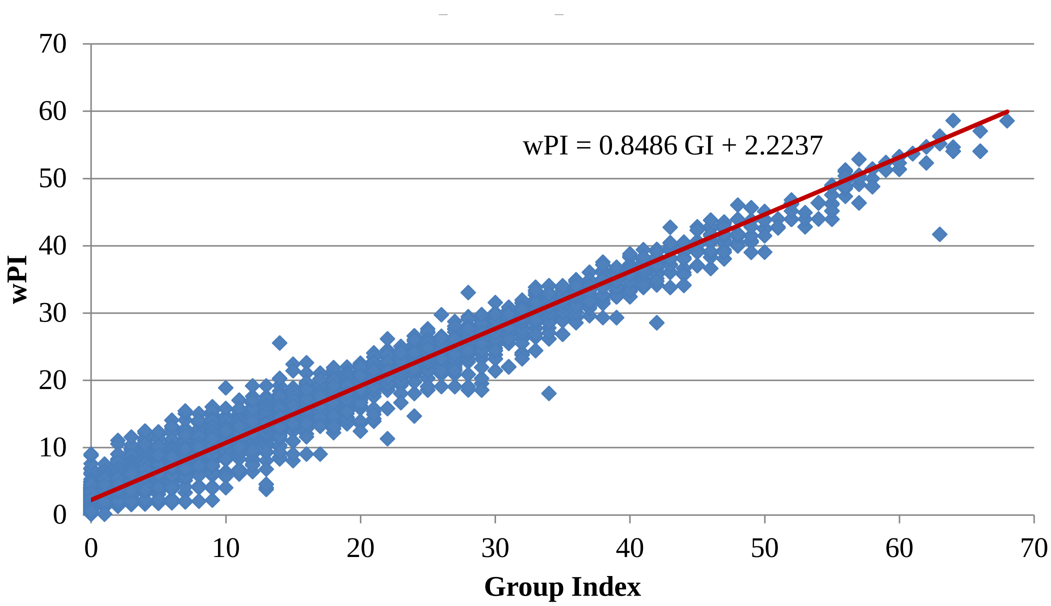



Figure 3-6 Relationship between the Group Index and the Weighted Plasticity Index

In order to have a complete database for this project (or for future projects) these two properties were calculated: the California Bearing Ratio (CBR) and the Resilient Modulus (MR).

The CBR is an empirical soil property that characterizes the strength of materials in subgrades and unbound material. This characteristic allows for the estimation of the resilient modulus by using the expression:

$$M_R(psi) = 2{,}555 \times CBR^{0.64} \qquad (3\text{-}9)$$

This expression is used in the ME-PDG Methodology (Witczak et al, 2001). CBR values can also be estimated based on index soil properties like Grain Size Distribution and Atterberg's Limits. USCS and AASHTO classifications are correlated to estimate typical CBR and MR values. However, one practical way is to use the grain size distribution.For coarse soils (with $wPI = 0$), the CBR value is referred to by the grain diameter at which 60% passes the grain size distribution ($D_{60}$), in millimeters. The formula in this case is:

$$CBR = 28.09(D_{60})^{0.358} \qquad (3\text{-}11)$$

This expression has two limitations: for soils with $D_{60}$ less than 0.01 mm, a CBR = 5 is used and for soils with $D_{60}$ greater than 30 mm, a CBR = 95 value is used. For fine soils (with wPI>0), the expression that is used is:

$$CBR = \frac{75}{1 + 0.728(wPI)} \qquad (3\text{-}12)$$

It should be realized that all of these conditions are approximations to the real measured laboratory value for either CBR or Mr. Their use should be confined to only level three applications of the design guide.

## 3.5 Summary

The database collection was a very important task for the development of the work presented in this thesis. The vast amount of data points collected contained a total of 36,394 different soils, with 4,518 items corresponding to non–plastic materials and 31,876 plastic soils. The database was collected by the National Conservation Resources Service for agricultural purposes and contains chemical, physical and engineering soil properties which can be used in a number of disciplines. The soils properties were obtained from studies developed during many years through the continental US, Alaska, Hawaii, and Puerto Rico. The database allowed for the estimation of parameters such as the wPI factor, Group Index, the Soil–Water Characteristic Curve fitting parameters and the Grain–Size Distribution fitting parameters.

Most of the properties were obtained directly from the laboratory or from field testing while other properties were estimated from correlations or estimations. Both sets of data or properties had some degree of uncertainty related to them. The uncertainty of the data can be attributed to several factors: First, the database was developed by collecting tests during a range of years (USDA–NRCS was established in 1935); second, uncertainty associated with environmental conditions and soil nature (samples were located all over the US territory); third, the tests were performed by following protocols and standards which are being constantly updated; and last, technological changes and advances in the field allowed for new data interpretations during more than 70 years the data has been collected.

In order to eliminate the variability encountered in the data, a moving average technique was employed, whereas the data was organized or sorted according to the geotechnical factor (predictor) that most affected the predicting variable. This process is commonly used when the database presents high variability in order to find the general trend of behavior (Graham, 1993).

It is important to emphasize that the vast database collected and presented as part of this thesis work was drawn directly from laboratory testing. It is perhaps the largest database of soil moisture retention curves available in the world. These facts allowed for optimal models to estimate the Soil–Water Characteristic Curve, as those presented in this work.

The process followed in this chapter had the objective of preparing a geotechnical database with the biggest quantity of data available for modeling. This database was used in correlations to create a new set of equations for the Soil-Water Characteristic Curve Parameter Two approaches were utilized to formulate these equations.

Table 3-4. Summarizes the process followed to prepare a complete database for this work and for future work. The Primary database was downloaded from the USDA–NRCS website which contains all the data used to interface with maps that are allowed for working in all the US areas for agricultural purposes. The Master database was condensed from the Primary database and contains exclusively engineering properties; this database was reduced to basically one with the largest amount of different soils. The Initial database was extracted from the Primary database and contains only the soil properties required for this work. This database presents the soil-properties and the original data from the Primary database. The complete database contains the initial database, as well as the soil–properties calculated such as the SWCC parameter, GSD parameters and particle size or D-values, computed at level 1 by using equations and correlations previously established in other research.

Table 3-4. Process Developed in Preparing the Database

| **Database** | **# Items** | **Obtained from** | **Contain** | **Soil-properties** |
|---|---|---|---|---|
| Main | 1,227,117 | USDA-NRCS | Entire database from US at SSURGO level | Contains more than 150 chemical, physical and geotechnical properties |
| Master | 36,462 | Main | Database with all the Engineering data | 52 soil properties to be used for this and future projects |
| Final | 36,462 | Master | Soil-properties required for this work | 18 soil properties selected from NRCS plus 19 properties estimated, for a total of 37 soil properties |

# CHAPTER 4

# VALIDATION OF AVAILABLE SWCC PREDICTION MODELS

## 4.1 Introduction

This chapter intends to confirm the validity of two available SWCC models that are based on index properties by using principles of statistics. In this manner, it is possible to evaluate the bias of the published models towards a rather limited database used in their development. A statistical analysis of errors will enable the study to reach this objective. This chapter intends to check two important projects previously developed at Arizona State University.

The first SWCC model to validate was proposed by Zapata in 1999. In her dissertation titled: "Uncertainty in Soil-Water Characteristic Curve and Impacts on Unsaturated Shear Strength Predictions." She presented two sets of SWCC fitting parameters (i.e. one set for plastic material and another set for non-plastic soils) derived from a regression analysis from a set of 190 laboratory tests. The SWCC model followed in this work corresponded to Fredlund & Xing, 1994.

The second SWCC model was developed in 2006 by an ASU research team as part of the NCHRP 1-40D project, titled "Models Incorporated into the Current Enhanced Integrated Climatic Model NCHRP 9-23 Project" which was developed for the National Cooperative Highway Research Program (Witczak at al, 2006). The model was a modification of the proposed equations by Perera, 2003 and had the main objective of validating the Enhanced Integrated Climatic

Model (EICM) to incorporate unsaturated soil properties and environmental effects in the overall pavement design procedure. EICM is the module that ASU research team included in the Mechanistic-Empirical Pavement Design Guide (MEPDG) version 0.7. As part of this project, a new set of models was presented for the $a_f$, $b_f$, $c_f$ and $h_r$ SWCC fitting parameters under the primary SWCC model published by Fredlund & Xing, 1994.

In short, the process developed in this chapter initially consisted in preparing a complete database with the information required: the database comprised geotechnical properties, such as wPI, % Grain Size Distribution parameters (passing a particular sieve), Particles Size diameters (called Dvalues), and the SWCC parameter predicted by the two models to be validated (Zapata's model and NCHRP 1-40D model). The database used in the validation process was explained in detail in Chapter 3 (Database Collection) and consisted of 36,462 soils (36,462 soils are plastic and 4520 soils are nonplastic). The next step was to calculate the volumetric water content for different suction values (1, 10, 100, 1,000 and 10,000 kPa) by using the two models. Finally, a statistical analysis of the errors was conducted and the results presented in tables and plots.

## 4.2 Validating Zapata's Model

### 4.2.1 Model for Plastic Soil.

The 190 data points used by Zapata, 1999, in developing her model were classified into two types according to Plasticity Index; 70 soils with plasticity index values greater than zero (plastic soils) and 120 soils with plasticity index values equal to zero (non-plastic soils). Soil properties obtained in the laboratory work allowed for an estimation of the best fitting SWCC parameters by using SoilVision® (software created by SoilVision System Ltd). This software estimated these parameters by fitting the primary SWCC model developed by Fredlund & Xing, 1994.

For plastic soils (soils with PI > 0), Zapata used the percent of Passing # 200 sieve and the Atterberg Limits, and specifically the Plasticity Index to find the models for the SWCC fitting parameters. Basically, the weighted plasticity index or wPI factor was the main geotechnical concept used in her model for this type of soil. wPI combines both properties the percent of Passing # 200 and the Plasticity Index, and it is defined as follows:

$$wPI = \frac{P200 \times PI}{100} \qquad (4\text{-}1)$$

Where:

P200 = Passing # 200 US Standard Sieve, expressed in percentage

PI = Plasticity Index, expressed in percentage

The equations proposed by Zapata are:

$$a_f = \frac{0.00364(wPI)^{3.35} + 4(wPI) + 11}{6.895} \qquad (4\text{-}2)$$

$$\frac{b_f}{c_f} = -2.313(wPI)^{0.14} + 5 \quad \text{(4-3)}$$

$$c_f = 0.0514(wPI)^{0.465} + 0.5 \quad \text{(4-4)}$$

$$\frac{h_r}{a_f} = 32.44e^{0.0186(wPI)} \quad \text{(4-5)}$$

The family of curves for plastic soils obtained by Zapata is shown in Figure 4-1.

Furthermore, a correlation to estimate the saturated volumetric water content, $\theta_s$, as a function of wPI was proposed:

$$\theta_s = 0.0143(wPI)^{0.75} + 0.36 \quad \text{(4-6)}$$

The validity of this equation was analyzed with the database available for this project. In order to accomplish this, the equation was statistically evaluated by calculating the mean algebraic and the mean absolute errors. Additionally, the adjusted coefficient of determination, $R^2$, and the Se/Sy parameter were computed in order to assess the accuracy of the equation (Hines & Montgomery, 1990)

The following statistical expressions were used:

Mean algebraic error: $$e_{a\lg} = \frac{\sum\left[\frac{(\theta_m - \theta_p)100}{\theta_m}\right]}{n} \quad \text{(4-7)}$$

Mean absolute error: $$e_{abs} = \frac{\sum \left[ \frac{(\theta_m - \theta_p)100}{\theta_m} \right]}{n} \quad \text{...................................(4-8)}$$

Sum of the squared error: $$S_e = \sqrt{\frac{\sum (\theta_m - \theta_p)^2}{n - p}} \quad \text{..................................(4-9)}$$

Mean of the squared error: $$S_e = \sqrt{\frac{\sum (\bar{\theta}_m - \theta_p)^2}{n - p}} \quad \text{..............................(4-10)}$$

Adjusted Coefficient of Determination: $R^2 = 1 - (S_e/S_y)^2$..................(4-11)

Where:

$\theta_m$ = Measured volumetric water content

$\theta_p$ = Predicted volumetric water content

$\bar{\theta}_m$ = Average measured volumetric water content

$n$ = Number of data points

$p$ = Number of parameters associated with the model

The statistical parameters found for the $\theta_s$ equation when using the database employed in this project are:

Database with n = 36,394

$e_{alg}$ = 98.87

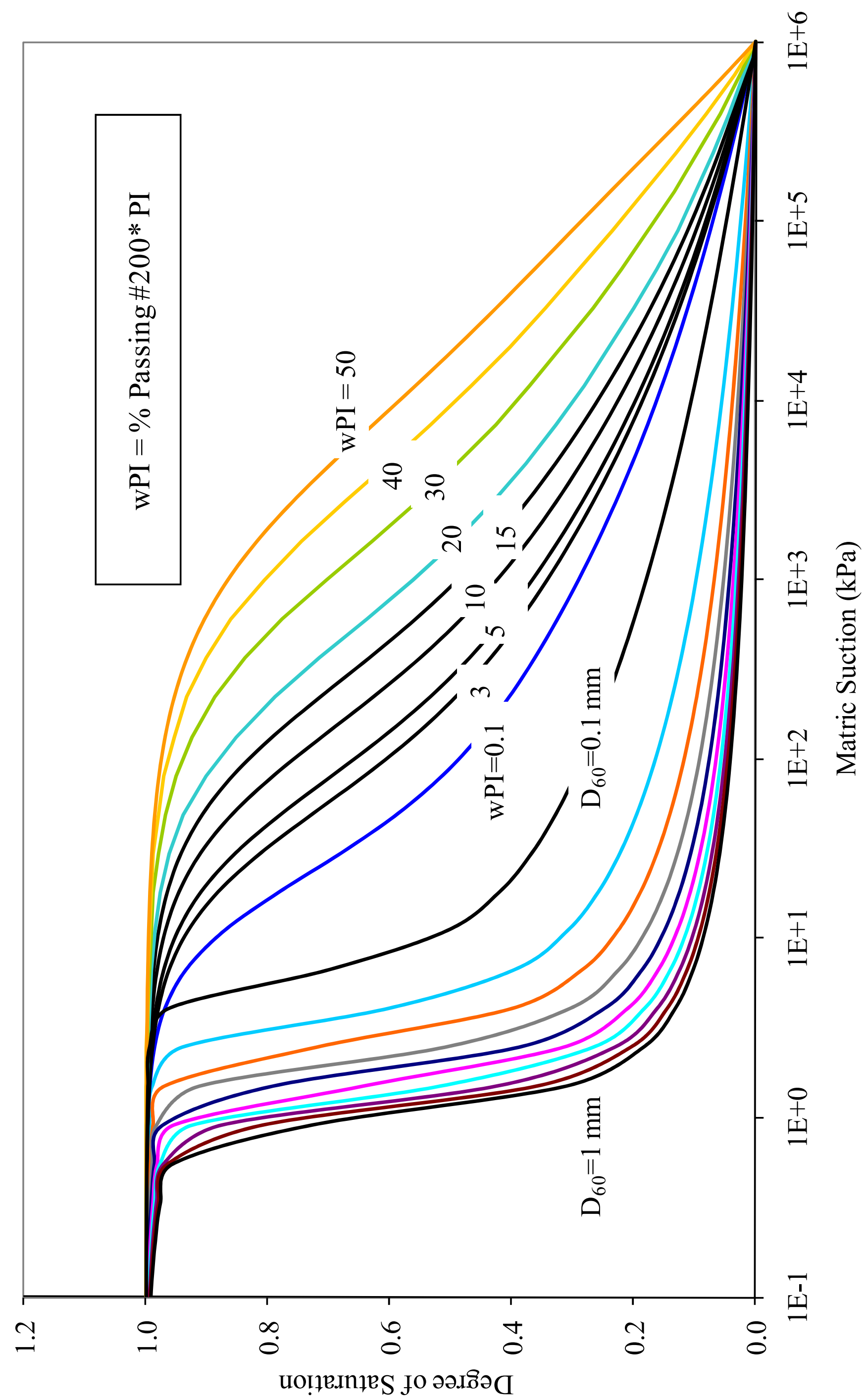



Figure 4-1. Family of SWCC's for Plastic Soils

$e_{abs} = 98.87$

$Se/Sy = 0.99$

$R^2 = 0.0224$

These results suggest a very weak, if at all, correlation. In order to find out if there is any correlation between wPI and $\theta_s$, the database was regressed. 36,394 points were used in the regression.

The following correlation was found:

$$\theta_s = 7.92(wPI)^{0.27} + 25 \qquad (4\text{-}12)$$

The plot obtained for this correlation is showed in Figure 4-2.

In order to eliminate some scatter, a moving average technique was employed in the analysis. This technique required the wPI values sorted from the smaller to the largest. Then every 300 points were averaged.

The relationship obtained confirmed the fact that there exists a correlation between volumetric water content and wPI. It also confirms the Zapata's correlation is only valid for the database used in her analysis, which is very limited.

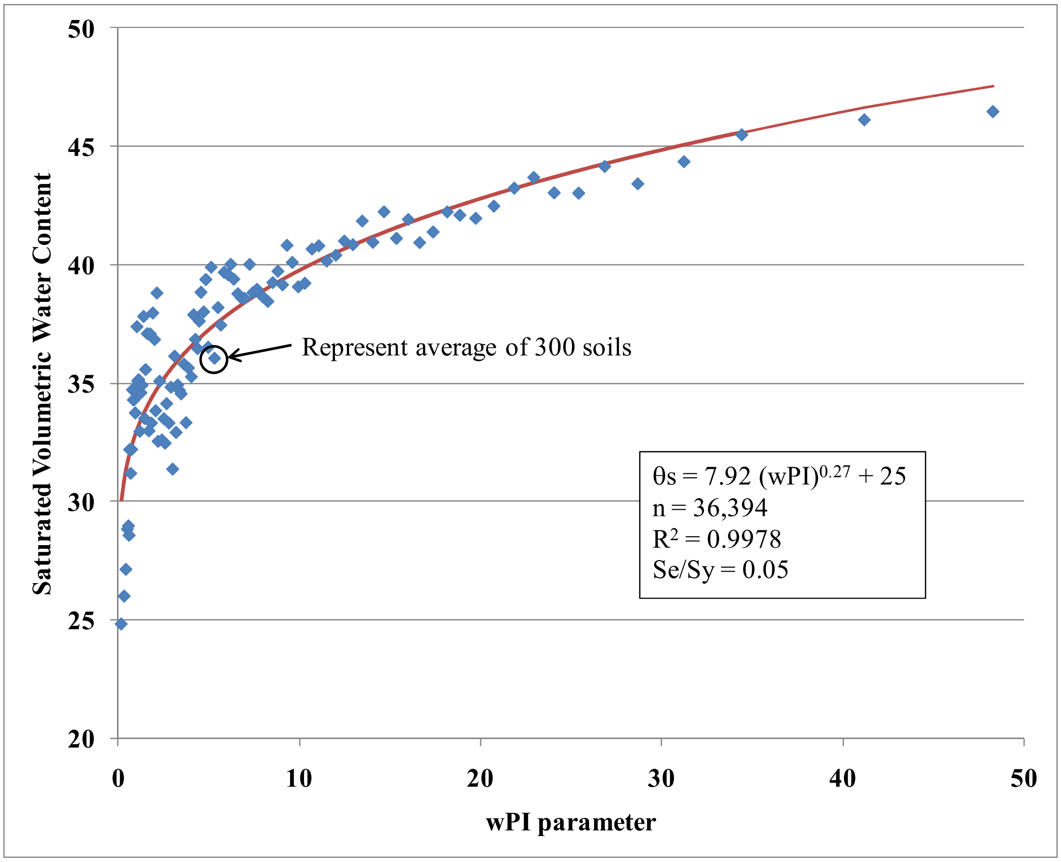



Figure 4-2. Relationship between Saturated Volumetric Water Content and wPI for Plastic Soils

### 4.2.2 Model for Non–Plastic Soil.

For non-plastic soils, Zapata used the Diameter $D_{60}$ as the main soil property or predictor to correlate with the SWCC parameters. The equations presented in her dissertation are:

$$a_f = \frac{0.8627(D_{60})^{-0.751}}{6.895} \quad \text{.....(4-13)}$$

$$\bar{b}_f = 7.5 \quad \text{..... (4-14)}$$

$$c_f = 0.1772\ln(D_{60}) + 0.7734 \text{ ..........................................................}(4\text{-}15)$$

$$\frac{h_r}{a_f} = \frac{1}{D_{60} + 9.7e^{-4}} \text{ ...................................................................}(4\text{-}16)$$

The family of curves for non-plastic soils obtained by using Zapata correlations is shown in Figure 4-3. The Combined families of curves for plastic and non-plastic soils were shown in Figure 4-4.

### 4.2.3 Zapata's Model Validation Analysis

In order to validate the model, the Fredlund & Xing, 1994, model was fitted to the matric suction data for each soil in the database. A comparative analysis was developed between the SWCCs fitted to the measured data and the SWCCs obtained with the models developed by Zapata, 1999. In order to estimate the errors, the measured volumetric water content, □m, was compared with the estimated volumetric water content, □p, by using Zapata's model. The analysis was performed by comparing these values at different suction values. In this manner, it was possible to assess the behavior of the model in a wide range of values.

The results of the error were assessed for both plastic and non-plastic soils. The results are shown in Table 4-1.

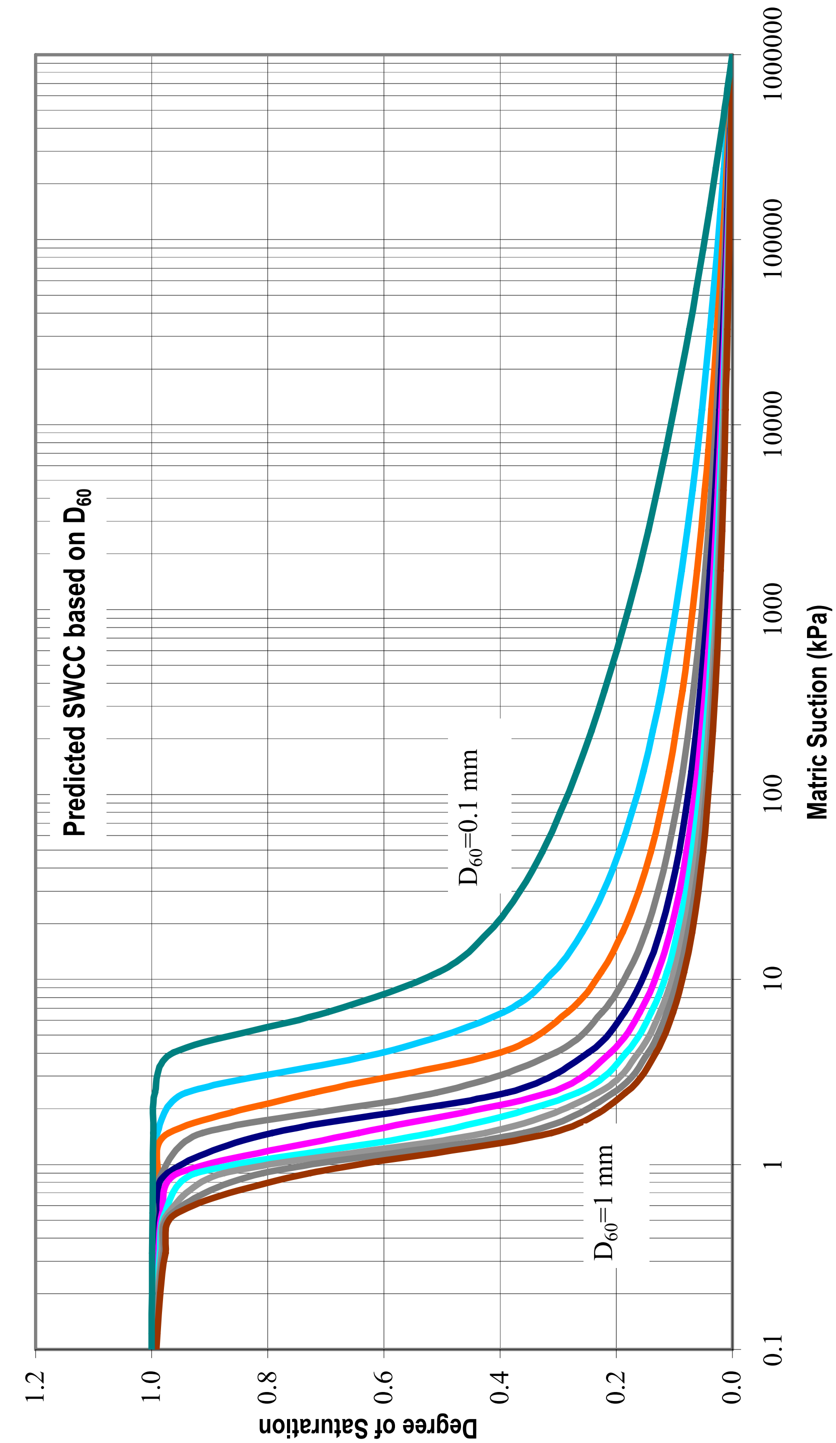



Figure 4-3. Family of SWCC's for Non-Plastic Soils

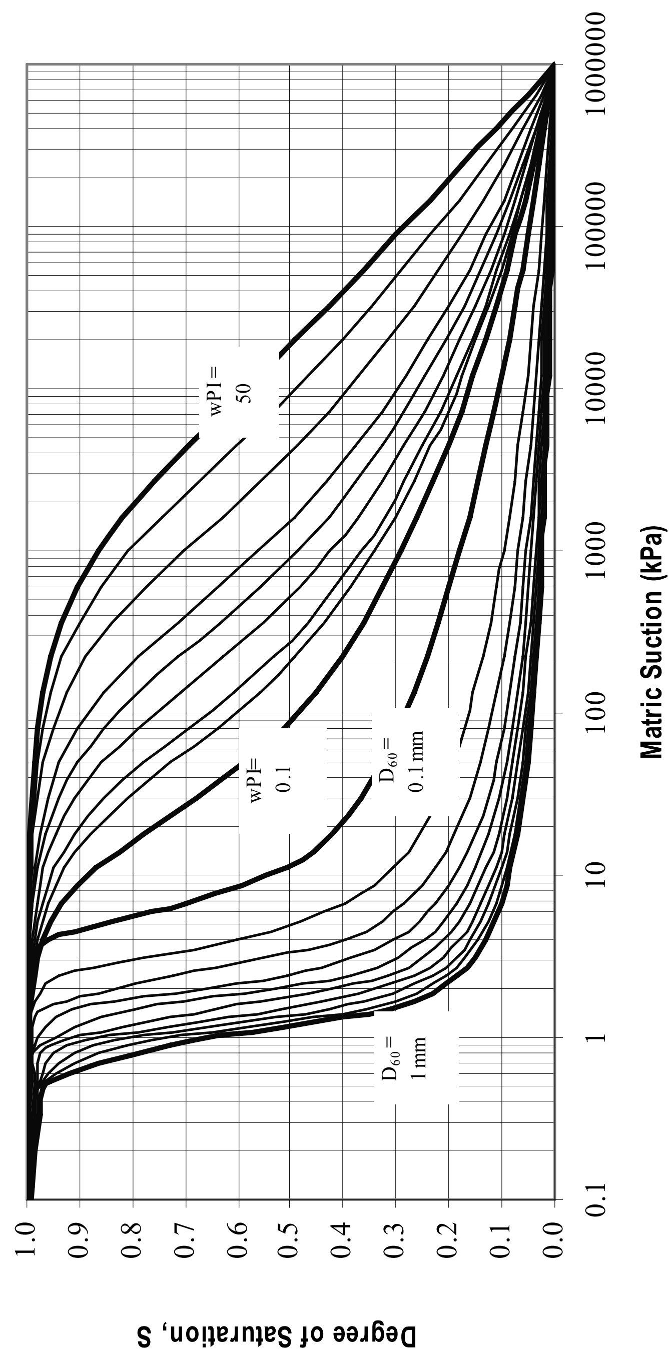



Figure 4-4. Combined Family of SWCC for both Plastic and Non-plastic Soils

Table 4-1. Comparative Analysis of Errors for SWCCs

**Analysis for Non Plastic Soils**

| Parameter | $\psi$ = 1 kpa | $\psi$ = 10 kpa | $\psi$ = 100 kpa | $\psi$ = 1,000 kpa | $\psi$ = 10,000 kpa |
|---|---|---|---|---|---|
| n | 4,518 | 4,518 | 4,518 | 4,518 | 4,518 |
| $e_{alg}$ | 26.88 | 67.84 | 62.18 | 61.00 | 59.05 |
| $e_{abs}$ | 38.22 | 73.80 | 68.62 | 68.70 | 69.23 |
| $S_e/S_y$ | 0.93 | 1.35 | 0.57 | 0.46 | 0.42 |
| $R^2$ | 0.1271 | -0.8119 | 0.6800 | 0.7872 | 0.8225 |

**Analysis for Plastic Soils**

| Parameter | $\psi$ = 1 kpa | $\psi$ = 10 kpa | $\psi$ = 100 kpa | $\psi$ = 1,000 kpa | $\psi$ = 10,000 kpa |
|---|---|---|---|---|---|
| n | 30,672 | 30,672 | 30,672 | 30,672 | 30,672 |
| $e_{alg}$ | -8.05 | -30.89 | -38.59 | -24.71 | -20.67 |
| $e_{abs}$ | 8.08 | 31.76 | 40.24 | 35.04 | 40.57 |
| $S_e/S_y$ | 0.42 | 1.16 | 1.01 | 0.63 | 0.62 |
| $R^2$ | 0.8262 | -0.3503 | -0.0248 | 0.6002 | 0.6110 |

Figures 4-5 through 4-9 show the plots measured versus predicted volumetric water content for plastic soils. These plots correspond to comparisons made at suctions of 1 kPa, 10 kPa, 100 kPa, 1,000 kPa, and 10,000 kPa, respectively.

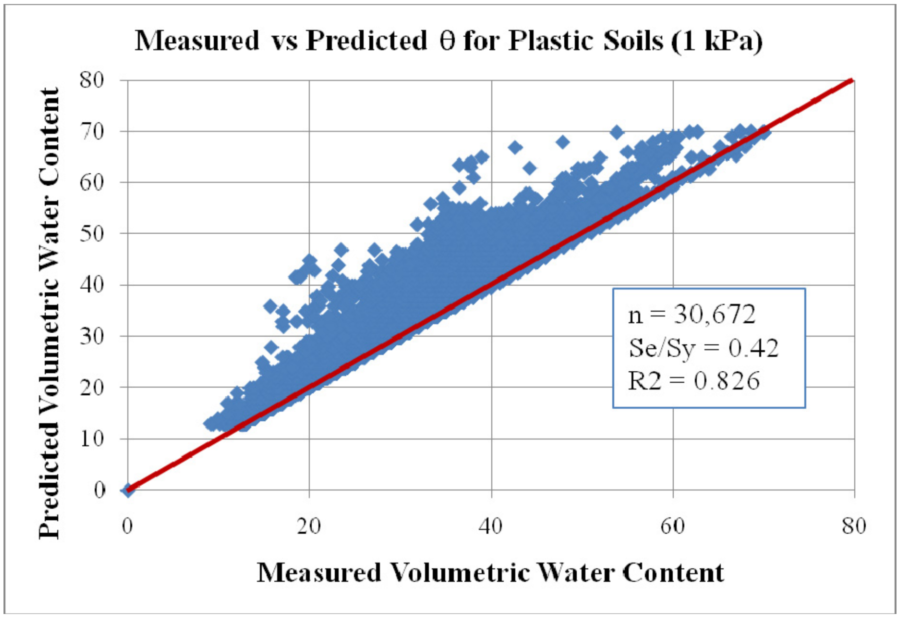



Figure 4-5. Measured vs. Predicted Volumetric Water Content based on Zapata's Model - Plastic Soils (Suction 1 kPa)

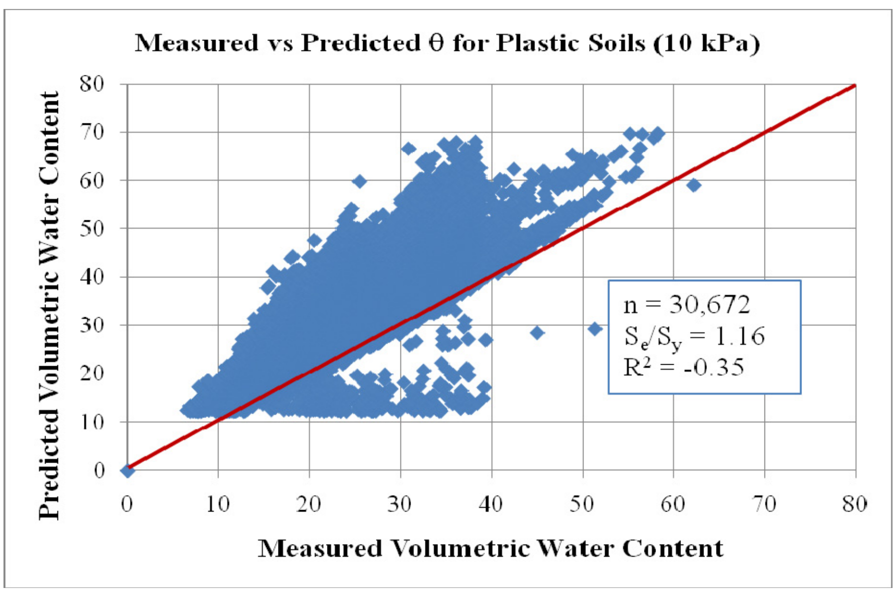



Figure 4-6. Measured vs. Predicted Volumetric Water Content based on Zapata's Model - Plastic Soils (Suction 10 kPa)

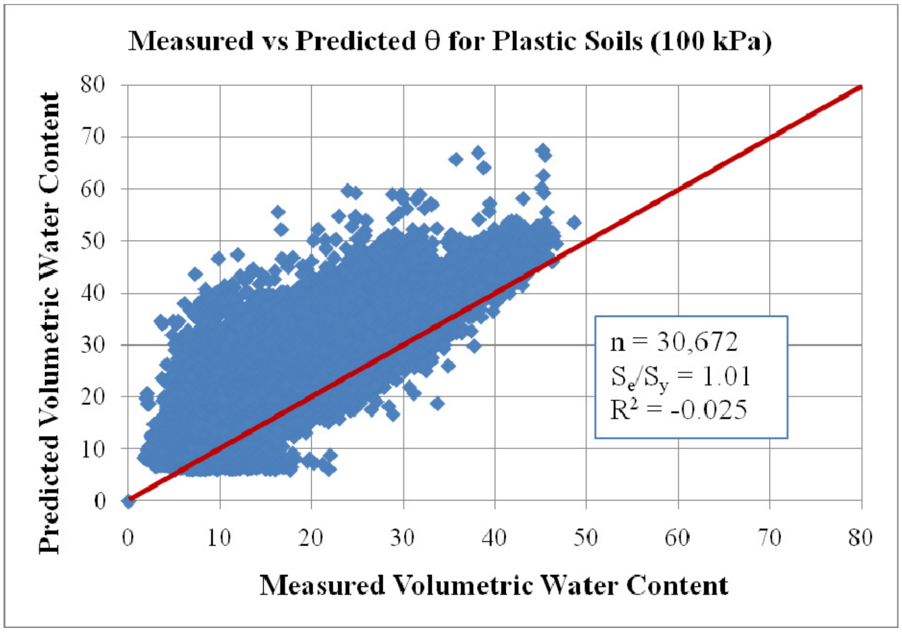



Figure 4-7. Measured vs. Predicted Volumetric Water Content based on Zapata's Model - Plastic Soils (Suction 10 kPa)

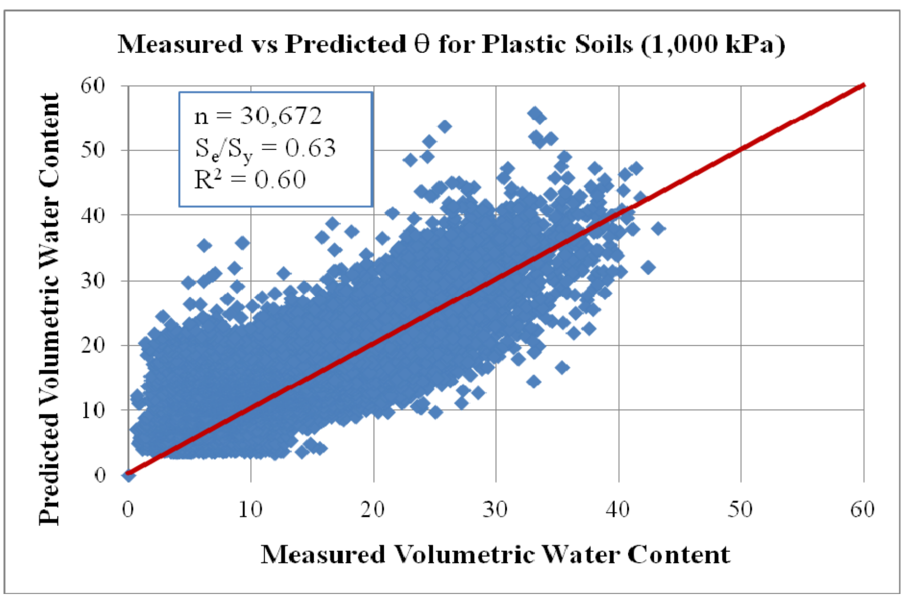



Figure 4-8. Measured vs. Predicted Volumetric Water Content based on Zapata's Model - Plastic Soils (Suction 10 kPa)

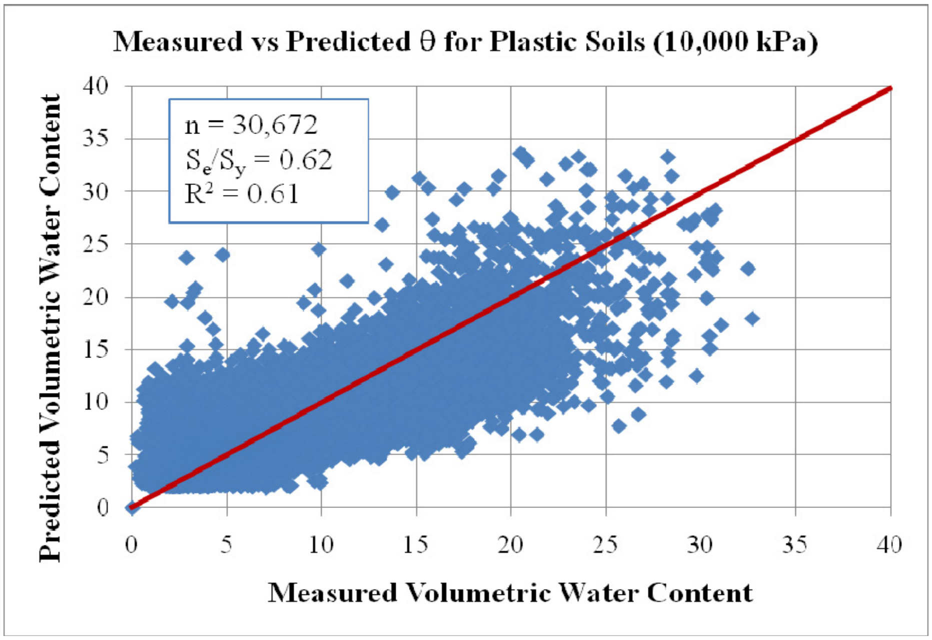



Figure 4-9. Measured vs. Predicted Volumetric Water Content based on Zapata's Model - Plastic Soils (Suction 10 kPa)

Figures 4-10 through 4-14 show the plots of measured versus predicted water content for non-plastic soils. These plots were developed for suctions of 1 kPa, 10 kPa, 100 kPa, 1,000 kPa, and 10,000 kPa.

