

Generalized Statistical Tolerance Analysis And Three Dimensional Model For
Manufacturing Tolerance Transfer in Manufacturing Process Planning

By

Nadeem Shafi Khan

A Dissertation Presented in Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy

Approved April 2011 by the
Graduate Supervisory Committee:

Patrick Phelan, Chair
Douglas Montgomery
Gerald Farin
Chell Roberts
Mark Henderson

ARIZONA STATE UNIVERSITY

May 2011

ABSTRACT

Mostly, manufacturing tolerance charts are used these days for manufacturing tolerance transfer but these have the limitation of being one dimensional only. Some research has been undertaken for the three dimensional geometric tolerances but it is too theoretical and yet to be ready for operator level usage. In this research, a new three dimensional model for tolerance transfer in manufacturing process planning is presented that is user friendly in the sense that it is built upon the Coordinate Measuring Machine (CMM) readings that are readily available in any decent manufacturing facility. This model can take care of datum reference change between non orthogonal datums (squeezed datums), non-linearly oriented datums (twisted datums) etc. Graph theoretic approach based upon ACIS, C++ and MFC is laid out to facilitate its implementation for automation of the model. A totally new approach to determining dimensions and tolerances for the manufacturing process plan is also presented. Secondly, a new statistical model for the statistical tolerance analysis based upon joint probability distribution of the trivariate normal distributed variables is presented. 4-D probability Maps have been developed in which the probability value of a point in space is represented by the size of the marker and the associated color. Points inside the part map represent the pass percentage for parts manufactured. The effect of refinement with form and orientation tolerance is highlighted by calculating the change in pass percentage with the pass percentage for size tolerance only. Delaunay triangulation and ray tracing algorithms have been used to automate the process of identifying the points inside and outside the part map.

Proof of concept software has been implemented to demonstrate this model and to determine pass percentages for various cases. The model is further extended to assemblies by employing convolution algorithms on two trivariate statistical distributions to arrive at the statistical distribution of the assembly. Map generated by using Minkowski Sum techniques on the individual part maps is superimposed on the probability point cloud resulting from convolution. Delaunay triangulation and ray tracing algorithms are employed to determine the assembleability percentages for the assembly.

DEDICATION

To my father Muhammad Shafi Khan, my mother Shahzadi Begum, my wife Nadia, my brother Saleem, my sister Gulnaz, my brother and sisters and to my kids Alina, Minahil, Aaiza, Daniyal and Sulemun.

ACKNOWLEDGMENTS

First of all, I would like to express my sincere gratitude to my advisor, Dr Patrick E. Phelan for his timely support and guidance. Also, I would like to thank Dr Mark Henderson, Dr Chell Roberts, Dr Gerald Farin, Dr Don Holcomb and Dr Douglas Montgomery for being on my PhD committee and taking the time to share their knowledge and experience with me. In addition, I would like to thank all my professors who taught me and/or supported me during my PhD. I also appreciate the financial help of all institutions in my endeavor for higher education.

I would also take this opportunity to thank all my fellow colleagues with whom I spent a lot of time studying and discussing research issues.

Finally, I would like to thank all of my family members and friends for their love and support.

TABLE OF CONTENTS

	Page
LIST OF TABLES.....	xiv
LIST OF FIGURES.....	xv
CHAPTER	
1 INTRODUCTION.....	1
1.1 Background.....	1
1.2 Problem Statement.....	9
1.3 Proposal outline.....	11
2 TOLERANCE TRANSFER IN MANUFACTURING.....	14
2.1 Rationale for tolerance transfer (Tolerance Control).....	14
2.2 Manufacturing Tolerance Chart: A definition.....	16
2.2.1 Types of tolerance charts.....	16
2.3 Use of 1-D Manufacturing Tolerance Charts.....	18
2.4 Stages involved in the development of Manufacturing Tolerance Charts.....	19
2.4.1 Blue print.....	19
2.4.2 Strip layout.....	19
2.4.3 Tolerance Chart.....	19
2.5 Pros of Manufacturing Tolerance Charts.....	20
2.6 Cons of Manufacturing Tolerance Charts.....	20
2.7 Overall analysis of the Manufacturing Tolerance Charts..	22

	Page
2.8	Limitations of Manufacturing Tolerance Charts.....22
2.8.1	Non-efficient use of Design tolerance range.....22
2.8.2	Non- proactivity of the method.....23
2.8.3	Limitation of use of equal bilateral tolerance system.....23
2.9	Related trends in tolerance control.....24
3	LITERATURE REVIEW: MATH MODELS.....25
3.1	Math models for tolerance representation.....25
3.1.1	Parametric models.....25
3.1.2	Variational Surface models.....30
3.1.3	Offset Zone model.....31
3.1.4	DOF model.....32
3.1.5	TTRS model.....33
3.1.6	Kinematic model.....36
3.1.7	Vectorial Model.....39
3.1.8	Multi-Variate Region model.....41
3.2	T-Map; An Introduction.....48
3.2.1	What is a T-Map?.....48
3.2.2	Basic impulse behind T-Map.....48
3.2.3	Conformance to ASME Y14.5 Standard.....49
3.2.4	Areal coordinates.....50
3.2.5	T-Maps for size.....53
3.2.6	T-map for form and orientation.....56

	Page
3.2.7	Examples of T-Maps for form and orientation.....57
3.2.8	Tolerance-map for a face with size and form tolerance: an example.....58
3.2.9	Material Condition.....59
3.2.10	Process of conforming of the T-Maps.....60
3.2.11	Accumulation T-Map.....60
3.2.12	Process of obtaining Accumulation T-Map.....60
3.2.13	Minkowski Sum.....60
3.2.14	Functional T-Map.....62
3.2.15	T-Map for frequency distribution generation.....63
3.2.16	Concluding remarks about T-Maps.....63
3.2.17	Pluses of the T-Map method.....64
3.2.18	Minuses of the T-Map method.....65
3.3	Other Approaches.....66
3.4	Model Comparisons.....66
3.5	Concluding Remarks for Math Model of Tolerance Representation.....68
4	THREE DIMENSIONAL MODEL FOR TOLERANCE TRANSFER IN MANUFACTURING PROCESS.....70
4.1	Background.....70
4.2	Requirement for datum change.....72
4.3	Reasons to avoid datum change.....72

	Page
4.4	When datum change is unavoidable.....73
4.5	Main Idea.....74
4.6	Methodology.....75
4.7	Mathematical details of tolerance transfer.....80
4.8	Case Study.....89
4.8.1	Inline datum change.....89
4.8.2	Inline with different azimuth datum change.....90
4.8.3	Twisted datum change.....92
4.8.4	Squeezed datum change.....93
4.9	Application of the mathematical model to various types of datums and the datum reference frames.....94
5	GRAPH THEORETIC APPROACH TO RELATIONSHIP REPRESENTATION AND TOLERANCE ANALYSIS AND SYNTHESIS FOR MANUFACTURING PROCESS PLANNING.....100
5.1	Introduction to graph theoretic approach.....100
5.2	Relationship between geometric features.....100
5.3	Geometric constraints as applicable to geometric features.....102
5.4	Tolerances applied to geometric constraints existing between geometric features.....103
5.5	Use of data structures.....104
5.6	Selection of data structure.....105

	Page	
5.7	Exploitation of the object oriented programming (OOP).....	105
5.8	Use of Microsoft Foundation Classes (MFC).....	106
5.9	Use of ACIS geometry kernel.....	106
5.10	Validation of the manufacturing process plans.....	106
6	THEORETICAL EXTENSIONS OF THE 3-D MODEL FOR TOLERANCE TRANSFER IN MANUFACTURING PROCESS PLANNING.....	109
6.1	Introduction.....	109
6.2	Background.....	109
6.3	Applicability of the manufacturing tolerance transfer model.....	111
6.4	Different types of tolerances used for same geometric variation.....	113
6.5	Case studies for manufacturing tolerance transfer arising from datum reference change.....	114
6.5.1	Dimensionality of the problem.....	114
6.5.2	Not limited to planar features.....	115
6.5.3	Interest in manufacturing process planning limited to tolerance analysis only.....	115
6.5.4	Sample Manufacturing Process plan extract #1.....	116
6.5.5	Sample Manufacturing Process plan extract #2.....	118

	Page
6.5.6	Sample Manufacturing Process plan extract #3.....119
6.5.7	Sample Manufacturing Process plan extract #4.....120
6.6	Case study for manufacturing tolerance transfer arising from different type of tolerance used in manufacturing process plan as compared to Design blue print.....121
6.6.1	Form controls.....121
6.6.1.1	Flatness.....123
7	USE OF THREE DIMENSIONAL TOLERANCE TRANSFER MODEL (3DTT)FOR DETERMINING MEAN DIMENSIONS AND TOLERANCES FOR MANUFACTURING PROCESS PLAN.....129
7.1	Introduction: Various Utilities based upon Three Dimensional Tolerance Transfer (3DTT) concept.....129
7.2	Preliminary Information (required to start).....130
7.2.1	Number of stack directions.....130
7.2.2	Types and accuracies of the tools and machines.....131
7.2.3	Design tolerances.....131
7.3	Beyond the final operation.....132
7.3.1	Mean dimensions.....132
7.3.2	Tolerances involved for the operation concerned.....133

	Page
7.4	Use of screw concept for representation of geometric variation.....134
7.5	Dimensionality of the geometric variation.....136
7.6	Information required to proceed with development of manufacturing process plan for tolerance analysis.....136
7.7	Case Study.....138
8	STATISTICAL TOLERANCE ANALYSIS.....156
8.1	Types of tolerance analysis.....156
8.2	Methods for Statistical Tolerance Analysis: Broad classification.....157
8.3	Methods for Statistical Tolerance Analysis: Individual methods/approaches.....158
8.3.1	Root Sum Squares Method.....159
8.4	Estimated Mean Shift Method.....161
8.5	Spotts' modified approach.....162
8.6	Bender's correction factor approach.....163
8.7	Mansoor's static mean shift model.....163
8.8	Gladman's dynamic mean shift model.....165
8.9	Desmond's probabilistic mean shift model.....166
8.10	Chase and Greenwood's iterative mean shift method.....167
8.11	Taguchi's method.....168
8.12	Hasofer- Lind reliability index method.....169

	Page
8.13	Evan’s numerical integration approach.....170
8.14	Monte Carlo simulation method.....171
8.15	Method of Moments.....172
8.15.1	First Four Moments of Statistical Distribution.....172
8.16	Mixed Moment + Monte Carlo Simulation method.....175
8.17	Need for Statistical tolerance analysis.....176
8.18	Types of Convolutions.....177
8.18.1	Convolution of two discrete random variables.....177
8.18.2	Convolution of two continuous random variables.....178
8.19	Sum of two independent uniform random variables.....178
9	PROBABILITY MAPS: A NEW STATISTICAL MODEL FOR NON LINEAR TOLERANCE ANALYSIS APPLIED TO RECTANGULAR FACES.....182
9.1	Introduction.....182
9.2	Purpose.....183
9.3	Background.....186
9.4	Prob Map for size tolerance on rectangular face.....188
9.5	Re-D Prob map for size tolerance specified on rectangular face.....192
9.6	4-D Prob Maps.....196
9.7	Measurement of three variables involved in construction of T-map.....200

	Page	
9.8	Extraction of the pass percentage of parts using prob Maps.....	201
9.9	Effect on pass percentage with specification of form tolerance.....	204
9.10	Effect on pass percentage with specification of orientation tolerance.....	209
9.11	Use of convolution for probability estimation of assembly statistical distribution from statistical distribution for the parts.....	211
9.12	Extraction of assembleability percentages for stack up of two parts.....	217
9.13	Conclusions.....	221
10	CLOSURE.....	223
11	REFERENCES.....	225
	APPENDIX: COMPUTER AIDED TOLERANCING.....	233
A.1	Software based tolerance analysis.....	234
A.2	Evaluation criteria for commercial tolerancing software.....	234
A.3	Commercial Tolerance Analysis Software Packages.....	235
A.4	A review of Software packages for Tolerance Analysis.....	236
	BIOGRAPHICAL SKETCH.....	243

LIST OF TABLES

Table	Page
3-1 Identification of the plane/points in the tolerance zone/T-Maps.....	55
3-2 Comparison of the different Math models	67
7-1 Worst case limits as derived from geometric tolerances specified in Designer blue print. Finishing operation dimension and tolerances as specified in Manufacturing Process plan is also shown.....	142

LIST OF FIGURES

Figure	Page
1-1	The relationship between tolerances and cost with respect to different machining operations. (Adapted from David G. Ullman, The Mechanical Design Process, 3 rd ed., McGraw-Hill, New York, 2003).....1
1-2	Role of tolerances in various stages of Product Life Cycle.....2
1-3	Change on the requirement of tolerances during various stages of Product Life Cycle.....3
1-4	Minkowski sum of two T-Maps (semi sections only)4
2-1	The reasons for the requirement of tolerance control.....16
2-2	Different types of Tolerance charts.....18
2-3	The documents involved in the construction of the manufacturing tolerance charts.....20
2-4	Pros and Cons of the Manufacturing Tolerance Charts.....21
3-1	(a) web (b) strut (c) plate (d) A Dimensioned & Toleranced Polygon.....26
3-2	Offset Zone shown on the CMM arm hinge.....31
3-3	Surface classes in the TTRS model.....33
3-4	Association and reclassification cases for TTRS.....35
3-5	Cylindrical slider joint and planar joint; a pictorial comparison.....37
3-6	3-D kinematic joints and their degrees of freedom.....38

	Page
3-7	The basic tetrahedron with the values of the basis points shown along with interaction and use of areal coordinates in the transformation from tolerance zone to T-Map.....52
3-8	Tolerance zone for size tolerance on a rectangular bar.....53
3-9	T-map for size tolerance on rectangular face. Octahedron containing the tetrahedron can also be seen.....54
3-10	T-Map for size tolerance specified on the round face.....56
3-11	Modification of the boundary of the T-Map by orientation tolerance.....57
3-12	The array of subsets for form (upper dices) and their companion possibilities for location (lower dices) within T-Map.....59
3-13	The effect of the material condition on the size of the T-Map.....59
3-14	Tolerance-Maps (a) for Part 1, (b) for Part 2, (c) for the assembly (Minkowski sum of (a) and (b)), (d) for the desired function, and (e) p'q' section of the fit of functional and accumulation T-Maps.....62
3-15	Frequency distribution of clearance corresponding to 3-D variation of target plane.....63
3-16	The pluses and minuses of the T-Map math model.....66
4-1	Algorithm for the mathematical model to cater for datum reference change and determination of manufacturing tolerances directly usable in manufacturing process plan.....77

		Page
4-2	The process flow for utility based upon Delaunay Triangulation to determine presence of a CMM reading of target feature to be inside tolerance zone.....	79
4-3	Coordinates of Design Tolerance zone for size tolerance specified on planar rectangular face.....	81
4-4	The end coordinates of the Manufacturing Tolerance Zone using Manufacturing Frame of Reference.....	84
4-5	Inline 1-D linear datum change.....	89
4-6	An example of inline 1-D linear datum change. The extreme end views (left and right) are the 2-D orthogonal views of the part shown in trimetric orientation in the center.....	90
4-7	A case of datum change involving different azimuth of datums for same target feature. Left most view gives the theoretical identification of the entities involved, central view is a 2-D view of the part shown in the trimetric view on the right.....	91
4-8	An example of datum change with different azimuth and offset involved. Target feature ‘T’ , Design datum ‘D’ and Manufacturing datum ‘M’ are identified. Remaining orthogonal and projected views are for clarifying geometry of part involved.....	92
4-9	The twisted case of datum reference.....	93
4-10	Twisted case of datum reference change with all three Euler angles involved	94

	Page
5-1	The planar part and the identified faces.....101
5-2	The associated G-graph for the part above. Face numbers are boxed while associated edges are represented by the continuous lines.....102
5-3	CF graph for the part under consideration.....103
5-4	A sample TCF graph shows how certain tolerances are defined as attributes of constraint that exists between geometric features.....104
5-5	CTF graph for the sample design blue print. Face f_4 is the target feature and face f_0 is the design datum feature.....107
5-6	Manufacturing Worst Case Tolerance Constraint Feature (MWCTCF) graph for given manufacturing process plan which is in need of validation.....108
6-1	Planar part with feature of interest f_4 dimensioned with face f_0 as the design datum in design blue print.....115
6-2	Tolerance analysis related details for the manufacturing plan #1 for feature of interest.....117
6-3	Tolerance analysis related details for the manufacturing plan #2 for feature of interest.....118
6-4	Tolerance analysis related details for the manufacturing plan #3 for feature of interest.....119
6-5	Tolerance analysis related details for the manufacturing plan #4 for feature of interest.....120

	Page	
6-6	Part under consideration and relevant feature is shown on left side. Top right shows top view. Bottom right shows tolerance zones involved.....	124
6-7	Calculation of parallelism tolerance limit as a replacement to flatness tolerance limit.....	128
7-1	Relevant details from Designer Blue print for part for case study.....	139
7-2	The part used for case study; the different faces are shown as identified..	140
7-3	Features involved and direction of cut in finishing operation of step 1.....	143
7-4	Features involved and direction of cut in finishing operation of step 2.....	144
7-5	Features involved and direction of cut in finishing operation of step 3.....	145
7-6	Features involved and direction of cut in finishing operation of step 4.....	146
7-7	Features involved and direction of cut in finishing operation of step 5.....	146
7-8	Features involved and direction of cut in finishing operation of step 6.....	147
7-9	End milling operation of face f_{10} with face f_8 as manufacturing datum (step 7).....	148
7-10	Edge milling of face f_{10} with face f_0 as the manufacturing datum (step 8).....	149
7-11	Shoulder milling of face f_8 with face f_0 as manufacturing datum (step 9).....	150
7-12	Face milling of face f_7 (step 10).....	151
7-13	Shoulder milling of face f_6 with face f_0 as manufacturing datum (step 11).....	152

	Page
7-14	Shoulder machining of face f_4 with face f_0 as manufacturing datum (step 12).....153
7-15	Shoulder machining of face f_2 with face f_0 as manufacturing datum (step 13).....154
7-16	Geometry of blank used for case study (step 14).....155
8-1	Different distribution types for various components contribute to assembly distribution.....175
8-2	Convolution of two uniform densities181
9-1	Re-D Prob maps for sections taken at intervals within range of total variation in plane tilt along x- axis (This figure is added to highlight bell shaped variation of Gaussian distribution for entire range of variation in plane tilt along x-axis).....193
9-2	Re-D Prob Map for section taken at Plane Tilt along x-axis =0.....195
9-3	Prob Map for size tolerance specified on rectangular face. Variables involved are size tolerance, plane tilt along x axis and plane tilt along y axis.....197
9-4	Another view of 4-D prob Map as given in figure 9-3. (Point to note is difference between the two views for additional information and pattern recognition).....198
9-5	Different views of 4-D prob Maps (a,b,c from Left to Right) (a). Looking from σ_2 side (b). Looking from σ_{8p} side (c). Looking from σ_{4p} side.....200

9-6	Algorithm break down of software developed for creation of prob Maps.....	205
9-7	(a to e from left to right) Floating of form subset within size T-map for rectangular face (size of figures reduced to save space; σ_2 , σ_1 , σ_{8p} and σ_{4p} are on top, bottom, left and right respectively).....	206
9-8	Effect of form refinement on size tolerance for pass percentage of parts manufactured (a to e from left to right top to bottom).....	208
9-9	Effect of orientation refinement to size tolerance for size T-map (orientation tolerance is 20% of plane tilt along y axis).....	210
9-10	Effect of tighter orientation tolerance on size T-Map as compared to scenario depicted in fig 9-9. (Orientation tolerance is 40% of the plane tilt along y axis).....	211
9-11	Two parts in stackup forming an assembly. Two dashed lines show extreme positions for part 1. Same does the dotted lines for part 2. L_1 and L_2 are characteristic lengths or dimensions for two parts respectively.....	212
9-12	Algorithm for code developed for determining pass percentages for assembly whose calculated statistical probability distribution is a convolution of part statistical distributions.....	216

- 9-13 (top) Half of the conformable q^{\prime} s sections of T-maps (on left) with its associated half section of accumulation T-map (on right) for parts in stack up in fig 9-11. (Bottom) Half of the conformable p^{\prime} s sections of T-maps (on the left) with its associated half section of accumulation T-map (on right) for parts in stack up in fig 9-11....218
- 9-14 Assembly Probability plot resulting from convolution of part probability distribution achieved for Minkowski Sum of part T-map for size tolerance specified on a rectangular face.....220

1 INTRODUCTION

1.1 Background

Parts are most unlikely to be manufactured to exact dimensions or geometric perfection under mass productions. A tolerance range defines the dimensional limits of variations that a feature on a part can have and still be considered interchangeable. It is generally believed that greater the tolerance range, there will be less rework or scrap of the parts and hence, cost savings. The inverse relationship of cost and tolerances is shown in the figure below:-

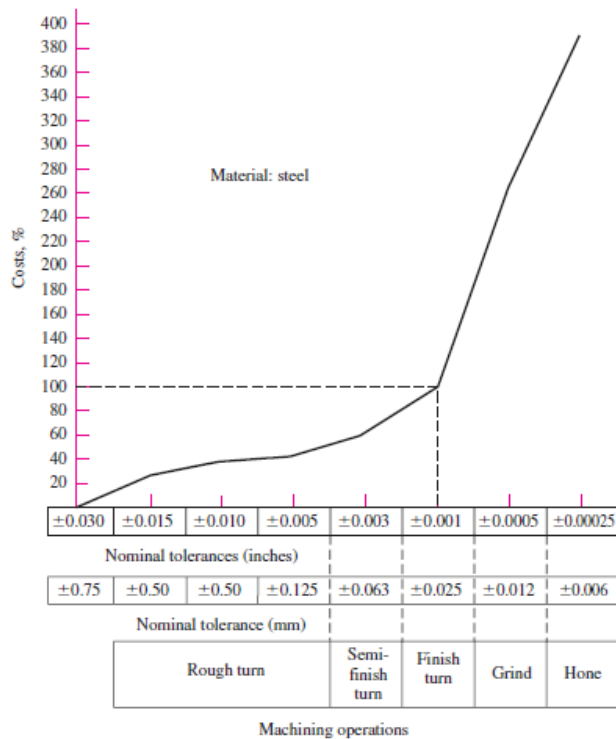


Figure 1-1 The relationship between tolerances and cost with respect to different machining operations. (Adapted from David G. Ullman, The Mechanical Design Process, 3rd ed., McGraw-Hill, New York, 2003.)

Geometric tolerances are the limits of manufacturing variations permitted by designers to ensure part function and assembly. Process planners are concerned with operation-based tolerance allocation to minimize manufacturing cost. Quality Assurance must verify that manufactured parts comply with the design specifications. Tolerances play an important role in all stages of product development as shown in the figure below:-

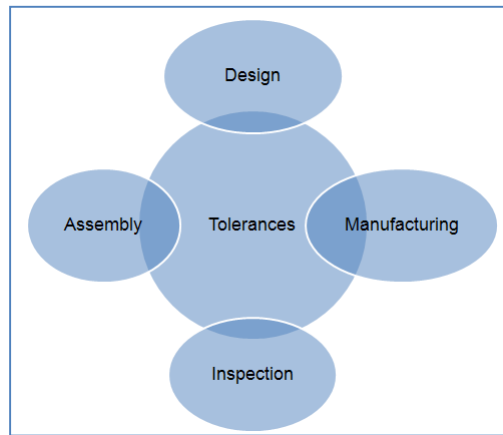


Figure 1-2 Role of tolerances in various stages of Product Life Cycle

Ideally tolerances should be as close to zero as possible but this generally requires more precise manufacturing processes at increased costs. Thus the challenge is to produce parts with as loose tolerance as possible, with minimal costs and yet ensure interchangeability. However, an insight view of the various stages of product life cycle show that different departments involved in the product development have contradicting requirement as shown below.

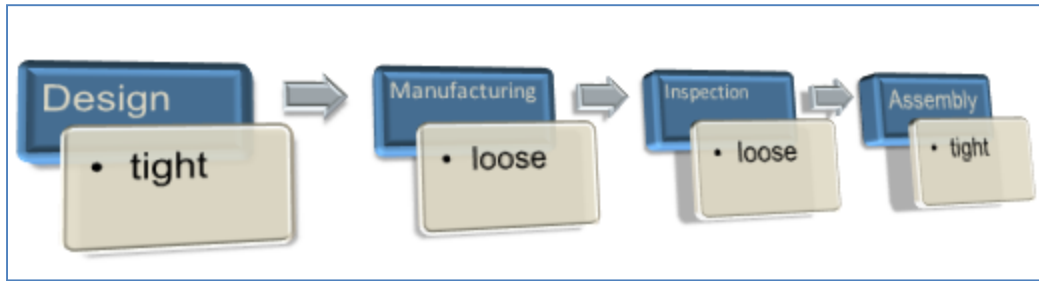


Figure 1-3 Change on the requirement of tolerances during various stages of Product Life Cycle

Product tolerances affect design, manufacturing, inspection, and assembly. However, tolerance assignment is one of the least well understood engineering tasks. A recent study [1] revealed that incorrect tolerance settings was one of the greatest causes of scrap, rework, and warranty returns. This is, in part, because Product Design Teams often pay a great deal of attention to ‘critical dimensions’ but pay little or no attention to tolerances of non critical dimensions, whose cumulative effect causes problems later in manufacturing.

The ultimate aim of the most of the manufacturing systems is to fabricate a product of consistent and repeatable quality at the minimum cost. This requires close coordination and communication between all engineering departments which include design, manufacturing and inspection etc. Lately the trend has been changed and the over the wall engineering has been replaced by Concurrent Engineering (CE) and Product Design Teams (PDTs). These teams closely monitor the various interactions of their design decisions as they affect product functions, manufacturability, service, reliability, and product quality.

Tolerance stackup indicates how individual processes or component feature tolerances can combine to affect a final assembly dimension. It is also known as 'propagation of error'. The ASU GD&T Math model is a bi-level model of geometric variation. In ASU T-Map model (explained later in detail), tolerance stackups are accomplished with the Minkowski Sum. The concept is introduced in the figure below and is covered in much detail in the sections to follow.

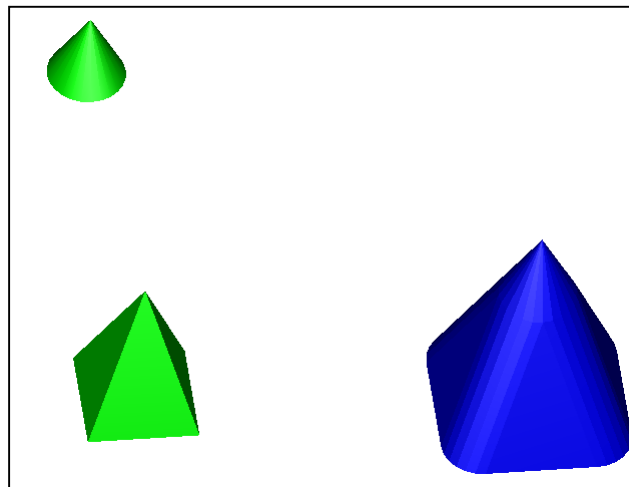


Figure 1-4 Minkowski sum of two T-Maps (semi sections only)

Traditionally, tolerance allocation has been based upon tabulated values, past design, rules of thumb, blanket tolerances and more recently, Computer Aided Design (CAD) system default settings. These methods are no longer in vogue due to heavy impact of accurately worked out tolerances on the production costs.

The main challenge in this scenario is the selection of an acceptable set of tolerances that will satisfy functional product requirement, manufacturing requirements, and quality requirements. In addition, with the adoption of ASME Y14.5M 1994 standard, Geometric Dimensioning and Tolerancing (GD&T),

proper choice of part dimension reference frames, and modeling of part feature dimensions can lead to vastly different assembly functions. GD&T is an ASME standard for geometric part representation, including nominal dimensions and tolerances. It is highly important as it provides a rigorous mathematical representation of feature dimensions and their tolerances.

ASME Y14.5M-2009[2] Standard provides a formal representation of tolerances and dimensional features. A standard is needed for unambiguous communication between design, manufacturing and inspection. The GD&T standard is a language for dimension and tolerance specification on engineering drawings. It follows a set of conventions and rules. It uses standard symbols, notations and syntax. It is part of the drafting standard. It is a feature-based specification standard (i.e. it treats surfaces, volumes, and similar geometric entities as continuous entities).

There are two main issues in tolerancing. The first one is called tolerance analysis which addresses the following question: - Given a set of individual component tolerances, what is the resulting assembly tolerance? The second is called tolerance synthesis or tolerance allocation in which the question of concern is: 'Given a required assembly tolerance, what should the component tolerances be?'

The main aim of tolerance analysis is the verification of the extent and nature of variation of an analyzed dimension or geometric feature of interest according to a given GD&T scheme. The source of the variation of the analyzed dimension is mostly from the accumulation of dimensional and/or geometric variations (other sources could be change in type of tool, machine, operator etc) in the tolerance chain. Following are relevant to the analysis:

1. **The contributors:** which are the dimensions or features that cause variations in the analyzed dimension?
2. **The sensitivities:** which is the relative contribution as compared with other contributors? Sometimes **percent contributions** are used for the same purpose.
3. **Worst case variations, statistical distributions and acceptance rates.**

All tolerance analysis approaches can be classified as following on the basis of:

1. **Dimensionality** e.g. 1-D, 2-D, 3-D or higher n-D.
2. **Acceptance rate** e.g. worst case (100 % acceptance rate) and statistical (less than 100 % acceptance rate)
3. **Type of variations** e.g. dimensional or conventional and dimensional and geometric according to the type of variations included.
4. **Scope** e.g. part level or assembly level.

Major analysis results of interest are:

1. Extreme values of independent variable
2. Frequency distribution of the dependent and independent variables

These results may be used in establishing the percentage of parts that would be acceptable for a desired specification.

In worst case analysis, 100% interchangeability is desired while in statistical tolerance analysis a predetermined percentage (less than 100%) of the assembly is acceptable. Relaxing the requirements for 100% interchangeability can cut cost if

the cost of rejected parts (out of tolerance assemblies or accumulated variation) is offset by the lower manufacturing cost of parts with less stringent tolerances.

The idea of statistical tolerancing emerges from the concept of total interchangeability of the parts. If two parts can be interchanged randomly within an assembly without affecting the function or quality of the assembly, they are considered interchangeable and hence, the same parts.

There are different methods used in tolerance analysis. These include but are not limited to:-

- a) Manual 1-D tolerance charts
- b) Linearized 2-D/3-D analysis
- c) Monte Carlo Simulation

A manual 1-D tolerance analysis chart is a manual book-keeping procedure for 1-D stack calculations. It can be applied to parts or assemblies. It can be applied to radial or linear direction. It can examine worst case only and different chart construction rules apply to different tolerance types.

For 2-D, 3-D analysis, A is a non-linear function 'f' of other variables d_1, d_2, d_n . Two common approaches for non-linear analysis are a) linearization of function f , leading to closed form solution and b) Monte Carlo simulation in which case, the results depend on the number of simulations.

Monte Carlo simulation is generally used in conjunction with parametric CAD. Each simulation represents one possible (and allowable) combination of d_i . In this process, random numbers are generated and values of d_i s are picked from pre-assigned values in tables which reflect the probability of each variable. In the

mean time, d_i values outside the tolerance range are discarded. Values of the desired variable A are calculated from the equation(s) relating A to the independent variables, using values d_i generated at random. After a large number of A values are obtained, simulation is stopped and distribution of A is determined which involves either tabulation or fitting to desired distribution.

It needs to be mentioned that determining assembly and component tolerances is mostly an iterative process. Typically, the required assembly tolerance is known and initial set of component tolerances are determined using traditional methods, such as handbooks or past designs. This is followed by a tolerance analysis to determine whether the initial component tolerance set results in an acceptable assembly tolerance. Often the need to go back and change specific component tolerances and recalculate the assembly tolerances arises. This process is repeated until a satisfactory set of tolerances capable of producing the desired yield has been established.

The emphasis has always been on providing a more rigorous mathematical representation of feature dimensions and their tolerances. This is motivated by increased inspection and the use of Coordinate Measuring Machines (CMMs). CMMs are high-precision, general-purpose, measuring instruments capable of measuring complex shapes. CMM consists of a contact probe and a mechanism to position the probe in three dimensions relative to surfaces and features of a part. They are finding increased use as on-line inspection machines in flexible manufacturing environments because CMMs may be programmed to measure a variety of dimensions automatically. CMMs and Boundary Representation

Computer Aided Design (B-Rep CAD) systems represent features by parametric models (i.e. a series of points and lines). The greater the number of points to model a feature, the closer the model approximates the feature.

The ASU GD&T Math model is a bi-level model of geometric variation. Global model portion relates all feature control frames to each other and models datum reference frames (DRF) and their precedence. It provides a basis for geometric validation of a D&T scheme and downstream tolerance analysis. On the other hand, local model portion includes modeling of the composite effect of all tolerances on a given feature. It clearly identifies interaction of size, form, orientation and position. All major characteristics of ASME GD&T standard Y14.5 M are embedded in the formulation. Its most promising benefit is that the relative volumes of regions can be used to study trade-offs in tolerance allocation. This research will provide an integrated view of 3-D manufacturing variations across all domains (design, manufacturing, inspection, quality etc) - the first such model ever developed. In the long run, it is envisioned that the math model and other concepts developed will ease the integration of GD&T standard Y14.5 with the inspection standard B 89.3.2 and CMM standard B 89.4.10.

1.2 Problem Statement

There are two problems that are intended to be investigated. First is the three dimensional model for manufacturing tolerance transfer. For the past three decades, industry professionals have been using the one dimensional tolerance charts for determining manufacturing tolerances during various phases of

machining of the components such that eventually, the tolerances as specified by the designer are met. Geometric tolerances have three dimensional associated tolerance zones and hence, it is very logical that affect of the specified tolerance will impact all the three dimensions. Some researchers have hesitantly and lately looked into the manufacturing process plans for three dimensional analysis but their technique appears to be purely theoretical and far from being practical. The intention of this research is to develop a model that is user friendly in the sense that it is based upon the inputs that are readily available to the manufacturing supervisor. Also, it is desired that the method should not be based upon the parameters for which the accuracy of determination is itself a big question. In addition, the orientation of the manufacturing datum with respect to the design datum is yet to be investigated in its role on the manufacturing tolerance transfer.

The second aspect of the research is to investigate the effect of individual part statistical distributions in the resulting statistical distributions of the assembly. Statistical tolerance analysis still remains limited to determining the moments of the distribution. Same moments can be attached to absolutely different statistical distributions (such as two parameter or four parameter etc) or two different types of distribution (such as standard or non standard etc). Hence, need for the use of the entire distribution and not only the moments of the distribution is deemed necessary for statistical tolerance analysis. Lately, some researchers have tried to employ certain assumed statistical distributions in their tolerance analysis. Monte Carlo method has been used for quite some time for tolerance analysis purposes but this method has its limitations and inaccuracies well known in the industrial

circles. The intent is to come up with a model in which assumed statistical distributions of any type could be used. Also, the interrelations of the various parameters that affect the pass percentage are yet to be taken care of.

1.3 Proposal outline

The various chapters in the proposal are arranged as mentioned below:-

Chapter 1 contains introduction which includes background such as tolerances, ASME standard, tolerance analysis, tolerance synthesis, Coordinate Measuring Machine (CMM) etc that is relevant to the research and are intended to familiarize the reader with basic areas that are of concern in this research.

Chapter 2 deals with the rationale of tolerance control and its relevance to this research. It explains and critically evaluates the tolerance control approaches in vogue and ends with an introduction to the currently researched tolerance control techniques.

Chapter 3 gives the details of the literature review of the math models for tolerance representation. It also includes the progress of the models which have also dealt with the tolerance analysis using the same model. The chapter concludes with a table comparing the different models for the various parameters of interest.

Chapter 4 presents the three dimensional model for tolerance transfer in manufacturing process planning. The idea and methodology behind the model is explained in detail followed by the case studies of various datum change scenarios

and different types of datum features to which the model can be successfully applied.

Chapter 5 presents the graph theoretic approach to relationship representation and tolerance synthesis for manufacturing process planning which contains some details about the use of ACIS, MFC, C++ and data structures which can be used in the computer implementation of the model.

Chapter 6 dilates about the possibility of using different types of geometric variations for representing a particular type of geometric variation. A relevant analytical formula for one such case has been derived and explained.

Chapter 7 lays down the step by step procedure for determining the dimensions and tolerances for the manufacturing process plan. A detailed case study is included.

Chapter 8 discusses various methods of statistical tolerance analysis which have been used/presented by various researchers over a period of time. This chapter also introduces convolution at the most elementary stage.

Chapter 9 presents a new statistical model for tolerance analysis based upon Joint Probability distribution of the trivariate normal distributed variables. The use of Delaunay triangulation and ray tracing algorithms is explained for the purpose of determining pass percentages of both part and assemblies. Convolution of two part distributions to arrive at the statistical distribution of the assemblies is also discussed.

Chapter 10 presents the closure of the document with the results achieved emphasized.

Appendix contains the literature review of the computer aided tolerancing field. This chapter contains the evaluation criteria for commercial tolerancing softwares and also critically reviews the progress and capability of each of the softwares.

2 TOLERANCE TRANSFER IN MANUFACTURING

Tolerance transfer (also called tolerance control) is the process of ensuring the achievement of designer's tolerances through the process of specifying intermediate level tolerances based upon the manufacturing process involved in the steps leading to the manufacturing of the part.

In other words, tolerance transfer is the set of practices that are used to ensure that parts are made within their specifications.

For this research, it is assumed that manufacturing will be done solely through machining. Hence, all other methods for manufacturing will not be discussed here.

2.1 Rationale for tolerance transfer (Tolerance Control)

Why is there a need to specify another set of tolerances to ensure achievement of the designer's tolerances?

This could be because of several reasons:-

1. There may be several processes occurring at inconsecutive order. Hence, the designer's final tolerance is to be broken down to tolerances for each individual intermediate steps.
2. The datums from which the designer specifies the final tolerance may not be available in the start of the process. Hence, different datums have to be specified than designer's specified datums.
3. The designer does not include stock control mean values and its tolerances which are unavoidable from the point of view of manufacturing.

4. The designer is not an expert in all of the processes or the machines that will be used for the manufacturing of the part. It is the manufacturing planners that will decide on which machines will be used to achieve the final shape dimensions.

5. In continuance of the above mentioned point, the designer's tolerance plan is general and is meant to be applicable under different circumstances such as:
 - a. Different sets of machines(based upon the accuracy levels of the machines),
 - b. Different level of expertise between a set of operators,
 - c. Different levels of processes (depending upon the process capability indices) etc.

The reasons for tolerance control / tolerance transfer are summarized below:-

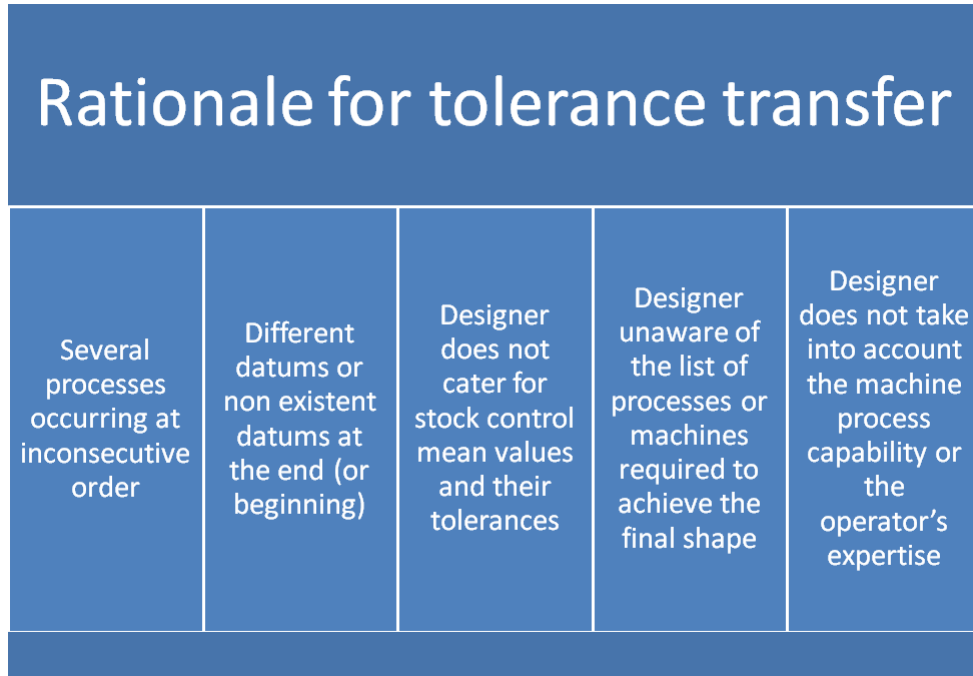


Figure 2-1 The reasons for the requirement of tolerance control

It is the job of the manufacturing planner to devise a plan which is specific to his/her floor scenario so that the ultimate mission of achieving the designer's tolerances is a success.

2.2 Manufacturing Tolerance Chart: A definition

A Manufacturing Tolerance Chart (MTC) is a type of a tolerance chart that graphically represents a process plan (limited to tolerance analysis purposes) and is used to manually control, the tolerance stack up when the machining of a component involves interdependent tolerance chains.

2.2.1 Types of tolerance charts

There are many types of tolerance charts in use these days. Also, tolerance charts are used in different ways. A few of these are mentioned below:-

1. Tolerance chart for calculating the tolerance in a stack up

This is the older type of tolerance chart that is very familiar and had been around for quite some time. This type of chart has been successfully automated.

2. Tolerance chart for analysis of existing mean dimensions and their tolerances to determine if the manufacturing of the part according to the blue print is viable

This tolerance chart presents a passive activity and is used mostly in the manufacturing environment only as a check.

3. Tolerance chart for evaluating working mean dimensions and the associated working tolerances required by a manufacturing process :
Manufacturing Tolerance Chart (MTC)

This is the type of chart that is of most interest over here. Although like anything else, it does have its limitations, but whenever a new blue print is received from the design department, the use of this tolerance chart is an option. [3]

The various types of tolerance charts are shown in the figure below:-

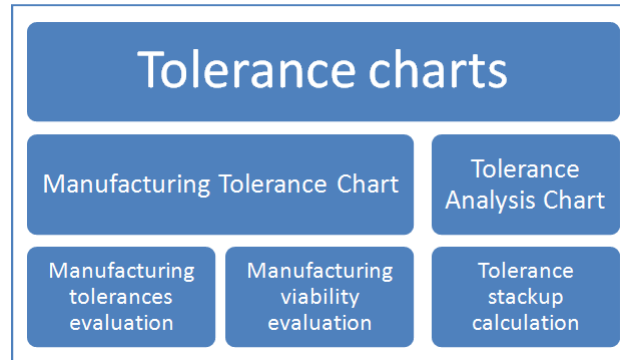


Figure 2-2 Different types of Tolerance charts

2.3 Use of 1-D Manufacturing Tolerance Charts

Use of 1-D Manufacturing Tolerance Charts has been in vogue since World War II and still, they are very common in the industry circles to evaluate the tolerances and the working dimensions at every step of the manufacturing process. All calculations are performed along a single axis or direction.

A Manufacturing Tolerance Chart is a graphical tabular tool that depicts the contributing individual machining cuts which combine to produce blue print dimensions. The process leads to a set of linear algebraic equations which are representative of the relationship between each desired blue print dimension and the individual contributing manufacturing operation. No doubt, it is a very handy tool for the process planners. That is why it is very common in industrial circles all around the globe. Tolerance control is inevitable these days for the production of high precision parts at low cost.

The detail about the chart is given in the following paragraphs.

