Capacity Planning, Production and Distribution Scheduling for a Multi-Facility and Multi-Product Supply Chain Network

by

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ABSTRACT

In today's rapidly changing world and competitive business environment, firms are challenged to build their production and distribution systems to provide the desired customer service at the lowest possible cost. Designing an optimal supply chain by optimizing supply chain operations and decisions is key to achieving these goals.

In this research, a capacity planning and production scheduling mathematical model for a multi-facility and multiple product supply chain network with significant capital and labor costs is first proposed. This model considers the key levers of capacity configuration at production plants namely, shifts, run rate, down periods, finished goods inventory management and overtime. It suggests a minimum cost plan for meeting medium range demand forecasts that indicates production and inventory levels at plants by time period, the associated manpower plan and outbound shipments over the planning horizon. This dissertation then investigates two model extensions: production flexibility and pricing. In the first extension, the cost and benefits of investing in production flexibility is studied. In the second extension, product pricing decisions are added to the model for demand shaping taking into account price elasticity of demand.

The research develops methodologies to optimize supply chain operations by determining the optimal capacity plan and optimal flows of products among facilities based on a nonlinear mixed integer programming formulation. For large size real life cases the problem is intractable. An alternate formulation and an iterative heuristic algorithm are proposed and tested. The performance and bounds for the heuristic are evaluated. A real life case study in the automotive industry is considered for the implementation of the proposed models. The implementation results illustrate that the proposed method provides valuable insights for assisting the decision making process in the supply chain and provides significant improvement over current practice.

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INTRODUCTION

In today's rapidly changing world and competitive business environment, firms face the challenge to constantly evaluate and build their production and distribution systems and strategies to provide the desired customer service at the lowest possible cost. Designing an optimal supply chain by optimizing supply chain operations and decisions is key to achieve these goals. Supply chains must operate such that long-term strategic objectives of the firm are met. In recent decades, research on supply chain network and operations design optimization has proven to convey significant reduction in supply chain costs and improvements in service levels by better aligning supply chain logistics flows with financial plans. Researchers have typically modeled supply chains based on the three levels of decisions which are long-term strategic, medium-term tactical, and short-term operational. Each level includes and focuses on different activities, some of them are shown in Figure 1.1.

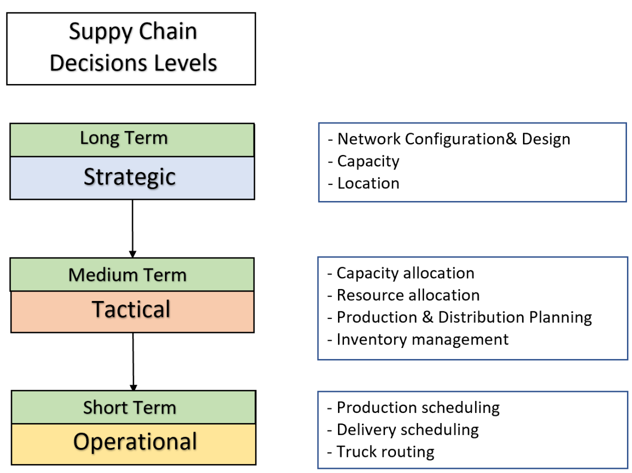


Figure 1.1: Supply Chain Decisions

Medium term tactical decisions are usually considered in the process of aggregate planning, in which overall operations of a firm including production levels, inventory levels, workforce schedule, overtime, and back ordering are optimized to satisfy fluctuating demand at a minimum cost. Those decisions are usually optimized each period of time and for each product or product family the firm produces over a medium-term planning horizon (typically 18 months). Aggregate Planning (AP) also attempts to balance capacity and demand to minimize cost. Therefore, it starts with demand and current capacity determination that is the number of units needed and the number of units that can be produced per time period respectively. Demand can be forecasted by sales deterministically or it can also be stochastic. Capacity can be composed of many components such as production rate, shifts, hours per shift and working days. Firms must decide whether to increase or decrease capacity to meet demand if the two are not in balance. On the other hand, they can also increase or decrease the demand as well to meet capacity. Firms have many options to adjust capacity such as scheduling overtime, building inventory and subcontracting. Demand can also be adjusted by varying prices, offering promotions, and by back ordering which all fall under demand smoothing techniques. The AP process includes determination of unit costs consisting of logistics cost, fixed and variable labor cost, inventory holding cost and shortage cost. Many other considerations are incorporated in AP such as organizational policies which govern the use of overtime, inventory levels and stability of workforce. Research and practice have shown that the most successful and useful mathematical approach for AP is LP/MIP which has been used in many applications. Other approaches such as control theory and simulation were also used for AP.

1.1 Research Focus

Medium term tactical decisions constitute the focus of this research. Specifically, models and solution methodologies for the capacity planning and scheduling problem are investigated in this research. First, the production scheduling for a multi-facility system producing multiple products is studied. In Figure 1.2, an example of a typical multistage supply chain is shown, where raw material is acquired from different suppliers and transported to multiple production facilities to produce various products to satisfy the demand of multiple markets.

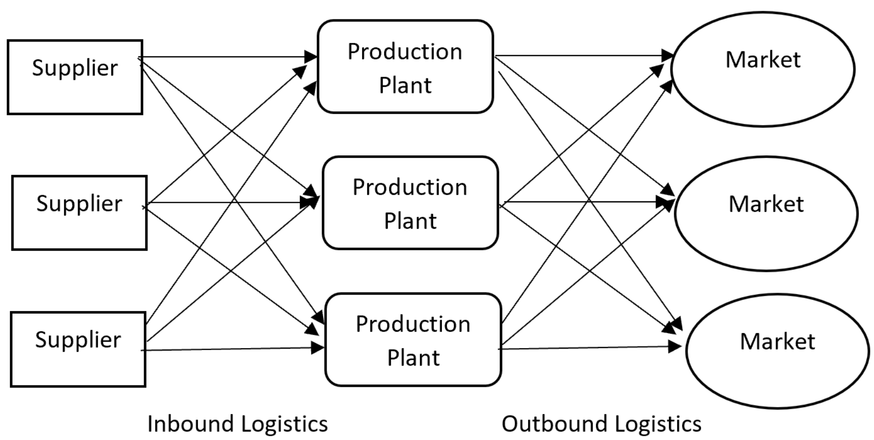


Figure 1.2: Multistage Supply Chain Network

For this system optimal production quantity of each product at each facility, the required schedule for shifts and overtime to accommodate planned production, shipments to demand markets and the resultant costs for each facility in each period over the planning horizon is to be obtained. Companies continuously seek innovations in their decision-making processes to decrease their operational costs. They have multiple options for setting capacity levels and then operationally using that capacity to meet demand. But flexibility is limited by tooling and other strategic capital investments that limit the options available at each facility.

Strategic capital investments are assumed to be already made but medium and short range tactical and operational decisions (options) are available. The proposed model in this research incorporates tactical shift scheduling, workforce level adjustments and the possibility of short term shut down periods in production plants as a tool to save on variable labor cost in periods where demand is less than capacity. Industries are constantly seeking cost-effective solutions to increase flexibility. In this research re-rating the production line is used as a tool to provide additional flexibility of the production line. Re-rating which is related to line speed can lead to a better response to demand changes especially with the uncertain environment of the supply chain networks. As the implementation of shift and run rate changes requires planning and training, the time delay and productivity impacts of any planned changes are incorporated in the model.

Another problem of interest in this research is price elasticity of demand of substitutable products which is used as a tool for demand smoothing. Demand response is usually studied in economics to match supply with demand. An important consideration in the analysis of supply chains is to express the relationship between the price of the product and demand for the product. This relationship is modeled using the concept of price elasticity of demand. The effects of the elasticity in price on production and inventory decisions is further studied in this research. Moreover, product substitutability by price change creates a challenge for capacity-constrained firms. In some periods demand might be high for some products with insufficient capacity at the plants. Slightly increasing the price of that product can help shift the demand to other substitutable products with excess capacity. Varying the selling price is an option for manufactures where increasing capacity is expensive or infeasible in the short term to achieve a balance between capacity and demand.

In this research, a series of mathematical models of increasing complexity is formulated to be able to suggest a minimum cost plan for meeting medium range demand forecasts that indicates production levels at plants by time period, the associated manpower plan (shifts and overtime) and outbound shipments. Pricing decisions are also included in the formulation to suggest a maximum profit plan. The research develops methodologies to optimize supply chain operations by determining the optimal flows of products among facilities based on a nonlinear, mixed integer programming formulation which has been proven to be a powerful tool to model such problems. In this research solution algorithms for the proposed mixed integer nonlinear mathematical models are also suggested. The performance and bounds for the algorithms are evaluated. A real-life case study of a major vehicle manufacturer in the US is studied for the implementation of the models.

A signicant amount of work has been done on models for supply chains. However, none of the existing models consider detailed system re-rating and down periods as tools to provide flexibility to the production line to respond to moderately high-volume durable goods demand changes. Re-rating the production line is a key issue and available option for the system studied. The model presented in this research is unique in the way capacity is defined, since it integrates the major levers of capacity: shifts, run rate, down periods and overtime. Existing research lacks the granularity to define production capacity at production plant. Results presented in this research proves the value of this integration. There also has been limited research on the price elasticity of demand in supply chain networks. Some literature work has been done to investigate pricing strategies in supply chains but few of them incorporates price elasticity of demand and its effect on other operational decisions in the supply chain. Efficient algorithms are crucial to be able to solve supply chain models especially with today's complex real-life problems.

Thus, there is a need for computationally efficient algorithms for such complex systems. The primary goal of the proposed research is to formulate the capacity planning problem with shifts, rate and down periods that suggest minimum cost at the production plants. The model is developed to analyze overall supply chain operations of a manufacturing system of durable goods. Efficient algorithm for solving those model is also suggested to demonstrate applicability of the proposed model. In addition of expanding the scope of models previously appearing in the literature to include options heuristically employed in production, intuitive guidelines on how to use available flexibility options inherent in multifacility networks and their imputed value are developed. Research results demonstrate the savings potential obtained by modeling supply chain operations with the determination of tactical production and distribution allocations. The implementation results illustrate that the proposed methods provide valuable insights for assisting the decision-making process in the supply chain. The proposed research provides a new approach for modeling tactical supply chain decisions with which different operating strategies and scenarios can be evaluated.

1.2 Major Contributions

The major contributions of this dissertation are listed below:

1. Integrated tactical and operational decision support tool for multi-facility and multi-product manufacturing firms capable of determining optimal capacity and production plans jointly based on effective capacity configuration including run rate, shifts, down periods, finished goods inventory management and overtime in a multi-period setting.

2. A flexible capacity configuration model for a multiplant, multiproduct production system with the ability of frequent capacity adjustment in the short term to adapt to changes in customer needs and to minimize logistics cost.

3. Resource constrained comprehensive cost optimization model suitable for capital intensive manufacturers of moderate volume durable goods faced with high capacity and signicant labor costs.

4. A computationally efficient, near-optimal iterative heuristic algorithm for solving the optimal capacity optimization model.

5. Development and testing of a rolling schedule scheme for model implementation.

6. Development and investigation of a model incorporating demand and supply factors through the addition of product pricing with both direct and indirect demand impacts.

7. Demonstration of implementation feasibility and cost savings of the proposed model and heuristic algorithm on a large, real-life problem for validation.

1.3 Research Outline

In the next chapter, related literature is reviewed with focus on most relevant research. The models and solution methods developed to date are discussed in Chapter 2. An integrated capacity planning model is presented in Chapter 3 with detailed model formulation and model discussion. Solution methods for the capacity planning model are proposed. In Chapter 3, a case study is investigated to implement the proposed model. Numerical results are shown and analysed for the case study with concluding remarks. The model is extended to investigate the value in investing in production flexibility. This model is presented in Chapter 4.