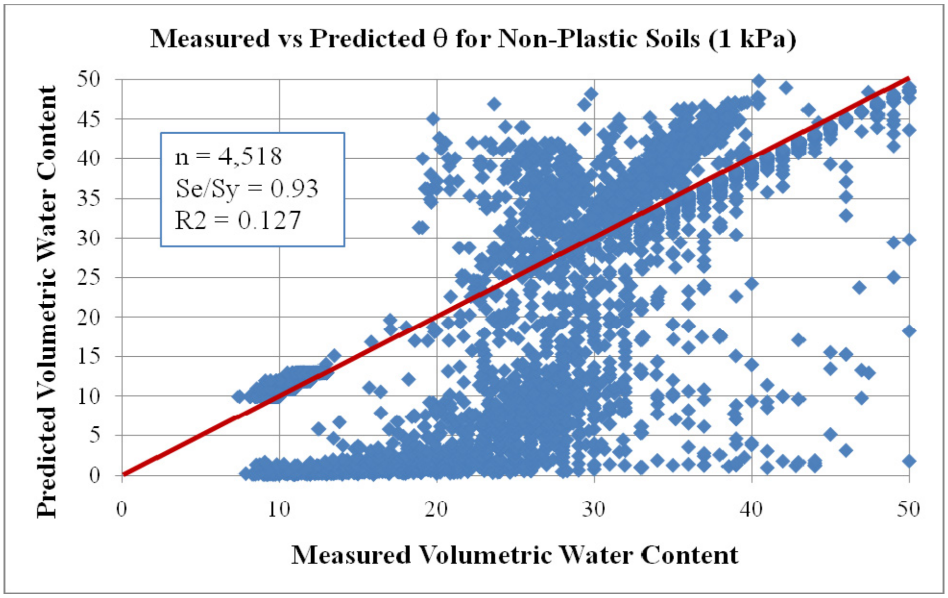



Figure 4-10. Measured vs. Predicted Volumetric Water Content based on Zapata's Model for Non–Plastic Soils (Suction 1 kPa)

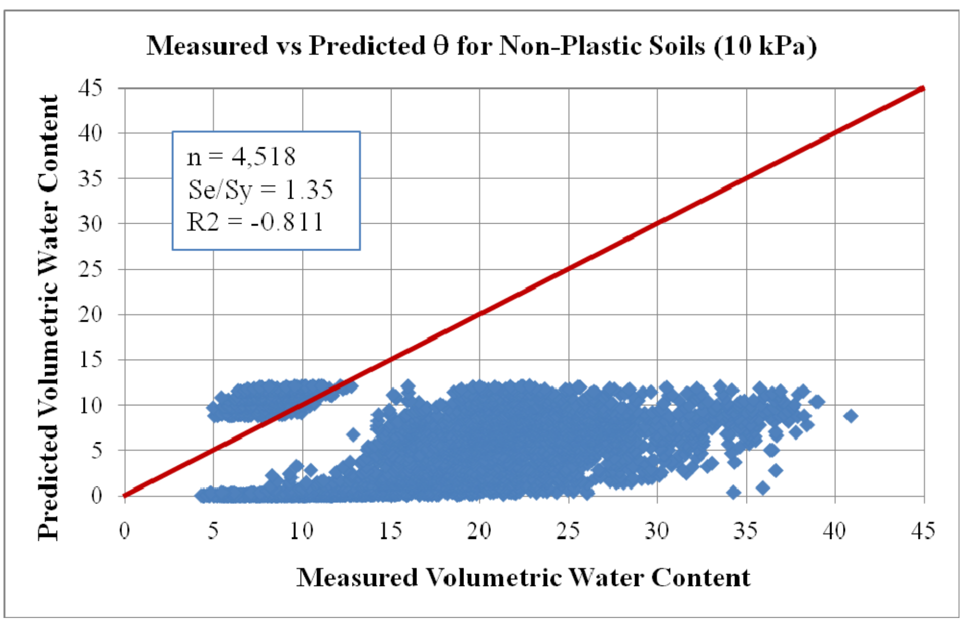



Figure 4-11. Measured vs. Predicted Volumetric Water Content based on Zapata's Model for Non–Plastic Soils (Suction 10 kPa)

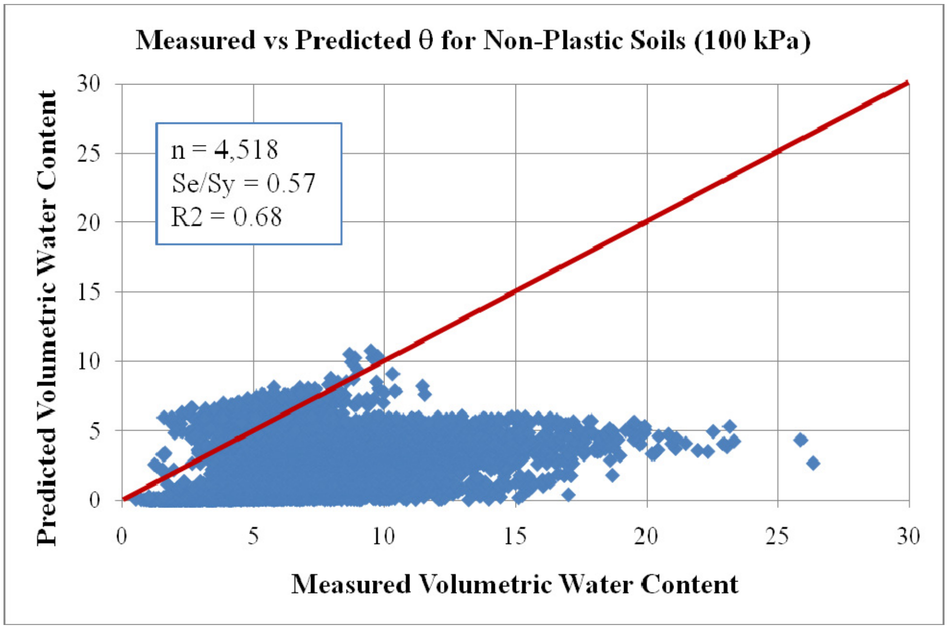



Figure 4-12. Measured vs. Predicted Volumetric Water Content based on Zapata's Model for Non–Plastic Soils (Suction 100 kPa)

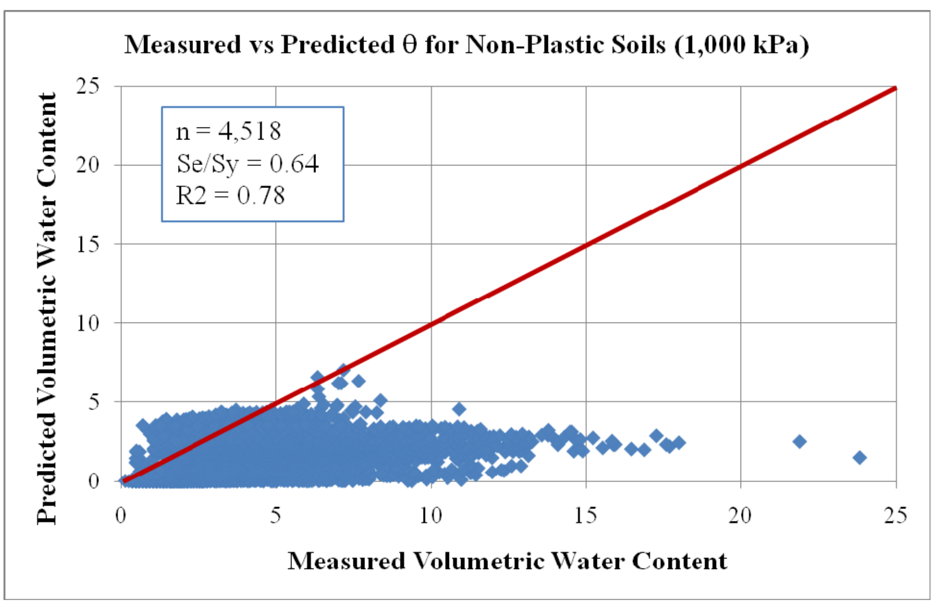



Figure 4-13. Measured vs. Predicted Volumetric Water Content based on Zapata's Model for Non–Plastic Soils (Suction 1,000 kPa)

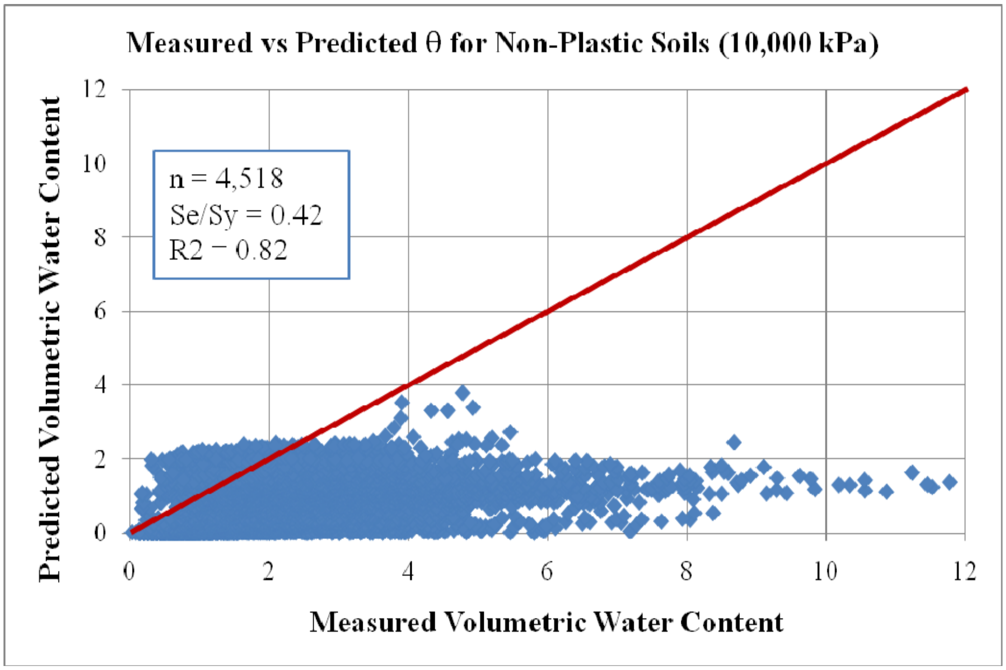



Figure 4-14. Measured vs. Predicted Volumetric Water Content based on Zapata's Model for Non–Plastic Soils (Suction 10,000 kPa)

### 4.3 Validating the MEPDG Model (Witczak et al, 2006)

The model developed under the NCHRP 1-40D project made used of a database of 217 data points; 154 data points corresponded to non-plastic soils and 63 corresponded to plastic soils. These data were obtained by combining the soil used by Zapata in 1999 and a database obtained under the NCHRP 9-23 project titled Environmental Effects in Pavement Mix and Structural Design Systems, Houston, Mirza, & Zapata (2006).

The development of the SWCC prediction equations presented in the NCHRP 1-40D project was performed in a similar manner as those developed by

Zapata (1999). However, a greater number of soils were included in an attempt to find correlations.

From the total database, 52 plastic soils were corrected by volume change. This correction was necessary because the change in volume due to the suction applied create errors in the SWCC, especially in the residual condition of the SWCC (high suction levels) where the function is very sensitive to changes in the density undergone by plastic soils. The pressure plate used to obtain the SWCC for these soils allows volume change measurements in the determination of the SWCC. The density is calculated at each point of the test and, therefore, the volume change correction is possible.

The procedure used to achieve the volume change correction is not included in this work, but it was clearly explained in both projects: Zapata, 1999 and NCHRP 9-23 project (Witczak et al., 2006). To estimate the corrected volumetric water content due to changes in density was given by the following expression:

$$\theta_{w\text{-}corr} = \frac{G_s w}{1 + e} \quad \text{.....(4-17)}$$

Where:

$\theta_{w\text{-}corr}$ = Corrected volumetric water content

Gs = Specific gravity of solids

w = Gravimetric water content

e = Void ratio

In addition, the NCHRP 1-40D predictive equations did not consider the effect of hysteresis. This project assumes that the difference between the wetting curve and the drying curve would be very insignificant. Furthermore, solute suction is not considered in this project and therefore, only matric suction was measured.

The analysis to predict the set of SWCC parameters was made separately for plastic soils and non-plastic soils, and the final equations presented in that project are given in the following section.

### 4.3.1 Predictive Equations for Fredlund and Xing SWCC Parameters for Non-Plastic Soils.

For non-plastic soils, the following equations were proposed by Witczak et al., 2006 (MEPDG model), to find the $a_f$ parameter:

$$a_f = 1.14a - 0.5 \quad \text{(4-18)}$$

$$a = -2.79 - 14.1\log(D_{20}) - 1.9\times10^{-6} P_{200}^{4.34} + 7\log(D_{30}) + 0.055D_{100} \quad \text{(4-19)}$$

Where:

$$D_{100} = 10^{\left[\frac{40}{m_1} + \log(D_{60})\right]} \quad \text{(4-20)}$$

$$m_1 = \frac{30}{[\log(D_{90}) - \log(D_{60})]} \quad \text{.......................................................(4-21)}$$

$a_f$ = SWCC fitting parameter

$D_{20}$ = Grain diameter corresponding to 20% of passing by weight, in mm

$D_{30}$ = Grain diameter corresponding to 30% of passing by weight, in mm

$D_{60}$ = Grain diameter corresponding to 60% of passing by weight, in mm

$D_{90}$ = Grain diameter corresponding to 90% of passing by weight, in mm

$P_{200}$ = Percent passing U.S. standard sieve #200

To find the $b_f$ parameter:

$$b_f = 0.936b - 3.8 \quad \text{.......................................................................(4-22)}$$

Where:

$b_f$ = SWCC fitting parameter

$D_{10}$ = Grain diameter corresponding to 10% of passing by weight, in mm

$$b = \left\{5.39 - 0.29\ln\left[P_{200}\left(\frac{D_{90}}{D_{10}}\right)\right] + 3D_0^{0.57} + 0.021P_{200}^{1.19}\right\} m_1^{0.1} \quad \text{................(4-23)}$$

$$D_0 = 10^{\left[\frac{-30}{m_2} + \log(D_{30})\right]} \quad \text{.....................................................................(4-24)}$$

$$m_2 = \frac{20}{[\log(D_{30}) - \log(D_{10})]} \quad \text{.......(4-25)}$$

To find the $c_f$ parameter:

$$c_f = 0.26e^{0.758c} + 1.4D_{10} \quad \text{.......(4-26)}$$

Where:

$c_f$ = SWCC fitting parameter

$$c = \log\left(m_2^{1.15}\right) \text{-} \left(1 - \frac{1}{b_f}\right) \quad \text{.......(4-27)}$$

The SWCC fitting parameter $h_{rf}$ was defined as a constant:

$$h_{rf} = 100 \quad \text{.......(4-28)}$$

These equations have the following constraints:

If $a_f < 1$, then $a_f = 2.25\ P_{200}^{0.5} + 5$

$0.3 < b_f < 4$

### 4.3.2 Predictive Equations for Fredlund and Xing SWCC Parameters for Plastic Soils.

For plastic soils, the following equations were proposed by Witczak et al., 2006 (MEPDG model):

$$a_f = 32.835\{\ln(wPI)\} + 32.438 \text{ ....................} (4\text{-}29)$$

$$b_f = 1.421(wPI)^{-0.3185} \text{ ....................} (4\text{-}30)$$

$$c_f = -0.2154\{\ln(wPI)\} + 0.7145 \text{ ....................} (4\text{-}31)$$

$$h_{rf} = 500 \text{ ....................} (4\text{-}32)$$

Where:

$wPI$ = weighted Plasticity Index as defined before

The constraints required for these equations are:

If $a_f < 5$, then $a_f = 5$

If $c_f < 0.01$, then $c_f = 0.03$

For the special case where wPI is less than 2 for plastic soils, a weighted average is used for the $a_f$ parameter. For $a_f$ parameter the following model was proposed:

$$a_{f\,avg} = a_{fn} + \frac{wPI}{2}\left(a_{fp} - a_{fn}\right) \text{ ....................} (4\text{-}33)$$

Where:

$a_{f\,avg}$ = $a_f$ average

$a_{fn}$ = $a_f$ value using the model for non-plastic soils

$a_{fp}$ = $a_f$ value using the model for plastic soils

For the parameter $b_f$, $c_f$, and $h_{rf}$ equations 4-30 to 4-32 apply.

**4.3.3 MEPDG Model Validation Analysis.**

Similar to the analysis used in the validation of Zapata's model, the validation of the MEPDG models was performed by a comparative analysis between the measured volumetric water content and the predicted. The measured volumetric water content were found by fitting the Fredlund & Xing, 1994, model to measured data. Several suction values that cover a wide range of suctions were chosen for the comparison with the predicted volumetric water content. The statistical equations 4-12 through 4-16, presented previously, were used for the error analysis.

The results of the error analysis were independently evaluated for non-plastic and plastic soils. The results are shown in Tables 4-2 and 4-3, respectively.

Table 4-2. Error Analysis for Non-Plastic Soils

| Parameter | ψ = 1 kpa | ψ = 10 kpa | ψ = 100 kpa | ψ = 1,000 kpa | ψ = 10,000 kpa |
|---|---|---|---|---|---|
| n | 4,487 | 4,487 | 4,487 | 4,487 | 4,487 |
| $e_{alg}$ | 4.38 | 4.86 | -30.54 | -22.07 | -11.02 |
| $e_{abs}$ | 15.92 | 30.87 | 62.08 | 66.17 | 70.56 |
| $S_e/S_y$ | 0.63 | 1.21 | 1.38 | 1.24 | 1.21 |
| $R^2$ | 0.5971 | -0.4666 | -0.9105 | -0.5284 | -0.4528 |

Table 4-3. Error Analysis for Plastic Soils

| Parameter | ψ = 1 kpa | ψ = 10 kpa | ψ = 100 kpa | ψ = 1,000 kpa | ψ = 10,000 kpa |
|---|---|---|---|---|---|
| n | 30,561 | 30,561 | 30,561 | 30,561 | 30,561 |
| $e_{alg}$ | -5.08 | -23.57 | -45.27 | -47.27 | -46.83 |
| $e_{abs}$ | 7.62 | 28.19 | 58.80 | 74.54 | 81.67 |
| $S_e/S_y$ | 0.41 | 1.10 | 1.60 | 1.56 | 1.43 |
| $R^2$ | 0.8329 | -0.2008 | -1.5690 | -1.4296 | -1.0584 |

Figures 4-15 through 4-19 show the plots of measured versus predicted volumetric water content for plastic soils. These plots represent comparisons at suctions of 1 kPa, 10 kPa, 100 kPa, 1,000 kPa, and 10,000 kPa.

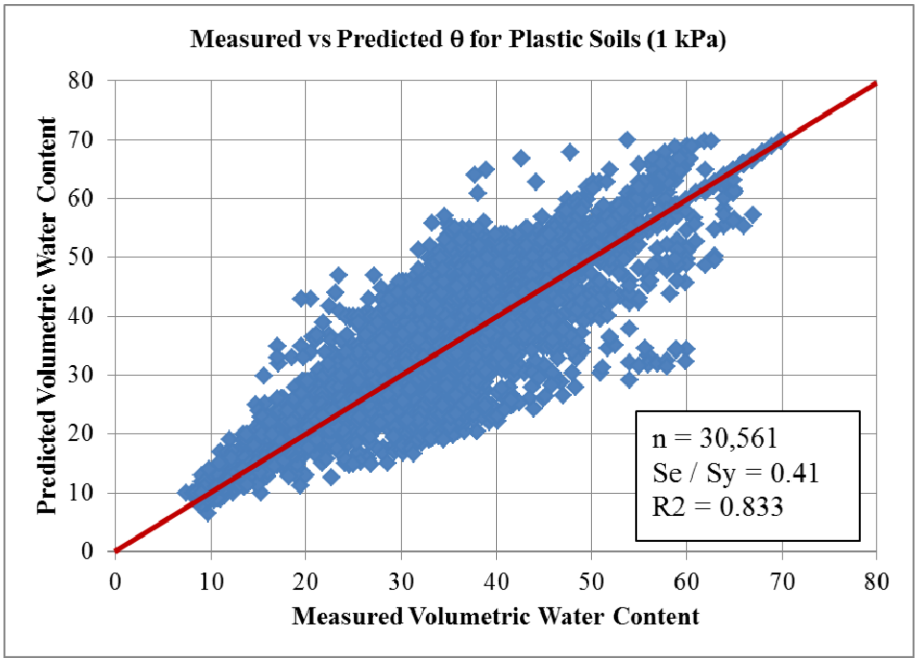



Figure 4-15. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Plastic Soils (Suction 1 kPa)

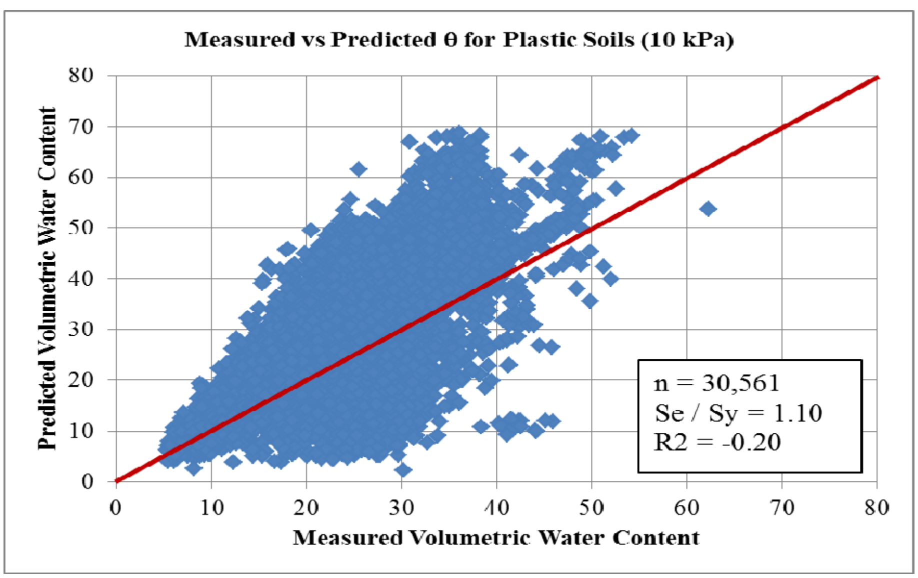



Figure 4-16. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Plastic Soils (Suction 10 kPa)

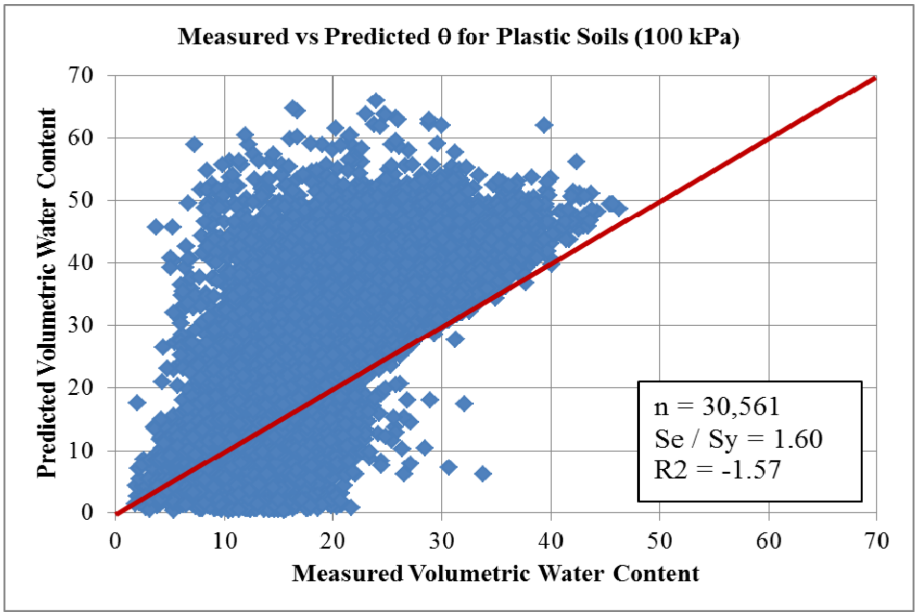



Figure 4-17. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Plastic Soils (Suction 100 kPa)

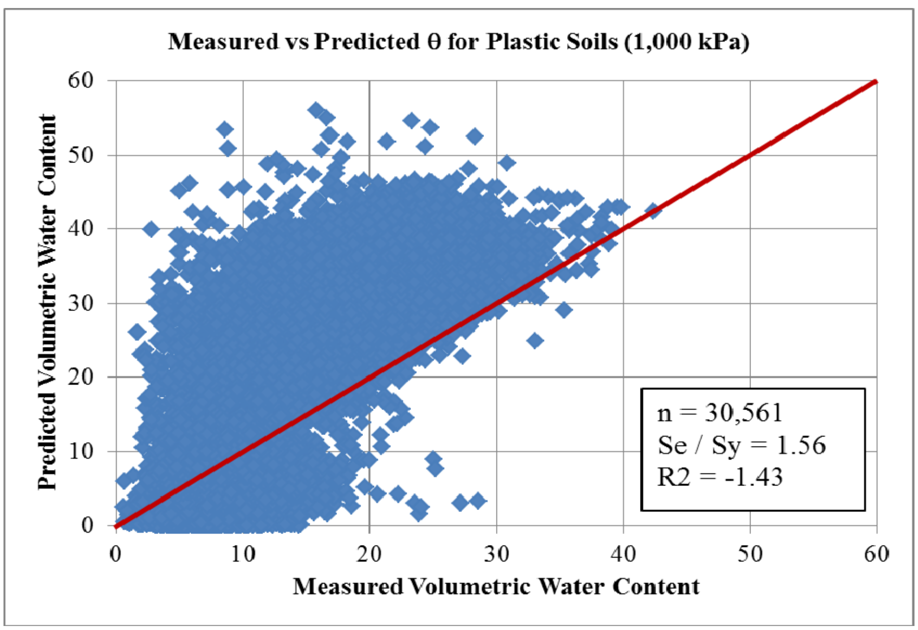



Figure 4-18. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Plastic Soils (Suction 1,000 kPa)

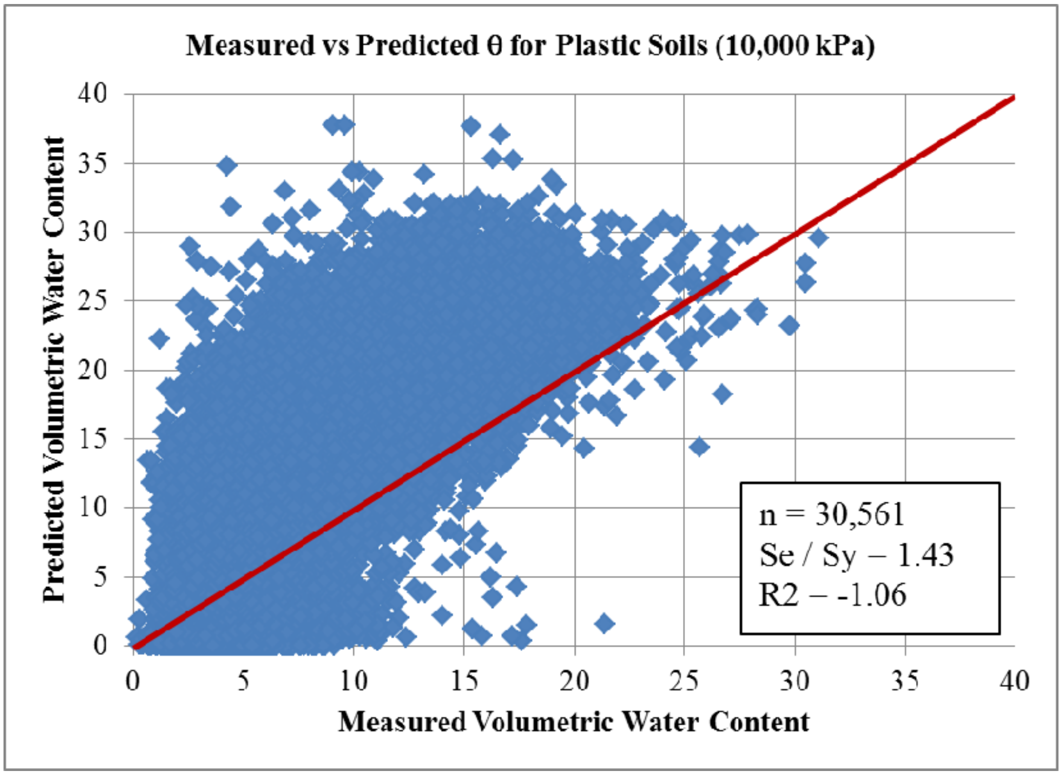



Figure 4-19. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Plastic Soils (Suction 10,000 kPa)

Figures 4-20 through 4-24 show the plots Measured versus Predicted for non-plastic soils. These plots were developed for suctions of 1, 10, 100, 1,000, and 10,000 kPa. In this way, it is possible to evaluate the range of suctions.

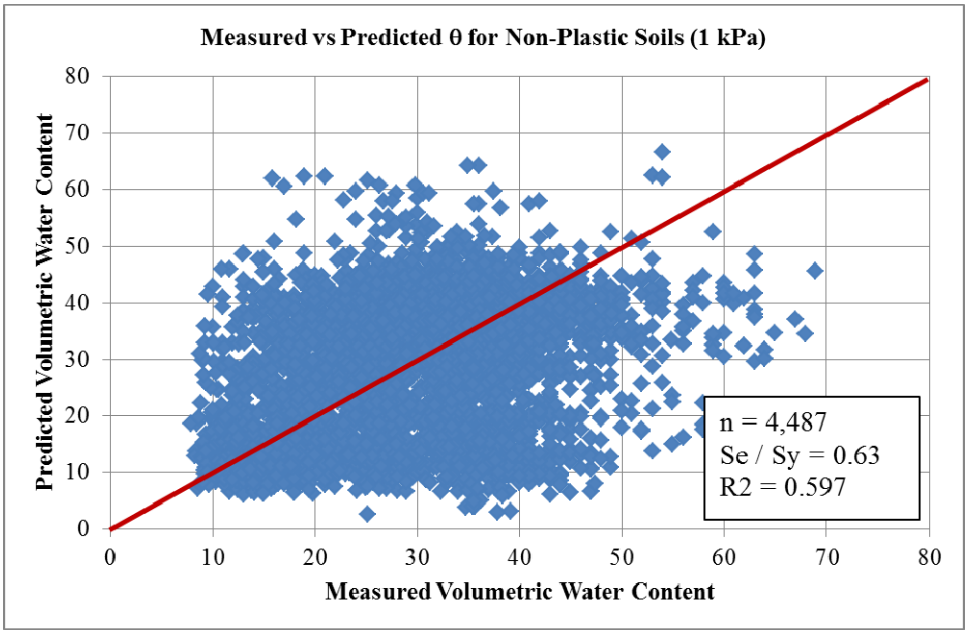



Figure 4-20. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Non–Plastic Soils (Suction 1 kPa)

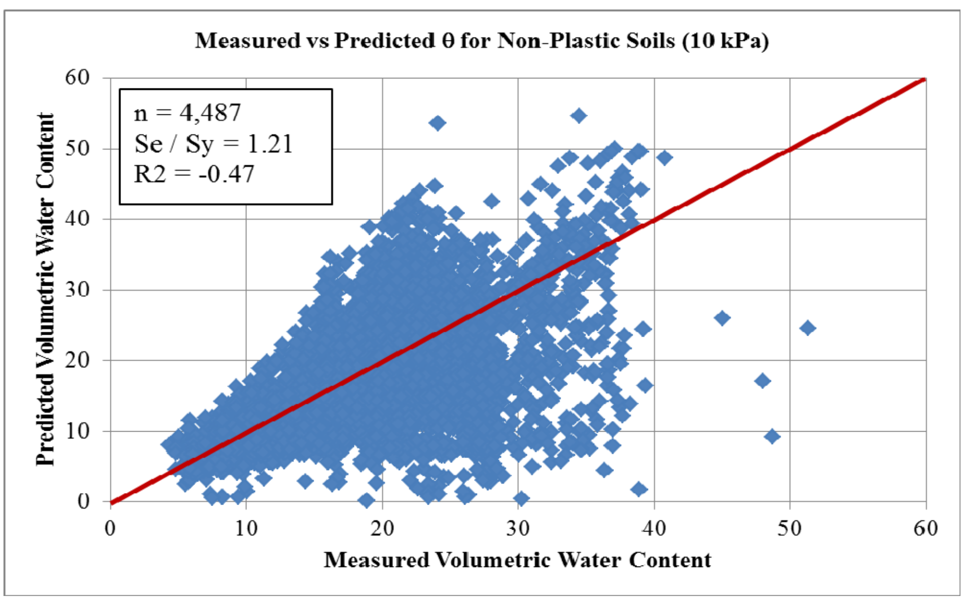



Figure 4-21. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Non–Plastic Soils (Suction 10 kPa)

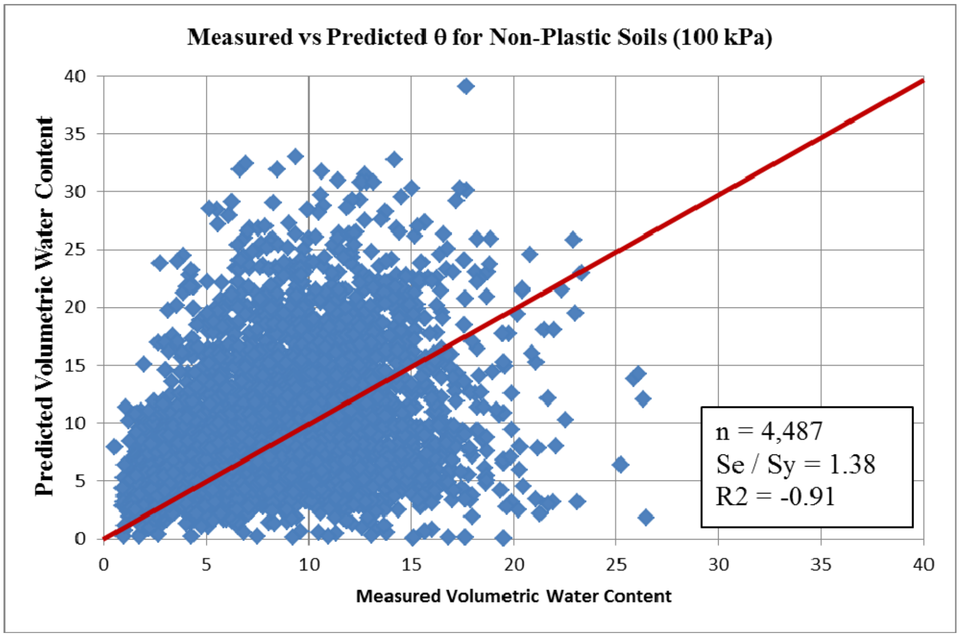



Figure 4-22. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Non–Plastic Soils (Suction 100 kPa)

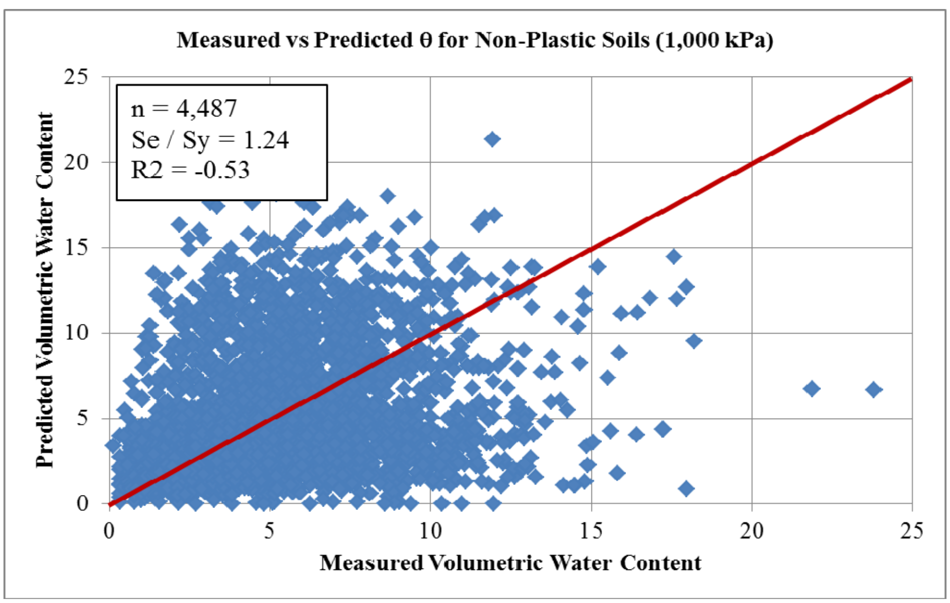



Figure 4-23. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Non–Plastic Soils (Suction 1,000 kPa)

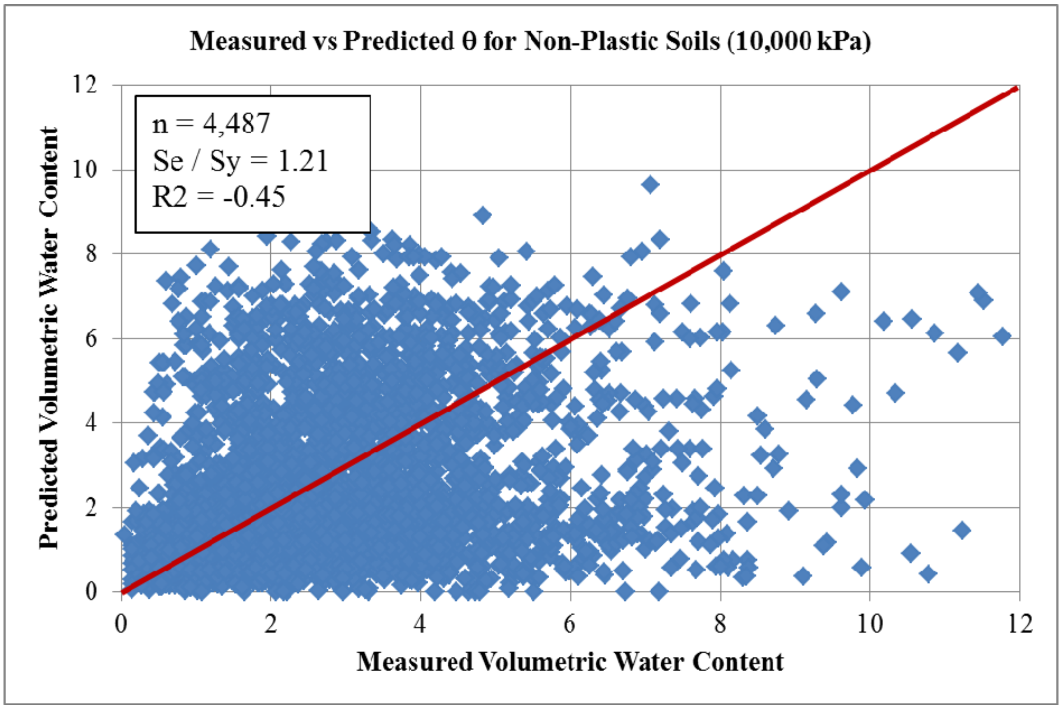



Figure 4-24. Measured vs. Predicted Volumetric Water Content based on MEPDG Model for Non–Plastic Soils (Suction 10,000 kPa)

## 4.4 Summary

Table 4-1 shows the errors found for the validation of Zapata's models for plastic and non-plastic soils. The validation was performed at different suction levels: 1, 10,100, 1,000 and 10,000 kPa. For non-plastic soils, the $R^2$ values ranged between 68% and 82%. Relatively good predicted water contents were found for suction values higher than 100 kPa. For plastic soils, the highest $R^2$ (82%) was found at suction values lower than 1 kPa and relatively acceptable $R^2$ (60%) was found for suction values higher than 1,000 kPa.

The figures 4-5 through 4-14 show the graphs Measured versus Predicted of volumetric water content values for granular and fine-grained soils separately. These figures include all the predicted volumetric water contents obtained at 1, 10, 100, 1,000 and 10,000 kPa of suction. The error analyses for these figures are summarized in Table 4-1.

Figure 4-25 and 4-26 show the measured versus predicted volumetric water content values obtained by using the model proposed by Zapata, 1999, for plastic and non-plastic soils, respectively. These figures include all the predicted water contents estimated at suctions of 1, 10, 100, 1,000 and 10,000 kPa. For plastic soils, the model developed by Zapata, 1999, although it presented an overall $R^2$ of 0.70, it was found to be biased towards overprediction for most of the data points. For non-plastic soils, the Zapata's model presents a different behavior, in which most of the data points were underpredicted and yielded a low overall $R^2$ value of 0.40.

In general, the models proposed by Zapata, 1999, present acceptable errors considering that it was developed 10 years ago with few data points, when compared to the vast database used in this project.

Tables 4-2 and 4-3 show the error analysis performed for the MEPDG models for non-plastic and plastic soils, respectively. For non-plastic soils, an $R^2$ value of 60%, which was considered to be acceptable, was found only for suctions values lower than 1 kPa. Similarly, for plastic soils, the highest $R^2$ value (83%) was found for suction values lower than 1 kPa.

The figures 4-15 through 4-24 show the graphs Measured versus Predicted of volumetric water content values for granular and fine-grained soils separately. These figures include all the predicted volumetric water contents obtained at 1, 10, 100, 1,000 and 10,000 kPa of suction. The error analyses for these figures are summarized in Table 4-1.

Figures 4-27 and 4-28 show the measured versus predicted volumetric water content values obtained by using the MEPDG model for plastic and non-plastic soils, respectively. These figures include all the predicted water contents obtained at 1, 10, 100, 1,000 and 10,000 kPa of suction. It was observed that for plastic soils, the volumetric water content was consistently overestimated and yielded an $R^2$ of 0.49. However, for non-plastic soils, the MEPDG model presented an acceptable prediction of volumetric water content with an $R^2$ value equal to 0.91.

In general the MEPDG models presented acceptable estimations considering the amount of data analyzed. The MEPDG model can be considered to be a better model for non-plastic soils, while the model proposed by Zapata, 1999 can be considered to perform better for fine-grained materials.