2.4 Stages involved in the development of Manufacturing Tolerance Charts

There are three pieces of document that are involved in the development of the manufacturing tolerance charts:-

2.4.1 Blue print

This is the document which shows the final shape and the final dimensions along with the related tolerances as set out by the Design personnel.

2.4.2 Strip layout

This document has all the operations listed out along with a short description of the operation, name of the machine involved, along with the figure of the part with different operations shown along with the affected features.

2.4.3 Tolerance Chart

This is the final product and it contains the working mean dimensions for each operation, along with the working tolerances. It also contains the stock removal means and the related tolerances. It also shows the balanced dimensions along with the related tolerances and also shows the blue print dimensions and the resultant dimensions along with the related tolerances.

The various documents involved in the construction of the manufacturing tolerance charts are shown below:-

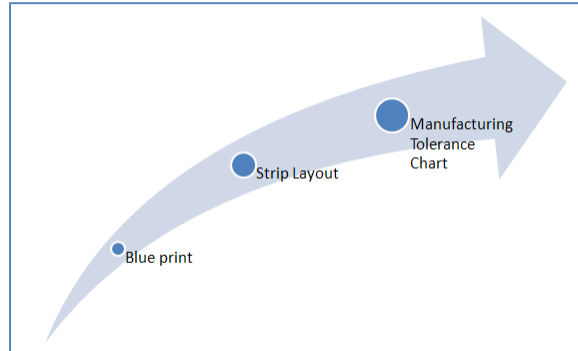


Figure 2-3 The documents involved in the construction of the manufacturing tolerance charts

2.5 Pros of Manufacturing Tolerance Charts

These are listed below:-

1. Unavoidable from manufacturing planner's point of view
2. Easy to make and use
3. Flexibility and adaptability

These can not only cover 1-D but also 2-D and 3-D (may be) scenarios (with limitations).

4. Open to different methods of tolerance allocation

The manufacturing tolerance charts are not limited to any one type of method for tolerance allocation.

2.6 Cons of Manufacturing Tolerance Charts

These are listed below:-

1. Chart specific to a situation

Each manufacturing tolerance chart is specific to one particular situation and a slight change in one parameter (mean dimension or the tolerance) can lead to entire new sets of calculations and hence, a new manufacturing tolerance chart.

2. Selection of one basic mean dimension a must

To start with, at least one basic mean dimension out of the ones given in the blue print (not necessarily the same value) has to be selected to proceed with the manufacturing tolerance chart.

3. Ineffective use of entire tolerance range

A major setback of the manufacturing tolerance charts is that it is, in most cases, unable to use the entire tolerance range given by the Designer. In other words, the resultant tolerances, in most cases, are smaller than the ones specified by the Designer.

The pros and cons of the manufacturing tolerance chart are summarized in the figure below:-

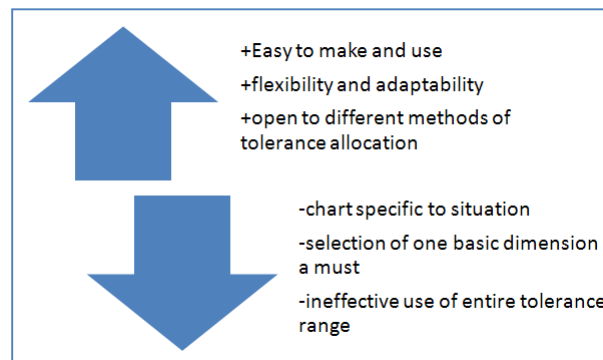


Figure 2-4 Pros and Cons of the Manufacturing Tolerance Charts

2.7 Overall analysis of the Manufacturing Tolerance Charts

When only one part is being made, the machinist zeros out each completed feature and uses the zeroed out feature as a datum for machining the next feature. In this way, the tolerance stack ups are bypassed. However, when fabricating parts in a lot, the datum surfaces have to be set up by the production engineer based on a selection of fixture locating surfaces and depending on cutting tool design layout decisions.

With Numerically Controlled (NC) machining, reduced tolerance stackup can be achieved by the following:-

1. Machining cuts as per the blue print
2. Eliminating manual control of machine decision affecting the cut
3. Reducing the number of locating surface changes
4. Reducing the number of attendant fixturing

However, not all tolerance stackups will be eliminated by using NC machines.

2.8 Limitations of Manufacturing Tolerance Charts

For limitations, the capabilities of the system are relevant while for Cons, undesirable elements of the system are relevant. The limitations are listed below:-

2.8.1 Non-efficient use of Design tolerance range

The main limitation of Manufacturing Tolerance is that although it can ensure that design tolerances are met, it is unable to ensure that the entire tolerance range as specified by the designer will be utilized by the manufacturing plan.

2.8.2 Non- proactivity of the method

Additionally, Manufacturing Tolerance Charts are not proactive. A Manufacturing Tolerance Chart is only ready to be made when certain of the engineering decisions have already been made. These include but are not limited to:-

1. The machine selection for each operation
2. The sequence of operations to be performed
3. The selection of the locating or datum surfaces
4. The dimensioning patterns for the cuts to be made in each operation
5. The selection of the type of tooling to be used for each operation

When these decisions are such that these will not turn out the most efficient tolerance values, then ends can be mended with use of higher accuracy tools than what are used normally. This leads to higher production cost.

2.8.3 Limitation of use of equal bilateral tolerance system

This limitation does not have any profound effect while using the manufacturing tolerance charts in the deterministic mode but it is feared that on exploring the statistical mode of the manufacturing tolerance charts, this limitation might prove significant.

2.9 Related trends in tolerance control

Sequential Tolerance Control (STC) is the process in which the results from the earlier operations are used to locate appropriate set points for later operations during the execution of a process plan.

In probabilistic search method for Sequential Tolerance Control, Nelder-Mead downhill simplex method is used to optimize an estimation of the expected process yield. This technique is as effective as Sphere-Fitting methods for normally distributed process deviations, but for skewed distributions, the probabilistic search method has yielded better set points than previous methods in different research studies.

3 LITERATURE REVIEW: MATH MODELS

This chapter deals with the explanation of the math models. The math models discussed over here are not only the ones that are used for tolerance representation solely but also, the ones that have been advanced into the tolerance analysis phase as well.

3.1 Math models for tolerance representation

3.1.1 Parametric models

Parametric model consists of a set of correlated mathematical relations in which diverse situations are defined by means of varying the values of a set of fixed coefficients (parameters). Other commonly used names for it are relational model or constraint based model. However, in all, tolerances are represented as +/- variations in the dimensions. In parametric model, constraints are solved by assigning values to the model variables sequentially where each assigned value is computed iteratively.

Hillyard et al [4] proposed a system that specified geometric constraints between part co-ordinates so that possible variations were restricted by a range given by certain particular tolerances.

In order to carry out dimensioning, they regarded an object as an engineering frame structure whose members and joints correspond to the edges and vertices of the object. The members were initially of unconstrained length and all the joints were pin- joints. Adding a stiffener to the frame was considered adding a dimension to the object. Various types of stiffeners and their combinations were

defined depending on the dimensions which they fixed e.g. a distance, an angle or a plane.

All stiffeners carried some information such as separation and a real number. Separation was stored as a unit vector with a magnitude while real number was for tolerance. They successfully showed that small geometric variations could be related to dimensional variations through a 'rigidity matrix'. The results were shown for polytopes in 1-D, 2-D and 3-D and it was expected that these results could be extended to spaces of higher order depending on the degrees of freedom.

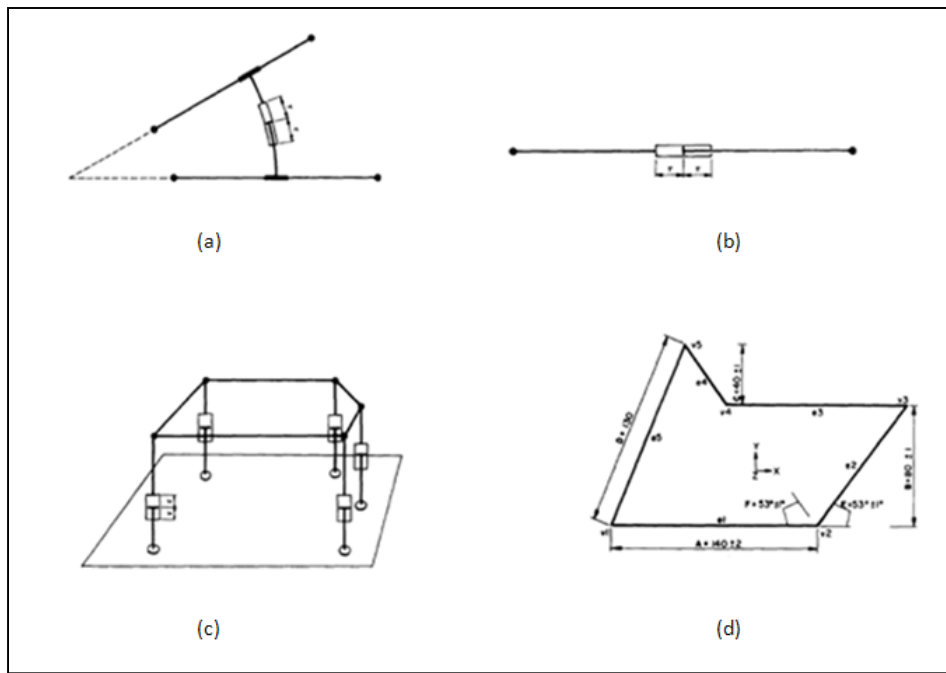


Figure 3-1 (a) web (b) strut (c) plate (d) A Dimensioned and Toleranced Polygon [4]

Hillyard and Braid [5] further enhanced the idea by visualizing the data structure defining an object as a pin-jointed, infinitely elastic wire frame covered all around by elastic membranes. They regarded shape descriptions as dynamic mechanism

rather than as static entities. They promised that engineering designers would be able to question the description to explore derived geometric quantities and production engineers will be able to explore the tolerance information required for manufacture planning.

Gossard and Light's work [6] paved the way for the generalization of the model by providing novel mathematical and geometrical tools to geometric representations. They used three-dimensional constraints between the characteristic points to identify an object's geometry while the alteration of the geometry was achieved by altering one or more constraints. A matrix method was used for the shape determination of the part through simultaneous solution of constraint equations.

Simply said, the parametric models of Hillyard and Braid represent distance relations between points, lines and planes. Hence, the CAD model is driven by key dimensions. The relations were represented as algebraic equations that could be solved sequentially or simultaneously. The sequential solution was limited to uncoupled equations. The tolerances were added as +/- variations of the dimensions.

Gossard and Light [7] depicted a fundamental approach to adapt a geometric model, a procedure for significant reduction of the number of constraint equations to be solved and the effect of sparse matrix methods in reducing the time required to solve the equations.

The constraints were represented analytically by nonlinear equations of the following form

Equation 3-1

$$F_i(x, d) = 0 \quad i = 1, 2, \dots, m$$

Where d = the vector of dimensional values

x = the geometry vector

m = the number for constraints

Using Newton-Raphson method, change in the geometry vector for each iteration is found by solving the matrix equation:

Equation 3-2

$$J \Delta x = r$$

Where J = is the Jacobian, a $(m \times n)$ matrix containing the partial derivatives of each constraint equation with respect to each degree of freedom

While Δx = the vector of displacements, given by

Equation 3-3

$$\Delta x = \{\Delta x_1, \Delta x_2, \dots, \Delta x_n\}^T$$

And r is the vector of residuals, given by

Equation 3-4

$$r = \{-F_1, -F_2, \dots, -F_m\}^T$$

The sparse matrix methods can be used for the solution since the Jacobian is a sparse.

Harwell Subroutine Library (a collection of Fortran 77 and 95 codes that address core problems in Numerical Analysis) enables to have the solution in $O(n) + O$

(τ) time where the size of the matrix is $n \times n$ and τ represents the number of non zeros. This is a substantial savings over the Gaussian reduction method which takes $O(n^3)$ time.

Although, their effort was mainly concentrated on the definition and modification of geometric model, they suggested that the method can be used for direct analysis of a tolerancing scheme.

They pointed out that system could compute the maximum variation in dimension (tolerances) in order to satisfy the specified tolerances. They suggested using this by the designer for having a quantitative basis for specifying tolerances. Gossard et al [8] explained a technique to automatically translate changes in dimensional values into related changes in geometry and topology.

As the model is based on satisfying constraint through explicit sequencing, parametric models are rapid. As parametric models work with variation of dimensional parameters and not the tolerance zone, none of the geometric tolerances can be implemented; similarly, neither Datum Reference Frames (DRFs) nor directed datum-target relations have been incorporated. Parametric models are unable to discriminate between different types of variations/tolerances.

The approach does not model the tolerance zone.

The model has not been completely successful in the tolerance analysis of 3-D profiles due to partial success in solving constraint equations simultaneously in 3-D and the equations are normally written for the length of straight line, limiting application usually to polyhedral parts only.

3.1.2 Variational Surface models

In this model, tolerances are associated with surface definitions spanned by the model variables. With the change of model variables, the boundary surface of a variational part is permitted to vary independently. Variational surfaces are used to calculate the positions of the vertices and edges.

Turner et al [9, 10] proposed that each surface is varied independently by changing the parameter values. These parameter values are in turn used to calculate NURBS/ B-Spline surface coefficients. Alternatively, each surface is broken into several small patches and each patch is fitted with a standard higher order patch. The approach was applied to solving the problems of eliminating rigid – body motion, handling incidence and tangency constraints and modeling form variations. No relation could be established between the parameters of the higher order surfaces and the standard tolerance classes. Also, this model is not very efficient with highly non-linear relationships and is computationally expensive. Still, this model has also been used for automated tolerance analysis.

Roy et al [11] applied a computational scheme for geometric tolerance representation and interpretation on polyhedral objects. Variations were applied to a part model by varying each surface's model variables which were in turn constrained by relations derived from tolerance zones.

Yau [12] compared the measurement data with a nominal CAD model and hence, offered a CAD model-based approach for examining form tolerances using non – uniform rational B-splines (NURBS). Since coordinate measuring machines (CMM) are more flexible in measuring dimensions and evaluating tolerances,

integration of CAD and CMM is an important aspect of overall manufacturing process and quality assurance. The classical methods generally construct substitute geometric features from the measurement data. However, he evaluated these features from tolerances by making a comparison of the measurement data with a nominal CAD model.

3.1.3 Offset Zone model

Offset zones are obtained by offsetting the nominal boundary of the part by an amount equal to the tolerance specified on either side of the nominal. In worst case analysis, offsets for the maximal and minimal objects are achieved. Tolerance zone is the region in between these zones and this is where the frontier of the part must lie.

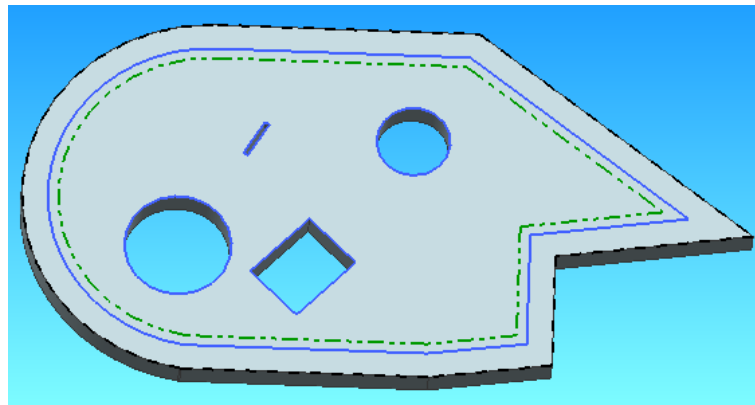


Figure 3-2 Offset Zone shown on the CMM arm hinge. The continuous line (blue) marks the ideal boundary while the dashed line (black) shows a positive (increased material) offset. The inner most dashed-dotted line (green) shows a negative (decreased material) offset.

Requicha [13] while proposing this model, pointed to tolerance specification as a collection of geometric constraints on an object's surface features which are in turn referred to as two dimensional subsets of the objects' boundary. He [14] disproved the CSG's incapacity to deal with features and tolerances. Requicha mainly employed Minkowski operations for carrying out offset activities.

A trivially diverse version of the model can be attributed to Jayaraman and Srinivasan [15, 16] which they called Virtual Boundary Requirements (VBR) (half space) approach. They introduced the concept of Conditional Tolerance Zones (CTs) which were generated thru the offsets of half spaces. The model is useful for measurements by Coordinate Measuring Machine (CMM), Process planning and control and statistical tolerancing. However, CTs cannot be explicitly derived from half spaces and this work is currently restricted to a few specified features.

Offset zone model is not conformant to Y14.5 as it is not capable of handling interaction and coupling of various tolerances since it requires separate tolerance zones for each type of tolerances on the same feature. Datum Reference frames cannot be modeled also as is the tolerance applied to derived features e.g. mid planes and axis.

3.1.4 DOF model

In DOF model, the primitive geometric entities e.g. points, lines, planes are treated as if they were rigid bodies with degree of freedom (DOF). The global GD&T model developed at ASU is based on this approach. The idea was first pioneered by Kramer [17] once he defined a general purpose symbolic system to

reason out assemblies based on degrees of freedom and relative constraints. Bernstein and Preiss [18] described the same idea independently.

3.1.5 TTRS model

In TTRS model, Clement et al [19] determined seven elementary surfaces including planes, cylinders, spheres etc which were unchanged by any displacement and rotation. For these surfaces which were called ‘Technologically and Topologically Related Surfaces’ (TTRS), they specified 28 different geometric relationships corresponding to 44 reclassification cases and the remaining degree of freedom for each combination. A TTRS is a pair of surfaces that represents a unique part or product and is associated by functional relations. For each tolerance related to these TTRS, the tolerance zone was represented by a torsor which is a six dimensional vector containing three displacement and three rotation values.




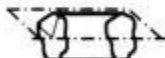
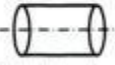
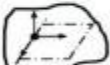

Surface Classes	Degrees of freedom	Surface Classes	Degrees of freedom
 1- Spherical surface	3 0 translation 3 rotations	 5- Surface of revolution	1 0 translation 1 rotation
 2- Planar surface	3 2 translations 1 rotation	 6- Prismatic surface	1 1 translation 0 rotation
 3- Cylindrical surface	2 1 translation 1 rotation	 7- Complex surface	0 0 translation 0 rotation
 4- Helical surface	1 1 combined translation and rotation		

Figure 3-3 Surface classes in the TTRS model [19]

Desrochers, A. & Clement, A [20] discussed that any part can be represented as a tree formed of succession of binary surfaces. They represented each surface association as a TTRS object by a set of minimum geometric datum elements (MGDE). This MGDE is basically a system of minimum datum- reference frames needed for each type of tolerance. However, there was no mean of distinguishing between the various types of variations and also, there was no consideration for datum precedence. Clement et al (1997) & Srinivasan (1999) also introduced the concept of Minimum Reference Geometrical Elements (MRGE) which are the situation elements e.g. Bezier's surface polygon or a cylinder axis for specifying the relative position for surface A in relation to surface B.

	7- Complex surface	6- Prismatic surface	5- Surface of revolution	4- Helical surface	3- Cylindrical surface	2- Planar surface	1- Spherical surface
7- Complex surface							
6- Prismatic surface							
5- Surface of revolution							
4- Helical surface							
3- Cylindrical surface							
2- Planar surface							
1- Spherical surface							

Legend :

- Lines coaxiality or points concentricity.
- Lines parallelism.
- Plane - line perpendicularity.

Figure 3-4 Association and reclassification cases for TTRS [21]

Based on TTRS model, Salomons et al [22] developed special software to aid the designer in analyzing and specifying the tolerances. In this tool, sets of equations were generated, based upon the number of points at which quality of the assembly is judged. This number is in turn dependent on the nature of surface association. Most critical direction of assembly which was called Virtual Plan Fragment Direction (VPFD) is determined using Virtual Plan Fragment Table, similar to the plan fragment table used in DOF analysis.

B. Anselmetti [23] proposed an approach based on the expertise of the designer called, 'method CLIC' (French abbreviation) which is in accordance with ISO [24] and ASME standard.

Desrochers [25] applied the tolerance transfer techniques to TTRS model and simulated tolerance chains or stack up which were generated according to paths or loops on the TTRS tree. Tolerance transfers are inevitable when design specifications cannot be achieved directly in one single machining operation. In terms of TTRS representation, tolerance transfer will ensue from the difference between the TTRS design and process plan trees.

3.1.6 Kinematic model

In the kinematic model, the degrees of freedoms are used but now these degrees of freedom are associated with different types of kinematic joints.

Rivest, Clement and Morel [26] were the first to pinpoint the kinematic facet to tolerance analysis. They proposed kinematic formulation of full 3-D geometric and dimensional tolerances.

In this model, a kinematic link is utilized between a tolerance zone and its datum feature. It treats the mating conditions as corresponding kinematic joints and variations in certain non-fixed components of the six degrees of freedom of the joint are consumed to incorporate the geometric tolerances.

Leo Joskowicz, Elisha Sacks, and Vijay Srinivasan [27] documented a general method for worst-case limit kinematic tolerance analysis in which the tolerance specifications on the part were utilized for computing the range of variation in the kinematic function of a mechanism. They called it 'Kinematic Tolerance Space'

which is a model of kinematic variation. They derived properties of this space which is an expression of the relationship between the nominal kinematics of mechanism and their kinematic variations. An efficient kinematic tolerance space computation program for planar pairs with two degrees of freedom is developed based on these properties. It claimed capturing both the quantitative and qualitative variations in the kinematic function due to part variations. This method is all inclusive for all types of mechanisms with parametric or geometric part tolerances.

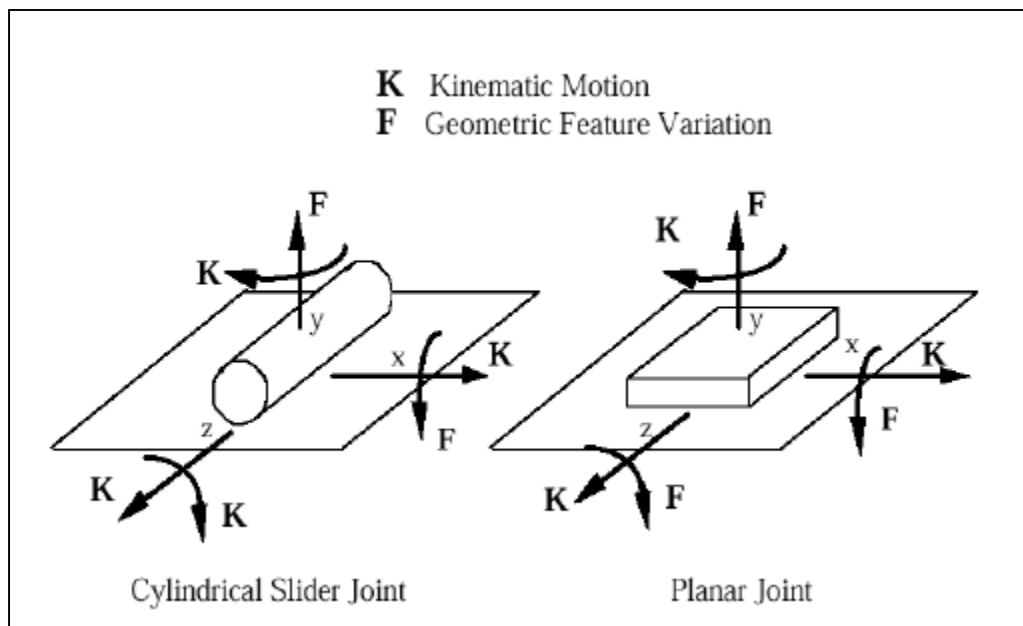


Figure 3-5 Cylindrical slider joint and planar joint; a pictorial comparison [28]

Kyung and Sacks [29] explored the non linear side of this work. In their case, the part profiles consisted of line and circle segments instead of planar pairs. A vector of tolerance parameters with range limits is used to parameterize the part shapes and motion axes. This work analyzed the system in two steps. The first step

involves the construction of the contact zones which are generalized configuration spaces that bound the worst –case kinematic variation of the pairs over the tolerance parameter range. In the second step, composing of the zones is carried out by bounding the worst-case system variation at designated configurations.

Chase et al employed this model in the study of tolerance analysis and synthesis by representing contacts within mechanisms using kinematic connections. They established vector loops around a functional requirement which led to a matrix of connectivity. This system was initially limited to 2-D but later, Chase, Gao and Maglegy [28] enhanced its applicability to 3-D by exploiting Hessian matrices. The **Hessian matrix** is the square matrix of second-order partial derivatives of a function.

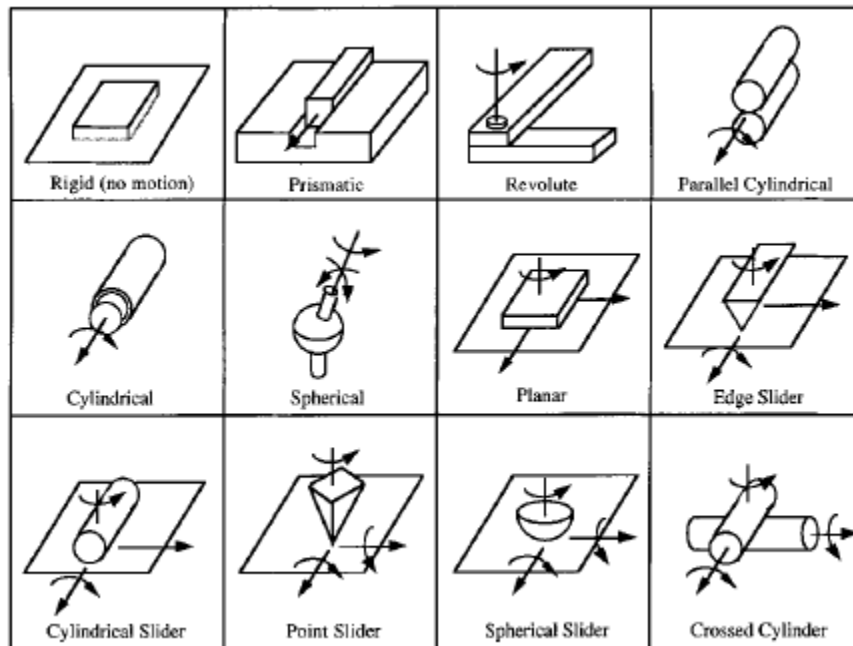


Figure 3-6 3-D kinematic joints and their degrees of freedom [28]

Laperriere and Lafond [30] presented the model that associated a set of six virtual joints, to every pair of functional elements in a tolerance chain. These virtual joints were used to furnish for position and orientation tolerance on two functional elements within the same part. The resulting six equations relate the new position and orientation of a point of interest in the chain (in Cartesian space) to the small dispersions of the functional elements of chain (in joint space).

Kinematic model cannot integrate Rule #1. However, it can be utilized to depict floating zones. Form tolerances cannot be built-in. They have not been made to hold information such as DRFs. This model cannot be extended to combine the interaction of geometric variations with size dimensions. This model is yet to be ASME Y14.5 Standard compliant. The model has not been shown to cater for directed datum target relations. However, the effects of bonus and shift can be positively included as well as datum precedence.

3.1.7 Vectorial Model

In this model, the position, orientation, form and size tolerance for a part are represented by four vectors. Two parameters are coupled to each vector to represent a) nominal state and b) variation. A real surface is defined by the vectorial addition of the nominal states and the variations. Writz [31] was the first one to use this approach. This model is also known as Vectorial Dimensioning & Tolerancing (VD&T) model.

Georg Henzold [32] carried out the comparison of vectorial tolerancing and conventional tolerancing. Martinsen [33] presented examples of application of vectorial tolerancing to manufacturing systems. Krimmel and Martinsen [34]

showed the application of vectorial tolerancing to analyze the interface between the forging process and the machining process. Bialas [35] identified problems like definition of co-ordinate systems or interference of size with form/position tolerances. Bialas, Humienny and Kiszka [36] discussed problems during conversion of ISO-tolerances related to planes and cylinders.

One of the biggest advantages of this model is that it can be used to pinpoint the erroneous manufacturing method based upon the defects in the surface generated. However, form tolerances cannot be adequately represented and need an extra column in the table or have to be specified exclusively on the drawing. VD&T model does not however, contain any information about the local sizes of the part. Additionally, it is cumbersome to stipulate conditions such as envelope condition, maximum material requirement or least material requirement.

Desrochers [37] presented the matrix approach for tolerance representation using homogenous transforms. This approach is intimately coupled to the notion of constraints. This model has been successfully employed in the demonstration of clearances and the transfer of tolerances. Desrochers [38] suggested combining the two existing models: Jacobian model which is based on the infinitesimal modeling of open kinematic chains in robotics and the tolerance zone representation model, using small displacement screws and constraints to establish the extreme limits of variation of point and surfaces.

3.1.8 Multi-Variate Region model

In this approach, geometric entities (planes, lines, circle, cylinder etc) are mapped to a hypothetical vector space defined by the basis points or lines corresponding to geometric variations.

The most developed model under this approach is the ASU local level model called a T-Map as its flag star. In order to understand the local model, more clearly, it will be worthwhile to look at the various aspects of the T-Map first.

The important characteristics of the t-map are listed below:-

- a) -The overall dimensions of the T-Map are dependent on the values of the tolerances applied.
- b) -Its shape is dependent on the shape of the feature and the type of tolerance. Hence, for rectangular shaped feature, the T-Map is a right- rhombic dipyramid, for a square, it is right square dipyramid while for a round feature, it is a dicone.
- c) -Each T-Map is oriented in the point space w.r.t. another T-Map depending on the orientation of the feature that it represents.
- d) -Accumulation of variations can be represented by Accumulation T-Map which is a Minkowski sum of two or more T-Maps.

- e) -For the target feature in a part/assembly, a functional T-Map is created which assumes perfect manufacture (zero tolerance) for the parts / features in the stack up (except for the target feature).

The idea of T-Maps was proposed for the first time in a NSF proposal by Davidson and Shah in 1998. Mujezinovic (MS thesis 1999) developed 3-D T-Maps for regular block and regular cylinder with size, form and orientation tolerances applied. Orientation tolerance has the effect of chopping the T-Map for size in all orientation directions while form tolerance causes the T-Map to vary in its scale which gives a series of T-Maps. Tolerance allocation for an assembly of rectangular or cylindrical parts was also modeled along with the stack up. This work also included the development of stack up for an assembly of part with offset along with some basic idea about the use of statistics with T-Map. He demonstrated the use of the model to distinguish the effects of different datum sequencing on the allocation of orientation tolerances.

Davidson and Shah (2002) developed T-Map for lines (axis of cylindrical feature) with position tolerance. This T-Map was developed using five screws with different orientation and position. These five screws represented the variations in position of the axis. Tab and slot case was dealt by Davidson and Shah (2003) with material modifiers applied at the part level.

Bhide (MS thesis 2002) further enhanced the 5-D model for the axis of cylindrical feature by including the orientation and form (straightness) to the already existing T-Map for position tolerance. He especially discussed the interaction of the

tolerances and its effect on the T-Maps for two coaxial holes and a slotted block with two pairs of coaxial holes. This work also included study for T-Map for pin and hole (cylindrical surface) having size, position, form (cylindricity) and material modifiers. He also developed T-Map for cylindrical holes for various tolerance zone shapes. He also investigated the effect of selection of datum and the sequence of datums on the shape of the resulting T-Map. He further extended the model for stack up and tolerance allocation for position in concentric bearings and stepped shaft.

Ameta (MS Thesis 2004) created T-Map for different specifications on angled faces. He carried out a comparison of the permitted variations with different specifications on angled face by comparing the volumes of the corresponding T-Maps. He performed tolerance analysis on an assembly with three parts stacked vertically using T-Maps for angled faces. This work also includes combination of features to control the invariant degrees of freedom of each feature.

Ameta also created a point line cluster and utilized it for analyzing the picture frame assembly. Volumes of the corresponding T-Maps were used to compare the different specifications on a picture frame part. This work included the creation of a 6-dimensional T-Map for a point- line – plane feature cluster.

Singh (MS thesis 2006) extended the T-Map model to include feature patterns such as a pattern of holes, pins, tabs and slots. He used T-Maps for representing the variational possibilities in (a) a one-dimensional pattern of multiple tabs and slots (e.g. the linear array in a piano hinge or an angular array to align parts that

are intended to be co-axial and (b) two – dimensional patterns of pins and holes that are intended to engage (e.g.) an integrated circuit and its plug –in base.)

He also incorporated the effect of composite tolerancing for the pattern of features on T-Maps. He developed T-Maps for both composite feature control frames and two single-segment feature control frames. The effects of including an additional datum in the lower (secondary) feature control frame and the addition of a datum with MMC specifier to the T-Map is also considered.

Ameta (PhD thesis 2006) presented a probability model for conducting statistical tolerance analysis and allocation using functional T-Maps and allocation T-Maps (as defined earlier). He determined a functional surface based upon the geometry of the target feature and a specific value of the dimension of interest which intersects the accumulation T-Map. This results into the common points between the functional surface and the accumulation T-Map which provides a measure of all variational possibilities of the parts, which will result into the specific value of the dimension of interest. A probability density function is achieved by choosing different values of the dimension of interest.

To complete the contribution of Arizona State University's bi-level model, advances on the global model are mentioned below.

The global model is based on a dimension and tolerance graph which is a data structure to inter-relate all feature control frames on a part or assembly. The geometric entities and their attributes are represented by the set of nodes in the

graph. The dimensions along with their dimensional tolerances are represented as the set of arc between the nodes. The dimensions include both specified and implied dimensions (such as parallel & perpendicularity). When geometric tolerances with respect to datum entities are represented then the arcs in the graph are directed. The rationale for D&T is through grouping geometric entities in “clusters.” Entities that are mutually completely constrained are organized into clusters.

Shah and Zhang [39] separated linear variations from angular variation in the global model for GD&T. Three basic geometric elements (points, lines, planes) and three features of size (parallel faces, sphere, and cylinder) were considered in the underlying model. This model is fully consistent with Y14.5 M standard and accounts for datum precedence also.

Kandikjian, Shah and Davidson [40] combined the constrained entities into progressively expanding clusters which were used to represent Datum Reference Frames, constraint groups of geometric entities, patterns, or the entire part. The method presented is ISO/ANSI/ASME compliant and can handle special pattern and profile entity relations. Ramaswamy (2000) used the model to develop a GD&T advisor system. It basically interacts with the user and gives feedback viz a viz specification and validation of the tolerancing scheme as judged from the ASME Y14.5 -1994 and good practice rules perspectives.

Wu (2002) refined the global model and adapted it for Computer Aided Tolerancing (CAT) from the point of view of assisting in tolerance specification, validation and analysis. The computer model was developed in the form of an

attributed graph. She also developed a DOF symbolic mechanism which validates the DRFs and pinpoints the conflicting controls.

Wu carried out a study of two algorithms for computing the Minkowski Sum of the convex polyhedral in 3-D space (3-D polytopes). One was based on convex hull and the other on slope diagrams. Convex hull algorithm as found in the literature was very costly while in the existing slope diagram algorithm, in order to merge the slope diagrams of the two operands, operation of stereographic projection from 3-D to 2-D was required.

She improved the computation time and complexity for both algorithms and the computational accuracy of the slope algorithm. This was achieved by using a pre-sorting procedure before constructing a convex hull for the convex hull based Minkowski Sum algorithm and using vector operations to find the interrelations between points, arc, and regions on a unit sphere for the slope diagram algorithm.

Shen (2005) investigated the current tolerance analysis methods and developed a set of computer –aided tolerance analysis tools, i.e. automated tolerance charting, 3-D feature variation and T-Maps based tolerance analyses. He studied the representation and automatic creation of the global model, which he described as a superset constraint-tolerance –feature graph base GD&T model. He carried out the automation of the manual tolerance chart based method.

Shen’s work also includes the study of the 3-D feature variation base tolerance analysis, which was carried out to perform tolerance analysis by simulating 3-D

geometric variations. He carried out the development of a generic and robust Minkowski Sum framework for Minkowski sum operations, development of the modeling functions to the different T-Maps, display of higher dimensional T-Maps and case studies for T-Maps based worst-case analysis. Finally, he offered recommendations to the designers about the suitability of a method for a type of a problem after conducting a comparative study of the tolerance analysis methods (i.e. charting, 3-D parametric and T-Maps based).

Somewhat analogous idea to T-Map was used by Zou and Morse [41] and was modified to call a 'gap space' model. They used the model to carry out assembleability analysis. In this model, gaps are used to simulate the mating relation between features. To identify the necessary and sufficient condition for assembleability, a graph is generated and a set of fitting conditions is discovered. A test of the relationship between the tolerance region and the assembly region in the assembly space is generated and executed. They carried out the worst case and statistical tolerances along with identifying over constrained assemblies based on the relationship between the fitting conditions.

Turner and Wozny [42] presented the approach in which instances of the toleranced part is mapped to points in normalized vector space over the real numbers. This approach was successfully exploited to automate tolerance analysis and tolerance synthesis using solid modeling technology. Turner et al [43] introduced M-space which is a succinct representation of both dimensional and geometric tolerances and is still standard compliant. It was shown that M-space

theory is highly effectual in development of efficient tolerancing algorithms and solution of such problems.

T-Maps are discussed in greater detail in following sections.

3.2 T-Map; An Introduction

3.2.1 What is a T-Map?

A tolerance Map (T-Map) is a hypothetical Euclidean point space which is a one to one map of the geometric tolerance zone.

A tolerance zone is the actual zone of space that includes all the possible variations for the target feature. The size and shape of the T-Map reflects all variational possibilities that can be taken up by a target feature. The above mentioned variations arise due to the specification of the various tolerances on various features of interest. [44, 45, 46 and 47]

3.2.2 Basic impulse behind T-Map

The basic impulse behind the creation and development of T-Maps appears to be to make more visible and mathematically representable the relationships between dimensions and tolerances and between the different classes of geometric tolerances. This makes the relationships more understandable and easily adaptable into CAD software. Another important emphasis is to make it conformable to ASME Y 14.5 standard.

3.2.3 Conformance to ASME Y14.5 Standard

In order to achieve the conformance to the standard, the model has to satisfy the following criteria:-

- i. Each tolerance class is represented by a region or zone whose shape is dependent on the type of tolerance and the type of tolerated feature. Rule #1, material condition and value of the tolerance control the size of the tolerance zone while the datums and the type of tolerance controls the orientation of the zone.
- ii. Rule #1 provides the opportunity for the tradeoff between the size and form for a size specific tolerance zone. This rule says that the size limit specifies the extent to which variations in form and size are permitted.
- iii. All Dimensioning and Tolerancing relations are 1-D i.e. datum-to-target. This statement however is not applicable to size tolerance.
- iv. The coordinate direction of control is determined by the order of datum precedence.
- v. The concept of floating zones is catered. Floating zones as the name suggests are the tolerance zones that float in another tolerance zone. Examples are the form tolerance zone which has floating position and orientation within the size zone and the orientation tolerance zone that floats only in position within the size zone.

- vi. The concept of bonus tolerances makes it possible to trade position variation for size variation. In addition, position tolerance zones may also ‘shift’ with the datum under certain material conditions giving rise to shift tolerances.
- vii. Tolerances can be applied to both resolved entities i.e. axes, mid-planes and to boundary elements such as faces and edges.

To sum up, in order for the model to be consistent with the ASME Y14.5 standard for T-Map,

- a. Distinct representations should occur for tolerances on size, form, orientation and location
- b. The above should hold even when applied to the same feature
- c. Distinct shapes and sizes should occur for different sequence of datums
- d. Distinct dimensions and shapes should occur for tradeoffs (coupling) between tolerances, such as between size and position or size and form.

3.2.4 Areal coordinates

The T-Maps are generated on the basis of the use of areal coordinates. A small discussion of areal coordinates follows.

In the entire literature on T-Maps, areal coordinates are referenced again and again [48, 49]. Areal coordinates are the generalization of the Barycentric coordinates.

Barycentric coordinates for a triangle represent the value of the masses positioned at the three vertices of the triangle. Any point 'P' within the triangle can be represented by a linear combination of the three barycentric coordinates. This means that the values of the masses at the three vertices of the triangle could be adjusted such that a particular position 'P' is occupied within the triangle. However, the point P is not restricted to within the triangle and all points within the plane containing the triangle can be represented by the barycentric coordinates. The point 'P' is referred to as the geometric centroid of the three masses.

Also, the barycentric coordinates t_1 , t_2 and t_3 are proportional to the areas of the triangles A_1A_2P , A_3A_2P and A_3A_1P where A_1 , A_2 and A_3 are the vertices of a triangle.

Barycentric coordinates are homogenous, so

Equation 3-5

$$(t_1, t_2, t_3) = (\mu t_1, \mu t_2, \mu t_3)$$

for $\mu \neq 0$.

Barycentric coordinates can be normalized so that they represent the actual areas of the sub triangles. Such coordinates are known as the normalized barycentric coordinates.

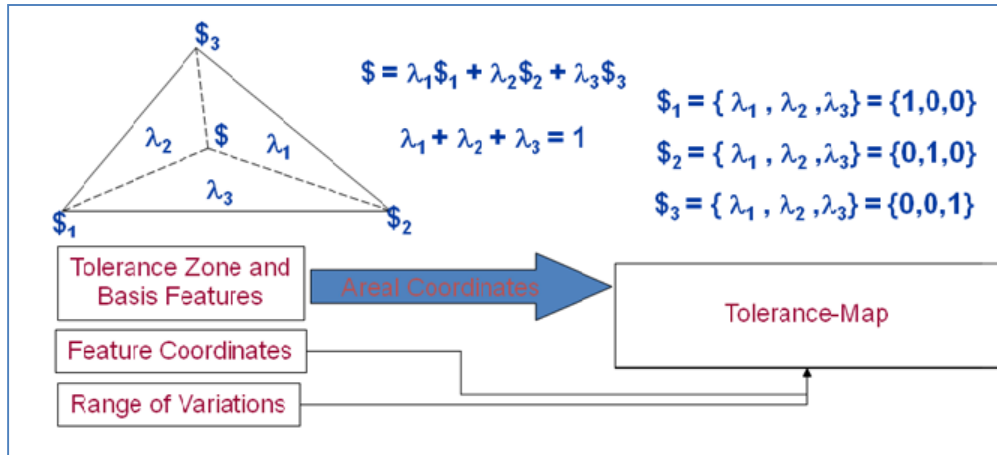


Figure 3-7 The basic tetrahedron with the values of the basis points shown along with interaction and use of areal coordinates in the transformation from tolerance zone to T-Map

Hence,

Equation 3-6

$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

In barycentric coordinates, a line has a linear homogenous equation. For example, a line joining points, (r_1, r_2, r_3) and (s_1, s_2, s_3) has equation:

Equation 3-7

$$\begin{vmatrix} r_1 & r_2 & r_3 \\ s_1 & s_2 & s_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{vmatrix} = 0$$

Areal coordinates are the barycentric coordinates normalized so that they become the areas of the triangles, PA_1A_2 , PA_2A_3 and PA_3A_1 normalized by the area of the triangle $A_1A_2A_3$.

The concept of areal coordinates is not limited to 2-D or 3-D only and can be extrapolated for higher dimensions.

3.2.5 T-Maps for size

The 3-D T-Map for a rectangular face is shown in the figure 3-9. It is a one to one mapping of the tolerance zone (tolerance zone shown in figure 3-8). This tolerance zone exists at the end of the rectangular bar of length L and cross sectional dimensions $d_y \times d_x$. It is imperative that all points lie within the planes σ_1 and σ_2 and within the rectangular limits of the face.