An alternative formulation for the capacity planning problem appears in Chapter 5 with comparison to the original model presented in Chapter 3. In Chapter 6, a pricing model is proposed. Detailed problem formulation as well as numerical results are presented. Demonstration on a case study also appear in Chapter 6. Finally, a summary of the research is provided in Chapter 7, including research goals, significant key findings, and conclusion. Future extensions to be explored for the research are described in Chapter 7.

Chapter 2

LITERATURE REVIEW

Supply chain network design and operations optimization are important research topics in the field of operations research. They have been extensively employed to improve the performance of manufacturing systems. These topics are integrated in this dissertation. In this chapter existing research on models for capacity allocation and production scheduling for multi-facility, multi-product manufacturing systems are reviewed. Strategic problem of allocating products to an existing network of production facilities and the tactical problem of using that set of capacitated facilities to plan production are highlighted first. Medium term tactical decisions are usually considered in the process of aggregate planning, in which overall operations of a firm such as production levels, inventory levels, workforce schedule are optimized to satisfy fluctuating demand at a minimum cost. In this literature review, the development of aggregate planning is presented including approaches and solution techniques. Papers discussing elasticity of demand and price response of substitutable goods are also reviewed.

As the literature is very extensive over an extended period, attention is to be given to seminal contributions and those most related to this dissertation, i.e. capacity allocation and production scheduling for highly capitalized facilities producing durable goods with significant labor and logistics costs.

2.1 The Development of Aggregate Planning

Aggregate planning is a complex optimization problem since it includes grouping of many interacting variables which need to be optimized to respond to fluctuating demand with minimum cost. A variety of research papers modeled the AP problem using deterministic models, others used stochastic models. Models for AP can be further classified according to the objective function, either single or multiple objective. Nam and Logendran (1992) provided a literature survey covering the development of AP since 1950. They identified the most frequently used approaches for AP such as linear programming, goal programming and simulation methods. Limitations and advantages of each are also provided. In deterministic models all parameters such as demand and costs are assumed to be known. Mazzola *et al.* (1998), Sillekens *et al.* (2011) and Zhang *et al.* (2012) formulate single objective function deterministic models in their research papers. Many researchers have been increasingly using multiple objective functions in their models because in many real-world instances, problems involve multiple objectives. In addition to cost typical objectives include maximize profit, maximize customer service level, minimize shortages, and minimize changes in workforce level. Leung *et al.* (2003) and Leung and Chan (2009) considered multiple objective functions in their deterministic models. Model parameters are subject to uncertainty, therefore researchers have been developing models to account for uncertainty using the concepts of randomness and probability. Some stochastic models for AP are presented in Leung *et al.* (2007) and Jamalnia and Feili (2013).

Models for AP started with simple structures then developed to be more complex to accommodate the needs of our rapidly changing world. For example, firms start producing a variety of products in multiple manufacturing plants which are then sold in different demand markets.

Also, given the different financial conditions worldwide, it became necessary to include financial concepts in AP models to study the trade-off between expenditure and profitability. Tang *et al.* (2003) implemented those concepts in their AP model. They introduced financial constraints defined by an upper bound of the amount of capital level available at each period of the planning horizon.

A variety of solution approaches for AP models are found in the literature. Classical techniques such as heuristic, linear programming and goal programming are very common especially for deterministic models. Sadeghi *et al.* (2013) use goal programming to solve their multi-objective models. Simulation, stochastic programming and parametric programming approaches are used for stochastic models. The success of Simulation for AP is shown in Jamalnia and Feili (2013) where different strategies for the AP problem are evaluated for shop floor activities of a real-world manufacturing system. Mirzapour Al-e Hashem *et al.* (2013) and Jamalnia *et al.* (2019) papers are examples of using stochastic programming to solve AP problems. Some solution techniques for AP models work well only under linearity assumption (all constraints are linear). Linear programming for instance has only been successfully employed in these models when they are fully linear or sufficiently small that linearization techniques can be used as for instance on some Mixed Integer Programs (MIPs) that incorporate binary variables as indicators of setups or other changes. Others solution techniques have no guarantee on optimal solutions. Also, a few real-world AP problems are compatible with the assumption under each approach. Nam and Logendran (1992) discussed the weaknesses and strengths of the models reviewed in their paper.

Global optimization procedures have been developed to overcome the weaknesses of classical techniques stemming from computational intractability. Metaheuristics approaches are examples of a global optimization procedure which is a trend in the AP research area in recent years.

Simulation Annealing Algorithm, Genetic Algorithm and Neural Network are examples of metaheuristics. Ganesh and Punniyamoorthy (2005) use both GA and SAA to optimize continuous time production planning problem. Mehdizadeh *et al.* (2018) use GA for a bi-objective production planning optimization model. Researchers have provided directions for future work in the area of AP. Further work on pricing is suggested by Jamalnia *et al.* (2019) . Also, investigation of more stochastic models with both single and multiple objective is needed. Sadeghi *et al.* (2013). Other papers suggested further research on sustainability and flexible manufacturing.

2.2 Capacity Allocation and Production Distribution Planning

A vast body of literature exists on using operations research methods to model capacity allocation and production distribution planning. Johnson and Montgomery (1974) provide a condensed overview of production system models including dynamic and static up to that point. Then, as is still the case, planning models have to assume discrete periods for demand and production to fit an organizationally implementable decision hierarchy.

Geoffrion and Graves (1974) developed one of the first models for strategic network planning. They proposed a MIP model to minimize the cost of the multi-product production and distribution network. The model was solved using Bender's decomposition approach. However, the demand is considered to be deterministic and the model is a single period model.

One of the earliest capacity allocation models is due to Eppen *et al.* (1989). In the paper the authors discuss the capacity planning problem and develop a mathematical model based on scenario planning for General Motors.

The capacity planning problem is formulated as a multi-product, multi-plant and multi-period MIP problem that determines the optimal level of production capacity at production sites from different possible capacity configurations to maximize profit. Due to the uncertain demand, risk resulting from uncertain demand is incorporated in the model by adding risk constraints. The risk constraint is formulated as the expected downside risk of profit. The model can effectively generate several capacity configurations that can be investigated by decision makers.

Arntzen *et al.* (1995) propose a multi-period MIP for the global SC. This model optimizes decisions on product allocation and production, inventory and distribution with cost minimization objective. The model incorporates international aspects of SC such as duties and duties drawback. A numerical example from the digital equipment industry is presented to prove applicability of the model.

Jordan and Graves (1995) consider manufacturing flexibility in their proposed model. They propose a model to determine the optimal product flexibility needed to hedge against uncertain demand. This is achieved by assigning products to plants to maximize sales. They model this problem as a bi-partite graphing (chaining). This approach is implemented for the vehicle allocation at GM. However, their model considers a single level supply chain, deterministic demand and a static environment.

Bradley and Arntzen (1999) further show the value of integrating capacity and production decisions along with inventory considerations. The model developed in this paper considers different profitability measure that focuses on revenues and costs directly related to the production plant. The effects of demand variability are also addressed. The proposed model is implemented on two different manufacturing companies and the results showe that maximizing capacity utilization is not always a good option since it increases inventory levels.

In Karabakal *et al.* (2000) a simulation and analytical optimization model is developed to improve the vehicle distribution system of Volkswagen of America. The model tries to improve customer service and vehicle distribution cost by providing a first-choice vehicle with shorter lead time. The proposed model optimizes vehicle distribution through the supply chain which consists of multi-product, multiple distribution centers and multiple dealers by making decision on opening new locations of the distribution centers. The high number of alternatives on locating the new DC as well as the dynamic and stochastic nature of model parameters such as demand and cost urge the use of simulation. Simulation and analytical models are combined by an iterative procedure to solve the MIP model. The output of the MIP which is a location policy is then simulated to get better estimates of the model parameters. The MIP is then updated and resolved using the new parameters to obtain new location policy. Although a computational experiment shows that the proposed procedure converges in a reasonable time, convergence was not guaranteed. Also, inventory holding cost was not included in the formulation.

Another example of combining analytical models with simulation is presented in Gnoni *et al.* (2003) where a MILP model is combined with a simulation model to solve the lot sizing and scheduling problem of a multi-plant braking equipment manufacturing system subject to capacity constraints. Two production scenarios are considered: local and global strategies. In the local strategy each production plant is modeled separately, and the model optimizes each plant cost individually whereas in the global strategy it optimizes the overall cost of the manufacturing plants. Results from each strategy were compared with a reference production plan which included the setup, holding and delivery delay costs. A stochastic discrete event simulation was then constructed to study the behavior of each production plant.

The simulation model generates random demand and obtains machine availability, inventories and mean setup time at each period. The MILP is then solved. This process is repeated at each simulation run to estimate averages and confidence intervals on the solution obtained by the MILP. This iterative procedure for solving the lot sizing and scheduling problem generates a non-optimal feasible solution. Comparing the local and global strategies, results showed that the global strategy produces a slight increase in setup cost however it reduces the total cost and this reduction is specifically due to a reduction in the holding costs which overcomes the increase in the setup cost.

Yan *et al.* (2003) proposed a strategic supply chain design model that considers Bill of Materials (BOM) by adding a logical constraint. The inclusion of BOM in the formulation facilitates the selection of suppliers especially for manufacturers who assemble parts from different locations to build their final products. The proposed MIP model optimizes strategic decisions on supplier selection, opening production plants and DC. It also optimizes tactical decisions on distribution and shipments of products from production plants to DC and from DC to customers. These decisions are made to satisfy customer demand and minimize overall supply chain cost. New constraints regarding the BOM were included as logical constraints. These constraints were presented in the form of cardinality rules. In order to solve the MIP with the logical constraints they are first linearized by transforming them to the conjunctive normal form then expressed as linear inequalities using binary variables. For a large-scale problem, adding logical constraints increases the size of the problem which is computationally inefficient. Therefore, methods to reduce linear inequalities were discussed in an effort to speed up the computational time.

A real-life example was studied to show the effectiveness and convenience of the proposed formulation. Results show that ordering cost is an important factor in the supply chain which is driven by supplier selection.

Park* (2005) examined the advantage of integrating the assignment of product models to plants and the outbound shipment to customers. Computational tests included randomly generated problems with sizes up to five plants, 70 markets, five products and ten periods. The study found that the integrated problem produced superior results. The results show the potential for integrated models for multi-stage logistic with improvement in profit averaging approximately 4 percent. The percent improvement in profit for using an integrated model decreased as problem size increased with a 5.7 percent improvement for small problems and 2.2 percent improvement for large problems. Nonetheless it is meaningful to note that the integrated modeling of vehicle assignment and production increased profit primarily by reducing outbound logistics costs.

Santoso *et al.* (2005) developed a stochastic programming model to solve the supply chain design problem. The proposed two stage stochastic model considers a large scale network with a large number of scenarios for uncertain parameters. The paper first describes the deterministic model formulation for the problem then extends it to the stochastic setting. The deterministic model is a single-period MIP with objective to minimize the fixed cost of investment and the variable per unit operational cost subject to capacity and demand constraints. In the stochastic two stage model parameters such as demand, capacities and costs were considered to be uncertain with known joint distribution. In the first stage the stochastic model decision on whether or not to open a production plant is determined. The second stage determines the product flow between nodes in the network. In the objective function expectation of operational cost is used to model the randomness in cost.

Sample average approximation algorithm (SAA) together with Benders decomposition are used to solve the model. In order to improve the convergence of this algorithm, the paper presented some accelerating techniques such as the cutting planes and adding valid inequalities. The different accelerating schemes are compared based on CPU seconds required to solve the SAA problem to test the computational efficiency. The algorithm was implemented in CPLEX and C++. Results showed that the stochastic model provides smaller cost variability than the deterministic model.