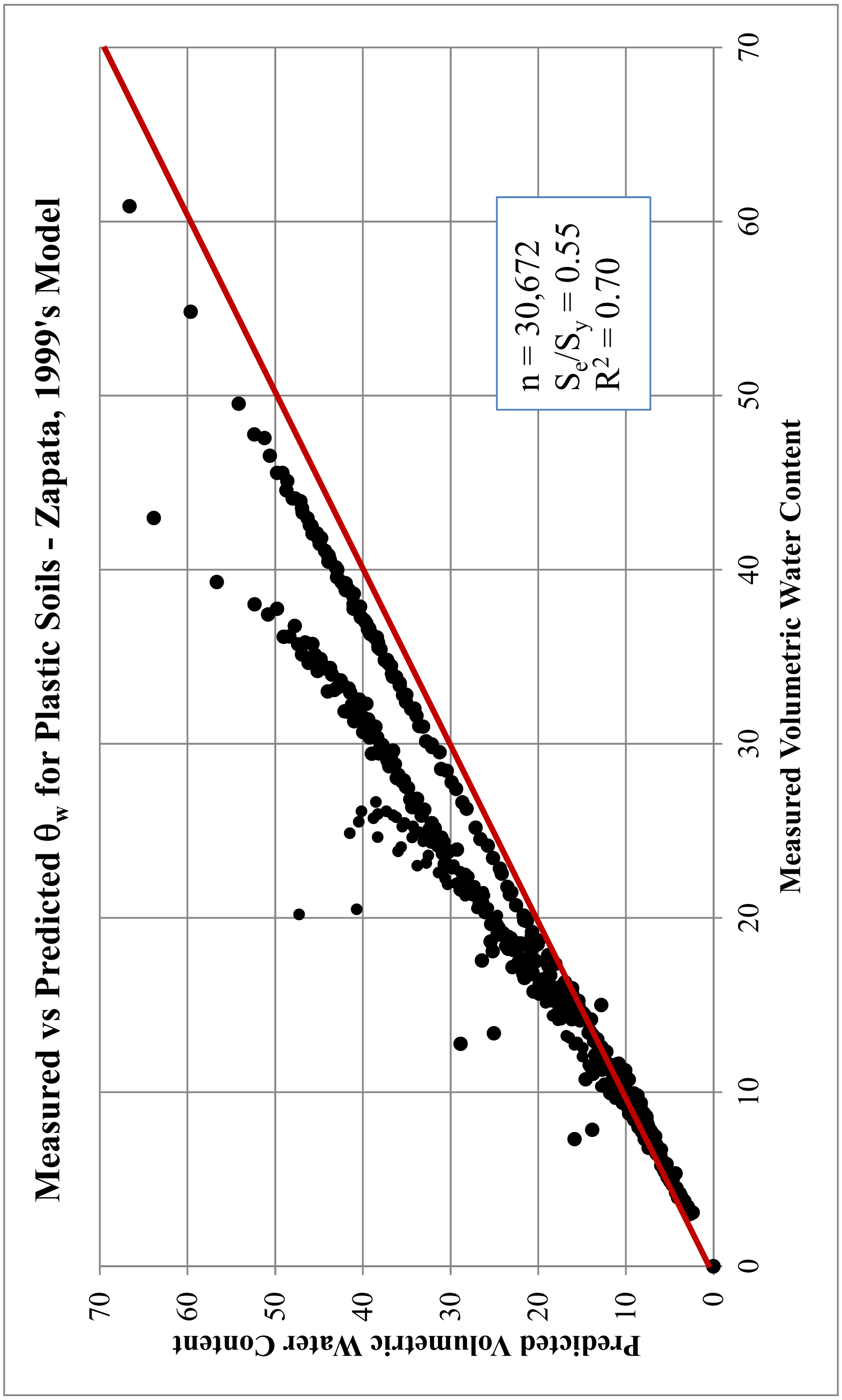



Figure 4-25 Measured versus Predicted Volumetric Water Content Using

Zapata Model for Plastic Soils

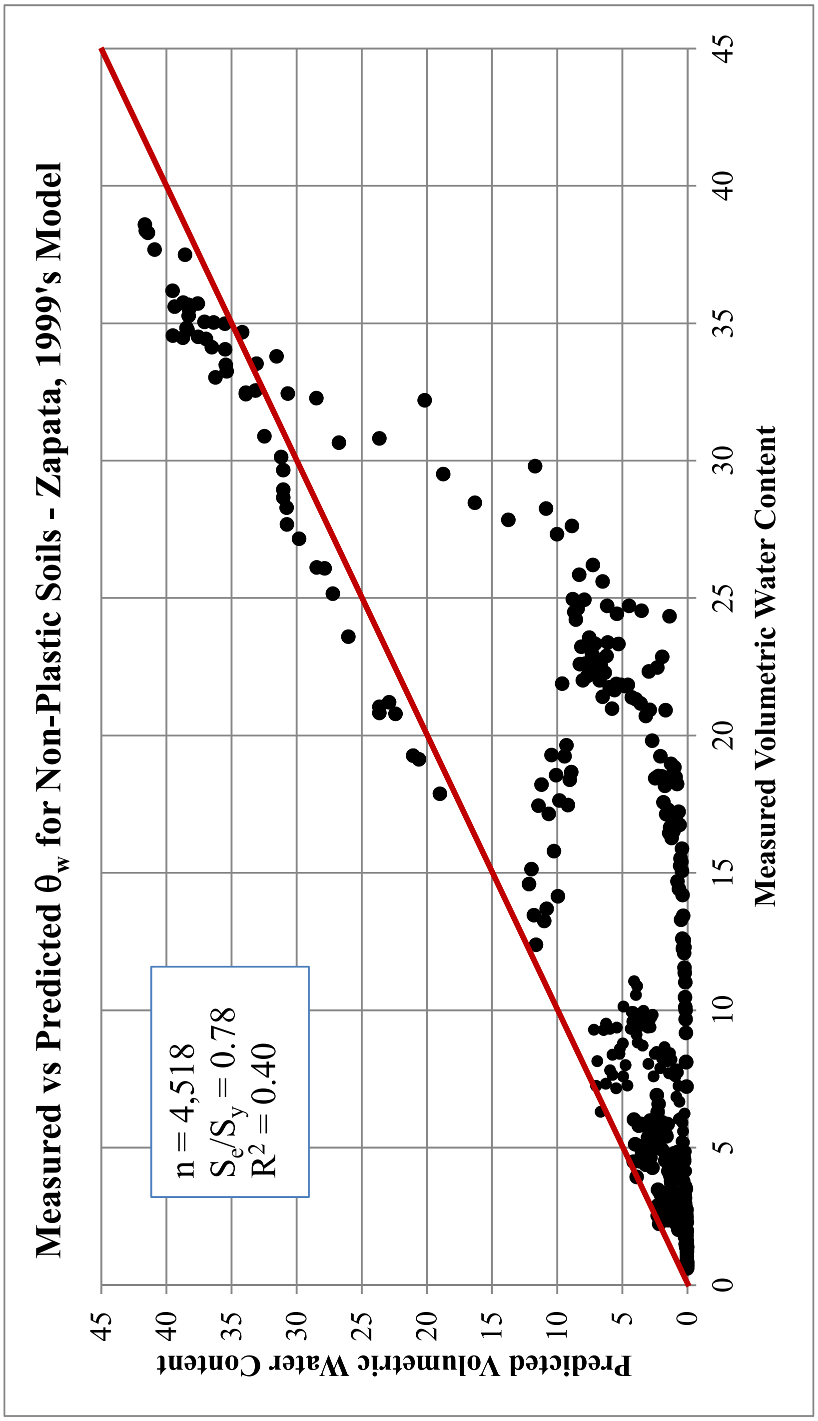



Figure 4-26 Measured versus Volumetric Water Content Using

Zapata Model for Non-Plastic Soils

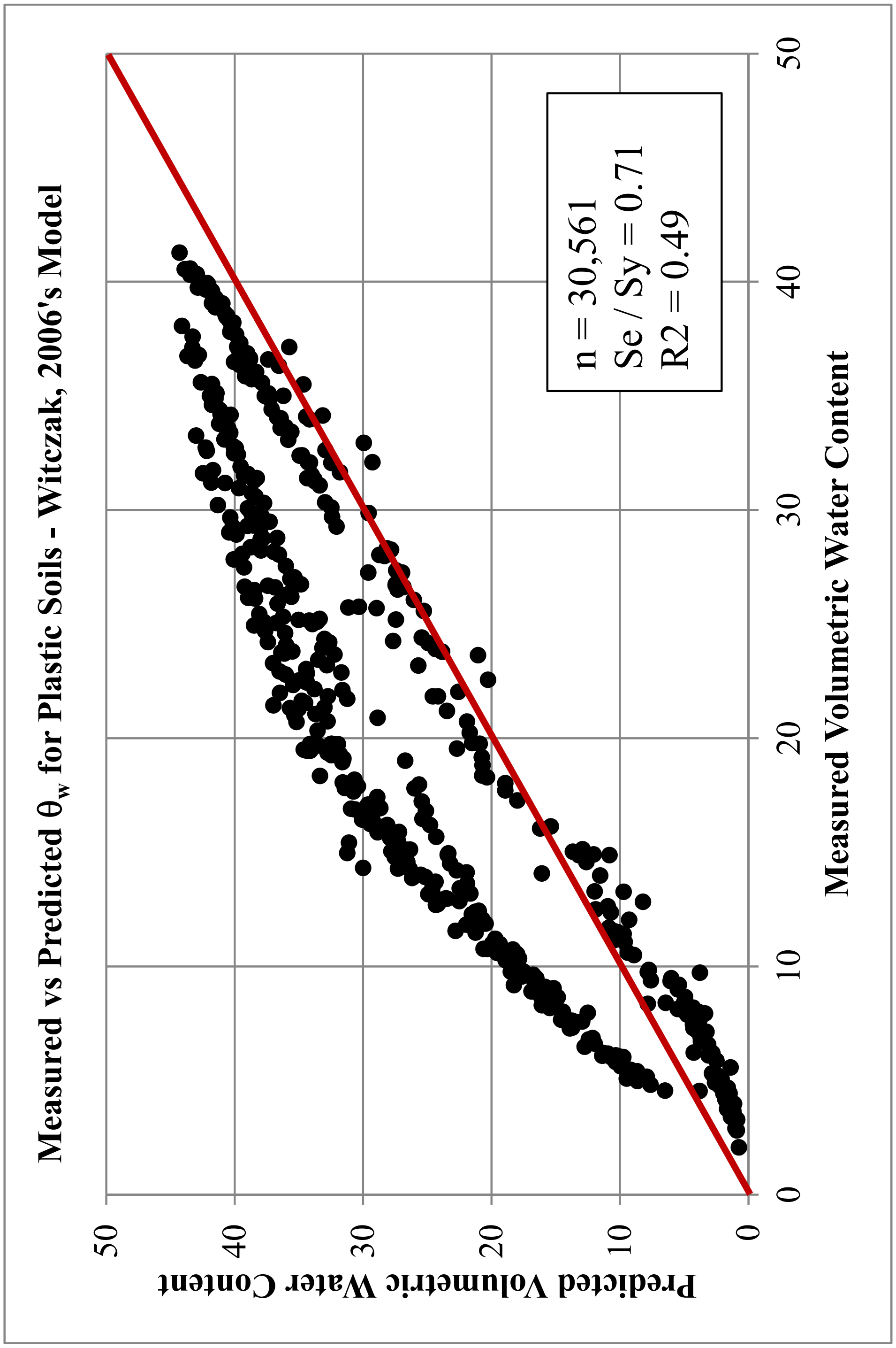



Figure 4-27 Measured versus Predicted Volumetric Water Content Using MEPDG Model for Plastic Soils

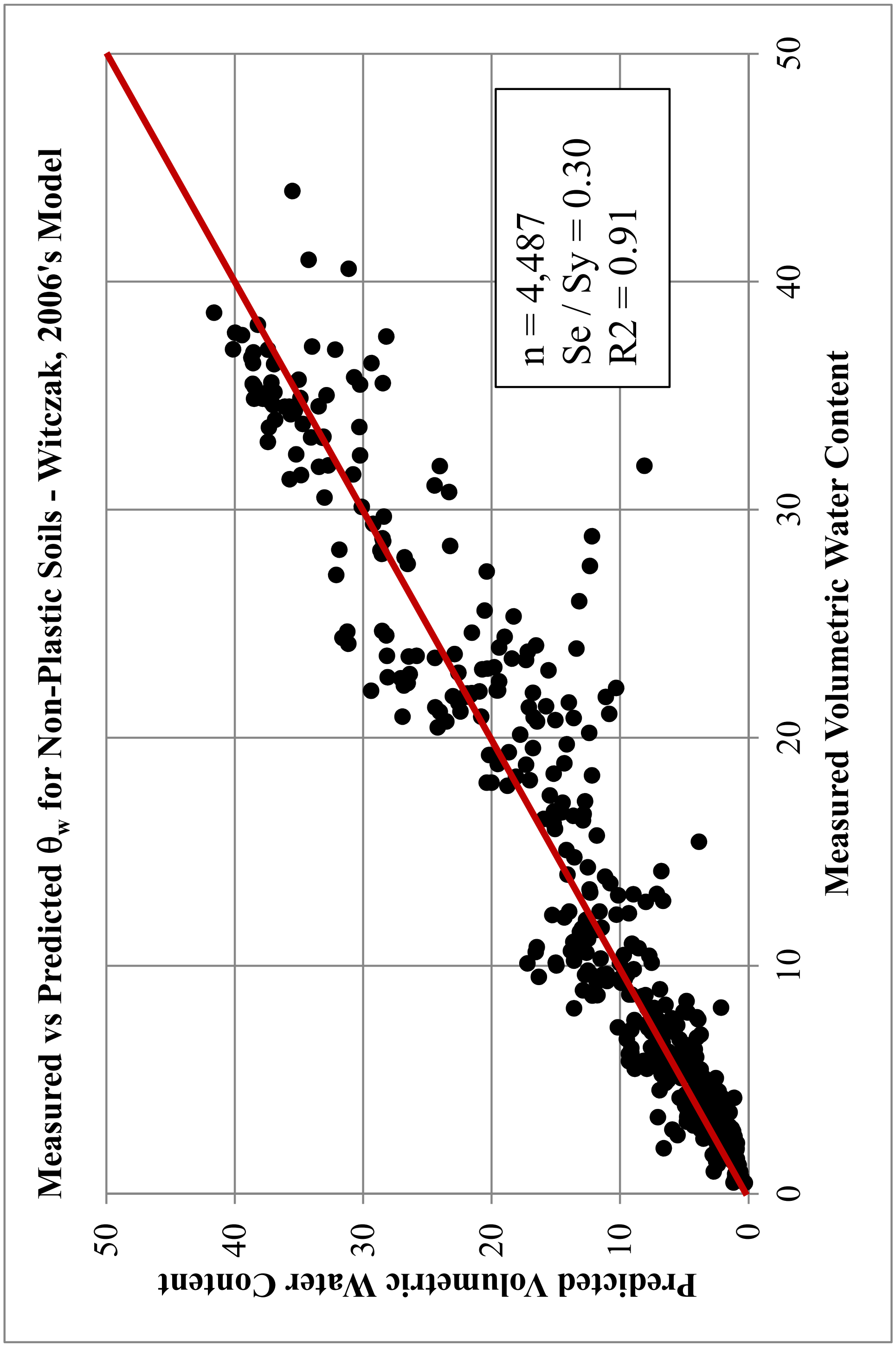



Figure 4-28 Measured versus Predicted Volumetric Water Content Using MEPDG Model for Non-Plastic Soils

# CHAPTER 5

## A NEW SWCC MODEL BASED ON SWCC PARAMETERS

### 5.1 Overview

The purpose of Chapter 5 is to propose an improved set of models for the SWCC parameters based on the equation given by Fredlund and Xing in 1994. This equation was shown in terms of volumetric water content in Chapter 2 (equation 2-12 and 2-13), but for other engineering purposes it can also be expressed in terms of degree of saturation as follows:

$$S(\%) = \frac{\theta_w}{\theta_s} = \left[1 - \frac{\ln\left(1 + \frac{\psi}{h_r}\right)}{\ln\left(1 + \frac{1{,}000{,}000}{h_r}\right)}\right]\left(\frac{1}{\left\{\ln\left[e + \left(\frac{\psi}{a_f}\right)^{b_f}\right]\right\}^{c_f}}\right) \quad \text{...........................(5-1)}$$

Where:

$S(\%)$ = Degree of Saturation, in Percentage

$\psi$ = Matric Suction in kPa

$a_f$, $b_f$, $c_f$, $h_r$ = SWCC Fitting Parameters

$\theta_w$ = Volumetric Water Content

$\theta_s$ = Saturated Volumetric Water Content

The parameters $a_f$, $b_f$, $c_f$ and $\psi_r$ were estimated, based on non-linear regression analysis, from laboratory measured values of suction and volumetric water content as indicated in Chapter 3 - Database Collection. The effect of the hysteresis is not considered in this analysis. Hysteresis is important in cases when the soil has air trapped or when the structure of the soil (pores and connectivity between pores) permits a different behavior of the soil under drying or wetting conditions. The data collected in this study was obtained from pressure plates, which is usually tested under drying conditions. The database used consisted of 36,394 soils obtained from the National Resources Conservation Service (NRCS) to work on a new model. 31,876 corresponding to plastic soils, 4,518 of non–plastic soils and 68 soils did not have enough information to be classified or defined.

The analysis was developed separately for plastic soils (fine grained soils) with wPI greater than zero and non–plastic soils (granular soils) with wPI equal to zero. The concept of wPI (previously explained) is a geotechnical expression where the Plasticity Index and the Gradation are directly involved in the analysis. The Weighted Plasticity Index usually called wPI is expressed as follows:

$$wPI = \frac{P_{200} \times PI}{100} \qquad (5\text{-}2)$$

Where:

$P_{200}$ = Material Passing # 200 US Standard Sieve, in Percentage

$PI$ = Plasticity Index, in Percentage

Once the database was divided according to wPI, the next step was to select soil properties most related to the moisture retention characteristic for each group of soils. This analysis was based on published work compile from several authors. The properties assessed by Zapata, 1999, Witczak et al. 2006, and some other studies by Fredlund, served as basis for the preliminary election of the properties.

For plastic soils, the properties considered into the analysis were: Group Index, the gradation available (percent passing #4, #10, #40, and #200), the total percent of clay (% of soil finer than 0.002 mm), Liquid limit, Plasticity Index and wPI. For non–plastic soils, the properties collected were the Group Index, the gradation (percent passing #4, #10, #40, and #200), the particle sizes ($D_{10}$, $D_{20}$, $D_{30}$, $D_{60}$, $D_{90}$), and the shape parameters $C_u$ and $C_c$. For both sets, volumetric water content values at 0.1, 0.33 and 15 bars of suction were available.

Group Index, GI, is an engineering parameter associated with AASHTO classification and used extensively for the analysis of pavement subgrades. The Group Index expression combines two important soil properties: gradation and consistency. The Group Index is expressed as:

$$GI = (P_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(P_{200} - 15)(PI - 10) \text{...........(5-3)}$$

Where:

$P_{200}$ = Percent Material Passing # 200 US Standard Sieve

*LL* = Liquid Limit

*PI* = Plasticity Index = *LL – PL*

On the other hand, the Weighted Plasticity Index or wPI, is a geotechnical property defined by Zapata, 1999 and shown in the Equation 5-2, which presents a narrow similarity with the Group Index. Both factors, wPI and GI, are functions of gradation ($P_{200}$) and consistency limits (LL and PL). Despite of this similarity and the close correlation between them, both indexes were considered in the statistical analysis of fine–grained soils. It is important to recognize that the Group Index is a factor that is easily recognized by the pavement design community and therefore, it is a good candidate for the application into the SWCC equation included into the MEPDG. The assessment of the predicted values versus the measured values was performed based on an "Error Analysis", (Zapata & Houston, 2008). The new models were analyzed through the following statistical concepts:

*Absolute Mean Error, $e_{abs}$*. This concept indicates how the predicted values are dispersed about the best fitting curve.

$$e_{abs} = \frac{100\left|\Sigma\frac{(y_m - y_p)}{y_m}\right|}{n} \qquad (5\text{-}4)$$

Where:

$y_m$ = Measured value

$y_p$ = Predicted value

$n$ = Number of data points

*Algebraic Mean Error*, $e_{alg}$, indicates how well the curve fit is centered on the data. A low value of $e_{alg}$ indicates a prediction well centered and with a very little bias. The sign of this factor describes the direction of the bias.

$$e_{alg} = \frac{100\sum\frac{(y_m - y_p)}{y_m}}{n} \quad \text{.....(5-5)}$$

Where:

$y_m$ = measured value

$y_p$ = predicted value

$n$ = number of data points

*Standard Error Divided by the Standard Deviation*, $S_e/S_y$. This ratio is an expression describing how spread out the data is,

$$S_e = \sqrt{\frac{\sum(y_m - y_p)^2}{n-p}} \quad \text{.....(5-6)}$$

$$S_y = \sqrt{\frac{\Sigma\left(y_m - \bar{y}_m\right)^2}{n-1}} \quad \text{.......(5-7)}$$

Where:

$p$ = number of parameters associated with the proposed functions

$\bar{y}_m$ = average of measured values

*The Adjusted Coefficient of Determination, $R^2$ (adjusted).* This coefficient defines how well the regressed predicted function approaches the measured data points,

$$R^2\left(Adjusted\right) = 1 - \left(\frac{S_e}{S_y}\right)^2 \quad \text{.......(5-8)}$$

As an overview, the procedure followed to find the new SWCC model is:

1. The database was classified according to the wPI property. Those soils with wPI>o were considered plastic soils, while the soils with wPI=0 were considered non–plastic.

2. For each type of soil, the measured SWCC parameters were treated as dependent variables and correlated with all the soil properties affecting the SWCC (independent variables). Arithmetic functions were considered as well as transformed functions, including squared values, log arithmetic values, natural log, arithmetic values and powered functions.

3. Each SWCC parameter was subjected to a statistical non–linear regression analysis against all possible combination of parameters. The adjusted R–square value, the algebraic mean error, the absolute mean error, the standard errors and the standard deviation were computed for each analysis.

4. Based on statistical, geotechnical and applicability considerations, the best model was chosen for each SWCC parameter.

5. Each Measured fitting parameter was compared to the Predicted value.

6. The predicted degree of saturation was obtained by fitting the predicted SWCC parameter to the Fredlund & Xing function.

7. Final model for plastic and non-plastic soil were proposed.

## 5.2 Database and Descriptive Statistics

From a total of 36,462 data, 36,394 soils or items were available for this project. Table 5-1 and Figure 5-1show the number of data points available for each type of soil. Most of the soils were found to be fine grained soils (classified from A-4 to A-7-6). In total, the database available consisted of 31,876 plastic soils (wPI > 0) and 4,518 non–plastic soils (wPI = 0).

Table 5-1. Database Available for Each Type of Soil

| **Type of Soil** | **Data Available** |
|---|---|
| A-1-a | 445 |
| A-1-b | 1335 |
| A-2-4 | 4256 |
| A-2-5 | 28 |
| A-2-6 | 878 |
| A-2-7 | 283 |
| A-4 | 12611 |
| A-5 | 174 |
| A-6 | 6237 |
| A-7-5 | 799 |
| A-7-6 | 4830 |
| | 31876 |

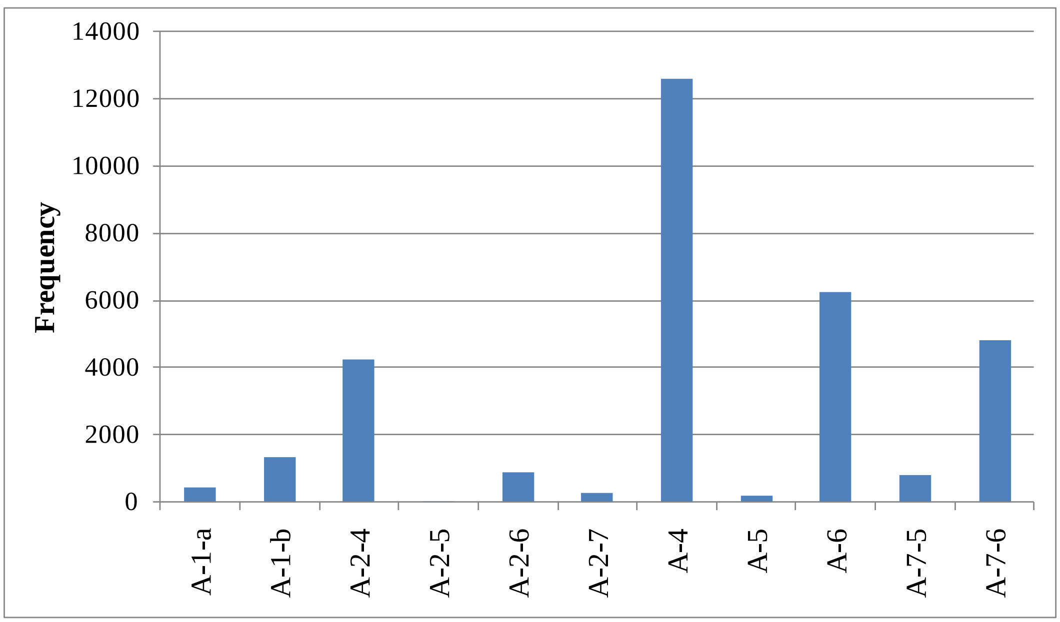



Figure 5-1. Graphical Representation of the Database Available for Each Type of Soil

A descriptive statistical analysis was performed on the available database for this project. The descriptive statistical analysis allowed for the preliminary assessment of the central tendency and variability of the database. This analysis was developed initially for the entire database, and included each soil property collected. Table 5-2 summarizes the data found for each parameter and includes the average, maximum, minimum value, as well as the median, mode, and standard deviation. The same analysis for selected parameters was developed separately for plastic and non–plastic soils and the results are shown on Tables 5-4 and 5-5. Only 12% of the items have available data for the volumetric water content at 10 kPa of suction, this created a problem because only two measure points were available to estimate the soil–water characteristic curve. However, in addition to these two points, the extremes of the SWCC function could be defined. For 100% of saturation or at very low suction the saturated volumetric water content was available. Also, a suction of 1,000,000 kPa can be assumed at zero degree of saturation (Fredlund & Xing, 1994). In this way, the regression analysis considered four points and in some instances, five points. A descriptive statistical analysis is presented in table 5.3 for selected properties. It can be seen that the property values cover a wide range of values.

Table 5-2. Descriptive Statistical Analysis on the Entire Database

| Statistical Parameter | Group Index | Passing #4 | Passing #10 | Passing #40 | Passing #200 | Passing 0.002 mm | Liquid Limit | Plasticity Index | wPI |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 5.7 | 84.6 | 80.0 | 69.4 | 51.4 | 21.4 | 32.0 | 10.3 | 6.8 |
| Median | 1.0 | 92.5 | 87.5 | 72.5 | 50.0 | 19.5 | 30.0 | 7.5 | 3.7 |
| Mode | 0.0 | 100.0 | 100.0 | 95.0 | 42.5 | 15.0 | 25.0 | 2.5 | 0.0 |
| Standard Deviation | 9.4 | 18.3 | 20.8 | 22.2 | 24.7 | 13.2 | 12.2 | 9.7 | 8.3 |
| Sample Variance | 88.3 | 335.5 | 433.7 | 493.5 | 611.0 | 175.4 | 149.9 | 95.0 | 68.6 |
| Kurtosis | 5.6 | 0.5 | -0.1 | -0.7 | -1.0 | 0.7 | 1.7 | 2.0 | 4.2 |
| Skewness | 2.3 | -1.2 | -1.0 | -0.6 | 0.0 | 1.0 | 1.1 | 1.4 | 2.0 |
| Range | 68.0 | 81.0 | 87.5 | 94.0 | 100.0 | 89.7 | 125.0 | 66.0 | 58.6 |
| Minimum | 0.0 | 19.0 | 12.5 | 6.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Maximum | 68.0 | 100.0 | 100.0 | 100.0 | 100.0 | 89.7 | 125.0 | 66.0 | 58.6 |
| Count | 36,394 | 36,462 | 36,462 | 36,458 | 36,455 | 36,461 | 32,494 | 36,400 | 36,394 |
| % data available | 99.8 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 89.1 | 99.8 | 99.8 |

Table 5-3. Descriptive Statistical Analysis on the Entire Database for Selected Soil Properties

| Soil Property | Unit | n | % Data | Max | Min | Average | Median | Mode | StDev |
|---|---|---|---|---|---|---|---|---|---|
| Top Depth of Layer | cm | 36,462 | 100 | 241 | 0 | 28 | 8 | 0 | 39 |
| Bottom Depth of Layer | cm | 36,462 | 100 | 254 | 2 | 67 | 41 | 152 | 58 |
| Thickness of the Layer | cm | 36,462 | 100 | 218 | 2 | 39 | 28 | 20 | 32 |
| Passing Sieve # 4 | % | 36,462 | 100 | 100 | 19 | 85 | 93 | 100 | 18 |
| Passing Sieve # 10 | % | 36,462 | 100 | 100 | 13 | 80 | 88 | 100 | 21 |
| Passing Sieve # 40 | % | 36,458 | 100 | 100 | 6 | 69 | 73 | 95 | 22 |
| Passing Sieve # 200 | % | 36,455 | 100 | 100 | 0 | 51 | 50 | 43 | 25 |
| Passing Sieve 0.002 mm | % | 36,461 | 100 | 90 | 0 | 21 | 20 | 15 | 13 |
| Saturated Hydraulic Conductivity | μm/s | 36,460 | 100 | 423 | 0 | 21 | 9 | 9 | 30 |
| Volumetric Water Content at Suction 10 kPa | % | 4,357 | 12 | 37 | 0 | 15 | 15 | 14 | 5 |
| Volumetric Water Content at Suction 33 kPa | % | 36,462 | 100 | 55 | 0 | 21 | 21 | 14 | 9 |
| Volumetric Water Content at Suction 1500 kPa | % | 36,462 | 100 | 42 | 0 | 12 | 10 | 7 | 7 |
| Saturated Volumetric Water Content | % | 36,462 | 100 | 70 | 10 | 37 | 38 | 41 | 10 |
| Liquid Limit | % | 32,494 | 89 | 125 | 0 | 32 | 30 | 25 | 12 |
| Plasticity Index | % | 36,400 | 100 | 66 | 0 | 10 | 8 | 3 | 10 |
| Elevation | m | 31,708 | 87 | 3,963 | -5 | 973 | 823 | 305 | 753 |
| Bedrock Depth - Minimum | cm | 10,218 | 28 | 202 | 0 | 54 | 41 | 77 | 36 |
| Water Table Depth - Annual - Minimum | cm | 10,065 | 28 | 168 | 0 | 51 | 46 | 0 | 42 |
| Water Table Depth - April - June - Minimum | cm | 9,363 | 26 | 168 | 0 | 51 | 46 | 0 | 42 |

Table 5-4. Summary of Descriptive Statistical Analysis on Fine–Grained Soils

| Statistical Parameter | Group Index | Passing #4 | Passing #10 | Passing #40 | Passing #200 | Passing 0.002 mm | Liquid Limit | Plasticity Index | wPI |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 6.5 | 85.0 | 80.6 | 71.5 | 55.9 | 23.6 | 32.5 | 11.8 | 7.7 |
| Median | 2.0 | 92.5 | 87.5 | 75.0 | 55.0 | 21.5 | 30.0 | 9.0 | 4.5 |
| Mode | 0.0 | 100.0 | 100.0 | 95.0 | 42.5 | 15.0 | 25.0 | 2.5 | 3.0 |
| Standard Deviation | 9.8 | 17.8 | 20.3 | 21.3 | 22.6 | 12.6 | 11.9 | 9.6 | 8.4 |
| Sample Variance | 95.9 | 318.6 | 410.8 | 453.7 | 511.2 | 159.7 | 141.2 | 91.2 | 70.8 |
| Kurtosis | 4.6 | 0.6 | 0.0 | -0.6 | -1.0 | 0.9 | 1.8 | 2.0 | 3.7 |
| Skewness | 2.1 | -1.3 | -1.1 | -0.6 | 0.0 | 1.0 | 1.3 | 1.4 | 1.9 |
| Range | 68.0 | 81.0 | 87.5 | 94.0 | 100.0 | 89.7 | 125.0 | 66.0 | 58.6 |
| Minimum | 0.0 | 19.0 | 12.5 | 6.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Maximum | 68.0 | 100.0 | 100.0 | 100.0 | 100.0 | 89.7 | 125.0 | 66.0 | 58.6 |
| Count | 31,876 | 31,944 | 31,944 | 31,940 | 31,937 | 31,943 | 31,854 | 31,882 | 31,876 |
| % data available | 99.8 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 99.7 | 99.8 | 99.8 |

Table 5-5. Summary of Descriptive Statistical Analysis on Granular Soils

| Statistical Parameter | Group Index | Passing #4 | Passing #10 | Passing #40 | Passing #200 | Passing 0.002 mm | Liquid Limit | Plasticity Index | wPI |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 0.3 | 81.9 | 76.0 | 54.5 | 19.7 | 5.8 | 9.2 | 0.0 | 0.0 |
| Median | 0.0 | 92.5 | 87.5 | 55.0 | 17.5 | 5.5 | 7.0 | 0.0 | 0.0 |
| Mode | 0.0 | 100.0 | 100.0 | 60.0 | 20.0 | 2.5 | 7.0 | 0.0 | 0.0 |
| Standard Deviation | 0.5 | 21.1 | 24.0 | 22.9 | 12.9 | 3.2 | 7.2 | 0.0 | 0.0 |
| Sample Variance | 0.2 | 446.2 | 578.0 | 522.3 | 166.4 | 10.0 | 51.4 | 0.0 | 0.0 |
| Kurtosis | 29.2 | -0.2 | -0.8 | -1.0 | 3.9 | 2.9 | 1.9 | 4,518.0 | - |
| Skewness | 2.6 | -1.0 | -0.8 | 0.0 | 1.6 | 1.2 | 0.9 | 67.2 | - |
| Range | 9.0 | 80.0 | 87.5 | 92.5 | 95.0 | 32.0 | 55.0 | 1.0 | 0.0 |
| Minimum | 0.0 | 20.0 | 12.5 | 7.5 | 0.0 | 0.5 | 0.0 | 0.0 | 0.0 |
| Maximum | 9.0 | 100.0 | 100.0 | 100.0 | 95.0 | 32.5 | 55.0 | 1.0 | 0.0 |
| Count | 4,518 | 4,518 | 4,518 | 4,518 | 4,518 | 4,518 | 640 | 4,518 | 4,518 |
| % data available | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 | 14.2 | 100.0 | 100.0 |

## 5.3 Correlations

### 5.3.1 Correlations for Fine Grained Soils.

There are several computer programs to statistically analyze the database. In this work, Microsoft® Excel, Statistica®5.5 and Minitab®15 were employed. There are advantages and disadvantages associated with each program, but all of them were used to complement, calculate, find better model predictors and check the results. These programs were employed to calculate the descriptive statistical parameters such as the average (arithmetic mean), the median, the mode, the standard deviation, the variance, the kurtosis, and the skewness. These programs were also used to define the possible statistical correlations between the SWCC parameters and the variables or soil properties considered in the analysis, to find the best predictors and finally to define the best models.

Table 5-6. Correlation Matrix for Fine–Grained Soils

| **Variables** | $a_f$ | $b_f$ | $c_f$ | $h_r$ |
|---|---|---|---|---|
| Group Index | 0.061 | 0.045 | -0.031 | 0.208 |
| Ln(GI+1) | 0.098 | 0.013 | -0.041 | 0.164 |
| LnGI+1^2 | 0.081 | 0.032 | -0.035 | 0.196 |
| Ln(GI+e) | 0.093 | 0.020 | -0.039 | 0.178 |
| LnGI+e^2 | 0.082 | 0.031 | -0.036 | 0.195 |
| sieveno4 | 0.024 | 0.004 | -0.016 | 0.059 |
| LnP4 | 0.023 | 0.003 | -0.016 | 0.054 |
| sieve10 | 0.028 | 0.005 | -0.016 | 0.065 |
| LnP10 | 0.028 | 0.003 | -0.016 | 0.059 |
| sieve40 | 0.057 | 0.012 | -0.015 | 0.087 |
| LnP40 | 0.053 | 0.008 | -0.015 | 0.076 |
| sieve200 | 0.088 | 0.024 | -0.010 | 0.111 |
| LnP200 | 0.086 | 0.017 | -0.010 | 0.094 |
| LnP200^2 | 0.088 | 0.020 | -0.010 | 0.100 |
| clay.002 | 0.063 | 0.023 | -0.051 | 0.201 |
| Lnclay | 0.071 | -0.006 | -0.058 | 0.158 |
| Lnclay^2 | 0.071 | 0.003 | -0.058 | 0.175 |
| ll_r | 0.056 | 0.032 | -0.039 | 0.197 |
| pi_r | 0.074 | 0.030 | -0.042 | 0.202 |
| clayPI | 0.047 | 0.053 | -0.034 | 0.222 |
| wPI | 0.072 | 0.041 | -0.034 | 0.207 |
| wPI^-2 | -0.020 | 0.003 | 0.003 | -0.019 |
| wPI^-1 | -0.068 | 0.008 | 0.023 | -0.065 |
| wPI^-0.5 | -0.093 | 0.005 | 0.035 | -0.105 |

Table 5-6. Correlation Matrix for Fine–Grained Soils (Cont'd)

| **Variables** | **$a_f$** | **$b_f$** | **$c_f$** | **$h_r$** |
|---|---|---|---|---|
| wPI^0.5 | 0.092 | 0.024 | -0.040 | 0.187 |
| wPI^2 | 0.032 | 0.059 | -0.023 | 0.203 |
| LogwPI | 0.101 | 0.007 | -0.041 | 0.150 |
| LogwPI^2 | 0.090 | 0.027 | -0.039 | 0.193 |
| P200LogwPI | 0.094 | 0.022 | -0.034 | 0.168 |
| LnwPI | 0.101 | 0.007 | -0.041 | 0.150 |
| LnwPI^2 | 0.090 | 0.027 | -0.039 | 0.193 |
| P200/PI | -0.043 | 0.028 | 0.058 | -0.084 |
| PI/P200 | 0.033 | 0.005 | -0.038 | 0.116 |
| P200^2PI | 0.063 | 0.046 | -0.029 | 0.203 |
| P200PI^2 | 0.039 | 0.057 | -0.025 | 0.209 |
| ksat_r | -0.082 | -0.006 | 0.008 | -0.065 |
| D10 | -0.019 | -0.003 | -0.005 | -0.007 |
| D20 | -0.018 | -0.002 | -0.001 | -0.013 |
| D30 | -0.018 | 0.000 | 0.004 | -0.018 |
| D60 | -0.006 | -0.002 | 0.008 | -0.028 |
| D90 | -0.023 | -0.003 | 0.012 | -0.044 |
| D100 | -0.003 | 0.002 | -0.003 | 0.001 |
| $C_u$ | 0.009 | -0.005 | 0.003 | -0.021 |
| $C_c$ | -0.007 | -0.002 | 0.002 | -0.010 |
| af | 1.000 | 0.094 | 0.011 | -0.046 |
| bf | 0.094 | 1.000 | 0.934 | -0.009 |
| cf | 0.011 | 0.934 | 1.000 | -0.006 |
| hr | -0.046 | -0.009 | -0.006 | 1.000 |

The values for the correlations obtained in Table 5-6 are considered low. Low correlation is caused usually by the high variability of the data. Some of the reasons of this variability were presented in Chapter 3.

Based on the results presented in Table 5-6, the parameter $a_f$ (related to the air entry value) presented the highest correlation (although poor) with the wPI and Group Index. The parameter $b_f$ presented the best correlation when related with the parameter $a_f$, while the parameter $c_f$ presented a very good correlation with $b_f$. Finally, the parameter $h_r$ showed some correlation with several variables, but in all cases yielded poor results.

This type of situation is analyzed by several researchers in different ways. For this particular work, the SWCC parameters for plastic soils were organized according to the wPI. The data points were grouped by taking 300 consecutive data points. A moving average and the median were calculated for each sub–group. These two statistical properties defined proper values to represent every sub–group of data. This procedure was used for each variable, except for the Group Index, in which case the sub–groups were formed by the group index number.

### 5.3.2 Correlations for Granular Soils.

In order to find the correlation matrix for granular or non-plastic material, the same considerations applied to the analysis done for fine–grained soil were use; however, the variables considered in the analysis were different. For these soils, the grain-size distribution parameter such as particle sizes, percent passing, the coefficient of uniformity, Cu, and the coefficient of curvature, Cc, were considered. Table 5-7 shows the correlation matrix for non-plastic material.

The parameter $a_f$ shows the best correlation when related with the percent passing #200 ($P_{200}$). Parameter $b_f$ shows a significant correlation with parameters $a_f$ and $c_f$. Parameter $c_f$ presents some correlation with $P_{200}$, Particle Size $D_{10}$ and $D_{90}$, and showed to be inversely proportional to $b_f$ parameter.

In summary, the results from the correlation matrix for granular material showed that the SWCC fitting parameters are not independent for each other.

Table 5-7. Correlation Matrix for Granular Soils

| **Variables** | **$a_f$** | **$b_f$** | **$c_f$** | **$h_r$** |
|---|---|---|---|---|
| Group Index | 0.135 | 0.081 | 0.032 | -0.021 |
| Ln(GI+1) | 0.170 | 0.097 | 0.041 | -0.027 |
| LnGI+1^2 | 0.124 | 0.075 | 0.029 | -0.019 |
| Ln(GI+e) | 0.163 | 0.094 | 0.039 | -0.026 |
| LnGI+e^2 | 0.148 | 0.087 | 0.035 | -0.023 |
| sieveno4 | -0.230 | -0.056 | -0.170 | 0.021 |
| Log P4 | -0.224 | -0.068 | -0.153 | 0.016 |
| LnP4 | -0.224 | -0.068 | -0.153 | 0.016 |
| sieve10 | -0.227 | -0.057 | -0.181 | 0.021 |
| Log P10 | -0.223 | -0.074 | -0.162 | 0.016 |
| LnP10 | -0.223 | -0.074 | -0.162 | 0.016 |
| sieve40 | -0.219 | -0.086 | -0.166 | 0.022 |
| Log P40 | -0.231 | -0.103 | -0.152 | 0.018 |
| LnP40 | -0.231 | -0.103 | -0.152 | 0.018 |

Table 5-7. Matrix of Correlation for Granular Soils (Cont'd)

| Variables | $a_f$ | $b_f$ | $c_f$ | $h_r$ |
|---|---|---|---|---|
| sieve200 | -0.199 | -0.249 | 0.047 | -0.014 |
| P200^ 0.5 | 0.318 | 0.281 | 0.035 | -0.010 |
| P200^2 | -0.087 | -0.188 | 0.088 | -0.025 |
| P200^3 | -0.019 | -0.141 | 0.102 | -0.030 |
| P200^4 | 0.016 | -0.110 | 0.104 | -0.032 |
| Log P200 | -0.307 | -0.288 | -0.016 | 0.003 |
| Log P200^2 | -0.277 | -0.279 | 0.005 | -0.004 |
| LnP200 | -0.297 | -0.285 | -0.008 | 0.000 |
| LnP200^2 | -0.271 | -0.277 | 0.009 | -0.005 |
| ll_r | -0.144 | -0.059 | -0.092 | 0.005 |
| pi_r | 0.039 | 0.038 | -0.006 | -0.004 |
| ksat_r | 0.177 | 0.209 | -0.040 | -0.018 |
| D10 | 0.207 | 0.221 | 0.023 | -0.011 |
| Log D10 | 0.232 | 0.229 | -0.002 | -0.034 |
| Ln D10 | 0.232 | 0.229 | -0.002 | -0.034 |
| D20 | 0.202 | 0.177 | 0.048 | -0.011 |
| Log D20 | 0.192 | 0.229 | -0.059 | -0.014 |
| Ln D20 | 0.192 | 0.229 | -0.059 | -0.014 |
| D30 | 0.192 | 0.132 | 0.054 | -0.008 |
| Log D30 | 0.166 | 0.179 | -0.025 | -0.003 |
| Ln D30 | 0.166 | 0.179 | -0.025 | -0.003 |
| D60 | 0.181 | 0.060 | 0.103 | -0.005 |
| Log D60 | 0.186 | 0.093 | 0.108 | 0.004 |
| Ln D60 | 0.186 | 0.093 | 0.108 | 0.004 |
| D90 | 0.210 | 0.066 | 0.129 | -0.011 |
| D90/D10 | -0.025 | -0.137 | 0.182 | -0.011 |
| P200 * D90 | -0.016 | -0.139 | 0.197 | -0.019 |
| Log D90 | 0.220 | 0.048 | 0.175 | -0.028 |
| Ln D90 | 0.220 | 0.048 | 0.175 | -0.028 |
| D100 | -0.043 | -0.028 | -0.016 | 0.014 |
| Cu | -0.043 | -0.104 | 0.102 | 0.002 |
| Log Cu | -0.123 | -0.170 | 0.061 | 0.035 |
| Ln Cu | -0.123 | -0.170 | 0.061 | 0.035 |
| Cc | -0.142 | -0.094 | -0.051 | 0.036 |
| Log Cc | -0.097 | -0.017 | -0.102 | 0.029 |
| af | 1.000 | 0.704 | -0.028 | -0.121 |
| bf | 0.704 | 1.000 | -0.464 | -0.108 |
| cf | -0.028 | -0.464 | 1.000 | 0.067 |
| hr | -0.121 | -0.108 | 0.067 | 1.000 |

## 5.4 Physical Significance of SWCC Parameters

It is important to understand the effect of each parameter into the shape of the SWCC given by Fredlund and Xing, 1994 (see equation 5-1). A sensibility analysis of each fitting parameter on the shape of the SWCC function I depicted in Figures 5-2 through 5-5.

Figure 5-2 shows four SWCCs, where the parameters $b_f$, $c_f$ and $h_r$ are fixed while the parameter $a_f$ is varying. This parameter $a_f$ is associated with the initial break of the SWCC, commonly known as the air–entry value. At this point, the air starts entering the soil filling up the larger pores. As the $a_f$ parameter increases, the matric suction increase. Fine grained soils have higher air–entry values than granular material, and therefore, fine grained soils require more pressure than granular soils to remove the same amount of water. It should also be notice that the suction at the inflection point correspond to the parameter $a_f$. This is an important observation because historically $a_f$ has been related to the air entry value and not with the inflection point of the SWCC.

In Figure 5-3 the parameter $b_f$ varies while the other parameters remain fixed. This graph clearly shows that $b_f$ parameter is intimately related with the slope of the SWCC. The higher the bf value, the steeper the SWCC becomes.