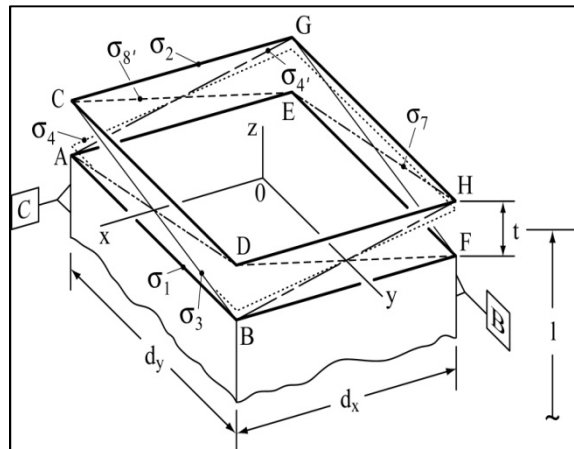


Figure 3-8 Tolerance zone for size tolerance on a rectangular bar

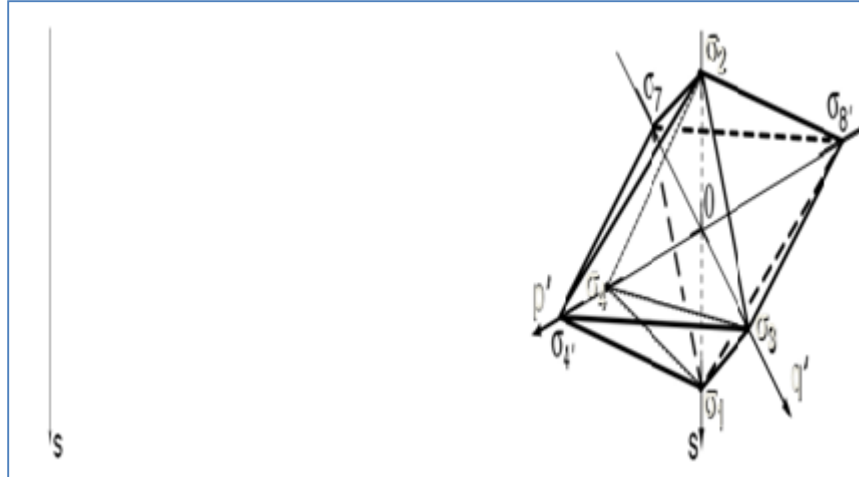


Figure 3-9 T-map for size tolerance on rectangular face. Octahedron containing the tetrahedron can also be seen.

Measures along the s-axis of the T-Map represent parallel variations of the plane negatively along the z-axis in the tolerance zone while the p`- and q`- axes represent the orientational variations of the plane about the y and x-axes, respectively.

To construct the T-Map, initially the four planes $\sigma_1, \dots, \sigma_4$ are identified, that cover the entire tolerance zone and later, these four planes appear as four points in the T-Map. The following table gives the vertices of the T-Map that lie on the corresponding planes of the tolerance zone:-

<u>Planeα</u>	<u>Points-on-the-planeα</u>
σ_1^α	ABFE α
σ_2^α	CDHG α
σ_3^α	CBFG α
σ_7^α	ADHE α
σ_8^α	CDEF α
σ_4^α	ABHG α

Table 3-1 Identification of the plane/points in the tolerance zone/T-Maps

The premise behind the selection of the basis-points was to make the T-Maps consistent with the existing results by others, although some of these results were obtained by intuition only in addition to others through math models. The maximum distance between σ_1 and σ_2 is t . Any point on the line $O\sigma_3$ represents the orientation of a plane that contains O and is tilted at a certain angle that is dependent on its position along the $O\sigma_3$ line. If it is at O along the $O\sigma_3$ line, then the tilt is zero. If it is at σ_3 along the same line, then the tilt is maximum permissible. σ_4 is the plane for which the orientation of σ_4' is reduced from td_y/d_x to t . σ_4 is identified only for the purpose of identifying the reference tetrahedron inside the T-Map.

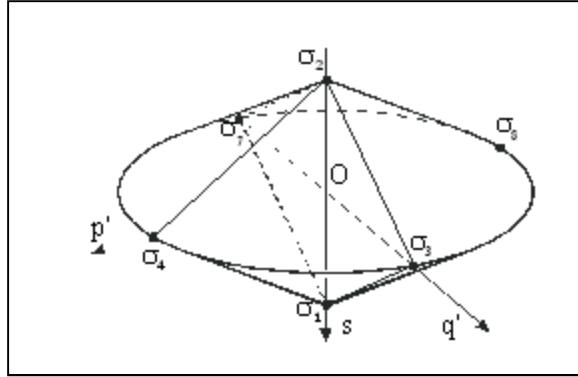


Figure 3-10 T-Map for size tolerance specified on the round face

3.2.6 T-map for form and orientation

Mujezinovic et al (2001) showed the sensitivity of the T-Map to the precedence (ordering) of datum reference frames. For this, they defined floating zones and internal sub-sets. Floating zone for form and orientation are contained within the tolerance zone for size. They can freely move within the size tolerance zone and occupy any position within. On the other hand, internal subsets are the T-Maps that are internal to the T-Maps for size. T-Map for the size is achieved by taking the Minkowski sum of the internal subset for form and the other subset for displacement of the warped surface.

In order for catering the orientation tolerance, a subset is designated to represent orientation tolerances. The net effect is that the regions add to a T-Map that is smaller than, and has a different shape from that for size.

3.2.7 Examples of T-Maps for form and orientation

3.2.7.1 Tolerance-map for a face with size and orientation tolerance:

Parallelism

If a parallelism tolerance is specified for the target face with respect to the datum A, this would cause a control of orientations of the target face with respect to the x- and y-axes. Points along the p`q` plane of the T-Map map the angular variation of the target face about x- and y-axis. The T-Map gets truncated along p` and q` beyond t_A ”, when the allowable orientations of the target plane about x- and y-axis are limited by t_A ”.

The result for the T-Map for circular bar with size and orientation tolerance is shown in the figure below:-

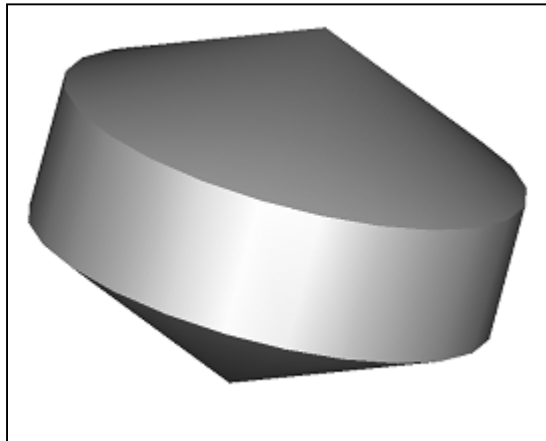


Figure 3-11 Modification of the boundary of the T-Map by orientation tolerance

3.2.7.2 Tolerance-map for a face with size and orientation tolerance:

Perpendicularity

Similarly a perpendicularity tolerance can be specified for the target face with respect to the datum C. A control of orientations of the target face with respect to the y-axis will be implied by this perpendicularity refinement with respect to C. Now, the points along the p^{\prime} axis of the

T-Map maps the angular variation of the target face about the y-axis. The T-Map gets truncated along p^{\prime} axis beyond t_c^{\prime} once the allowable orientations of the target plane about y-axis are limited by t_c^{\prime} .

3.2.8 Tolerance-map for a face with size and form tolerance: an example

Internal sub-sets within a tolerance zone on either size or position are used to represent form variations (e.g. warp). Form tolerance t^{\prime} once applied limits the amount of warp. The T-Map for position (or size) tolerance t applied to the feature represents the combined form variation and companion possibilities for location of the warped feature at a particular instance. This is in observance of Rule # 1 of ASME Y14.5 Standard.

Figure below shows the tradeoff between the array of subsets for form and their companion locations within the T-Map of figure 3-12.

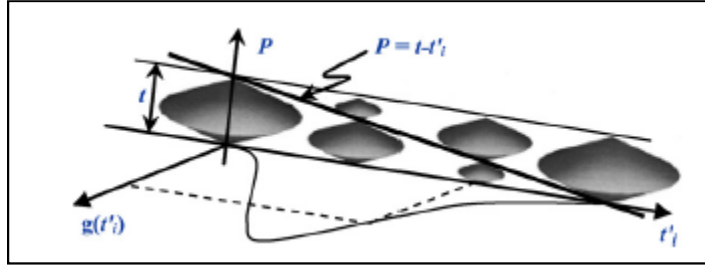


Figure 3-12 The array of subsets for form (upper dicones) and their companion possibilities for location (lower dicones) within the T-Map for **Figure 3-10**.

3.2.9 Material Condition

The modifier M for maximum material condition (MMC) implicitly applies a linear coupling between the tolerances on size and position. This means that the narrower the tab (within the specified tolerance τ), the greater the freedom (bonus tolerance) for its location (for a tab & slot assembly).

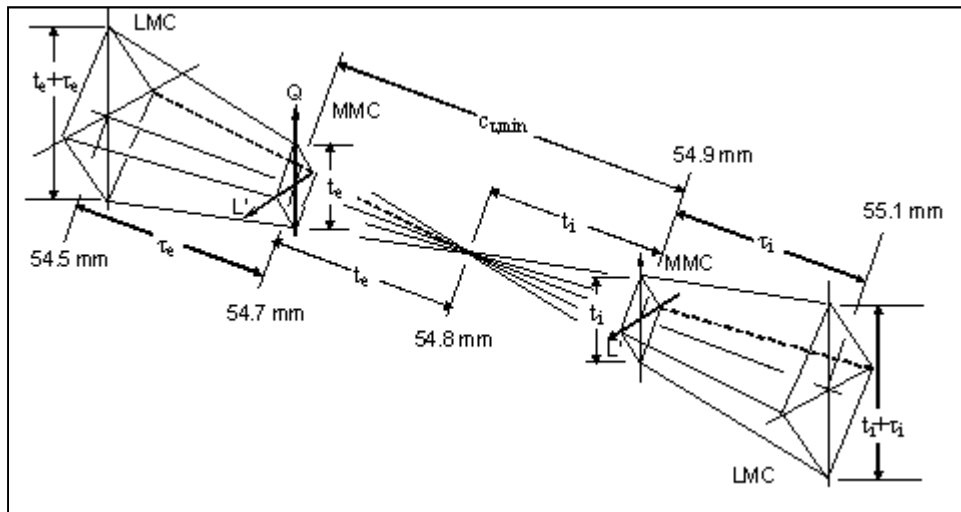


Figure 3-13 The effect of the material condition on the size of the T-Map.

Figure above shows the change in the size of the T-Map with change in material condition as specified by the material modifier M for Maximum Material Condition and L for least material condition.

3.2.10 Process of conforming of the T-Maps

In this process, all variations of every feature are represented as the variations of the target face.

3.2.11 Accumulation T-Map

After the conformed T-Map is obtained, the next step is the development of the accumulation T-Map. An accumulation T-Map is a T-Map which represents the accumulated variations of all the parts in the assembly at the target face.

3.2.12 Process of obtaining Accumulation T-Map

Conformed T-Maps for all the toleranced part-features are obtained for all parts that lie in the stack up. All these conformed T-Maps are then combined together through Minkowski sum to get the accumulation T-Map.

3.2.13 Minkowski Sum

Minkowski sum of two sets A and B is the sum of every element of a set A to every other element of set B. It is also known as ‘dilation’ or the binary dilation of A by B. Symbolically,

Equation 3-8

$$A + B = \{a + b | a \in A, b \in B\}$$

If the set A and set B have only one member, then it reduces to vector addition.

The algorithm for Minkowski sum in 2-D is fully developed as far as polygons are concerned. Only three algorithms for 3-D Minkowski sum have been proposed till now, and they can deal with planar faces only. The main steps involved in the different algorithms are:-

- 1.) To continuously locate the corresponding points(having the same tangent direction) on the profiles of two operands
- 2.) To vectorially add these two points to obtain the new point on the resultant profile.

The two main approaches for polygons in 2-D are:-

- a.) Slope Approach.

In this approach, the normal of the edges in two operands are sorted out based upon the polar angle and then concatenated one by one to get the resultant polygon.

- b.) Support Function Approach.

Distance of the point to the origin is described by the support function. Minkowski sum of two profiles is equal to the addition of the numerical values of the corresponding distances.

As mentioned earlier, all approaches in 3-D work on polyhedrons only.

- i.) Vectorial Approach

In this approach, the vectorial addition of every vertex in two operands is carried out followed by the convex hull of the point cloud.

ii.) Sub – interval Approach

In this approach, a feature volume is divided recursively into sub-intervals along n dimensions.

iii.) Slope Diagram Approach

In this approach, 3-D profile is transformed into 2-D followed by the sorting and relating of the two operands according to their normal.

3.2.14 Functional T-Map

It is a T-Map for an assembly with all parts perfect except the target feature. That means that only target feature will have tolerance specified while all other parts in the assembly will have only their characteristic dimension specified and no tolerance applicable to these remaining parts.

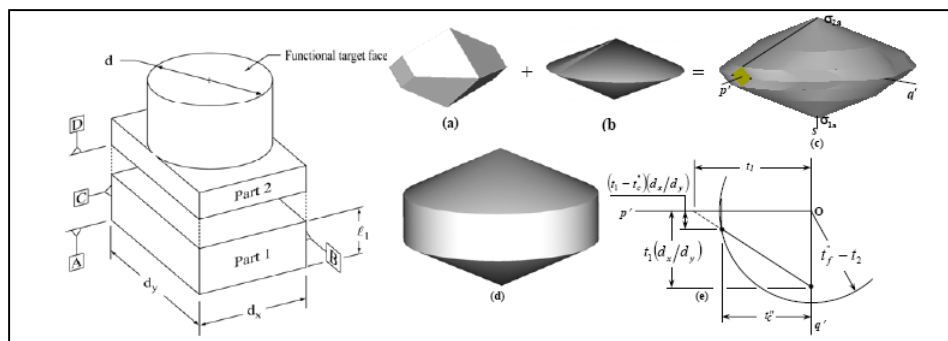


Figure 3-14 Tolerance-Maps (a) for Part 1, (b) for Part 2, (c) for the assembly (Minkowski sum of (a) and (b)), (d) for the desired function, and (e) p'q' section of the fit of functional and accumulation T-Maps

3.2.15 T-Map for frequency distribution generation

Frequency corresponding to a particular value of clearance is found by identifying all the variational possibilities of the feature in the tolerance-zone and the corresponding points in the T-Map which yield that same value of clearance between the target face and the datum face.

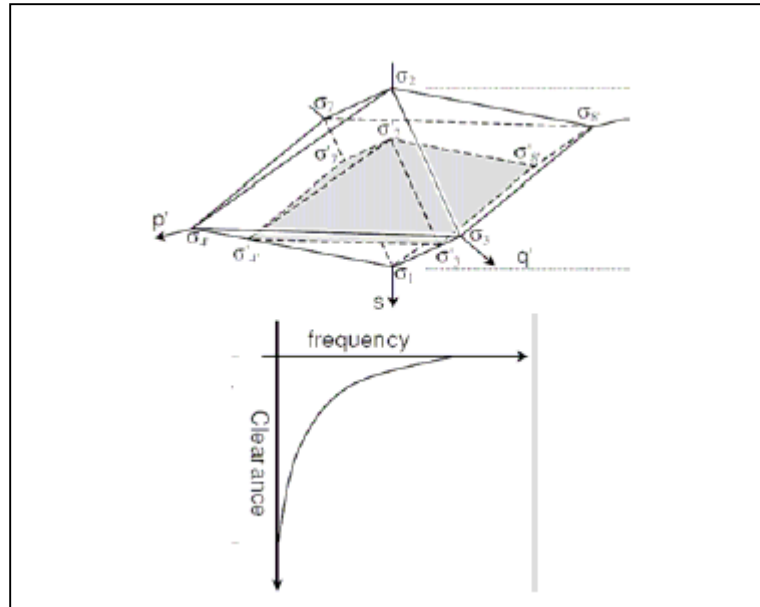


Figure 3-15 Frequency distribution of clearance corresponding to the 3-D variation of the target plane

3.2.16 Concluding remarks about T-Maps

T-Map has many uses that surpass the effectiveness of its competitors' models. It is able to find relationships among geometric tolerances which are yet to be reported by others e.g. non linear stack up relations. In addition, the model is able to provide non -linear symbolic relationships among tolerances that were previously available in qualitative form, with restricted demonstrated applications

and mostly proprietary. Examples are the relationships that provide edge for specifying tolerances with material modifiers instead of with RFS.

T-Maps are able to represent floating orientation zones. They can distinguish between ordering and choices of datums. They can effectively cater to bonus tolerances which is the linear coupling of size and position-tolerances that occur when material modifiers are incorporated in the tolerance representation. T-Maps can be used to derive new stack up conditions which are dependent on the sizes and shapes for the parts. Also, it provides quantitative geometric measure, i.e. volume (3-D) or content (in higher dimension) that can be used for comparison to statistical quality studies that employ rejection criteria.

To list, following are the pluses and minuses of the T-Map technology:-

3.2.17 Pluses of the T-Map method:-:-

1. T-Map can model all the 3-D variations of a feature, such as size, form, orientation and position, consistently with ASME Y14.5 standard.
2. All the interactions of the variations are completely and precisely modeled through T-Map.
3. All advanced concepts such as incorporation of rule #1, floating zones, bonus/shift tolerances, datum precedence, and material modifiers are viable.
4. Accumulation of various part tolerances in an assembly have been efficiently demonstrated.

5. T-Map is a mean to provide multiple stack up equations and metric measures which can be readily used by a designer in selecting optimal tolerances.
6. The analysis model in T-Map based method is independent of the user's choices, which means that the results by T-Map method will be same for a particular problem irrespective of how the analysis model is created.

3.2.18 Minuses of the T-Map method:-

1. The method is still under development.
2. Statistical approach with non uniform manufacturing variations has yet to be incorporated.
3. Visualization of higher dimensional T-Map is difficult. However, 2-D or 3-D cross sections are highly effective in revealing the intricacies of the tolerance interaction as well as the entire picture of tolerance situation.
4. The method has difficulty presenting results in the same format as other CAT packages.

The pluses and minuses of the T-Map are summarized in the figure below:-

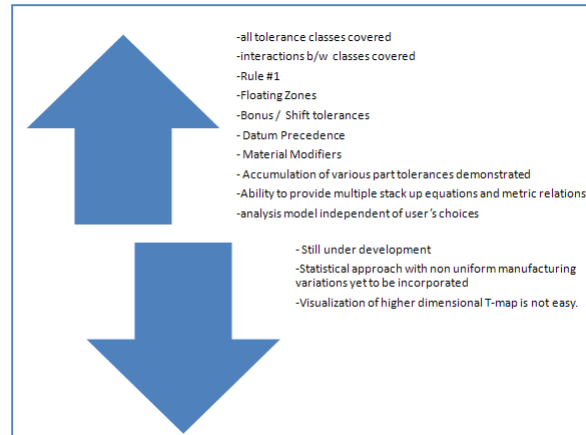


Figure 3-16 The pluses and minuses of the T-Map math model

3.3 Other Approaches

Under this approach, all the limited work done for the tolerance representation using lie algebra, interval arithmetic, tolerance graphs especially Bi-partite graphs etc. can be discussed. These mathematical theories undertake a lot of potential in the ease of tolerance depiction. However, this area has not been fully explored and there are still reservations whether these mathematic theories will eventually prove their worth in tolerance representation.

3.4 Model Comparisons

In order to compare the various models discussed above, it will be necessary to define some criteria on the basis of which each model can be evaluated and give a personal view about its pros and cons as compared to other models.

The various premises for model comparison are discussed below:-

- a) Completeness

The model is able to represent all the tolerance classes.

b) Applicability

The model can be applied to various uses such as in worst case or statistical analysis.

c) Complexity

How complex is the mathematics behind the model?

d) Compatibility

How far is the model compatible with the standards such as ASME Y14.5 or ISO etc? This could be further divided into the Maximum material condition, form tolerance, floating zones and datum precedence.

Based upon the above items of criteria, the different models can be formally compared in the form of chart as shown below:-

Model	Completeness	Applicability		Complexity	Compatibility with Y14.5			
		Worst Case	Statistical		Material Condition	Datum Precedence	Floating Zones	Form Tolerances
Parametric	Moderate	Yes	Poor	High	No	No	No	No
Variational Surface	Poor	Yes	Poor	Moderate	No	No	No	No
Offset zone	Poor	Yes	Poor	Moderate	No	No	No	No
DOF	Good	Yes	Good	Moderate	Yes	Yes	Yes	Yes
TTRS	Good	Yes	Moderate	Low	No	No	No	Yes
Kinematic	Moderate	Yes	Moderate	Moderate	No	No	No	No
Vectorial	Moderate	Yes	Moderate	Low	No	No	No	No
Multi-Variate Region	Good	Yes	Good	Moderate	Yes	Yes	Yes	Yes

Table 3-2 Comparison of the different Math models

3.5 Concluding Remarks for Math Model of Tolerance Representation

Ideally speaking, following are the characteristics desired in a math model of tolerance representation:-

1. Compatibility with Y14.5M

The model should be compatible with ASME Y14.5M standard.

2. Distinction between types of variations.

It should be able to distinct between the different types of variations.

3. Conformability with Rule # 1

The model should conform to Rule #1 of tolerance analysis.

4. Support for floating Tolerance Zones

The model should support floating tolerance zones.

5. Support for bonus tolerance zones

It should cater for bonus tolerances.

6. Accountability for Datum Shift

It should be capable to account for datum shift.

7. Support for 1-D datum to target relations

The model should support 1-D datum to target relations.

8. Representation of the effect of datum precedence

It should be able to represent the effect of datum precedence.

9. Detection of conflicting requirements on DOFs

The model should be smart enough to detect conflicting requirements on DOFs.

4 THREE DIMENSIONAL MODEL FOR TOLERANCE TRANSFER IN MANUFACTURING PROCESS PLANNING

This chapter presents the three dimensional model for tolerance transfer in manufacturing process planning. The chapter starts with the background of the research with respect to different types of datums used in the design, manufacturing and inspection circles. It is followed by a discussion upon the reasons for avoiding datum change. Also, discussed are the situations when datum change is unavoidable. This is followed by the main idea behind the creation of three dimensional model for datum reference change and then, the methodology used for this purpose is explained. Next, the mathematical details of the model are taken up. The chapter concludes with the case studies of the various datum change scenarios that have been successfully tackled with the mathematical model presented and finally, the various types of datum features to which this model can be applied, is discussed.

4.1 Background

Before explaining the real crux of this research, it would be worthwhile to go back to theory books and understand what is a datum? How many different types of datums are in use? etc. A theoretically exact point, line, axis or plane which indicates the origin of a specified dimensional relationship between a toleranced feature and a designated feature or a part is called a datum. From the above definition, it is clear that datums do not exist in reality. For this reason, a designated part feature serves as the datum feature. On the other hand, true

counterpart (the gage) of the designated part feature establishes the datum plane or axis. For the reasons of practicality, a datum is to be simulated by processing or inspection equipment, such as machine tables, surface plate, collets, gage surfaces etc.

Based on the use, there are several types of datums in use in the industry right now. These are Design datums, Operational (or Manufacturing) datums, Locating datums, Measuring or Inspection datums, Assembly datums etc. A design datum is a point, line or a surface in a design blue print, from which the position of another point, line or surface on the part is dimensioned. Sometimes more than one geometric entity may have the same design datum. Conversely, an entity may be defined by several design datums. Design datums and design dimensions and tolerances are laid out by the product designer based upon several reasons such as the condition in which the part is going to function, product appearance, rules of physics e.g. kinematics, dynamics etc and finally, customer's request. Operational or manufacturing datum is a geometric entity (point, line, surface etc) which is used to determine the position of the surface to be machined. The operational or manufacturing datums usually appear in an operational work piece sketch in an operation sheet. The manufacturing datums, manufacturing dimensions and tolerances are specified by the process planner.

Locating datum is a surface of a work piece which is used to define the proper position of the work piece in the direction of the manufacturing dimension on the work holder or the machine table for work piece set –up. A supporting locating datum determines the proper position of the work piece through the contact of the

locating datum with the corresponding surface on the work holder or machine table. On the other hand, a calibrating locating datum does the same job by calibration of the position of the locating datum.

4.2 Requirement for datum change

Now, after the explanation of the different types of datums, let's talk about the requirement for datum change. In actual industry practice, all efforts are made to ensure that maximum number of above mentioned datums coincide. This is in accordance with the Principle of coincidence of datums, generally talked about in the process planner's circles. That means that it is desired that operational (or manufacturing datum) be the same as the design datum or the inspection (or measuring datum) be the same as the design datum. There are several reasons for avoiding as much as possible the datum change. In addition, there are certain situations in which datum change is unavoidable. These are explained in the following paragraphs.

4.3 Reasons to avoid datum change

According to manufacturing industry current synopsis, there are three main types of wastes produced by the manufacturing processes. The biggest one of these is known as the process waste and these (from tolerance point of view) include the rejected parts which have their dimensions outside the accepted variation range. The second type of waste which is of the direct concern is the waste from startups, shutdowns, maintenance and other offhand operations. This includes removing the part from the machine to verify the tolerance range. This type of waste is the

main reason to avoid datum change in the manufacturing process planners circle. The reason for this is quite simple as no part can be positioned to its exact 100% previous position once removed. (The third type of waste is the utilities waste which results from the utility systems that are needed to power the manufacturing processes. This is not relevant to this research and is mentioned only for the reason of completeness.)

4.4 When datum change is unavoidable

Examples of the situations in which datum change is unavoidable is situation in which the designer has referred as datum a HVoF (High Value of Finish) feature which means that feature will be one of the last steps of the operation. Another reason could be that the particular datum is lying flat to a jig edge and hence, no manufacturing cut or measurement could be made from that edge. After the above discussion, it is quite clear that not only the datum change is unavoidable in certain cases; it is one of the major reasons for scraping of the parts. So, now that the datum change is likely to be there in a process plan, what steps are needed to cater for the datum change? Datum change leads to recalculation of dimensions and tolerances involved.

A linear datum change refers to the change between datums which lie in the same stackup (plus or minus) direction. These types of datum changes result in addition and subtraction of certain already known values to give new values of dimensions and tolerances. These datum changes will be referred to as inline datum changes from hence forth. However, the main topic of this research is non inline datum

change which is a totally new idea. In this research, full three dimensional datums are considered and datum change direction may or may not be in the direction of stackup calculation (plus or minus). These are referred to non inline offset datum changes. In addition, this research considers inaccuracies in the establishment of 3 dimensional coordinate systems and thus, not only datum changes between rotated and offsetted (in 3-D) coordinate systems (called twisted datum from hence forth) but also, datum changes between non orthogonal axis 3-D coordinate systems (called squeezed datum from hence forth) will also be considered.

4.5 Main Idea

The main idea of this research is to determine the transformation between a design datum and a manufacturing (or an operational) datum and to use it for determining the limits of the tolerance zone based upon design tolerances. This transformed tolerance zone is then used for determining the extreme limits of the manufacturing tolerances that can be used for the creation of that individual feature on the part. These could be directly used in the manufacturing process plan. This is shown in the figure 4-1. The second utility could be that the transformed tolerance zone that results is triangulated using Delaunay triangulation and then, any further reading by the CMM (Coordinate Measurement Machine) to be subjected to a test of whether that reading lies inside the transformed tolerance zone or not. If the reading from CMM lies within the transformed tolerance zone, than it should be acceptable at the stage of

manufacturing and will result in acceptable results at the inspection stage and hence, showing conformance to the limits imposed by the designer.

4.6 Methodology

The first and foremost thing that needs to be done is to specify as accurately as possible the two types of datums involved in the transformation. Each of the datum needs to be established as an independent coordinate system. For the establishment of a viable coordinate system, it is to be made sure that all the degrees of freedom are fixed. This will require a definition of a plane, followed by a line and a point. A plane requires at least three points to fully define it. Line could be defined with two points along an edge of the concerned datum. The point could be a midpoint of a line or the centre of a circle or a sphere. The point could also be defined by the intersection of a perpendicular edge. The selection of these datum features is in accordance with the ASME GD&T Y14.5 2009 standard definition of the primary, secondary and tertiary datums.

Apart from the fixation of all degrees of freedom for the two coordinate systems for the two types of datum, it is also to be checked whether the two systems are orthogonal or not? It is important for determining the accuracy of the transformation, although it will not affect the procedure for the extraction of the transformation. If the two coordinate systems representing design and manufacturing datums respectively are orthogonal themselves, then the transformation will be unique and exact. However, in cases where the two coordinate systems are not orthogonal, the code utilizes the goodness of fit

criterion. This criterion could be based upon several possible parameters but for this research, the standardized minimized value of the sum of the squares of the errors has been used which is given by the following equation:-

Equation 4-1

$$e' = e/d$$

Where ‘e’ is the minimized sum of squared errors and ‘d’ is given by the following equation:-

Equation 4-2

$$d = \sum_{j=1}^{n_c} \sum_{i=1}^{n_r} (X(i,j) - XX(i,j))^2$$

Where n_c is the number of columns and n_r is the number of rows of the matrix ‘X’.

In above equation, the matrix XX is defined as given below:-

Equation 4-3

$$\begin{bmatrix} \frac{\sum_{i=1}^{n_r} X(i,1)}{n_r} & \frac{\sum_{i=1}^{n_r} X(i,2)}{n_r} & \dots & \frac{\sum_{i=1}^{n_r} X(i,n_c)}{n_r} \\ \frac{\sum_{i=1}^{n_r} X(i,1)}{n_r} & \frac{\sum_{i=1}^{n_r} X(i,2)}{n_r} & \dots & \frac{\sum_{i=1}^{n_r} X(i,n_c)}{n_r} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\sum_{i=1}^{n_r} X(i,1)}{n_r} & \frac{\sum_{i=1}^{n_r} X(i,2)}{n_r} & \dots & \frac{\sum_{i=1}^{n_r} X(i,n_c)}{n_r} \end{bmatrix} \quad (i:k = 1, 2, \dots, n_r)$$

The math model could be used for determining the manufacturing tolerances that are required for building the manufacturing process plans. The algorithm is shown in the figure below.

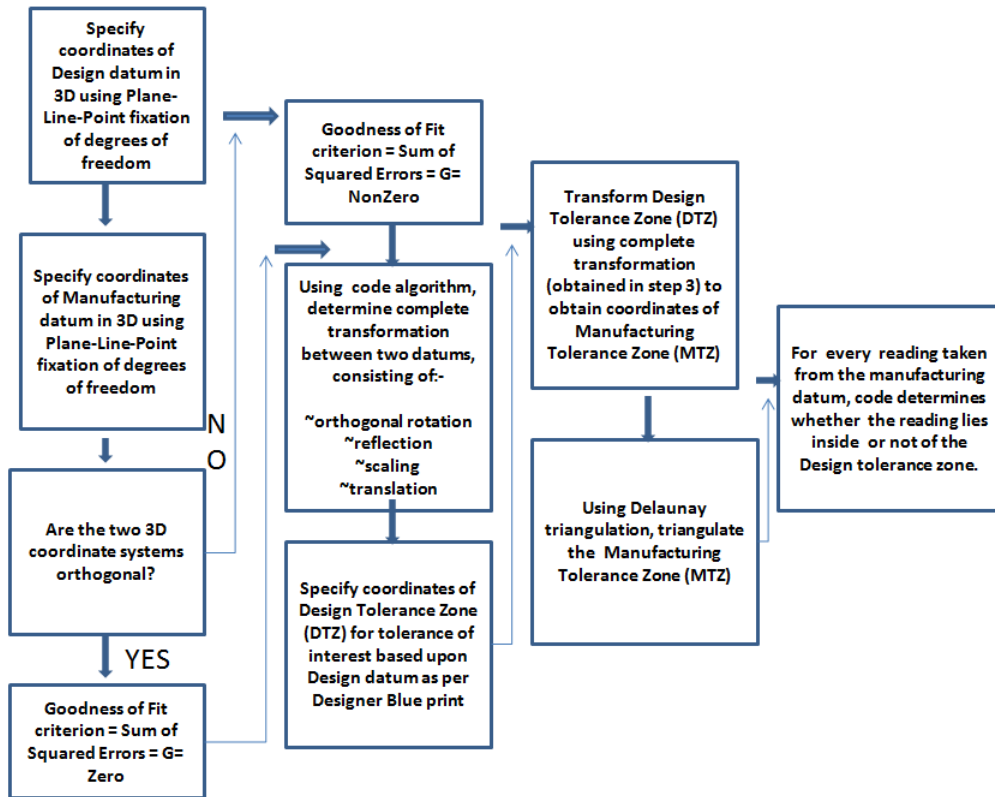


Figure 4-1 Algorithm for the mathematical model to cater for datum reference change and determination of manufacturing tolerances directly usable in manufacturing process plan.

In both cases i.e. orthogonal datums and non orthogonal datums, the transformation computed by the code consists of the orthogonal rotation. In addition, orthogonal reflection, scaling and translation values are also determined. Once the transformation has been achieved, the next step is to specify the

coordinates of the tolerance zone for the tolerance of interest from the Design blue print. This tolerance zone has been called Design Tolerance Zone (DTZ). This will be the easier part of the entire task as design datum and the Design Tolerance Zone (DTZ) are linearly linked.

After the determination of the Design Tolerance Zone (DTZ) coordinates, these are fed into the code. The code then determines the coordinates of the Manufacturing Tolerance Zone (MTZ). This is accomplished through reverse transformation of the Manufacturing Datum coordinates to the Design Datum coordinates and then incorporating the necessary transformation of the Design Tolerance Zone (DTZ) from the design datum. In all of these calculations, a close eye is kept on the goodness of fit criterion and results with any value of the parameter 'e' above the pre selected value can be disregarded. In such a case, the whole process is to be repeated from the start and all computations and reverse transformations accomplished till the time the value of the parameter 'e' is below the threshold value predetermined.

The threshold value of the standardized minimum value of error 'e' could be selected based upon several factors, such as the machining accuracy of the tool(s), machine allowance, a certain percentage of the ratio of the tolerance value to the mean dimension value, pass percentage of the parts based upon statistical sampling etc.

The model can be used for directly verifying the correctness of a CMM (Coordinate Measuring Machine) of the target feature to be within the design permitted variation range. This requires the use of Delaunay Triangulation to

triangulate the space within which the variations are allowed. The procedure is depicted in the figure below.

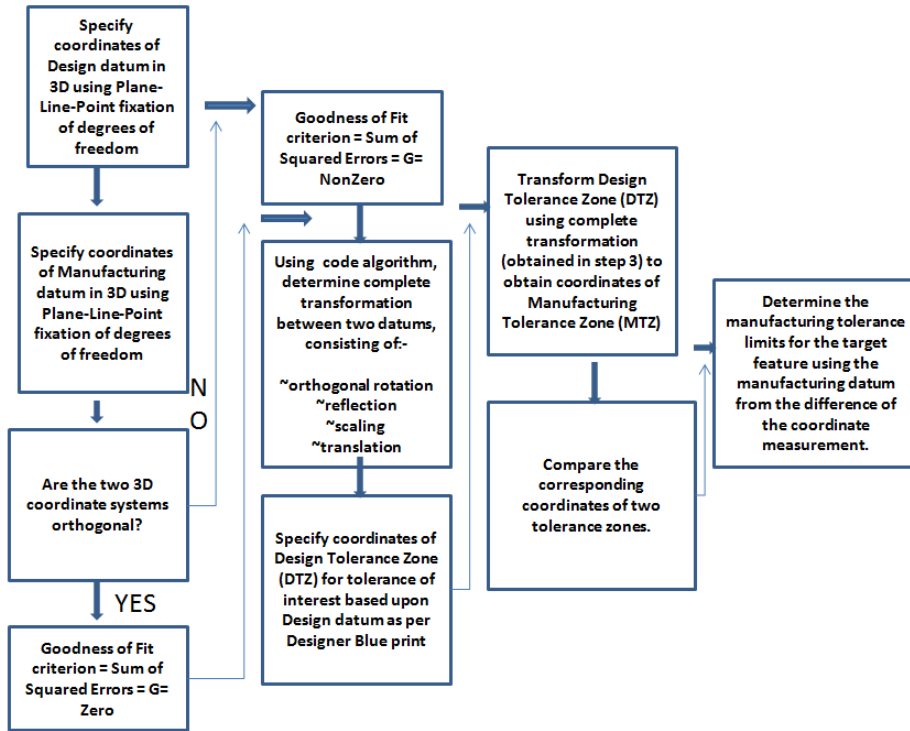


Figure 4-2 The process flow for the utility based upon Delaunay Triangulation to determine the presence of a CMM reading of the target feature to be inside the tolerance zone

Once the coordinates of the Manufacturing Tolerance Zone (MTZ) have been determined, the creation of the hypothetical Manufacturing Tolerance Zone (MTZ) requires a few more steps and certain considerations. One of the major considerations to take into account is the shape of the Manufacturing Tolerance Zone (MTZ). It is highly logical to assume that the contour (shape) of the Manufacturing Tolerance Zone will be exactly the same as that of the Design Tolerance Zone. It is to be remembered that all coordinates of the Design

Tolerance Zone and the Manufacturing Tolerance Zone are in the similar sequence and orientation for different involved geometric features such as entity face etc. This is crucial for obtaining similar contours (shapes) for both DTZ and the MTZ. However, in certain cases, exactly similar contours could not be achieved for the two tolerance zones. In such situations, decremented changes in the value of the coordinates of the manufacturing tolerance zone could be used to achieve similar contour as that of the design tolerance zone. Decremental changes in the value of the coordinates of the manufacturing tolerance zone will also be required in order to cater for the stock removal tolerances and the machine allowance along with the machining allowance for the tool.

4.7 Mathematical details of tolerance transfer

The topic of linear tolerance transfer is almost three decades old. It is probably due to the types of machines available at that time. However, now with turret machining, multi axis milling, water jet cutting, laser machining etc, the topic of non inline tolerance transfer is more appropriate. In this research, the emphasis has been on the non inline tolerance transfer. However, the same methodology can be used to deal with linear tolerance transfers. As mentioned earlier, the total transformation calculated includes subsets such as scaling, translation, orthogonal rotation and orthogonal reflection. While each of the above mentioned subsets would be having some value in the non inline case, orthogonal rotation and orthogonal reflection will have null values in the case of linear tolerance transfer.

For illustration purposes, let's consider only the Design Tolerance Zone (DTZ) of a size tolerance specified on the planar rectangular face. The coordinates of the DTZ in the design datum frame of reference are shown in the figure below.

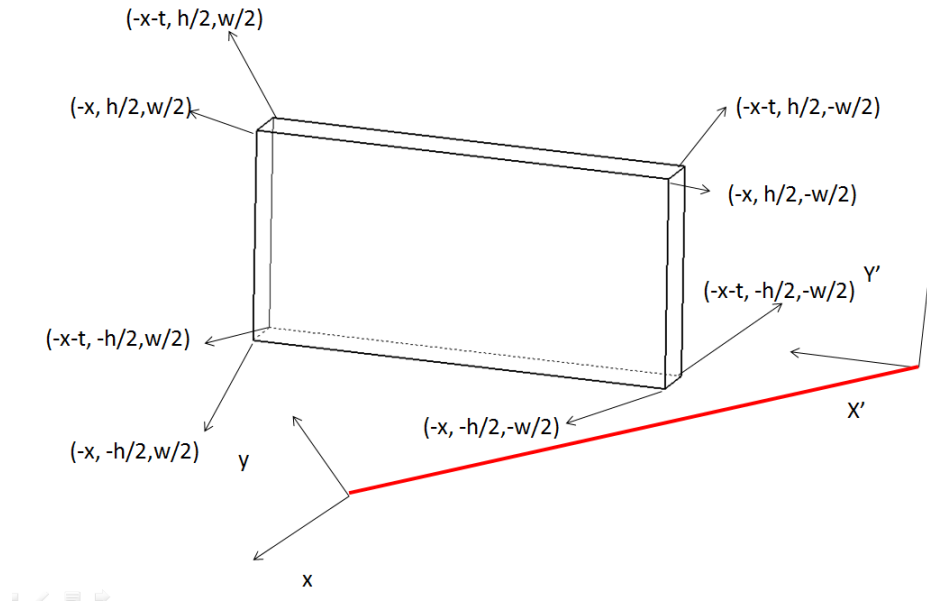


Figure 4-3 Coordinates of the Design Tolerance zone for size tolerance specified on planar rectangular face.

In figure 4-3, in addition to the coordinates, two frames of reference are also shown. The X, Y (and normal Z) is the design reference frame. The width 'w' is along the Z axis. The height 'h' is along the Y axis. The thickness of the tolerance zone is given by 't' which is along the X axis. The second frame of reference shown is the Manufacturing frame of reference which is oriented with respect to the manufacturing datum as the Design frame of reference was oriented with respect to the design datum. This second frame of reference is represented by 'X', 'Y' and normal 'Z'. Here it is made clear that two reference frames could be any orientation and any amount of translation along any axis may be involved.

As, mentioned earlier, the non inline transformation between the DFoR (Design Frame of Reference) and MFoR (Manufacturing Frame of Reference) may involve rotation, reflection and scaling in addition to translation. The three rotations along the x y and z axis respectively are defined in the matrix form as shown below.

Equation 4-4

$$\theta_{rotx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

Equation 4-5

$$\theta_{roty} = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}$$

Equation 4-6

$$\theta_{rotz} = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Any orientation in a 3-D frame of reference can be converted into three sequential rotations which is product of three matrices as mentioned in equations 4.4, 5 and 6. The result of the multiplication of these matrices in a sequence will result in the matrix given by equation 4.7.

Equation 4-7

$\theta_{rot_{xyz}}$

$$= \begin{bmatrix} \cos \beta \cos \gamma & \sin \alpha \sin \beta \cos \gamma + \cos \alpha \sin \gamma & \sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma \\ -\cos \beta \sin \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & \cos \alpha \sin \beta \sin \gamma + \sin \alpha \cos \gamma \\ \sin \beta & -\sin \alpha \cos \beta & \cos \alpha \cos \beta \end{bmatrix}$$

Sometimes, in order to achieve a particular orientation, reflection may be necessary. An example of reflection by an amount ' δ ' about the x axis is given by the following matrix:-

Equation 4-8

$$ref_{\delta,x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \delta & \sin \delta \\ 0 & \sin \delta & -\cos \delta \end{bmatrix}$$

These transformation matrices will be utilized to arrive at the end coordinates of the MTZ using the MFoR. The results are symbolized and depicted in the figure below.

In order to arrive at the end coordinates of the manufacturing tolerance zone, the origins of the two frames of reference i.e. DFoR and MFoR are to be specified in a common third frame of reference which will be called, henceforth, GFoR or the Global Frame of Reference. The associated derivation of the relations and matrices is included in the following paragraphs.

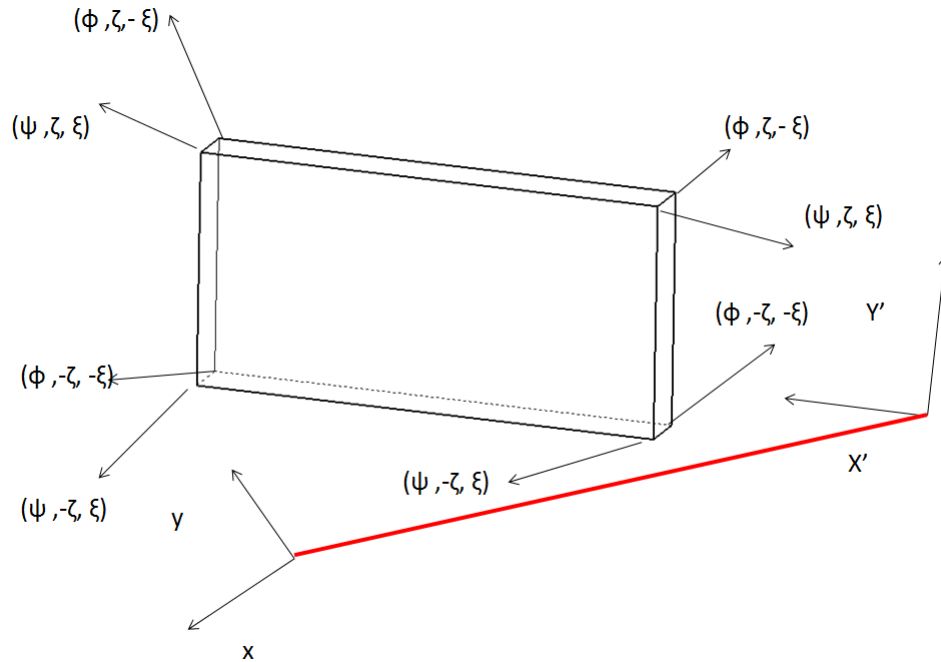


Figure 4-4 The end coordinates of the Manufacturing Tolerance Zone using the Manufacturing Frame of Reference.