Denton *et al.* (2006) proposed a MIP model to optimize IBM's semiconductor supply chain. Several heuristic algorithms are designed to provide near optimal solution to handle the complexity of the model. The proposed MIP model optimizes the production, inventory and shipments to minimize the cost subject to capacity, demand, and inventory constraint. Wafers are usually produced in batches; thus the discrete lot-sizing constraint defines rules on minimum quantities of a batch. Heuristics are used to solve the MIP model all of which are based on decomposition. The Pre-solve heuristic fixes some decision variables at each iteration using column pricing and then pre-solves the model. This procedure reduces the model size and as a result reduces the computational time of solving the problem. The Preemptive Priority Algorithm is another heuristic which is based on demand prioritization. It starts with grouping the demand into classes and then assigning a priority to each group such that demand groups are arranged from lower to higher priority. This algorithm iteratively solves and modifies the MIP which reduces computational time. Another heuristic algorithm is the lot-sizing algorithm which, while relevant for multiproduct environments with shared capacity and could be relevant for component plants, is less relevant for industries not producing in batches.

Fleischmann *et al.* (2006) present a MIP model to optimize strategic decisions of the product allocation to production sites in the global supply chain with investment in additional capacity. Information about investment budget and possible types of capacity expansion at each plant is considered in the model. The proposed model allows the decision makers to investigate many scenarios of product allocation in reasonable time. The 12-year planning horizon model tries to optimize the multi-period, multi-product and multi-site global supply chain decisions taking into account taxes and exchange rates. The objective maximizes the net present value of costs and investment. BMW has implemented the proposed model on a test data of 36 products and 6 production plants. Different strategies were generated and evaluated by comparing them with a reference strategy. Based on the results a cost savings of about five percent in investment and material, production and distribution costs is expected.

Li *et al.* (2009) presented a MIP model to optimize the capacity allocation of products to plants in an integrated supply chain. The large-scale model solves for multi-period, multi-product and involves multiple suppliers, multiple production plants, and multiple distribution centers. The model tries optimizing the overall profit of the supply chain subject to demand and capacity constraints. Heuristic algorithms were used to solve the MIP mode. Lagrangian relaxation was first used to get an upper bound on the objective function. A decomposition heuristic which is based on splitting the large problem in to small problems to reduce complexity was suggested. The allocation model was split in to three sub models: the production, the distribution and the raw material supply. The heuristic starts with solving each sub model separately and then the objective of the original problem can be obtained.

Results showed that the heuristic provides feasible solution to the problem and is computationally efficient but since it ignores interaction between the three operations it causes shortages and surpluses in some DCs. Therefore, an integrated heuristic algorithm was used to improve the solution. The integrated heuristic algorithm is based on the simultaneous consideration of capacity production and inventory. Computational results were presented in order to compare the two heuristic algorithms. The performance was compared on small, medium and large problems using CPLEX. Results showed that the computational time is high but was improved using the heuristic. Sensitivity analysis was performed to test the robustness of the model by considering variation in prices and capacity.

Bihlmaier *et al.* (2009) proposed a two-stage stochastic MIP to model flexibility by product allocation and workforce planning in the auto industry. The first stage determines strategic decisions of assigning products to plants. The second stage determines the tactical (Shift planning) decisions after the strategic decision have been realized. Capacity is modeled using different capacity stages or configurations. They first presented a deterministic model then it was extended to account for stochastic demand. Uncertain demand was assumed to have a known probability distribution. A heuristic algorithm inspired by the Bender's decomposition method is used to solve the model. The method was implemented on a real-world example from the automotive industry and the results showed that their stochastic mode can efficiently consider many scenarios of demand.

Researchers have been working for a number of years on the modeling and design of efficient distribution strategies. Nasiri *et al.* (2014) addressed the location–allocation problem and the distribution strategies in a multi-stage, multi-product SC. The proposed model integrates the production, distribution and inventory decision simultaneously using a top-down two-stage hierarchical approach.

In the first stage the facility location problem is solved, and network configuration is determined. In the second stage production and distribution plan is created at each facility to satisfy stochastic customer demand. LR is used to solve the problem combined with genetic algorithm. A test problem was created to implement the model, results showed that the proposed procedure is effective and efficient for different problem sizes.

Mariel and Minner (2015) proposed a MIP to model strategic capacity planning in a multi-stage multi-product, multi-period automotive SC. A feature in this model is considering the international aspects of automotive SC such as duties and duty drawbacks. Another feature is the adaptation of capacity using flexible capacity stages taking into account investment requirements. Results showed a cost reduction in the total SC cost when considering duties in the capacity planning model. Results also showed capacity adjustment in the form of volume and product flexibility causes cost reduction. Demand is deterministic, transportation cost was not taken into account.

The problem of interest addressed in this research has some relation to these strategic models but differs in some fundamental ways. The potential value of flexibility is of interest for use in determining the structure of future strategic planning models but it is not the intent of this research to allocate strategic capacity. Such decisions require significant cost and time for planning and implementation and thus are performed in conjunction with program planning on a 7 to 10 year basis rather than the 12 to 24 month medium term capacity planning decisions considered here. On the other hand, pricing decisions can be implemented in a shorter time frame and thus are considered instead for adjusting demand to capacity and for seeking logistics cost savings.

2.3 Demand Elasticity of Goods and Price Response

In order to improve supply chain performance, pricing decisions need to be incorporated together with production and distribution decisions of the supply chain. Varying the selling price of some products helps balancing demand and supply especially for manufacturers where it's expensive to expand the capacity. Moreover, diversity in market preferences have pushed firms to produce multiple substitutable products that can be distinguished by features and prices which increases the complexity of firm's decisions on setting products prices. A number of industries have used consumer choice modeling for modeling demand of multiple differentiated products to enhance their pricing strategies since it provides insights on the purchasing behavior of the buyer.

A research paper by Rusmevichientong *et al.* (2006) proposed a nonparametric formulation to model the multiproduct pricing problem at General Motors. The formulation was motivated by the consumer preference data available in the e-commerce websites. Modeling consumer purchase behavior was based on consumer requirements and budget constraints. The proposed model provides optimal pricing policy based on sample consumer data. In the paper, it has been shown that the number of required samples is a function of the number of products and for a sufficiently large sample the model provides a near optimal solution. The pricing strategy tries to set prices for products to maximize the expected revenue. The revenue function is set such that a choice function is incorporated. This choice function depends on information about consumer which is represented by a profile function that includes both budget as well as ordered list of recommended products. The revenue function also includes the set of possible prices of a product. Due to the complexity of the pricing problem, a heuristic algorithm was provided to facilitate solving the problem.

The proposed approach is used to set prices of vehicles at General Motors. In this experiment three pricing policies were compared based on the revenues.

A similar problem was addressed in Silva-Risso *et al.* (2008) where an incentive planning system that has been used by Chrysler to improve their pricing decisions is developed. Silva-Risso *et al.* (2008) however, developed his system using a hierarchical Bayes structure together with random-effect multinomial nested logit choice model. In the first step of the hierarchical Bayes structure the nested logit model is used to model consumer selection behavior. The nested logit model determines the probability that a consumer in a specific market chooses a vehicle at a specific time. This probability is a function of utility function of each vehicle. The utility function consists of different parameters related to market and transaction type. In the second step of the hierarchical Bayes structure a multivariate normal prior is specified for these parameters. Chrysler has implemented the suggested model for all their products and for a 12-month period as a result an increment in profit was reported.

Biller *et al.* (2005) advised a different dynamic pricing strategy for the automotive industry. The pricing problem was formulated by a multi-period mathematical model that incorporates pricing, production scheduling and inventory decisions under limited manufacturing capacity. The model tries to maximize the profit by making optimal decisions on the amounts to be produced and sold subject to the available capacity. Revenue is expressed as function of satisfied demand. Demand and price are assumed to have a linear relation. Therefore, product price is determined by the satisfied demand. The model is then converted to a network formulation to be able to solve it using min-cost flow algorithm. In the computational results the dynamic pricing model is compared with the fixed price model for different demand scenarios. Additionally, the impact of dynamic pricing on profit is studied. The model can be extended to a multi-product model sharing common capacity.

It also can be modified to include lead time and production set up cost. However, the model neither accounts for price competition nor does it include purchasing behavior of the consumer.

Hsieh *et al.* (2010) presented a price discount strategy for a two-stage supply chain consisting of a distributor who coordinates the retailers' orders and in which both are facing price-sensitive demand. In their numerical example they considered linear demand in price and constant demand elasticity in price. In the results they showed the effect of varying price elasticity and inventory holding cost rate on the distributor's average profit. Results demonstrate that when demand is more sensitive to the selling price, both the distributor's profit and the retailers' profits will decrease. It also implied that the price discount strategy is more beneficial to the distributor when customer demand is more sensitive to the selling price.

In Kaplan *et al.* (2011) a multi-stage, multi-product supply chain network with multi-period setting is investigated. They study sensitivity of product price with varying demands. First the concept of price elasticity of demand is discussed. Then the effects of the elasticity in price demand in production and inventory decisions are examined by formulating a mixed-integer nonlinear programming model. Solution strategy for the proposed model is also presented. Reformulation of the proposed model using linear approximation is also presented to overcome the local optimal solutions because the revenue function is not linear. Numerical results showed that increasing price elasticity of products results in an increase in the satisfied demand and as a result the profitability of the system increases. Also, the average price of a product is also affected by the capacity of the production plants.

Ray *et al.* (2005) proposed profit maximizing models for firms producing a single product with price-sensitive customer demand.

An EOQ model with price and order quantity as independent decision variables is proposed with both log-linear and linear demand functions of price. They also developed a mark-up pricing model where operating cost per unit is considered as well as production cost. Results of a numerical example showed that the optimal order quantity is not necessarily increasing in the setup cost when demand is highly elastic. They also suggest that aggressive pricing strategy is preferred to gain most profit especially for highly price sensitive and non-linear demand.

Levis and Papageorgiou (2009) presented a NLP model that maximizes profit by optimizing pricing policies as well as output levels for substitute products taking into account demand elasticity. They stated that an efficient pricing strategy should consider simultaneously costs, customers and competition. They first proposed a model for a single product production system and then extend it for multi-product production systems, producing substitutable products at different prices. In their model they consider price competition of two firms. Demand forecast is considered as well as outsourcing options. An iterative algorithm is proposed to optimize pricing decision between two competing firms. For the numerical example CONOPT NLP solver is used. A comparison among different cases is presented which provided insight about policies with highest profits.

2.4 Conclusion

Managing and optimizing supply chain operations is a complex task that has gained much attention from researchers. However, there is still a void in the existing literature, since most of the existing models address inventory management or production and distribution decisions separately. Therefore, an integrated model that considers production, scheduling, and distribution as well as inventory decisions, all at a detailed level, is needed.

There is also evidence that re-rating the production line has received little attention by researchers except for as a general workforce level decision which oversimplifies the issue of fixed and variable labor costs, a 24/7 time constraint, retooling/training costs and implementation time issues. Also, few of the models include the flexibility to schedule shifts and overtime in a multi-stage, multi-product network and in a multi-period setting. While some models consider stochastic factors such as demand uncertainty, even multi-period modes are solved at a single time period.

Pricing strategy is an important issue to the supply chain, especially when the price elasticity of demand is high, that is when the price has a significant effect on the product demands. Thus, how to make the correct pricing decisions is crucial when studying supply chain operations. Integrating pricing decisions together with all other supply chain operation decisions has received little attention by researchers and was ignored by many related works. The substitutability of products taking into account price elasticity of demand plays an important role in the optimization of supply chain systems. The proposed models in this research aim to fill the gap in the literature covering integrated models for production, capacity planning, scheduling, distribution as well as pricing in durable goods supply chain networks all modeled at a high level of fidelity recognizing physical, temporal and organizational constraints.