In Figure 5-4 the parameters $a_f$, $b_f$ and $h_r$ are fixed while the parameter $c_f$ changes. It can be observed that the parameter $c_f$ is related to the parameter $b_f$; because the slope of the SWCC is steeper as $C_f$ increases. Furthermore, the

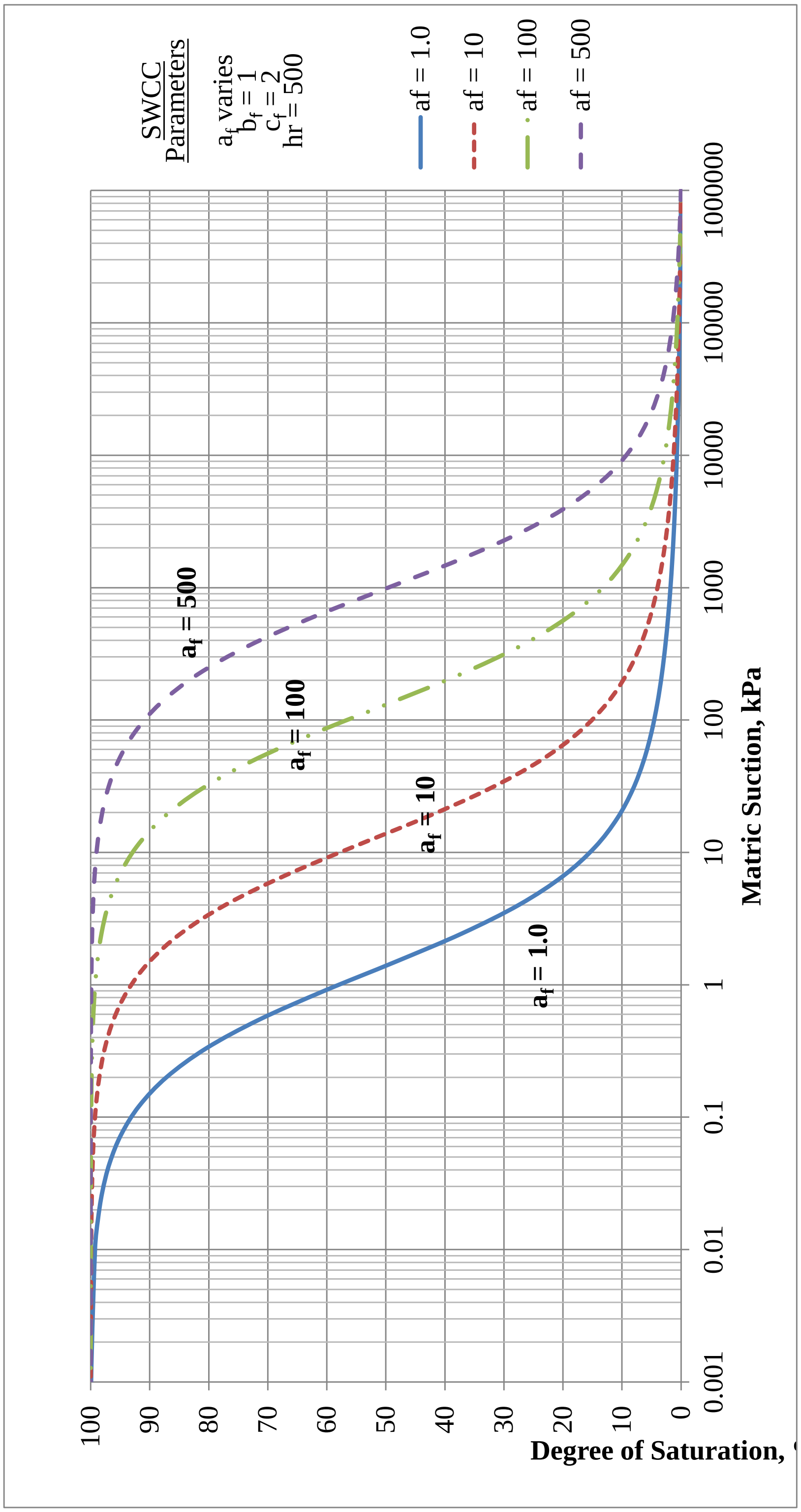



Figure 5-2. Changes in the SWCC Shape Due to Changes in the $a_f$ Parameter

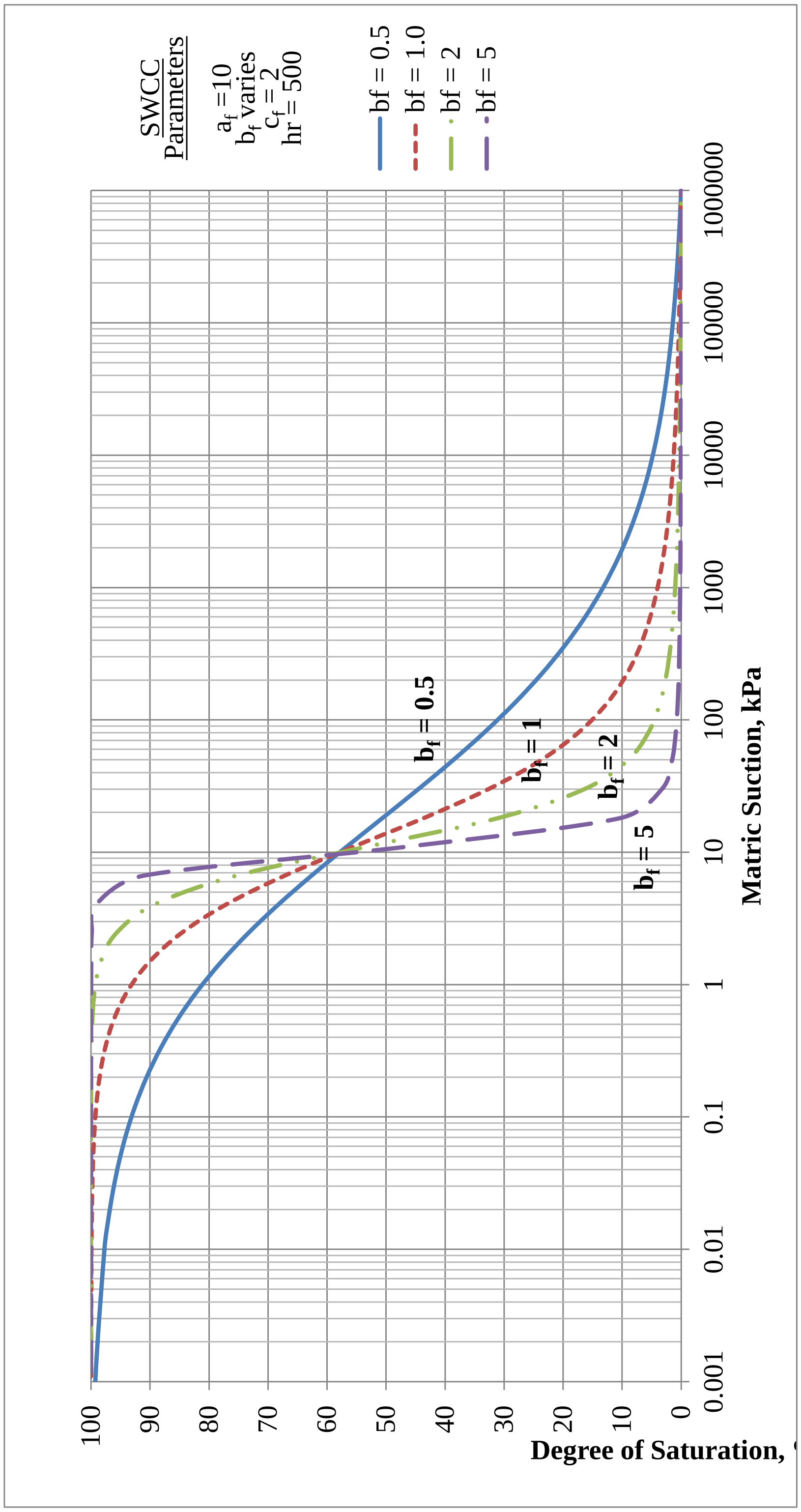



Figure 5-3. Changes in the SWCC Shape Due to Changes in the $b_f$ Parameter

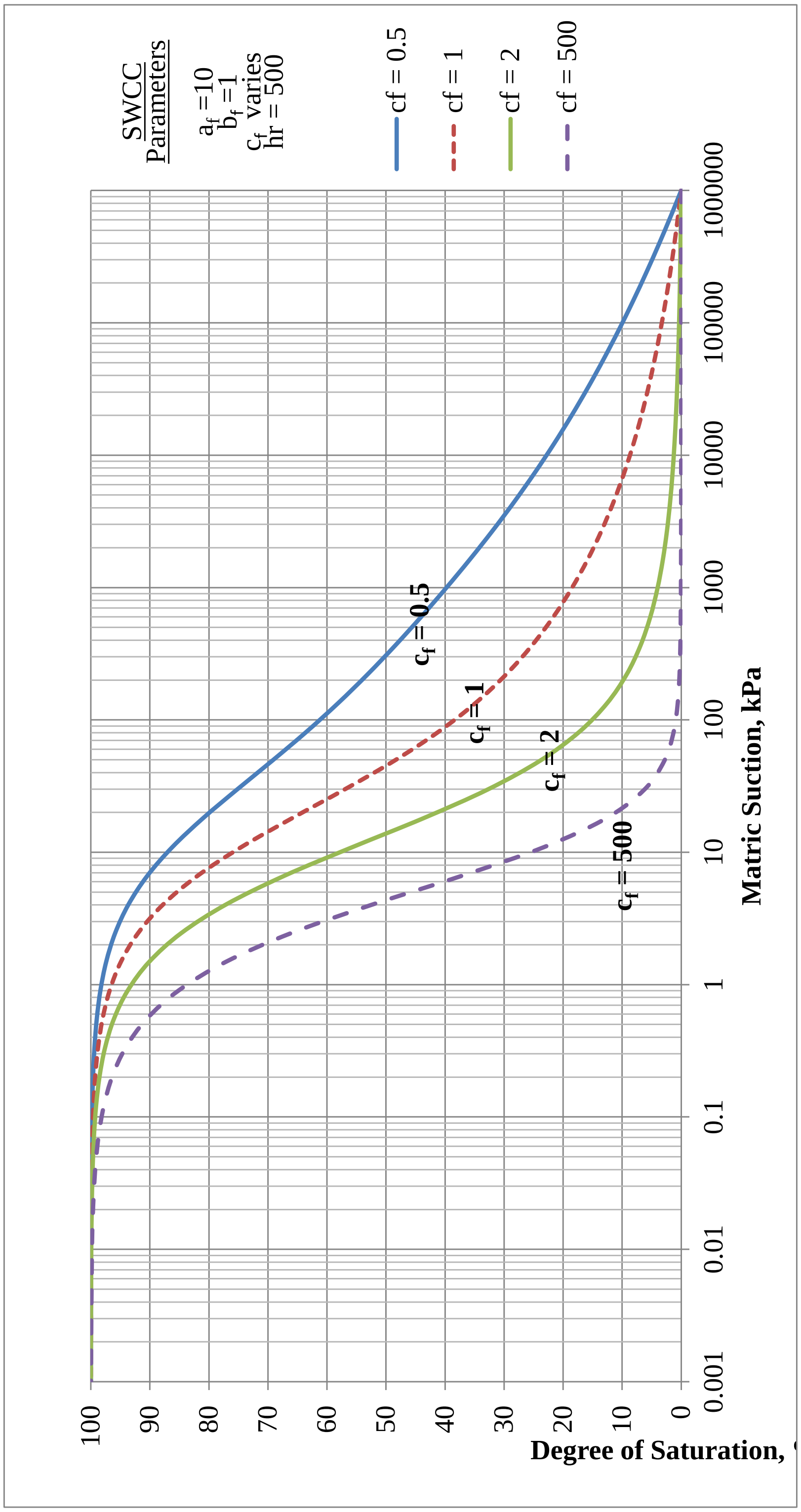



Figure 5-4. Changes in the SWCC Shape Due to Changes in the $c_f$ Parameter

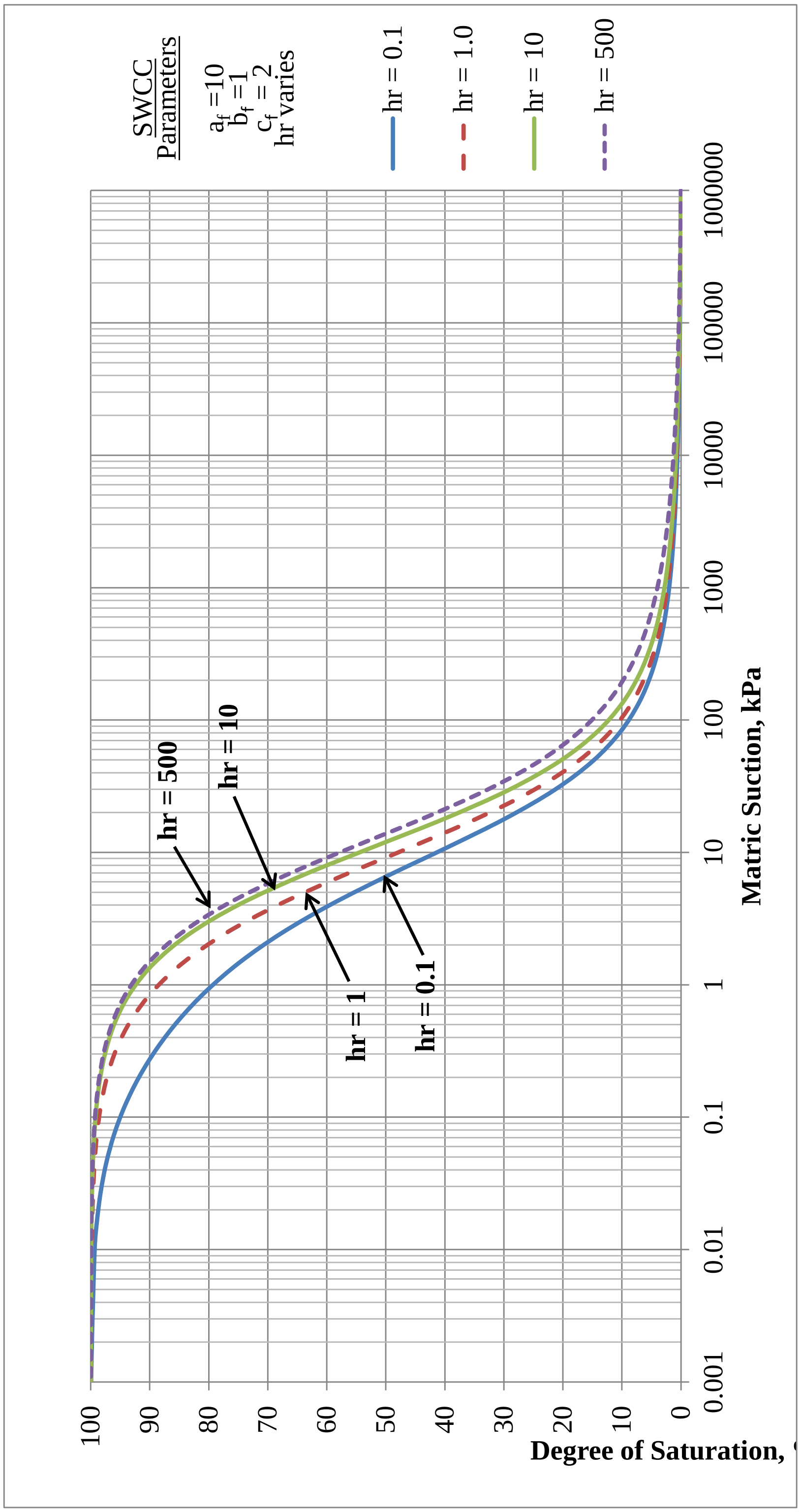



Figure 5-5. Changes in the SWCC Shape Due to Changes in the $h_r$ Parameter

residual degree of saturation decreases as $c_f$ parameter increases. Finally, Figure 5-5 shows the variation of the SWCC Shape when hr parameter varies while $a_f$, $b_f$, and $c_f$ remains constant. It can be seen that the sensibility of the SWCC due to changes in the hr parameter is relatively low when compared to the changes observed due to the variation of the other parameters.

### 5.5 SWCC Prediction Models for Fine–Grained Soils

Table 5-8 presents a summary of the models proposed for the SWCC parameters for fine–grained soils. A detailed process of how the models were obtained is presented in the next four sections.Briefly, the process used to find models is as follows. This process was used, in general to find the models for the four SWCC parameters, $a_f$, $b_f$, $c_f$ and $h_r$:

Step 1. *Choosing the Best Predictors*: Based on the results obtained with the correlation matrix, the best predictions were chosen to be correlated with the SWCC fitting parameter. The database was sorted by the best predictor from the lowest value to the highest value. The data points were grouped by taking 300 consecutive data points. Either the average or median was chosen to represent the values of the predictor as well as the value of the fitting parameter.

Step 2. *Regression Analysis*: The best predictors of each parameter were then use to regress several models. The statistical package MiniTab® 15 was used for the analysis. In some instances, the analysis was complemented with features from Statistica® 5.5 and Microsoft Excel®.

Table 5-8. Proposed Models for the SWCC Parameters for Fine–Grained Soils

| | |
|---|---|
| Modified Soil-Water Characteristic Curve Equation (Fredlund and Xing, 1994) | $S(\%) = C(\psi) \times \left[ \dfrac{1}{\left\{ \ln\left[ e + \left( \dfrac{\psi}{a} \right)^{b_f} \right] \right\}^{c_f}} \right]$ <br> $C(\psi) = 1 - \dfrac{\ln\left(1 + \dfrac{\psi}{h_r}\right)}{\ln\left[1 + \left(\dfrac{1{,}000{,}000}{h_r}\right)\right]}$ |
| SWCC Parameter af, kPa | $a_f = 10^{\left(0.69 - \frac{2.7}{1+e^{4-0.14GI}}\right)}$ |
| SWCC Parameter bf | $b_f = 10^{\left(\frac{0.78}{1+e^{6.75-0.19GI}}\right)}$ |
| SWCC Parameter cf | $c_f = 0.03 + 0.62 \times e^{\left(-0.82\left(\log a_f - 0.57\right)^2\right)}$ |
| SWCC Parameter hr, kPa | $h_r = 494 + \dfrac{660}{1 + e^{(4-0.19GI)}}$ |
| Where: <br> S = Degree of Saturation, % <br> $\psi$ = Matric Suction, kPa <br> $GI$ = Group Index | |

Non–linear regression analyses were performed for each combination of variables by using the least squared error criterion, in which the sum of the differences between the squares of measured values and estimated values are minimized. The regression results yielded the best model. A plot of measured versus predicted values is presented.

Step 3. *Error Analysis*: The error analysis was performed by using equations 5-4 through 5-8 to assess the accuracy of the model proposed. The data was analyzed and summary tables showing the errors found in each case, were created.

Step 4. *Final Assessment of the SWCC parameters Models*: A final assessment of the validity of the four models proposed (one for each parameter) was performed. In order to accomplish that, the predicted parameters were applied to the Fredlund and Xing equation and the predicted degree of saturation was obtained. This value was compared with the measured degree of saturation.

The following sections detail the steps follow to obtain the models.

### 5.5.1 Modeling SWCC parameter $a_f$.

Based on the correlation matrix shown in Table 5-6, the Group Index (GI) and the wPI parameters were chosen as the best predictor of the $a_f$ parameter, despite the low correlation observed. Following the moving average procedure described above, the data was grouped and the median and mean values were used in the regression analysis.

Several trial models were analyzed. The models were carefully chosen based on previous published corrections and the trends observed in the correction matrix. The mean and the median values were used; but in general, the best correlations were found to be related to the mean values.

Table 5-9 summarizes the best correlations found out of many trials. In general, the Group Index showed better performance as predictor than the wPI value. Also, the logarithmic of $a_f$ parameter was found to correlate better than the arithmetic value.

Based on the results shown in Table 5-9, the model proposed for this parameter is Model 3.

$$\log(a_f) = 0.69 - \frac{2.7}{1 + e^{(4-0.14GI)}} \quad \text{.......................................................(5-5)}$$

Where:

*GI* = Group Index, expressed in Chapter 3, equation 3-7 as follows:

$GI=(P_{200}-35)[0.2+0.005(LL-40)]+0.01(P_{200}-15)(PI-10)$

Where:

$P_{200}$ = Material Passing # 200 US Standard Sieve

*LL* = Liquid Limit

*PI* = Plasticity Index

Although this equation has not the best correlation, this model was selected for two simple reasons. First, the third order polynomial equations given by models 1 and 2 present maximum and minimum that implies the correlation will yield the same $a_f$ parameter value for different group indexes; and second, the expressions are rather complicated for the little gained of accuracy.

The plot of the model selected is shown in Figure 5-6. The statistical analysis yielded the following results:

Number of Data Points, $n$ = 31,835

Absolute Mean Error, $e_{abs}$ = 30.01

Algebraic Mean Error, $e_{alg}$ = -12.19

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.21

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.9552

The spreadsheet used to calculate the statistical errors is shown in Figure 5-7. This spreadsheet was used on a check for the R–squared calculation, by using a similar expression.

To evaluate de goodness of the model proposed (in other words the model accuracy), the Figure 5-8 present graphically the relationship between the measured values versus the predicted values.

Table 5-9. Summary of Trials Finding the Best Model (Parameter $a_f$)

| # | Model Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | $\log af = 0.00004GI^3 - 0.004GI^2 + 0.041GI + 0.469$ | 0.9633 | Average |
| 2 | $\log af = -0.1964Ln^3(GI+1) + 0.7766Ln^2(GI+1) - 0.7248Ln(GI+1) + 0.5549$ | 0.9579 | Average |
| 3 | $\log(af) = 0.69-2.7 / (1+EXP(4-0.14GI)$ | 0.9552 | Average |
| 4 | $\log(af) = 0.6338-2.6978 / (1+EXP(3.9958-0.1362GI)$ | 0.9549 | Average |
| 5 | $\log af = -0.0003GI^2 - 0.0433GI + 0.8487$ | 0.9392 | Average |
| 6 | $\log af = -0.5067Ln^2(GI+1) + 1.5514Ln(GI+1) - 0.2216$ | 0.9094 | Average |
| 7 | $\log af = 0.00004GI^3 - 0.0035GI^2 + 0.0298GI + 0.5147$ | 0.802 | Average |
| 8 | $\log af = 0.0003GI^2 - 0.0708GI + 1.0549$ | 0.7628 | Average |
| 9 | $\log af = -0.0729Ln^3(GI+1) + 0.0823Ln^2(GI+1) + 0.2902Ln(GI+1) + 0.3281$ | 0.7375 | Average |
| 10 | $\log af = -0.4317Ln^2(GI+1) + 1.2851Ln(GI+1) - 0.0623$ | 0.7311 | Average |
| 11 | $\log af = 0.00009wPI^3 - 0.0074wPI^2 + 0.1008wPI + 0.2317$ | 0.6129 | Average |
| 12 | $af = 0.00007GI^3 - 0.0045GI^2 - 0.0457GI + 4.7922$ | 0.5982 | Median |
| 13 | $\log af = 0.00008GI^3 - 0.0082GI^2 + 0.1591GI + 0.0602$ | 0.5964 | Median |
| 14 | $af = 0.0002GI^3 - 0.022GI^2 + 0.5614GI + 5.5527$ | 0.5815 | Average |
| 15 | $af = 0.00008wPI^3 - 0.0054wPI^2 - 0.0081wPI + 4.3602$ | 0.5793 | Median |
| 16 | $af = -9.5517\log^3(wPI) + 13.898\log^2(wPI) + 3.9952\log(wPI) + 1.6786$ | 0.5747 | Median |
| 17 | $af = -0.5543Ln^3(GI+1) + 2.0125Ln^2(GI+1) + 0.6174Ln(GI+1) + 4.1266$ | 0.5684 | Average |
| 18 | $af = 0.0004wPI^3 - 0.039wPI^2 + 0.9793wPI + 3.1511$ | 0.5615 | Average |
| 19 | $af = 0.0016wPI^2 - 0.1746wPI + 5.1796$ | 0.5528 | Median |
| 20 | $af = 0.0016\log^2(wPI) - 0.1746\log(wPI) + 5.1796$ | 0.5528 | Median |

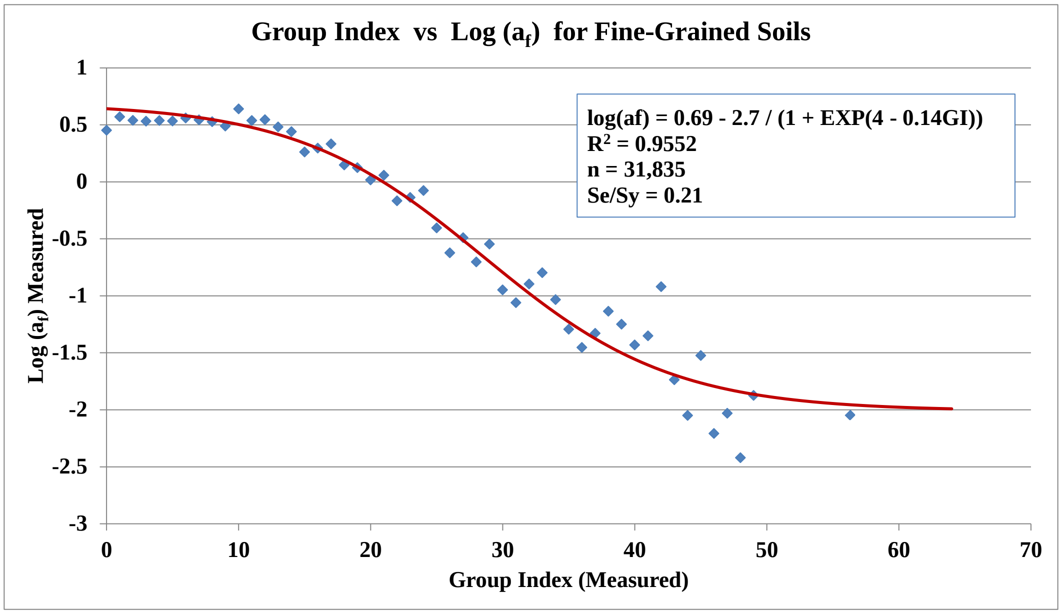



Figure 5-6. Model Predicting Parameter $a_f$ for Fine-Grained Soils

| | | | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_p)^2$ |
|---|---|---|---|---|---|---|---|
| Model: | | | $e_{alg}$ = | -12.19 | | | |
| $\log(a_f) = 0.69 - 2.7 / (1 + \exp(4 - 0.14GI))$ | | | $e_{abs}$ = | | 30.01 | | |
| | | | S = | | | 1.91 | 42.65 |
| | | | $n_{group}$ = | | | 51 | 51 |
| Checking Calculation | | | p = | | | 1 | 1 |
| $R^2$ adj = | 1-((SSE/(n-p))/(SST/(n-1))) | | Se = | | | 0.20 | |
| | | | Sm(Avrg) | | | | -0.4885 |
| SSE = | S $(Sm - Sp)^2$ | 1.91 | Sy = | | | | 0.92 |
| | | | | | | | |
| SST = | S $(Sm(avg) - Sm)^2$ | 42.65 | Se/Sy = | | | | 0.21 |
| | | | | | | | |
| $R^2$ adj = | 0.9552 | | $R^2$ = | 0.9552 | | | |

Figure 5-7. Spreadsheet Used in Calculating Errors and $R^2$ Values for the $a_f$ Model for Fine–Grained Soils

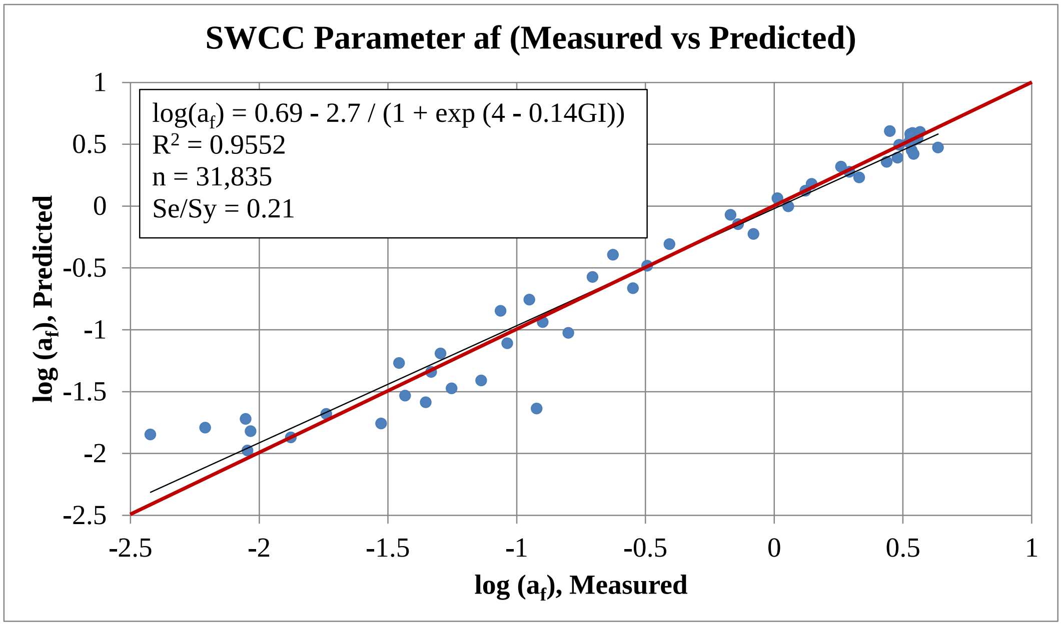



Figure 5-8. Measured versus Predicted Analysis for SWCC Parameter $a_f$

### 5.5.2 Modeling the SWCC parameter $b_f$.

Using the same procedure used for the analysis of parameter $a_f$, the model obtained for the SWCC parameter $b_f$ was based on correlations of this parameter with the soil properties that were consider the best predictors. Despite the low correlation observed (see Table 5-5) when the data treated independently, parameters such as wPI and GI were chosen as the best predictors of the bf parameter. Table 5-10 summarizes the results of non–linear regressions performed for several trials. The average values used corresponded to the mean of the values per Group Index number, in cases where the variable independent was different of GI the average correspond to the mean of 300 consecutive values, arranged in increasing order of wPI.

Table 5-10. Summary of Trials Performing in Finding the Best Model for the Parameter $b_f$ for Fine–Grained Soils

| # | Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | $\log(bf) = -0.00002wPI^3 + 0.0019wPI^2 - 0.0337wPI + 0.1353$ | 0.8652 | Avg 300 data points |
| 2 | log (bf) = 1/(1+EXP(5-(0.125*GI))) | 0.8612 | Average |
| 3 | log (bf) = 0.78/(1+EXP(6.75-(0.19*GI))) | 0.8509 | Average |
| 4 | $\log(bf) = -0.00003wPI^3 + 0.0025wPI^2 - 0.0465wPI + 0.1842$ | 0.7890 | Average |
| 5 | $bf = -0.00002GI^3 + 0.0021GI^2 - 0.0392GI + 0.1366$ | 0.7677 | Average |
| 6 | $bf = -0.0007wPI^3 + 0.0669wPI^2 - 1.2122wPI + 7.463$ | 0.6981 | Average |
| 7 | $bf = -0.0006wPI^3 + 0.0556wPI^2 - 0.8916wPI + 5.9533$ | 0.6358 | Avg 300 data points |
| 8 | $\log(bf) = -0.00004wPI^3 + 0.0035wPI^2 - 0.0663wPI + 0.2656$ | 0.4701 | Average |
| 9 | $bf = -0.001wPI^3 + 0.0851wPI^2 - 1.471wPI + 8.2345$ | 0.4008 | Average |
| 10 | $\log(bf) = -0.00003E\text{-}05wPI^3 + 0.0033wPI^2 - 0.0712wPI + 0.2753$ | 0.3319 | Median |
| 11 | $bf = -0.0007wPI^3 + 0.0636wPI^2 - 1.3822wPI + 6.5846$ | 0.2285 | Median |
| 12 | $bf = 0.00001af^3 - 0.0031af^2 + 0.235af + 3.0982$ | 0.0273 | Total |

The model proposed for this parameter is the Model 3 on Table 5-10:

$$\log(b_f) = \frac{0.78}{1 + e^{(6.75 - 0.19GI)}} \qquad \text{(5-6)}$$

Although this model did not yield the best correlation, it was selected for two reasons: The third–order polynomial equation, given by trial #1, presents maximum and minimum values which will yield the same bf parameter for different wPI values. On the other hand, the second expression (Trial #2) yields very high values for intermediate values of GI.

The plot of the model selected is shown in Figure 5-9. The statistical analysis of errors yielded the following values:

Number of Data Points, $n$ = 31,833

Absolute Mean Error, $e_{abs}$ = 21.31

Algebraic Mean Error, $e_{alg}$ = 87.04

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.39

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.8509

The spreadsheet used to calculate the statistical errors is shown in Figure 5-10. This spreadsheet allows for the check of the R–squared calculation by using a similar expression than that used by Statistica®.

Figure 5-11 presents the relationship between the measured $b_f$ parameters and the predicted values. This figure allows evaluating the goodness of the fit or accuracy of the model proposed for the $b_f$ parameter.

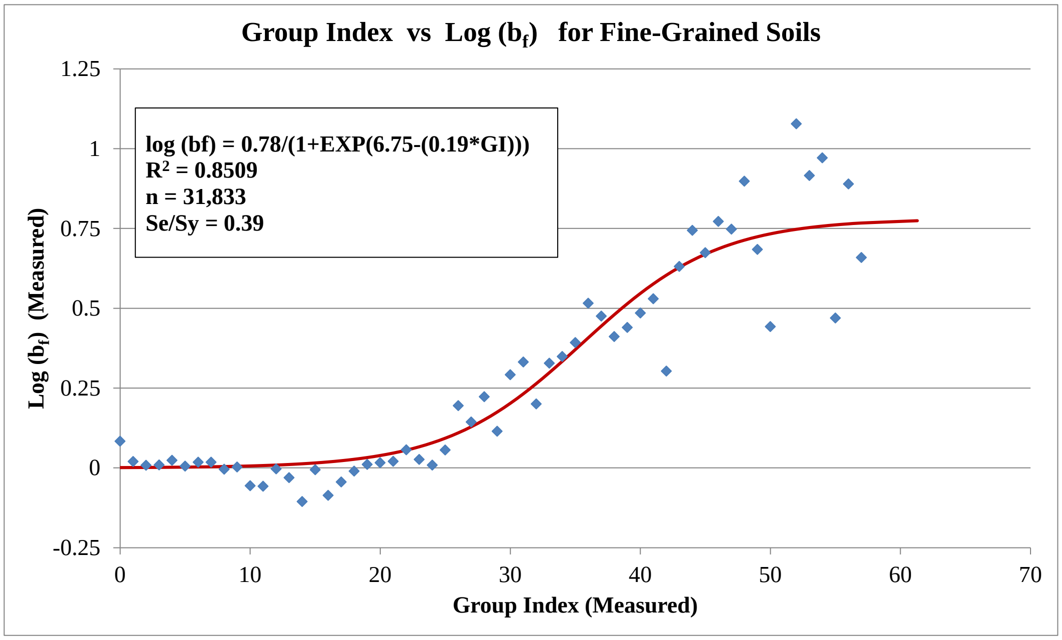



Figure 5-9. Model Predicting Parameter $b_f$ for Fine–Grained Soils

| | | | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_p)^2$ |
|---|---|---|---|---|---|---|---|
| Model: | | | $e_{alg}$ = | 21.31 | | | |
| log ($b_f$) = 0.78/(1+EXP(6.75-(0.19*GI))) | | | $e_{abs}$ = | | 87.04 | | |
| | | | S = | | | 1.09 | 7.32 |
| | | | $n_{group}$ = | | | 58 | 58 |
| Checking Calculation | | | p = | | | 1 | 1 |
| $R^2$ adj = | 1-((SSE/(n-p))/(SST/(n-1))) | | Se = | | | 0.14 | |
| | | | Sm(Avrg) | | | | 0.3060 |
| SSE = | S (Sm - Sp)$^2$ | 1.09 | Sy = | | | | 0.36 |
| | | | | | | | |
| SST = | S (Sm(avg) - Sm)$^2$ | 7.32 | Se/Sy = | | | | 0.39 |
| | | | | | | | |
| $R^2$ adj = | 0.8509 | | $R^2$ = | 0.8509 | | | |

Figure 5-10. Spreadsheet Used in Calculating Errors and $R^2$ Values for the $b_f$ Model for Fine–Grained Soils

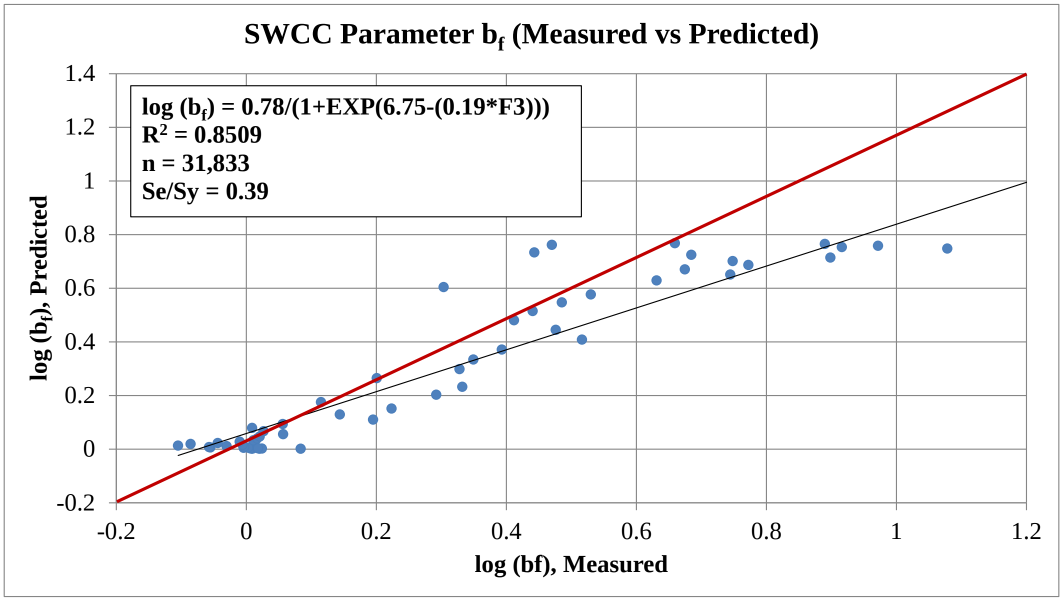



Figure 5-11. Measured versus Predicted Analysis for SWCC Parameter $b_f$

### 5.5.3 Modeling SWCC parameter $c_f$.

The trials shown in Table 5-11 reflect some of the attempt to find a good correlation. As shown by the correlation matrix (Table 5-6), the $c_f$ parameter seem to be highly correlate with parameter $b_f$. However, when the data was analyzed by the average of 300 consecutive data points, a strong correlation between the $a_f$ and the $c_f$ parameter were found; as well as an important correlation with the GI parameter.

Table 5-11. Summary of Trials Performed in Finding the Best Model for the Parameter $c_f$ for Fine-Grained Soils.

| # | Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | cf=0.08+(0.59*0.94$^{GI}$) | 0.9825 | Average per GI |
| 2 | cf=0.03+0.62*(exp(1)^(-0.82*((logaf-0.57)^2))) | 0.9215 | Avg of 300 data |
| 3 | cf=0.65*(exp(1)^(-0.65*((logaf-0.58)^2)))) | 0.9038 | Average |
| 4 | cf = -0.0302log$^3$(af) - 0.0992log$^2$(af) + 0.1907log(af) + 0.5281 | 0.8266 | Avg of 300 data |
| 5 | cf = 0.0007af$^3$ - 0.0208af$^2$ + 0.1651af + 0.2745 | 0.6777 | Avg of 300 data |

Based on the results from different trials, the model proposed for the parameter $c_f$ is the Model 2 presented in the Table 5-11:

$$c_f = 0.03 + 0.62 \times e^{\left(-0.82\left(\log a_f - 0.57\right)^2\right)} \quad \text{(5-7)}$$

Equation 5-7 was selected due to the convenience of being able to mathematically link the $c_f$ parameter with the $a_f$ parameter. On the other hand, equation 5-7 yields an acceptable adjusted $R^2$ value of 0.9215. The plot of the model selected is shown in Figure 5-12. The statistical analysis of errors yielded the following:

Number of Data Points, $n$ = 31,520

Absolute Mean Error, $e_{abs}$ = 10.25

Algebraic Mean Error, $e_{alg}$ = -1.41

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.28

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.9215

The spreadsheet used to calculate the statistical errors is shown on Figure 5-13. This spreadsheet allows for checking the R–squared calculation using a similar statistical expression as that used by Statistica®.

To evaluate de goodness of the fit or accuracy of the model proposed, the measured values were plotted against the predicted values as shown in Figure 5-14. The fine line shows a linear regression between the measured and predicted points, which indicates a relative unbiased prediction.

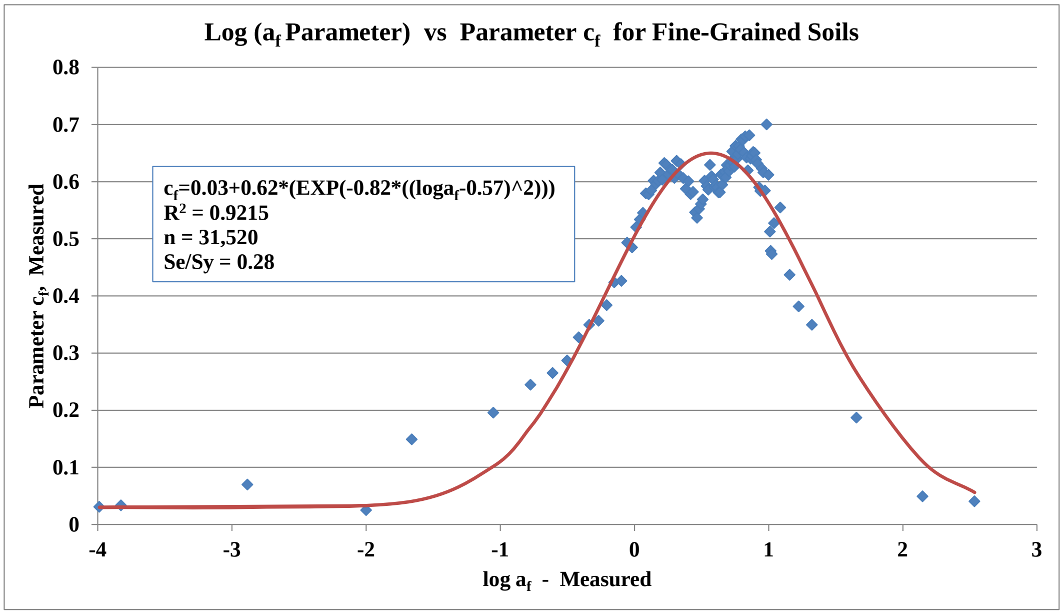



Figure 5-12. Model Predicting $c_f$ Parameter for Fine–Grained Soils

| | | | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_p)^2$ |
|---|---|---|---|---|---|---|---|
| Model: | | | $e_{alg}$ = | -1.41 | | | |
| $c_f$=0.03+0.62*(EXP(-0.82*((log$a_f$-0.57)^2) | | | $e_{abs}$ = | | 10.25 | | |
| | | | S = | | | 0.23 | 2.99 |
| | | | $n_{group}$ = | | | 107 | 107 |
| Checking Calculation | | | p = | | | 1 | 1 |
| $R^2$ adj = | 1-((SSE/(n-p))/(SST/(n-1))) | | Se = | | | 0.05 | |
| | | | Sm(Avrg) | | | | 0.5323 |
| SSE = | S (Sm - Sp)$^2$ | 0.23 | Sy = | | | | 0.17 |
| | | | | | | | |
| SST = | S (Sm(avg) - Sm)$^2$ | 2.99 | Se/Sy = | | | | 0.28 |
| | | | | | | | |
| $R^2$ adj = | 0.9215 | | $R^2$ = | 0.9215 | | | |

Figure 5-13. Spreadsheet Used in Calculating Errors and $R^2$ Values for the $c_f$

Model for Fine–Grained Soils

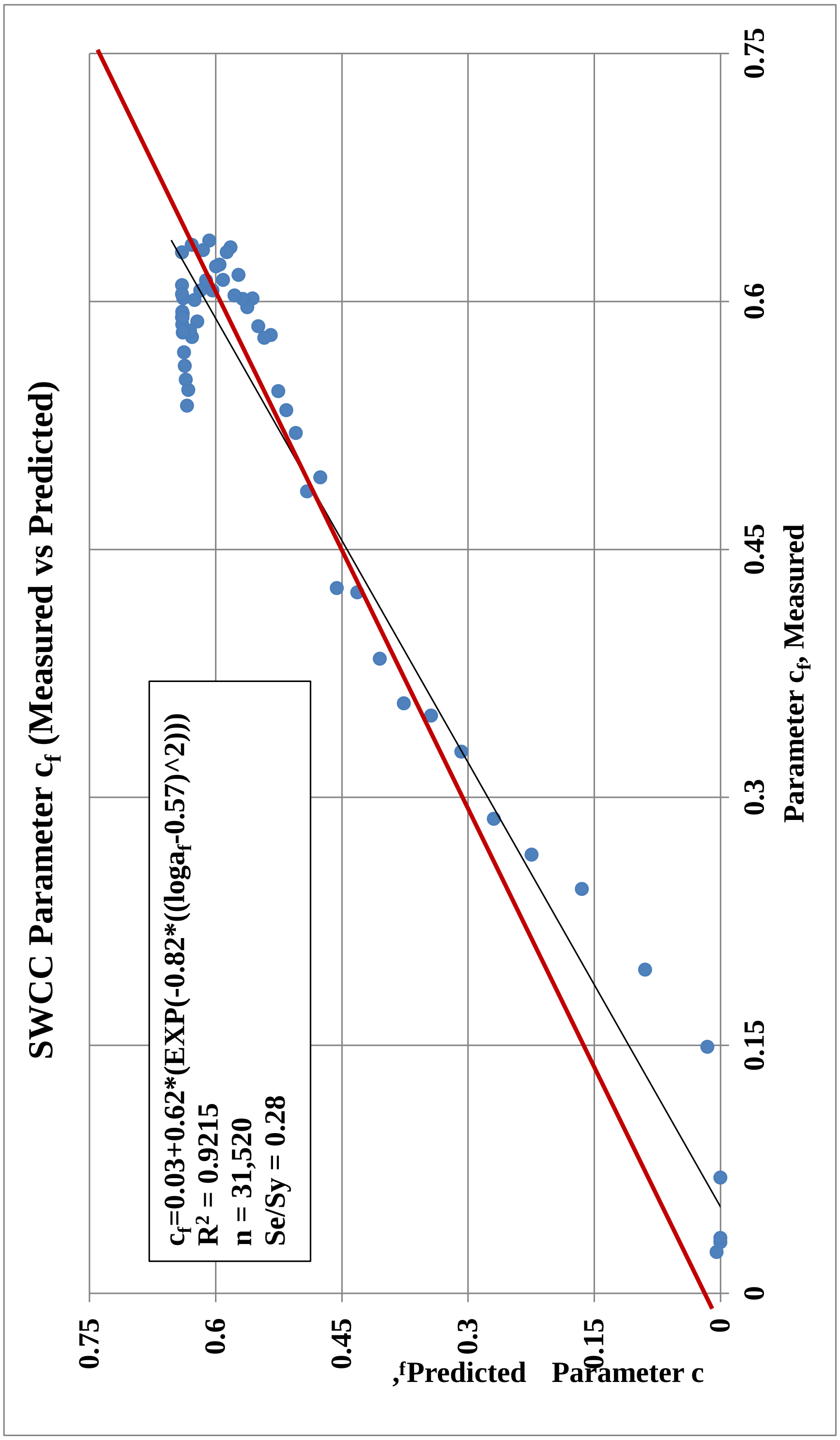



Figure 5-14. Measured versus Predicted Analysis for SWCC Parameter $c_f$

### 5.5.4 Modeling SWCC parameter $h_r$.

Table 5-12 summarizes the regression trials attempted to correlate the hr parameter with the GI value. Several trials were attempted using the wPI with no success. The average and media values as well as grouping 300 consecutive data points were attempted.