For the code to work, coordinates are needed for both type of datums involved i.e. Design datum and Manufacturing datum. It is to be remembered that each end coordinate is not individually required to be measurable as various pieces of information will be used about the shape and dimensions of the tolerance zone such as cuboids or rhombus and the width and height of the feature involved. This will help us in properly forming the tolerance zone involved and also, it is required from the mathematical dimensionality requirement of the code. To demonstrate the working of the code, first end coordinates of the Design datum with Global Frame of Reference (GFoR) as the origin are determined and the design datum feature as the plane feature of the Design Frame of Reference (DFoR). This will be fed to the code in form of an 8x3 matrix called 'D'. Next, end coordinates of the Manufacturing datum with Global Frame of Reference

(GFoR) as the origin are needed and the manufacturing datum feature as the plane feature of the Manufacturing Frame of Reference (MFoR). This will be fed to the code in the form of 8x3 matrix called 'M'. The code will then determine the optimal transformation between the two frames of reference based upon the standardized minimum square of errors. The transformation process is shown by the equation below

Equation 4-9

$$M = A * D * R + T$$

Where T is a translation matrix

A is the scaling matrix

R is the rotation matrix

Later the code employs these transformations in reverse direction (inverse transform) to find out the end coordinates of the Design datum. This should ideally and most uncommonly be the same as the matrix 'D'. However, since there is no surety that any coordinate system itself is orthogonal or not and depending upon the value of the goodness of fit criterion, it will be definitely different than the matrix 'D'. This resultant matrix is called Corrected Matrix 'C' and is calculated as shown in the equation below.

Equation 4-10

$$C = (M - T) * R^{-1}$$

Here it is to be kept in mind that R is itself the product of the rotation matrices along the three coordinate axis as laid out in the equation form in the dissertation.

Also, for the calculation of matrix C, scaling matrix has been assumed to be an identity matrix 'I'.

Design Tolerance Zone (DTZ) of the target feature is linearly oriented with respect to the design datum feature. Hence to arrive at the end coordinates of the DTZ as measured from the Manufacturing frame of reference, the end coordinates of the DTZ of the target feature is added and end coordinates of the specified Manufacturing datum is subtracted. This gives us the end coordinates of the DTZ of the target feature in the Manufacturing frame of reference which is given by the resultant matrix 'R' in equation 4.9.

Equation 4-11

$$R = C + D_z - E - M$$

Where 'D_z' is the end coordinates of the Design Tolerance Zone in Global Frame of Reference

and 'E' is the error term of the transformation

Also till now, for the sake of simplicity, it has been assumed that size of the manufacturing datum feature is the same as that of the design datum feature. In cases not confirming to such as situation, the code uses actual values of the scaling matrix which has been replaced by the identity matrix for above calculations. The most general solution with the actual scaling matrix incorporated is given by matrix 'G' as calculated below.

Equation 4-12

$$G = (A^{-1} * (M - T) * R^{-1}) + D_z - E - M$$

(equation 4-11). This is presented at the end of the chapter.

$x_{11} - m_{11} - d_{11} + (m_{13} - t_{13}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{11} - t_{11}\cos\beta\cos\gamma + m_{12} - t_{12}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{12} - m_{12} - d_{12} + (m_{12} - t_{12})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{11} - t_{11})(\cos\beta\sin\gamma) + (m_{13} - t_{13})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{13} - m_{13} - d_{13} - (m_{12} - t_{12})(\sin\alpha\cos\beta) + (m_{11} - t_{11})(\sin\beta) + (m_{13} - t_{13})\cos\alpha\cos\beta$
$x_{21} - m_{21} - d_{11} + (m_{23} - t_{23}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{21} - t_{21}\cos\beta\cos\gamma + m_{22} - t_{22}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{22} - m_{22} - d_{12} + (m_{22} - t_{22})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{21} - t_{21})(\cos\beta\sin\gamma) + (m_{23} - t_{23})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{23} - m_{23} - d_{13} - (m_{22} - t_{22})(\sin\alpha\cos\beta) + (m_{21} - t_{21})(\sin\beta) + (m_{23} - t_{23})\cos\alpha\cos\beta$
$x_{31} - m_{31} - d_{11} + (m_{33} - t_{33}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{31} - t_{31}\cos\beta\cos\gamma + m_{32} - t_{32}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{32} - m_{32} - d_{12} + (m_{32} - t_{32})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{31} - t_{31})(\cos\beta\sin\gamma) + (m_{33} - t_{33})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{33} - m_{33} - d_{13} - (m_{32} - t_{32})(\sin\alpha\cos\beta) + (m_{31} - t_{31})(\sin\beta) + (m_{33} - t_{33})\cos\alpha\cos\beta$
$x_{41} - m_{41} - d_{11} + (m_{43} - t_{43}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{41} - t_{41}\cos\beta\cos\gamma + m_{42} - t_{42}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{42} - m_{42} - d_{12} + (m_{42} - t_{42})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{41} - t_{41})(\cos\beta\sin\gamma) + (m_{43} - t_{43})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{43} - m_{43} - d_{13} - (m_{42} - t_{42})(\sin\alpha\cos\beta) + (m_{41} - t_{41})(\sin\beta) + (m_{43} - t_{43})\cos\alpha\cos\beta$
$x_{51} - m_{51} - d_{11} + (m_{53} - t_{53}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{51} - t_{51}\cos\beta\cos\gamma + m_{52} - t_{52}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{52} - m_{52} - d_{12} + (m_{52} - t_{52})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{51} - t_{51})(\cos\beta\sin\gamma) + (m_{53} - t_{53})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{53} - m_{53} - d_{13} - (m_{52} - t_{52})(\sin\alpha\cos\beta) + (m_{51} - t_{51})(\sin\beta) + (m_{53} - t_{53})\cos\alpha\cos\beta$
$x_{61} - m_{61} - d_{11} + (m_{63} - t_{63}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{61} - t_{61}\cos\beta\cos\gamma + m_{62} - t_{62}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{62} - m_{62} - d_{12} + (m_{62} - t_{62})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{61} - t_{61})(\cos\beta\sin\gamma) + (m_{63} - t_{63})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{63} - m_{63} - d_{13} - (m_{62} - t_{62})(\sin\alpha\cos\beta) + (m_{61} - t_{61})(\sin\beta) + (m_{63} - t_{63})\cos\alpha\cos\beta$
$x_{71} - m_{71} - d_{11} + (m_{73} - t_{73}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{71} - t_{71}\cos\beta\cos\gamma + m_{72} - t_{72}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{72} - m_{72} - d_{12} + (m_{72} - t_{72})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{71} - t_{71})(\cos\beta\sin\gamma) + (m_{73} - t_{73})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{73} - m_{73} - d_{13} - (m_{72} - t_{72})(\sin\alpha\cos\beta) + (m_{71} - t_{71})(\sin\beta) + (m_{73} - t_{73})\cos\alpha\cos\beta$
$x_{81} - m_{81} - d_{11} + (m_{83} - t_{83}(\sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma) + m_{81} - t_{81}\cos\beta\cos\gamma + m_{82} - t_{82}(\cos\alpha\sin\gamma + \sin\alpha\sin\beta\sin\gamma) + \sin\alpha\sin\beta\cos\gamma)$	$x_{82} - m_{82} - d_{12} + (m_{82} - t_{82})(\cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - (m_{81} - t_{81})(\cos\beta\sin\gamma) + (m_{83} - t_{83})(\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma)$	$x_{83} - m_{83} - d_{13} - (m_{82} - t_{82})(\sin\alpha\cos\beta) + (m_{81} - t_{81})(\sin\beta) + (m_{83} - t_{83})\cos\alpha\cos\beta$

= [R].....(Equation4-14)

4.8 Case Study

4.8.1 Inline datum change

This is the most common type of datum change that has been dealt with in so many publications and books for many years now. The purpose of dealing with this case is to just show how trivially it deals with this issue and then more complex issues are later on dealt with.

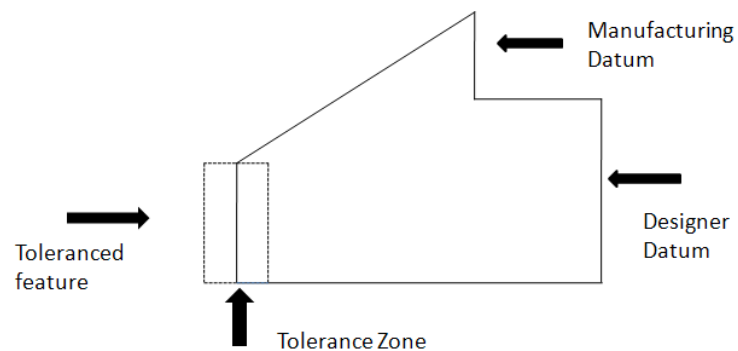


Figure 4-5 Inline 1-D linear datum change

Figure shows the theoretical setup for the case of the positional tolerance as specified on the toleranced feature with respect to two datums; one used by the designer and called the Design datum and other used by the process planner and is called the Manufacturing datum. For the above theoretical case, let's take one specific example. The figure of the part is shown below.

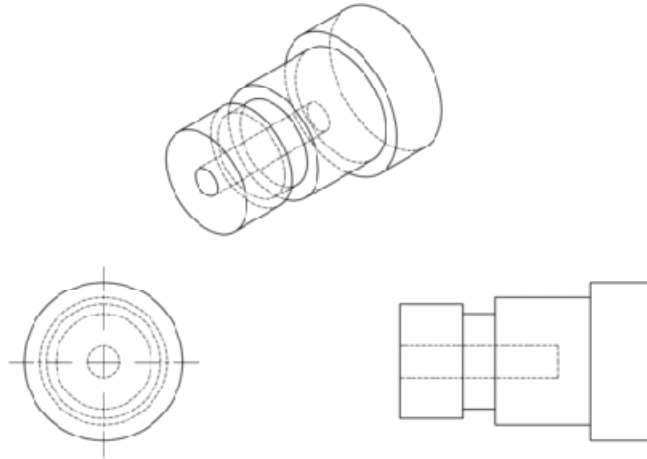


Figure 4-6 An example of inline 1-D linear datum change. The extreme end views (left and right) are the 2-D orthogonal views of the part shown in trimetric orientation in the center.

This is the trivial case of the linear stack up and it involves the change in datum for the positional tolerance specified on the target feature. The target feature, designer datum and the manufacturing datum could be any of the parallel faces and the procedure mentioned in this research can cater to all such situations successfully. From the mathematical model discussion as given previously, it is clear that only the translational matrix will be relevant in this case while the rotation matrix will be an identity matrix.

4.8.2 Inline with different azimuth datum change

Another case could be the target feature inline to both design and manufacturing datum but the two datums are having different azimuth in addition to not being inline to each other.

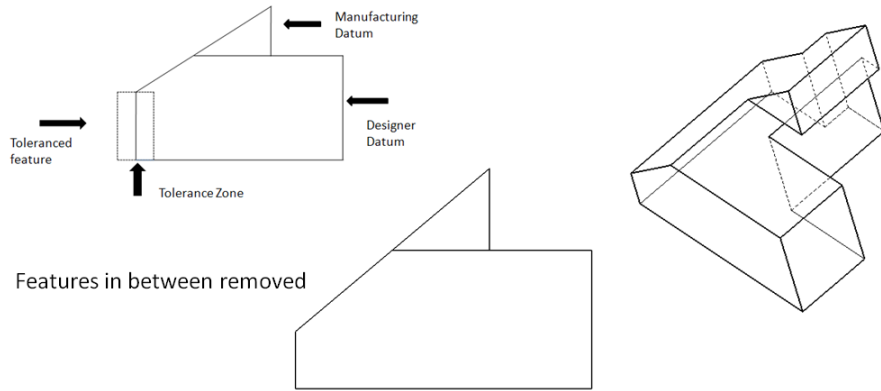


Figure 4-7 A case of datum change involving different azimuth of datums for same target feature. Left most view gives the theoretical identification of the entities involved, central view is a 2-D view of the part shown in the trimetric view on the right.

More interesting, there could be a case of the manufacturing datum feature whose plane normal is although parallel to the plane normal of the design datum, but the two normals are never coincident in the entire features. This case is shown in the figure below:-

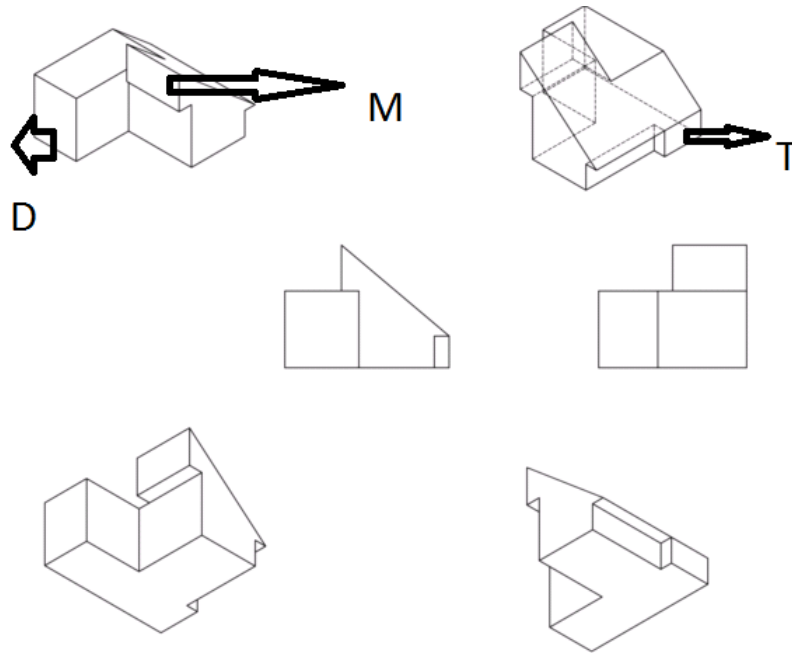


Figure 4-8 An example of datum change with different azimuth and offset involved. The target feature ‘T’ , Design datum ‘D’ and Manufacturing datum ‘M’ are identified. The remaining orthogonal and projected views are for clarifying the geometry of the part involved.

Also you can see that the target feature is in line with the designer datum feature while no material exists in front of the manufacturing datum in the line of the direction of the target feature. Here D represents Designer Datum, M stands for Manufacturing Datum and T stands for Target feature on which tolerance is specified.

4.8.3 Twisted datum change

The next example could be of a twisted datum change between the Designer blue print and the manufacturing supervisor process plan. In this case, rotation of the axis system is involved in addition to the azimuth, offset and translation. Thus, in

this case, the rotation transformation matrix is any 3x3 matrix other than an identity matrix. This is shown in the figure 4-9. The next case shows the scenario when all three rotations in addition to the translations and scaling is involved. This scenario is presented in figure 4-10.

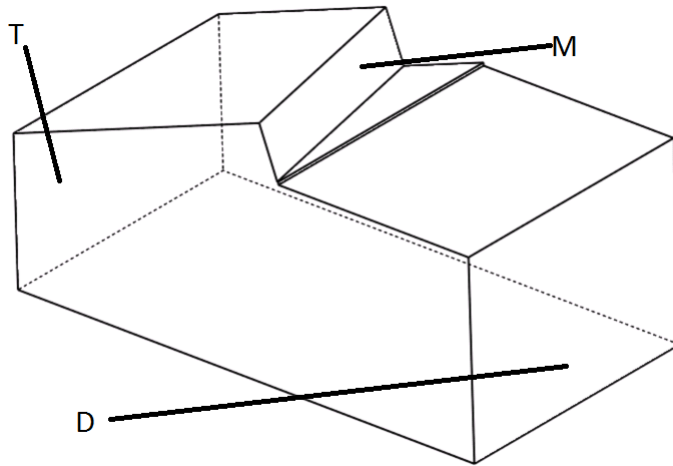


Figure 4-9 The twisted case of datum reference change

4.8.4 Squeezed datum change

It is to be remembered that all these scenarios of datum reference change may or may not have squeezed datums involved. This will be the situation when the two frames of reference are not themselves orthogonal. Hence, this situation could apply to any of the cases that are discussed earlier.

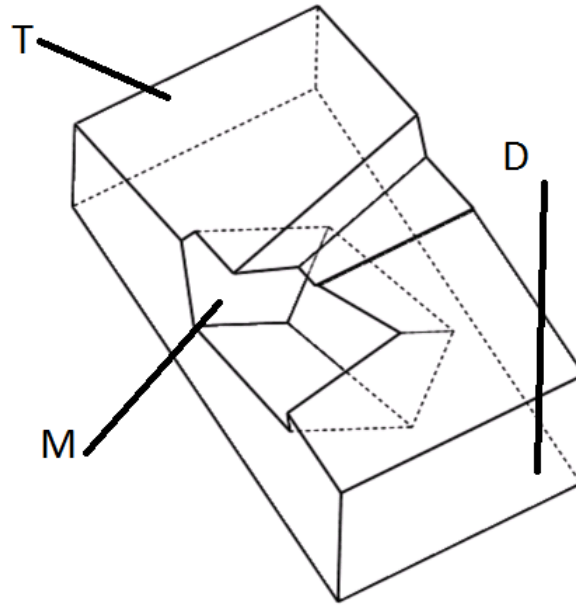


Figure 4-10 Twisted case of datum reference change with all three Euler angles involved

4.9 Application of the mathematical model to various types of datums and the datum reference frames.

This math model is capable of dealing with all types of datums commonly used in tolerancing the part by the designers and the manufacturing process planners alike. Also, the math model and the associated code has been applied for determining manufacturing tolerances involving various types of datums as specified in ASME GD&T standard (Y14.5 – 2009). The different types of datum features for which this model (in addition to the planar datum features already demonstrated) can be used, are listed below:-

- i) Width as a datum feature. This is a relatively new datum feature in which two opposed parallel surfaces are simultaneously used for identifying another parallel surface which is equidistant from the two

opposed surfaces. A common example of this kind of datum feature is the CTP (Central Theoretical Plane). It is called theoretical as it is not a physical entity. CTP used in the case of tab and slot is one of the common uses. This kind of datum feature has three unconstrained degrees of freedom i.e. two translations and one rotation.

- ii) Spherical (or conical or cylindrical) datum feature. Spherical datum feature is used to establish a spherical datum feature simulator that involves the creation of the datum centre point and constraining three translational freedoms. Conical datum feature is required to establish a conical datum feature simulator that identifies a datum axis and an associated datum point. This results in only one unconstrained degree of freedom which is rotational. Cylindrical datum feature types establish cylindrical datum feature simulators which have one rotational and one translational degree of freedom unconstrained.
- iii) Linearly extruded datum features. These are the datum features which could involve any planar profile linearly extruded to form a geometric feature. These are one of the types which are comparatively not easy to handle analytically. However, one of the biggest pros of this mathematical model and the associated code is that it is fully developed to handle such features just like any other feature. Datum feature simulator for this type involves the creation of a datum plane and a datum axis. Datum plane is the symmetry plane which could be perpendicular to the direction of extrude. Datum axis is the direction of

extrude. This type of datum feature has only one translational degree of freedom unconstrained.

- iv) Complex datum features. Any datum feature type which is not planar and not covered in the above list will be called complex datum feature. For the datum feature simulator for this type, datum point needs to be defined in addition to previously mentioned datum plane and datum axis (for previous case). This is a fully constrained feature with all three translational and rotational degrees of freedom constrained.

5 GRAPH THEORETIC APPROACH TO RELATIONSHIP REPRESENTATION AND TOLERANCE ANALYSIS AND SYNTHESIS FOR MANUFACTURING PROCESS PLANNING

5.1 Introduction to graph theoretic approach

After a close look at all the key aspects of tolerance analysis and synthesis, it is clear that the whole world of tolerance analysis and synthesis revolves around the following:-

- a. Geometric features
- b. Geometric constraints
- c. Geometric tolerances
- d. Manufacturing worst case limits

These aspects and their relationship can be explained in a better way by using pictorial graphs for visualization purposes and by using graph data structures for automating the process of tolerance analysis and synthesis. In this section, pictorial graphs are used to explain the process and then the use of graph data structures and object oriented programming is touched upon to explain how the process of automation of tolerance analysis and synthesis can be facilitated by the use of these programming techniques.

5.2 Relationship between geometric features

Relationships between the geometric features can be represented by a G-graph. G-graph is the most basic graph to which other graphs are attached to achieve other

detailed graphs as explained later. The G-graph can consist of faces, edges, wires, bodies or lumps etc as recognized in detail in the publication documents of the geometric kernels. However, interest will be limited to those entities which are required for manufacturing tolerance analysis and synthesis.

The part under consideration and its associated G-graph are shown in the figures below.

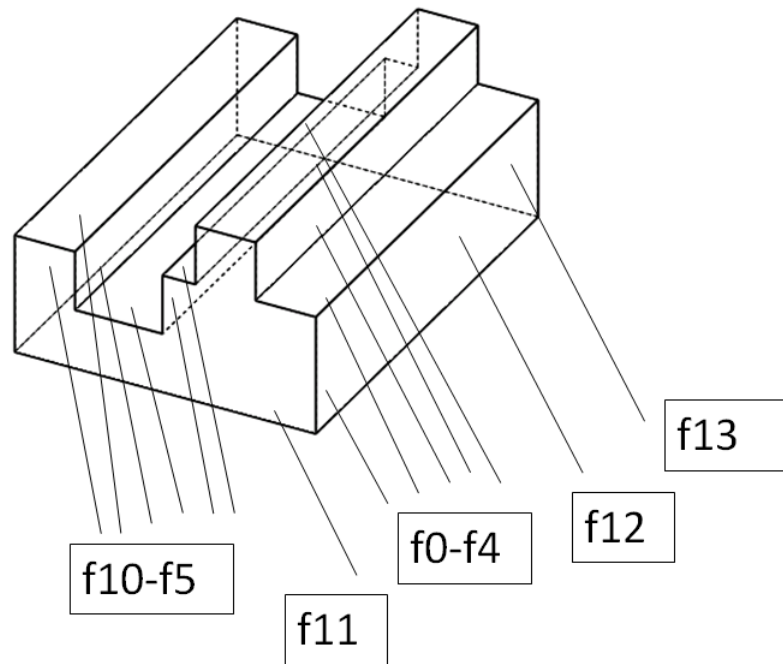


Figure 5-1 The planar part and the identified faces

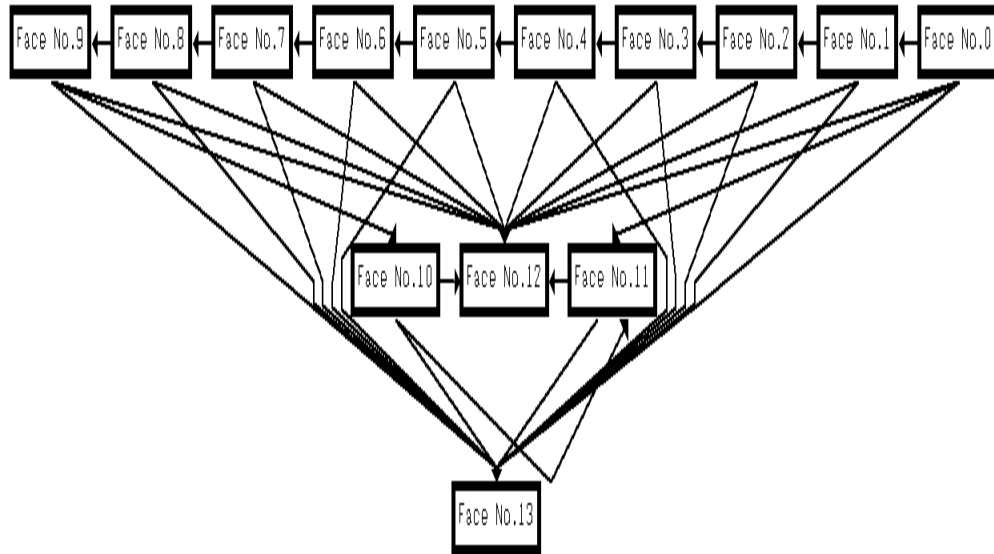


Figure 5-2 The associated G-graph for the part above. Face numbers are boxed while the associated edges are represented by the continuous lines.

5.3 Geometric constraints as applicable to geometric features

Geometric constraints such as parallel, perpendicular etc are relations that relate the geometric features. In a Constraint Feature graph or the CF- graph, the geometric features are the vertices of the graph while the geometric constraints are represented by the edges that link the vertices. For pictorial graphs such as the one shown in the next figure, the different types of the constraints can be shown by different line type such as dashed, dotted etc. Further refinements of the line type could be the weight of the line type or the color of the line (for color graphs only).

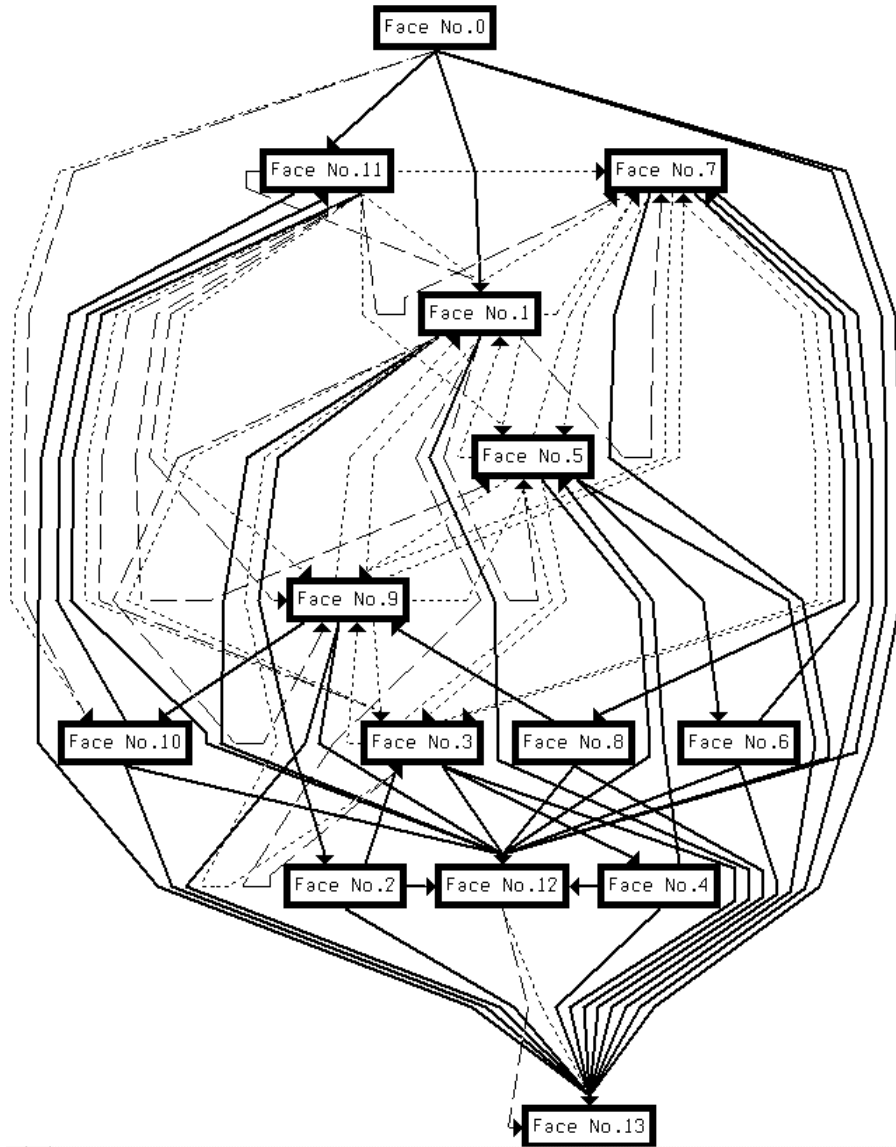


Figure 5-3 CF graph for the part under consideration.

5.4 Tolerances applied to geometric constraints existing between geometric features

Geometric tolerances can be regarded as the refinements of the geometric constraints that exist between the geometric entities. These can be made part of

the graph by defining them as attributes so that they are associated to the constraint that exists between the geometric entities.

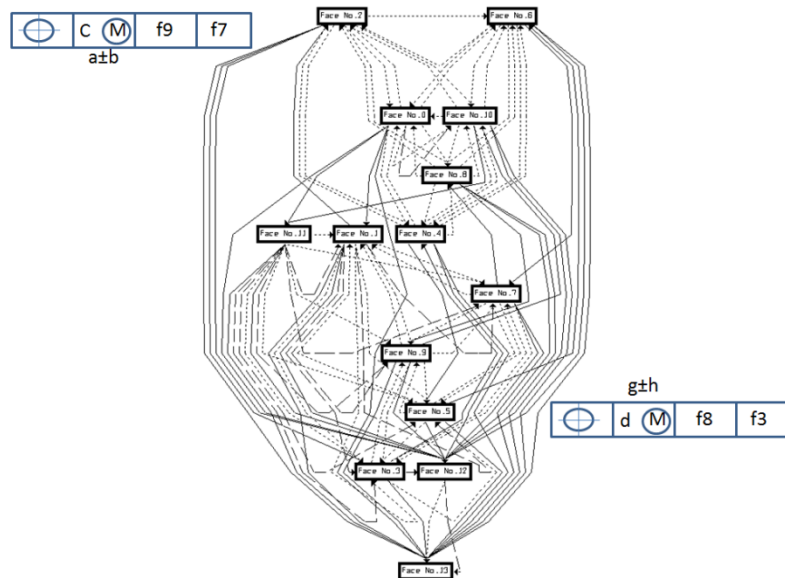


Figure 5-4 A sample TCF graph shows how certain tolerances are defined as the attributes of the constraint that exists between the geometric features.

5.5 Use of data structures

The data structure is a particular way of storing and organizing the data or information in the computer so that it can be used efficiently for analysis purposes. From the previous figures, it is evident that all this information needs to be organized in an efficient way so that it can be used for an efficient analysis in the least amount of time. The most important information from the manufacturing tolerance transfer analysis and synthesis point of view is the relationship between the target and datum for both the design blue prints and manufacturing process plans. Other pieces of information that are relevant to the analysis is the shape of the geometric feature (applicability of the tolerance type to the geometric feature

type), material condition of the target feature and primary, secondary and tertiary datums, tolerance chain,(consisting of stack direction, far and near side normal, reference point) etc.

5.6 Selection of data structure

Based upon the understanding of the mechanics of the manufacturing tolerance transfer analysis and synthesis, the doubly linked list is the most suitable data structure to store the information of the geometric constraints and the geometric tolerances as far as the target and datum features are considered; both from the designer and manufacturing supervisor point of view. In a doubly linked list, each node has two link fields, one linking in the forward direction and the other linking in the backward direction. This is especially useful for tolerance chain detection.

5.7 Exploitation of the object oriented programming (OOP)

There is no need to introduce or explain the benefits of object oriented programming over here. The intent is to explain how these special characteristics of the OOP can be utilized for manufacturing tolerance transfer analysis and synthesis. The basic features such as abstraction and encapsulation need not be mentioned. The pointers could be used for relating target features to the datum features both for design and manufacturing. The concept of inheritance can be used for deriving classes from the base class such as the Parallel constraint class from the base geometric constraint class. Operator overloading can be used to utilize same functions for design and manufacturing whenever the need arises.

5.8 Use of Microsoft Foundation Classes (MFC)

In a graphic user interface environment, it is all the messages that are crucial to its working. These messages include the location of the mouse which is the most basic information passed through a message. Basic message handling such as this is a feature of the visual C++ IDE. MFC is a collection of C++ classes that provide an object oriented framework that can be utilized to create windows applications. MFC helps you create dialog boxes, text boxes, radio controls, and check boxes and helps you in working with icons, cursors and bitmaps. One of the other most important use of the MFC is the creation of the tree view controls along with the status bars, mouse hover messages etc.

5.9 Use of ACIS geometry kernel

ACIS is the object oriented (written in C++) three dimensional geometric modeling engine which is the product of Spatial Technology Inc (Spatial). All information relevant to the geometric features such as their shape, dimensions (height, width, length etc as applicable) are provided by the ACIS kernel. Hence, it is used in the construction of the CF graph and all other graphs built on top of it such as the TCF graph and Manufacturing Worst Case Tolerance Feature Graph (MWCTCF).

5.10 Validation of the manufacturing process plans

For the purpose of the validation of the manufacturing process plans, first the TCF graph for the designer blue print is to be constructed. It will be searched for any geometric feature that has been differently dimensioned in the manufacturing

process plan as compared to the designer blue print. The next figure shows the specific dimensions with the target feature and the datum feature indentified (red line).

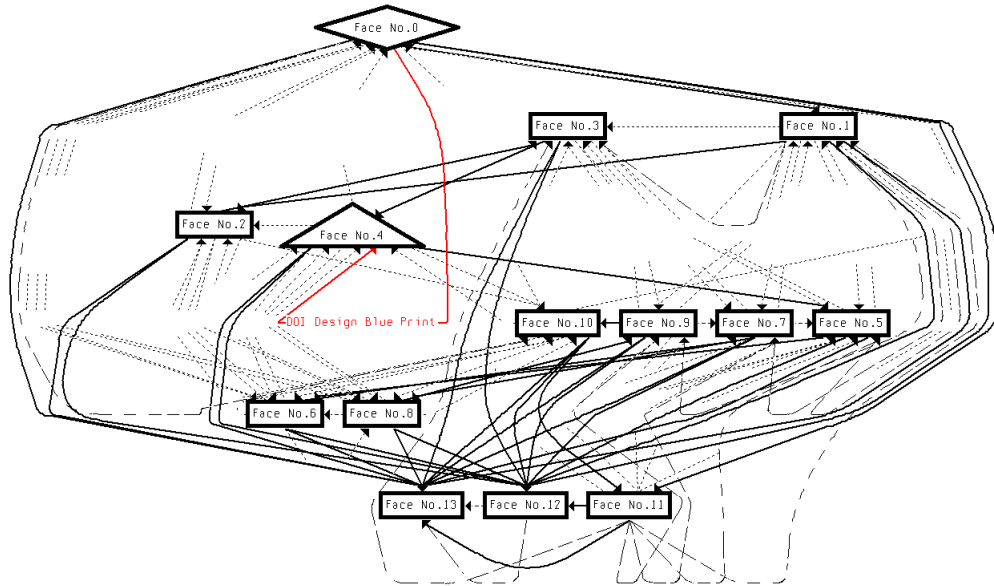


Figure 5-5 TCF graph for the sample design blue print. Face f_4 is the target feature and face f_0 is the design datum feature.

Another graph is then created which is called the Manufacturing Worst Case Tolerance Feature Graph (MWCTCF). This is based upon the manufacturing process plan which is in need of verification. With the target feature same as the one identified earlier, the tolerance chain is traversed to reach to the design datum feature. This portion of the tolerance chain is highlighted in green in the next figure.

Finally, based upon the stack calculations and tolerance accumulation (positive or negative) as applicable, the validity of the relations (as mentioned in the

theoretical portions of the model) will determine the validity of the manufacturing process plans.

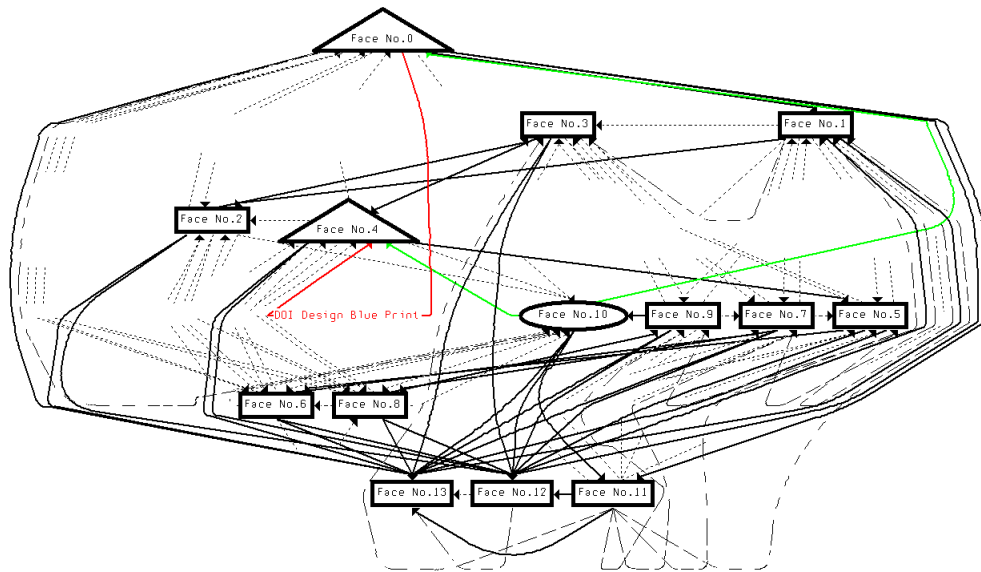


Figure 5-6 Manufacturing Worst Case Tolerance Constraint Feature (MWCTCF) graph for the given manufacturing process plan which is in need of validation.

6 THEORETICAL EXTENSIONS OF THE 3-D MODEL FOR TOLERANCE TRANSFER IN MANUFACTURING PROCESS PLANNING

6.1 Introduction

In this chapter, various reasons for the requirement of tolerance transfer in manufacturing process planning are discussed. Various options for tolerance types for a given geometric variation are explained. More than one interpretation of the tolerance type is forwarded as an example to justify the replacement of the tolerance type with another geometric tolerance. This is followed by few case studies for tolerance transfer with datum reference change involved. Lastly, as an example, it is shown how parallelism tolerance can be used to replace the flatness tolerance mentioned in the design blue print. This is backed up by the justification of the legality of such a change in the tolerance type used. An analytical formula is derived and presented for this case at the end.

6.2 Background

For the purpose of this research, it is assumed that the manufacturing process plan is provided. It means that the details of the different processes are included. Each process description will contain the features involved in the machining operation, the locating features, the fixtures used, associated dimensions and tolerances etc. The overall process plan will also enlist the sequence in which the various processes proceed.

The utility of this three dimensional tolerance transfer model arises from the fact that not all features are dimensioned and toleranced from the same datum as given by the Design Blue print. The manufacturing supervisors when making the process plan change datums for GD&T details of a particular feature for their ease of use, efficient use of machine, availability of jigs and fixtures, minimization of part wastes and overall cost reduction. Thus, with the datum change, the desired dimension and tolerance on a feature of interest might not be directly available on the Manufacturing Process Plan as it is on the Design Blue Print. The new model, as presented in this research, will enable tolerance analysis which will determine if the dimensions and tolerances mentioned in the Manufacturing Process Plan will be able to meet the Design Blue Print dimensions and tolerances eventually.

An important question in this regard is about the domain of the types of tolerances involved in the creation of this model for tolerance transfer. Mostly, in industry, manufacturing tolerance charts are in use even in these days. These charts deal with conventional plus / minus tolerances mostly. Some variations of the manufacturing tolerance charts use geometric tolerances. Not all types of geometric tolerances are directly dealt with. However, in this new approach of manufacturing tolerance transfer, all geometric tolerances will be dealt with. For the feature of interest, it can be shown that if different geometric tolerances are applied in the Design Blue Print, these can be resolved, represented and evaluated in the Manufacturing Process Plan.

6.3 Applicability of the manufacturing tolerance transfer model

On considering the tolerances minutely and looking at how the specification of the different tolerances affects the tolerance analysis, it will be clear that all tolerances specified do have an associated tolerance zone or they affect the tolerance zone of another geometric tolerance for which it acts as a refinement. Based upon the above observation, the case of a specific geometric tolerance zone can be resolved in two different ways:-

- a. the specified geometrics tolerance might have been replaced with the same type of geometric tolerance but with a different datum (could be primary, secondary or tertiary as applicable).
- b. the specified geometric tolerance might have been replaced with a different type of tolerance which covers the same worst case limit.

In second case, a special consideration is to be given to the type of tolerance specified by the Designer in the Designer Blue Print. For example, if the position tolerance is replaced by the worst case size limit, it will entail that two separate interpretation of tolerance zones need to be satisfied;

- a. as a boundary limiting the movement of the surface of a feature of size (FOS)
- b. as a three dimensional volume in space coordinates representing limits of the movement of the axis of a feature

As can be seen, in the first case, the surface is involved while in the second case, it is the axis which is being considered. This axis could be the axis of the hole or pin for cylindrical features and is the Central Theoretical Plane (CTP) for the

planar features such as tab and slot. Both the concepts could be shown to be equivalent. However, the boundary concept is more commonly used as it represents functional requirements of the mating parts and is also, more flexible of the two concepts discussed here.

In dealing with manufacturing process plans, the major reason for existence for this model of tolerance transfer is the datum reference change. Depending upon the type of the geometric tolerance, there could be two cases:-

1. It could be the change in the sole datum required for the geometric tolerance specification or
2. Change in the role of the datum for the specification of the geometric tolerance

While case # 1 has been discussed earlier, case # 2 is described here for better understanding.

If multiple datums are required for the specification of the geometric tolerance, then two different types of situations can be visualized:-

1. Change in the order of the datums i.e. the order of precedence for the datums
2. Replacement of the datum reference frame with datums not included in the design datum reference frame for the feature of interest.

However, it remains to be specified that reason for existence of the requirement for tolerance transfer is not limited to datum(s). Other reasons are listed below:-

1. Change in the tolerance type for representing the geometric variation from Design Blue Print to the Manufacturing Process Plan.

2. Change in the Material condition for the datum.

6.4 Different types of tolerances used for same geometric variation

An important point to consider is multiple geometric tolerances for the feature of interest. If multiple geometric tolerances are specified for the feature of interest in the design blue print, it seems appropriate that same or similar type of geometric tolerances be specified in the Manufacturing Process plan. A good justification for this is based upon the ASME Y14.5 GD&T standard which specifically requires that each tolerance specified on the feature should be individually respected for the part to be usable. In the above lines, the words ‘same or similar’ have been intentionally used. Same, here refers to exactly the same type of geometric tolerance while similar, here refers to a different type of geometric tolerance which serves the same purpose as the original geometric tolerance in the Design Blue print.

For similar types of tolerances, the example of location control for coaxial features of size can be cited. Coaxial features of Size are two (or more) features of size, normally diameters, whose axes are coincident. In cases, when it is desirable to permit the feature of size to depart from the coaxial condition, three types of geometric controls can be effectively used. These are:-

1. Tolerance of position
2. Run out
3. Concentricity

Out of the above three, tolerance of position is the most liberal and least expensive coaxial control. This is the reason for it being most widely used in such situations. Run out is a compromise between tolerance of position and concentricity in such situations as it is a tighter and more costly control than position tolerance. Of all, concentricity is the most stringent and expensive control. It is also most difficult to verify.

6.5 Case studies for manufacturing tolerance transfer arising from datum reference change

Before discussing these cases one by one, let's make certain things clear.

6.5.1 Dimensionality of the problem

The model for tolerance transfer in manufacturing process planning as presented in this research is three dimensional. This is the reason that all figures show 3-D view of the parts involved. Every machining cut is assumed to have some amount of error in accuracy which will cause other dimensions to be affected which are not intended as far as the machining cut is concerned. Hence, every machining cut will entail tolerances to be affected three dimensionally. This model considers the effect of these errors in machining and hence, apart from affect on tolerances in the direction of the cut, projections as determined mathematically in other directions are included in the other associated dimensions.