Chapter 3

INTEGRATED CAPACITY PLANNING MODEL

The objective of this research is to develop efficient optimization models for the capacity planning and production and distribution scheduling problem of a multi-product, multi-plant supply chain network for durable consumer goods. The research methodology is based on mathematical programming techniques especially MIP models and solution approaches. A new model characteristic that incorporates planning and scheduling options not previously discussed in the literature is presented in this chapter. It also addresses the issue of how to use production flexibility when some products can be made in multiple plants. In this chapter, a nonlinear, MIP model for the research problem is defined. A detailed mathematical formulation is presented as well as model assumptions. Solution methodologies are then discussed. The proposed model is then applied to a real-world example to illustrate it's applicability. Finally, computational results and concluding remarks are presented.

3.1 Problem Statement

In this problem, a multi-facility production system that produces multiple products to satisfy demand of multiple markets from the set of capacitated production plants is considered. The system consists of inbound shipments to production plants, product assembly and outbound shipment to destination markets. A schematic of the system is shown in Figure 3.1 .

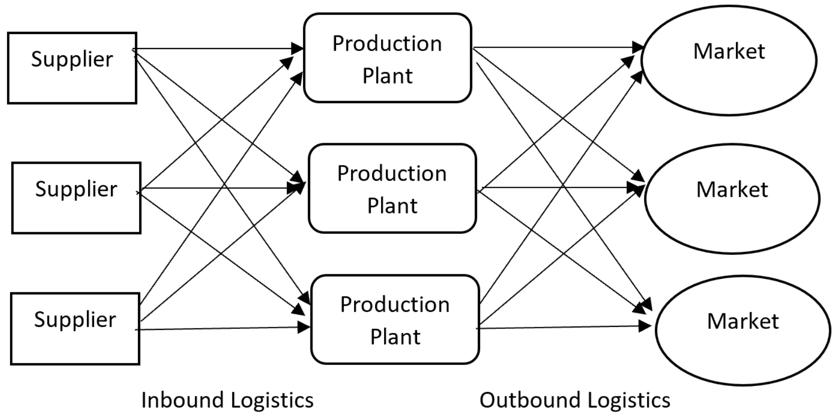


Figure 3.1: The Inbound-Outbound Production System

An integrated mathematical model is formulated to suggest a minimum cost plan for meeting medium range demand forecasts. The model determines the optimal production levels of each product type at each plant during regular time and overtime given the capacity. The model also determines the associated manpower plan which includes the schedule for shifts and overtime to accommodate planned production. Shipments to demand markets and the resultant costs for each facility in each period over the planning horizon are also determined. The proposed model has the flexibility to add or eliminate shifts at plants at any time period considering the cost for changing the capacity level and the loss in productivity that may occur when new shift is added.

The model is also designed to determine the optimal planned shutdown periods for each production plant taking into account potential savings in labor cost during down periods. Scheduled downtime results in partial variable labor cost savings. The model also determines the run rate at production plants. The possibility to re-rate the production line which is related to line speed is also investigated. Line speed measured by run rate (number of units produced per hour) may be varied subject to a known upper bound on run rate based on physical human and technology limits and experience with impact on product quality.

Re-rating the line provides some additional flexibility to respond to demand changes but requires significant planning and training. Production line rating is modeled as the choice among several rates for a line recognizing that rates are unlikely to change significantly from current values. Implementation time required to change the rate is taken into account. Given the cost and time required to reconfigure the line, re-rating is best applied for persistent minor changes on demand. Shift changes are needed for larger systematic changes in demand.

The proposed model also determines inventory levels of each product at each time period limited by lower and upper bounds. Penalty cost is included in the objective function whenever inventory level deviates from the bounds. This will try to keep the inventory within the limits without causing infeasibility if the model cannot naturally stay within the bounds. Additionally, the model allows shortages resulting from unmet demand taking in to account maximum allowable shortages of each product at each time and each market. The following cost factors are included in the formulation:

- Inbound supply logistics cost by product type and plant;
- Outbound supply logistics cost by product type and origin (plant)-destination (market region);
- Shortage cost (lost profit of unmet demand approximately 50% of product price);
- Inventory holding costs;
- Penalty cost if inventory level deviates from bounds;
- Fixed investment and operational cost per shift;

- Fixed plant labor and operation cost per shift;
- Variable labor cost per shift for each plant;
- Overtime premium measured as a percentage addition to regular time pay rate;
- Cost to add and ramp up a new shift or eliminate an existing shift;
- Cost to change the production rate to/ from a level.

The following Technological data are also considered:

- Demand forecast for each product by market;
- Current and allowed run rates by production plants and run rate change lead time;
- Products that can be produced at each production plant (also called "system flex");
- Overtime and other scheduling limitations;
- Productivity factors for overtime, shifts and shift startup;
- Fixed and variable labor costs by plant and shift including impact of down weeks and under production;
- Target inventory levels and allowable ranges;
- Minimum shift lead time;
- Product mix requirements determined by equipment capacity;

The following assumptions are made:

- The set of product allocations to production plants is given. Not all allowable allocations must be utilized but no additional allocations are considered in the model. Thus, investment cost is assumed fixed and exogenous to the model.

- Demand for each product type in each market each period is known deterministically over the planning horizon or through forecast. In practice only the first month's demand is known precisely.

- Cycle time is given for the plant based on an upper percentile of the slowest product and the product mix and sequencing are such that that cycle time can be met.

- Cycle time (inverse of run rate) limits are known by plant.

- Run rate can be varied at most once a year and only be plus/minus 5%, 10% or 20% of current rate based on observed past practice.

- A lead time is required to change the run rate, nominally assumed to be six months.

- Adding or deleting a shift requires a known lead time, nominally set to three months.

- Inventory accumulation is to be generally avoided by matching production to demand but is allowable at production plants. Initial values are known.

- The minimum inventory and maximum inventory for each product at each time is limited by organizational policy.

- Inventory deviation penalty per unit is a percentage of product book value.

- The minimum period for assigning production volume is four-weeks/one month.

- Inbound logistics cost per product-plant combination is constant over the time horizon and known by assuming existing suppliers will be used or suppliers will relocate with linear cost contracts.

- Outbound logistics cost per product-destination pair is constant over the time horizon.

- Shift labor is composed of a fixed (primarily salary) workers and variable (hourly) workers.

- Labor cost at a plant consists of fixed plant labor, fixed shift labor and variable shift labor. Fixed cost exists if a shift is active. Variable cost per hour is directly proportional to the line rating.

- Production rate for a product at a plant may also be limited as a percentage of total production or a maximum rate for that product in addition to total plant capacity.

- Shifts one and two have eight productive hours per day but the third shift adds only an additional 6.5 productive hours. Overtime is limited to a maximum total activation of 22.5 hours per day with additional limits on length of full overtime usage.

- Weekend shifts may be used but there exists a limit on the amount of overtime that may be used in any consecutive two months (periods) based on labor rest requirements and organizational policy.

- Rate change cost up or down is independent of number of shifts and is based on cost for required tooling adjustment.

3.2 Notation

Indices and sets:

- $\mathcal{I} : \{1 \cdots, i, \cdots I\}$, set of products
- $\mathcal{J} : \{1 \cdots, j, \cdots J\}$, set of plants
- $\mathcal{M} : \{1 \cdots, m, \cdots M\}$, set of markets
- $\mathcal{T} : \{1 \cdots, t, \cdots T\}$, set of time periods
- $\mathcal{R} : \{1 \cdots, r, \cdots R\}$, set of feasible run rates measured as proportional changes from the nominal rate for a specific plant
- $\mathcal{S} : \{1, 2, 3\}$, set of shifts
- $\mathcal{F}_j$, flexibility set, representing products that can be made at plant j

Parameters:

- α_j: overtime premium rate on variable labor at plant j for each product produced during over time
- β_r: ratio of run rate level r w.r.t the starting rate at a production plant, $\beta_r \in \{0.8, 0.9, 0.95, 1.0, 1.05, 1.1, 1.2\}$
- γ_{sjt}: proportion of capacity loss when shift s is added to the plant j at time t
- $\underline{\eta_{it}}$: lower inventory limit based on total demand of product i at time t
- $\overline{\eta_{it}}$: upper inventory limit based on total demand of product i at time t
- $\overline{\rho}_{ij}$: maximum percentage of production of product i at plant j a teach period
- w_{jt}: available weeks for plant j at period t

- wd_{jt}: available workdays for plant j at period t
- d_{imt}: demand of product i at market m during period t
- h_{sj}: effective working hours for shift s at plant j
- l^S: lead time needed to schedule a change of shift (nominally set to 3 months)
- l^R: lead time needed to schedule a change of rate (nominally set to 6 months)
- $\overline{o}_s$: maximum allowable overtime (in hours) for shift s
- o'_{jt}: percentage of over time capacity to limit over time usage in plant j in t
- $\overline{r}_{ij}$: maximum average production rate for product i at plant j
- $\overline{u}_{im}$: maximum shortages allowed for product i at market m in any time period
- c^{in}_{ij}: unit inbound cost for product i at plant j, including component costs and shipping cost
- c^{out}_{ijm}: unit outbound cost for shipping product i to market m from plant j
- c^u_{im}: unit shortage cost for product i at market m
- c^f_{sj}: fixed labor cost for shift s at plant j
- c^v_{rj}: variable regular time labor cost per shift at plant j with run rate r
- c^o_j: overtime labor cost per unit at plant j
- $c^{\Delta I}_i$: penalty cost per unit for inventory deviation from limits
- $c^{r+(-)}_j$: fixed cost to increase (decrease) the rate at plant j
- $c^{l+(-)}_{sj}$: fixed cost to add(delete) shift s at plant j

- c_{rj}^{ds}: proportion of variable labor cost saved during a down week at plant j with run rate r

- c_j^{ro}: cost to reopen plant j (increasing shifts from zero to one or more)

- c_i^I: unit inventory holding cost for product i per time period

- r_j^0: initial run rate of plant j

- I_{ij}^0: initial inventory of product i at plant j

- I_i^e: desired inventory of product i at the end of the planning horizon

- L_{sj}^0: binary indicator if initial shift s is scheduled for plant j

- R_{rj}^0: binary indicator if initial run rate level r is selected at plant j

- $\overline{Z}_{jt}$: required minimum number of down weeks for plant j at period t

Decision Variables:

- X_{ijt}: units of product i produced in plant j during regular time at period t

- O_{ijt}: units of product i produced in plant j during overtime at period t

- O_{jt}^C: overtime capacity of plant j at period t, internal variable, not used in actual runs

- S_{ijmt}: units of product i shipped to market m from plant j in period t

- I_{ijt}: units of product i in inventory at plant j at end of period t

- ΔI_{it}: units of inventory deviation from target inventory of product i during period t

- I_{it}^+: units of inventory deviation above upper limit of product i during period t

- I_{it}^{-}: units of inventory deviation below lower limit of product i during period t

- U_{imt}: units shortage of product i at market m during period t

- Z_{jt}: number of scheduled down weeks at plant j in time t

$$\boldsymbol{L_{sjt}} = \begin{cases} 1, & \text{if shift } s \text{ is scheduled in plant } j \text{ at period } t \\ 0, & \text{if otherwise} \end{cases}$$

$$\boldsymbol{L_{sjt}^{(+)(-)}} = \begin{cases} 1, & \text{if shift } s \text{ is added(closed) in plant } j \text{ at period } t \\ 0, & \text{if otherwise} \end{cases}$$

$$\boldsymbol{R_{rjt}} = \begin{cases} 1, & \text{if run rate level } r \text{ is scheduled in plant } j \text{ at period } t \\ 0, & \text{if otherwise} \end{cases}$$

$$\boldsymbol{R_{jt}^{(+)(-)}} = \begin{cases} 1, & \text{if run rate is increased(decreased) in plant } j \text{ at period } t \\ 0, & \text{if otherwise} \end{cases}$$

$$\boldsymbol{RO_{jt}} = \begin{cases} 1, & \text{if plant } j \text{ is reopened at period } t \\ 0, & \text{if otherwise} \end{cases}$$

3.3 Integrated Capacity Planning, Production Scheduling and Distribution Model with Shift schedule, Run Rate and Down Periods

In this model three key levers representing binary decisions: shift schedule (regular shift), down periods and run rate are integrated to determine capacity. In each time period and for each plant's production plan the model determines optimal values of the key decisions as well as production volume, shipments, inventories, and shortages for each product type.