Table 5-12. Summary of Trials finding the best model (Parameter $h_r$)

| # | Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | hr = 494 + 660 / (1 + exp(1)^(4 - 0.19GI)) | 0.9041 | Avg of 300 data |
| 2 | hr = $-0.0082GI^3 + 0.5996GI^2 + 4.342GI + 496.67$ | 0.8775 | Avg of 300 data |
| 3 | hr = $-0.0012GI^3 + 0.0082GI^2 - 0.2679GI + 499.65$ | 0.8392 | Avg of 300 data |
| 4 | hr = 9.0009GI + 570.31 | 0.4563 | Average |
| 5 | hr = $0.0169GI^3 - 2.0349GI^2 + 65.747GI + 238.02$ | 0.4314 | Average |
| 6 | hr = $0.1449GI^2 - 6.5654GI + 546.35$ | 0.4188 | Median |

The model proposed for the hr parameter is Model 1 shown in Table 5-12:

$$h_r = 494 + \frac{660}{1 + e^{(4-0.19GI)}} \qquad \text{(5-8)}$$

This equation was selected because it has the best coefficient of determination and behaves asymptotically on the extremes. The plot of the model selected is shown in Figure 5-15. The statistical analysis of errors yields the following results:

Number of Data Points, $n$ = 31,839

Absolute Mean Error, $e_{abs}$ = 4.25

Algebraic Mean Error, $e_{alg}$ = -1.39

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.31

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.9041

The spreadsheet used to calculate the statistical errors is shown on Figure 5-16. This spreadsheet allows for checking the R–squared calculation using a similar expression as that used by Statistica®.

To evaluate de goodness of the fit or accuracy of the model proposed for the hr parameter for fine–grained soils the measured versus the predicted values were plotted as shown in Figure 5-17. It can be seen that the model is unbiased, which is reflected in the low $e_{alg}$ found.

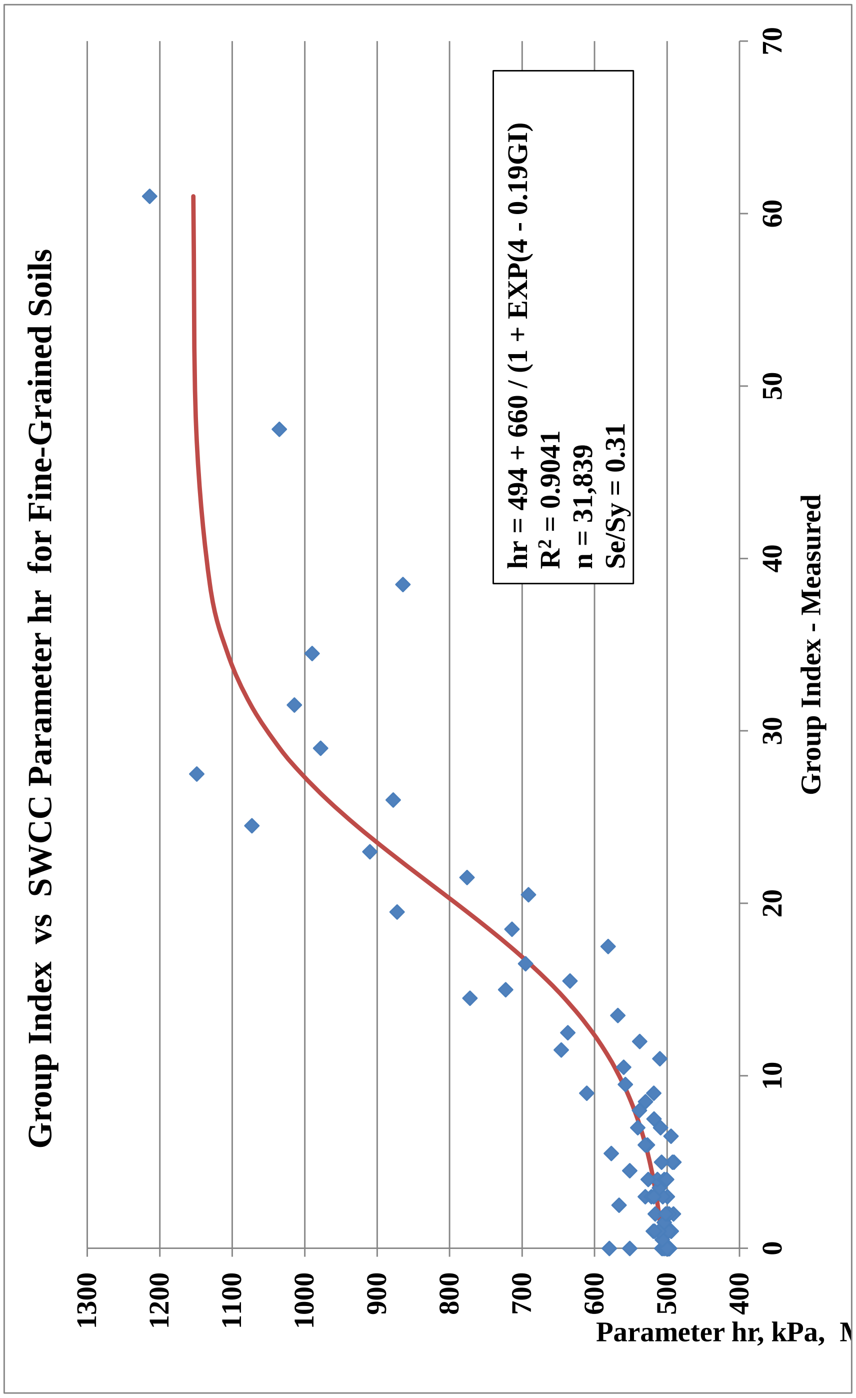



Figure 5-15. Model Predicting $h_r$ Parameter for Fine-Grained Soils

| | | | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_p)^2$ |
|---|---|---|---|---|---|---|---|
| Model: | | | $e_{alg}$ = | -1.39 | | | |
| hr = 494 + 660 / (1 + EXP(4 - 0.19GI) | | | $e_{abs}$ = | | 4.25 | | |
| | | | S = | | | 272,094.83 | 2,837,347.14 |
| | | | $n_{group}$ = | | | 107 | 107 |
| Checking Calculation | | | p = | | | 1 | 1 |
| $R^2$ adj = | 1-((SSE/(n-p))/(SST/(n-1))) | | Se = | | | 50.66 | |
| | | | Sm(Avrg) | | | | 578.2088 |
| SSE = | S (Sm - Sp)$^2$ | 272,094.83 | Sy = | | | | 163.61 |
| | | | | | | | |
| SST = | S (Sm(avg) - Sm)$^2$ | 2,837,347.14 | Se/Sy = | 1 - (Se/Sy)$^2$ | | | 0.31 |
| | | | | | | | |
| $R^2$ adj = | 0.9041 | | $R^2$ = | 0.9041 | | | |

Figure 5-16. Spreadsheet Used in Calculating Errors and $R^2$ Values for the $h_r$ Model for Fine–Grained Soils

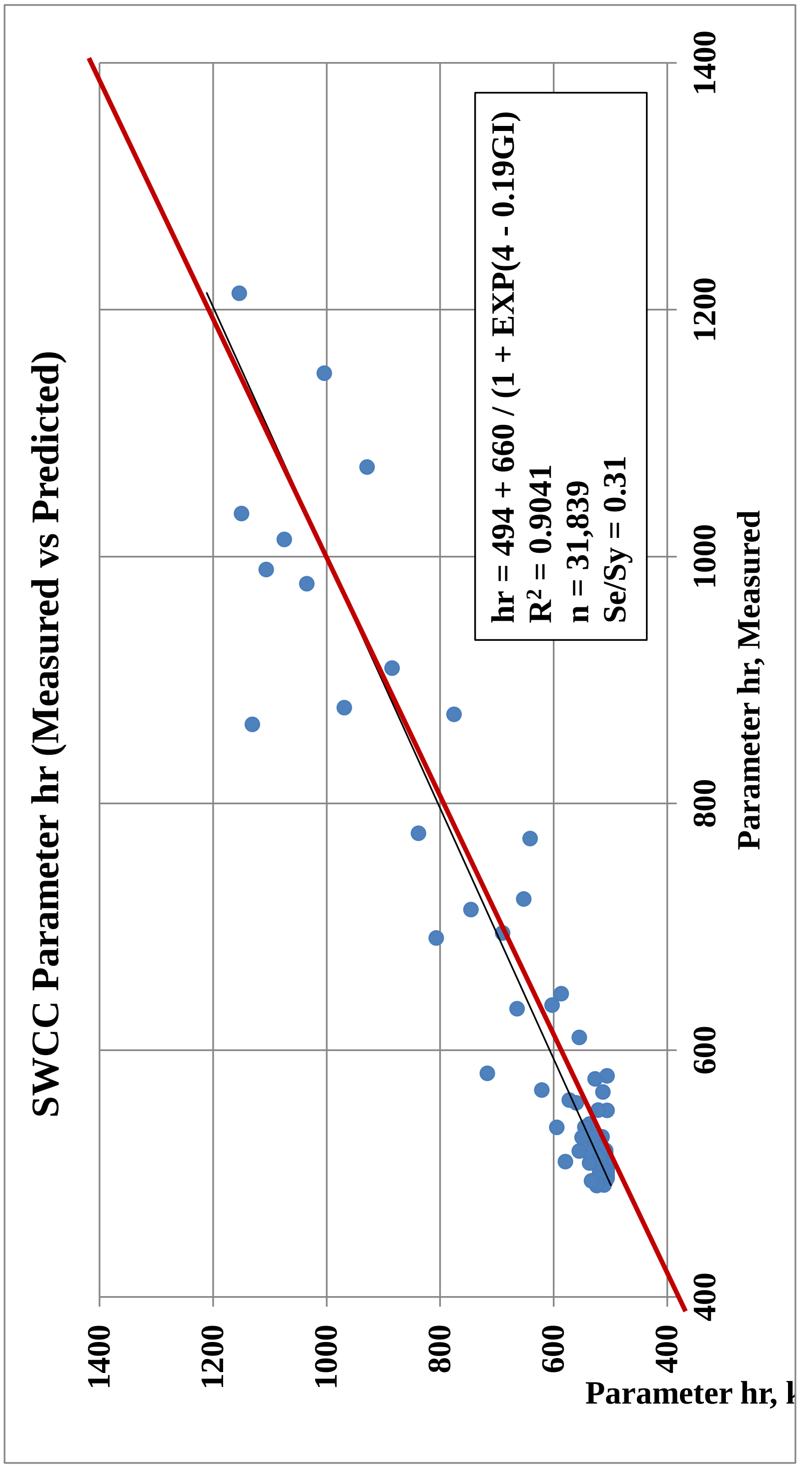



Figure 5-17. Measured versus Predicted Analysis for SWCC Parameter $h_r$

A final checking was performed on the database to validate the data information associated with the plastic limit and saturated volumetric water content.

Figure 5-18 shows the Shrinkage Curve. In this curve are indicated the limits of consistency when the soil is drying process. The point when immediately the soil begins the desaturation (or when the curve starts) is considered the plastic limit. This point is associated with the Air Entry Value (AEV) (Fredlund, et al., 2011)

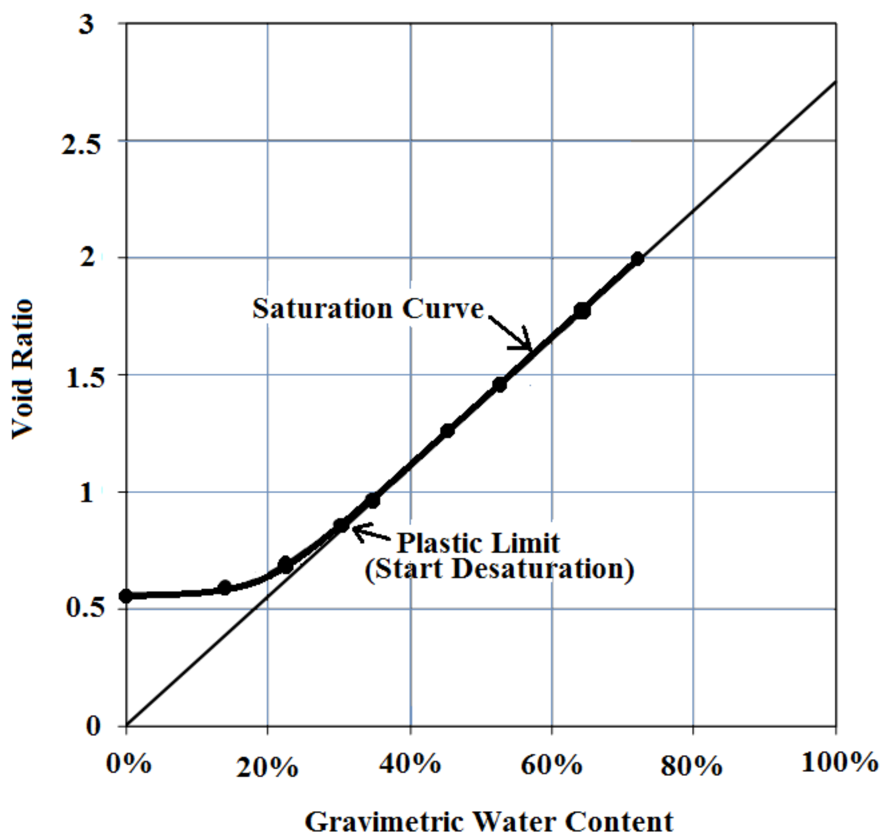



Figure 5-18 Curve Gravimetric Water Content vs Void Ratio

A statistical analysis of the differences between the plastic limit and the saturated volumetric water content can be observed in the histogram shown in Figure 5-19. For plastic soils the values for the saturated volumetric water content and the plastic limit should be really close.

The statistical analysis was performed on a range of data-points with the 95% of confidence considering a normal distribution like can be observed in Figure 5-19. The data points out of this range can be considered suspicious and should not be included in any regression analysis.

A difference between the plastic limit and the saturated volumetric water content greater than 10 is affecting the validity of the model and makes the fit and data suspicious. At this point, further attempt to find a better model for plastic material should be consider the following:

Eliminate soil data that look suspicious. The comparison of plastic soil and saturated volumetric water content is a starting step that can help in the elimination process.

The initial SWCC fitting parameters for the Fredlund and Xing model should be analyzed carefully. $a_f$ parameter for plastic materials should be higher than 10, because a regression analysis with low values of $a_f$ create in SWCC a bimodal shape. A low value of $a_f$ force the curve to have an initial decreasing and the same time to approach to the given points of water content at 33 and 1500 kPa. This effect can be observed in the soil 1 in Figures 5-36 and 5-37

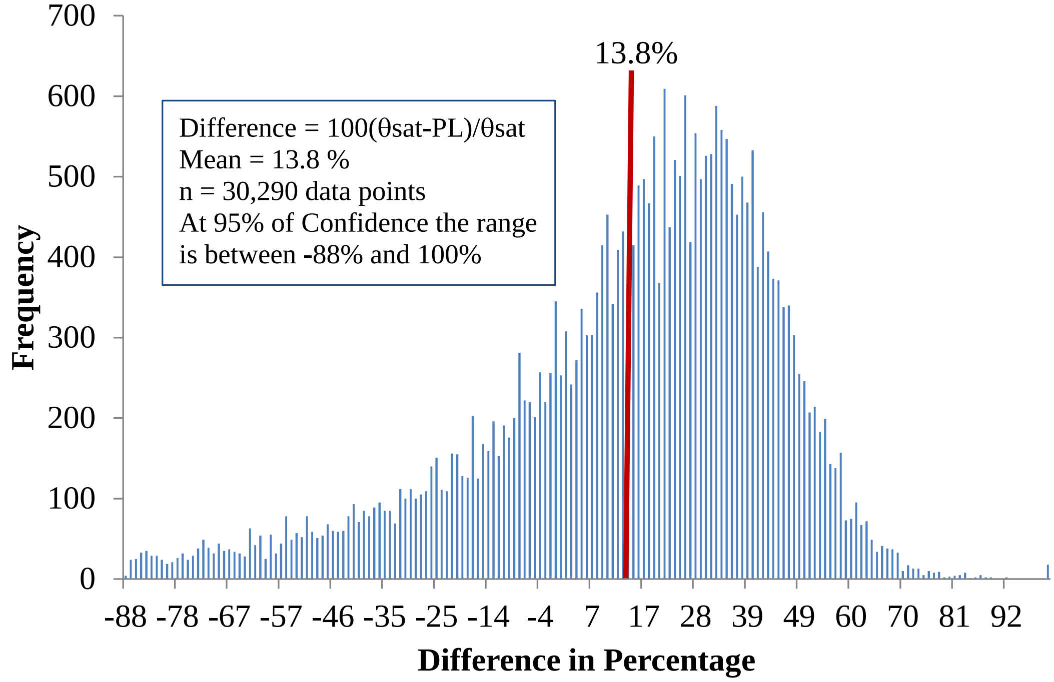



Figure 5-19 Histogram Showing Differences between Plastic Limit and Saturated Volumetric Water Content

## 5.6 SWCC Prediction models for Granular Soil

Table 5-13 presents a summary of the SWCC parameters models proposed for granular or non-plastic soils. The complete analysis is presented in the following four sections. These parameters apply to the Soil–Water Characteristic Curve (equation 5-1) defined by Fredlund & Xing, 1994.

The equation for $a_f$ is a function of Particle Size $D_{10}$. The effective particle size $D_{10}$ has been related to the coefficient of permeability in the past

(Hazen, 1911) and therefore, it seems logical that it correlates well with moisture retention characteristic. It can be seen from the correlation matrix (Table 5-7) that the correlations among the parameters yielded the highest correlation values. The SWCC parameter $b_f$ was found to correlate with $a_f$ and parameter $c_f$ is inferred from $b_f$ parameter. The dependency between $b_f$ and $c_f$ with $a_f$ is found to be convenient because it eliminates the possibility of not getting a continuous SWCC function once the fitting parameters are put together in the Fredlund & Xing equation. Finally, the parameter $h_r$ yielded a constant value of 100.

The process followed to estimate the fitting parameter for granular soils was the same used for fine–grained soils. Refer to section 5.5 for details.

Table 5-13. Proposed Models for the SWCC Parameters for Granular Soils

| | |
|---|---|
| Modified Soil-Water Characteristic Curve Equation (Fredlund and Xing, 1994) | $S(\%) = C(\psi) \times CF(\psi) \times \left[ \dfrac{1}{\left\{ \ln \left[ e + \left( \dfrac{\psi}{a} \right)^{b_f} \right] \right\}^{c_f}} \right]$ <br> $C(\psi) = 1 - \dfrac{\ln\left(1 + \dfrac{\psi}{h_r}\right)}{\ln\left[1 + \left(\dfrac{1{,}000{,}000}{h_r}\right)\right]}$ |
| SWCC Parameter af, kPa | $a_f = -967.21 D_{10}^{2} + 218.37 D_{10} - 2.7$ <br> Constraint: if $D_{10} < 0.020$, then $a_f = 1.28$ |
| SWCC Parameter bf | $b_f = 10^{\left(-0.0075 a_f^{3} + 0.1133 a_f^{2} - 0.3577 a_f + 0.3061\right)}$ |
| SWCC Parameter cf | $c_f = 0.0058 a_f^{3} - 0.0933 a_f^{3} + 0.4069 a_f + 0.3481$ |
| SWCC Parameter hr, kPa | $h_r = 100$ |
| Where: <br> S = Degree of Saturation, % <br> $\psi$ = Matric Suction, kPa <br> $D_{10}$ = Grain Diameter at 10% Passing by Weight | |

### 5.6.1 Modeling the SWCC Parameter $a_f$ for Granular Soils.

The model obtained for the SWCC parameter $a_f$ is given as a function of granular soils properties found to be the best predictors according to results presented in Table 5-7 and explained in section 5.3.2. Table 5-14 summarizes some of the trials used to estimate the best model. The variables used in the regressions were the effective particle size $D_{10}$, percentage passing sieve #200 and the grain–size distribution shape parameters Cu (Coefficient of uniformity) and Cc (Coefficient of curvature). These geotechnical properties were considered in previous studies such as the dissertation written by Zapata, 1999, and Perera, 2003.

The database used for the non–linear regression analysis consisted of 4,485 data points. After the database was sorted from minimum to maximum value of the SWCC parameter, the average of 50 consecutive points was obtained. Groups of 50 data points were desired due to the fact that the database corresponding to granular materials was smaller than the database obtained for fine–grained soils.

From the results presented in Table 5-14, it can be seen that the expression makes use of all the soils properties found to be good predictors of parameters af. Expression given by trial 2 does not consider the Cu and Cc properties and has a light low R2. Equation 3 uses only the particle size $D_{10}$ as independent variable, and yet keeps a $R^2$ value of 0.72. The expression shown for trials 4 through 8 are rather complex and do not improve the correlations.

Table 5-14. Summary of Trials Use in Finding the Best Model for Parameter $a_f$ for Granular Soils

| # | Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | af=1.574+0.1296P200+0.0442D90+0.3962D60+1.2088D30-24.6642D20+145.7467D10+1.5322logD90+2.5409logD10-12.2653/cc+95.1692/cu | 0.7420 | Avg of 50 data |
| 2 | af = 1.1319 + 0.1032P200 + 0.0357D90 + 0.4277D60 + 0.035D30 - 20.5359D20 + 136.7032D10 + 1.8858*logD90 + 2.1319logD10 | 0.7377 | Avg of 50 data |
| 3 | af = -967.21D102 + 218.37D10 - 2.7006 | 0.7167 | Avg of 50 data |
| 4 | af = 1.13 + 0.103 P200 + 0.0357 D90 + 1.89 log(D90) + 0.43 D60 + 0.0 D30 - 20.5 D20 + 137 D10 + 2.13 log(D10) | 0.7120 | Avg of 50 data |
| 5 | af = 0.28 + 0.149 P200 + 0.0028 D90 + 2.05 log(D90) + 1.67 D60 - 9.2 D30 - 10.1 D20 + 142 D10 + 2.29 log(D10) - 0.000020 Cu + 0.00133 Cc | 0.7070 | Avg of 50 data |
| 6 | af = - 1.51 + 0.060 P200 + 0.0485 D90 + 1.30 log(D90) - 1.29 D60 + 10.5 D30 - 29.8 D20 + 130 D10 + 1.59 log(D10) + 10.4 log(D60) - 11.9 log(D30) + 5.7 log(D20) | 0.7050 | Avg of 50 data |
| 7 | af = - 3.42 + 0.123 P200 + 0.0074 D90 + 1.51 log(D90) - 0.26 D60 + 5.5 D30 - 26.5 D20 + 137 D10 + 1.82 log(D10) - 0.000028 Cu + 0.00249 Cc + 14.5 log(D60) - 20.6 log(D30) + 10.9 log(D20) | 0.7010 | Avg of 50 data |
| 8 | af = 11.6 - 0.010 P200 + 1.03 log(D90) + 3.57 log(D10) + 13.6 log(D60) - 29.9 log(D30) + 21.3 log(D20) | 0.6530 | Avg of 50 data |

The model proposed for the parameter $a_f$ is the Model 3 presented in Table 5-14:

$$a_f = -967.21 D_{10}^{\ 2} + 218.37 D_{10} - 2.7 \text{ ......................................................(5-9)}$$

Constraint: if $D_{10} < 0.020$, then $a_f = 1.28$

Figure 5-20 shows the plot of the selected model. The error analysis yielded the following results:

Number of Data Points, $n$ = 4,485

Absolute Mean Error, $e_{abs}$ = 170.62

Algebraic Mean Error, $e_{alg}$ = -139.48

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.53

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.7167

The spreadsheet used to calculate the statistical errors is shown in Figure 5-21.

The goodness of the model proposed 5-9 is evaluated and illustrated in Figure 5-22 where the measured values of $a_f$ are plotted versus the predicted values.

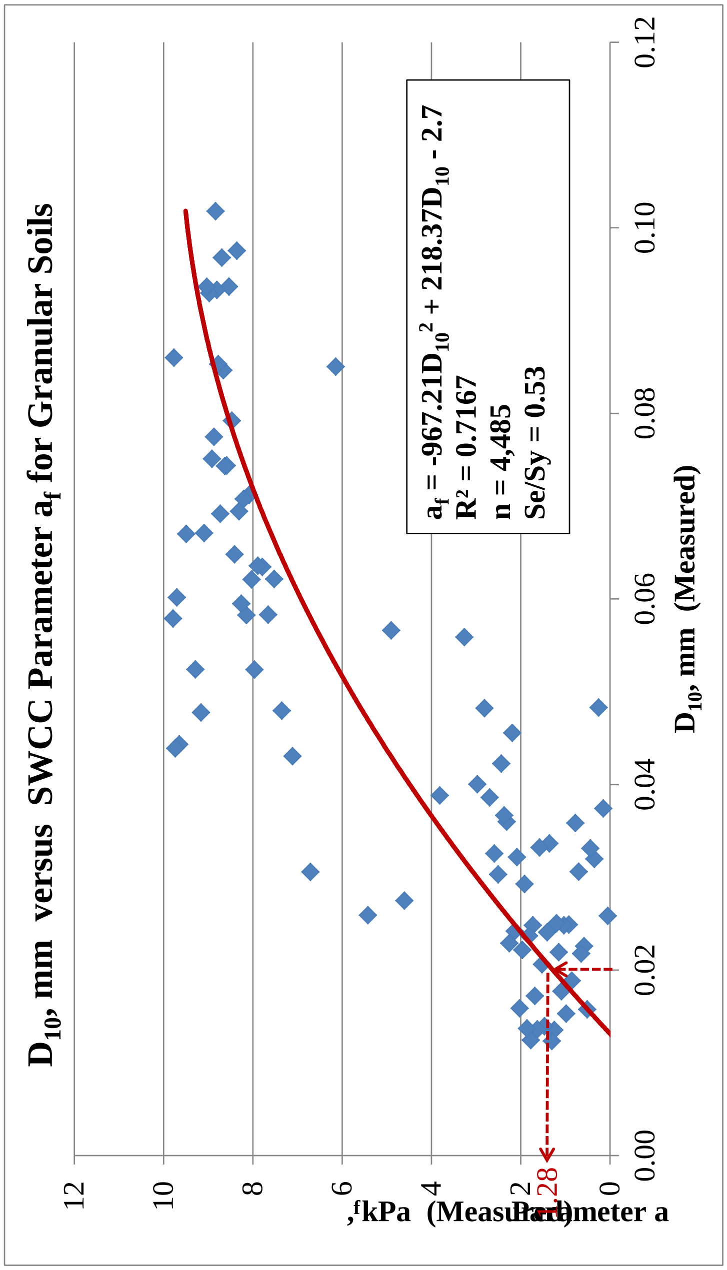



Figure 5-20. Model Predicting Parameter af for Granular Soils

| | | | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_m)^2$ |
|---|---|---|---|---|---|---|---|
| Model: | | | | | | | |
| $a_f = -967.21D_{10}^2 + 218.37D_{10} - 2.7006$ | | | $e_{alg}$ = | -139.48 | | | |
| | | | $e_{abs}$ = | | 170.62 | | |
| | | | Σ = | | | 310.50 | 1,096.16 |
| | | | $n_{group}$ = | | | 90 | 90 |
| | | | p = | | | 1 | 1 |
| $R^2$ adj = | 1-((SSE/(n-p-1))/(SST/(n-1))) | | Se = | | | 1.87 | |
| | | | Sm(Avrg) | | | | 4.7697 |
| SSE = | S (Sm - Sp)$^2$ | 310.50 | Sy = | | | | 3.51 |
| | | | | | | | |
| SST = | S (Sm(avg) - Sm)$^2$ | 1,096.16 | Se/Sy = | | | | 0.53 |
| | | | | | | | |
| $R^2$ adj = | 0.7167 | | $R^2$ = | 0.7167 | | | |

Figure 5-21. Spreadsheet Used in Calculating Errors and R2 Values for the $a_f$ Model for Granular Soils

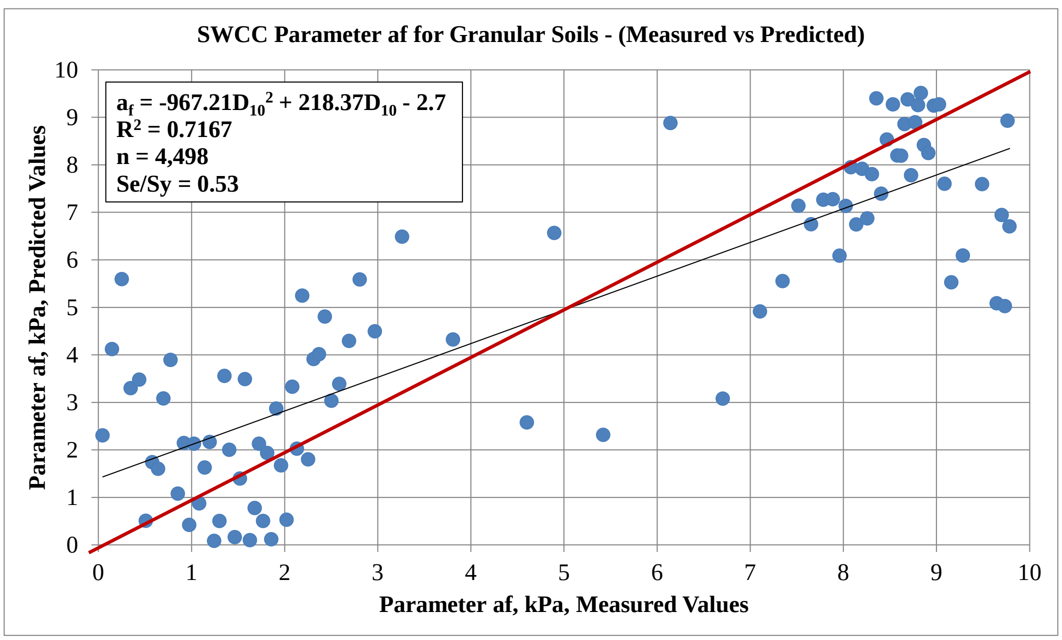



Figure 5-22. Measured versus Predicted Analysis for SWCC Parameter $a_f$

### 5.6.2 Modeling the SWCC parameter $b_f$.

The first step taken in order to analyze the $b_f$ parameter for granular materials was to choose the best soil properties predictors from the matrix of correlation shown in Table 5-7 and explained in detail under section 5.3.2. Table 5-15 shows some of the trials used in calculating the R–squared. During this process the database was worked in two ways; initially, all the data was used, then the average/median values of grouped parameters were analyzed.

Table 5-15. Summary of Trials Used in Finding the Best Model for Parameter $b_f$

| # | Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | $\log(b_f) = -0.0075af^3 + 0.1133af^2 - 0.3577af + 0.3061$ | 0.9668 | Avg of 50 data points |
| 3 | $\log(b_f) = 0.1384\log^3(af) + 0.548\log^2(af) + 0.1755\log(af) - 0.0216$ | 0.7067 | Avg of 50 data points |
| 4 | $bf = -0.075af^3 + 1.0141af^2 - 2.653af + 2.619$ | 0.6418 | Avg of 50 data points |
| 5 | $bf = 0.7458\log^3(af) + 3.4994\log^2(af) + 2.1039\log(af) + 0.7301$ | 0.4655 | All data points |

The best model corresponded to that obtained by trial # 1, which presented the highest $R^2$. The model proposed for the $b_f$ parameter is:

$$\log b_f = -0.0075 a_f^{\,3} + 0.1133 a_f^{\,2} - 0.3577 a_f + 0.3061 \quad \text{..................................(5-10)}$$

The graph corresponding to this model is given in Figure 5-23.Even though third order polynomials present inflection points that might not reflect the measure data, in this case it is a good representation of the data obtained.

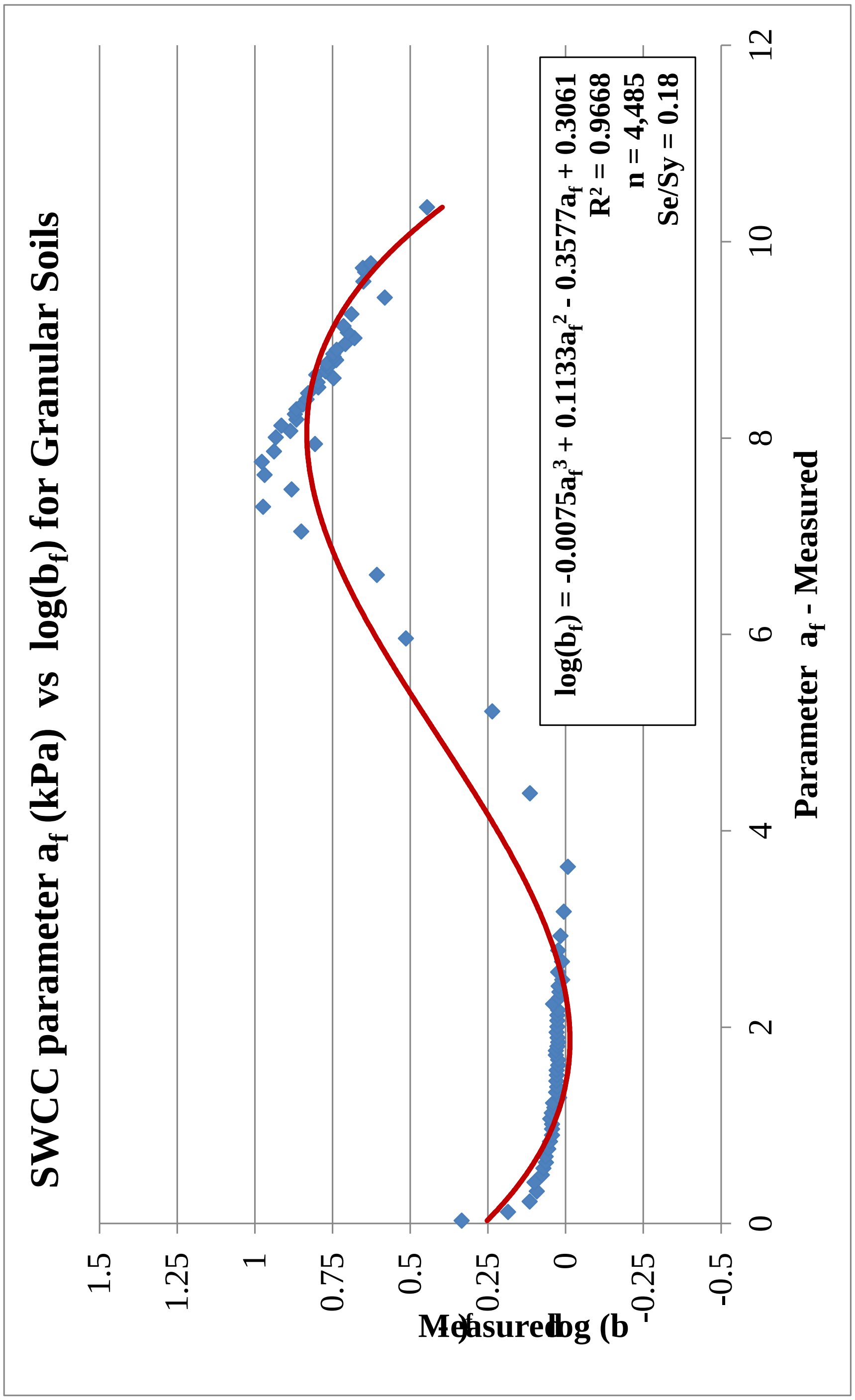



Figure 5-23. Model Predicting Parameter $b_f$ for Granular Soils

Equations 5-4 through 5-8 were used in the error analysis, which gave the following results:

Number of Data Points, $n$ = 4,497

Absolute Mean Error, $e_{abs}$ = 89.80

Algebraic Mean Error, $e_{alg}$ = 25.73

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.18

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.9668

The spreadsheet used to calculate the statistical errors is shown in Figure 5-24.

| | | | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_m)^2$ |
|---|---|---|---|---|---|---|---|
| Model: | | | | | | | |
| $\log(b_f) = -0.0075a_f^3 + 0.1133a_f^2 - 0.3577a_f + 0.$: | | | $e_{alg}$ = | 25.73 | | | |
| | | | $e_{abs}$ = | | 89.80 | | |
| | | | S = | | | 0.41 | 12.26 |
| | | | $n_{group}$ = | | | 90 | 90 |
| Checking Calculation | | | p = | | | 1 | 1 |
| $R^2$ adj = | 1-((SSE/(n-p-1))/(SST/(n-1))) | | Se = | | | 0.07 | |
| | | | Sm(Avrg) | | | | 0.3779 |
| SSE = | S $(Sm - Sp)^2$ | 0.41 | Sy = | | | | 0.37 |
| | | | | | | | |
| SST = | S $(Sm(avg) - Sm)^2$ | 12.26 | Se/Sy = | | | | 0.18 |
| | | | | | | | |
| $R^2$ adj = | 0.9668 | | $R^2$ = | 0.9668 | | | |

Figure 5-24. Spreadsheet Used in Calculating Errors and $R^2$ Values for $b_f$ Model in Granular Soils

Figure 5-25 presents the relationship between the measured values versus the predicted values in order to evaluate de goodness of the model proposed.

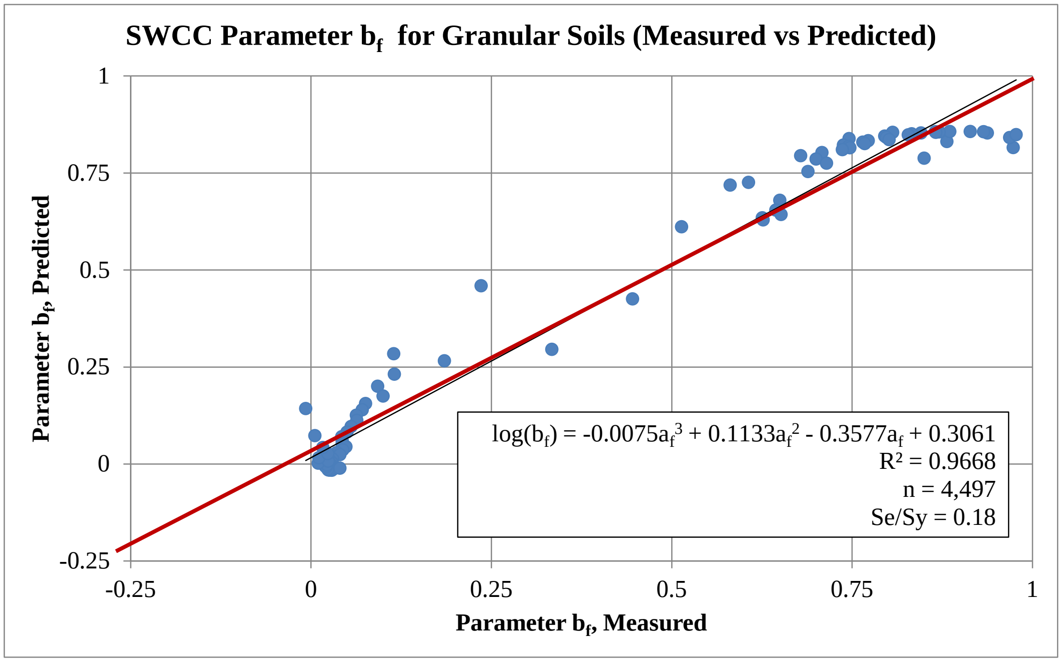



Figure 5-25. Measured versus Predicted Analysis for SWCC $b_f$ Parameter

### 5.6.3 Modeling the SWCC parameter $c_f$.

The correlation matrix yielded $a_f$ parameter as the best predictor for granular or non-plastic soils for $c_f$ parameter. The results of several correlation trials are shown in Table 5-16.

The model proposed for this parameter is the Model 1 from Table 5-16:

$$c_f = 0.0058a_f^3 - 0.0933a_f^2 - 0.4069a_f + 0.3481 \quad \text{..............................(5-11)}$$

Table 5-16. Summary of Trials Used in Finding the Best Model for Parameter $c_f$

| # | Equation | $R^2$ | Type of Data |
|---|---|---|---|
| 1 | cf= 0.0058af3 -0.0933af3 + 0.4069af + 0.3481 | 0.8735 | Avg of 50 data points |
| 2 | $cf = 2.8649\ e^{-1.227\ bf}$ (bf < 3) | 0.7978 | Avg of 50 data points |
| 2 | $cf = 1.9\ bf^{-0.58}$ (bf >= 3) | 0.6395 | Avg of 50 data points |
| 3 | $cf = -0.00008bf^3 + 0.0031bf^2 - 0.0675bf + 0.9272$ | 0.5054 | Avg of 50 data points |
| 4 | $cf = 0.0003bf^3 - 0.0086bf^2 + 0.0348bf + 0.7391$ | 0.2224 | All data points |

The model selected is shown in Figure 5-26. The statistical analysis of errors yielded the following results:

Number of Data Points, $n$ = 4,450

Absolute Mean Error, $e_{abs}$ = 4.74

Algebraic Mean Error, $e_{alg}$ = 1.55

Standard Error divided by the Standard Deviation, $S_e/S_y$ = 0.36

Adjusted Coefficient of Determination, $R^2$ *(adjusted)* = 0.8735

The spreadsheet used to calculate the statistical errors is shown on Figure 5-27. The goodness of fit or accuracy of the model can be visualized in figure 5-28, where the measured $c_f$ parameter is presented against the predicted value. The results yielded an unbiased model.