6.5.2 Not limited to planar features

Although, planar features have been used to demonstrate the details of the three dimensional tolerance transfer model, the model itself is all encompassing and can be applied to all types of features.

6.5.3 Interest in manufacturing process planning limited to tolerance analysis only

This research's focus is on tolerance analysis and only the details relevant to this aspect of the manufacturing process planning are presented.

Let's consider the case of the Design blue print for the part shown in the figure below:-

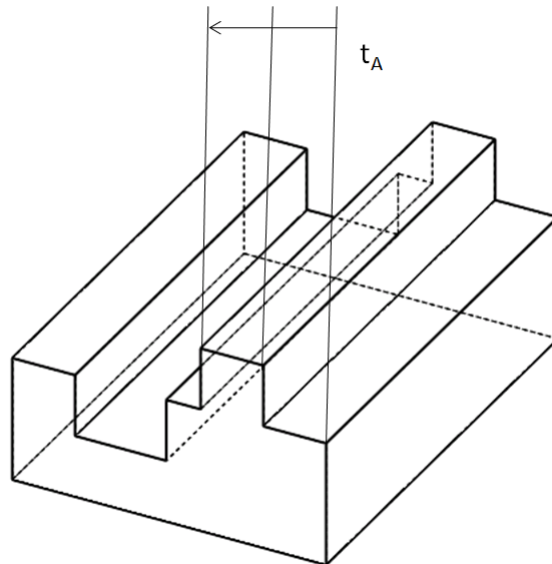


Figure 6-1 Planar part with feature of interest f_4 dimensioned with face f_0 as the design datum in the design blue print.

All unnecessary details that are not relevant to the discussion over here are omitted. This means that although a complete Design blue print is being reviewed, the feature of interest has been identified and only the information relevant to that feature is being displayed. Here the feature of interest is f_4 and is toleranced with feature f_0 as the datum. The concerning mean dimension is 'A' and associated design tolerance is ' t_A '. This means that although other features have their dimensions and tolerances specified, they have been ignored to save clutter. The feature of interest has been selected based upon the criteria for the tolerance transfer in manufacturing process planning which is listed below:-

1. Different datum has been used for the specified tolerance in Design blue print as compared to the Manufacturing process plan.

Other reasons for the selection of the feature of interest for the tolerance transfer in manufacturing process planning have been identified earlier and discussed in detail and will not be repeated over here. Although, they would be mentioned summarily once the relevant case is under discussion.

The manufacturing plan should have the tolerance for the feature of interest less than or equal to A; otherwise the manufacturing plan needs a review.

Four different situations have been envisaged for the case discussed above and are represented by four different Manufacturing Process plans as discussed in the following paragraphs.

6.5.4 Sample Manufacturing Process plan extract #1

Let's consider the first case which is depicted in figure 6-2 below.

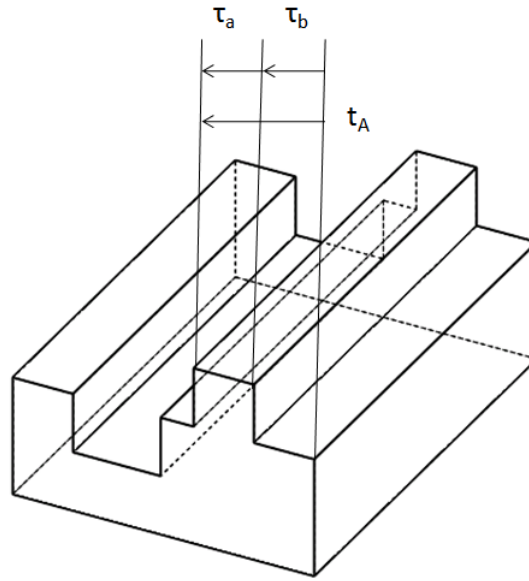


Figure 6-2 tolerance analysis related details for the manufacturing plan #1 for the feature of interest.

The arrows show the individual machining cuts made during the processes. The arrow head represents the target feature while the arrow tail represents the datum feature or the reference feature.

Considering the manufacturing process plan with the associated Geometric Dimensioning and Tolerancing reveals that feature of interest which is f_i ($f_i = f_4$ in this case) has been dimensioned with face f_0 as the reference in Designer blue print. However the same feature of interest has been dimensioned in this manufacturing process plan with respect to face f_2 as the reference. This reference face f_2 has been in turn dimensioned with face f_0 as the reference. The corresponding tolerances are respectively ' τ_a ' and ' τ_b '. The rules of manufacturing imply that the accumulated tolerance in the manufacturing process plan for the

feature of interest should be less than or equal to the design tolerance. This is mathematically expressed as follows:-

Equation 6-1

$$\tau_a + \tau_b \leq t_A$$

6.5.5 Sample Manufacturing Process plan extract #2

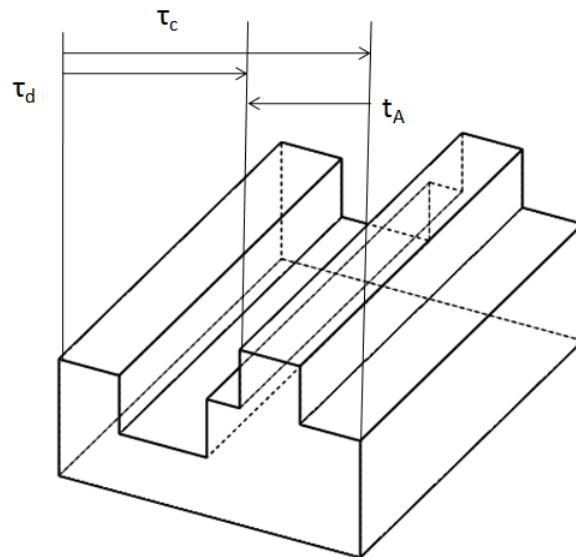


Figure 6-3 Tolerance analysis related details for the manufacturing plan #2 for the feature of interest.

The second manufacturing process plan is depicted in the figure above. Again the design tolerance for the feature of interest f_i ($i=4$ again) for this case is t_A with the associated mean dimension A . However, in this process plan, the feature of interest f_4 is being dimensioned with f_{10} as the manufacturing datum. This means that the target feature f_4 will be machined with f_{10} as the datum. However, this feature f_{10} is also being used as a datum for feature f_0 which is the design datum.

In this case, if the mean dimensions will be maintained, then the following relation should hold:-

Equation 6-2

$$\tau_d - \tau_c \leq t_A$$

6.5.6 Sample Manufacturing Process plan extract #3

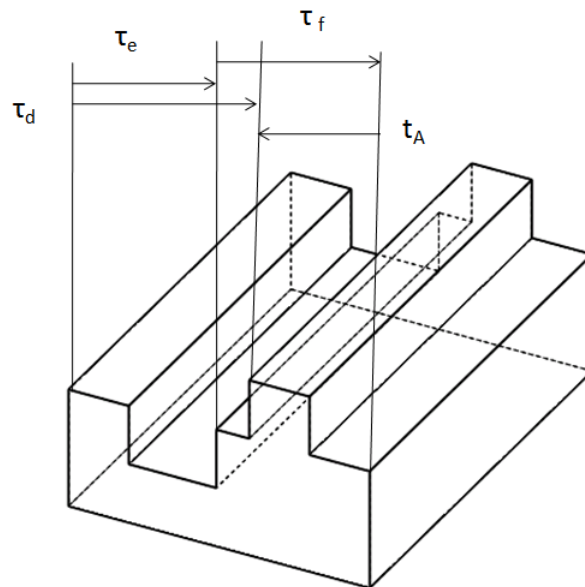


Figure 6-4 tolerance analysis related details for the manufacturing plan #3 for the feature of interest.

In this manufacturing plan, the feature of interest f_4 has been dimensioned with face f_{10} as the manufacturing datum. This feature f_{10} has also been used as a manufacturing datum for face f_6 . The face f_0 has been dimensioned in this manufacturing plan with face f_6 as the manufacturing datum. This face f_0 is the design datum which has been used for the dimensioning of the target feature (f_4 in

this case). For this manufacturing process plan to be acceptable for the specific case of the feature of interest f_4 , the following relation should be valid:-

Equation 6-3

$$\tau_d - \tau_e - \tau_f \leq t_A$$

6.5.7 Sample Manufacturing Process plan extract #4

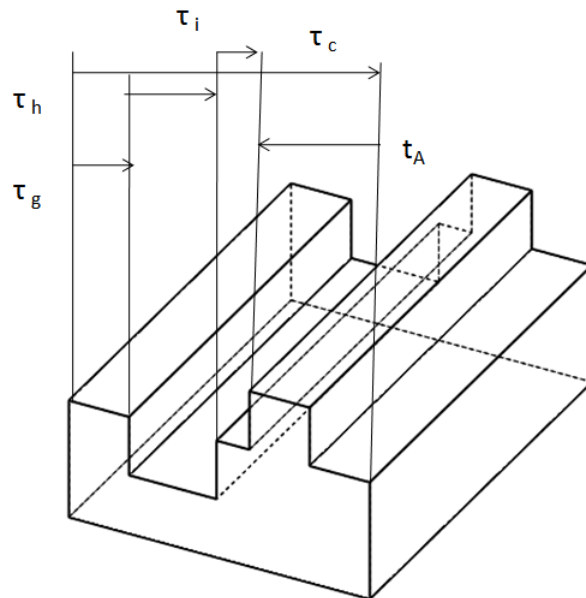


Figure 6-5 tolerance analysis related details for the manufacturing plan #4 for the feature of interest.

In this depiction of manufacturing process plan, target feature f_4 is dimensioned with feature f_6 as the datum which in turn acts as a target feature with face f_8 as the datum. This feature f_8 is in turn dimensioned with face f_{10} as the target feature. Face f_{10} serves as a manufacturing datum for the feature f_0 which is the design datum for the feature of interest in the design blue print. The validity of the

manufacturing process plan as far as the target feature of interest is concerned will be governed by the following relation:-

Equation 6-4

$$\tau_g + \tau_h + \tau_i - \tau_c \leq t_A$$

Before moving to the next section, there are certain points that need to be cleared. First of all, these figures do not show the complete details of the manufacturing plan. Instead, they only show the details for the feature of interest from the tolerance analysis point of view. Also, it is to be kept in mind that once it is stated that this feature 'x' is dimensioned with feature 'y' as the manufacturing datum, then that means that machining supervisors are instructing the shop operators to use feature 'y' as the operational datum for machining the target feature. For more clarity, please refer to the details about the meaning of the arrow head and the arrow tail viz-a-viz manufacturing process plan.

6.6 Case study for manufacturing tolerance transfer arising from different type of tolerance used in manufacturing process plan as compared to Design blue print.

6.6.1 Form controls

In general, form controls refine (reduce) or expand the shape of the tolerance zone for a feature-of-size in the event that functionality requirements are not met , satisfied or applicable as set out by Rule #1. This GD&T rule which is also known as the Envelope rule or the Taylor's rule, says that for features of size, where only a size dimension is specified, the surfaces shall not extend beyond a boundary

(envelope) of perfect form at Maximum Material Condition (MMC). It is to be kept in mind that Rule # 1 applies to all features-of-size on a drawing. It is like an invisible control that always applies to all features-of-size unless there is another geometric tolerance that overrides it. Rule #1 applies to all individual features-of-size only and there are only two exceptions to this rule: -

- a. non rigid parts
- b. stock sizes.

It is to be kept in mind that Rule #1 and its associated size dimension are interrelated.

When the form of an object is under consideration; it could be the flatness of its surface, the roundness of its circular section, or the degree of straightness of the line elements of the object. Form control is used to define the shape of a feature in relation to itself. Hence form controls never use a datum.

So, if no datums are involved, could manufacturing tolerance analysis be carried out for parts in which feature of interest have form control applied on it? As mentioned previously, the justification of the creation of this model for manufacturing tolerance transfer could be based upon the following scenario i.e. specified geometric tolerance might have been replaced with a different type of tolerance which covers the same worst case limit.

Now, within the form tolerance control, let's see which types of tolerances are there and what are the alternate tolerance specifications for each.

6.6.1.1 Flatness

A flat surface from the tolerance point of view means that all of its elements lie in a single true plane. The amount by which the surface elements are permitted to vary from this true plane gives us the flatness tolerance value. A flatness tolerance zone is generated by the distance between two parallel planes. The theoretical plane is established by the three high points of the considered surface. The second plane is parallel to this theoretical plane and is offset by the amount of the tolerance value. All points of the considered surface must be found to be between these two planes.

An important point to consider is the refinement of the implicit flatness control based upon Rule #1 by the explicit flatness tolerance control specified by the geometric tolerance. Whenever Rule #1 applies to a planar feature-of-size, an automatic flatness control is created for both the involved surfaces; i.e. the target surface and the reference surface. This implicit flatness control is the result of the interrelationship of Rule # 1 (perfect form at MMC) and the size dimension. As the feature of size departs from MMC to LMC, a flatness error, equal to the amount of the departure is permitted.

However, when this implicit flatness control is not sufficient to satisfy the functional requirement of the part surface, an explicit flatness control in the form of flatness tolerance needs to be added. Although, it is to be kept in mind that this flatness tolerance will never over ride Rule # 1 but it refines (reduces) the maximum allowable flatness error of the surface.

In addition to the implicit flatness control of Rule # 1, there are other geometric controls that can affect flatness of a surface indirectly. These are:-

1. Parallelism
2. Perpendicularity
3. Angularity
4. Profile
5. Total Run out

Let's take the case of flatness tolerance as applied in the figure below:-

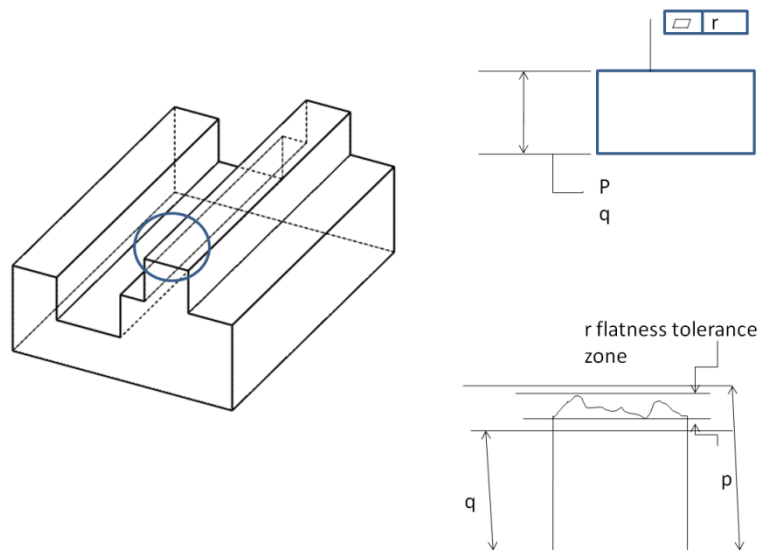


Figure 6-6 The part under consideration and the relevant feature is shown on the left side. Top right shows the top view. Bottom right shows the tolerance zones involved.

Implicit flatness tolerance zone as given by the Rule # 1 has a height of 'd' (= p-q) as given by the difference of the boundary envelope and Least Material Condition (LMC) limit. On the top of that, 'r' (unit) flatness tolerance has been

specified by the designer. This flatness tolerance specification gives a tolerance zone which floats within Rule # 1 limit.

This is the part of the specification as given in a sample Designer Blue print. Let's now consider the same situation as covered by the manufacturing supervisors in the manufacturing Process plan. Leaving aside other GD&T related information on the part which is not directly related to the feature of interest and is not contributing to the tolerance analysis that is intended to be carried out based upon manufacturing process plan 'A', it is observed that the manufacturing supervisors have specified a parallelism tolerance on the feature of interest.

Before the modalities of the tolerance analysis based upon the manufacturing process plan are discussed, it needs to be seen that such a specification is legal, practical and common or not? As far as legality is concerned, it is 100% legal as it meets the legal specification criteria of parallelism tolerance. The criteria cannot be fully discussed but it mainly emphasizes that

1. A datum reference should be used in the feature control frame.
2. The parallelism control should be mainly specified on a feature of size. In some cases, if it is not specified on the feature of size, it is still legal specification if no modifiers such as Regardless of Feature Size (RFS) , LMC or MMC, are used.
3. Tolerance value should be a refinement of the other geometric tolerance. In this case, parallelism tolerance is the refinement of the tolerance zone set out by Rule #1

There is no doubt that the replacement of the flatness tolerance by parallelism is practical as both of the tolerances in this case could be verified with almost the same ease. The last thing to look for is that whether this type of replacement tolerance common in the industry or not. No perfect comments could be forwarded on this as no one person or agency can claim that he /she /it has the knowledge of all types of cases for parallelism or flatness specification; whatever be his/her /their area of specialization and tenure of experience.

There is a major difference between the tolerance zones of parallelism tolerance and flatness tolerance for the same feature of size for identical value of tolerance. It is that flatness tolerance zone is composed of two parallel planes which are not necessarily parallel to the reference plane for the size tolerance, while in the case of parallelism tolerance, the two parallel planes are to be necessarily parallel to the reference plane for the size tolerance. So once the replacement of the flatness tolerance by parallelism tolerance is under consideration, an interesting case arises which is discussed below.

In the figure above, size tolerance zone along with the flatness tolerance zone is shown. The parallelism tolerance should always be greater than or equal to the flatness tolerance. This is determined by the angle of tilt of the flatness tolerance zone parallel planes. This tilt of the flatness tolerance zone has not been individually considered anywhere till now. This tilt is important as it also determines the maximum flatness tolerance refinement that can be applied to size tolerance. It can be seen that if this tilt is minimal, then flatness tolerance as close to the size tolerance could be applied. However, if this tilt is significant, then the

amount of flatness tolerance that could be applied as a refinement to the size tolerance will be greatly reduced. This idea is not discussed in detail as to how much this reduction is dependent on the amount of tilt but this idea is extremely helpful in understanding how much should be the value of the parallelism tolerance once it is applied as a replacement to the flatness tolerance.

In the figure below, a 2-D cross section of the tolerance zone for parallelism tolerance is shown. If 'θ' is the angle of the tilt of the parallel planes for flatness tolerance, and d_y is the length of the tolerance zone in this view, and t_f is the amount of flatness tolerance, then the amount of the parallelism tolerance 'τ_p' required for the replacement of the flatness tolerance is given by :-

Equation 6-5

$$\tau_p = \tan \theta (d_y + t_f \sin \theta) + t_f \cos \theta$$

To verify the relationship, it can be seen that for the case of zero tilt of the planes of the flatness tolerance, the parallelism tolerance value comes out to be equal to the flatness tolerance value.

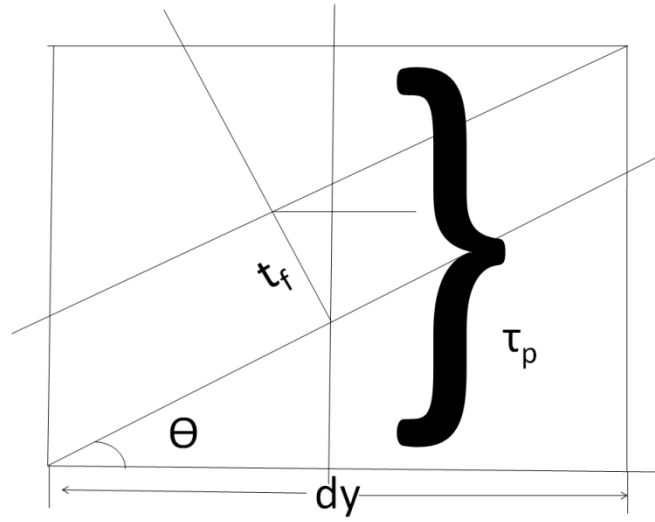


Figure 6-7 Calculation of the parallelism tolerance limit as a replacement to flatness tolerance limit.

7 USE OF THREE DIMENSIONAL TOLERANCE TRANSFER MODEL (3DTT) FOR DETERMINING THE MEAN DIMENSIONS AND TOLERANCES FOR THE MANUFACTURING PROCESS PLAN

7.1 Introduction: Various Utilities based upon Three Dimensional Tolerance Transfer (3DTT) concept

This paper is based on the research that shows how 3DTT is used to determine the dimensions and tolerances for the process plan. This is in series of how the 3DTT could be used for various purposes related to manufacturing operations and manufacturing tolerances. The use of 3DTT can be summarized as following:-

- a. For determining manufacturing tolerances for Non inline datum reference change.
- b. For working out tolerances and dimensions for implied cases between features which are specifically toleranced in Designer Blue Print and not in Manufacturing Process Plan.
- c. For checking the validity of the manufacturing process plan
- d. For determining the dimensions and tolerances of the manufacturing process plan
- e. Finally, graph theoretic approach to model the manufacturing process plan to be used for tolerance analysis and synthesis.

Here the intent is to discuss the details involved for the task mentioned as 'd' above.

7.2 Preliminary Information (required to start)

Before beginning to work upon the manufacturing process plan, the Designer Blue print is to be fully read and understood. Any special stipulations as mentioned by the designer are to fully borne in mind.

7.2.1 Number of stack directions

Next thing is to see in how many dimensions (spatial), the designer has specified dimensions and tolerances. This may initially seem to be a trivial thing in most circumstances but the approach presented here is intended to be all encompassing as much as possible based upon knowledge of the manufacturing operations and the manufacturing environment. If a certain series of dimensions and tolerances have been specified in such a way that they are forming a stack up but this direction does not coincide with the three primary dimensions of the part which roughly define its width, length and height, then this dimension is to be treated separately. This could be a case in comparatively complex parts but is mentioned here for completeness.

Here the researchers would like to take the opportunity to clarify that only the details of the process plan which are relevant to the tolerance analysis and synthesis will be dealt over here. This means that factors such as degree of cut, material of the part, etc which do not have direct bearing on the tolerance analysis and synthesis will not be discussed. However, factors such as the number of axis of the machine, the tool type, whether stops are used or not etc are considered relevant and will be included in discussion.

7.2.2 Types and accuracies of the tools and machines

Every manufacturing supervisor is (or is supposed to be) well aware of the accuracies of the tools and machines in the inventory he/she holds in the shop and thus, he/she lays down the process plan according to these tolerance values in mind.

7.2.3 Design tolerances

An important premise on which the new technique which is being presented over here is based is the viability of the Design tolerances. The manufacturing process planner before starting to work on the details of the process plan has to comment or give verdict to the viability of the design tolerances specified. If any of the tolerances needs to be reworked out, the process planner is always welcomed in the industry to discuss and sort the things out.

In the first step of working on the actual manufacturing process plan, the supervisor is to determine what tolerances he would be able to maintain against the tolerances given by the Designer.

For each final manufacturing tolerance (or the Designer blue print tolerance), factors such as the following are to be kept in mind:-

1. Industry standard for those types of operations
2. Machine capability and the process capability
3. History based data to support
4. The variation between the actual feature and the idealized feature for the features in concern for the specific operation.

So, to reiterate, the manufacturing supervisor will use the same dimensions as the ones used by the designer and for tolerances, based upon the factors discussed above, the manufacturing tolerances will be less than or equal to the design tolerances.

7.3 Beyond the final operation

For all the operations between the final operation and the first operation, the following philosophy will be the guiding factor for determining the dimension and tolerances for each individual operation.

‘Each previous operation will have the mean dimension that will represent the sum/difference of the mean dimensions of all the features that are involved in the dimension loop in that particular direction with an addition or subtraction of the tolerance accumulation.’

The above philosophy has two main issues of concern:-

7.3.1 Mean dimensions.

For every dimension of interest, there will be two features involved. One is called the datum and the other is called target. The definition of these features is as per the GD&T standard Y14.5 and will not be explained here any further.

For every dimension of interest, the dimension loop will be traversed which will start from the target feature on a previous step and will go to the last feature in the direction opposite to in which the datum feature is located (e.g. let’s assume this is Left Hand Side (LHS)). From the last feature, the loop will continue to the first feature on the right hand side and from there, the loop will again continue in the

left direction and will end at the datum feature. As can be seen later, in this technique, there are no charts involved, and no balance dimensions need to be calculated. This technique is logically based upon mathematical intuition and bears all relevant rules in this regard. All dimensions will have an attached sense of direction depending upon their direction of travel and the logical sum of these values with their symbols will provide for a resultant value.

7.3.2 Tolerances involved for the operation concerned

While catering for all types of tolerances involved for the operation concerned, there are several factors that need to be taken into consideration such as:

- a. Tool holder
- b. Work piece holder

There could be several sources of variations introduced which can result into broader range of tolerances to be satisfied. These are listed below:-

- i. Variations due to the part holder deviating from its nominal position for the concerned setup within the operation under consideration
- ii. Variations due to the varying location of the surface operated in the machining operation with regard to its nominal position on the part holder for the concerned setup
- iii. Variations introduced due the varying interface between the work piece and the work holder for the concerned setup within the manufacturing operation

- iv. Variations due to work piece offset from its nominal position for the concerned setup within the manufacturing operation

All such variations that are relevant to the setup within the manufacturing plan for a certain manufacturing operation will be three dimensional and have to be first calculated as accurately as possible and then catered in the calculation for the dimension and tolerances of the previous step within the manufacturing operation.

In order to calculate and include these three dimensional variations, the intent is to use the screw concept. The screw concept is not new in general and the details of the screw theory could be found in the following references. (44, 45)

As can be seen, the screws could be of at least three types namely screw, twist and wrench. However, for tolerance analysis purposes, attention will be restricted to screws only.

7.4 Use of screw concept for representation of geometric variation

The screw concept lets us model the variation in space based upon three Euler angles and three lateral translations along the three orthogonal axes. An important point to note is that it is just the variation and no original or actual entity is involved. The reader and the user of the screw concept has to keep the original type of feature and its location and other parameters relevant to his/her work in his/her mind and then apply the screw parameters to arrive at the new position or location of the feature involved. An easier way to tackle this situation is to assume that the nominal position is at the origin of the axis system along which the screw coordinates are identified.

Other researchers have used concepts like Small Displacement Torsor (SDT) etc for such purposes. However, these approaches are still in theoretical phase and far from being ‘practicable and applied’ in nature. The major drawback of these approaches is that although they delve into such depth but eventually, they are specifying these angles and lateral translations manually and the errors and variations involved in the calculation of these parameters of critical interest are not taken into account.

The novelty of the research presented here is that it does not stop at the level at which other researchers gave up but came up with a technique which will cater for the uncertainty and inaccuracy at this level also.

Euler angles and lateral translations are calculated based upon two point clouds for two features between which the variation is to be calculated. It is a general technique and can be applied in various situations within varying environments. Values of these parameters which give the least value for the criterion selected are used for all calculations. In this research, the standardized minimized value of the sum of the squares of the errors has been used. For details, please refer to the chapter 4 in this regard.

The variations from nominal geometry for different cases will be represented as a screw. This will give us three tolerances in three orthogonal directions and hence, the contribution of the variations in all the relevant directions will be accounted for.

7.5 Dimensionality of the geometric variation

Why is there a need to account for tolerances in all the three directions? This is explained in the following.

Every normal and common machining cut involves a surface which is most commonly a planar surface. Exceptions to the planar curves could be such as cylindrical surface. In turning of the rotational parts, depth of the cut and longitudinal travel of the cut is involved. However, in milling of the planar surfaces, three dimensions may be simultaneously involved such as depth, width and length of the slot. While in all of these cases, one or more dimension may be the driving or the critical dimension while the other may or may not be relevant in certain cases. However, the above is justifiable in general.

7.6 Information required to proceed with the development of the manufacturing process plan for tolerance analysis

The previous section discussed how to calculate the various variations which result due to setup and manufacturing operations that are present between the current step and the theoretical previous step. Once, all the tolerances have been calculated, it is time to determine the mean dimension for the previous step.

The general methodology based upon the philosophy mentioned above is presented here. The overall dimensional loop relations have to be taken into account. In addition to the tolerances that are to be catered for, the variations caused by the setup and manufacturing operations will also be involved in the

calculation of the mean dimension. Some other role playing entities have been determined to be:-

- a. Stock removal
- b. Direction of cut
- c. Type of machining cut such as end milling, facing, edging, shouldering etc.

Stock removal is the amount of material that is removed depending on the dimensions of the tool tip which plays the critical role in machining operation. When the direction of the cut remains unchanged and the cut is a finishing cut, stock removal is not to be exclusively included in the mean dimension calculation. However, when the cut is physically reducing the overall size of the work piece, then for all prior operations, stock removal values will be used in the calculation of the mean dimensions of the prior operations.

Another important rule which has been dealt with by other researchers is the concept of the increasing operation and the decreasing operation. For the pair of features of interest, an operation which increases the lateral dimension between them is an increasing operation. The converse is true for the decreasing operation. Hence, there are three main types of entities that will determine the mean dimension of the previous operation:-

- a. Mean dimensions of the features involved in the dimension loop
- b. Tolerances of the features involved in the dimension loop. This will also include the tolerances because of setup and location datum changes.

- c. Stock removal between the current operation and the previous operation.

The role of this entity will be determined as per the rule mentioned above.

The overall sum of these entities (with direction of cut accounted for) will give the mean dimension of the previous operation. The process will be repeated till it is time to do calculations for the first operation.

7.7 Case Study

To demonstrate the theory explained in the previous sections, the manufacturing process plan for the part shown in the figure will be constructed. It is to be recalled that interest in the manufacturing process plan is limited to the manufacturing tolerances and their analysis and synthesis, and hence, details such as planning for a specific process, type of tool, setup configuration etc will not be discussed in detail. It will be assumed that the manufacturing supervisors have already taken decisions about which operation will follow which operation, the machine involved and the required setup and this information will be used in conjunction with the designer blue print to determine the mean dimensions and tolerances from the final operation to the first operation.

For the part chosen, it is clear that this is a planar feature part and hence will require milling operations to create it. In most publications, rotational parts have been used. In rotational parts, machining operation involves only one surface while in milling, it could be anything between one and three surfaces such as for creating a slot through grooving, three surfaces are involved.

Relevant details from Designer Blue Print are shown in the following figure. It

can be seen that geometric tolerances such as position, form and orientation tolerances are applied to different features. In addition, size tolerances are also applied to Features of Size (FOS).

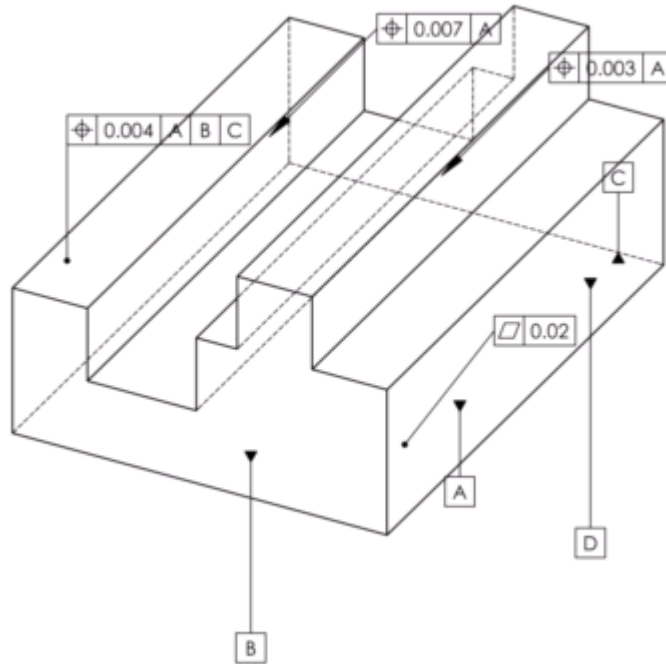


Figure 7-1 Relevant details from Designer Blue print for the part for case study

First of all, let's identify the geometric features involved. The faces of the part are identified and marked in the figure below.

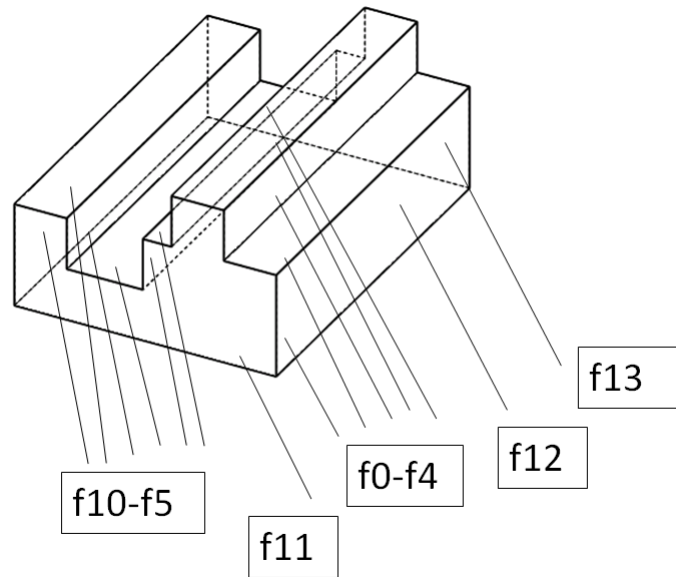


Figure 7-2 The part used for case study; the different faces are shown as identified

Before proceeding any further, all geometric tolerances have to be converted to equivalent worst case limits expressed in symmetric bilateral form. The position tolerance has been specified for the tab consisting of two parallel faces i.e. f_2 and f_4 alongwith the intermediate perpendicular face f_3 . The position tolerance is specified to CTP (Central Theoretical Plane) and is a means to exercise control on the physical features that define the CTP. The worst case limits equivalent to position tolerance will give the location of the faces f_2 and f_4 with respect to face f_0 and this will also determine the width of the face f_3 .

Similarly, position tolerance has been specified for the slot feature consisting of two parallel faces f_6 and f_8 alongwith the intermediate perpendicular face f_7 . The worst case limits equivalent to position tolerance will give the location of the faces f_6 and f_8 with respect to face f_0 and this will also determine the width of the

face f_7 .

In addition, form tolerance has also been specified. Also, orientation tolerance can also be seen in the designer blue print. However, these tolerances are not specifically catered for in the manufacturing process plan and by ensuring worst case limits for other geometric tolerances, these will be automatically taken care of.

As mentioned earlier, the manufacturing supervisors always start from the final operation and then traverse their way back (on the manufacturing process plan) to the first operation in sequential steps. Also, based upon the knowledge of the working environment which includes the accuracy of the machines, the condition of the tools etc, manufacturing supervisors start with an initial guess of the tolerances that they think are such that these could be maintained successfully. However, they might need to make some adjustments based upon the overall stack-up results as the values have to be chosen in such a way that they satisfy not only the tolerances for the step but also ensure that tolerances in the stack-up are also within the designer limit. As far as mean dimensions are concerned, there is no variation permitted and these values are adopted as they are. For the part, the worst case limits for the Designer blue print and the final dimensions and tolerances as given by the manufacturing process planner for the case in hand is shown in the table below:-

<i>S. No</i>	<i>Target feature</i>	<i>Datum feature</i>	<i>Designer Blue Print</i>		<i>Manufacturing Process Plan</i>	
			<i>Mean</i>	<i>Tolerance</i>	<i>Mean</i>	<i>Tolerance</i>
			<i>Dimension</i>		<i>Dimension</i>	
1	f_{10}	f_0	5.0000	0.0012	5.0000	0.0010
2	f_8	f_0	4.0000	0.0042	4.0000	0.0040
3	f_6	f_0	3.0000	0.0090	3.0000	0.0080
4	f_4	f_0	2.0000	0.0110	2.0000	0.0100
5	f_2	f_0	1.0000	0.0320	1.0000	0.0300

Table 7-1 Worst case limits as derived from geometric tolerances specified in Designer blue print. Finishing operation dimension and tolerances as specified in Manufacturing Process plan is also shown.

As can be seen from the above table, all manufacturing tolerances are less (could be equal to, at the best) than design tolerances. Before proceeding ahead, let's define some of the terms for easy identification and later usage. τ_{si} stands for tolerance for the step i and similarly, δ_{si} for the mean dimension for the step i . $\Sigma\tau_{si}$ stands for tolerance accumulation for the step i . D_i and T_i stand for mean dimension and tolerance as given in the manufacturing process plan for the final finishing operations. Also, the geometric feature before the arrow head (as read from left to right) in the figures is the datum entity while the other one is the target entity.

Now let's turn attention to the final finishing operation (step 1) as casted out by the manufacturing process planner. The figure below lays down the strip layout

for the finishing (grinding) operation for the features involved.

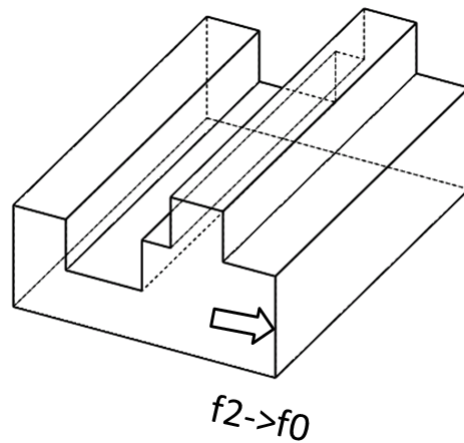


Figure 7-3 The features involved and the direction of cut in the finishing operation of step 1

Let's consider the operation as shown in the figure above. It is the grinding of the face f_0 and dimensions and tolerances controlled from face f_2 . The dimensions and tolerances are adopted from the table above as it is exactly the same operation as given in the designer blue print except that the datum and target features are switched.

Equation 7-1

$$\tau_{s1} = T_5$$

Equation 7-2

$$\delta_{s1} = D_5$$

Hence mean dimension and tolerance for this step is 1.0000 ± 0.0300 units.

Let's move to the next (previous machining) step (step 2) as shown in the figure below:-

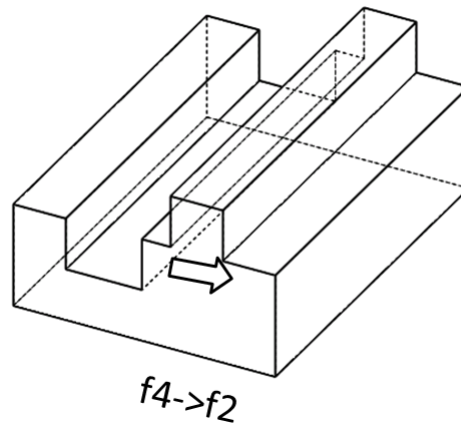


Figure 7-4 The features involved and the direction of cut in the finishing operation of step 2

This is a finishing (grinding) operation with f_2 as target and f_4 used as a datum. The mean dimension is calculated by forward difference of D_4 and D_5 where D_4 and D_5 are the mean dimensions in fourth and fifth row from manufacturing Process plan columns in table 7-1 above.

Equation 7-3

$$\delta_{s2} = D_4 - D_5$$

This gives the mean dimension for this step as 1.0000. The manufacturing tolerance for this step is calculated by the backward difference of the tolerances of the above mentioned rows in the table 7-1.

Equation 7-4

$$\tau_{s2} = T_5 - T_4$$

Thus, for this step, the manufacturing dimension and tolerance are 1.0000 ± 0.0200 units.

For the next step (step 3), please refer to the figure below:-

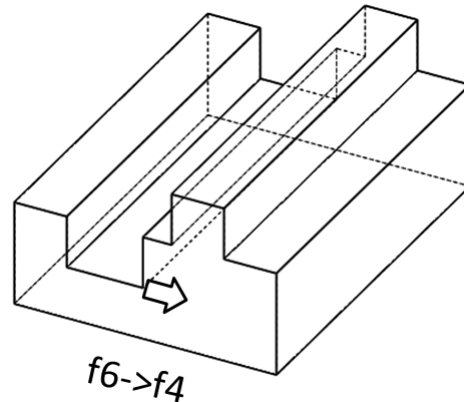


Figure 7-5 The features involved and the direction of cut in the finishing operation of step 3

This is a finishing (grinding) operation with face f_4 as target and f_6 as datum. The mean dimension for this step is the forward difference of mean dimension in rows 3 and 4 as given by the manufacturing process planner.

Equation 7-5

$$\delta_{s3} = D_3 - D_4$$

This gives the mean dimension for this step as 1.0000. The manufacturing tolerance for this step is calculated by the backward difference of the tolerances of the above mentioned rows in the table 7-1.

Equation 7-6

$$\tau_{s3} = T_4 - T_3$$

Hence, manufacturing tolerance for this step is $0.0100 - 0.0800 = 0.0200$. Hence, mean dimension and manufacturing tolerance for step 3 comes out to be $1.0000 \pm$

0.0200.

In the same footsteps, the mean dimension and tolerances for the steps shown in the figures below are computed and come to be 1.0000 ± 0.0040 (step 4) and 1.0000 ± 0.0030 (step 5) respectively.

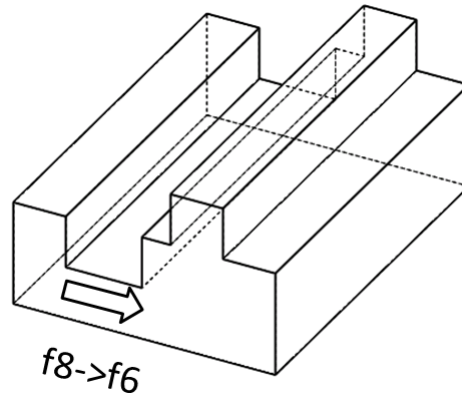


Figure 7-6 The features involved and the direction of cut in the finishing operation of step 4

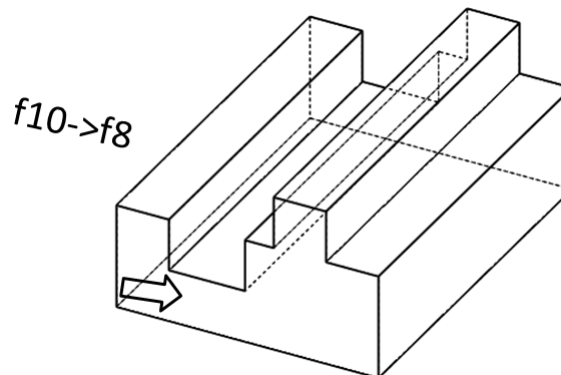


Figure 7-7 The features involved and the direction of cut in the finishing operation of step 5

The next (previous machining) step (step 6) is shown in the figure below which is

face milling of the faces, f_9 , f_7 , f_5 , f_3 and f_1 respectively.

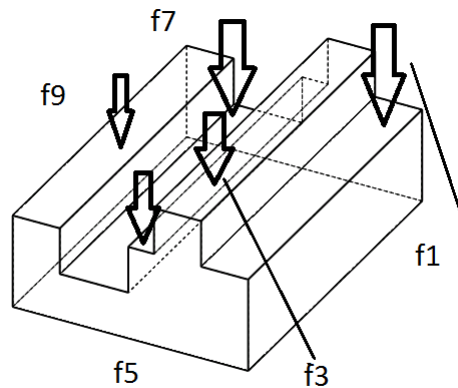


Figure 7-8 The features involved and the direction of cut in the finishing operation of step 6

The next (previous machining) step (step 7) is important as it requires change in setup. Here, the research team would like to clarify that there is always(or mostly) more than one way of creating the part and in this publication, interest is restricted to one process plan which is assumed to be provided by the manufacturing supervisor. All calculations are dependent on the details of this plan and the values for the same final dimensions and tolerances will change if the plan is changed.

The next (previous machining) step (step 7) as shown in the figure below is an end milling operation of the face f_{10} with f_8 as the manufacturing datum.