The objective function in equation (3.1) accumulates costs of inbound logistics to production plants required to support production during regular and over time, outbound logistics from production plants to markets, lost profit from unmet demand or cost of supplying demand from a plant outside the model. The objective function also includes inventory carrying cost from holding finished products at the plants as well as penalty cost if inventory level exceeds the pre-defined bounds. Inventory holding and penalty costs are both based on the book value of the product. Variable and fixed labor cost for shifts utilized are also counted. Fixed labor cost is incurred if a shift is schedule for supervision. Variable labor cost per shift is proportional to the run rate selected at the production plant. Overtime labor cost is calculated based on labor cost per unit and overtime pay rate premium. The cost to add or eliminate shifts and to re-rate the production line are also counted which are the cost associated with changing capacity level at the production plants. A fixed cost for plant re-opening is also included whenever shift schedule changes from no shifts to at least one shift. This cost is included to limit the occurrences of re-opening since it is rare and expensive in practice. Finally, savings from scheduled down periods is calculated. At each down period, savings are calculated as a percentage of variable labor cost per shift.

All costs are summed overall production plants and time periods in the planning horizon and the model is solved to minimize the total cost.

$$\text{Minimize: Total Cost} = \sum_{i \in F_j} \sum_{j=1}^{J} c_{ij}^{in} . \sum_{t=1}^{T} (X_{ijt} + O_{ijt}) + \sum_{i \in F_j} \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{t=1}^{T} (c_{ijm}^{out} . S_{ijmt})$$
$$+ \sum_{i=1}^{I} \sum_{m=1}^{M} \sum_{t=1}^{T} (c_{im}^{u} . U_{imt}) + \sum_{i \in F_j} \sum_{j=1}^{J} \sum_{t=1}^{T} (c_i^I . I_{ijt}) + \sum_{i=1}^{I} \sum_{t=1}^{T} (c_i^{\Delta I} . \Delta I_{it})$$
$$+ \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} (c_{sj}^{f} . L_{sjt}) + \sum_{r=1}^{R} \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=1}^{T} (c_{rj}^{v} . L_{sjt} . R_{rjt})$$
$$+ \sum_{i \in F_j} \sum_{j=1}^{J} \sum_{t=1}^{T} ((1 + \alpha_j) . c_j^o . O_{ijt}) + \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} ((c_{sj}^{l+} . L_{sjt}^{+}) + (c_{sj}^{l-} . L_{sjt}^{-}))$$
$$+ \sum_{r=1}^{R} \sum_{j=1}^{J} \sum_{t=1}^{T} ((c_j^{r+} . R_{jt}^{+}) + (c_j^{r-} . R_{jt}^{-})) + \sum_{j=1}^{J} \sum_{t=1}^{T} (c_j^{ro} . RO_{jt})$$
$$- \sum_{r=1}^{R} \sum_{s=1}^{S} \sum_{j=1}^{J} \sum_{t=1}^{T} (c_{rj}^{ds} . c_{rj}^{v} . R_{rjt} . L_{xjt} . Z_{jt} / w_{jt}) \tag{3.1}$$

The set of constraints are formulated to ensure the feasibility of the solution. Demand constraint shown in equation (3.2), ensures that demand is met for each product in each demand market in each period or the shortage is counted. Setting $U_{imt} = 0$ would eliminate any shortages. Shipments are aggregated from all plants equipped to produce a particular product. Plants priorities by products occur naturally based on inbound logistics, out bound logistics, and labor efficiency at the plant.

$$\sum_{j=1}^{J} S_{ijmt} + U_{imt} = d_{imt} \quad \forall i \in I, m \in M, t \in T \tag{3.2}$$

Inventory is assumed to be stored at the production plant. Constraints (3.3) and (3.4) balance the inventory by tracking inventory levels for each product at each plant and ensure adequate production to meet shipments.

Constraints (3.5) and (3.6) provide lower and upper bounds on the amount of inventory that are desired for each product at each time period. Constraint (3.7) sums the amount of inventory deviation (above and below limits) from each product at each period. Constraint (3.8) sets the required ending inventory quantities of each product at each plant at the end of the planning horizon.

$$X_{ijt} + O_{ijt} + I_{ij}^{0} = \sum_{m=1}^{M} S_{ijmt} + I_{ijt} \quad \forall i \in F_j, j \in J, t = 1 \tag{3.3}$$

$$X_{ijt} + O_{ijt} + I_{ij(t-1)} = \sum_{m=1}^{M} S_{ijmt} + I_{ijt} \quad \forall i \in F_j, j \in J, t > 1 \tag{3.4}$$

$$\sum_{j=1}^{J} I_{ijt} \geq \underline{\eta_{it}}. \sum_{m=1}^{M} d_{imt} - \sum_{j=1}^{J} I_{ijt}^{-} \quad \forall i \in F_j, t \leq T-1 \tag{3.5}$$

$$\sum_{j=1}^{J} I_{ijt} \leq \overline{\eta_{it}}. \sum_{m=1}^{M} d_{imt} + \sum_{j=1}^{J} I_{ijt}^{+} \quad \forall i \in F_j, t \leq T-1 \tag{3.6}$$

$$\Delta I_{it} = \sum_{j=1}^{J} (I_{ijt}^{+} + I_{ijt}^{-}) \quad \forall i \in F_j, t \in T \tag{3.7}$$

$$\sum_{j=1}^{J} I_{ijt} = I_i^e \quad \forall i \in I, t = T \tag{3.8}$$

Constraint (3.9) ensures sufficient capacity to execute the planned regular time production. In ensuring adequate capacity the constraints identify the use of shifts at each plant in each period as well as run rate (units per hour). Some loss in productivity in the period in which a shift is added is also counted. In this constraint production during planned down periods is prevented. To do that, number of down days (wd_{jt}) is subtracted from the available work days (wd_{jt}).

Note that while this level of detail is needed for accurately portraying regular time capacity, the use of the run rate, shift schedule and down period levers introduces significant nonlinearity of the binary variables into the model.

$$\sum_{i \in F_j} X_{ijt} \leq \sum_{r=1}^{R} (\beta_r . r_j^0 . R_{rjt}).(wd_{jt}.(1 - Z_{jt}/w_{jt})). \sum_{s=1}^{S} ((h_{sj}.(L_{sjt} - \gamma_{sjt}.L_{sjt}^{+}))$$

$$\forall j \in J, t \in T \tag{3.9}$$

Constraint (3.10) prevents a shift change in the first period. In general, shift schedules for initial periods can be fixed in the model based on the implementation lead time. Indeed, only preplanned shift changes are allowed in initial periods prior to the implementation parameter number of periods. Initial shift usage, L_{sj}^0, is assumed to be known. Constraints (3.11) and (3.12) provide consistency that earlier shifts will always be preferred and serve to eliminate symmetric solutions in the model. Constraint (3.13) forces $L_{sjt}^{+} = 1$ when adding shift s at plant j in period t and $L_{sjt}^{-} = 1$ when eliminating a shift. Constraint (3.14) is a balance constraint to track shift changes between periods. Constraint (3.15) limits the frequency of shift changes to be at least the planning time such that no more than one change is allowed over any l^s length of time periods. An additional parameter could be specified to separate frequency of allowed shifts and lead time.

$$L_{sjt} = L_{sj}^0 \quad \forall s \in S, j \in J, t = 1 \tag{3.10}$$

$$L_{2jt} \leq L_{1jt} \quad \forall j \in J, t \in T \tag{3.11}$$

$$L_{3jt} \leq L_{2jt} \quad \forall j \in J, t \in T \tag{3.12}$$

$$L_{sjt} - L_{sj}^0 - L_{sjt}^{+} + L_{sjt}^{-} = 0 \quad \forall s \in S, j \in J, t = 1 \tag{3.13}$$

$$L_{sjt} - L_{sj(t-1)} - L_{sjt}^{+} + L_{sjt}^{-} = 0 \quad \forall s \in S, j \in J, t > 1 \tag{3.14}$$

$$\sum_{t' \in [t-l^s+1,t]} (L^+_{sjt'} + L^-_{sjt'}) \leq 1 \quad \forall j \in J, t \geq l^s \tag{3.15}$$

Constraint (3.16) defines over time capacity limit which allows overtime in the first and second shifts as a percentage of a shift if scheduled. Overtime is allowed for Saturday and Sunday also as a percentage of a shift if scheduled. If a third shift is scheduled there will be no overtime allowed during the Monday to Friday work week. Maximum overtime hours per shift is defined using $\overline{o}_s$ which is a parameter that can be calculated using percentages of shifts depending on organizational rules for over time. Constraint (3.17) limits the amount of overtime allowed in each plant in consecutive periods using over time limit (o'_{jt}). For example, a maximum of 50 % of over time in every two consecutive time periods can be allowed by setting $o'_{jt} = 0.5$. This implements the organizational limit on long term over time usage. In calculating production during over time, run rate and down periods variables are included.

$$\sum_{i \in F_j} O_{ijt} \leq \sum_{r=1}^{R} (\beta_j . r^0_j . R_{rjt}).(wd_{jt}.(1 - Z_{jt}/w_{jt})). \sum_{s=1}^{S} (\overline{o}_s . L_{sjt}) \quad \forall j \in J, t \in T \tag{3.16}$$

$$\sum_{i \in F_j} (O_{ijt} + O_{ij(t+1)}) \leq o'_{jt}.(O^C_{jt} + O^C_{j(t+1)}) \quad \forall j \in J, t \geq 1 \tag{3.17}$$

$$O^C_{jt} = \sum_{r=1}^{R} (\beta_{rj} . r^0_j . R_{rjt}).(wd_{jt}.(1 - Z_{jt}/w_{jt})). \sum_{s=1}^{S} (\overline{o}_s . L_{sjt}) \quad \forall j \in J, t \in T \tag{3.18}$$

The $\overline{o}_s$ values are constructed to implement the allowable overtime. For instance, during a one shift operation overtime is restricted to four hours per day Monday through Friday plus up to 12 hours each on Saturday and Sunday. For the second shift the incremental overtime is smaller since there is a maximum operation of 22.5 hours per day.