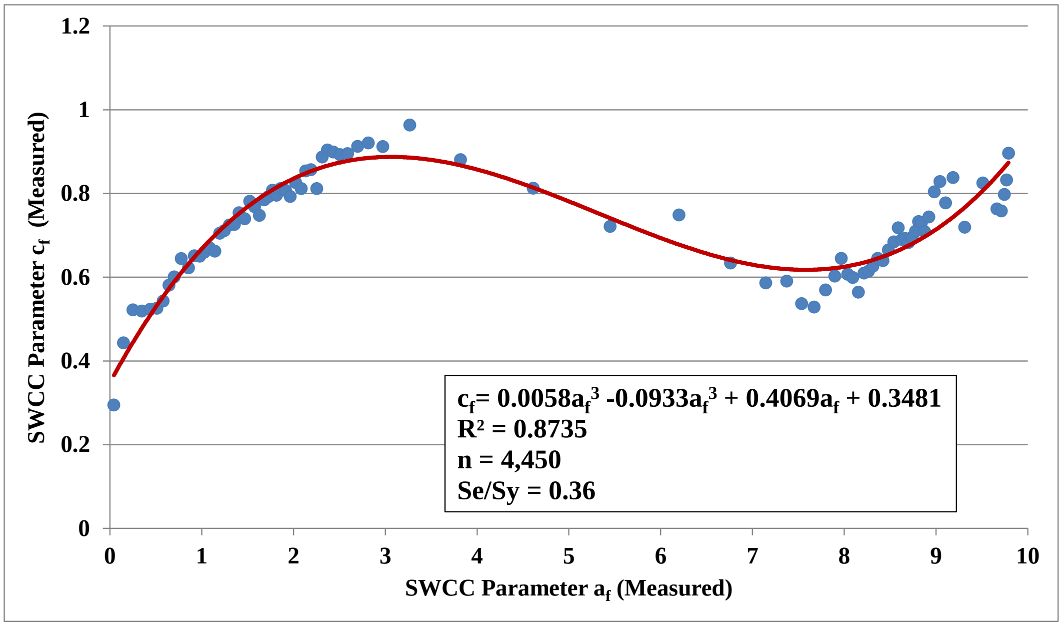



Figure 5-26. Model Predicting Parameter $c_f$ for Granular Soils

| | | $e_{alg}$ | $e_{abs}$ | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_m)^2$ |
|---|---|---|---|---|---|
| Model: | $e_{alg}$ = | 1.55 | | | |
| $c_f = 0.0058a_f^3 - 0.0933a_f^2 + 0.4069a_f + 0$ | $e_{abs}$ = | | 4.74 | | |
| | S = | | | 0.17 | 1.38 |
| | $n_{group}$ = | | | 89 | 89 |
| | p = | | | 1 | 1 |
| $R^2$ adj = 1-((SSE/(n-p))/(SST/(n-1))) | Se = | | | 0.04 | |
| | Sm(Avrg) | | | | 0.6994 |
| SSE = S (Sm - Sp)$^2$ 0.17 | Sy = | | | | 0.13 |
| SST = S (Sm(avg) - Sm)$^2$ 1.38 | Se/Sy = | | | | 0.36 |
| $R^2$ adj = 0.8735 | $R^2$ = | 0.8735 | | | |

Figure 5-27. Spreadsheet Used in Calculating Errors and $R^2$ Value for $c_f$ model for Granular Soils

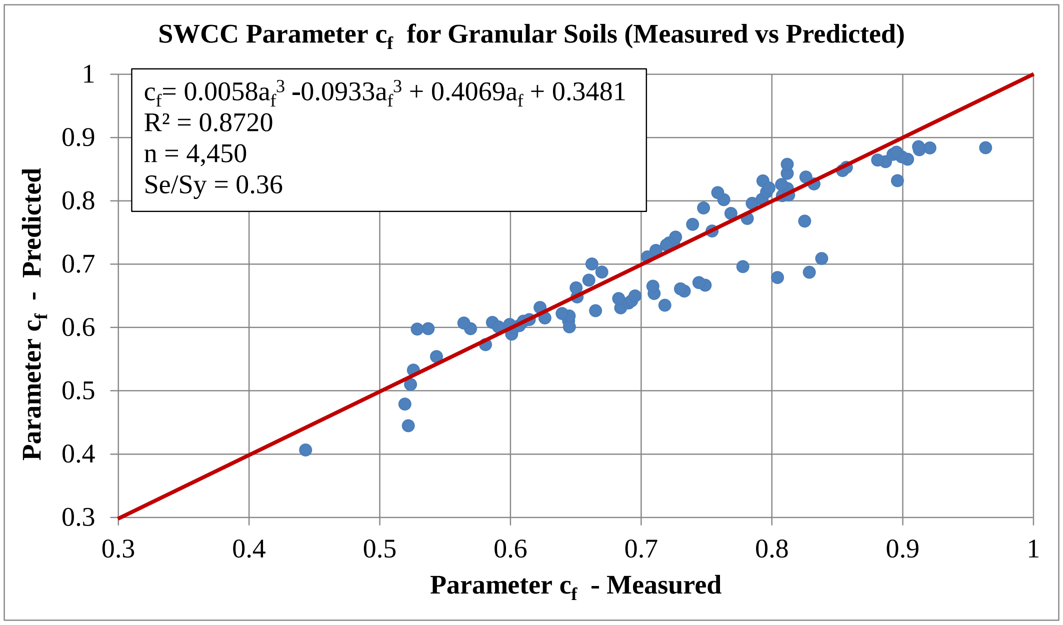



Figure 5-28. Measured Versus Predicted for SWCC Parameter $c_f$

### 5.6.4 Defining the SWCC parameter $h_r$.

The statistics for the measured hr parameter are as follows:

Average = 100.17

Mode = 100.10

Median = 100.4

Standard Deviation = 1.68

Given these results, it is appropriate to use a value equal to 100 to represent this parameter. It is important to notice that in order to fit the SWCC equation to the measured data points, the optimization process requires to assign initial values to af, bf, cf and hr parameters. For the hr parameter for granular or non-plastic materials, an initial value of 100 was assigned. This implies that the shape of the SWCC is basically independent of this parameter. Therefore a constant is proposed:

$h_r = 100$ ........................................................................................(5-12)

The histogram of measured $h_r$ parameters illustrates the reasonableness of this selection.

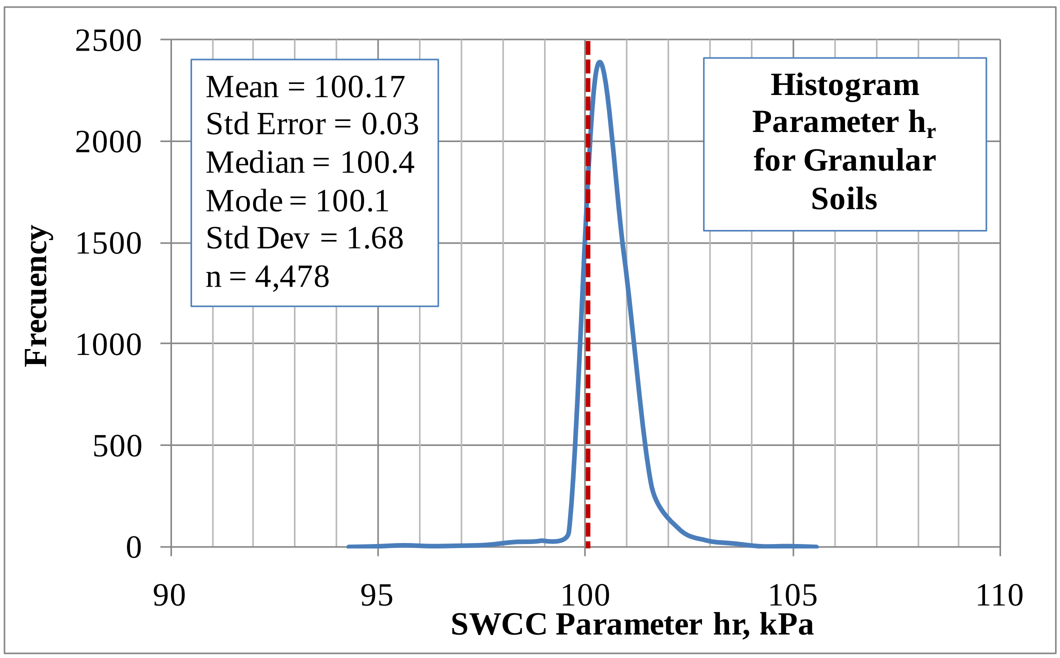



Figure 5-29. Histogram of Parameter $h_r$

## 5.7 Measured versus Predicted Degree of Saturation

To assess the validity of the model proposed independently for each parameter, a comparison between measured versus predicted degree of saturation is presented. The measure values used in the analysis correspond to the saturation at 33 kPa and 1,500 kPa. These values were calculated based on the volumetric water content directly obtained from testing. On the other hand, the predicted degree of saturation at 33 kPa and 1,500 kPa of suctions were estimated from the Fredlund & Xing equation (eq. 5-1) by fitting the predicted equations proposed.

The first analysis was developed for fine–grained soils. Figure 5-30 shows the graph for Measured versus Predicted degree of saturation, while Figure 5-31 presents the spreadsheet used for the error analysis. As it can be seen the combined equations yielded an $R^2$ of 0.56.

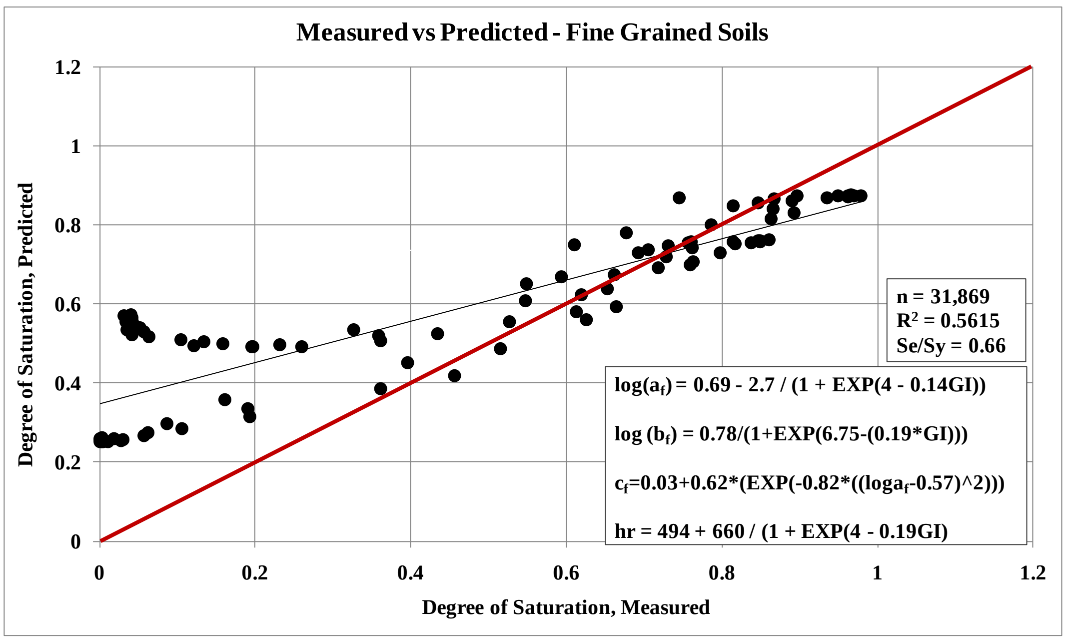



Figure 5-30. Measured vs Predicted Degree of Saturation for Fine–Grained Soils

| | SS err | SS tot | | | $e_{alg}$ | $e_{abs}$ | (Se) | (Sy) |
|---|---|---|---|---|---|---|---|---|
| | $(S_m - S_p)^2$ | $(S_m - S_{m(avg)})^2$ | | | $100*(S_m - S_p)/S_m$ | - | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_m)^2$ |
| $\Sigma$ = | 6.13 | 13.98 | | $e_{alg}$ = | -25.48 | | | |
| | | | | $e_{abs}$ = | | 25.52 | | |
| Sm(avg) = | 0.5176 | | | $\Sigma$ = | | | 6.13 | 13.98 |
| $n_{group}$ = | 102 | | | $n_{group}$ = | | | 102 | 102 |
| p = | 1 | | | p = | | | 1 | 1 |
| Se = | 0.25 | | | Se = | | | 0.25 | |
| | | | | Sm(avg) = | | | | 0.5176 |
| Sy = | 0.37 | | | Sy = | | | | 0.37 |
| | | | | | | | | |
| Se/Sy = | 0.66 | | | Se/Sy = | | | | 0.66 |
| | | | | | | | | |
| $R^2$ (adj) = | 0.5615 | Predicted | | $R^2$ (adj) = | 0.5615 | | | |

Figure 5-31. Spreadsheet Used in Calculating the Error Analysis for Measured versus Predicted Degree of Saturation for Fine-Grained Soils

The analysis for granular soils followed the same for fined–grained soils. The predicted values of degree of saturation were calculated at suctions of 33 and 1,500 kPa using the Fredlund & Xing equation and the predicted equation found for the fitting parameters for granular or non-plastic materials. These predicted values were compared to the measured values of degree of saturation available in the database. Figure 5-32 shows the plot of Measured versus Predicted degree of saturation values for granular soils, for suctions of 33 kPa and 1,500 kPa together. Figure 5-33 shows the spreadsheet used for the calculations of errors.

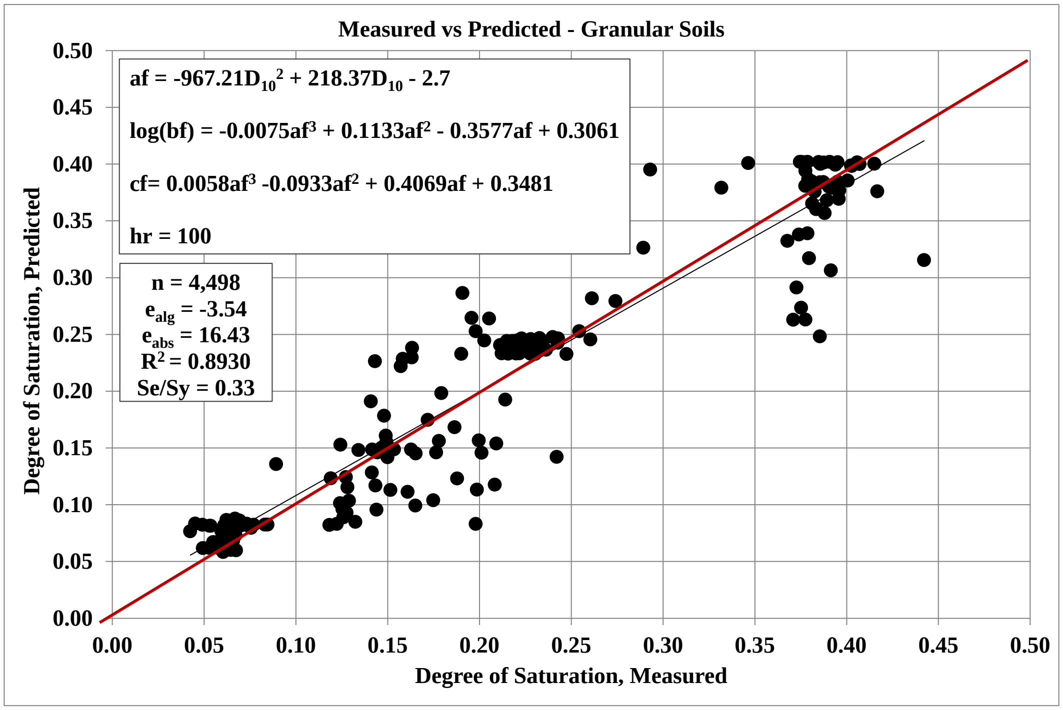



Figure 5-32. Measured versus Predicted Degree of Saturation for Granular Soils

| | SS err | SS tot | | | $e_{alg}$ | $e_{abs}$ | (Se) | (Sy) |
|---|---|---|---|---|---|---|---|---|
| | $(S_m - S_p)^2$ | $(S_m - S_{m(avg)})^2$ | | | $100*(S_m - S_p)/S_m$ | - | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_m)^2$ |
| Σ = | 0.28 | 2.58 | | $e_{alg}$ = | -3.54 | | | |
| | | | | $e_{abs}$ = | | 16.43 | | |
| Sm(avg) = | 0.2137 | | | S = | | | 0.28 | 2.58 |
| u = | 180 | | | $n_{group}$ = | | | 180 | 180 |
| e = | 1 | | | p = | | | 1 | 1 |
| Se = | 0.04 | | | Se = | | | 0.04 | |
| | | | | Sm(avg) = | | | | 0.2137 |
| Sy = | 0.12 | | | Sy = | | | | 0.12 |
| | | | | | | | | |
| Se/Sy = | 0.33 | | | Se/Sy = | | | | 0.33 |
| | | | | | | | | |
| $R^2$ (adj) = | 0.8930 | Predicted: | | $R^2$ (adj) = | 0.8930 | | | |

Figure 5-33. Error Analysis for Degree of Saturation for Granular Soils

## 5.8 Procedure to Estimate the SWCC from the Proposed Models

The following is the procedure to obtain the SWCC for Fine-Grained (plastic) and Granular (non-plastic) soils, based on the work presented in this Chapter.

Step 1. Obtain the Grain–Size Distribution and the Atterberg's limits (liquid limit, *LL* and plasticity limit, *PL*)

Step 2. Calculate the Plasticity Index $PI = LL - PL$

Step 3. Calculate the Group Index, *GI*, by using equation 5-3

Step 4. Calculate Weighted Plasticity Index, *wPI* by using equation 5-2

Step 5. Plot the Grain–Size Distribution Curves

Step 6. Obtain the Particle Size, $D_{10}$, (see Figure 5-34)

Step 7. Define the model to be used. For soil with wPI > 0, the soil is categorized as plastic. If the wPI = 0, the soil is categorized as non–plastic or granular. If the soil is plastic, continue with step 8. If the soil is granular or non–plastic soils, continue with step 17

Step 8. Calculate the SWCC parameter $a_f$ according to equation 5-5. The parameter $a_f$ is a function of the Group Index *GI*

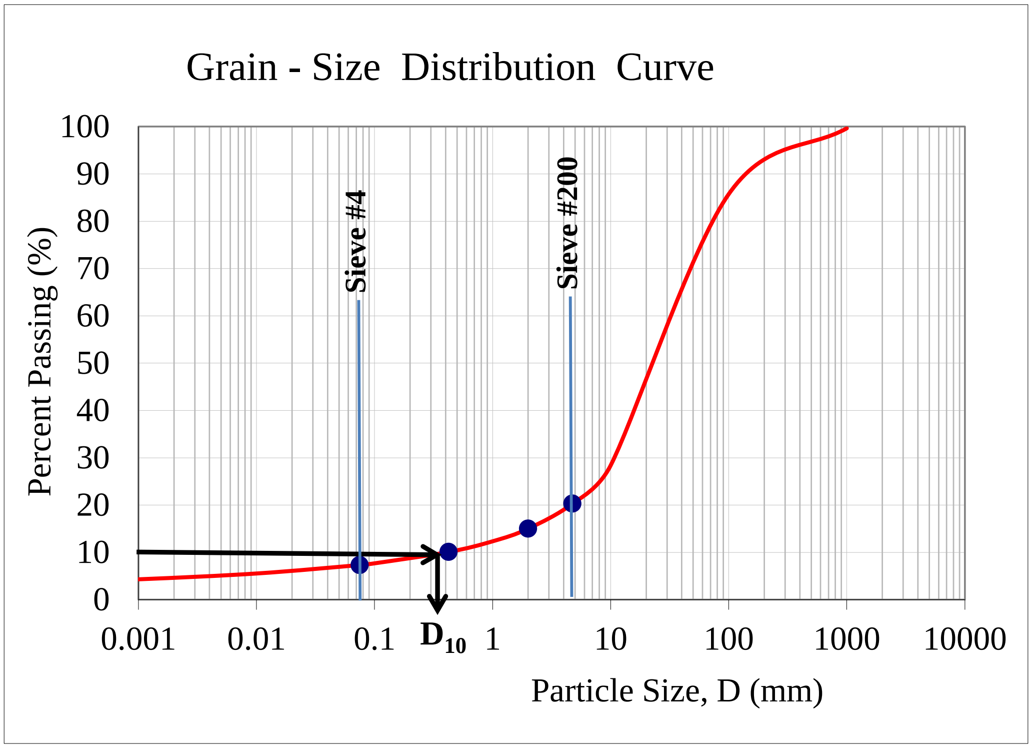



Figure 5-34. Calculating $D_{10}$ from the GSD curve

Step 9. Calculate the SWCC parameter $b_f$ according to equation 5-6. The parameter $b_f$ is a function of the Group Index, *GI*

Step 10. Calculate the SWCC parameter $c_f$ according to equation 5-7. The parameter $c_f$ is a function of the SWCC parameter $a_f$, calculated in step 8

Step 11. Calculate the SWCC parameter $h_r$ according to equation 5-8. The parameter $h_r$ is a function of Group Index, *GI*

Step 12. Calculate the degree of saturation, *S(%)*, according to equation 5-1, which correspond to the SWCC function defined by Fredlund & Xing, 1994

Step 13. Plot the soil–water characteristic curve

Step 14. If the particle size $D_{10}$, calculated in step 6, is less than 0.020, use $a_f$ = 1.28. Otherwise, go to step 15

Step 15. Calculate the SWCC parameter $a_f$ according to equation 5-9. In this equation the parameter $a_f$ is a function of particle size $D_{10}$

Step 16. Calculate the SWCC parameter $b_f$ according to equation 5-10. The parameter $b_f$ is a function of SWCC parameter $a_f$ calculated in steps 17/18

Step 17. Calculate the SWCC parameter $c_f$ according to equation 5-11. The parameter $c_f$ is a function of SWCC parameter $a_f$ obtained in steps 17/18

Step 18. Use $h_r$ = 100

Step 19. Calculate the degree of saturation, *S%*, using the Fredlund & Xing equation 5-11 and the fitting parameters estimated in steps 15 to 18

Step 20. Plot the soil–water characteristic curve

The procedure outlined before is summarized in Figure 5-35.

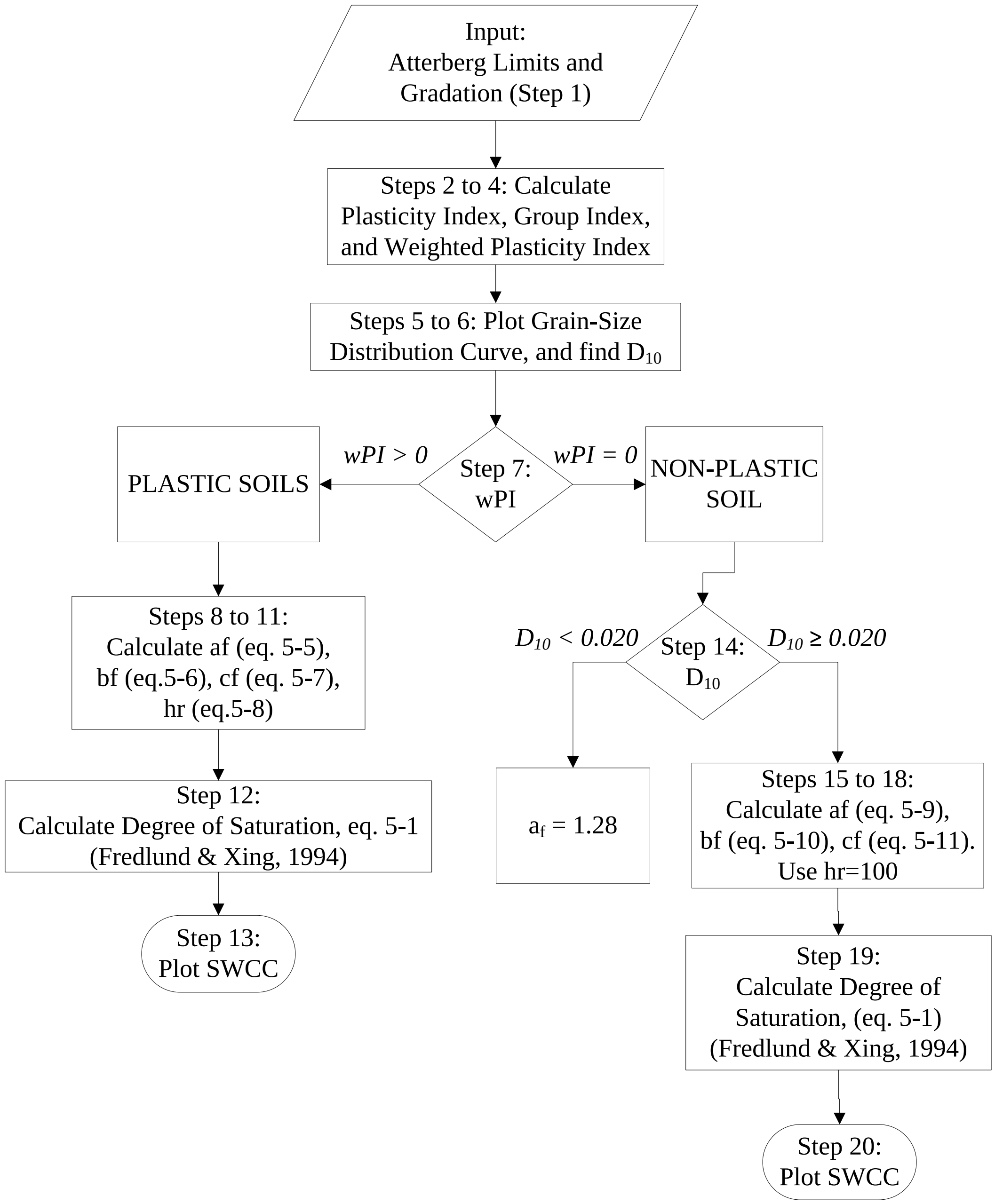



Figure 5-35. Approach to Estimate the SWCC Based on Statistical Correlation of Fredlund & Xing Parameters with Soil Index Properties

## 5.9 Summary

Tables 5-8 and 5-13 present the proposed models for the SWCC parameters for the Fredlund and Xing equation, for plastic and non-plastic soils, respectively. The models proposed for plastic soils were estimated in function of the Group Index, which is in turn a function of passing sieve #200, liquid limit and plasticity index. On the other hand, the models proposed for non-plastic soils were estimated as function of the particle diameter $D_{10}$.

The procedure followed to develop this Chapter 5 had the following order:

- The database was classified according to the wPI property.
- The measured SWCC parameters were correlated with all the soil properties affecting the SWCC.
- Each SWCC parameter was subjected to a statistical non–linear regression analysis against all possible combination of parameters, followed of a statistical error analysis.
- Based on statistical, geotechnical and applicability considerations, the best model was chosen for each SWCC parameter.
- Each Measured fitting parameter was compared to the Predicted value.

- The predicted degree of saturation was obtained by fitting the predicted SWCC parameter to the Fredlund & Xing function.

- Final model for plastic and non-plastic soil were proposed.

The models proposed in Chapter 5 to estimate the SWCC parameters, present the following advantages:

- The models proposed can be implemented in the Enhanced Integrated Climatic Model (EICM), which is incorporated in the Mechanistic–Empirical Pavement Design Guide (MEPDG). The format required allows including the SWCC fitting parameters.

- The database is vast. With more than 31,000 data points for plastic soils and 4,500 data points for non-plastic soils, it contains the most important Soil Index Properties obtained directly from laboratory testing or in the field. This database can be considered the largest in the world containing unsaturated soil properties.

- The models proposed are very simple to be implemented. For plastic soils, the Atterberg's Limits and the Passing US sieve #200 are needed as input parameters; while for non-plastic materials, only the particle diameter $D_{10}$ is needed. These are parameters commonly used by practicing engineers and therefore, this model becomes an excellent candidate for practical applications.

The approach and the models proposed in this chapter have the following limitations:

- The models proposed for the SWCC fitting parameters were estimated independently of each other. That makes it difficult to control the shape of the Soil–Water Characteristic Curve.

- Due to the tremendous amount of data points in the database, a moving average estimate of the parameters during the statistical analysis was necessary. The fine-grained material database was sorted according to the wPI and the data were averaged in groups of 300 consecutive data points.  Groups of 50 consecutive data points were used for non-plastic soils, based on the $D_{10}$ parameter. While working with the moving average allows finding a clear tendency of the data and hence, better correlations; the variability gets somehow masked within each range. A deeper study of variability should be performed.

When comparing the model for plastic soils found by Zapata, 1999 (Figures 4-25 and 4-26) with the model proposed in this work (Figures 5-30 and 5-32), it can be seen that the $R^2$ improved from 0.70 to 0.81; while the $R^2$ improved from 0.40 to 0.89 for non-plastic materials. Even though the $R^2$ improved marginally for plastic soils, it can be observed that the Zapata's model is biased towards overprediction for most of the dataset. For non–plastic soils the results obtained with the Zapata's model are underpredicting most of the data points.

On the other hand, when comparing the model proposed in this work (Figure 5-30) with the plastic model used in the MEPDG model, it can be seen that the $R^2$ improved greatly from 0.49 to 0.81. For non-plastic or granular soils, the $R^2$ for both models are somewhat similar. Even though the non–plastic model from the MEPDG model is almost similar than the model proposed in this work, the later model is much simpler and easier to implement because it only depends on one gradation parameter, the particle size at 10% Passing or $D_{10}$.

It can be concluded that the new models proposed in this Chapter 5 will enhanced the prediction of the SWCC and therefore, it is recommended to consider applying them in practical applications.

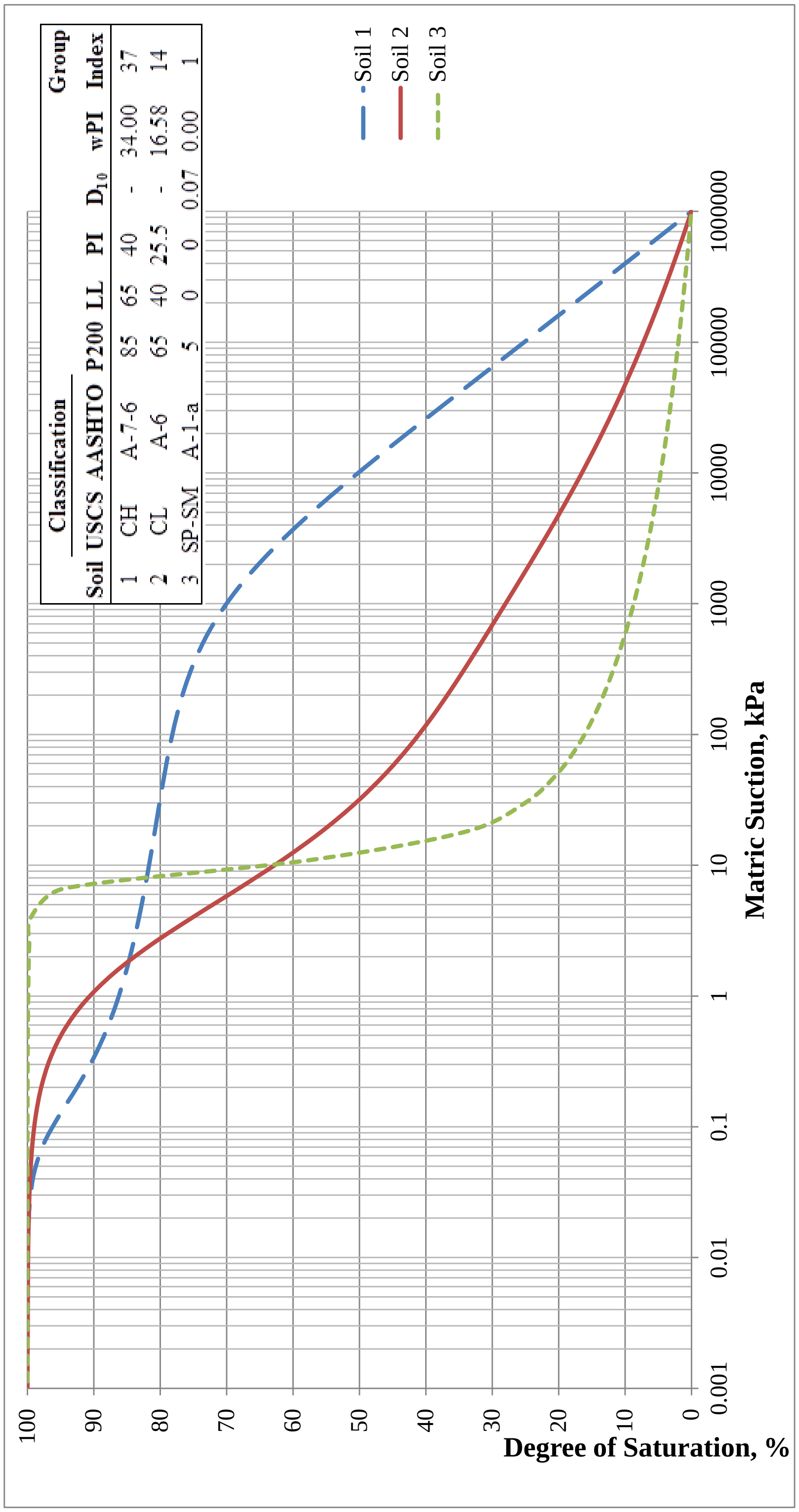
As an example of the predicted SWCC based on the models proposed in Chapter 5, the soil index properties shown in Table 5-17 were used to calculate the SWCC fitting parameters. The three soil–water characteristic curves obtained are shown in Figure 5-36.

Table 5-17. Soil Index Properties for Three Soils Taken from the Database

| | Classification | | | | | | | Group | Predicted SWCC Parameters | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Soil | USCS | AASHTO | P200 | LL | PI | $D_{10}$ | wPI | Index | af | bf | cf | hr |
| 1 | CH | A-7-6 | 85 | 65 | 40 | - | 34.00 | 37 | 0.0709 | 2.1570 | 0.0849 | 1,102.1 |
| 2 | CL | A-6 | 65 | 40 | 25.5 | - | 16.58 | 14 | 1.8436 | 1.0489 | 0.6047 | 691.5 |
| 3 | SP-SM | A-1-a | 5 | 0 | 0 | 0.07 | 0.00 | 1 | 8.0760 | 7.1855 | 0.6041 | 100.0 |

In Figure 5-36, it can be observed that the SWCC for fine-grained material loses its sigmoidal shape. This this result might represent a “dual porosity” for soils that are highly plastic (Zhang & Chen, 2005). The first air–entry value (i.e.0.05 kPa) might be associated with a macro–porosity while the second air–entry value (i.e. about 1,000 kPa) might be associated with a micro–porosity of the soil.  This is reasonable given the fact that the measurements of suctions were obtained from natural clods and not from slurries.

Another important aspect that should be considered is the initial fitting parameters when making analysis regressions. In this work was used 10 kPa as the initial $a_f$ parameter for plastic and non–plastic materials. This initial value for the parameter $a_f$ is low and creates a change in the sigmoidal shape for the SWCC when the curve is forced also to approximate the water content at 33kPa and 1500 kPa. Figure 5-37 illustrate the effect on the shape of the SWCC when the fitting is changed by a low value of the $a_f$ parameter.




| Soil | Classification USCS | Classification AASHTO | P200 | LL | PI | $D_{10}$ | wPI | Group Index |
|---|---|---|---|---|---|---|---|---|
| 1 | CH | A-7-6 | 85 | 65 | 40 | - | 34.00 | 37 |
| 2 | CL | A-6 | 65 | 40 | 25.5 | - | 16.58 | 14 |
| 3 | SP-SM | A-1-a | 5 | 0 | 0 | 0.07 | 0.00 | 1 |

Figure 5-36. Examples of SWCCs Using the Model Proposed

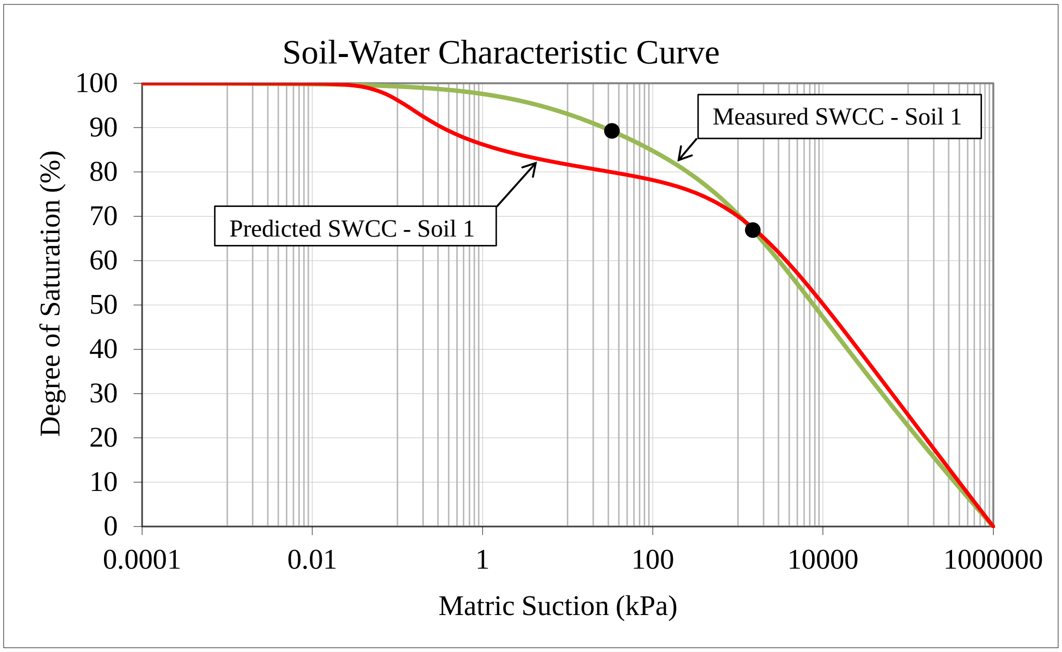



Figure 5-37 Effect of a Low Initial $a_f$ Parameter Value in the SWCC

# CHAPTER 6

# SWCC MODEL PREDICTED FROM GRAIN–SIZE DISTRIBUTION

## 6.1 Introduction

In Chapter 5, it was proven that the Soil–Water Characteristic Curve (SWCC) can be predicted by independently predicting of fitting parameters of the SWCC function. Even though the prediction using this procedure yielded acceptable results, there is a concern about discontinuities in the function due primarily to the fact that most of the fitting parameters are independent from each other, particularly the ones defined for fine-grained materials. In order to avoid the uncertainty associated with the statistical process, a new model for the Soil–Water Characteristic Curve based on the entire Grain–Size Distribution (GSD) function is presented in this chapter. This new model is founded on two equations, the SWCC function given by Fredlund & Xing (1994) and the GSD function that can be also represented by a model as that given by Fredlund et al (2002). The Soil–Water Characteristic Curve is primary associated with two concepts: The Pore–Size Distribution and the Capillary Pressure. The soil is considered as a group of spherical pores interconnected and therefore, the pore–size distribution is directly related to the grain–size distribution. The Capillary Pressure is the tension into the soil–water that allows the water to flow upward from the static groundwater table. This pressure is commonly called Matric Suction $(u_a - u_w)$, where $u_a$ is the pore–air pressure and $u_w$ is the pore–water pressure. In this work, the retention characteristic of the soil is assumed to be primarily related to matric suction, and therefore, the osmotic suction is considered to have a negligible effect.

The objective of this chapter is to present a method to correlate the Soil–Water Characteristic Curve with the Grain–Size Distribution. In this process, some assumptions are considered:

- The solids of the soil have spherical shape (soil–texture is not considered).
- Stress history is not considered (test were performed at zero overburden pressure).
- The hysteresis effect is not considered.
- It is assumed that each suction value is associated with one pore–size or grain–size.

The last assumption is perhaps the most important. The Soil–Water Characteristic Curve equation proposed by Fredlund & Xing, 1994, is directly linked to the pore–size distribution; the function is a sigmoid, which is the same shape followed by the Grain–Size Distribution. The GSD model proposed by Fredlund et al., 1997 is based on models developed by Wagner & Ding, 1994, which are based in a modified lognormal distribution. When the distribution is plotted, in cumulative way, it presents also a sigmoidal shape. This principle allows relating both equations.

The soil–suction and its corresponding water content should be fit properly to the SWCC equation. In order to fit the suction–moisture pair of values to obtain the SWCC, the least squared error were minimized. The objective function to minimize, in terms of degree of saturation is as follows:

$$ObjectiveFunction = \Sigma\left[S_{meas} - S_{pred}\right]^2 \qquad (6\text{-}1)$$

Where:

$S_{meas}$ = Measured Degree of Saturation

$S_{pred}$ = Predicted Degree of Saturation

For the case of the GSD equation, the best function is obtained in the same way. That is the least squared error minimized by comparing the percent passing measured versus the predicted values:

$$ObjectiveFunction = \Sigma\left[\%Pass_{meas} - \%Pass_{pred}\right]^2 \qquad (6\text{-}2)$$

Where:

$\%Pass_{meas}$ = Measured percent of passing

$\%Pass_{pred}$ = Predicted percent of passing

Since both functions have a range between 0% and 100%, that allows a one–to–one comparison by normalization

The following is an overview of the procedure follow to estimate the SWCC function based on the grain–size distribution:

1. The fitting parameters for the SWCC function were obtained by minimizing the squared errors between measures versus predicted degree of saturation values. The Fredlund & Xing, 1994 equation was used in this process.

2. The fitting parameters for the grain–size distribution function were obtained by minimizing the squared errors between measured versus predicted % Passing values.

3. For values of % Passing that corresponded to the same value of degree of saturation, the corresponding particle size and suction values were obtained and compared.

4. Correlations between the particle size and suctions values were developed based on other simple index properties such as the wPI factor.

The Weighted Plasticity Index usually abbreviated wPI is:

$$wPI = \frac{P_{200} \times PI}{100} \quad \text{.........................................................................(6-3)}$$

Where:

$P_{200}$ = Material Passing # 200 US Standard Sieve, expressed in %

*PI* = Plasticity Index, expressed in %

This process was followed for all the 33,210 soils available in database. The soil properties collected in database are presented in Table 6-1 for Meegernot soil.