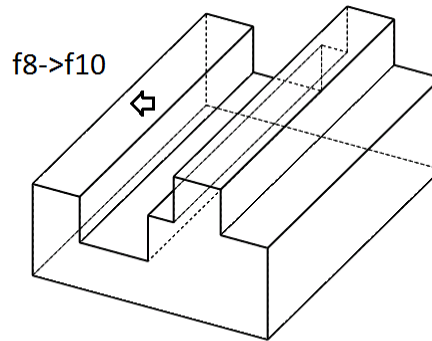


Figure 7-9 End milling operation of face f_{10} with face f_8 as manufacturing datum (step 7)

First of all, tolerance for this step needs to be assigned. This tolerance value will depend upon how much the material is removed during this end milling operation. Also, it is be noticed that direction of machining is changed. The tolerance based upon the manufacturing supervisor advice is assumed to be $\tau_{s7} = 0.0040$ units. (inches, millimeters etc). Hence, the mean dimension for this step is calculated as the mean dimension of the previous step plus tolerance accumulation of this step and the previous step.

Equation 7-7

$$\delta_{s7} = \delta_{s6} + \sum \tau_{s7}$$

Where

Equation 7-8

$$\sum \tau_{s7} = \tau_{s6} + \tau_{s7}$$

Thus, mean dimension for this step is $1.0000+0.0030+0.0040= 1.0070$ units.

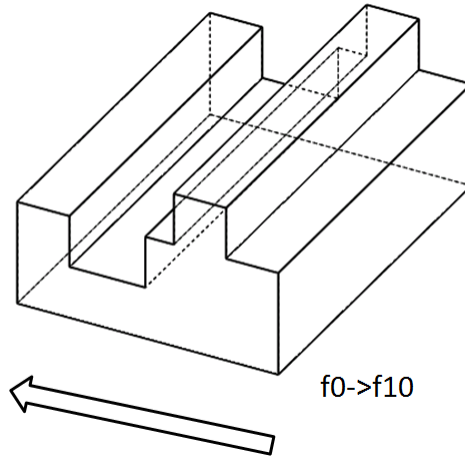


Figure 7-10 Edge milling of face f_{10} with face f_0 as the manufacturing datum (step 8)

The above figure shows the next (previous machining) step (step 8). It is the edge milling of face f_{10} with face f_0 as the datum. Comparing this step and the previous step, the same target face i.e. f_{10} is involved. The mean dimensions cannot be calculated at this point and this step will be considered later. The tolerance for this step is however, assigned as 0.0030 units. The next figure shows the next (previous machining) step (step 9) in the manufacturing process plan.

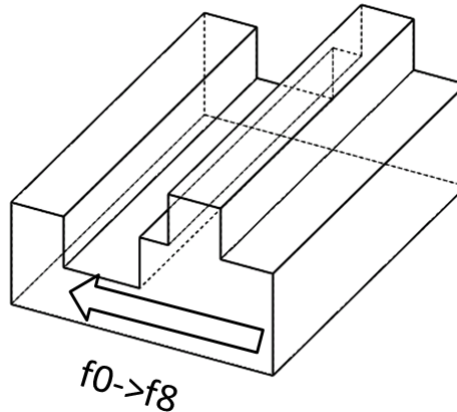


Figure 7-11 Shoulder milling of face f_8 with face f_0 as manufacturing datum (step 9)

In this step, f_0 is the manufacturing datum while f_8 is the target feature. The tolerance for this step is assigned as 0.0050 units. Tolerance accumulation for this step (step 9) is given by the following equation:-

Equation 7-9

$$\sum \tau_{s9} = \tau_{s9} + \tau_{s7} + T_1$$

This comes out to be 1.0000 units. The mean dimension of this step is calculated as follows:-

Equation 7-10

$$\delta_{s9} = D_1 - \delta_{s6} + \sum \tau_{s9}$$

This comes out to be $5.0000 - 1.0070 + (0.0010 + 0.0040 + 0.0050) = 4.0030$ units.

Tolerance accumulation for step 8 is now calculated as the tolerance for steps 9 and 7 and ofcourse step 8.

This is given mathematically as following:-

Equation 7-11

$$\sum \tau_{s8} = \tau_{s9} + \tau_{s7} + \tau_{s8}$$

This comes out to be $0.0050+0.0040+0.0030$ (assigned) = 0.0120 units . The mean dimensions for step 8 is the mean dimension of step 7 + mean dimension of step 9 + tolerance accumulation calculated earlier as given by the following equation:-

Equation 7-12

$$\delta_{s8} = \delta_{s7} + \delta_{s9} + \sum \tau_{s8}$$

This comes out to be $1.0070+4.0030+0.0120 = 5.0220$ units.

The face milling involved in step 10 for the slot geometry is shown in the figure below.

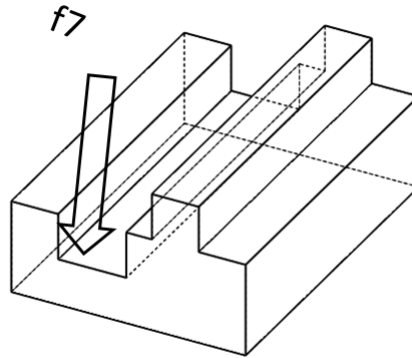


Figure 7-12 Face milling of face f_7 (step 10)

The shoulder machining involved in step 11 is shown in the figure below.

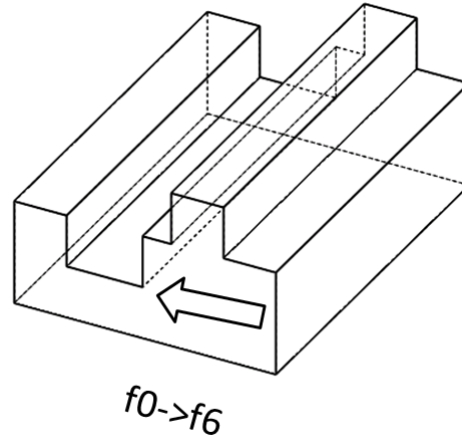


Figure 7-13 Shoulder milling of face f_6 with face f_0 as the manufacturing datum (step 11)

The tolerance accumulation for this step involves the tolerances of steps 9, 7, 5 and 4 along with ofcourse step 11 i.e.

Equation 7-13

$$\sum \tau_{s11} = \tau_{s9} + \tau_{s7} + \tau_{s5} + \tau_{s4}$$

This comes out to be $0.0050+0.0040+0.0030+0.0040+0.0050$ (assigned) = 0.0210 units. The mean dimension for this step is calculated as shown below:-

Equation 7-14

$$\delta_{s11} = \delta_{s7} + \delta_{s9} - (\delta_{s5} + \delta_{s4}) - \sum \tau_{s11}$$

This is calculated as $4.0030 + 1.0070 - (1.0000 + 1.0000) - 0.0210 = 2.9890$ units.

The shoulder machining involved in step 12 is shown in the figure below.

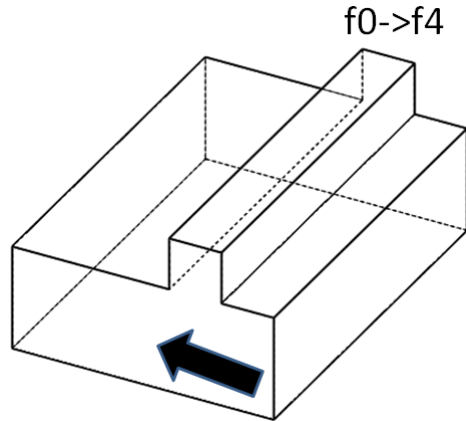


Figure 7-14 Shoulder machining of face f_4 with face f_0 as manufacturing datum (step 12)

The assigned tolerance for this step is assumed to be same as previous step and is equal to 0.0050 units. The tolerance accumulation is given by the following expression=

Equation 7-15

$$\sum \tau_{s12} = \tau_{s12} + \tau_{s9} + \tau_{s7} + \tau_{s5} + \tau_{s4} + \tau_{s3}$$

Once calculated, it comes out to be $0.0050 + 0.0050 + 0.0040 + 0.0030 + 0.0040 + 0.0020 = 0.0230$ units.

The mean dimension for this step is calculated as follows:-

Equation 7-16

$$\delta_{s12} = \delta_{s7} + \delta_{s9} - (\delta_{s5} + \delta_{s4} + \delta_{s3}) - \sum \tau_{s12}$$

The above expression gives the mean dimension for this step as $4.0030 + 1.0070 - (1.0000 + 1.0000 + 1.0000) - 0.0230 = 1.9870$ units.

The shoulder machining involved in the actual first step of the manufacturing process plan (step 13) is shown in the figure below:-

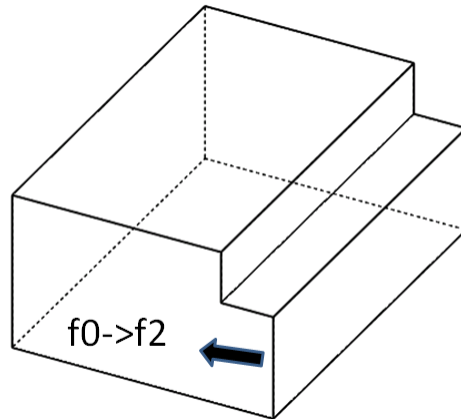


Figure 7-15 Shoulder machining of face f_2 with face f_0 as the manufacturing datum (step 13)

The tolerance accumulation for this step is given by the expression:-

Equation 7-17

$$\sum \tau_{s13} = \tau_{s13} + \tau_{s12} + \tau_{s9} + \tau_{s7} + \tau_{s5} + \tau_{s4} + \tau_{s3}$$

This is calculated as follows: - 0.0050 + 0.0050 + 0.0050 + 0.0040 + 0.0030 + 0.0040 + 0.0020 + 0.02 = 0.0430 units.

The mean dimension for this step is given by the expression as follows:-

Equation 7-18

$$\delta_{s13} = \delta_{s7} + \delta_{s9} - (\delta_{s5} + \delta_{s4} + \delta_{s3} + \delta_{s2}) - \sum \tau_{s13}$$

The mean dimension for the step 13 comes out to be 4.0030 + 1.0070 - (1.0000 + 1.0000 + 1.0000 + 1.0000) - 0.0430 = 0.9670 units.

This leads to the starting point of the manufacturing process plan for tolerance analysis and synthesis purposes which is a blank as shown in the figure below.

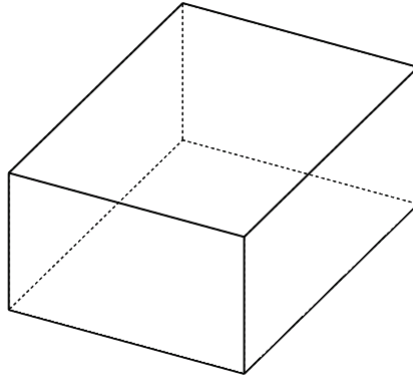


Figure 7-16 Geometry of the blank used for the case study (step 14)

8 STATISTICAL TOLERANCE ANALYSIS

8.1 Types of tolerance analysis

There are mainly two types of analyses [50] for assembly tolerances which are explained below:-

a. Worst case

A worst-case analysis is usually conducted by maximizing or minimizing the chain of dimensions in the stack up equations. For one dimensional case,

Equation 8-1

$$T_Y = \sum T_i$$

While in the multi dimensional case,

Equation 8-2

$$T_Y = \sum \left(\left| \frac{\delta f}{\delta X_i} \right| T_i \right)$$

where 'f' is the assembly function that is a function of 'X_i', the nominal component dimensions. The partial derivatives represent the sensitivity of the assembly tolerance to variations in individual component dimensions.

b. Statistical tolerance analysis

In statistical tolerance analysis, a relationship like the one below is normally used:-

Equation 8-3

$$Y = f(X)$$

Where 'Y' is the response (a characteristic such as a gap or functional characteristic or clearance) of the assembly and $X = \{X_1, X_2, X_3, \dots, X_n\}$ gives the values of the characteristics (situation deviations or the intrinsic deviation) of the parts that make the assembly. These 'X_s' are continuous random variables and could be mutually dependent. The function f is the assembly response function. This function could be an implicit analytic function or explicit analytic function or could require complex engineering calculations or experimental setup or simulation.

8.2 Methods for Statistical Tolerance Analysis: Broad classification

Several methods exist for solving the above computational problem. These can be broadly classified into the following main classes:-

- a. Linear Propagation
- b. Non-linear propagation
- c. Linearized form for non linear propagation
- d. Numerical Integration
- e. Monte Carlo simulation

If the assembly response function is a linear analytic function, linear propagation could be used. However, application of linear propagation to a non linear function could be disastrous. In that case, use of non-linear propagation is highly recommended and may be the only resort in certain cases. Linearization of the non linear propagation is only recommended once it is applicable and logically

viable and acceptable within its domain. Within the non-linear propagation class, if the extended Taylor series is used, then the function 'f' should be available in analytic form.

If the function is not available in analytical form, then the suitable options are numerical integration and Monte Carlo simulation. Within the numerical integration class, if the quadrature technique is utilized than the function needs to be defined in an approximate form. However, Monte Carlo is the most widely used technique for statistical tolerancing. The best part about Monte Carlo is that it can be applied under very general settings and there is no limit to the amount of precision that can be achieved. In general, Monte Carlo simulation could be used in any situation in which the remaining three could be used and can provide more precise results.

The variance reduction techniques of **correlation and importance sampling** can greatly reduce the number of samples required for achieving a particular accuracy.

8.3 Methods for Statistical Tolerance Analysis: Individual methods / approaches

The main problem in statistical tolerance analysis is to estimate the moments of the assembly function based upon the moments of the component functions. In other words, the distribution of the independent component functions is known but the distribution of the assembly function is unknown. The Assembly response function Y is expressed as a function of component dimensions X_1, X_2, \dots, X_n as below:-

Equation 8-4

$$Y = f(X_1, X_2, \dots, X_N)$$

There are several methods that will be discussed here. However, there are several assumptions that are common to each. Each method assumes that six sigma range of each component distribution covers its tolerance range. Another important assumption is that each distribution is assumed to be symmetric about its mean which in turn is the nominal value of this parameter. However, under various situations, these assumptions are relaxed which is covered by shifts and drifts of the distribution as explained in the following paragraphs.

8.3.1 Root Sum Squares Method (RSS)

This method is also known as the linear stack-up method. Its applicability criteria are simple; use it when the assembly function is expressible as a linear combination of the component parameters as given below:-

Equation 8-5

$$Y = a_0 + a_1 X_1 + a_2 X_2 + \dots + a_n X_n$$

Where 'Y' is the assembly response function, 'X_i's' are the component response functions and 'a_i's' are the constants.

According to this method, the first two moments that is the mean and the variance of the assembly response function are dependent on the component parameters as shown below:-

Equation 8-6

$$\mu_Y = a_0 + a_1 \mu_1 + a_2 \mu_2 + \cdots + a_n \mu_n$$

Equation 8-7

$$\sigma_Y^2 = a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 + a_3^2 \sigma_3^2 + \cdots + a_n^2 \sigma_n^2$$

Here, 'μ' represents mean and 'σ²' = variance. All component functions are independent. The assembly function is assumed normal. That is supposed to be universally true if the component functions are all normal. However, in case of some or all non normal component functions, the assembly response function can be assumed approximately normal in case of large number of component parameters. The above decision is a direct consequence of Central Limit Theorem.

What happens if the function as given by equation 8-4 is just marginally linear? The solution suggested by Evans [51] is to expand the assembly response function into a Taylor series and neglecting all terms except the constant and linear terms. However, this formulation will only work within the range within which the linearization is a good approximation of the actual response function. Evans also proposed a slight modification of the above for the assembly response functions that are not available in analytical form. (See [51] for details)

What to do in case the assembly response function is neither linear nor approximately linear; however, it is available in analytical form? The answer is to use the extended Taylor series approximation as suggested by Tuckey and presented by Evans. This method is tedious largely due to the computational

effort required for the partial derivatives of the Taylor series expansion of the response function, which requires nontrivial algebraic manipulation. In order to reduce the computational complications and at the same time, providing reasonable results, Cox (1979) proposed a modification suggesting Taylor series expansion only up to 2nd order terms.

Another question arises: what if the component distribution has a wider span than its tolerance limit? This would result into a truncated normal distribution due to the rejection of the out of tolerance components. Croft [52] approximated this truncated normal distribution with a rectangular distribution. There are three things worth mentioning: first; rectangular distribution assumes equal distribution in the range, second; the rectangular distribution has greater concentration on the extremes as compared to truncated normal and hence, will give a pessimistic value. Lastly, rectangular distribution ignores any skewness in the real distribution which will give an optimistic value.

Croft hoped that this blend of optimism and the pessimism will help in portraying the true real situation. Based on the results, it can be seen that Croft's method gives better results for smaller number of components (less than 20). However, for larger number of components, the values are identical for the Croft's method and the linear stack up.

8.4 Estimated Mean Shift Method

This method [53] goes by this name since it is required that the designer must estimate the bias for each component in an assembly. It is done by defining a zone

that contains the probable location of the mean of a typical batch of parts. This zone is called the midpoint tolerance zone. It is expressed as a fraction of the specified tolerance range for the part dimension. This value between 0.1 and 1 is known as the mean shift factor and is represented by m_i for the i^{th} component. The assembly tolerance is given by the relation:-

Equation 8-8

$$T_Y = m_1 \left(\frac{\partial f}{\partial X_1} \right) T_1 + m_2 \left(\frac{\partial f}{\partial X_2} \right) T_2 + \dots + m_N \left(\frac{\partial f}{\partial X_N} \right) T_N + \sqrt{(1 - m_1)^2 \left(\frac{\partial f}{\partial X_1} \right)^2 T_1^2 + (1 - m_2)^2 \left(\frac{\partial f}{\partial X_2} \right)^2 T_2^2 + \dots + (1 - m_N)^2 \left(\frac{\partial f}{\partial X_N} \right)^2 T_N^2}$$

This method keeps the estimate of the mean of the component distribution flexible. By defining the zone for the mean, it accounts for the shifts in the distribution which are so common in actual manufacturing.

Stefano [54] provided a systematic approach to evaluate the mean shift factor after taking principal factors such as mean shift ratio, confidence level, tolerance assortment between the component dimensions and the number of dimensions of the assembly, into consideration.

8.5 Spotts’ modified approach

Spotts [55] suggested the equation below which is based upon practical experience. He suggested taking the average of the worst case (WC; method of extremes) and RSS. It was based upon the idea that since WC is very pessimistic and RSS is too optimistic, there is a lot more chance that their average is going to be closer to reality.

Equation 8-9

$$\Delta Y = \frac{1}{2} \left[\sum_{i=1}^n x_i + \sqrt{\sum_{i=1}^n x_i^2} \right]$$

Where ΔY is the variation in assembly dimension and x_i is the variation in the i^{th} component dimension.

8.6 Bender's correction factor approach

Normally the observed assembly tolerance is more than what is suggested by RSS method. In order to rectify the situation, Bender [56] (1962, 1968) came up with the idea of having a corrective multiplication factor. Bender suggested a value of $\alpha = 1.5$ and later, Gladman suggested a value between 1.4 and 1.8. 'Z' is the number of standard deviations, ' ΔY ' is the variation in assembly dimension and ' x_i ' is the variation in the i^{th} component dimension in the equation below:-

Equation 8-10

$$\Delta Y = \sum_{i=1}^n \alpha_i x_i + \frac{Z}{3} \sqrt{\sum_{i=1}^n (1 - \alpha_i)^2 x_i^2}$$

8.7 Mansoor's static mean shift model

For this model of tolerance analysis [57], following are the assumptions:-

- a. The processes are always able to meet the specified tolerances i.e. the specified design tolerance is always greater than or equal to the natural process tolerance.
- b. Components are normally distributed.

This model assumes the worst possibility of mean shift, which is a conservative assumption. Theoretically, the spread of the distribution is calculated as +/- 3.09 times or 6.18 times the standard deviation of the distribution. Mansoor's model assumes that there is a shift in all or some component distributions and he suggests the spread of the assembly distribution by giving an additive correction factor which is equal to the sum of the shifts for component distributions.

So

Equation 8-11

$$\Delta Y = \pm 3.09 \sigma$$

Or

Equation 8-12

$$Y = 6.18 \sigma$$

In case of shifts in component distributions, the above formulae are amended by adding a corrective factor,

Equation 8-13

$$\Delta Y = \sum_{i=1}^n x_i \pm 3.09 \left[\sum_{i=1}^n \sigma_i^2 \right]^{1/2}$$

Or

Equation 8-14

$$Y = 2 \sum_{i=1}^n x_i \pm 6.18 \left[\sum_{i=1}^n \sigma_i^2 \right]^{1/2}$$

8.8 Gladman's dynamic mean shift model

The assumptions for this model [58] are:-

- a. The component distributions are uniformly spread over the entire range of Design tolerance
- b. The component distributions are approximately normal.

Additionally, the process standard deviation is given by

Equation 8-15

$$\sigma_v = \rho_v(2t)$$

Where ρ = design factor for each component tolerance.

And the natural process tolerance is represented by

Equation 8-16

$$2t = 6\sigma_v$$

The above two equations result in:-

Equation 8-17

$$\rho_v = 1/6$$

This model calculates the variability of the assembly response in three steps:-

- a. It calculates the design factor for each component dimension as follows ($\rho_v = 1/6$ as from above) :

Equation 8-18

$$\rho_i = \pm 1/C_p \sqrt{[\rho_v^2 + 1/3 (C_p - 1)^2]}$$

Where C_p is the Process capability index.

- b. It determines the standard deviation for each component dimension using the following formula:-

Equation 8-19

$$\sigma_i = \rho_i(2t_i)$$

- c. It estimates the variability of the assembly dimension using the RSS equation :-

Equation 8-20

$$\sigma_Y = \sqrt{\sum_{i=1}^n a_i^2 \sigma_i^2}$$

8.9 Desmond's probabilistic mean shift model

This model [59] uses the in-between approach to Mansoor's (worst possible mean shift) and Gladman's model (uniformly distributed mean shift). The variation in the assembly dimension is given by:-

Equation 8-21

$$\Delta Y = \sqrt{\sum_{i=1}^n (\delta_i - t_i)^2} + \sqrt{\left[\sum_{i=1}^n t_i^2 \right]}$$

This is a slight modification to the expression as given by Mansoor.

8.10 Chase and Greenwood's iterative mean shift method

Chase and Greenwood [53] came up with this iterative method which tries to match the desired assembly limits by making small adjustments to the component mid points which in turn are obtained from component tolerances using first order approximation. This method is applicable to assemblies involving only the simple dimension chains.

This method is based upon the expectation that in case the tolerance build-up on assembly dimension is approximately correct, then both desired limits of assembly dimension can be obtained by shifting the midpoint value of the components. The required shift in the assembly dimension is given by the following expression:-

Equation 8-22

$$\Delta Y_{shift} = \left. \frac{(Y_{high} - Y_{low})}{2} \right|_{Desired} - \left. \frac{(Y_{high} - Y_{low})}{2} \right|_{Iteration\ 1}$$

As per Chase and Greenwood, the shift in assembly dimension is related to the shifts in component midpoint as per the following equation:-

Equation 8-23

$$\Delta Y_{shift} = \frac{\delta Y}{\delta X_{01}} \Delta X_{01} + \frac{\delta Y}{\delta X_{02}} \Delta X_{02}, \dots + \frac{\delta Y}{\delta X_{0N}} \Delta X_{0N}$$

The component midpoint shift can be proportional to each component tolerance. Thus each component midpoint shift can be a certain percentage of its tolerance as shown in the following equation:-

Equation 8-24

$$\Delta X_{0i} = SGN(\Delta Y)SGN\left[\frac{\delta Y}{\delta X_{0i}}\right]f\delta_i$$

Thus finally, an expression for the shift in assembly dimension can be expressed as a function of component tolerance as shown below:-

Equation 8-25

$$\Delta Y_{shift} = \sum_{i=1}^n \frac{\delta Y}{\delta X_{0i}} SGN(\Delta Y)SGN\left[\frac{\delta Y}{\delta X_{0i}}\right]f\delta_i$$

The sensitivities are updated by using the new component mid-point values and the method proceeds iteratively until there is a satisfactory agreement between the desired and the actual limits of the assembly dimension.

8.11 Taguchi's method

A '3-level factorial experiment' is used where the mean μ_i , and $\mu_i \pm \sqrt{\frac{3}{2}} \sigma_i$ are the three levels [60]. The assembly response function is evaluated at all 3^n combination of the levels of each component. Once these 3^n values of Y have been obtained, all the required moments of the assembly response function can be found using standard statistical formulae.

The great benefit of this method is that it works even if you do not know the assembly response function. However, it requires that all the component functions be statistically independent. Further it assumes a normal distribution for the component functions.

D'errico and Zaino [61] compared the results for the calculation of moments for the assembly response function with the results from a simulation consisting of 250,000 observations and found that this method needs to be modified in order to get better results for the fourth moment as well. The method recommended by them is known as '**Modified Taguchi method**' and it uses weights of 1/6, 4/6 and 1/6 to the high, center, and low levels respectively.

D'errico and Zaino further observed that with increasing value of n, the number of evaluations required by the '3- level factorial experiment' becomes unmanageable. For this, they recommend a threshold value of 10 to switch over from '3 – level factorial experiment' to 'fractional factorial experiment' (or orthogonal arrays).

The best thing about the modified Taguchi method is that it is not limited to normal distributions only; rather different values of the weights can be used to simulate any non-normal distribution. Additionally, the number of levels can also be increased.

8.12 **Hasofer- Lind reliability index method**

In this method, the probability of successful assembly or the yield is based upon the Hasofer – Lind reliability index [62]. The main steps involved in this method are given below:-

- a. Each of the random variables is transformed into the standard normal variable using the moments of the component distributions.

b. Each design requirement can be represented by a failure surface in the space of the individual variables. Certain coordinate transformation operations(as many as required) are carried out such that a new coordinate space is obtained in which the above mentioned standard normal random variable are independent with zero means and unit variances.

c. The minimum distance beta between the origin and the failure surface is used to estimate the failure probability. Limits of this probability are found by using a hyper plane at distance beta from the origin and the hyper sphere of radius beta with its center at the origin.

d. Yield probability is unity minus the above mentioned failure probability.

8.13 Evan’s numerical integration approach

For any assembly function $Y = f(X_1, X_2, \dots, X_n)$ with each component function having a known distribution w_i [63], the expected value of the function f is given by the integral $I\{f\}$ as below:-

Equation 8-26

$$I\{f\} = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} f(X_1, X_2, \dots, X_n) w_1(X_1) w_2(X_2) \dots \dots \dots w_n(X_n) dX_1 dX_2 \dots \dots \dots dX_n$$

This integral can be approximated by the quadrature expression $Q\{f\}$ as indicated by the following expression:-

Equation 8-27

$$Q\{f\} = G \cdot f(\mu_1, \mu_2, \dots, \mu_n) + \sum_i H_i \left[\frac{h(\mu_i + \alpha_i^+ \sigma_i)}{\alpha_i^+} - \frac{h(\mu_i + \alpha_i^- \sigma_i)}{\alpha_i^-} \right] \\ + \sum_{i < j} P_{ij} \left[\frac{h(\mu_i + \alpha_i^+ \sigma_i, \mu_j + \alpha_j^+ \sigma_j)}{\alpha_i^+ \alpha_j^+} - \frac{h(\mu_i + \alpha_i^- \sigma_i, \mu_j + \alpha_j^+ \sigma_j)}{\alpha_i^- \alpha_j^+} \right. \\ \left. - \frac{h(\mu_i + \alpha_i^+ \sigma_i, \mu_j + \alpha_j^- \sigma_j)}{\alpha_i^+ \alpha_j^-} - \frac{h(\mu_i + \alpha_i^- \sigma_i, \mu_j + \alpha_j^- \sigma_j)}{\alpha_i^- \alpha_j^-} \right]$$

Where the first four moments of the component functions have been utilized for calculating the constants G , H_i , P_{ij} , α_i^+ , α_i^- , α_j^+ and α_j^- .

The above formulation is specifically applicable to the case when the component functions are assumed to be normally distributed. Evans [64] considers the issue of error formulation resulting in due to approximation of the integral through the quadrature formula.

Evans [65] considered the case when the above stipulation was not valid. However, it is not very important for current research although it might be important for people working in pure statistics or other fields.

8.14 Monte Carlo simulation method

It is the simplest and the most popular method for non linear statistical tolerance analysis. [66]

The general and simplified description of the method is given below:-

Random values are generated for each component using the specified component statistical distribution. These values are used for calculating the value of the

response function for each set of component values. In this way, sample values are generated for the response function which can be further used for calculating the moments of this population using the standard statistical formulae. For greater accuracy, it should be ensured that a non-repeating sequence for the base random number generator is used.

The advantages of using Monte Carlo simulation is that it can be used for all types of distributions for the components i.e. it is not restricted to only normal distributions. Also, since the response function is calculated by simulation, it can easily handle both the linear and non linear cases.

The main disadvantage is that a varying sample size will give different values of the assembly rejects. This means that the result will vary each time with the change in sample size. Also, since the random number generator requires a starting or a seed value, the results may vary depending on different seeds.

8.15 Method of Moments

This method is based upon the first four moments of the statistical distribution. So, before explaining the method, a small explanation of these moments is given below:-

8.15.1 First Four Moments of Statistical Distribution

1. The first moment of a statistical distribution is commonly known as mean or average and is an indicator of the central tendency of the data. It is given by the following equation:-

Equation 8-28

$$\mu_1 = E(x)$$

Where μ_1 is the mean, 'E' represents the expected value, and the variable concerned is 'x'.

2. The second moment of a statistical distribution is commonly known as the variance and is an indicator of the scatter or dispersion of the data. It is given by the following equation:-

Equation 8-29

$$\mu_2 = E(x - \bar{x})^2$$

Where μ_2 is the variance and \bar{x} denotes the sample mean for the variable 'x'.

3. The third moment of a statistical distribution is the skewness of the distribution and is an indicator of the symmetry or otherwise of a distribution about any value. It is given by the following formula:-

Equation 8-30

$$\mu_3 = E(x - \bar{x})^3$$

Where μ_3 represents the skewness.

4. The fourth moment of a statistical distribution is the Kurtosis of a distribution and is an indication of the nature of flatness of the distribution. It is given by the following equation:-

Equation 8-31

$$\mu_4 = E(x - \bar{x})^4$$

Where μ_4 represents the kurtosis.

It is to be kept in mind that all these are the central moments.

There could be 'n' number of moments.

All these moments are independent of origin as these are functions of the differences

$$x - \bar{x}$$

However, they all depend upon scalar parameter such as unit of variable. Thus for the same variable such as μ_3 or μ_4 , there will be different values depending upon units used.

To cater to different units used, there are indices of skewness or kurtosis, which are not discussed or defined here as these are beyond the scope of this research.

Now, coming back to the 'Method of Moments'; in this method, the statistical moments of the component distributions and the first and second partial derivatives of the assembly function are used to find the first four moments of the assembly distribution. These four moments are subsequently utilized to find the parameters of a general distribution e.g. Pearson system, Johnson system or the Lambda distribution.

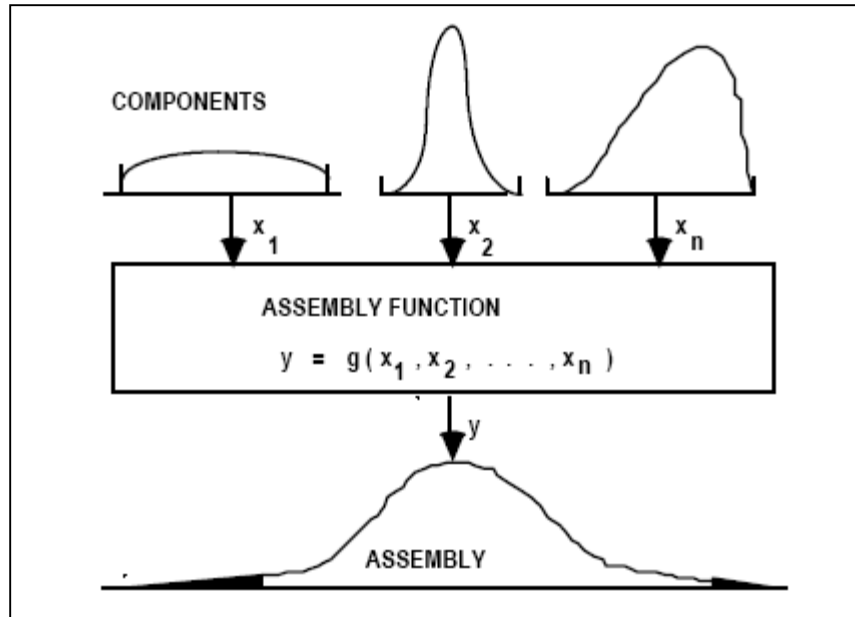


Figure 8-1 Different distribution types for the various components contribute to the assembly distribution

Eventually, the number of rejects is found by using the standard tables, numerical integration or algebraic equations. The computational process involves numerical derivatives and series summation.

8.16 Mixed Moment + Monte Carlo Simulation method

This method is a mixture of the two methods mentioned above. The benefit of it is that it requires a moderately complicated program and has a moderate computation time as compared to the above two methods. In the first part of the method, Monte Carlo simulations are used to generate a smaller number of assembly values with sample sizes ranging from 1000 to 5000. The statistical moments of the assembly distribution are calculated from these resultant assembly dimensions which in turn are used for estimation of the fraction of the rejects. The

main advantage of this method is that it bypasses the numerical derivatives and series summation. Also, since the sample size is small, the computation time is reduced as compared to standard Monte Carlo Simulation.

In the following, requirement of statistical tolerance analysis along with the concept of convolution is discussed.

8.17 Need for Statistical tolerance analysis

The worst case analysis for 100% interchangeability within the assembly has already been discussed. As opposed, in the statistical tolerance analysis, interest resides in a certain success rate of assembly rather than 100 % success rate that was used in the worst case analysis.

Consider an assembly of two parts with two dimensions of interest. For part A's dimension of interest (likewise for part B dimension of interest), there will be an associated frequency distribution and the corresponding probability distribution. Initially, consider that both parts (A's and B's) dimensions have a normal or Gaussian distribution. Later on, other types of distribution as well as justification of their use based upon some established reasoning will be considered. Also, it will be the desire to find out and present the error of fit for each standard distribution that is fitted to a particular distribution (in the statistical validation portion of proposed research plan).

Once an assembly C that is a combination of the parts A and B, is created, then the question arises what will be the distribution of the size dimension for assembly C.

If parts A and B are randomly chosen, then it will ensue that there is no covariance. Covariance can exist if every assembly C is ensured to consist of part A made from the same machine as that for part B etc. Absence of covariance leads to the conclusion about their independence.

In the absence of covariance, whenever two parts combine together (each dimension of interest having their own probability distribution), then the assembly will have a probability distribution that is the convolution of the two constituent probability distributions.

8.18 Types of Convolutions

Speaking of convolution, there could be two cases:-

8.18.1 Convolution of two discrete random variables

Suppose X and Y are two independent discrete random variables with distribution functions $d_1(x)$ and $d_2(x)$ respectively. The convolution of these results into a distribution function

$d_3 = d_1 * d_2$ and is given by

Equation 8-32

$$d_{3(i)} = \sum_k d_1(k) \cdot d_2(i - k)$$

For $i = \dots, -2, -1, 0, 1, 2, \dots$ and $k \leq i$ The resulting function $d_3(x)$ is the distribution function of the random variable $Z = X + Y$.

8.18.2 Convolution of two continuous random variables

Suppose X and Y are two independent continuous random variables with distribution functions $d_1(x)$ and $d_2(y)$ respectively. The convolution of these results into a distribution function

$d_3 = d_1 * d_2$ and is given by

Equation 8-33

$$(d_1 * d_2)(z) = \int_{-\infty}^{+\infty} d_1(z - y)d_2(y) dy$$

Or

Equation 8-34

$$(d_1 * d_2)(z) = \int_{-\infty}^{+\infty} d_2(z - x)d_1(x) dx$$

8.19 Sum of two independent uniform random variables

Take the case of two uniformly distributed variables. i.e.

Equation 8-35

$$d_1(x) = d_2(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

For the sum $Z = X + Y$, the density function for the sum is given by

Equation 8-36

$$(d_1 * d_2)(z) = d_3(z) = \int_{-\infty}^{+\infty} d_1(z - y)d_2(y) dy$$

Since,

Equation 8-37

$$d_2(y) = \begin{cases} 1 & \text{if } 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Thus Equation 8.36 becomes,

Equation 8-38

$$d_3(z) = \int_0^1 d_1(z - y) dy$$

The above integral is equal to 1 only when

Equation 8-39

$$0 \leq z - y \leq 1$$

Which can be reduced to the following thru steps

- i. by subtracting z everywhere,

Equation 8-40

$$-z \leq -y \leq 1 - z$$

And finally,

- ii. multiplying by -1 everywhere, which changes the inequalities direction, and gives

Equation 8-41

$$z - 1 \leq y \leq z$$

This leaves with two scenarios (for non zero value)

Case 1.

For

Equation 8-42

$$0 \leq z \leq 1$$

Equation 8.38 becomes

Equation 8-43

$$d_3(z) = \int_0^z dy = z$$

Case 2.

For

Equation 8-44

$$1 < z \leq 2$$

Equation 8.38 becomes

Equation 8-45

$$d_3(z) = \int_{z-1}^1 dy = 2 - z$$

To conclude, all the results are listed below:-

Equation 8-46

$$d_3(z) = \begin{cases} z & \text{if } 0 \leq z \leq 1 \\ 2 - z & \text{if } 1 < z \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

The result of the convolution of two uniformly distributed densities is shown below:-

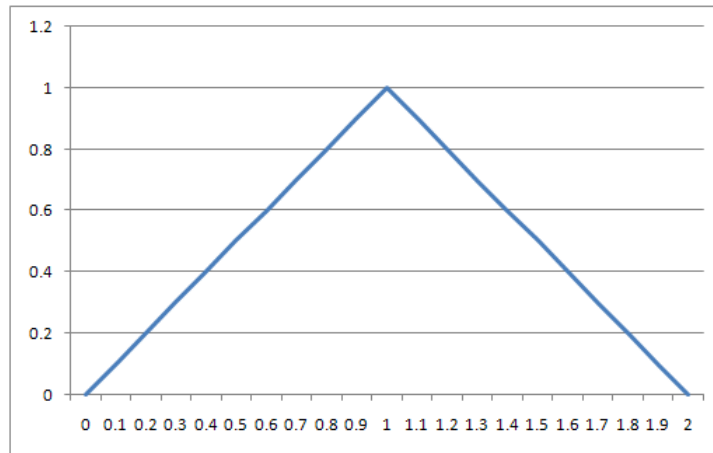


Figure 8-2 Convolution of two uniform densities

9 PROBABILITY MAPS: A NEW STATISTICAL MODEL FOR NON LINEAR TOLERANCE ANALYSIS APPLIED TO RECTANGULAR FACES

9.1 Introduction

A new statistical model for the tolerance analysis based upon joint probability distribution of the trivariate normal distributed variables involved in the construction of Tolerance-maps (T-Maps) for rectangular face is presented. Central to the new model is a Tolerance-Map (T-Map). It is the range of points resulting from a one-to-one mapping from all the variational possibilities of a perfect-form feature, within its tolerance-zone, to a specially designed Euclidean point-space. The model is fully compatible with the ASME/ANSI/ISO Standard for geometric tolerances. In this research, 4-D probability Maps (prob Maps) have been developed in which the probability value of a point in space is represented by the size of the marker and the associated color. Additionally, 3-D prob Maps (3-D cross sections of the 4-D prob Maps at pre specified values) are used to represent the probability values of two variables at a time for a constant value of the third variable on a plane. Superposition of the probability point cloud with the T-Map clearly identifies which points are inside and which are outside the T-Map. This represents the pass percentage for parts manufactured with the statistical parameters such as mean and standard deviation as of the assumed trivariate probability distribution. The effect of refinement with form and orientation tolerance is highlighted by calculating the change in pass percentage with the pass

percentage for size only. Delaunay triangulation and ray tracing algorithms have been used to automate the process of identifying the points inside and outside the T-Map. Proof of concept software has been implemented to demonstrate this model and to determine pass percentages for various cases. The model is further extended to assemblies by employing convolution algorithms on two trivariate statistical distributions to arrive at the statistical distribution of the assembly. Accumulation T-Maps generated by using Minkowski Sum techniques on the T-Maps of the individual parts is superimposed on the probability point cloud resulting from convolution. Delaunay triangulation and ray tracing algorithms are employed to determine the assembleability percentages for the assembly.

9.2 Purpose

Traditional tolerance analysis techniques refer to the practices that help in how tolerances are developed in most of the western countries. In contrast, experimental techniques are most widely used in Japan. Standard worst-case approach is used to add or subtract all the extreme (maximum and minimum) tolerances that are associated with the nominal set point for each component. The worst-case tolerance stack up results in the assembly at either its largest or smallest allowable dimension. The worst-case method does not take into account the laws of probability---at least not in a realistic sense. Over the period, it has been proved that worst-case methods should be used sparingly because the tolerance stack up process used is not representative of the way tolerances build up in the probabilistic environment in which the parts are made and assembled. It

is because it is very unlikely that all the components manufactured are at their maximum or minimum tolerance levels at the same time. The only instance when such an analysis is unavoidable when the assembly is made up of a very few parts that have a critical (safety or customer preference) interface with some other product feature that cannot be allowed to interfere or be spaced too far apart. The real world of component manufacturing is highly influenced by the laws of probability, random chance and special causes. In other words, it is not possible to create component exactly on target every time. All parts manufacturing processes result in a distribution of output that is spread around the targeted output specification. It is because every process contains an inherent variability. In addition, special cause variability could also be present. This is any external or deteriorative source that moves the process from a state of random variation to a new non-random state of variability that is beyond what is caused due to natural (random) events. Examples could be batch-to-batch variability, damaged or worn tools, contaminated raw material, and numerous other noises.

The Root Sum of Squares (RSS) approach is used to account for the low likelihood of all dimensions occurring at their extreme limits simultaneously. In processes where mean shift is suspected, Motorola's Dynamic Root Sum Square approach could be efficiently employed. There is a static version (as compared to the dynamic version) of the method also available, which is more useful in cases involving sustained process mean shift. All the methods mentioned above do not use the distributions as such and are more dependent on the moments of the distribution. For example, two distributions could have same first and second

moments but one distribution may belong to four parameter family of continuous probability distribution while other is only a two parameter distribution. Overall probability as given by the two distributions will be quite different generally and significantly different in certain regions of the range of the variables involved. Another example could be the difference in standard and non standard distributions for same mean and variance values.

In this publication, a new approach is used to carry out statistical tolerance analysis. This approach uses the entire distribution and not dependent solely on the moments of the distribution. The model presented here determines the overall distributions based upon the distributions of the involved variables. Joint probability distributions are used to generate point clouds for parts while convolution algorithms have been used for generating probability point clouds for assemblies. Also, it is the first time in mechanical tolerance analysis that such techniques as Delaunay triangulation and ray tracing techniques have been used to determine the pass percentages for the parts and assembleability percentages for the assemblies.

Section 3 provides the review of those publications in which related research work had been carried out. Section 4 explains briefly the two methods for visualizing the probability point cloud. It also has a brief theoretical introduction of Joint probability distribution concept. Section 5 walks the reader through the Re-D (Reduced Dimension) method of visualization and also cover theoretical background for bivariate normal distributions. Section 6 explains the Hi-D (Higher Dimension) method of visualization along with details on trivariate

normal distributions. Section 7 discusses the viability of the measurement of the variables involved in the analysis. Use of the prob Maps for the extraction of pass percentages is detailed in section 8. Section 9 tackles the effect on the pass percentage when form refinement is additionally specified. Section 10 does the same for orientation refinement. Use of convolution techniques to arrive at the assembly statistical distributions using trivariate normal distributions of the parts is explained in section 11. Section 12 explains the extraction of assembleability percentages for stack up of two parts along with overview of future work and work in progress research activities which is followed by conclusions in section 13.