Constraint (3.19) limits the maximum number of shortages (unmet demand) by product in each market and time period.

$$U_{imt} \leq \overline{u}_{im} \quad \forall i \in I, m \in M, t \in T \tag{3.19}$$

Run rate for each plant each period is modeled as the choice of among a set of discrete values. Allowable percentage increments are nominally set to 5%, 10% or 20% but in general are indexed by r, where r values are as follows:

$$r = \begin{cases} 1, & \text{Initial run rate} \\ 2, & \text{Initial run rate -5\%} \\ 3, & \text{Initial run rate -10\%} \\ 4, & \text{Initial run rate -20\%} \\ 5, & \text{Initial run rate +5\%} \\ 6, & \text{Initial run rate +10\%} \\ 7, & \text{Initial run rate +20\%} \end{cases}$$

Constraint (3.20) ensures the choice of exactly one run rate for each period at each plant at each time period. No run rate change for the first period is enforced in constraint (3.21). Constraint (3.22) and (3.23) account for changes in run rate between periods. Constraint limits the number of run rate changes using run rate lead time (l^r). Due to planning and training time, run rate changes require an implementation lead time set by l^r. The number of run rate changes over the planning horizon can also be limited by constraint (3.24).

$$\sum_{r=1}^{R} R_{rjt} = 1 \quad \forall j \in J, t \in T \tag{3.20}$$

$$R_{rj1} = R^0_{rj} \quad \forall r \in R, j \in J \tag{3.21}$$

$$\sum_{r=1}^{R} (\beta_{rj}.R_{rjt}) - (\beta_{rj}.R_{rj(t-1)}) \leq \max\{\beta_{rj}\}.R^+_{jt} \quad \forall j \in J, t \in T \tag{3.22}$$

$$\sum_{r=1}^{R} (\beta_{rj}.R_{rj(t-1)}) - (\beta_{rj}.R_{rjt}) \leq \max\{\beta_{rj}\}.R^-_{jt} \quad \forall j \in J, t \in T \tag{3.23}$$

$$\sum_{t' \in [t-l^R+1,t]} (R^+_{jt'} + R^-_{jt'}) \leq 1 \quad \forall j \in J, t \geq l^R \tag{3.24}$$

Constraint (3.25) states that down periods can be scheduled only if at least first shift is scheduled. Note that fixed labor cost is incurred but a certain portion of variable labor cost is saved during down periods. Constraint (3.26) implements the required scheduled down weeks. These implement planned down weeks for model changes, plant vacations and similar organizational actions.

$$Z_{jt}/w_{jt} \leq L_{1jt} \quad \forall j \in J, t \in T \tag{3.25}$$

$$Z_{jt} \geq \overline{Z}_{jt} \quad \forall j \in J, t \in T \tag{3.26}$$

In constraint (3.27) the occurrences of re-open the plant are counted. To reopen a plant at time t, at least one shift should be added at t and no shift scheduled at time $t-1$, i.e., it should be closed at $t-1$. The first shift is used only due to the previously defined shift consistency rule that earlier shifts are scheduled first.

$$L_{1jt} - L_{1j(t-1)} \leq RO_{jt} \quad \forall j \in J, t > 1 \tag{3.27}$$

Constraint (3.28) limits maximum percentage of production of a product at a plant. These constraints may be used to limit production rate of a particular product that has a limited feeder rate for parts.

Constraint (3.29) limits the maximum regular time production of a product at a plant based on its limiting tool capacity.

$$X_{ijt} + O_{ijt} \leq \overline{\rho_{ij}}.\sum_{i \in F_j}(X_{ijt} + O_{ijt}) \quad \forall i \in I, j \in J, t \in T \tag{3.28}$$

$$X_{ijt} \leq \overline{r}_{ij}.wd_{jt}.\sum_{s=1}^{S}((h_{sj}.(L_{sjt} - \gamma_{sjt}.L^{+}_{sjt})) \quad \forall i \in I, j \in J, t \in T \tag{3.29}$$

Continuous and integer variables are defined in the following constraints:

$$X_{ijt}, O_{ijt}, I_{ijt}, \Delta I_{it}, I^{+}_{ijt}, I^{-}_{ijt}, S_{ijmt}, U_{imt} \geq 0 \quad \forall i \in F_j, j \in J, m \in M, t \in T \tag{3.30}$$

$$L_{sjt}, L^{+}_{sjt}, L^{-}_{sjt}, R_{rjt}, R^{+}_{jt}, R^{-}_{jt}, RO_{jt} \in \{0, 1\} \quad \forall s \in S, j \in J, t \in T, r \in R \tag{3.31}$$

$$Z_{jt} \in N \quad \forall j \in J, t \in T \tag{3.32}$$

3.4 Model Discussion

The formulation described by equations (3.1) to (3.32) forms a MINLP. The objective function is not linear since the variable labor cost and down period savings are formulated as a product of binary variables and as a product of binary and integer variables respectively. Some of the constraints are also nonlinear. Nonlinearity comes from the capacity constraint and overtime capacity constraints as well where the nonlinearity is in the form of the product of binary, another binary and integer variable. These constraints can be linearized as shown later. All other constraints are linear. Based on initial results, solving the model is expected to be computationally prohibitive especially for large scale problems. Also, MIP solver is not very efficient in terms of computational time for problems of the desired size. Thus, several approaches are proposed to solve the model. A potentially global optimal approach and a heuristic approach are proposed each of which is described in the next two sections of this chapter.

3.4.1 Model Linearization

In this approach the model is incrementally linearized by linearizing the nonlinear constraints, taking into account the different forms of nonlinearity in the model. The following results are employed to linearize the different forms of nonlinearity. Linearization of the product of two binary variables and the linearization of the product of a binary and continuous variables are discusses next.

If $z = x.y$ where z, x, y are binary, then the nonlinear product can be expressed as:

$$z \leq \; x \tag{3.33}$$

$$z \leq \; y \tag{3.34}$$

$$z \geq \; x + y - 1 \tag{3.35}$$

$$0 \leq z \leq \; 1 \tag{3.36}$$

Constraints (3.33) and (3.34) ensure that z will be zero if either x or y are zero. Constraint(3.35) ensures that z will be one if both x and y are one. Note that z is relaxed to be continuous to reduce the number of binary variables. With this setting z is will automatically take values of either zero or one. Note that each product of binary variables requires the addition of a new binary variable.

To linearize the product of integer or continuous variables with binary variables the standard linear McCormick envelopes McCormick (1976) is used. Suppose $z = A.x$ where x is binary and A is a continuous (or integer) variable that is bounded below by zero (i.e. $A \in R_+$) and above by $\bar{A}$, then the nonlinear product can be expressed as:

$$z \leq \; \bar{A}.x \tag{3.37}$$

$$z \leq \; A \tag{3.38}$$

$$z \geq \; A - (1 - x).\bar{A} \tag{3.39}$$

$$z \geq \; 0 \tag{3.40}$$

Constraint (3.37) ensures that if x is zero then z will be zero as well. But if x is one, z is ensured to be less than $\bar{A}$ the upper bound which is tightened in constraint (3.38). Constraint (3.39) ensures that z will be at least as much as A when $x = 1$. The non-linear terms of the model are described below as well as their linearization.

Variable labor cost shown in equation (3.41) is formulated as a product of two binary variables therefore, the first result discussed above is used for linearization. To distinguish variables from parameters, red font is used for variables.

$$\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=1}^{T}(c_{rj}^{v}.L_{sjt}.R_{rjt})(\text{Non-linear-Quadratic}) \tag{3.41}$$

A new variable RL_{rsjt} is defined, where $RL_{rsjt} = R_{rjt}.L_{sjt}$ and the constraint is reformulated as shown in equation (3.42) and constraints (3.43), (3.44), (3.45), (3.46) are added.

$$\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=1}^{T}(c_{rj}^{v}.RL_{rsjt})(\text{linear}) \tag{3.42}$$

$$RL_{rsjt} \leq R_{rjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.43}$$

$$RL_{rsjt} \leq L_{sjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.44}$$

$$RL_{rsjt} \geq R_{rjt} + L_{sjt} - 1 \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.45}$$

$$0 \leq RL_{rsjt} \leq 1 \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.46}$$

Down savings shown in equation (3.47) is in the form of the product of two binary variables and an integer variable. The product of two binary is also binary therefore, the second result discussed above is used for linearization.

$$\sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{rj}^{ds}.c_{rj}^{v}.R_{rjt}.L_{sjt}.Z_{jt}/w_{jt})(\text{Non-linear-Cubic}) \tag{3.47}$$

A new variable RLZ_{rsjt} is defined, where $RLZ_{rsjt} = R_{rjt}.L_{sjt}.Z_{jt}$ and the constraint will be reformulated as follows:

$$\sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{rj}^{ds}.c_{rj}^{v}/w_{jt})(RLZ_{rsjt})(\text{linear}) \tag{3.48}$$

In this case $\bar{A} = 4$, since Z_{jt} is integer taking values 0,1,2,3,4. The following constraints are added:

$$RLZ_{rsjt} \leq 4.R_{rjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.49}$$

$$RLZ_{rsjt} \leq 4.L_{sjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.50}$$

$$RLZ_{rsjt} \leq Z_{jt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.51}$$

$$RLZ_{rsjt} \geq Z_{jt} - 4.(2 - R_{rjt} - L_{sjt}) \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.52}$$

$$RLZ_{rsjt} \geq 0 \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.53}$$

The cubic capacity constraint shown in equation (3.54) is cubic as well therefore, the second result is used for linearization. First the constraint is expanded as shown in equation (3.55) to facilitate linearization.

$$\sum_{i \in F_j} X_{ijt} \leq \sum_{r=1}^{R}(\beta_r.r_j^0.R_{rjt}).(wd_{jt}.(1 - Z_{jt}/w_{jt})).\sum_{s=1}^{S}((h_{sj}.(L_{sjt} - \gamma_{sjt}.L_{sjt}^{+}))$$
$$\forall j \in J, t \in T \tag{3.54}$$

$$\sum_{i \in F_j} X_{ijt} \leq (\sum_{r=1}^{R}(\beta_r.R_{rjt}).wd_{jt}.\sum_{s=1}^{S}(h_{sj}.(L_{sjt})) - (\sum_{r=1}^{R}(\beta_r.R_{rjt}).wd_{jt}.Z_{jt}.\sum_{s=1}^{S}(h_{sj}.(L_{sjt}))$$
$$-(\sum_{r=1}^{R}(\beta_r.R_{rjt}).wd_{jt}.\sum_{s=1}^{S}(h_{sj}.(L_{sjt}^{+})) + (\sum_{r=1}^{R}(\beta_r.R_{rjt}).wd_{jt}.Z_{jt}.\sum_{s=1}^{S}(h_{sj}.(L_{sjt}^{+}))$$
$$\forall j \in J, t \in T \tag{3.55}$$

The previously introduced variables RL_{rsjt} are used, where $RL_{rsjt} = R_{rjt}.L_{sjt}$ and ZRL_{rsjt} where $ZRL_{rsjt} = Z_{jt}.R_{rjt}.L_{sjt}$. Additional new variable ZRL^{+}_{rsjt} is defined, where $ZRL^{+}_{rsjt} = Z_{jt}.R_{rjt}.L^{+}_{sjt}$ and RL^{+}_{rsjt}, where $RL_{rsjt} = R_{rjt}.L^{+}_{sjt}$. The constraint is reformulated in equation (3.56) and the set of equations from (3.57) to (3.65) are added.

$$\sum_{i \in F_j} X_{ijt} \leq (\sum_{r=1}^{R}\sum_{s=1}^{S}(\beta_r.h_{sj}.RL_{rsjt}).wd_{jt} - (\sum_{r=1}^{R}\sum_{s=1}^{S}(\beta_r.\gamma_{sjt}.RL^{+}_{rsjt}).wd_{jt}) - (\sum_{r=1}^{R}\sum_{s=1}^{S}(\beta_r.h_{sj}.ZRL_{rsjt})) + (\sum_{r=1}^{R}\sum_{s=1}^{S}(\beta_r.\gamma_{sjt}.ZRL^{+}_{rsjt})) \quad \forall j \in J, t \in T \quad \text{(linear)} \tag{3.56}$$

$$RL^{+}_{rsjt} \leq R_{rjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.57}$$

$$RL^{+}_{rsjt} \leq L^{+}_{sjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.58}$$

$$RL^{+}_{rsjt} \geq R_{rjt} + L^{+}_{sjt} - 1 \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.59}$$

$$0 \leq RL^{+}_{rsjt} \leq 1 \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.60}$$

$$ZRL^{+}_{rsjt} \leq 4.R_{rjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.61}$$

$$ZRL^{+}_{rsjt} \leq 4.L^{+}_{sjt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.62}$$

$$ZRL^{+}_{rsjt} \leq Z_{jt} \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.63}$$

$$ZRL^{+}_{rsjt} \geq Z_{jt} - 4.(2 - R_{rjt} - L^{+}_{sjt}) \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.64}$$

$$ZRL^{+}_{rsjt} \geq 0 \quad \forall r \in R, s \in S, j \in J, t \in T \tag{3.65}$$

Over time capacity constraint shown in equation (3.66) is linearized similar to capacity constraint since their formulation is very similar.

$$\sum_{i \in F_j} O_{ijt} \leq \sum_{r=1}^{R} (\beta_r . r_j^0 . R_{rjt}).(wd_{jt}.(1 - Z_{jt}/w_{jt})). \sum_{s=1}^{S} (\bar{o}_s . L_{sjt}) \quad \forall j \in J, t \in T$$

$$\text{(Non-linear-Cubic)} \tag{3.66}$$

3.4.2 Iterative Fix and Re-solve Heuristic for Solving the Model

A decomposition approach is proposed to overcome the computational complexity of the problem. The integrated model is decomposed based on the three key binary variables: Shifts, Run Rate,and Down Periods. Each of these represents a stage in an iterative solution procedure. In the three-stage Fix and Re-solve iterative approach optimal values of a binary variable set is obtained assuming the other two sets are known. The full model is then solved iteratively by setting one set of variables active at a time then using the output to the next stage. This approach will overcome the non-linearity of the model since at each iteration two binary variables are set as parameters and solve for the third. An illustration of this approach is shown in Figure 3.2.