Table 6-1 Soil Properties for the Meegernot Soil

| Soil Property | Unit | Item |
|---|---|---|
| Component Name | - | Meegernot |
| AASHTO Classification | - | A-1-a |
| AASHTO Group Index | - | 0 |
| Unified Soil Classification System | - | GP |
| Top Depth of Layer | cm | 147 |
| Bottom Depth of Layer | cm | 168 |
| Thickness of the Layer | cm | 21 |
| Passing # 4 | % | 20 |
| Passing # 10 | % | 15 |
| Passing # 40 | % | 12.5 |
| Passing # 200 | % | 5 |
| Passing Sieve 0.002 mm | % | 7.5 |
| Liquid Limit | % | 22.5 |
| Plasticity Index | % | 2.5 |
| Weighted Plasticity Index | % | 0.125 |
| Specific Gravity | g/cm3 | n/a |
| Saturated Hydraulic Conductivity (Ksat) | mm/s | 91.7432 |
| Volumetric Content of Soil Water Retained at a Tension of 1/10 bar | % | 5.1 |
| Volumetric Content of Soil Water Retained at a Tension of 1/3 bar | % | 3.7 |
| Volumetric Content of Soil Water Retained at a Tension of 15 bars | % | 1.3 |
| Elevation | m | 2469 |

## 6.2 Calculating Suctions from the Soil–Water Characteristic Curve

This step consisted in finding, for every soil in the database, the suctions values corresponding to several degrees of saturation (5%, 10%, 15%, and so on 100%). In order to achieve this, the measured data was fitted to the Fredlund and

Xing equation, and the SWCC parameters $a_f$, $b_f$, $c_f$, and hr were obtained. That allowed for the development of the entire SWCC function

The process to calculate the suction values required using either the Solver or Goal Seek utilities available in Excel®. These utilities were necessary because the suction is the independent variable into the Fredlund & Xing's equation (6-4), and in order to mathematically solve for matric suction as a function of the degree of saturation is really quite complicated:

$$S(\%) = \frac{\theta_w}{\theta_s} = \left[1 - \frac{\ln\left(1 + \frac{\psi}{h_r}\right)}{\ln\left(1 + \frac{1{,}000{,}000}{h_r}\right)}\right]\left(\frac{1}{\left\{\ln\left[e + \left(\frac{\psi}{a_f}\right)^{b_f}\right]\right\}^{c_f}}\right) \qquad \text{.............................(6-4)}$$

Where:

*S(%)* = Degree of Saturation, in percentage

$\psi$ = Matric Suction, in kPa

$a_f$ , $b_f$ , $c_f$ , $h_r$= SWCC Fitting Parameters, $a_f$ and $h_r$ in kPa

$\theta_w$ = Volumetric Water Content

$\theta_s$ = Saturated Volumetric Water Content

Equation (6-4) represents a sigmoidal model as shown in Figure 6-1:

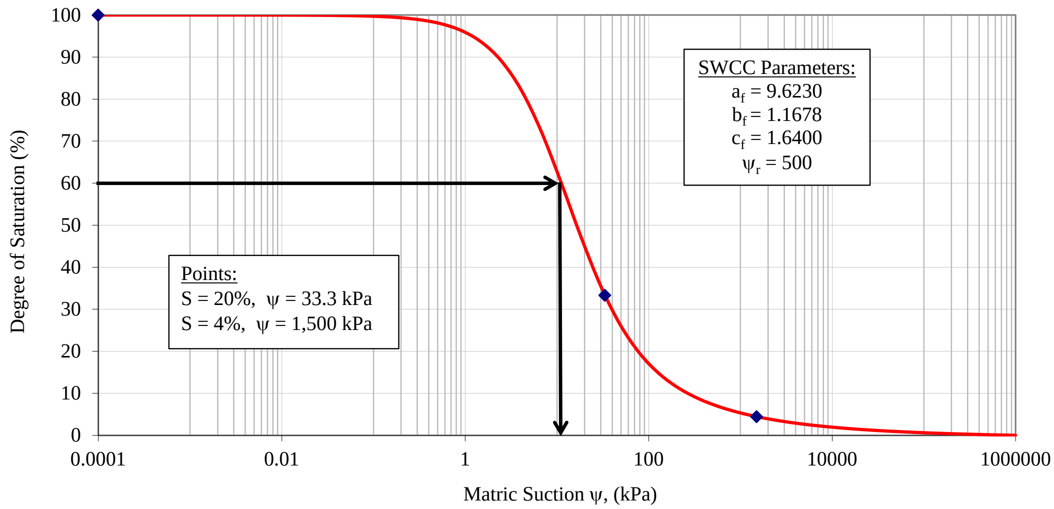
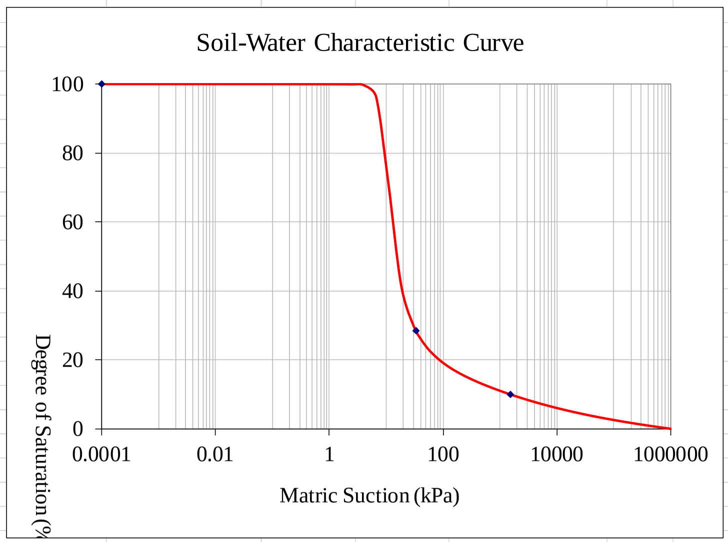



Figure 6-1. Soil–Water Characteristic Curve

The procedure followed to find the SWCC parameters was explained in detail in Chapter 3 under section 3.3.1. The spreadsheet shown in Figure 6-2 was used to estimate first the SWCC parameters and then to calculate the data shown in Table 6-2 by using Solver or Goal Seek Functions in Excel®.

| Suction | | Vol. w/c | Sat vol w/c | Degree Saturation |
|---|---|---|---|---|
| (Bar) | (kPa) | (%) | (%) | (%) |
| 0.1 | 10 | | 13 | |
| 0.33 | 33.3 | 3.7 | | 28.5 |
| 15 | 1500.0 | 1.3 | | 10.0 |

| SWCC Parameters | | initial SWCC | | |
|---|---|---|---|---|
| $a_f$ | 9.0869 | 10 | | Objective Function |
| $b_f$ | 5.6921 | 1 | | 2.67671E-11 |
| $c_f$ | 0.6236 | 2 | | |
| hr | 500.0 | 500 | | |
| | | | | |
| ψ | θ | $S_{measured}$ | $S_{predicted}$ | Constraints |
| Suction (kPa) | Vol. Water Content | | | |
| 0.0001 | 13.0 | 100.0 | 100.0 | 0.000 |
| 33.3333 | 3.7 | 28.5 | 28.5 | 0.000 |
| 1,500.0000 | 1.3 | 10.0 | 10.0 | 0.000 |




| Graph | |
|---|---|
| Suction, kPa | Saturation, % |
| 0.0001 | 100.00 |
| 0.001 | 100.00 |
| 0.01 | 100.00 |
| 0.016 | 100.00 |
| 0.027 | 100.00 |
| 0.045 | 100.00 |
| 0.074 | 100.00 |
| 0.122 | 100.00 |
| 0.201 | 99.99 |
| 0.331 | 99.99 |
| 0.546 | 99.99 |
| 0.900 | 99.98 |
| 1.484 | 99.96 |
| 2.447 | 99.92 |
| 4.034 | 99.67 |
| 6.651 | 96.24 |
| 10.966 | 70.88 |
| 18.080 | 42.13 |
| 29.810 | 30.12 |
| 49.148 | 24.09 |
| 81.031 | 20.34 |
| 133.5973 | 17.68 |
| 220.2647 | 15.62 |
| 363.155 | 13.91 |
| 598.7414 | 12.41 |
| 987.1577 | 11.05 |
| 1627.548 | 9.80 |
| 2683.373 | 8.65 |
| 4424.134 | 7.58 |
| 7294.164 | 6.60 |
| 12026.04 | 5.69 |
| 19827.59 | 4.86 |
| 32690.17 | 4.08 |
| 53896.98 | 3.36 |
| 88861.1 | 2.69 |
| 146507.2 | 2.07 |
| 241549.5 | 1.48 |
| 398247.8 | 0.93 |
| 656599.7 | 0.41 |
| 1000000 | 0.00 |

| Graph | |
|---|---|
| Suction, kPa | Saturation, % |
| 0.00 | 100.00 |
| 7.04 | 95.00 |
| 8.14 | 90.00 |
| 8.96 | 85.00 |
| 9.68 | 80.00 |
| 10.38 | 75.00 |
| 11.09 | 70.00 |
| 11.86 | 65.00 |
| 12.74 | 60.00 |
| 13.77 | 55.00 |
| 15.07 | 50.00 |
| 16.79 | 45.00 |
| 19.26 | 40.00 |
| 23.14 | 35.00 |
| 30.04 | 30.00 |
| 44.70 | 25.00 |
| 85.66 | 20.00 |
| 261.55 | 15.00 |
| 1,500.08 | 10.00 |
| 18,158.64 | 5.00 |

Figure 6-2 Spreadsheet for Calculating the SWCC Parameters

Table 6-2 Calculating Suction Values from the Degree of Saturation

| Degree of Saturation, % | Suction, kPa |
|---|---|
| 100 | 0.00 |
| 95 | 7.05 |
| 90 | 8.15 |
| 85 | 8.97 |
| 80 | 9.69 |
| 75 | 10.39 |
| 70 | 11.10 |
| 65 | 11.87 |
| 60 | 12.74 |
| 55 | 13.77 |
| 50 | 15.07 |
| 45 | 16.79 |
| 40 | 19.26 |
| 35 | 23.14 |
| 30 | 30.04 |
| 25 | 44.70 |
| 20 | 85.66 |
| 15 | 261.55 |
| 10 | 1,500.08 |
| 5 | 18,158.64 |

## 6.3 Calculating Particle Diameter from the Grain–Size Distribution

In order to calculate the particle diameter at each percent passing from the grain–size distribution, a model presented by Fredlund et al., 2002. In their article: "Use of the Grain–Size Distribution for Estimation of the Soil–Water Characteristic Curve". This model is shown in equation 6-5:

$$P_p(d) = \frac{1}{\ln\left[\exp(1) + \left(\frac{g_a}{D}\right)^{g_n}\right]^{g_m}} \left[1 - \left[\frac{\ln\left(1 + \frac{D_r}{D}\right)}{\ln\left(1 + \frac{D_r}{D_m}\right)}\right]^7\right] \quad \text{..........................(6-5)}$$

Where:

$P_p(D)$ = percent passing a particular grain–size $D$

$g_a$ = fitting parameter related to the initial break point in the grain–size curve

$g_n$ = fitting parameter related to the steepest slope of grain–size curve

$g_m$ = fitting parameter related to the curvature (fine section) of grain–size curve

$D$ = particle diameter (mm)

$D_r$ = residual particle diameter (mm), related to the fines

$D_m$ = minimum allowable particle diameter (mm)

The last part of the equation corresponds to a correction factor which adjusts the extremes of the model properly.

In order to define the GSD function for each soil of the database, it was necessary to estimate the fitting parameters. This process was explained in detail in Chapter 3 section 3.3.2.

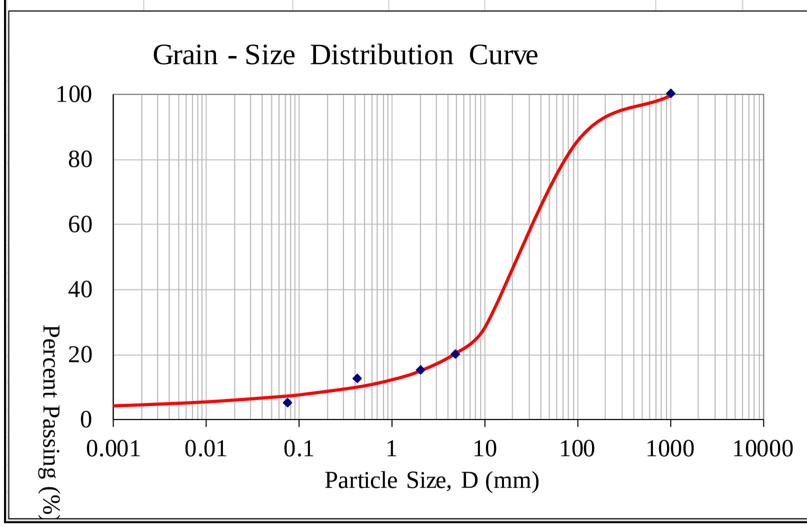
| Particle Size | | Passing |
|---|---|---|
| # | (mm) | (%) |
| 4 | 4.750 | 20.0 |
| 10 | 2.000 | 15.0 |
| 40 | 0.425 | 12.5 |
| 200 | 0.075 | 5.0 |

| | GSD parameters | initial |
|---|---|---|
| ga | 62.8595 | 1 |
| gn | 1.7000 | 0.5 |
| gm | 1.0716 | 0.5 |

Objective Function

11.46316221

| $D_{measured}$ Diameter (mm) | Sieve # | $P_{meas}$ % Passing | $P_{pred}$ % Passing | Constraints |
|---|---|---|---|---|
| 1,000 | | 100.0 | 99.6 | 0.000 |
| 4.750 | 4 | 20.0 | 20.3 | 0.000 |
| 2.000 | 10 | 15.0 | 15.0 | 0.000 |
| 0.425 | 40 | 12.5 | 10.1 | 0.000 |
| 0.075 | 200 | 5.0 | 7.3 | 0.000 |




| Graph | |
|---|---|
| Particle Size (mm) | Passing (%) |
| 0.0001 | 3.49 |
| 0.001 | 4.32 |
| 0.01 | 5.54 |
| **0.075** | **7.34** |
| 0.1 | 7.69 |
| **0.425** | **10.10** |
| 1 | 12.35 |
| **2** | **15.01** |
| **4.75** | **20.32** |
| 10 | 28.39 |
| 100 | 85.73 |
| 1000 | 99.64 |

| Table | |
|---|---|
| Particle Size (mm) | Passing (%) |
| 36,923.075 | 100.00 |
| 202.717 | 95.00 |
| 128.664 | 90.00 |
| 96.358 | 85.00 |
| 77.011 | 80.00 |
| 63.597 | 75.00 |
| 53.473 | 70.00 |
| 45.377 | 65.00 |
| 38.626 | 60.00 |
| 32.815 | 55.00 |
| 27.682 | 50.00 |
| 23.051 | 45.00 |
| 18.799 | 40.00 |
| 14.844 | 35.00 |
| 11.139 | 30.00 |
| 7.685 | 25.00 |
| 4.562 | 20.00 |
| 1.995 | 15.00 |
| 0.405 | 10.00 |
| 0.004 | 5.00 |

Figure 6-3 Spreadsheet Used to Find the GSD Fitting Parameters

The spreadsheet shown in Figure 6-3 was used to estimate the GSD parameters $g_a$, $g_n$,$g_m$ and consequently to calculate the values shown in Table 6-3 by using the Goal Seek function in Microsoft Excel®. Two parameters needed to be assumed in the process: the residual particle diameter (*Dr*) and the minimum allowable particle diameter (*Dm*). The values assumed were 0.001 mm and 0.00001 mm, respectively.

Table 6-3 Calculated Particle Diameter from the Percent Passing Values

| **Passing (%)** | **Particle Size, D (mm)** |
|---|---|
| 100 | 36,923.136 |
| 95 | 202.718 |
| 90 | 128.664 |
| 85 | 96.358 |
| 80 | 77.011 |
| 75 | 63.597 |
| 70 | 53.473 |
| 65 | 45.376 |
| 60 | 38.626 |
| 55 | 32.816 |
| 50 | 27.682 |
| 45 | 23.051 |
| 40 | 18.799 |
| 35 | 14.843 |
| 30 | 11.139 |
| 25 | 7.685 |
| 20 | 4.562 |
| 15 | 1.995 |
| 10 | 0.405 |
| 5 | 0.004 |

Once the suction (kPa) was obtained from the degree of saturation (%) based on the SWCC, and the particle size (mm) obtained from the percentage passing (%) base on the GSD; the following step consisted in relating the suction values with the particle diameter values. Figure 6-4 shows the relationship between these values for one specific soil. The values are summarized in Table 6-4.

It is important to note that the relationship between particle diameter and suction values was possible because the SWCC was obtained in terms of degree of saturation. In this case, the degree of saturation, which ranges between 0% and 100% can be normalized or scale to the same range of variation of the % passing in the grain–size distribution curve. The same process used to obtain the relationship shown in Figure 6-4 for one soil was used to obtain the same relationship for every soil available in the database. This analysis was possible due to the creation of a program Macro in Excel®. Note that there is a clear connection between the particle diameter and the suction value due to the relationship between the grain–size distribution and the pore–size distribution (PSD) of the soil.

Table 6-4 Relationship between Suction Values versus Particle Size

| **Particle Size, D (mm)** | **Suction, $\psi$ (kPa)** |
|---|---|
| 36,923.14 | 0.00 |
| 202.72 | 7.05 |
| 128.66 | 8.15 |
| 96.36 | 8.97 |
| 77.01 | 9.69 |
| 63.60 | 10.39 |
| 53.47 | 11.10 |
| 45.38 | 11.87 |
| 38.63 | 12.74 |
| 32.82 | 13.77 |
| 27.68 | 15.07 |
| 23.05 | 16.79 |
| 18.80 | 19.26 |
| 14.84 | 23.14 |
| 11.14 | 30.04 |
| 7.68 | 44.70 |
| 4.56 | 85.66 |
| 1.99 | 261.55 |
| 0.41 | 1,500.08 |
| 0.00 | 18,158.64 |

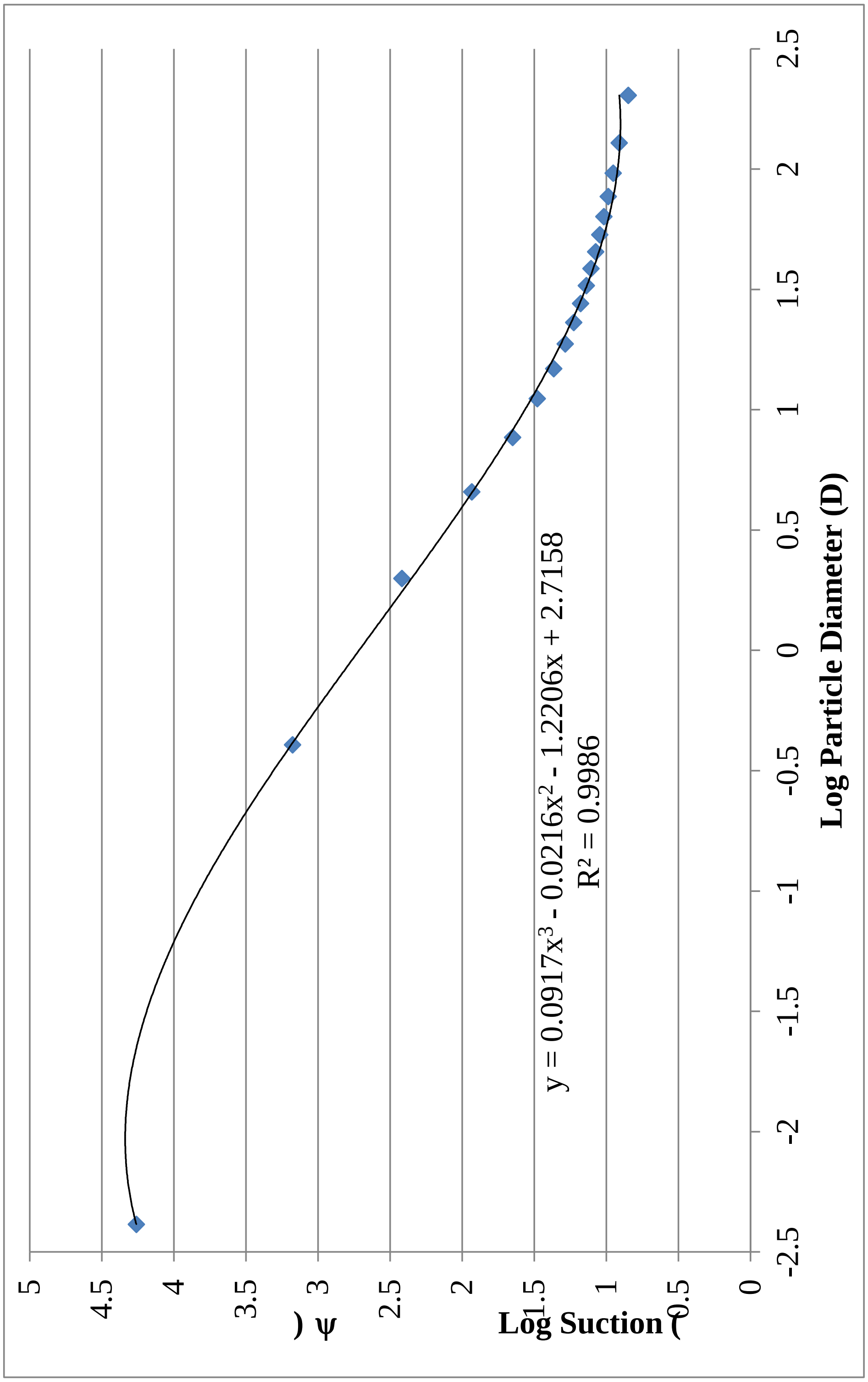



Figure 6-4 Log Suction versus Log Particle Size for One Soil

## 6.4 Ranges of wPI and Statistical Information

The database used in developing this model consisted of more than 660,000 soils as shown in Table 6-5. Given the large amount of data available it was deemed necessary to subdivide it and grouped the soils based on a soil property representative of the moisture retention characteristic of the soil. The weighted plasticity index (wPI) was proven to be significantly related to the SWCC (Zapata, 1999) and therefore it was selected as the basis for the grouping process. Furthermore, several authors have attempted to relate the SWCC with the GSD (Arya and Paris, 1981; Fredlund et al., 1997) without success for all ranges of soil encountered in the field. This might be due to the variability associated with the porous materials, and therefore, a process that group soils with relatively the same characteristics would eliminate some of this variability.

Table 6-5 presents the data divided by wPI ranges along with the number of soil in each range and statistic. Note that at higher values of wPI, the variability inferred from higher values of standard deviation and variance, is high.

Table 6-5 Database Divided by wPI Ranges and Statistics Associated with Each Range of Values

| items in file | Range | Count | Average | Median | Mode | St dev | Var |
|---|---|---|---|---|---|---|---|
| 86,100 | wPI = 0 | 4305 | 0.00 | 0.00 | 0.000 | 0.000 | 0.000 |
| 79,260 | 0 < wPI ≤ 1 | 3,963 | 0.67 | 0.69 | 0.750 | 0.221 | 0.049 |
| 85,680 | 1 < wPI ≤ 2 | 4,284 | 1.48 | 1.49 | 1.063 | 0.297 | 0.088 |
| 111,160 | 2 < wPI ≤ 4 | 5,558 | 2.95 | 3.00 | 3.000 | 0.575 | 0.331 |
| 126,480 | 4 < wPI ≤ 8 | 6,324 | 5.73 | 5.63 | 4.500 | 1.146 | 1.313 |
| 64,640 | 8 < wPI ≤ 12 | 3,232 | 9.88 | 9.75 | 10.500 | 1.170 | 1.370 |
| 38,520 | 12 < wPI ≤ 16 | 1,926 | 13.98 | 13.95 | 13.500 | 1.132 | 1.282 |
| 27,000 | 16 < wPI ≤ 20 | 1,350 | 17.97 | 18.00 | 18.000 | 1.131 | 1.279 |
| 34,320 | 20 < wPI ≤ 30 | 1,716 | 24.18 | 24.00 | 2.744 | 7.530 | 22.500 |
| 9,160 | 30 < wPI ≤ 40 | 458 | 33.97 | 33.75 | 2.631 | 6.921 | 34.000 |
| 1,880 | 40 < wPI ≤ 60 | 94 | 45.54 | 43.73 | 4.933 | 24.336 | 42.750 |
| 664,200 | | 33,210 | | | | | |

## 6.5 Calculating the Models for Each Range

Several plots like the one shown in Figure 6-4 were processed to get an idea of the best models to fit the particle diameter versus suction data. These plots showed that a third order polynomial model had the highest correlation as implied from the high $R^2$ values obtained (0.96 and higher). This process was performed by using Excel® and Minitab® 15 software packages. Having defined the type of model to use for the non–linear regression analysis and the ranges based on wPI values, the following step was to find the constants that yielded the highest $R^2$ values. Statistica® 5.5 was used for the determination of the best model. Figure 6-5 is an example of a spreadsheet used in the estimation for only one range of wPI values. The same process was repeated for all the selected ranges.

The general form of the model used to relate particle diameter and suction values is as follows:

$$\log\psi = k_1 + k_2 \log D + k_3 \log D^2 + k_4 \log D^3 \text{ .......................................... 6-6}$$

Where:

$\psi$ = Suction in kPa

$D$ = Particle Diameter in mm

$k_1$, $k_2$, $k_3$, $k_4$ = Regression Constants

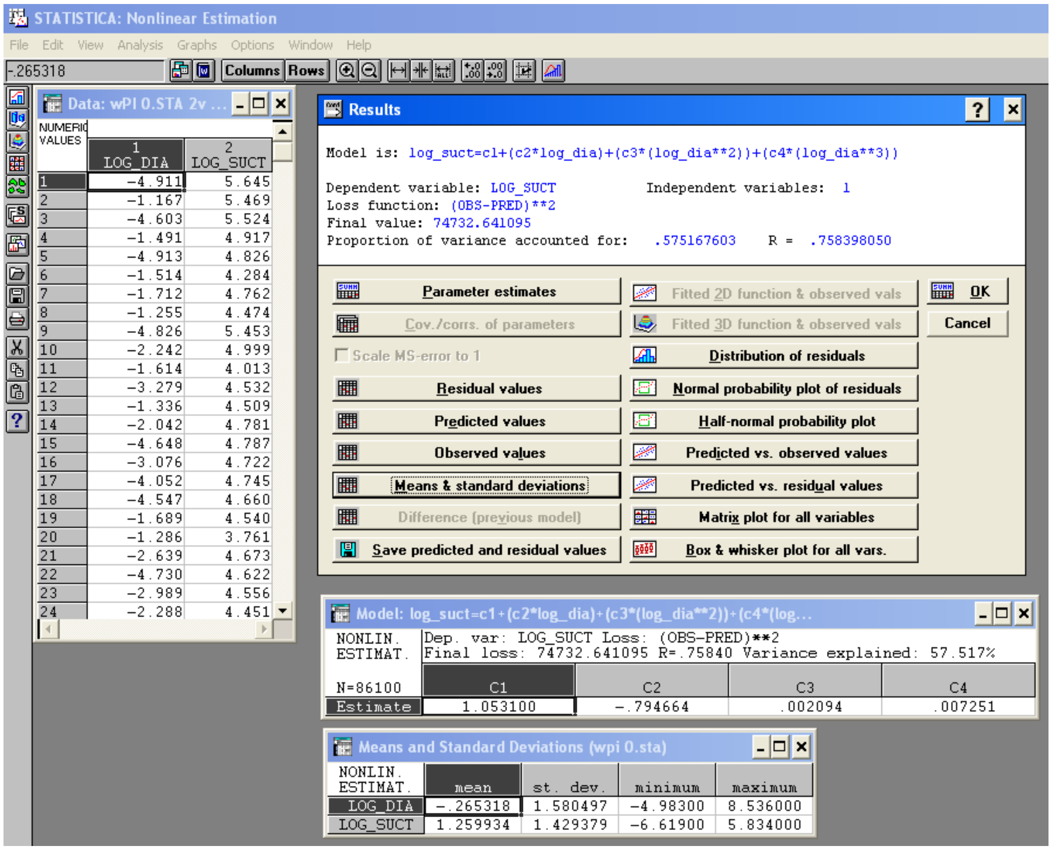



Figure 6-5 Spreadsheet from Statistica® Used in Estimating the Best Models between Particle Size and Suction Values

A summary of the regression constants found is presented in Table 6-6 for each range of wPI values selected.

Table 6-6 Summary of Fitting Parameters Found for the Correlation between Log of Particle Size Versus log of Suction for each Range of wPI Values

| **wPI Ranges** | | | | | | | |
|---|---|---|---|---|---|---|---|
| **From** | **To** | **n** | $k_{1\ avg}$ | $k_{2\ avg}$ | $k_{3\ avg}$ | $k_{4\ avg}$ | $R^2$ |
| 0 | 0 | 4305 | 1.05310 | -0.79466 | 0.00209 | 0.00725 | 0.5752 |
| 0 | 1 | 3963 | 1.29284 | -0.84537 | -0.01175 | 0.00658 | 0.6904 |
| 1 | 2 | 4284 | 1.08684 | -0.72871 | -0.01610 | 0.00461 | 0.6206 |
| 2 | 4 | 5558 | 1.28120 | -0.64835 | -0.01631 | 0.00112 | 0.6429 |
| 4 | 8 | 6324 | 1.09260 | -0.77777 | -0.01294 | 0.00620 | 0.6131 |
| 8 | 12 | 3232 | 1.20260 | -0.84218 | -0.02023 | 0.00813 | 0.5882 |
| 12 | 16 | 1926 | 1.18353 | -0.86519 | -0.01715 | 0.00848 | 0.6064 |
| 16 | 20 | 1350 | 0.97223 | -0.96512 | -0.01304 | 0.01141 | 0.5934 |
| 20 | 30 | 1716 | 0.60121 | -1.06883 | -0.01141 | 0.01260 | 0.5637 |
| 30 | 40 | 458 | 0.52383 | -1.16329 | -0.03659 | 0.01030 | 0.5532 |
| 40 | 60 | 94 | 0.88131 | -1.37328 | -0.08086 | 0.01249 | 0.5029 |

## 6.6 Plotting the Models

The relationship between suction and particle diameter was plotted for each range of wPI values as shown in Figure 6-6. At higher ranges of suction (small particle sizes) the curves essentially merged. However, as the suction decreases (large particle sizes) a dependency of the relationship on wPI can be noted. This is particularly true for particle size greater than 1 mm. In order to assess the dependency of this relationship on wPI, the $k_{i\ avg}$ constants obtained at a wPI corresponding to the mid-point of each range were plotted against wPI values. These relationships can be seen in Figure 6-7. It can be seen that particularly constants $k_1$ and $k_2$ present high correlation with wPI. Constants $k_3$ and $k_4$ are also correlated to lower degree.

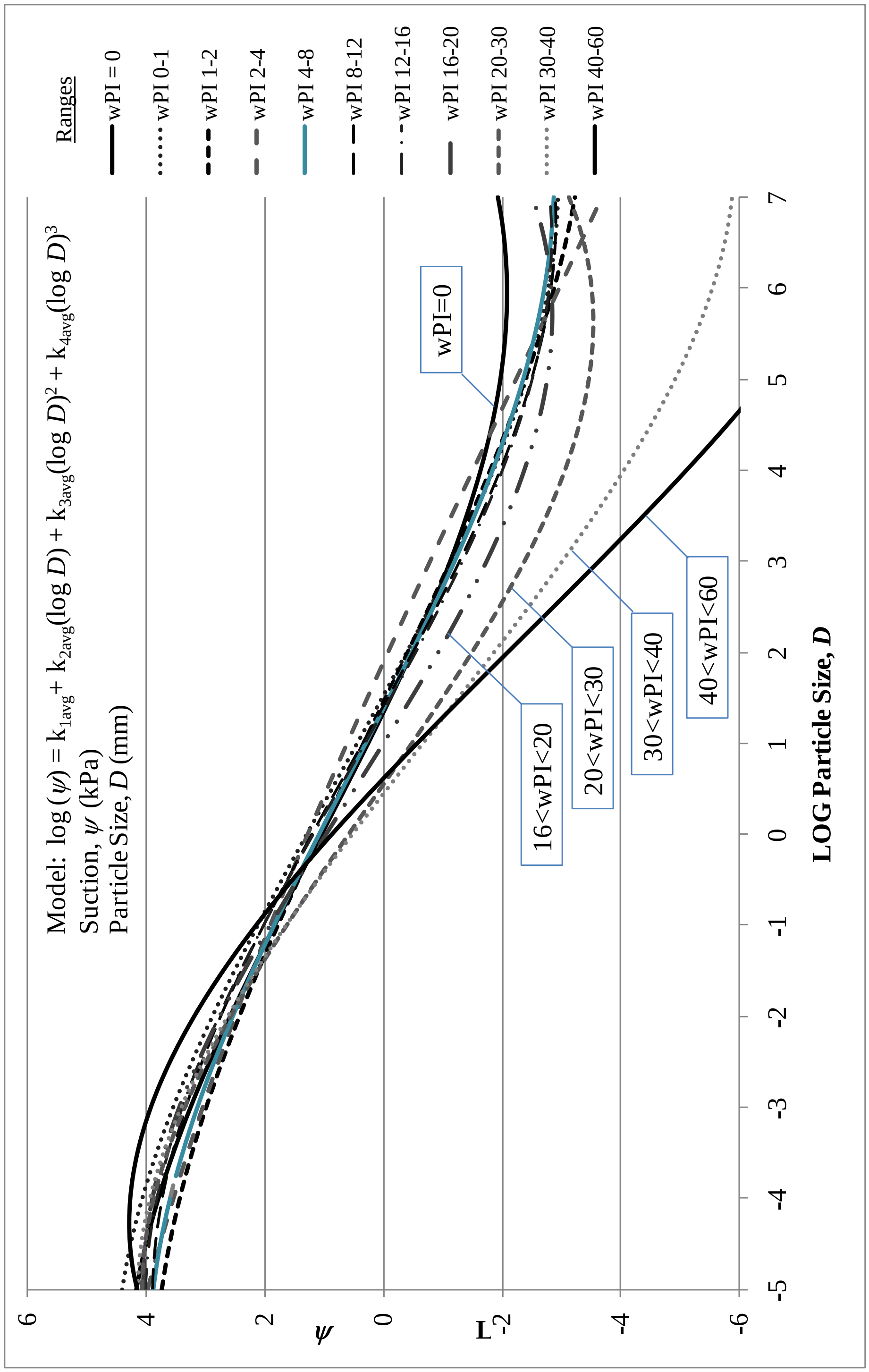



Figure 6-6 Plot of Log Suction versus Log Particle Size

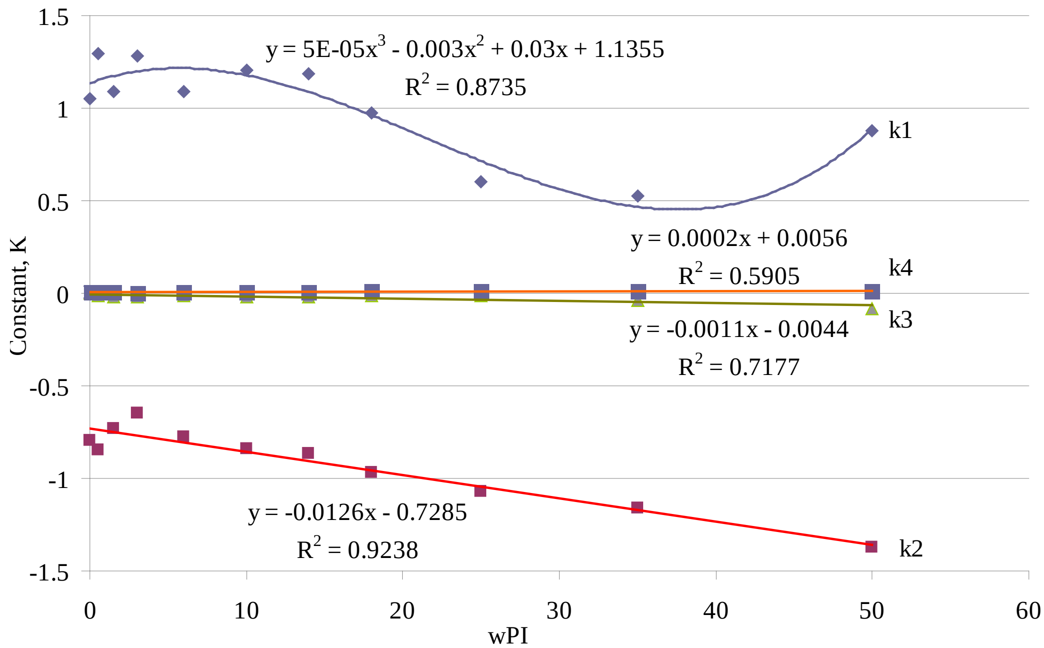



Figure 6-7 Relationship between the Constant Values and the wPI

Based on these observations, the following equations are proposed for constants $k_1$, $k_2$, $k_3$ and $k_4$:

$$k1 = 0.00005\,(wPI)^3 - 0.003\,(wPI)^2 + 0.03\,(wPI) + 1.1355 \quad \text{6-7}$$

$$k2 = -\,0.0126\,(wPI) - 0.7285 \quad \text{6-8}$$

$$k3 = -0.0011(wPI) - 0.0044 \quad \text{6-9}$$

$$k4 = 0.0002(wPI) + 0.0056 \quad \text{6-10}$$

Equation 6-7 yielded an $R^2$ of 0.87 while equation 6-8 yielded an $R^2$ of 0.92. Furthermore, equation 6-9 and equation 6-10 yielded $R^2$ values of 0.72 and 0.59, respectively.

Substituting equation 6-7 to 6-10 in to the equation 6-6, we obtain:

$$\log \psi = 0.00005\,(wPI)^3 - 0.003\,(wPI)^2 + 0.03\,wPI + 1.1355 - (0.0126\,wPI + 0.7285)\log D - (0.0011wPI + 0.0044)\log D^2 + (0.0002wPI + 0.0056)\log D^3 \quad \text{6-11}$$

This equation is valid for plastic and granular materials.

For non-plastic granular materials, the equation gets reduced to:

$$\log \psi = 0.0056\log D^3 + 0.0002wPI\log D^3 - 0.0044\log D^2 - 0.0011wPI\log D^2 - 0.7285\log D - 0.0126\,wPI\log D + 0.00005\,(wPI)^3 - 0.003\,(wPI)^2 + 0.03\,wPI + 1.1355 \quad \text{6-12}$$

Figure 6-8 presents a family of curves representatively the relationship given by equation 6-11

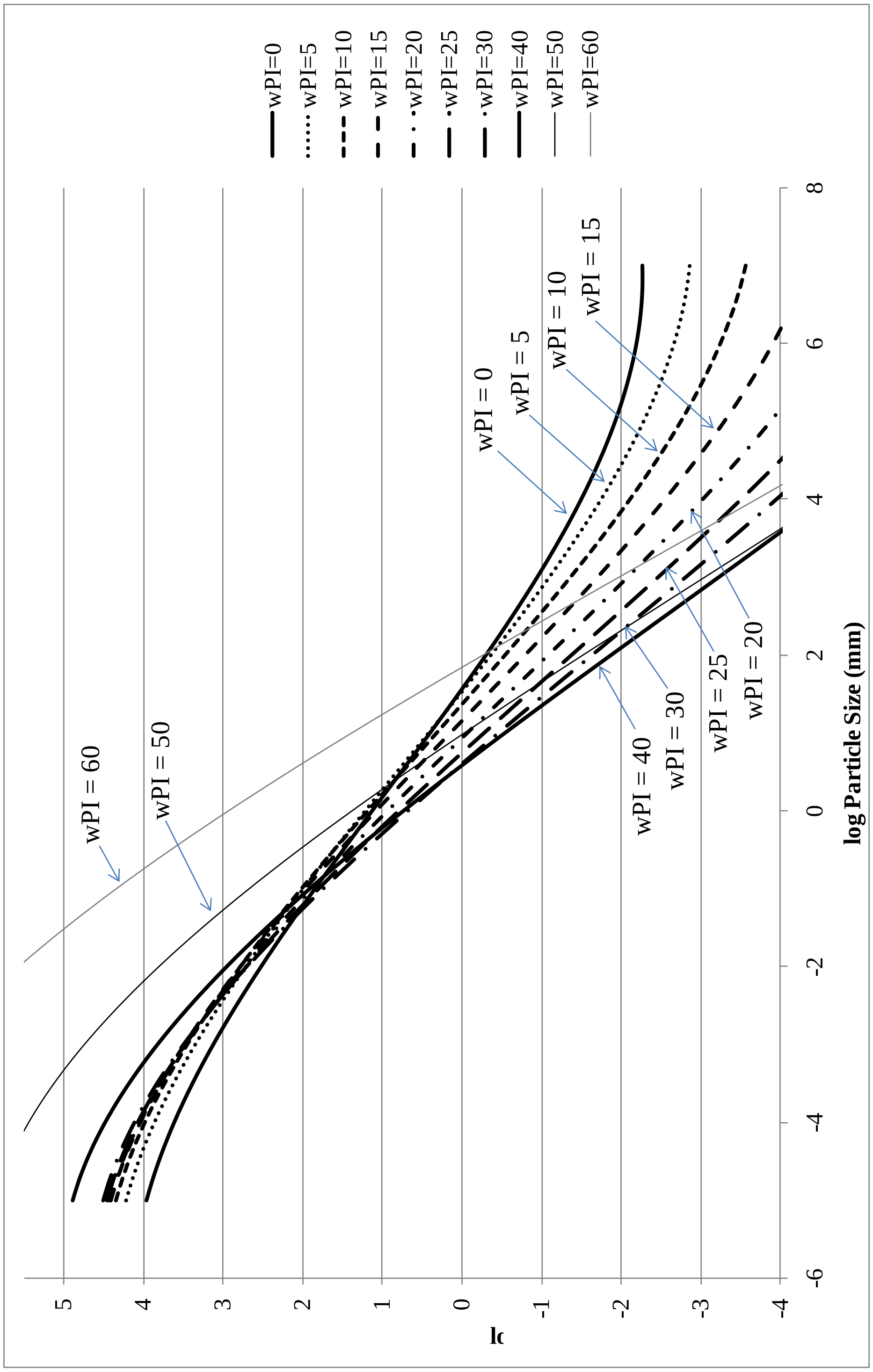



Figure 6-8 Suction as a Function of Particle Diameter and wPI

| | SS err | SS tot | SS reg | | $e_{alg}$ | $e_{abs}$ | (Se) | (Sy) |
|---|---|---|---|---|---|---|---|---|
| | $(S_m - S_p)^2$ | $(S_m - S_{m(avg)})^2$ | $(S_p - S_{m(avg)})^2$ | | $100*(S_m - S_p)/S$ | - | $(S_m - S_p)^2$ | $(S_{m(avg)} - S_m)^2$ |
| Σ = | 3,358.71 | 18,047.62 | 9,095.74 | $e_{alg}$ = | -27.10 | | | |
| | | | | $e_{abs}$ = | | 143.04 | | |
| Sm(avg) = | 2.3567 | | | S = | | | 3,358.71 | 18,047.62 |
| υ = | 6,642 | | | $n_{group}$ = | | | 6,642 | 6,642 |
| e = | 1 | | | p = | | | 1 | 1 |
| Se = | 0.71 | | | Se = | | | 0.71 | |
| | | | | Sm(avg) = | | | | 2.3567 |
| Sy = | 1.17 | | | Sy = | | | | 1.65 |
| | | | | | | | | |
| Se/Sy = | 0.61 | | | Se/Sy = | | | | 0.43 |
| | | | | | | | | |
| $R^2$ = | 0.8139 | | | $R^2$ = | 0.8139 | | | |

Figure 6-9 Error Analysis

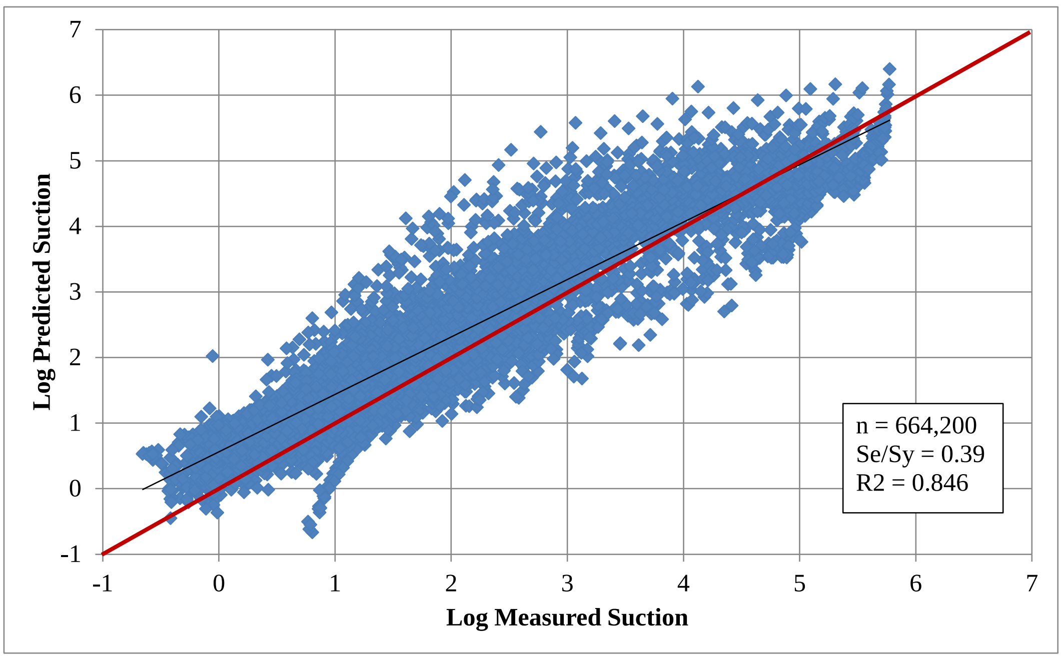



Figure 6-10 Comparison of Measured versus Predicted Suction

### 6.7 Assessment of the Model for Fine–Grained Materials

The families of curves for granular and fine–grained soils by using the approach indicated in this Chapter 6 are shown in Figure 6-11 and 6-12.

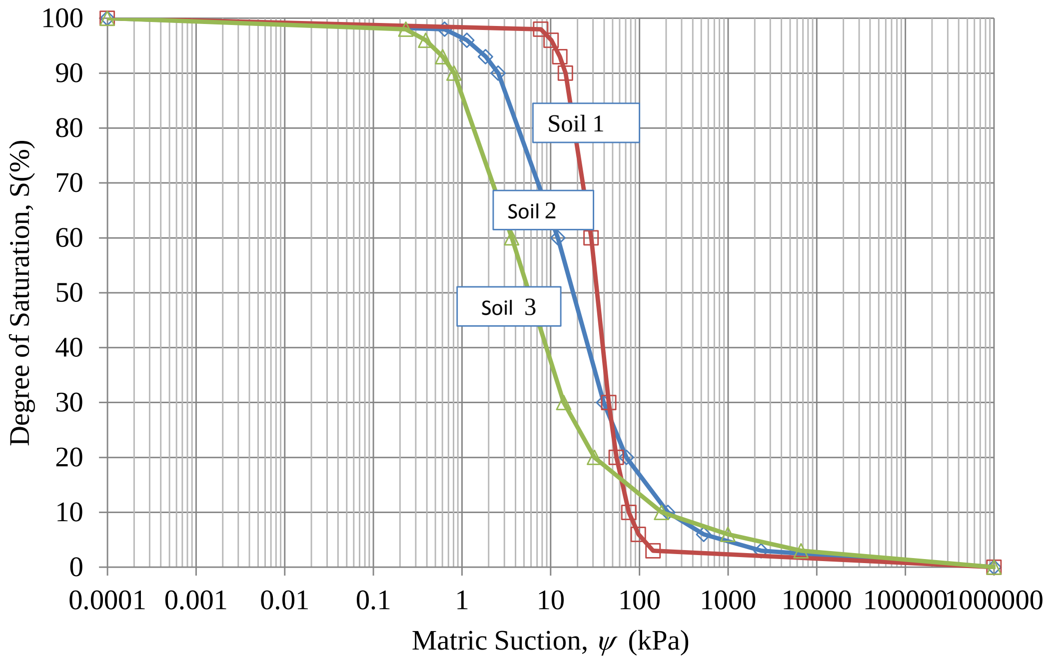



Figure 6-11 Family of Curves for Granular Materials

It is clear that the model doesn't perform well for suction values greater than about 10,000 kPa when the material is fine–grained, mostly. The reason for this is the fact that the particle diameter on the lower part is not accurately determined. Equation 6-5 that defines the grain–size distribution makes use of two parameters: $D_r$ and $D_m$ which correspond to the residual particle diameter and the minimum allowable particle diameter respectively.