9.3 Background

Rules for specifying and interpreting geometric tolerances can be found in the ASME Y14.5M standard [2] and its counterpart the ISO 1101 Standard [24]. One of the earlier researchers in the field of statistical tolerancing is undoubtedly Mansoor [57]. Mansoor's method assumes component dimensions follow normal distribution and obtains the resultant assembly tolerance by using the root sum of squares (RSS) method. The most simple and widely used method for statistical tolerance analysis is the root sum squares (RSS) method. Parkinson [60] later generalized the technique and used it for optimization of dimensional tolerances. Bjorke [67] developed a similar model based on beta distribution, although he assumed that the resultant dimension based on the linear stackup would follow a normal distribution. This leads to serious inaccuracies especially if the part count

in the assembly is low. O'Connor and Srinivasan [68] developed the concept of Distributed Function Zone (DFZ) as an aid to statistical tolerancing. In this approach, the pass percentage of a population of parts is determined by requiring that a pair of specified non-standard distribution functions bound the distribution function of relevant values of the parts.

Many researchers have widely used Monte Carlo simulation for statistical tolerance analysis such as [69]. In this method, a random number generator is used to simulate the geometric variations for the components. Using the assembly function, these values are combined to determine the resulting influence on some clearance or gap dimension. Many of the commercial systems (e.g. Team Centre VisVSA (Visual Variance Simulation Analysis)) [70, 71] utilize Monte Carlo simulations for statistical tolerance analysis. However, over the period of time, simulation has been found to be very slow and computationally expensive. It appears that these could be regarded as analysis tool but their use for tolerance synthesis is economically limited.

Fitting of a multidimensional Gaussian probability density function to the multidimensional variational possibilities was investigated by Whitney et al. [72]. Using transformation matrices as proposed in above referred publication, Lee and Yi[73] applied these results to obtain a statistical representation of tolerances for evaluating clearances in assemblies.

Ameta G., Davidson J. and Shah J. used tolerance maps (T-Maps) for generating frequency distribution of 1-D clearance and allocate tolerances [74 and 75]. Multivariate statistical analysis was used to rapidly explore potential chemical

markers for the discrimination between raw and processed radix [76]. Koksal G. and Fathi Y. used statistical tolerancing in designed experiments in a noisy environment [77]. Choudhary A. suggested a statistical tolerancing approach for design of synchronized supply chains [78]. Gonzalez I. and Sanchez I. utilized statistical tolerancing to come up with an innovative methodology to allocate optimal statistical tolerances to dependent variables, in cases where the dependence structure could be estimated from the manufacturing process [79]. Dantan J. and Qureshi A. presented a new mathematical formulation of worst case and statistical tolerance analysis based on quantified constraint satisfaction problems utilizing Monte Carlo simulations [80]. Bruyere J., et al applied statistical tolerance analysis on a bevel gear employing tooth contact analysis and Monte Carlo simulation [81]. Ramaswami H. and Acharya et al recognized the need for multivariate statistical tolerance analysis for sampling uncertainties in geometric and dimensional errors for circular features [82]. They used Exploratory Factor Analysis to arrive at six dimensional performance metric vectors to quantify the difference between the true value of the errors and the value evaluated using the sample.

9.4 Prob Map for size tolerance on rectangular face

As mentioned earlier in the previous paragraphs, the simplest T-Map which is the T-Map for size tolerance specified on the rectangular face has three variables; namely size tolerance t , plane tilt along x axis and plane tilt along the y axis. To visualize, interpret and analyze the statistical probability distribution for all the

design points that are represented inside the T-Map, one has to consider statistical probability distribution for each of the variables involved in the construction of the T-Map. In other words, if there is statistical probability distribution associated with each of these variables, then the simultaneous behavior of these constituent statistical probability distributions will be necessary for the visualization, interpretation and analysis of the probabilities associated to every point represented inside the T-Map.

In this regard, this research uses and incorporates the concept of joint probability distribution to arrive at the statistical probability distribution for all the design points represented inside the T-Map. Joint probability distribution gives the simultaneous behavior of two or more variables that have a statistical probability distribution of their own.

The explanation of the theory behind the joint probability distribution follows in the coming paragraphs.

In the study of probability, for two continuous random variables A and B under consideration, the probability distribution of both random variables considered simultaneously is given by the Joint probability distribution of A and B .This can be written as $f_{A,B}(a, b)$.For any region R in R^2 space,

Equation 9-1

$$P((A, B) \in R) = \iint_R f_{A,B}(a, b) dx dy$$

In the case of only two variables involved in the Joint probability distribution, then it is called bivariate distribution. It is called multivariate distribution in case of three or more variables involved.

Conditional probability is the probability of one variable when the probability of the other variable is known. Marginal probability distribution is the individual probability distribution of a random variable whose joint probability distribution with one or more random variable exists. The joint probability distribution for two variables can also be written in terms of conditional distributions

($f_{B|A}(b|a)f_{A(a)}$ and $f_{A|B}(a|b)f_{B(b)}$ represent the conditional distribution of B given A=a and of A given B=b respectively) and marginal distributions ($f_{A(a)}$ and $f_{B(b)}$ represent the marginal distribution for A and B respectively) as

Equation 9-2

$$f_{B|A}(b|a)f_{A(a)} = f_{A|B}(a|b)f_{B(b)} = f_{A,B}(a, b)$$

As already explained, out of the assortment of T-Maps that have been discovered so far, the most simple is the T-Map for size tolerance specified on the rectangular face. However, even the simplest T-Map uses three variables for its construction and the probability value will be the fourth entity in the proposed Prob map. So let's consider the joint probability concept for three or more variables.

The joint probability distribution for two random variables could be extended to three variables (infact n variables) by adding them with the identity sequentially as

Equation 9-3

$$f_{A_1, \dots, A_n} (a_1, \dots, a_n) = f_{A_n | A_1, \dots, A_{n-1}} (a_n | a_1, \dots, a_{n-1}) f_{A_1, \dots, A_{n-1}} (a_1, \dots, a_{n-1})$$

Thus, in actuality, the simplest Prob Map will be 4 dimensional. Two methods for the display of these Prob Maps have been proposed in this research. One is called the Re-D Prob map and the other is called Hi-D Prob map. These are explained in the following paragraphs.

In Re-D (Reduced Dimension) Prob maps, only three out of the four entities are catered. In other words, one needs to select two variables out of three variables (size, plane tilt along x axis and plane tilt along y axis). For example, if plane tilt along x axis equal to a certain constant value such as a_0 is selected, then one can achieve a 3-D view of the 4-D statistical probability distribution plot. However, this will only be the state for one particular instance when the plane tilt along x axis is equal to a_0 . To get the complete picture, one needs to have a series of such 3-D probability plots. In the Re-D Prob Maps, the color of the point within the plot shows the value of the probability for that combination of the two variables.

In Hi-D (Higher Dimension) Prob Maps, for every combination of the three variables i.e. size tolerance, plane tilt along x axis and plane tilt along y axis, the probability value is shown by the Prob Map. In such a case, the color of the point within the plot and the size of the point reflect the value of the probability for the combination of the three variables involved.

9.5 Re-D Prob map for size tolerance specified on rectangular face

The T-map for the size tolerance specified on rectangular face was developed in Shah et al [48,49]. Here three variables are involved i.e. size tolerance, plane tilt along x axis and plane tilt along y axis. The two approaches that have been described earlier in this publication will now be applied to the situation in hand.

For the Re-D method, in the series of plots shown below, the third variable has been used for sectioning the 4-D plot to get a 3-D plot. All the Prob maps above are generated for the bivariate normal distribution (reduced) and are only applicable for the points in the plane of size tolerance and the plane tilt along Y axis. The bivariate normal probability distribution has the following density function

Equation 9-4

$$P(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{z}{2(1-\rho^2)}\right\}$$

Where z (a convenience variable) and ρ (correlation of x_1 and x_2 and V_{12} is the covariance) are given respectively by :-

Equation 9-5

$$z = \frac{(x_1 - \mu_1)^2}{\sigma_1^2} - \frac{2\rho(x_1 - \mu_1)(x_2 - \mu_2)}{\sigma_1\sigma_2} + \frac{(x_2 - \mu_2)^2}{\sigma_2^2}$$

And

Equation 9-6

$$\rho = \text{cor}(x_1 - x_2) = \frac{V_{12}}{\sigma_1\sigma_2}$$

For Re-D prob Maps, a series of plots were generated for the entire range of variables (size tolerance and plane tilt along y axis) at a certain fixed value of third variable (plane tilt along x axis). A selected lot out of the series of such Re-D prob Maps is shown in figure 9-1.

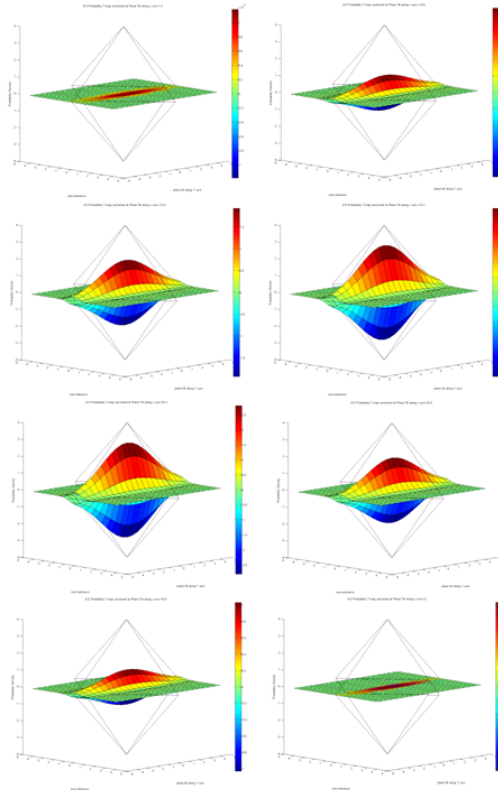


Figure 9-1 Re-D Prob maps for sections taken at intervals within the range of total variation in plane tilt along x- axis (This figure highlights the bell shaped variation of the Gaussian distribution for the entire range of the variation in plane tilt along x-axis.

The plots have two active variables i.e. size tolerance and plane tilt along y axis. Equation 9-4 could have been used for generating the probability point involving

two variables. However, instead, equation 9-7 has been used for generating the probability point cloud. The difference between the Re-D and Hi-D method is that although the same expression of trivariate normal distribution is used, in Re-D, the expression given by equation 9-7 is evaluated several times, each time with a different constant value of the third variable (plane tilt along x axis in this case).

In constructing these Prob Maps, the range of variation for size tolerance was assumed to be from -0.04 to 0.04 (e.g. mm), for plane tilt along x axis, it was -0.02 to 0.02 (e.g. radians) and for plane tilt along y axis, it was -0.05 to 0.05 (e.g. radians). These values were selected from a set of viable correlated range of values for the variables. Any other value could also have sufficed. The black line depicts the boundary of the T-Map for the size tolerance specified on the rectangular face. The Prob Map is symmetric in all directions as it is expected (because of the symmetric nature of the T-Map).

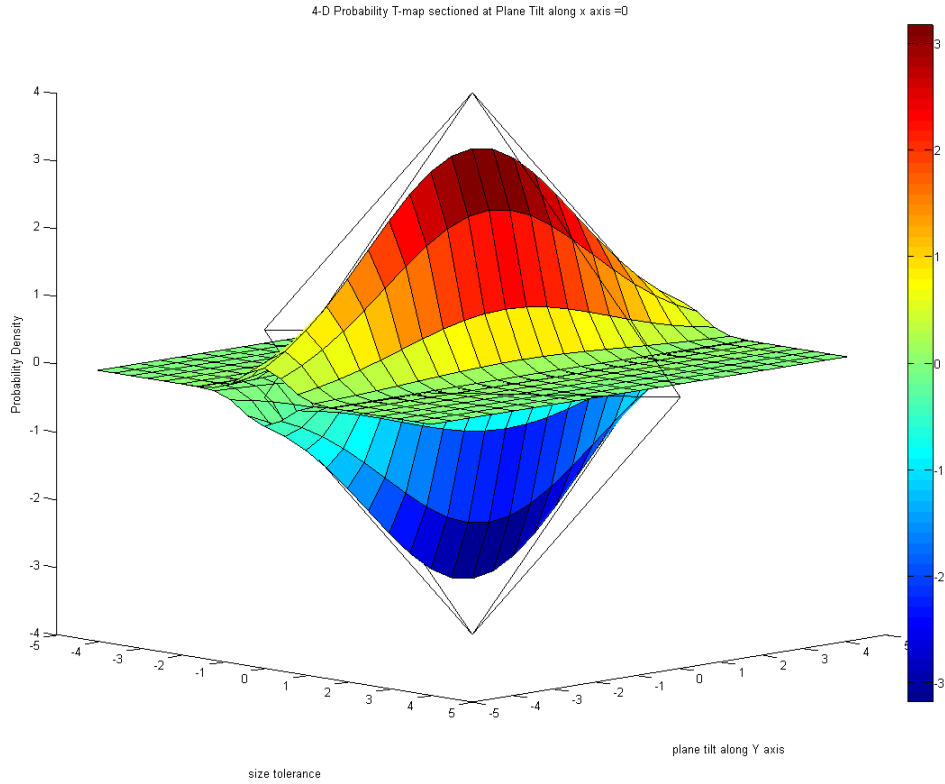


Figure 9-2 Re-D Prob Map for section taken at Plane Tilt along x-axis =0

The size of all the plots in figure 9-1 is kept intentionally small to save on space. Figure 9-2 is a full size Re-D prob Map for plane tilt along x axis equal to zero. It is to be made clear that Re-D method gives a 3-D probability plot which is a cross section (or series of cross sections) of the full 4-D probability plot once the size tolerance on a rectangular face is under consideration. Re-D Prob Maps can be used for calculating pass or reject percentages for those parts which are represented within the plane of the two variables used for constructing the prob Map.

9.6 4-D Prob Maps

4-D prob Maps are created using Hi-D method of visualization as explained earlier. For the Hi-D method, the probability distribution for each of the three variables, i.e. size tolerance, plane tilt along x axis and plane tilt along y axis is given by their joint probability distribution which is given below:

Equation 9-7

$$P(x_1, x_2, x_3) = \frac{e^{-w}}{2 \sqrt{2} \pi^{3/2} \sqrt{1 - (\rho_{12}^2 + \rho_{13}^2 + \rho_{23}^2) + 2 \rho_{12} \rho_{13} \rho_{23}}}$$

Where 'w' is a convenience variable and is given by:-

Equation 9-8

$$w = x_1^2 (\rho_{23}^2 - 1) + x_2^2 (\rho_{13}^2 - 1) + x_3^2 (\rho_{12}^2 - 1) + 2 [x_1 x_2 (\rho_{12} - \rho_{13} \rho_{23}) + x_1 x_3 (\rho_{13} - \rho_{12} \rho_{23}) + x_2 x_3 (\rho_{23} - \rho_{12} \rho_{13})]$$

And ρ_{ij} is the correlation between the three variables.

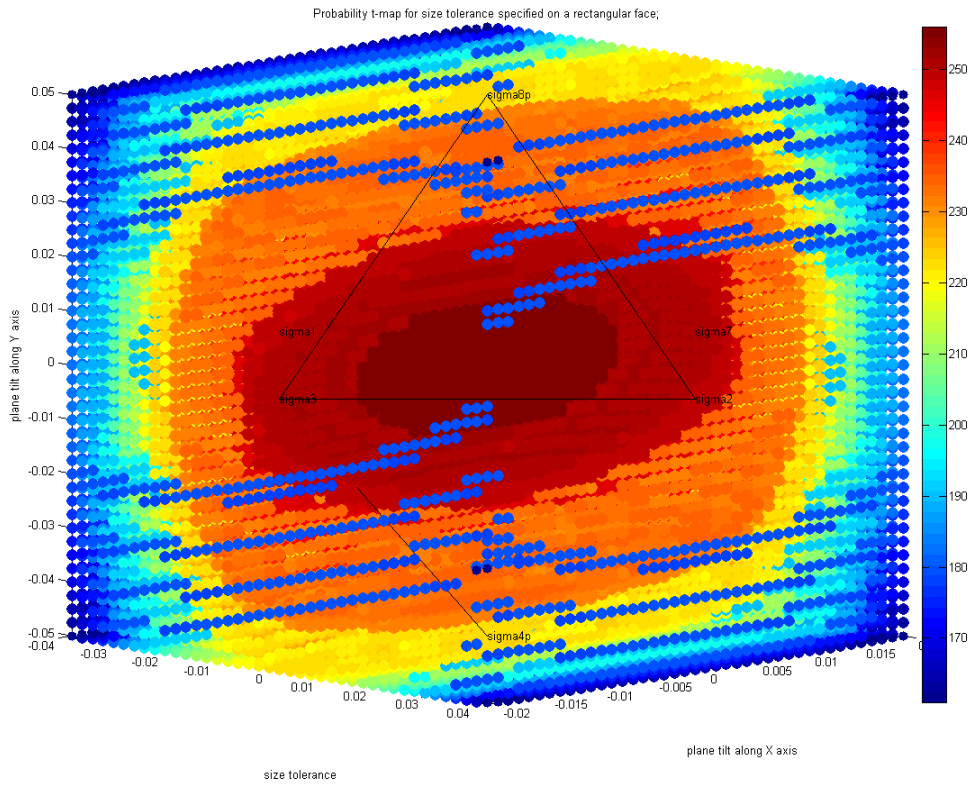


Figure 9-3 Prob Map for size tolerance specified on rectangular face. The variables involved are the size tolerance, plane tilt along x axis and plane tilt along y axis.

In figure 9-3, the 4-D Prob Map for the size tolerance specified on the rectangular face is shown. The three variables involved are the size tolerance, plane tilt along x axis and plane tilt along y axis. The fourth parameter which is the probability density function value is depicted by the size of the dot and its normalized density value is shown with the color of the dot. $\sigma_1 - \sigma_2$ is the axis along which the size varies. Similarly, $\sigma_3 - \sigma_7$ shows the variation for the plane tilt along x axis while $\sigma_{4p} - \sigma_{8p}$ caters to the plane tilt along the y axis. The range of variations for each

of the three variables is shown in the plots. Again, size tolerance is varying from -0.04 to 0.04 units (e.g. mm) and plane tilt along x axis varies from -0.02 to 0.02 units (e.g. radians) while plane tilt along y axis varies in between -0.05 to 0.05 units (e.g. radians). Figure 9-4 is just another view of the 4-D prob Map and it is revealing a totally new picture of how the probability values are distributed along the three axes.

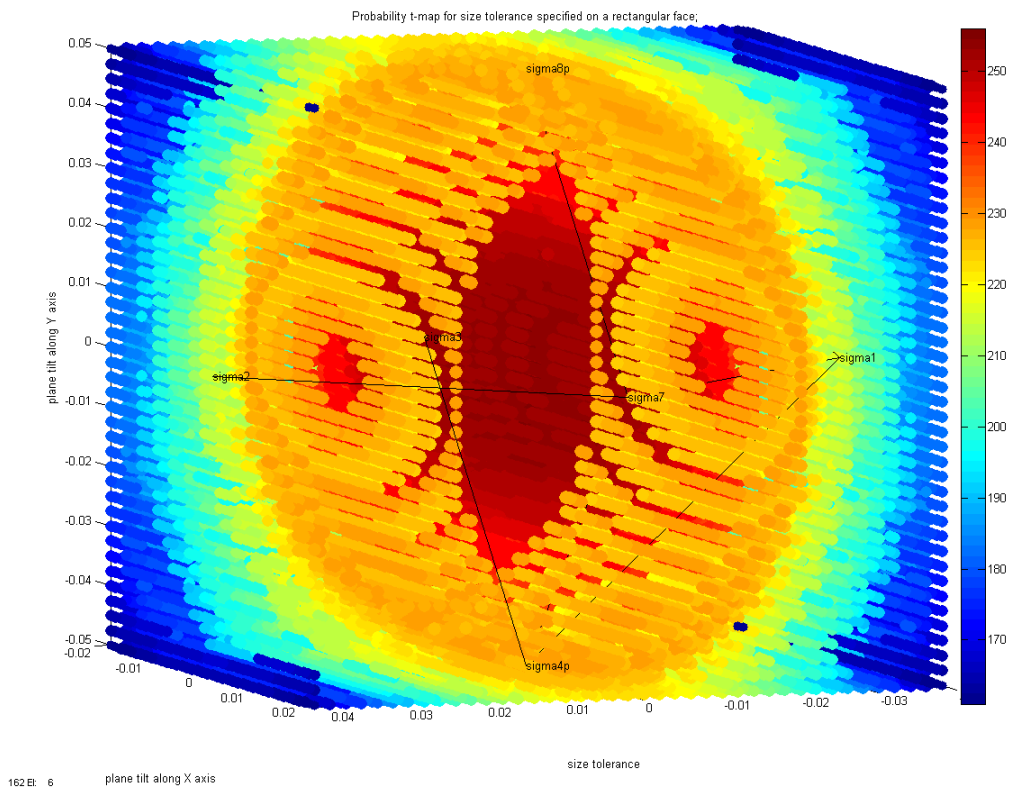


Figure 9-4 Another view of the 4-D prob Map as given in fig 9-3. (Point to note is the difference between the two views for additional information and pattern recognition)

The three different orientations of the Prob Map are depicted in the figures 9-5a, 9-5b and 9-5c. These figures show the utility of the Prob Maps to show the

probability distribution from any desired point of view. Figure 9-5a shows the orthographic view of the Prob Map looking from σ_2 side. As can be seen, the probability values have a greater bias along the σ_3 - σ_7 axis which is the axis along which the plane tilt along the x-axis varies. The difference between the figure 9-5b and 9-5c is that although, both the views contain only the size tolerance and the plane tilt along x axis, figure 9-5b is the top view with σ_{8p} axis coming towards the viewer and figure 9-5c is the bottom view with σ_{4p} coming towards the viewer.

Figure 9-5b shows σ_{8p} coming out of the plane of the paper. It clearly shows that the trend of the bias of the probability values is not uniformly distributed along the $\sigma_2 - \sigma_1$ axis. In fact, it is only restricted to the zero value of the variable only. It also confirms that this trend is uniformly distributed not only about the $\sigma_3 - \sigma_7$ axis but extends well beyond the accepted range also. In figure 9-5c, σ_{4p} is coming out of the plane of the paper. It shows that the trend is not symmetric along the $\sigma_{4p} - \sigma_{8p}$ axis. Also, it can be seen that in addition, these Prob Maps can also be used to observe the pattern for two dimensional cross sections for the probability distribution as shown in figure 9-5b and figure 9-5c. All the different views presented here are meant to show the range of maneuverability that the viewer/user can achieve while having access to the 4-D Prob Map.

Now the question arises: Are these really 4 dimensional? Before this question is answered, let's answer this question: are these 4 dimensional geometries? The answer is simply 'No'. The answer

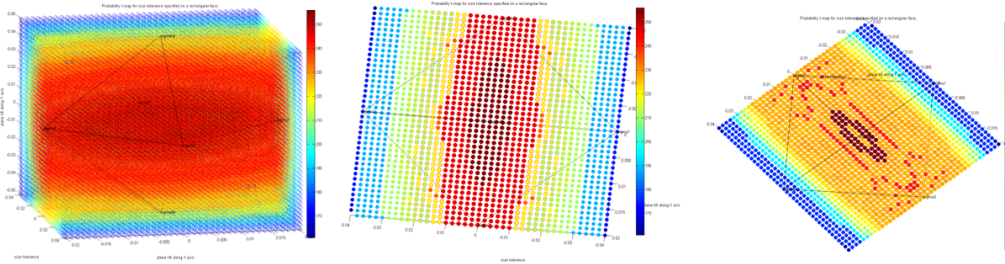


Figure 9-5 Different views of the 4-D prob Maps (a,b,c from Left to Right) (a). Looking from σ_2 side (b). Looking from σ_{8p} side (c). Looking from σ_{4p} side

to the first question is that these are representation of data which has a dimension equal to 4. 4-D prob Maps are in actuality 3-D solids which are representing 4 data variables at a time. This is also to differentiate them from 3-D surfaces such as Re-D probability Maps. The coining of this term is fully consistent with the present trend in exploratory data analysis. Another similar (but limited in utility) example of visualizing high-dimensional data is by using a glyph.

9.7 Measurement of three variables involved in the construction of T-map

As mentioned earlier, the simplest T-map which is the one for size tolerance specified on the rectangular face has three variables of interest; namely, size tolerance, plane tilt along x-axis and plane tilt along y-axis. The question now arises: are these variables separately measurable? The answer is yes. The size of the part is measured by the point farthest away from the reference plane; if only one measurement is being made. However, if a bunch of points is obtained on the target surface using the Coordinate Measuring Machine (CMM), then a plane could be fitted to those points, based upon the different criteria such as Least

Square Fit, Chebychev fit, or one sided fit. Whatever be the method of fitting the plane to the point cloud obtained at the target face, a single plane will be achieved. In highly improbable cases, that plane will be parallel to the reference plane which is used for the calibration of the CMM. In all other possible cases, that plane will be at a certain orientation with respect to the x-axis and a certain orientation with respect to the y-axis. The largest measurement along the third dimension will be used for size determination. So once, the size of the part is concluded using point cloud thru plane fitting, for every size value of the part, there will be an associated value of the plane tilt along x-axis and plane tilt along y-axis.

9.8 Extraction of the pass percentage of parts using prob Maps

Every point represented in the 4-D prob Map has a probability value assigned to it. This final probability value is the Joint probability density function value based upon the simultaneous probabilities of three variables that are involved in the construction of the T-Map for size tolerance specified on rectangular face. However, as can be seen from the figures 9-3, 9-4 and 9-5 (a-c) for 4-D prob Map, not all the points reside inside the T-Map. Only those points, which are inside the T-Map, will contribute to the pass percentage of the parts that will be accepted based upon the probability distribution of each of the three contributing variables including size tolerance.

In order to determine if a point is inside any surface, the concept called ray tracing, which is quite frequently used in computer science has been utilized for

the prob Maps. In ray tracing, a directed ray (one whose direction and starting point is known) is theoretically fired towards the point of interest. When the ray intersects the point, the location of the point inside or outside the surface is determined, by how many times the ray has already entered or exited through the bounding surface. For this to be useful, the entire surface is triangulated or in other words, the entire surface is decomposed into numerous triangles. For volumes in 3-D, tetrahedrons are used. When volumes are involved, the method becomes more complicated as normals of tetrahedrons are also involved. Breaking up of the surface or volumes requires Delaunay triangulation. The method of Delaunay triangulation assumes that any surface or volume can be represented fully by a series of triangles or tetrahedrons respectively.

The 4-D prob Map and the associated code developed in this research is able to give the pass percentage for any value of mean , standard deviation , and range for all three variables. For the purpose of this research, normal or Gaussian distribution has been used. However, method can be easily adapted for any other type of assumed distribution for three variables. Although this research team thinks that the assumption of Gaussian distribution is fully justified especially basing the premise on central limit theorem in theory of probability.

Breakdown of the algorithm used for the software developed is shown in figure 9-6. Initially, type of the statistical distribution is to be specified for each of the involved variables. This can be assumed on the basis of empirical or historical knowledge or it can be estimated from the sample drawn from the population of the parts. If the first option is pursued, then the assumption of the characteristic

parameters for that distribution is also to be assumed. The second approach will give estimates of the characteristic parameters which will depend on the confidence level also.

The next step is to clarify that whether the involved variables are independent or not. Specific tests can be carried out (as specified in books on probability and statistics) to test how much is the degree of correlation between the variables. Depending on the values of covariance estimated, it is up to the user to decide whether to declare them as independent or dependent variables. If the variables are dependent, the software requires the specification of the correlated standard deviations. These standard deviations could be set to zero in cases where the involved variables are independent.

Next step is to determine the probability values using the expressions for joint probability distributions as given by equation 9-7. These values are displayed as a probability point cloud. This is followed by generating the 3-D geometric model of the T-Map which is superimposed on the probability point cloud. Next, it needs to be determined that which points of the point cloud are inside the T-Map. These are the points which represent parts acceptable to the designer or the user after manufacturing. This requires that the bounding surface of the T-Map be triangulated. For this, Delaunay triangulation is used. The next step is the use of ray tracing algorithms to find out which points of the probability point cloud represent acceptable parts. The probability value of the acceptable parts is used finally to arrive at the pass percentage of the parts acceptable after manufacturing.

The code, as already mentioned, is fully adaptable (the number of variables involved can be increased to beyond three) and can be easily used to study various parameters of interest. In conventional statistical tolerance analysis, for example, the interest is to find out the sensitivities of the various parameters involved. This could be a matter of less than a second with this code for 4-D prob Map.

For example, the pass percentage for the given case of size tolerance specified on rectangular face with zero means for each of the three variables gives a pass percentage of 87.19%. However, if the mean is changed slightly (say by 0.001) for each of the three variables one at a time while keeping the mean and other parameters such as standard deviation, range of variation etc constant for the other variables gives the pass percentages of 87.10% , 86.96% and 87.11% respectively. Similarly, for the same amount of change in the standard deviation of three variables , one at a time, while keeping the mean, standard deviation and range of variation etc unchanged for the other variables gives the pass percentage of 95.65%, 96.90% and 94.49% respectively.

9.9 Effect on the pass percentage with the specification of form tolerance

A flatness control tolerance zone is two parallel planes separated by the flatness tolerance value. A flatness control is always applied to planar surfaces. Hence, it can never use an MMC or LMC modifier. Flatness is a separate requirement if placed by the designer and is verified separately from the size tolerance and Rule #1 requirement. Rule # 1 of the standard implies that for features of size, in cases where only a size tolerance is specified, the surfaces shall not extend beyond a

boundary (envelope) of perfect form at MMC. Hence, it can be seen that Rule # 1 is an indirect flatness control. In fact, flatness effects of Rule # 1 are never inspected as they are a result of the boundary and size limitations.

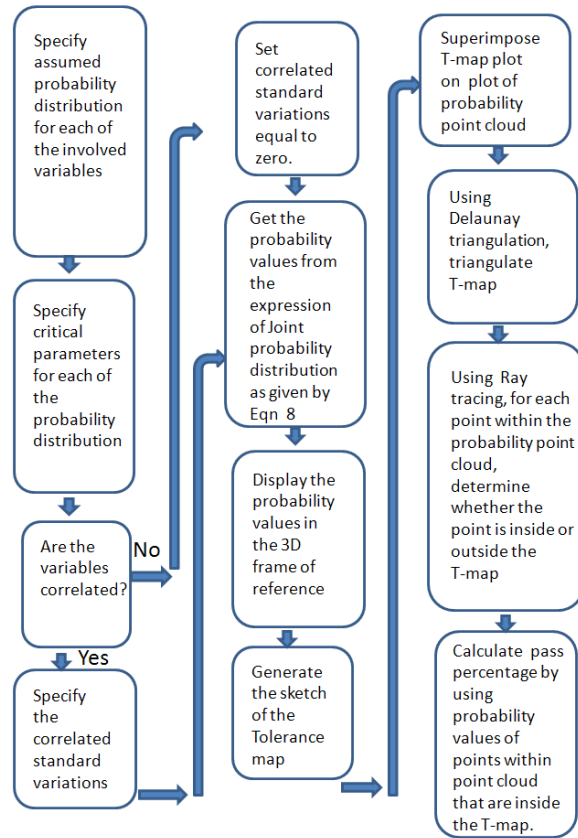


Figure 9-6 Algorithm break down of the software developed for the creation of prob Maps

For round faces, the effect of form tolerance on the T-Map for size tolerance has been explained in [48]. Effect of form tolerances on T-Map for size tolerance on a rectangular face will be discussed here for the very first time. It can be seen that form tolerance will have a smaller T-Map that will reside inside the overall T-Map for size tolerance. In other words, although, form tolerance will have its own

unique tolerance zone and hence corresponding T-Map, it will not affect the worst case boundary and hence the overall T-Map for size tolerance is still applicable. Still, since the feature on which form tolerance has been specified needs to satisfy both requirements for size and form, there could be a separate pass percentage for parts in such a situation.

T-Map for form tolerance will float inside the T-Map for size. It can be anywhere inside the T-Map for size but its orientation with respect to the orientation of the size T-Map will remain same. This has been shown in figure 9-7.



Figure 9-7 (a to e from left to right) Floating of the form subset within the size T-map for rectangular face (the size of the figures reduced to save space; σ_2 , σ_1 , σ_{8p} and σ_{4p} are on top, bottom, left and right respectively)

There could be several possibilities for the location of the form subset inside the T-Map for size. The final pass percentage for each of the case will be different. However, if the symmetry between two locations of the form T-Map exists, then the pass percentages can also be same. It is because of the trivariate normal distribution which is symmetric about the means. Figure 9-7a is mirror image of figure 9-7c and hence, the effect of the form tolerance specification on the pass percentage will almost be same. Same is the case as shown in figures 9-7b and 9-7d. In figure 9-7e, the form subset is exactly located in the centre of the T-Map

for size tolerance. However, the effect of the form tolerance specification between figure 9-7a (or 9-7c), 9-7b (or 9-7d) and 9-7e will be totally different and will depend on the statistical parameters used for the selected statistical distribution for the three variables involved in the construction of the size T-Map.

Using the code developed in this research, one can find out how and by what amount the pass percentage will be affected when form tolerance refinement is used in addition to the size tolerance. Five sample cases are discussed and evaluated using the code and reproduced below: For one such sample case, with trivariate normal distribution, the size tolerance T-Map will restrict the pass percentage to 91.71 % for a given range of three variables involved in the T-Map for size tolerance for rectangular face. However, if form tolerance equal to 83.33% of size tolerance is additionally specified, then the pass percentage will be restricted to 81.06% (figure 9-8a). However, if the form tolerance is made finer and it is now set to 50% of the size tolerance, the pass percentage will be reduced to 41.55% (figure 9-8b). Also, if the amount of form tolerance is kept same along with other parameters constant, but the form tolerance subset is shifted by a unit amount (unit amount remains constant for the next three cases) along the size tolerance axis within the size T-Map, it was observed that the pass percentage was changed to 37.81% (figure 9-8c). For form tolerance of 50% size tolerance and unit offset along plane tilt (along x-axis) axis, the pass percentage reduces to 33.94% (figure 9-8d). For the fifth case, form tolerance is 50% of the size tolerance and unit offset is now along the plane tilt (along y-axis). This results into pass percentage of 39.34% (figure 9-8e)

It is to be remembered again that the percentage calculations were carried out with almost 69000 points within the probability point cloud. However, the plots below were generated with a less dense formulation to illustrate the change depicted in each of these figures.

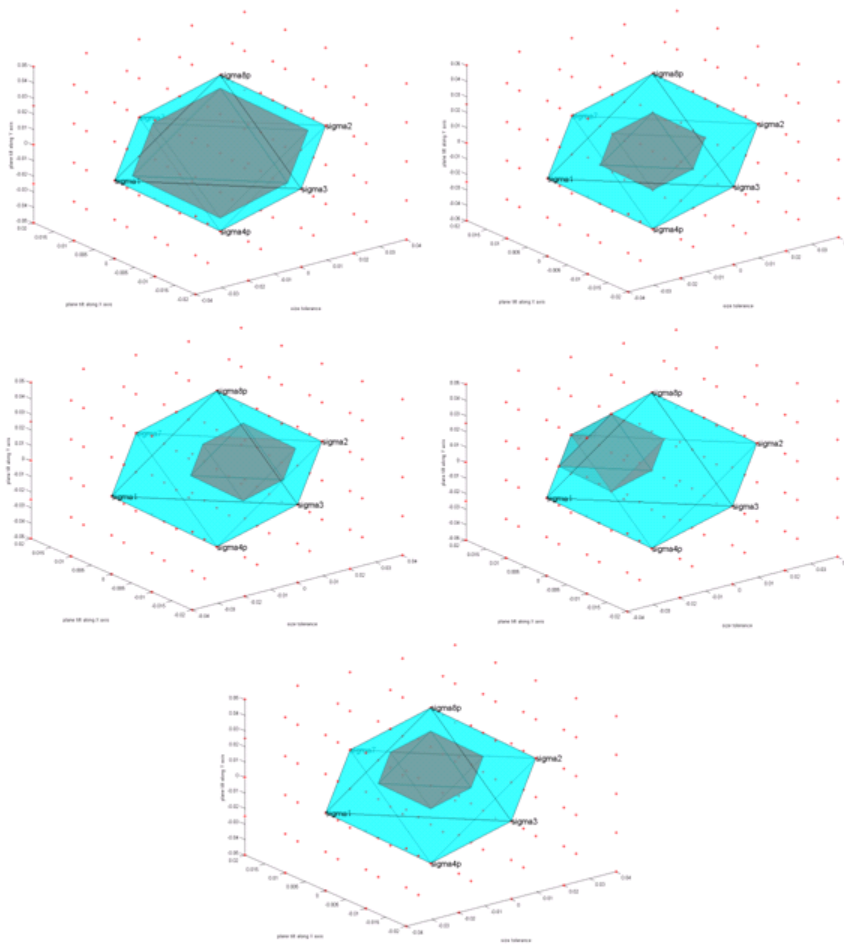


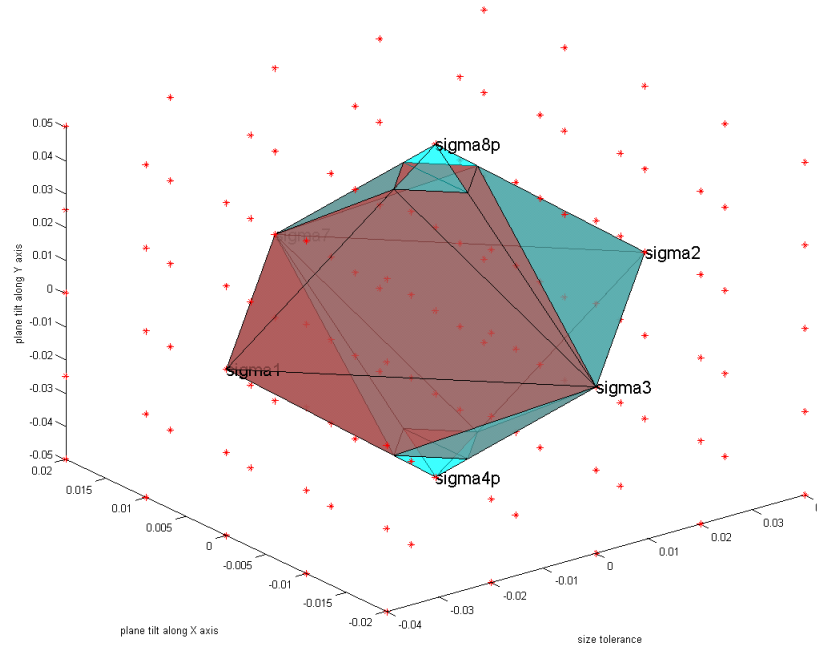
Figure 9-8 Effect of form refinement on size tolerance for pass percentage of the parts manufactured (a to e from left to right top to bottom) (a). Form tolerance specification of exactly 83.33% of size and no offset along any axis (b). Form tolerance specification of exactly 50% of size and no offset along any axis (c). Form tolerance specification of exactly 50% of size and unit

offset along size tolerance axis only (d). Form tolerance specification of exactly 50% of size and unit offset along Plane tilt (along x-axis) axis only (e). Form tolerance specification of exactly 50% of size and unit offset along Plane tilt (along y-axis) axis only

9.10 Effect on the pass percentage with the specification of orientation tolerance

In order to demonstrate the effect of orientation tolerances on the pass percentage of the parts which already have size tolerance specified, the interest will be limited to perpendicularity tolerance refinement only. The T-Maps for size and the floating subsets for orientation for polygonal faces have been discussed in detail in [48,49]. The refinement by orientation tolerances further restricts the pass percentage of the parts. The code developed in this research has been used to calculate the effect for the case given in the figures 9-9 and 9-10.

The pass percentage for only the size tolerance specified on the rectangular face for a given set of parameters has been calculated by the code and is found out to be 95.75%. However, when orientation tolerance (perpendicularity) of value equal to 20% of the size tolerance is specified, the pass percentage is reduced to 71.71% (refer figure 9-9). If the orientation tolerance refinement is doubled, then the pass percentage is reduced to 44.75 % (refer figure 9-10).



**Figure 9-9 Effect of orientation refinement to size tolerance for size T-map
(orientation tolerance is 20% of the plane tilt along y axis)**

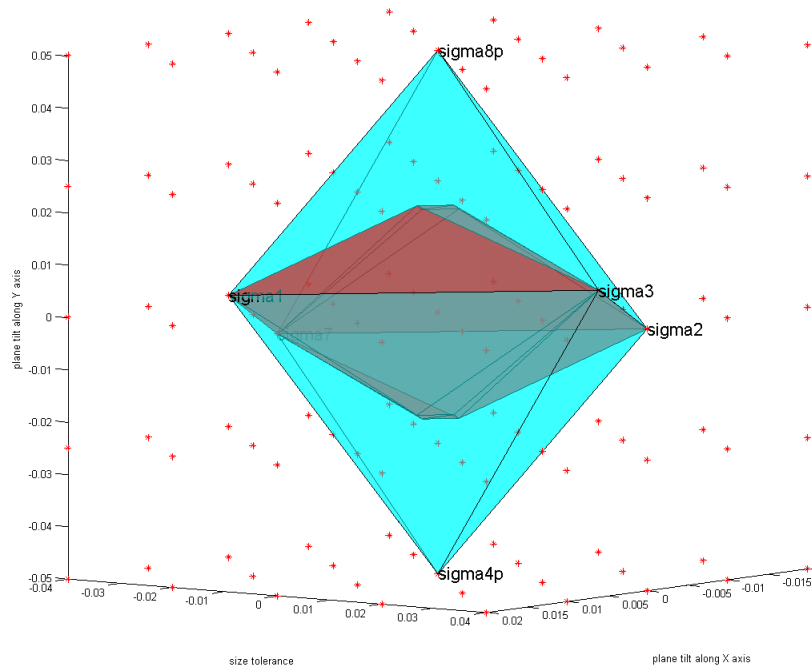


Figure 9-10 Effect of tighter orientation tolerance on a size T-Map as compared to scenario depicted in fig 9-12. (Orientation tolerance is 40% of the plane tilt along y axis)

For the 4-D prob Maps as shown in the figures above, a convenient yet accurate decomposition of the entire domain was suggested. These values have been calculated with 68921 points. However, to exactly show which points are inside and which are outside the T-Map, a lesser dense formulation has been used for the figures above.

9.11 Use of convolution for probability estimation of assembly statistical distribution from statistical distribution for the parts

Let us consider two parts that are in linear stack up as shown in the figure 9-11. Both the parts have size tolerance specified on the rectangular faces. For the T-

map for each individual part, there will be a dipyrmaid, the construction of which is covered in detail in [49]. For the construction of each dipyrmaid, the three variables involved will be size tolerance, plane tilt along x-axis and plane tilt along y-axis. So in all, there are six variables involved. Employing Minkowski sums, the two T-Maps will be used to arrive at the accumulation T-Map for the assembly. A brief discussion of the Minkowski sum is given in the following paragraph.

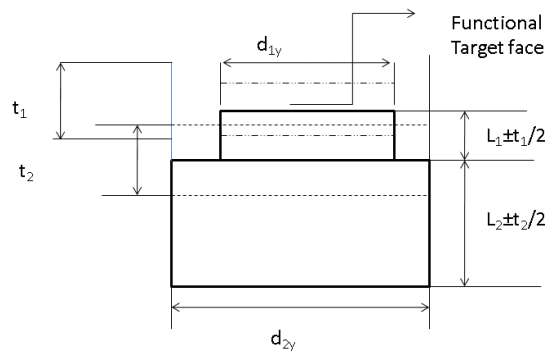


Figure 9-11 Two parts in stackup forming an assembly. The two dashed lines show the extreme positions for part 1. Same does the dotted lines for part 2. L_1 and L_2 are the characteristic lengths or dimensions for the two parts respectively.