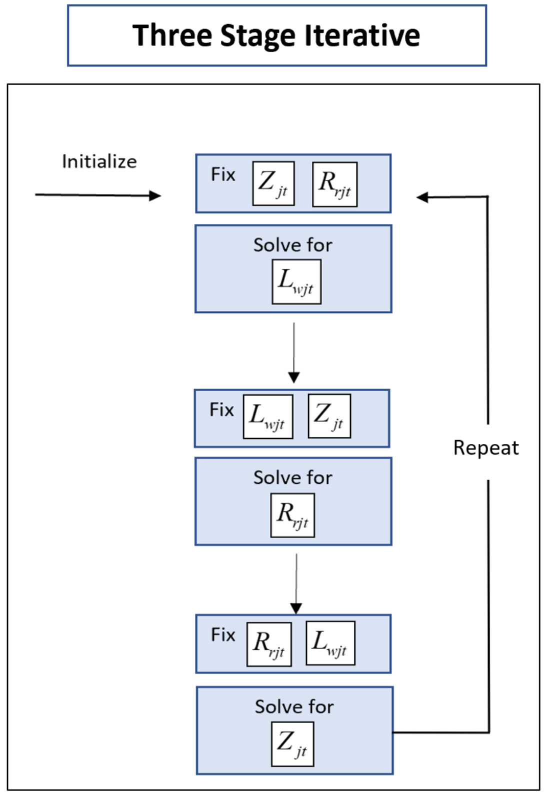


Figure 3.2: A Schematic of the Three-Stage Fix and Re-solve Heuristic Approach

In the first stage the Down Period Z_{jt} and Run Rate R_{rjt} variables are fixed to initial values and the model is solved for optimal shift schedule L_{sjt} for each plant at each time period assuming that down periods and run rates are known (set to be parameters of the model). The model also determines all other decisions such as production X_{ijt}, O_{ijt}, inventory I_{ijt} and shipments S_{ijmt}. In the second stage, shift schedule L_{sjt} is fixed to the current optimal values from the first stage, Down Period Z_{jt} is still fixed to the initial value and Run Rate R_{rjt} is set to be active. The optimal values are determined and fed to the third stage in which, the model solves for Down Period Z_{jt} with R_{rjt} and L_{sjt} taking current optimal values from the previous stages. This procedure continues until no change in the optimal values of the binary variables is observed and the objective value remains the same.

This approach reduces the complexity of the problem in two ways. First, at each iteration a linear model is solved since some of the binary variables which are causing nonlinearity are set to be parameters. Second, at each iteration less complicated model is solved since only the constraints related to the active binary variables are active. All constraints related to only the fixed binary variables are not active anymore.

As shown in Algorithm 1 two out of the three discrete capacity schedule variables are fixed and the model is solved to optimality or until an allowed time limit. Then the new set of variables are fixed and the model is re-solved for the third. At each step the model finds the values for the free set of binary and integer variables, its associated decision variables such as shift or run rate changes, inventory, distribution and overtime plan. This process continues until the objective value (and variables) remain the same. Note that the solution obtained at each iterative stage is either an improvement in the objective or terminates with no change. Gap tolerance is nominally set to 0.1% and a time is can be allowed for each iteration if the desired gap is not achieved. This process continues until the gap between the objective value of two iterations is with in a pre-defined error (ε). Note that the optimal solution is either increasing at each of the three stages iteration or unchanged. As there are a finite number of binary and integer assignments, the process will converge given that the algorithm is stopped when ever a solution is repeated. Note that an initial solution is required denoted as (R_0, L_0, D_0). This initial solution can be obtained by setting down weeks to the required down weeks (due to holidays, maintenance or upgrading) and all shifts and rates to the initial schedule.

Algorithm 1 Fix and Re-Solve Heuristic Algorithm

```
z* ← +∞; ε ← 1e0;                  ▷ Ititialize optimal value and gap
n ← 0;                             ▷ n is the number of times solution remain the same
z ← z(R_0, L_0, D_0);              ▷ initialalize shifts, rate, down
loop
    if z − z* < ε then             ▷ check if the gap is reached
        z* ← z; n ← 0;             ▷ set counter to zero, update objective
    else
        n ← n + 1
        If n ≥ 4 break;            ▷ break the loop
    end if
    fix Z, R;                      ▷ Fix down weeks and run rate
    solve;                         ▷ Solve the model for L
    z ← Objective;                 ▷ set objective value
    if z − z* < ε then             ▷ Check if the pre-defined gap is reached
        z* ← z; n ← 0;
    else
        n ← n + 1
        If n ≥ 4 break;
    end if
    fix Z, L;                      ▷ Fix down weeks and shifts
    solve;                         ▷ Solve the model for R
    z ← Objective;
    if z − z* < ε then
        z* ← z; n ← 0;
    else
        n ← n + 1
        If n ≥ 4 break;
    end if
    fix R, L;                      ▷ Fix run rate and shifts
    solve()                        ▷ Solve the model for Z
    z ← Objective;
end loop
```

3.5 Case Study

In this section a real-life vehicle production supply chain network is investigated to illustrate the applicability of the proposed optimization model. Background and data for the case study are given first. Then the results are presented and discussed. Sensitivity analysis is also conducted. Several alternative scenarios of the problem are further investigated and compared with original model solution.

3.5.1 Background

A supply chain network of a major vehicle producer in the US is considered. The vehicle production firm is a multinational firm associated with manufacturing, distributing and marketing of automobiles and its components. It manufactures and sells a variety of vehicle models such as SUV, trucks, vans and cars. Their network consists of multiple assembly plants producing multiple vehicle types (nameplates). The system modeled consists of inbound part and subassembly shipment to the assembly plants, assembly, and outbound shipment to destination markets. A schematic of the system is shown in Figure 3.3.

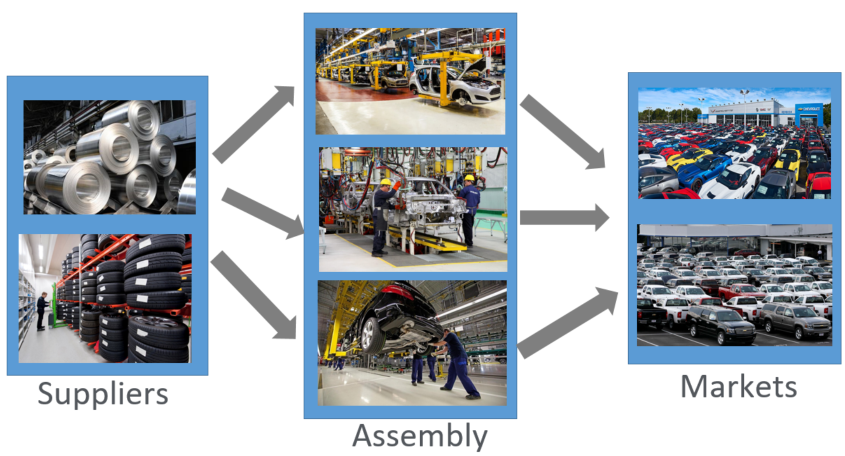


Figure 3.3: Material Flow of the Vehicle Manufacturer

For this case study the following assumptions are made:

- Investments have been made that define the set of vehicle configurations that can be produced at each assembly plant.
- The model is currently restricted to North American plants and markets but can be easily extended to a broader network if data is available.
- Shift change lead time is three months.
- Rate change lead time is six months.
- Maximum allowable over time in every two consecutive months is 50%.
- There are 5 working days a week.
- Run rates can be varied within known limits, in particular an upper bound exists on run rates due to quality, tooling and elemental operation time constraints.

3.5.2 Data

For this problem an average of 59 (I) nameplates, 17 (J) North America plants, 3 (S) shifts maximum, 24 (T) monthly time periods, 7 (R) possible run rates and 98 (M) markets matching the states/provinces of North America are considered. The network is shown on the map in Figure 3.4. Assembly plants are located in the red bullets and demand markets are located in the blue bullets. Because the data used in for the case study is confidential actual model names are not used. Instead they are labeled with numbers from 1 to 59.

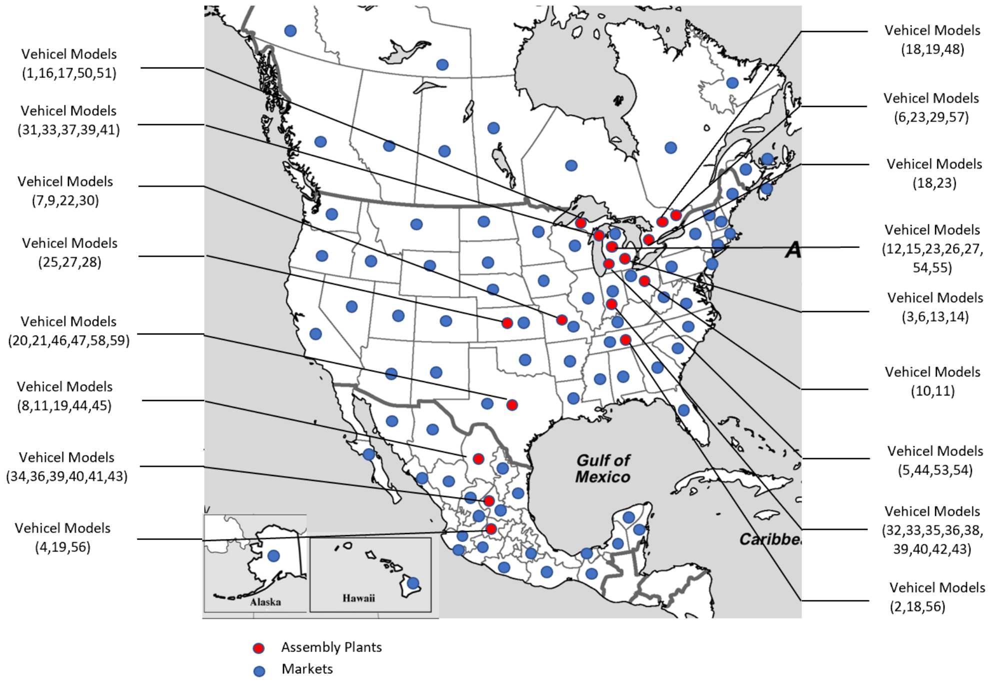


Figure 3.4: Production Network of the Vehicle Manufacturer

3.6 Case Study Results and Discussion

Prior to linearization, the MIP model for the case has $JT(3S + R + 3) = 7,752$ binary variables and $IT(5J + 1 + JM + M) = 2,619,600$ continuous variables and $JT = 408$ integer variables. There are about $T(3I + 9J + 1.16RJ + 2IJ + IJM) + SJ(1.33 + R)) + 3RJ = 2,730,947$ constraints. After linearization additional $4RJST = 34,272$ continuous variables and $16RJST = 137,088$ constraints are defined. In practice the problem is significantly smaller since only the feasible nameplate/plant combinations are necessary. At a maximum average of 5 feasible nameplates per plant, this reduces the IJ problem characteristics by a factor of at least 10. AMPL with CPLEX solver is used to solve the model using the two approaches discussed in the previous section. Initial experimentation indicated that solving the full mode is computationally inefficient even after linearization.