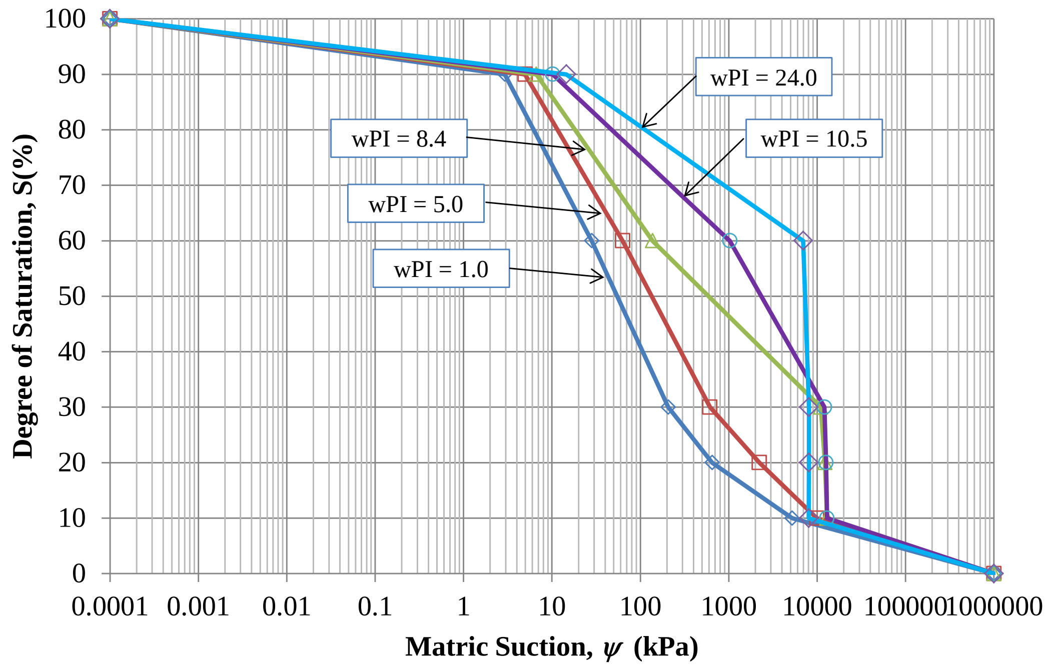



Figure 6-12 Family of Curves for Fine–Grained Materials

The assumed values for these two parameters were 0.001 mm and 0.00001 mm, respectively. These parameters appear to be underestimated for the fine–grained materials and they should be probably not constant values but rather a function of the soil plasticity.

In order to check if indeed these parameters were affecting the prediction of the SWCC, an example that included 5 different soils is presented below. First, the GSD curve was fitted to the measured values and a value of $10^{-14}$ mm for $D_m$ was used this time. A new relationship between particle diameters and suctions was obtained as shown in Figure 6-13. The equation that best represents this relationship is:

$$\log\psi = 5.797 - \left(\frac{6.774}{0.83 + e^{-0.634\log D}}\right) \quad \text{6-13}$$

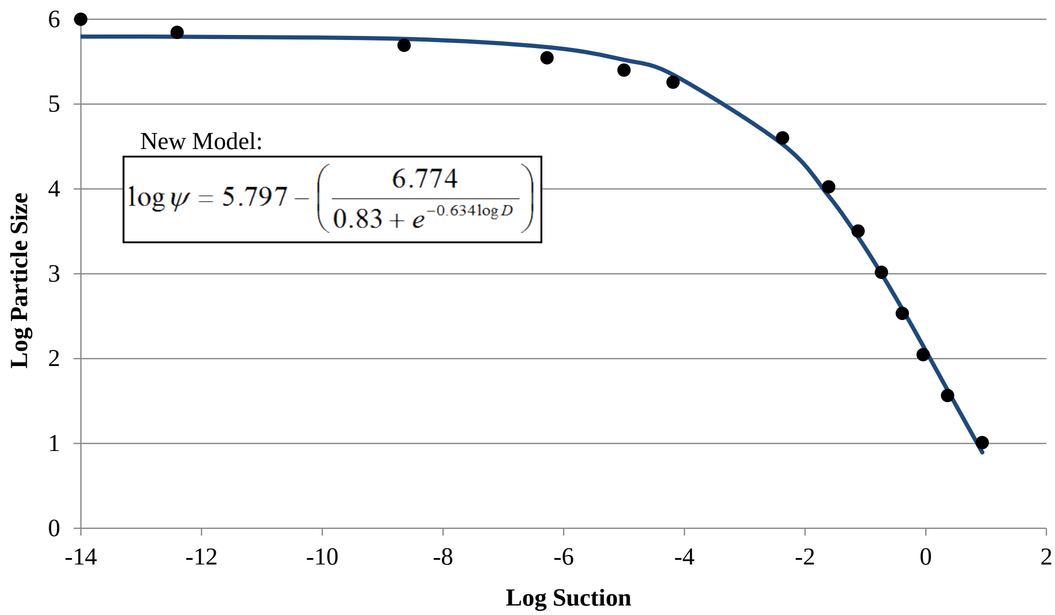



Figure 6-13 Example Relationship Log Suction vs Log Particle Size

Just as an example, Equation 6-13 used in the example before. These soils were used to estimate the SWCC for the five soils with wPI values ranging from 1 to 24. The grain–size distribution is presented in Figure 6-14. Based on the GSD, the SWCC presented in Figure 6-14 were obtained. Note how the prediction improves dramatically.

It is then recommended to perform the analysis for all the soils in the database to obtain a more reliable model.

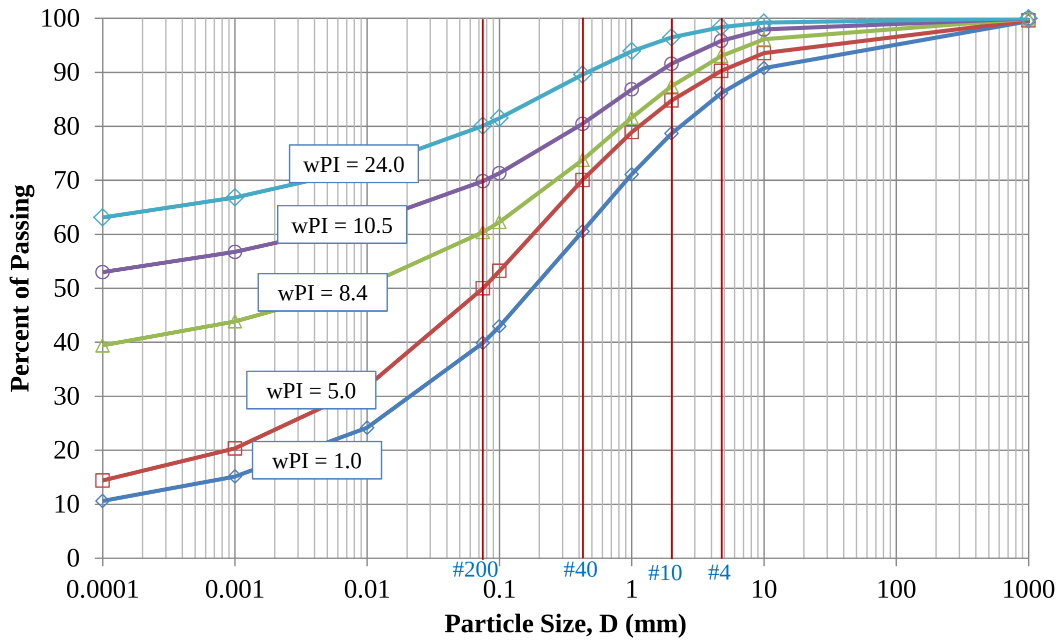



Figure 6-14. Grain–Size Distribution Example

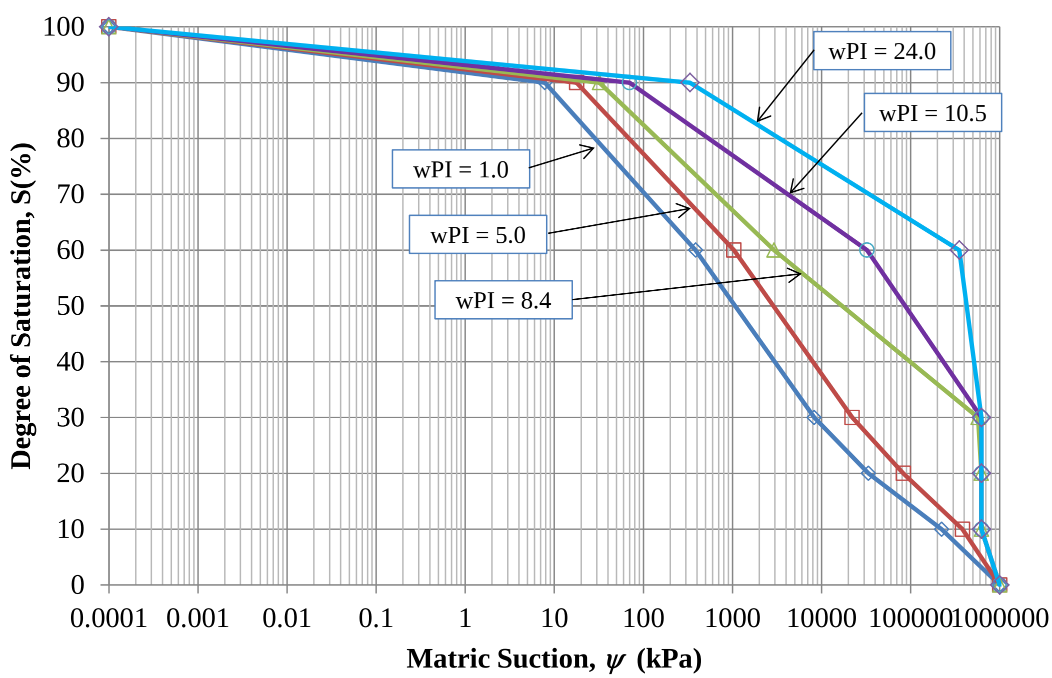



Figure 6-15. Soil–Water Characteristic Curve Example

## 6.8 Implementing the SWCC model based on GSD

The Procedure to estimate the SWCC based on the entire GSD is summarized as follows:

Step 1: Obtain the grain–size distribution of the soil and Atterberg limits.

Step 2: Calculate the Weighted Plasticity Index, wPI.

Step 3: Estimate the particle diameter, *D* values for different values of % passing: 5%, 10%, 15%, 20%, 25%, 30%… 95%, 100%. These values can be obtained by fitting the gradation data to the sigmoid function presented by Fredlund et al., 2002 (equation 6-5) and then using the Goal Seek function to solve for D. alternatively, the particle diameter values can be found by simply reading them off the GSC graph.

Step 4: Estimate the suction values by using equation 6-11 for the same degree of saturation equivalent to the % passing for which the particle diameter *D* was calculated in step 3.

Step 5: Plot the suction values versus degree of saturation pair of values found in step 4 and fit the SWCC Fredlund & Xing Function by using a non-linear regression package such as Solver in Excel®.

Figure 6-16 illustrates the procedure described to estimate the SWCC from the Grain–Size Distribution

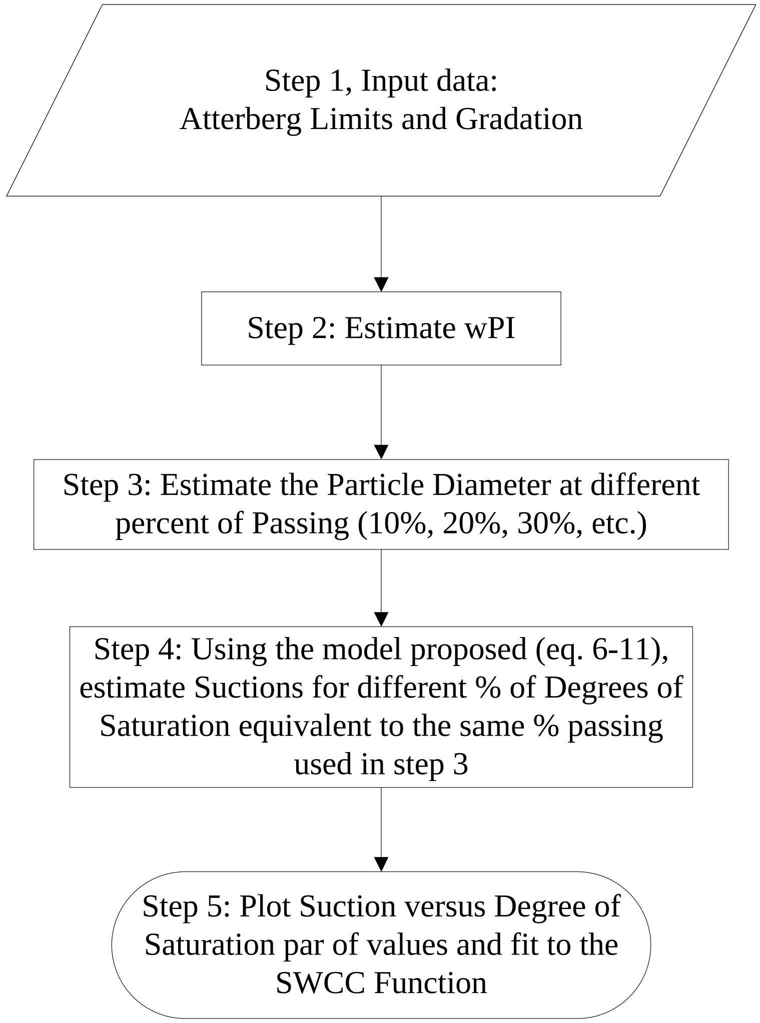



Figure 6-16. Approach to Estimate the SWCC based on the Grain–Size Distribution.

## 6.9 Summary

An approach to estimate the SWCC function based on the grain–size distribution of the soil has been proposed in Chapter 6. This approach relied on 33,210 soil–water characteristic curves that corresponded to 664,200 data suction points measured by the Natural Resources Conservation Service. The procedure described in Figure 6-1 explains the process to estimate the SWCC from the Atterberg limits and the plasticity index. The model proposed in Equation 6-11 is used to estimate the suction values for the same degree of saturation equivalent to the % passing for which the particle diameter *D* is defined. This model expresses the suction as a function of the wPI and the particle diameter, D.

The advantages of the proposed model include:

- The model makes use of the complete grain–size distribution data.
- The model produce a continuous SWCC function as opposed to the method proposed in Chapter 5.
- The model is based on the physical concept that relates the grain–size distribution to the pore-size distribution of the soil.
- The model is based on simple and routinely measured soil index properties such as gradation and Atterberg limits.
- Its implementation is straight forward.

A disadvantage of the model lies on the fact that the database provided only 2 to 3 measured data points and therefore, in order to fit a complete SWCC function, the author had to rely on the extremes of the function. That is, two extra data points were included in the fitting process: 100% saturation was assigned to a very low suction value and 0% saturation was assumed to occur at 1,000,000 kPa.

When comparing the model for plastic soils found by Zapata, 1999 (Figures 4-25 and 4-26) with the model proposed in this work (Figure 6-10), it can be seen that the $R^2$ improved from 0.70 to 0.81; while the $R^2$ for non-plastic soils improve from 0.40 to 0.81. On the other hand, when comparing the model proposed in this Chapter 6 (Figure 6-10) with the MEPDG model (Figure 4-27 and 4-28), it can be seen that the $R^2$ improved greatly from 0.49 to 0.81 for plastic soils. For non–plastic soils, the $R^2$ values were somewhat similar.

The new models proposed in this Chapter 6 based on the entire GSD did not perform well for soils with high plasticity. New equations for $k_1$, $k_2$, $k_3$ and $k_4$ parameters should be found. The procedure used in Chapter 6 looks promising and can be used to repeat the analysis once all the problems have been recognized.

# CHAPTER 7

# SIGNIFICANCE OF THE STUDY

## 7.1 Conclusions

This thesis work proposes a new set of models for the prediction of the SWCC fitting parameters based on the equation given by Fredlund and Xing in 1994. These models were estimated by following two different approaches. The first approach, explained in detail in Chapter 5 was based on a statistical regression analysis from values of matric suction and water content data points found in a large database maintained by the National Resources Conservation Service (NRCS). The second approach, presented in Chapter 6, proposes a model based on the Grain–Size Distribution, which is based on the relationship between the GSD and the Pore–Size Distribution (PSD) of the soil. Both methods have practical application. The first method can be directly implemented in the Enhanced Integrated Climatic Model (EICM), which is the model that incorporates environmental effects in the estimation of the resilient modulus of unbound materials. The EICM is an important component of the new Mechanistic–Empirical Pavement Design Guide (MEPDG).

A summary of the conclusions from each chapter is presented below.

### 7.1.1 Conclusions Chapter 3 – Database Collection.

The database collection was a very important task for the development of the work presented in this thesis. The vast amount of data points collected contained a total of 36,394 different soils, with 4,518 items corresponding to non–plastic materials and 31,876 plastic soils. The database was collected by the National Conservation Resources Service for agricultural purposes and contains chemical, physical and engineering soil properties which can be used in a number of disciplines. The soils properties were obtained from studies developed during many years through the continental US, Alaska, Hawaii, and Puerto Rico. The database allowed for the estimation of parameters such as the wPI factor, Group Index, the Soil–Water Characteristic Curve fitting parameters and the Grain–Size Distribution fitting parameters.

Most of the properties were obtained directly from the laboratory or from field testing while other properties were estimated from correlations or estimations. Both sets of data or properties had some degree of uncertainty related to them. The uncertainty of the data can be attributed to several factors: First, the database was developed by collecting tests during a range of years (USDA–NRCS was established in 1935); second, uncertainty associated with environmental conditions and soil nature (samples were located all over the US territory); third, the tests were performed by following protocols and standards which are being constantly updated; and last, technological changes and advances in the field allowed for new data interpretations during more than 70 years the data has been collected.

In order to eliminate the variability encountered in the data, a moving average technique was employed, whereas the data was organized or sorted according to the geotechnical factor (predictor) that most affected the predicting variable. This process is commonly used when the database presents high variability in order to find the general trend of behavior (Graham, 1993).

It is important to emphasize that the vast database collected and presented as part of this thesis work was drawn directly from laboratory testing. It is perhaps the largest database of soil moisture retention curves available in the world. These facts allowed for optimal models to estimate the Soil–Water Characteristic Curve, as those presented in this work.

### 7.1.2 Conclusions Chapter 4 – Validating Existing Models.

Based on the database collected and described in detail in Chapter 3, the validation of two existing models to estimate the SWCC was possible. The analysis is presented in Chapter 4. The models corresponded to those proposed by Zapata, 1999, and the MEPDG model (Witczak et al., 2006). These validations were statistically calculated separately for fine–grained plastic soils and for granular non–plastic materials.

Table 4-1 shows the errors found for the validation of Zapata's models for plastic and non-plastic soils. The validation was performed at different suction levels: 1, 10,100, 1,000 and 10,000 kPa. For non-plastic soils, the $R^2$ values ranged between 68% and 82%. Relatively good predicted water contents were found for suction values higher than 100 kPa. For plastic soils, the highest $R^2$ (82%) was found at suction values lower than 1 kPa and relatively acceptable $R^2$ (60%) was found for suction values higher than 1,000 kPa.

Figure 4-25 and 4-26 show the measured versus predicted volumetric water content values obtained by using the model proposed by Zapata, 1999, for plastic and non-plastic soils, respectively. These figures include all the predicted water contents estimated at suctions of 1, 10, 100, 1,000 and 10,000 kPa. For plastic soils, the model developed by Zapata, 1999, although it presented an overall $R^2$ of 0.70, it was found to be biased towards overprediction for most of the data points. For non-plastic soils, the Zapata's model presents a different behavior, in which most of the data points were underpredicted and yielded a low overall $R^2$ value of 0.40.

In general, the models proposed by Zapata, 1999, present acceptable errors considering that it was developed 10 years ago with few data points, when compared to the vast database used in this project.

Tables 4-2 and 4-3 show the error analysis performed for the MEPDG models for non-plastic and plastic soils, respectively. For non-plastic soils, an $R^2$ value of 60%, which was considered to be acceptable, was found only for suctions values lower than 1 kPa. Similarly, for plastic soils, the highest $R^2$ value (83%) was found for suction values lower than 1 kPa.

Figures 4-27 and 4-28 show the measured versus predicted volumetric water content values obtained by using the MEPDG model for plastic and non-plastic soils, respectively. These figures include all the predicted water contents obtained at 1, 10, 100, 1,000 and 10,000 kPa of suction. It was observed that for plastic soils, the volumetric water content was consistently overestimated and yielded an $R^2$ of 0.49. However, for non-plastic soils, the MEPDG model presented an acceptable prediction of volumetric water content with an $R^2$ value equal to 0.91.

In general, the MEPDG models presented acceptable estimations considering the amount of data analyzed. The MEPDG model can be considered to be a better model for non-plastic soils, while the model proposed by Zapata, 1999 can be considered to perform better for fine-grained materials.

### 7.1.3 Conclusions Chapter 5 – Approach 1 to Predict the SWCC.

Tables 5-8 and 5-13 present the proposed models for the SWCC parameters for the Fredlund and Xing equation, for plastic and non-plastic soils, respectively. The models proposed for plastic soils were estimated in function of the Group Index, which is in turn a function of passing sieve #200, liquid limit and plasticity index. On the other hand, the models proposed for non-plastic soils were estimated as function of the particle diameter $D_{10}$.

The models proposed in Chapter 5 to estimate the SWCC parameters, present the following advantages:

- The models proposed can be implemented in the Enhanced Integrated Climatic Model (EICM), which is incorporated in the Mechanistic–Empirical Pavement Design Guide (MEPDG). The format required allows including the SWCC fitting parameters.

- The database is vast. With more than 31,000 data points for plastic soils and 4,500 data points for non-plastic soils, it contains the most important Soil Index Properties obtained directly from laboratory testing or in the field. This database can be considered the largest in the world containing unsaturated soil properties.

- The models proposed are very simple to be implemented. For plastic soils, the Atterberg's Limits and the Passing US sieve #200 are needed as input parameters; while for non-plastic materials, only the particle diameter $D_{10}$ is needed. These are parameters commonly used by practicing engineers and therefore, this model becomes an excellent candidate for practical applications.

The approach and the models proposed in this chapter have the following limitations:

- The models proposed for the SWCC fitting parameters were estimated independently of each other. That makes it difficult to control the shape of the Soil–Water Characteristic Curve.

- Due to the tremendous amount of data points in the database, a moving average estimate of the parameters during the statistical analysis was necessary. The fine-grained material database was sorted according to the wPI and the data were averaged in groups of 300 consecutive data points. Groups of 50 consecutive data points were used for non-plastic soils, based on the $D_{10}$ parameter. While working with the moving average allows finding a clear tendency of the data and hence, better correlations; the variability gets somehow masked within each range. A deeper study of variability should be performed.

When comparing the model for plastic soils found by Zapata, 1999 (Figures 4-25 and 4-26) with the model proposed in this work (Figures 5-30 and 5-32), it can be seen that the $R^2$ improved from 0.70 to 0.81; while the $R^2$ improved from 0.40 to 0.89 for non-plastic materials. Even though the $R^2$ improved marginally for plastic soils, it can be observed that the Zapata's model is biased towards overprediction for most of the dataset. For non–plastic soils the results obtained with the Zapata’s model are underpredicting most of the data points.

On the other hand, when comparing the model proposed in this work (Figure 5-30) with the plastic model used in the MEPDG model (Figure 4-27), it can be seen that the $R^2$ improved greatly from 0.49 to 0.81. For non-plastic or granular soils, the $R^2$ for both models are somewhat similar. Even though the non–plastic model from MEPDG models is almost similar than the model proposed in this work, the later model is much simpler and easier to implement because it only depends on one gradation parameter, the particle size at 10% Passing or $D_{10}$.

It can be concluded that the new models proposed in this Chapter 5 will enhanced the prediction of the SWCC and therefore, it is recommended to consider applying them in practical applications.

As an example of the predicted SWCC based on the models proposed in Chapter 5, the soil index properties shown in Table 7-1 were used to calculate the SWCC fitting parameters. The three soil–water characteristic curves obtained are shown in Figure 7-1.

Table 7-1. Soil Index Properties for Three Soils Taken from the Database

| | Classification | | | | | | | Group | Predicted SWCC Parameters | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Soil** | **USCS** | **AASHTO** | **P200** | **LL** | **PI** | **$D_{10}$** | **wPI** | **Index** | **af** | **bf** | **cf** | **hr** |
| 1 | CH | A-7-6 | 85 | 65 | 40 | - | 34.00 | 37 | 0.0709 | 2.1570 | 0.0849 | 1,102.1 |
| 2 | CL | A-6 | 65 | 40 | 25.5 | - | 16.58 | 14 | 1.8436 | 1.0489 | 0.6047 | 691.5 |
| 3 | SP-SM | A-1-a | 5 | 0 | 0 | 0.07 | 0.00 | 1 | 8.0760 | 7.1855 | 0.6041 | 100.0 |

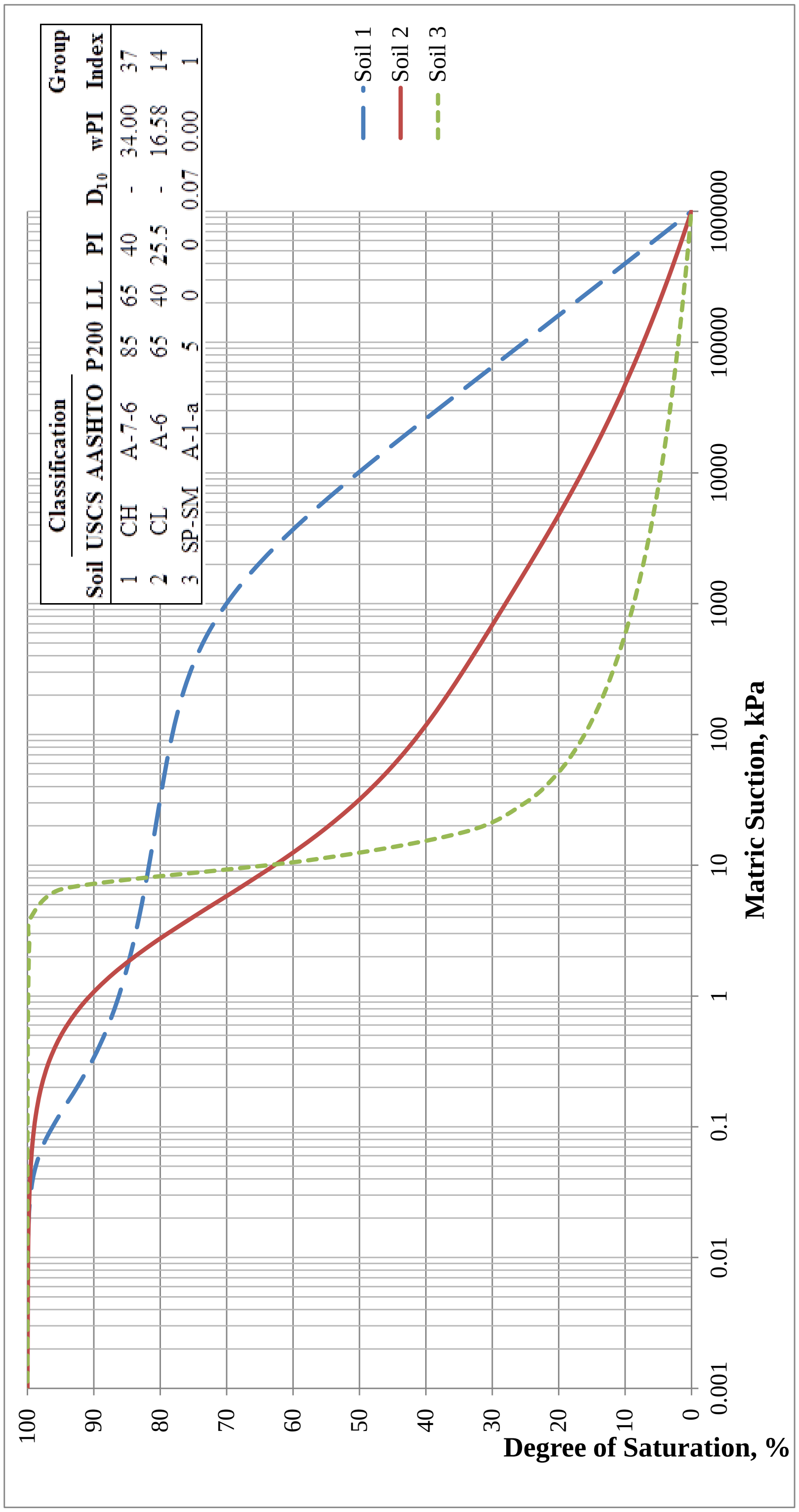



Figure 7-1. Examples of SWCCs Using the Model Proposed

In Figure 7-1, it can be observed that the SWCC for fine-grained material loses its sigmoidal shape. This result might represent a "dual porosity" for soils that are highly plastic (Zhang & Chen, 2005). The first air–entry value (i.e.0.05 kPa) might be associated with a macro–porosity while the second air–entry value (i.e. about 1,000 kPa) might be associated with a micro–porosity of the soil. This is reasonable given the fact that the measurements of suctions were obtained from natural clods and not from slurries.

### 7.1.4 Conclusions Chapter 6 – Approach 2 to Predict the SWCC.

An approach to estimate the SWCC function based on the grain–size distribution of the soil has been proposed in Chapter 6. This approach relied on 33,210 soil–water characteristic curves that corresponded to 664,200 data suction points measured by the Natural Resources Conservation Service. The procedure described in Figure 6-1 explains the process to estimate the SWCC from the Atterberg limits and the plasticity index. The model proposed in Equation 6-11 is used to estimate the suction values for the same degree of saturation equivalent to the % passing for which the particle diameter *D* is defined. This model expresses the suction as a function of the wPI and the particle diameter, D.

The advantages of the proposed model include:

- The model makes use of the complete grain–size distribution data.
- The model produce a continuous SWCC function as opposed to the method proposed in Chapter 5.

- The model is based on the physical concept that relates the grain–size distribution to the pore-size distribution of the soil.
- The model is based on simple and routinely measured soil index properties such as gradation and Atterberg limits.
- Its implementation is straight forward.

A disadvantage of the model lies on the fact that the database provided only 2 to 3 measured data points and therefore, in order to fit a complete SWCC function, the author had to rely on the extremes of the function. That is, two extra data points were included in the fitting process: 100% saturation was assigned to a very low suction value and 0% saturation was assumed to occur at 1,000,000 kPa.

When comparing the model for plastic soils found by Zapata, 1999 (Figures 4-25 and 4-26) with the model proposed in this work (Figure 6-10), it can be seen that the $R^2$ improved from 0.70 to 0.81; while the $R^2$ for non-plastic soils improve from 0.40 to 0.81. On the other hand, when comparing the model proposed in this Chapter 6 (Figure 6-10) with the MEPDG model (Figure 4-27 and 4-28), it can be seen that the $R^2$ improved greatly from 0.49 to 0.81 for plastic soils.  For non-plastic soils, the $R^2$ values were somewhat similar.

The new models proposed in this Chapter 6 based on the entire GSD did not perform well for soils with high plasticity. New equations for $k_1$, $k_2$, $k_3$ and $k_4$ parameters should be found. The procedure used in Chapter 6 looks promising and can be used to repeat the analysis once all the problems have been recognized.

Similarly to Chapter 5, it can be concluded that the new models proposed in this Chapter 6 will enhanced the prediction of the SWCC and therefore, it is also recommended to consider applying them in practical applications.

## 7.2 Application

One of the most important applications for the models proposed in this thesis work is in pavement design; where the unsaturated soil mechanics plays an important role in the performance of the pavement structure, mainly on the resistance and deformation of the soil. These characteristics of the soil are mainly due to variations in matric suction, which could take place due to changes on external conditions especially due to presence of water, changes in temperature, depth of the ground water table, external loads, etc. This relationship between the matric suction and the amount of water into the soil has been considered in the Enhanced Integrated Climatic Model (EICM) as part of the Mechanistic–Empirical Pavement Design Guide (MEPDG).

## 7.3 Recommendations for Further Research

This work was focused on soil properties which are affecting the SWCC such as the volumetric water content for suctions of 10, 33 and 1,500 kPa, the percentage of passing sieve # 200, and the Atterberg's limits. The database initially considered in this thesis has more physical and engineering properties which could be used to establish more estimations or correlations.

Many studies relate the soil–water characteristic curve with other soil–properties to obtain unsaturated soil property functions. The SWCC is related with the grain–size distribution as applied in Chapter 6. Furthermore, the SWCC is related with shear strength parameters to predict shear strength functions; and it is also related to other hydraulic properties to obtain the water seepage constitutive function. The database contains enough information to attempt correlations that include the SWCC, the saturated hydraulic conductivity and other properties in order to estimate the unsaturated hydraulic conductivity function.

In summary, unsaturated soil mechanics allows relating constitutive relationships (water seepage, air flow, heat flow, shear strength and volume–mass change), and compaction properties with the soil–water characteristic curve and this database could be used to obtain other estimations.

RERERENCES

ASTM Standard D 5298–10, 2010, “Standard Test Method for Measurement of Soil Potential (Suction) Using Filter Paper,” ASTM International, West Conshohocken, PA, 2010, DOI: 10.1520/D5298–10, www.astm.org.

Arya, L.M., & Paris J.F. (1981) “A Physicoempirical Model to Predict the Soil Moisture Characteristic from the Particle–Size Distribution and Bulk Density data, Soil Science Soc. American Journal, Vol. 45, pp. 1023–1030

Bishop, A. W., 1959, “The principle of effective stress”, Norwegian Geotechnical Institute, Vol. 39, pp. 859–863.

Darter, M.I., Mallela, J., Titus-Glover, L., Rao, C., Larson, G., Gotlig, A., Von Quintus, H., Khazanovich, L., Witczak, M., El-Basyouny, M., El-Badawy, S., Zborowski, A. and Zapata, C. (2006). Research Results Digest 308: Changes to the Mechanistic-Empirical Pavement Design Guide Software through Version 0.900, National Cooperative Highway Research Program, Transportation Research Board of the National Academies, September 2006. 22 pages.

Feng, G., Sharratt, B., Vaughan, J., & Lamb, B. (2009) “A Multiscale Database of Soil Properties for Regional Environmental Quality Modeling in the Western United States”, Journal of Soil and Water Conservation, Vol. 64, No. 6, December 2009, pp 363–373.

Fredlund, D.G. (2006). “Unsaturated Soil Mechanics in Engineering Practice”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 132, No. 3, pp. 286–321.

Fredlund, D.G., & Morgenstern, N.R. (1976) “Constitutive relations for volume change in unsaturated soils”, Canadian Geotechnical Journal, V. 13, No. 3, pp. 261–276.

Fredlund, D.G., & Morgenstern, N.R. (1977) “Stress State Variables for Unsaturated Soils”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 130, No. GT5, May 1977, pp. 447–466.

Fredlund, D.G. (l979), “Second Canadian Geotechnical Colloquium: Appropiate Concepts and Technology for Unsaturated Soils”, Canadian Geotechnical Journal, Vol. 169, No. 1, pp. 121–139.

Fredlund, D.G., & Rahardjo, H. (1993). Soil Mechanics for Unsaturated Soils. John Wiley & Sons, Inc., New York, NY.

Fredlund, D.G., & Xing, A. (l994), “Equations for the Soil–Water Characteristic Curve”, Canadian Geotechnical Journal, Vol. 31, No. 3, pp. 521–532.

Fredlund, D.G. (1995). “The Prediction of Unsaturated Soil Functions Using the Soil–water Characteristic Curve. In Proceedings of the Bengt Broms Symposium in Geotechnical Engineering”, 13–16 Dec., 1995, Singapore, World Scientific, River Edge, NJ, pp. 113–133.

Fredlund, D.G. (1981). “Seepage in Saturated Soil”, 13–16 Dec., 1995, Singapore, World Scientific, River Edge, NJ, pp. 113–133.

Fredlund, M.D., Fredlund, D.G., & Houston, S.L., Houston, W. (2003). “Assessment of Unsaturated Soil Properties for Seepage Modeling through Tailings and Mine Wates”, Proceedings of the Tenth International Conference on “Tailings and Mine Wastes”, Vail, Colorado, U.S., pp. 149 – 157, Oct. 12-15

Fredlund, M.D., Fredlund, D.G., & Wilson, G.W. (1997). “Prediction of the Soil–Water Characteristic Curve from Grain–Size Distribution and Volume–Mass Properties”, Third Brazilian Symposium on Unsaturated Soils, Rio de Janeiro, Brazil, 1977

Fredlund, M.D., Fredlund, D.G., & Wilson, G.W. (2000). “An Equation to Represent Grain–Size Distribution”, Canadian Geotechnical Journal, Vol. 37, No. 4, pp. 817–827

Fredlund, D.G., Stone, J., Stianson, J., Sedgwick, A., (2011) “Obtaining Unsaturated Soil Properties for High Volume Change Materials”, Golder Associated Ltd. & Total E.P. Canada

Fredlund, M.D., Wilson, G.W., & Fredlund, D.G. (2002). “Use Of The Grain–Size Distribution For Estimation Of The Soil–Water Characteristic Curve”, Canadian Geotechnical Journal, Vol. 39, No. 5, pp. 1103–1117

Gitirana Jr. G., & Fredlund, D.G. (2004). “Soil–Water Characteristic Curve Equation with Independent Properties”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 130, No. 2, pp. 209–212

Graham, R.C. (1993) “Data Analysis for the Chemical Science: A guide to statistical techniques”, VCH Publishers Inc., New York, NY

Gupta, S.C., & Larson, W.E. (1979). “Estimation Soil–water Retention Characteristics from Particle–size Distribution, Organic Matter Percent, and Bulk Density", Water Resources Research Journal, Vol. 15, No. 6, pp. 1633–1635

Hazen, A. (l911). "Discussion of Dams on Sand Foundation", A.C. Koenig Transactions, ASCE, Vol. 73, pp. 199–203.

Hines, W.W., & Montgomery, D.C. (1990). Probability and Statistic in Engineering and Management Science, John Wiley & Sons, Inc., Canada, 732 p

Houston, W.N., Mirza, M.W. and Zapata, C.E. (2006). "Environmental Effects in Pavement Mix and Structural Design Systems. Calibration and Validation of the ICM Version 2.6. NCHRP 9-23 project". Preliminary Draft. Final Report - Part 2. Submitted to National Cooperative Highway Research Program. (Under panel revision).

Leong, E.C., & Rahardjo, H. (1997). "Review of Soil–Water Characteristic Curve Equations", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 123, No. 12, pp. 1106 – 1117

Perera, Y. Y. (2003). Moisture Equilibria Beneath Paved Areas. Ph. D. Dissertation, Arizona State University, Tempe, United States.

Richards, L. A. (1931). Capillary conduction of liquids through porous mediums. In Black, C. A. (Ed.). Methods of Soil Analysis, Part 1: Physics (pp. 318–333).

Saxton, K.A., & Rawls, W.J. (2006). "Soil Water Characteristic Estimates by Texture and Organic Matter for Hydrologic Solutions", Soil Science Society of America Journal, Vol. 70, pp. 1569–1578

Sillers, W.S., & Fredlund, D.G. (2001). "Statistical Assessment of Soil–water Characteristic Curve Models for Geotechnical Engineering", Canadian Geotechnical Journal, Vol. 38, No. 6, pp. 1297–1313

Soil Survey Division Staff, (1993) Soil Survey Manual—United States Department of Agriculture handbook no. 18 (rev. ed.): [Washington, D.C.], U.S. Department of Agriculture, 437 p.

Soil Survey Staff, (1994) Keys to Soil Taxonomy (6th ed.): [Washington, D.C.], U.S. Department of Agriculture, Soil Conservation Service, 306 p.

Soil Survey Staff, (1993) National Soil Survey Handbook, title 430 VI (issue 3): [Washington, D.C.], U.S. Department of Agriculture, Soil Conservation Service [variously paged].

Soil Survey Staff, (1975) Soil taxonomy—A Basic System of Soil Classification for Making and Interpreting Soil Surveys, agriculture handbook no. 436: [Washington, D.C.], U.S. Department of Agriculture, Soil Conservation Service, 754 p.

Taylor, J.K. (1987). Quality Assurance of Chemical Measure, Lewis Publishers, 1987, 328 p. (QD51.T38 1987)

Terzaghi, K. (1943). “Theoretical Soil Mechanics”, John Wiley & Sons, Inc., New York, NY

U.S. Department of Agriculture, National Resources Conservation Service, Soil Data Mart, Order U.S. Soil Map (STATSGO) Data. June 2008. Retrieved from: http://soildatamart.nrcs.usda.gov/USDGSM.aspx

Van Genutchen, M.Th. (1980). “A Closed–Form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils”. Soil Science Society of American Journal, Vol. 44, No. 5, pp. 892-898.

Wagner, L.E., Ding, D. (1994). “Representing Aggregate Size Distributions as Modified Lognormal Distribution”, Soil and Water Division of American Society of Agricultural Engineers, Vol. 37, No. 3, pp. 815-821

Witczak, M.W., Zapata, C.E. and Houston, W.N. (2006). Models Incorporated into the Current Enhanced Integrated Climatic Model: NCHRP 9-23 Project Findings and Additional Changes after Version 0.7. Final Report. Project NCHRP 1-40D. Inter Team Technical Report. Arizona State University. Tempe.

Zapata, C.E. (1999). Uncertainty in Soil–Water Characteristic Curve and Impacts on Unsaturated Shear Strength Predictions. Ph. D. Dissertation, Arizona State University, Tempe, United States.

Zapata, C.E. (2010). Research Results Digest 347: A National Catalog of Subgrade Soil–Water Characteristic Curves and Selected Soil Properties for Use with the MEPDG. National Cooperative Highway Research Program, Transportation Research Board, of the National Academies. ISSN 0077-5614. ISBN: 978-0-309-09929-5. Library of Congress Control Number 2008924251. pp. 62.

Zapata, C.E., Andrei, D., Witczak, M.W., & Houston, W.N. (2007) “Incorporation of Environmental Effects in Pavement Design”, International Journal of Road Material and Pavement Design, Vol. 8, No. 4, 2007, pp. 667–693

Zapata, C.E., Houston, W.N. (2008). “NCHRP Report 602, Project 9-23: Calibration and Validation of the Enhanced Integrated Climatic Model for Pavement Design”, National Cooperative Highway Research Program. Transportation Research Board, Washington D. C.

Zapata, C.E., Houston, W.N., Houston, S.L., & Walsh, K.D. (1999). “Soil–Water Characteristic Curve Variability”, Department of Civil and Environmental Engineering, Arizona State University.

Zhang, L., & Chen, Q. (2005). “Predicting Bimodal Soil–Water Characteristic Curves”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 131, No. 5, pp. 666-670