Minkowski sum of two sets A and B is the sum of every element of a set A to every other element of set B. It is also known as ‘dilation’ or the binary dilation of A by B. Symbolically,

Equation 9-9

$$A + B = \{a + b | a \in A, b \in B\}$$

If the set A and set B have only one member, then it reduces to vector addition. For details, refer to previous chapters.

Now from statistical analysis point of view, the first part's tolerance zone has three variables involved and each of them have an assumed statistical probability distribution. Same is true for the second part in the assembly. As previous, use of joint probability distribution concept is used to determine the probability distribution of a point represented inside the T-Map. For the two parts in the assembly, now this means that there are two joint probability distributions, with a total of six variables. To arrive at the statistical distribution of the assembly, in this research, the concept of convolution of the probability distribution has been used. Convolution is a well know concept and the details of the mathematical process could be found in [83, 84]. A brief introduction of the concept is given in the following paragraph.

Convolution, in layman terms, is an operation in mathematics, carried out on two functions which results into a third function. More technically, it is defined as the integral of the product of the two functions after one is reversed and shifted. For details, refer to previous chapters.

Convolution is a mathematical process and will involve two integral equations. This is an idea applied in mechanical tolerance analysis for the first time. The result of the operation will be a set of probability values that will dictate the chances that an assembly with certain value of the variables will be manufactured.

A separate code has been developed for determining assembly probability values based upon the probability distributions for parts in the assembly. The algorithm for the code is shown in figure 9-12.

First of all, all the parts in the stackup (for this example, it is only two. However, the code has been designed in such a way that it can cater for more parts in the stackup), the type of statistical probability distribution is to be assumed. This assumption could be based upon empirical knowledge or based upon the results of a sample out of the population. Next step is the specification of critical parameters for the assumed statistical probability distribution. The number of critical parameters could be different for each of the assumed probability distribution. These are also commonly known as the moments of the distribution. Examples of these moments of distribution are mean, variance (or instead, standard deviation is commonly used), kurtosis and skewness. The values of these may be assumed or estimated in various ways. For example, mean could be the desired nominal value or the population mean estimated from the sample mean.

The analysis will give noticeably different results depending upon whether the variables involved are dependent or independent. Software caters for this by having correlated standard deviations specified. Once all above information is fed in, the software calculates the Joint probability distribution values based upon equation 9-7 as mentioned earlier in the publication.

The same process is repeated for the second part in the stackup. The two joint probability distributions are then convoluted to arrive at the probability distribution for the assembly. The final values are then displayed as a point cloud.

On the other hand, a size T-Map is constructed for each part depending upon the amount of size tolerance and the geometry of the target feature. For the assembly, Minkowski sum is used to arrive at the accumulation T-Map. The plot of this accumulation map is then superimposed upon the point cloud. The next step is to determine how many points out of the probability point cloud are inside the T-Map. The points inside the accumulation T-Map represent those assemblies which will be acceptable to the designer after being manufactured.

In order to judge numerically whether a point is inside the boundary of the accumulation T-Map, Delaunay triangulation for the accumulation T-Map is desired. The philosophy behind the above mentioned process and why it is required theoretically has been mentioned in the earlier paragraphs. Finally ray tracing algorithm will exactly identify which points out of the entire point cloud are inside the boundary of the accumulation T-Map.

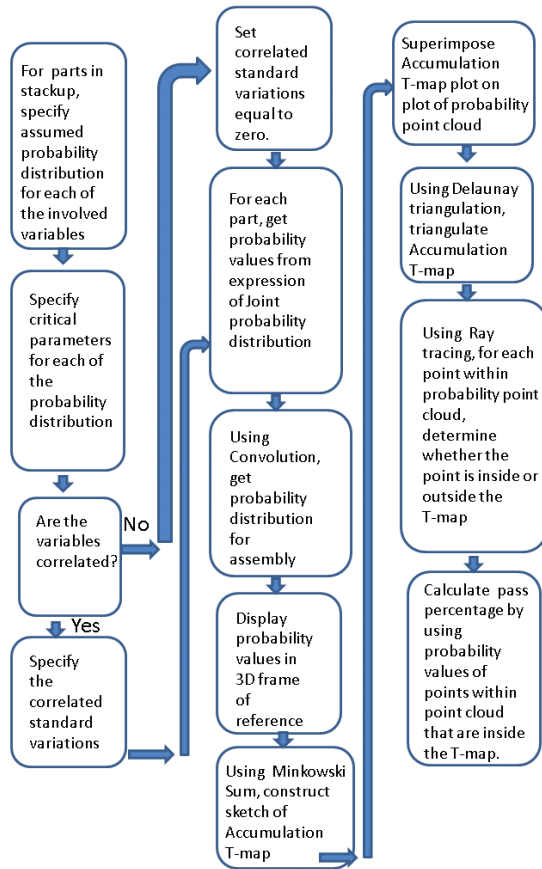


Figure 9-12 Algorithm for the code developed for determining the pass percentages for the assembly whose calculated statistical probability distribution is a convolution of the part statistical distributions.

After the points have been sorted out to which are in and which are outside the accumulation T-Map, percentage values of (not the number of points) the aggregated probability values will determine the statistical pass percentage for the points that will be functionally fit. It needs to be reemphasized that convolution of part distribution functions to arrive at the assembly distribution functions will only work if the involved functions are independent. The software has the flexibility for catering for dependent functions and variables but in order to arrive

at the assembly distribution function in such cases, certain changes to the code will need to be made to make it mathematically and logically viable.

9.12 Extraction of assembleability percentages for stack up of two parts

For each of the two parts involved in the stack up as shown in figure 9-11, there will be a separate T-Map similar to the one shown in figure 3-9. It should be recalled that these T-Maps are for size tolerance specified on rectangular face. When the two parts are involved in a stack up, then Minkowski sum is used to arrive at the accumulated T-Map based upon the individual T-Maps. The details of Minkowski sum have been covered in the previous section. Accumulated T-Map is the result of the Minkowski sum followed by the convex hull. The details are shown in the figure below.

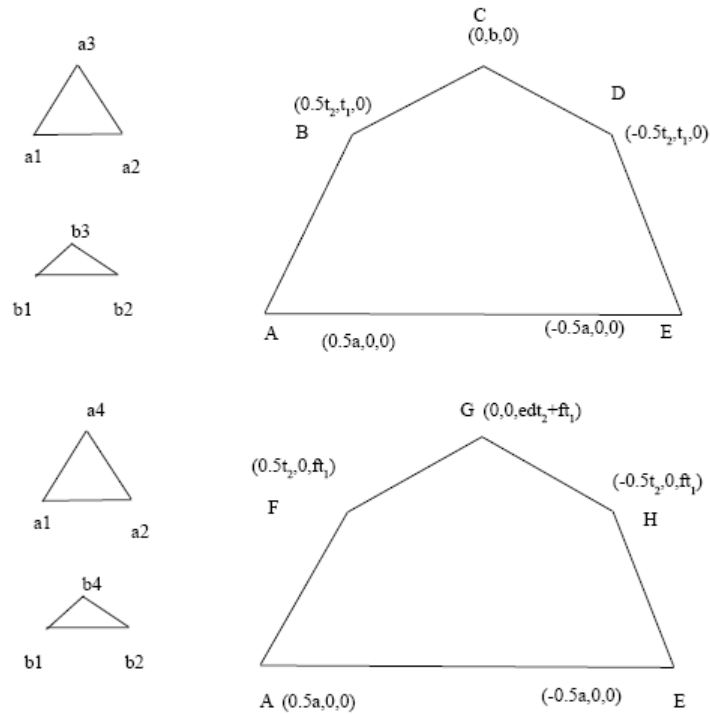


Figure 9-13 (top) Half of the conformable $q`$ s sections of the T-maps (on the left) with its associated half section of the accumulation T-map (on the right) for parts in stack up in fig 9-11. (Bottom) Half of the conformable $p`$ s sections of the T-maps (on the left) with its associated half section of the accumulation T-map (on the right) for parts in stack up in fig 9-11 .

The detail of the figure above is given in the following paragraph. $\Delta a_1 a_2 a_3$ is the half of the conformable $q`$ s-sections of the T-Map for part 1 while $\Delta b_1 b_2 b_3$ is the half of the conformable $q`$ s-sections of the T-Map for part 2. The Minkowski sum followed by the convex hull of these triangles gives the half section of the accumulation map which is given by the polygon with vertices ABCDE. The vertices of the polygon are given in the three dimensional coordinate system for the three variables of interest, i.e. size tolerance, plane tilt along x-axis and plane

tilt along y-axis. In the half section of the accumulation T-Map for q's -section, $a = t_1 + t_2$ while $b = t_1 + d t_2$ whereas $d = (d_{1y}/d_{2y})$. In a similar manner, the bottom three polygons refer to the p's -section. $\Delta a_1 a_2 a_4$ is the half of the conformable p's-sections of the T-Map for part 1 while $\Delta b_1 b_2 b_4$ is the half of the conformable p's-sections of the T-Map for part 2. The Minkowski sum followed by the convex hull of these triangles gives the half section of the accumulation map which is given by the polygon with vertices AFGHE. The vertices of the polygon are given in the three dimensional coordinate system for the three variables of interest, i.e. size tolerance, plane tilt along x-axis and plane tilt along y-axis. Here, $e = (d_{1y}/d_{2x})$ and $f = (d_{1y}/d_{1x})$. The determination of the vertices of the half sections of the accumulation T-Map is necessarily required for input to the code developed.

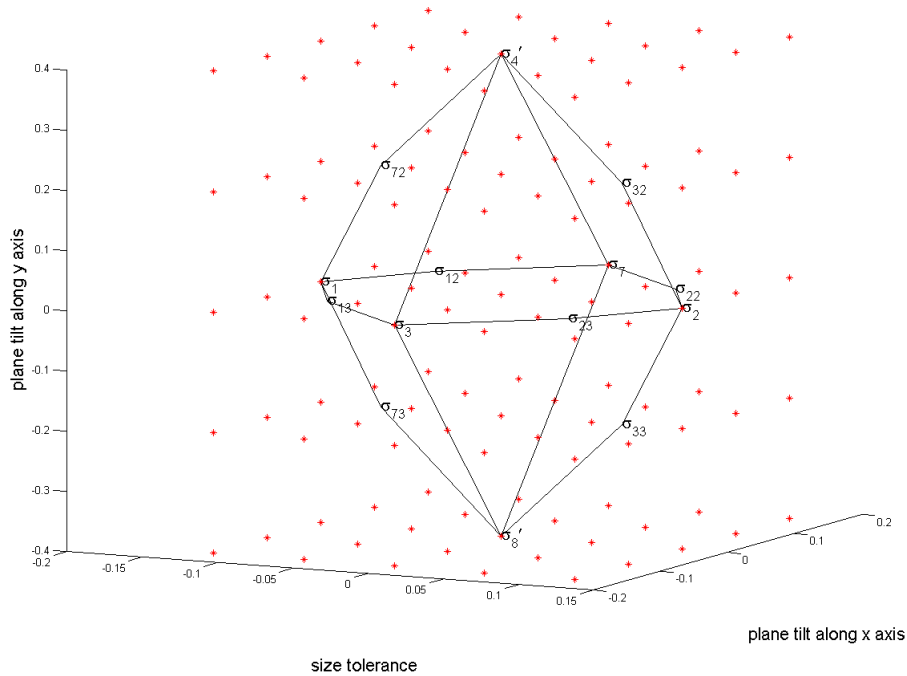


Figure 9-14 Assembly Probability plot resulting from the convolution of part probability distribution achieved for Minkowski sum of part t-map for size tolerance specified on a rectangular face

As can be seen from the figure above, the accumulation map which is the result of the Minkowski sum of the individual part T-Maps has additional vertices in addition to a larger enclosed volume. These additional vertices arise due to the conformance of the larger T-Map as per the ratios of the sides of two parts $d = (d_{1y}/d_{2y})$. Also, it needs to be mentioned that for all values of the variables, the constraint $d_y > d_x$ has been maintained. Also, the values are calculated for almost 69000 points. However, the plots have been generated with lesser dense point cloud for visualization purposes. For a specific case of part 1; size tolerance varying from -0.4 to +0.4, plane tilt along x axis varying from -0.06 to 0.06, and plane tilt along y axis varying from -0.15 to 0.15, and for part 2; size tolerance

varying from -0.08 to 0.08, plane tilt along x axis varying from -0.10 to 0.10 and plane tilt along y axis varying from -0.25 to 0.25, the pass percentage for the assembly distribution as arrived from the convolution of the two part distributions estimated based upon the points within the accumulation T-Map arrived from the Minkowski sum is 85.72%. For all other values remaining same, and changing the mean size tolerance by one hundredth of the unit (e.g. mm) for part 2, gives us a pass percentage of 85.42%. Similarly, if all other values remain same while the standard deviation for size tolerance is increased by 10 times, gives us a pass percentage of 84.39%.

Until now, some initial results have been found by use of the above techniques. Future work may involve evaluating the pass percentages for orientation tolerances specified on the two parts along with multiple orientation tolerances using different datums and evaluation of the effect of the form tolerances on the parts for assembly pass percentages.

9.13 Conclusions

A new statistical model for non-linear tolerance analysis is presented in this research. This model has the flexibility of catering to joint probability distribution of any dimension. In this chapter, the model has been applied to trivariate normal statistical distribution. The model is not limited to any particular type of statistical distribution. Even, it is not limited to use of standard distributions only. A test case has been adopted from the ASU Tolerance Maps model to judge the pass percentages of the manufactured parts. Other means of judging the pass

percentages could also be employed. The three variables of interest in the tolerance maps are not independent. However, the method proposed in this publication, in its current state, can be used for independent variables also. The model is capable of not only aiding in the determination of the pass percentages but a reverse approach could also be employed such as for a given pass percentages, what are the characteristics of the statistical distribution that should be maintained during the manufacturing process. The model has been used to arrive at the pass percentage of parts in stack up using convolution of the part distributions and accumulation T-Maps arrived at Minkowski sums of the part T-Maps. The model uses Delaunay triangulation and ray tracing algorithms to determine the percentage of points within the point cloud that are inside a certain volume. The model, as presented, could be used for determining pass percentages even for non-regular or curved volumes. The process of triangulation could become very cumbersome when the volume for determining the pass percentage is irregular. The method presented and the code developed in this research could be directly used for identifying the points (generated from a point cloud using a Coordinate Measuring Machine) which are inside a Tolerance Map (T-map) or Inspection Map (i-Map).

10 CLOSURE

This research has presented new approaches in manufacturing tolerance transfer and statistical tolerance analysis. The non inline three dimensional model for tolerance transfer caters for all geometric tolerances. It is capable of predicting manufacturing tolerances in highly advanced manufacturing environments such as five axis milling. In other lesser advanced machines, as well, it is always the desire of the manufacturing supervisors that the work piece be not removed from the machine if at all possible. This model is based upon Coordinate Measuring Machine readings and hence can be used to check for tolerances, no matter what be the orientation of the work piece is.

Graph theoretic approach presented in this research is not totally new but once based upon the mathematical model, it will highly efficient to translate a three dimensional variation into its projection along the designated axes. This approach is novel to the manufacturing tolerance transfer field in particular. Use of the geometric kernel ACIS in a C++ Object Oriented environment will be a building block for this approach. Use of Microsoft Foundation Class will greatly expedite the implementation process for this technique.

The approach to determining the manufacturing tolerances in a reverse order in the manufacturing process plan is based upon determining the mathematical relationships and is more easily adaptable to the three dimensional geometric tolerances.

The new statistical model for tolerance analysis uses the entire statistical distribution and not only the moments of the statistical distribution. Additionally,

the use of the concept of joint probability for the multivariate distribution is totally new for mechanical tolerance analysis purposes. This model is further raised to higher level by incorporating the convolution of the multivariate distributions to arrive at the statistical distribution of the assemblies. Graphical presentation of the point cloud in three dimensional space is highly efficient in determining the pass percentages of the parts as well as assemblies.

This research can be extended to various types of probability distributions such as Weibull, Rayleigh etc will be considered. This is closer to the actual state of manufacturing and hence, the results obtained will be more reliable. The success rates of the assembly will be more accurately forecasted and will eventually result into lesser failure rates. This will cause lesser scrap, rework and waste of efforts which will eventually drive the costs lower. The ultimate result will be greater quality at reduced cost.

This task of the research is totally new and immensely challenging. None of the researchers in the field of tolerancing have pursued the proposition to the extent as mentioned here. The results of this research will be highly valuable for industries who sublet lot of parts for manufacturing to smaller and more focused manufacturers (focus from the point of manufacturing only). These industries normally assemble these parts after receiving them from the vendors. Such an analysis as suggested in this research task will aid the managers and top executive bosses to make small changes in the tolerancing area to achieve smaller material waste and item returns.

11 REFERENCES

1. Richard J. Gerth, 'Tolerance analysis: a tutorial of current practice', 'Advanced Tolerancing Techniques', edited by Hong-Chao Zhang, 1997, John Wiley & Sons, inc.
2. ASME Standard, 2009, "Dimensioning and Tolerancing ", ASME Y14.5 M-2009, American Society Of Mechanical Engineers, New York.
3. Wade, O., 'Handbook of Mechanical Engineer'.
4. Hillyard, R. C., and Braid, I. C., 1978, 'Analysis of dimensions and tolerances in computer-aided mechanical design', *Computer-Aided Design*, 10(3):161–166.
5. Hillyard R.C. & Braid I.C., 'Characterizing non-ideal shapes in terms of dimensions and tolerances', *International Conference on Computer Graphics and Interactive Techniques, Proceedings of the 5th annual conference on Computer graphics and interactive techniques*, 1978, Pages: 234 - 238
6. Lin, V. C., Gossard, D. C., and Light, R. A., 1981, 'Variational geometry in computer-aided design', *Proc. Of Siggraph, Comp. Gr.* 15(3):171–177.
7. Light, R. A., and Gossard, D. C., 'Modification of geometric models through variational geometry', *Computer-Aided Design*, 14(4):209–214.
8. Gossard, D. C., Zuffante, R. P. and Sakurai, H., 'Representing dimensions, tolerances, and features in MCAE systems', *IEEE Comp. Gr. and Appl.* 8(2):51–59.
9. Suvajit Gupta, Joshua U. Turner, 'Variational Solid Modeling for Tolerance Analysis', *IEEE Computer Graphics and Applications*, Vol. 13, No. 3, pp. 64-74, May/June, 1993
10. Turner, J.U., 1990, 'Exploiting solid models for tolerance computations', in *Geometric Modeling for product Engineering*, (M.J Wozny, J.U. Turner and K.Preiss, eds.), pp. 237- 258, North Holland
11. Roy U, Lib U, 'Representation and interpretation of geometric tolerances for polyhedral objects. II. Size, orientation and position tolerances', *Computer-aided design*, vol. 31, no 4, pp. 273-285, April, 1999.

12. Hong-Tzong Yau, 'A model-based approach to form tolerance evaluation using non-uniform rational B-splines', *Robotics and Computer-Integrated Manufacturing* Volume 15, Issue 4, August 1999, Pages 283-295
13. Aristides A. G. Requicha, 'Toward a Theory of Geometric Tolerancing', *The International Journal of Robotics Research*, Vol. 2, No. 4, 45-60 (1983)
14. Requicha, A. Chan, S. 'Representation of geometric features, tolerances, and attributes in solid modelers based on constructive geometry', *IEEE Journal of Robotics and Automation*, 1986, Volume: 2, Issue: 3, pp 156- 166.
15. Jayaraman, R., and Srinivasan, V., 1989, 'Geometrical Tolerancing 1: Virtual Boundary Conditions', *IBM J. Res. Dev.*, 33(2), pp. 90–104.
16. Srinivasan, V., Jayaraman, R., 1989, 'Geometrical Tolerancing 2: Conditional Tolerances', *IBM J. Res. Dev.*, 33(2), pp. 105–124
17. Kramer, G.A., 1992, 'Solving Geometric Constraint System: A Case Study in Kinematics', MIT Press.
18. Bernstein, N., and Preiss, K., 1989, 'Representation of tolerance information in solid models', DE-Vol.
19. Clément A., Desrochers A., Riviere A., 1991, 'Theory and Practice of 3-D tolerancing for assembly', *Proceedings, Second CIRP seminar on Comp. aided Tolerancing*, Penn State.
20. Desrochers, A. Clement, A. *INTERNATIONAL JOURNAL OF ADVANCED MANUFACTURING TECHNOLOGY*, 1994, VOL 9; NUMBER 6, pages 352
21. D.Gaunet, '3-D functional tolerancing & annotation: CATIA tools for Geometrical product', *Selected Conference Papers for the 7th CIRP International Seminar on Computer –Aided Tolerancing*, France, April 2001.
22. O. W. Salomons, F. J. Haalboom, H. J. Jonge Poerink, F. van Slooten, F. J. A. M. van Houten and H. J. J. Kal, *Computers in Industry* Volume 31, Issue 2, 1 November 1996, Pages 175-186
23. B. Anselmetti, K. Mawussi, 'Tolérancement fonctionnel d'un mécanisme : identification de laboucle de contacts', *IDMME*, May 14-16 2002 Clermont-Ferrand, France.

24. International Organization For Standardization ISO 1101, 1983, "Geometric Tolerancing _Tolerancing Of Form, Orientation, Location And Run-Out __Generalities, Definitions, Symbols And Indications On Drawings.
25. A Desrochers, S Verheul – 'Global Consistency of Tolerances', Proceedings of the 6th CIRP, 1999
26. Louis Rivest, Clement Fortin, Claude Morel, 'Tolerancing a solid model with a kinematic formulation', Computer-Aided Design, Volume 26, Issue 6, June 1994, Pages 465-476.
27. Leo Joskowicz, Elisha Sacks, and Vijay Srinivasan, 'Kinematic tolerance analysis', Computer-Aided Design, Volume 29, Issue 2, February 1997, Pages 147-157.
28. J Gao, KW Chase, SP Magleby, 'Generalized 3-D tolerance analysis of mechanical assemblies with small kinematic adjustments', IIE Transactions, 1998 – Springer
29. Min-Ho Kyung, Elisha Sacks, 'Nonlinear kinematic tolerance analysis of planar mechanical systems', Computer-Aided Design, Volume 35, Issue 10, September 2003, Pages 901-911
30. Laperriere and Lafond, 'Tolerance analysis and synthesis using virtual joints', 6th CIRP International Seminar on computer Aided Tolerancing, 24-25 April, 2001
31. Wirtz, A., 1989, 'Vectorial Tolerancing', International Conference on CAD/CAM and AMT, CIRP Session on Tolerancing for Function in a CAD/CAM Environment, Vol. 2, Israel, Dec. 11-14.
32. G Henzold - Proceedings of the 1993 International Forum on Dimensional ..., 1993 - American Society of Mechanical Engineers
33. Martinsen, K., 'Vectorial tolerancing for all types of surfaces', In Proceedings of 19th ASME Design Automation Conference, Albuquerque, 1993, Vol. 2 (ASME Press).
34. Krimmel and Martinsen, 'Industrial application of Vectorial Tolerancing to improve clamping of forged work pieces in machining', Proceedings of 6th International CIRP seminar, 1999.
35. Bialas, 'Transformation of Geometrical Dimensioning and Tolerancing (GD&T) into Vectorial Dimensioning and Tolerancing (VD&T)', ISO Document, 1997.

36. Bialas, Humienny and Kiszka, 'Relations between ISO 1101 Geometrical Tolerances and Vectorial Tolerances – Conversion Problems', Proceedings of 5th CIRP International Seminars on Computer Aided Tolerancing, 1997, pp 37-48.
37. Desrochers 'A matrix approach to the representation of tolerance zones and clearances', International journal of advanced manufacturing technology, yr:1997 vol:13 iss:9 pg:630
38. Desrochers , 'Application of a Unified Jacobian—Torsor Model for Tolerance Analysis', Journal of Computing and Information Science in Engineering -- March 2003 -- Volume 3, Issue 1, pp. 2-14
39. Shah, J. J., and Zhang, B., 1992, 'Attributed graph model for geometric tolerancing', Proc. of 18th ASME Design Automation Conf., Scottsdale, ASME Press, pp. 133–139.
40. Kandikjian T., Shah, J.J, and Davidson, J.K, 'A mechanism for validating dimensioning and tolerancing schemes in CAD systems', Computer-Aided Design, 33, 721-737, 2001.
41. Z Zou, EP Morse, 'Assembleability Analysis Using Gap Space Model for 2-D Mechanical Assembly', Proceedings of the 7th Design for Manufacturing Conference, 2002 - coe.uncc.edu
42. Turner J.U.,Wozny M, 'A Mathematical Theory of Tolerances", in Geometric Modeling for CAD Applications', Wozny, McLaughlin, Encarnacao (eds), Elsevier Publ., 1988.
43. Turner, J. U., and Wozny, M. J., 1990, 'The M-space theory of tolerances,' in B. Ravani, ed., Proc. of 16th ASME Design Automation Conf., ASME Press, pp. 217–225.
44. Ameta, G, 'PhD thesis', Arizona State University
45. Shen, Z, 'PhD thesis', Arizona State University
46. Mujezinovic, A, 'M.S. thesis', Arizona State University
47. Wu, Y, 'PhD thesis', Arizona State University
48. Mujezinovic, A., Davidson, J., and Shah, J., 'A new mathematical model for geometric tolerances as applied to round faces', Journal of Mechanical Design, December 2002, volume 124, Issue 4, 609(14 pages)

49. Mujezinovic, A., Davidson, J., and Shah, J., 'A new mathematical model for geometric tolerances as applied to polygonal faces', *Journal of Mechanical Design*, May 2004, volume 126, Issue 3, 504(15 pages)
50. Swami and Turner, 'Review of statistical approaches to tolerance analysis', *Computer Aided Design* Volume 27 Number 1 January 1995
51. Evans, 'Statistical tolerancing: The state of the art, part I: Background', *Journal of Quality Technology*, 1974, VOL 6, issue 4, pg 188
52. Crofts, J 'Assembly tolerance problem', *Engineer*, Vol 206, (Dec 1958), pp 918-922.
53. Chase. K W and Greenwood, W H 'Design issues in mechanical tolerance analysis', *Manufacturing Rev.*, Vol 1, No 1, (1988), pp 50-59.
54. Stefano, 2003, 'Tolerance analysis and synthesis using the mean shift model', *Journal of mechanical engineering science*, 217: 149-159
55. Spotts, M F 'An application of statistics to the dimensioning of machine parts' *J. Eng. Industry*, (Nov 1959), pp 317-322.
56. Bender Jr, A 'Statistical tolerancing as it related to quality control and the designer', *SAE Transactions*, Vol 77, (May 1968), pp 1965-1971.
57. Mansoor, E M 'The application of probability to tolerances used in engineering designs', *Proc. Inst. Mech. Eng.*, Vol 178, No 1, pp 29-51.
58. Gladman, C A 'Applying probability in tolerance technology', *Trans. Inst. Eng. Australia Mech. Eng.*, Vol ME5 No 2, (1980), pp 82-88.
59. Desmond, D. J. and Setty, C. A., 1962, 'Simplification of selective assembly', *International Journal of Production Research*, 1(3), 3± 18.
60. Parkinson, D B 'The application of reliability methods to tolerancing', *Trans. ASME 1. Mech. Des.*, Vol 104, No 3, (1982), pp 612-618.
61. D'Errico, J R and Zaino Jr, N A 'Statistical tolerancing using a modification of Taguchi's method', *Technometrics*, Vol 30, No 4, (1988), pp 397-405
62. Hasofer, A. M., and Lind, N. (1974), "An exact and invariant first-order reliability format." *J. Engg. Mech.*, ASCE, 100(1), 111-121.
63. Evans. D H, 'An application of numerical integration techniques to statistical tolerancing', *Technometrics*, Vol 9, No 3, (1967), pp 441-456.

64. David H. Evans, 'An Application of Numerical Integration Techniques to Statistical Tolerancing, II: A Note on the Error', *Technometrics*, Vol. 13, No. 2 (May, 1971), pp. 315-324
65. Evans, D H 'An application of numerical integration techniques to statistical tolerancing. Part III: General distributions', *Technometrics*, Vol 14, No I, (1972), pp 23-35
66. Shen, Z., 'Software Review – Tolerance Analysis with EDS/VisVSA', *Journal of Computing and Information Science in Engineering*, 3(1), pp 95-99
67. BJORKE, O., 1989, "Computer Aided Tolerancing", ASME Press, New York, NY.
68. O'Connor M., Srinivasan V., 1997, "Composing Distribution Function Zones For Statistical Tolerance Analysis", *Proceedings Of 5th CIRP International Seminar On Computer Aided Tolerancing (CAT)*
69. Grossman D., 1976, "Monte Carlo Simulation Of Tolerancing In Discrete Parts Manufacturing And Assembly", *Research Report, STAN-CS-76-555*, Computer Science Department, Stanford University, Stanford, CA.
70. VisVSA Solutions Training Manual, Version 1.3
71. VisVSA online help, version 4.0
72. Whitney D., Gilbert O., Jastrzebski, M., 1994, "Representation Of Geometric Variations Using Matrix Transforms For Statistical Tolerance Analysis In Assemblies, ", *Res. Eng. Des.*, 6, Pp. 191-210
73. Lee S., And Yi C., 1998, "Statistical Representation And Computation Of Tolerance And Clearance For Assembleability Evaluation", *Robotica*, 16, Pp. 251-264
74. Ameta G., Davidson J. And Shah J., 2010, "Influence Of Form on Tolerance-Map-Generated Frequency Distributions for 1D Clearance In Design", *Precision Engineering*, Volume 34, Issue 1, Pp. 22 -27.
75. Ameta G., Davidson J. And Shah J., 2007, "Using Tolerance –Maps To Generate Frequency Distributions Of Clearance And Allocate Tolerances For Pin-Hole Assemblies", *Journal Of Computing And Information Science In Engineering*
76. Song-Lin L., Jing-Zheng S., Et Al., 2010, "A Novel Strategy To Rapidly Explore Potential Chemical Markers For The Discrimination Between Raw And

Processes Radix Rehmanniae By UHPLC_TOFMS With Multivariate Statistical Analysis”, Journal Of Pharmaceutical And Biomedical Analysis.

77. Koksall G. And Fathi Y., 1998, “Design Of Economical Noise Array Experiments for A Partially Controlled Simulation Environment”, Computers and Industrial Engineering.

78. Choudhary A., 2006, “A Statistical Tolerancing Approach for Design of Synchronized Supply Chains”, Robotics And Computer-Integrated Manufacturing.

79. Gonzalez I., And Sanchez I., 2009, “Statistical Tolerance Synthesis with Correlated Variables”, Mechanism And Machine Theory.

80. Dantan J., And Qureshi A., “Worst Case and Statistical Tolerance Analysis Bases On Quantified Constraint Satisfaction Problems and Monte Carlo Simulation”, Computer Aided-Design.

81. Bruyere J., Et Al., 2007, “Statistical Tolerance Analysis of Bevel Gear by Tooth Contact Analysis And Monte Carlo Simulation”, Mechanism And Machine Theory.

82. Ramaswami H., Acharya S. Et Al., “2006, “A Multivariate Statistical Analysis of Sampling Uncertainties In Geometric And Dimensional Errors For Circular Features”, Journal Of Manufacturing Systems.

83. Bracewell R., 1965, "Convolution" and "Two-Dimensional Convolution.", Ch. 3 in “The Fourier Transform and Its Applications”, New York: McGraw-Hill, pp. 25-50 and 243-244.

84. Hirschman I., and Widder D., 1955, “The Convolution Transform”, Princeton, NJ: Princeton University Press.

85. Wu, Y., Shah, J., Davidson, J., ‘Rationalization and computer modeling of GD&T classes’, proceedings of ASME DETC 2002, Montreal Canada.

86. Shen, Z., Shah, J., Davidson, J., ‘Automation of linear tolerance charts and extension to statistical tolerance analysis’, ASME CIE conference.

87. Chiesi, F., Governi, L., ‘Tolerance Analysis with eM-TolMate’, Journal of Computing and Information Science in Engineering, Vol 3(1), pp 100-105.

88. <http://www.3ds.com/products/catia>.

89. http://www.plm.automation.siemens.com/en_us/products/nx/

90. <http://www.ptc.com/products/proengineer/>

91. http://www.plm.automation.siemens.com/en_us/products/nx/

APPENDIX A
COMPUTER AIDED TOLERANCING

A.1 Software based tolerance analysis

These days, there are many commercial and non-commercial software packages that are available to ease out the process of tolerance design practice.

A.2 Evaluation criteria for commercial tolerancing software

The different characteristics that can be used for comparing the commercially available tolerancing software are listed below:

1. Ease of Use

The software should be easy to use and user friendly.

2. Compatibility with CAD

The software should work directly with the CAD geometry.

3. Number of inputs required

The number of additional inputs for tolerance analysis should be minimal after the CAD geometry has been realized within the software.

4. 3-D capability

In addition to 1-D and 2-D tolerances, it will be nice to see how the software handles 3-D problems.

5. Geometric Tolerance handling

In addition to plus and minus tolerances, the software should handle geometric tolerances as well. Ideally, ASME Y14.5 should be fully applicable and implementable within the software.

6. Restriction on number of tolerances specified

There should not be any limit on the maximum or minimum number of tolerances specified.

7. Efficiency in modeling

The software should efficiently model all assembly level relationships.

8. Assumptions made

The software should not make too much simplifying assumptions that can ruin the analysis and result in inaccurate results.

A.3 Commercial Tolerance Analysis Software Packages

The following is the list of Tolerance Analysis capable packages that have been around for some time:-

- 1) I-Deas.(Geometric and Dimensional parametric analysis; linearized and non -linear 2-D only), Pro/Engineer, 2) VSA (3-D Monte Carlo), 3) Mechanical Advantage), 4) Analytix), 5) Design View, 6) Mechanical Engineering Workbench, 7) VarTran, 8) CeTol six sigma, 9) ToleranceCalc, 10) MechTOL, 11)VALISYS or eM-TolMate, 12) MITCalc, 13) Tolerance Stackup software toolset from Advanced Dimensional Management LLC, 14) CRYSTAL BALL, 15) 3-DCS and 1-DCS, 16) TOLTECH, 17) TOLCON, 18) CATS, 19) CATIA 3-D FDT (Functional Dimensioning and Tolerancing), 20) Unigraphics/ Quick Stack.

The detail about the capabilities of some of these softwares (major ones only) is discussed in the following paragraphs.

A.4 A review of Software packages for Tolerance Analysis

There are two key aspects of CAT systems: how the mathematical model is created, and how it is solved. VisVSA follows the parametric approach in building models of geometric variations. Also, VisVSA imports geometry from CAD system [70, 71] via its own proprietary file format *.jt. Neither Geometric Dimensioning & Tolerancing information nor mating conditions can be transferred from CAD files into *.jt files. Also, constraint information is lost while importing.

VisVSA uses an abstraction of the geometry and selected dimensions and tolerances for analysis. For example, a cylinder is represented by 'point (for center) + vector (for axis) + radius (dimension) + length (dimension). Similarly, a planar feature is represented by 'point (for a point on the plane) + vector (for normal to the plane) + width (dimension) + length (dimension). These abstracted objects are termed 'features' by VisVSA. The various features supported by VisVSA are point, plane, pin, hole, tab, and slot. The measurements involved are point to point, point to line, point to plane, Gap/Flush, angle, Maximum or Minimum virtual clearance.

The type of results that VisVSA output include statistical distribution (nominal, mean, standard deviation), contributors and corresponding contribution percentage, etc. VisVSA predicts the amount of variation on the basis for Monte

Carlo simulation. Statistical distributions available are normal or Gaussian (default), uniform, extreme or Pearson distribution.

VisVSA handles geometric tolerances by actually moving/deforming a feature according to tolerances specified with the help of geometric solver. So if a point is defined on pin surface and that pin has a size and location tolerance, then VisVSA will actually vary size and location of that pin (within the bounds of tolerances), and determine (via its geometric solver) where the user-defined point lies in model space for that particular simulation.

Tolerance validation involves the following items [85]:-

1. The tolerance specified should commensurate with the target entity type.
2. A DRF should be capable of controlling the desired variation.
3. The datum members in a datum reference frame (DRF) should have the correct entity type.
4. A material modifier can only be specified to a feature of size for a straightness tolerance, a positional tolerance, an angularity tolerance, a parallelism tolerance, or a perpendicularity tolerance.
5. Tolerance refinement relation must be maintained for the tolerances specified on the same target entity e.g. size tolerance > location tolerance > orientation tolerance > form tolerance.

VisVSA provides support for validating the completeness, appropriateness, legality of tolerance specifications etc. It does check for tolerance refinement relationship but it does not check for DRF validation. Some ASME tolerances classes are not available in VisVSA such as concentricity tolerance, or composite

tolerance, or datum target points, or pattern tolerance. To incorporate dimensional and geometric tolerances, VisVSA applies transformations to theoretically perfect feature within the zones which are simulated by putting limits on rotation and translation parts of the transformation matrix based on feature dimensions and tolerance values. Additionally, bonus and shift tolerances are properly taken care of by VisVSA.

VisVSA depends on the constraint solver Conjoin [86] for aligning 3-D parts and modeling assembly operations. However it is important that actual assembly sequence should be used when building up the model. If the parts don't arrive in proper locations on import, then VisMockup 3-D alignment should be used to align correctly. However, when specifying alignment constraints, VisVSA does not give explicit feedback regarding the constraint conditions, over constraint/under-constraint condition or constraint/unconstraint degree of freedom.

To conclude, VisVSA permits the user to develop a 3-D procedural point model by defining one point at a time. It has been integrated with most CAD systems. However VisVSA performs some validation but not completely. In VisVSA, all tolerances are classified into four classes i.e. size, location, orientation and form. However, only one tolerance from amongst the tolerances belonging to the same tolerance class can be applied to one target.

VALISYS (or eM-TolMate) [87] is another commercial computer-aided tolerance analysis tool which is embedded in four major CAD systems, CATIA[88], UG[89], Pro/E[90], and SDRC[91]. The basic features supported are plane, pin

(cylindrical, tapered, and threaded), hole (cylindrical, tapered, and threaded), point, tab, slot, constant profile surface, constant cross section, sphere, surface of revolution, general 3-D surface etc. Edge features for thin walled parts is also available. Also derived features such as line of intersection between a plane and parallel cylinder, centroid of several points, or best fit line between several points etc.

In VALISYS, there are three proprietary parametric constraint solvers: Least Square Method, Datum Method and High Point Method. The User is unable to choose, instead, the system chooses on the basis of the peculiarity of the problem, (e.g. isostatic or hyper static constraint scheme). The discussion of the type of constraint scheme is beyond the scope of this research.

Various types of measurements are supported in VALISYS such as linear distance, angle, clearance, virtual size and also user –defined measurement. VALISYS features an internal programming language, the VCL (Valisys Control Language), to create user –defined assembly operations or measurements).

The inferred limits of each measurement can be set to be based on one of the following statistical estimate: Normal (upper and lower limits of variation symmetrically inferred from simulated mean against a desired interval of confidence of a Gaussian distribution fitting the simulated histogram), Pearson (upper and lower limits inferred based on a Pearson distribution fitting the simulated histogram) or Actual (the lowest and highest number simulated by VALISYS, no statistical extrapolation is made).

In VALISYS, the system warns the user about any lack of completeness or ambiguity in tolerancing scheme (e.g. loop, unreferenced datum etc). There are two kinds of results: the variation analysis, which computes statistical parameters and reports the overall variation range of each measurement, and the contributor analysis, which determines the sources of variation and present this information in a sorted list. With this information the user can conduct comparative ‘what –if’ studies, optimize tolerances and assembly methods and eliminate costly ‘trial and error’ studies on the shop floor.

3-DCSTM – is a tolerance simulation tool that permits modeling of the effect of variations on an assembly and testing of alternative tolerancing. During Tolerance analysis using Monte Carlo simulation, the user has the option of selecting from the following distribution types: Normal, Weibull, Uniform or User supplied. The software is capable of Pareto Analysis to identify the critical features and the sensitivity analysis as well. However, the software does not take care of geometric tolerances fully.

Mechanical Advantage and the Analytix are declarative model based tolerance analysis software packages and they perform the tolerance analysis by varying the individual dimensions as input. Both packages support the use of normal distributions only for their Linearized statistical analysis. Mechanical Advantage uses the Newton Raphson iterative solver for the solution of the constraint equations. Analytix on the other hand, solves the equations analytically, a few at a time, by deriving a sequence of construction operations for computing the geometry. This procedure has proved more robust than the iterative solver.

However, both these packages use zero default tolerances when default tolerances are not specified. This can cause significant sources of variation that can be overlooked.

CATS (Computer Aided Tolerancing System) developed by ADCATS (Association for Development of computer Aided Tolerance System) at Brigham Young University (BYU) and Texas Instruments (TI) carries out both tolerance analysis and tolerance synthesis. Actually, most of the tolerance analysis softwares can be customized to give tolerance synthesis results by the user itself.

CE/TOL models the assembly mating relationships with kinematic joints. A vector loop is detected and a transformation matrix of small displacements is constructed for tolerance analysis. Certain GD&T validations have been implemented such as validating a DRF and the type of tolerance zone. Geometric tolerances are accounted by means of zero length vectors \pm tolerance whose orientations depend on the type of kinematic joint, introduced into the vector loop. Three types of tolerance analysis are available: Worst Case (WC), Root Sum Square (RSS) and Motorola Six Sigma. No option for automatic tolerancing is available. It also does not have an automated optimization method. It uses a set of weight factors, which the user chooses manually for every component tolerance. Later, the inbuilt tolerance synthesis algorithm automatically redistributes the tolerances according to the selected weight factors.

CATIA 3-D FDT (Functional Dimensioning & Tolerancing) is based upon the TTRS (Topologically and Technologically Related Surfaces) model. It provides for automatic tolerancing but this option is not available when tolerancing within

a part. Based upon the seven classes of elementary surfaces, all possible associations have been analyzed which reveals 28 cases of surface association and 44 cases of tolerancing. Thus there are a finite number of tolerance cases, and the model can provide a tolerancing scheme for each type for surface association.

This system is capable of only doing worst case analysis. Upon specification of the tolerances, the software automatically creates the equation system and solves it, with respect to the constraints. The results can be the min max of a stack dimension or the feasibility of an assembly. For inspection, Dassault Systems have developed a partnership program called Component Application Architecture (CAA) V5. [88]

Unigraphics / Quick Stack is a simple tolerance stack up analysis tool providing the minimum and maximum variation in an assembly and identifying key contributions to out of tolerance conditions. Maple has the capability to do 1-D worst case tolerance analysis.

After going through the relevant literature, it has become evident that none of the above mentioned tolerancing softwares do any type of tolerance transfer. Also, it is unknown if any of these softwares do statistical tolerance analysis with multivariate distributions.

BIOGRAPHICAL SKETCH

Nadeem Shafi Khan was born in Karachi, Pakistan. He attended Saint Mary's Cambridge School, Rawalpindi from 1972-1983 and attended Sir Syed College, Rawalpindi till 1985, when he joined Pakistan Air Force College, Sargodha for commission in Aeronautical Engineering branch. He joined Pakistan Air Force Academy in December 1986, from where he graduated as Flying Officer with Bachelor in Engineering degree with honors (in Aerospace from NED University of Engineering & Technology, Karachi) in 1990. He has served in various appointments on different weapon systems such as American F-16, French Mirages, Chinese F-7 and F-7P. He completed several professional courses on Pratt & Whitney F100 engine. He completed his Masters in Business Administration (MBA) from AIO University, Pakistan in 2002. He did his masters in Aerospace Engineering (Aero Structures) in 2003 from National University of Science & Technology, Pakistan. He did another Masters in Aerospace Engineering (Design) from Georgia Institute of Technology, Atlanta, GA. He came to Arizona State University in fall 2007. He is currently a serving Squadron Leader (Major) in Pakistan Air Force. He is a student member of American Society of Mechanical Engineers.