However, solving the model with the heuristic iterative approach seems to be more computationally efficient. Next, results for the vehicle manufacturer case study using both approaches are presented with detailed discussion and comparison. Because data for the case study is confidential, actual cost is not shown, instead percentages are used. All experiments were run on an Intel(R) core(TM) i7-8550U CPU @ 1.80 GHz and 8 GB RAM machine.

3.6.1 Potentially Global Optimal Approach Results

Solving the linearized model for the case study is computationally complex given the size of the problem. Solving the model with the full size problem is first attempted. The model cannot be solved optimally with in 24 hours. To improve computation time, problem is divided into smaller size sub problems which can be easier to solve. Problem separation is based on decomposing the set of plants and products into non-overlapping sets based on common vehicle models each plant can produce. Each plant set producing similar products can be seen as a problem and can be solved separately. This will allow solving smaller size problems which will improve computation time. The separated problems are shown in Figure 3.5. The size of each problem in terms of number of plants and products is listed in Table 3.1. Computational time to solve each problem as well as optimality gap are also presented in Table 3.1. Optimality gap is a measure of the quality of the current solution. In MIP, the gap measures how far is the current best integer solution is a way from the best possible lower bound on the objective function. Initially, the LP relaxation is set to be the lower bound of the objective value, then as the solution improves a better bound is found. A value of less than or equal to 0.1% is usually considered as an optimal solution. Therefore, the gap tolerance is set to 0.1%. Solution time is set to 24 hours. Note that for Cars problem solution is not optimal since within 24 hours the reported gap is 3.26%.

The other four sub-problems are solved optimally within reasonable time. All Sub-problems are solved using AMPL with CPLEX solver.

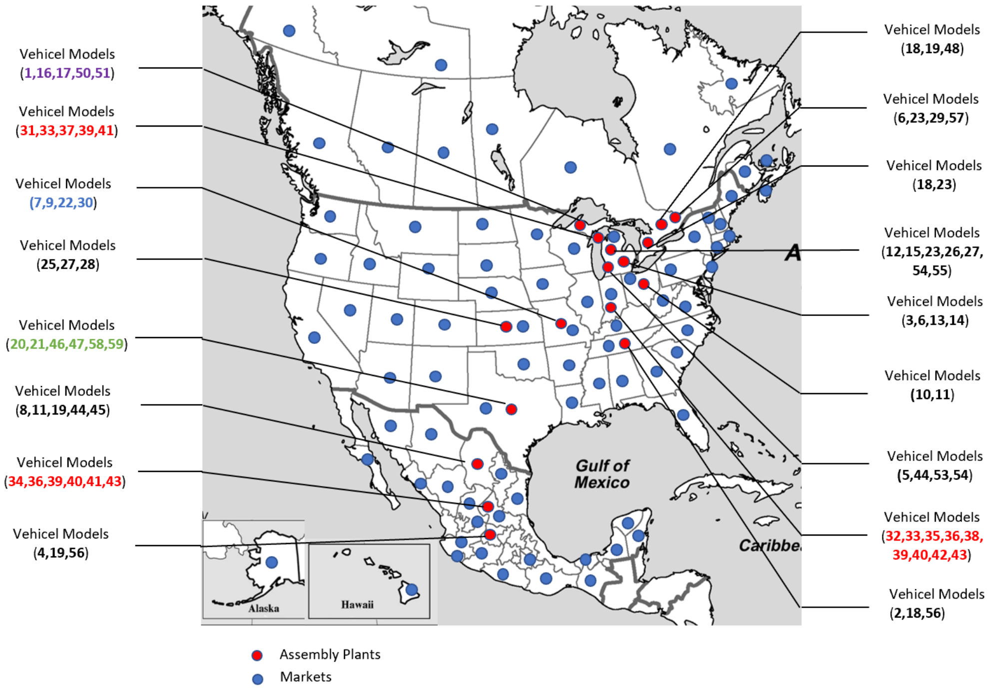


Figure 3.5: Problem Separation of the Vehicle Production Network

	Size (Plants,Vehicles)	CPU	Gap
SUV	(1,6)	7(s)	0.01%
Mid-size SUV	(1,5)	33.5(s)	0.10%
Vans	(1,4)	13.4(s)	0.08%
Trucks	(3,13)	9.5(min)	0.10%
Cars	(11,31)	24(hr)	3.26%*

Table 3.1: Results of the Separated Problems Using Potentially Global Optimal Approach

The total cost is aggregated for all five problems to represent the total cost of the full problem. The breakdown of the optimal costs in percentage is given in Figure 3.6.

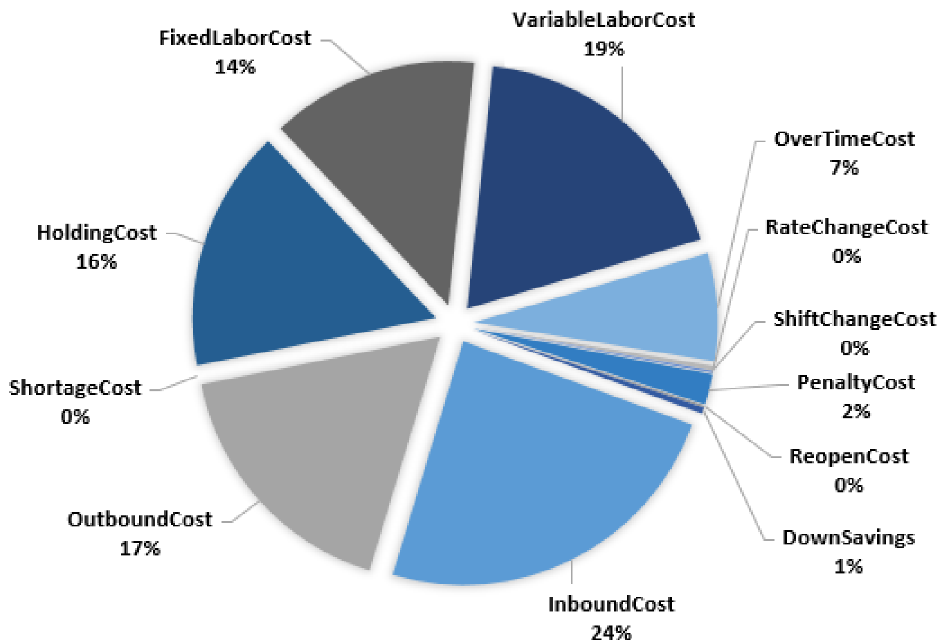
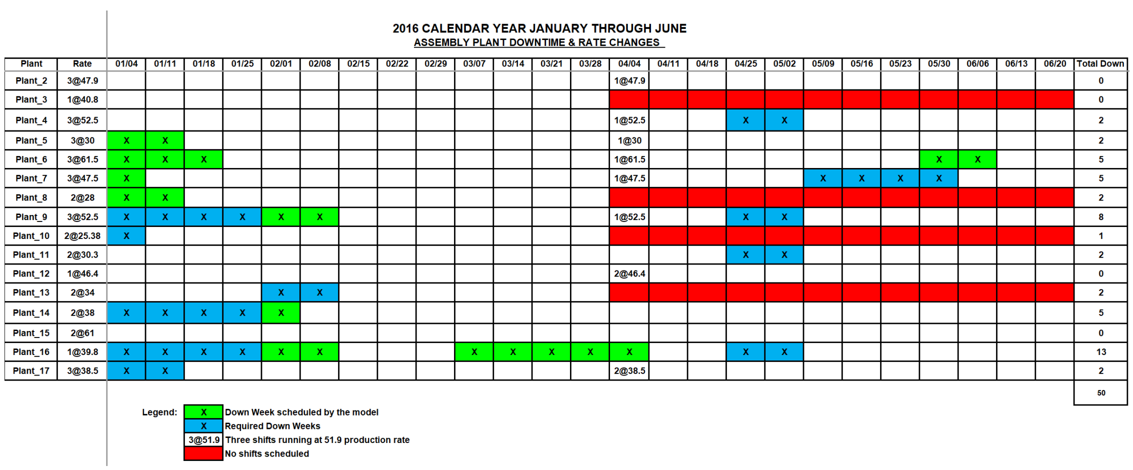


Figure 3.6: Breakdown of the Optimal Cost Obtained by the Potentially Global Optimal Approach

From Figure 3.6 it is observed that inbound cost, labor cost and holding cost are major costs of the total cost. Production plant cost is due to logistics cost, labor cost and inventory holding cost. Therefore, the company can gain potential saving by further analyzing and studying these costs. For example, finding other ways to ship and transport raw materials and finished goods. The optimal capacity plan for assembly plants including the optimal shift schedule, run rate and down periods is given in Figures 3.7, 3.8, 3.9, and 3.6.1. All other optimal decisions for a sample plant is shown in Figure 3.10 including capacity, production during regular and over time, shipments, and shortages by months.

In analysing the results, it is observed that all options to regulate capacity (shift, rate, down period, and over time) have been employed. Run rate is increased to the maximum allowable level with removal of some shifts in most production plants. In a few plants the run rate is decreased to the minimal level. This adjustment of run rate provides the most flexibility to capacity. It is further observed that with the fastest rate, the model adjusts to small demand changes using overtime production. Large changes in demand is captured by addition of shifts. The model tries to satisfy demand from the cheapest plant if extra capacity exists. If it is not economical to operate a plant, a single shift and slowest rate are selected to cut production. In some cases plants are closed (i.e. no shifts are scheduled). Due to large fixed labor cost of regular time production, overtime capacity is usually utilized before adding shifts.


2016 CALENDAR YEAR JANUARY THROUGH JUNE

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	01/04	01/11	01/18	01/25	02/01	02/08	02/15	02/22	02/29	03/07	03/14	03/21	03/28	04/04	04/11	04/18	04/25	05/02	05/09	05/16	05/23	05/30	06/06	06/13	06/20	Total Down
Plant_2	3@47.9														1@47.9												0
Plant_3	1@40.8																										0
Plant_4	3@52.5														1@52.5			X	X								2
Plant_5	3@30	X	X												1@30												2
Plant_6	3@61.5	X	X	X											1@61.5								X	X			5
Plant_7	3@47.5	X													1@47.5					X	X	X	X				5
Plant_8	2@28	X	X																								2
Plant_9	3@52.5	X	X	X	X	X	X								1@52.5			X	X								8
Plant_10	2@25.38	X																									1
Plant_11	2@30.3																	X	X								2
Plant_12	1@46.4														2@46.4												0
Plant_13	2@34					X	X																				2
Plant_14	2@38	X	X	X	X	X																					5
Plant_15	2@61																										0
Plant_16	1@39.8	X	X	X	X	X	X				X	X	X	X	X			X	X								13
Plant_17	3@38.5	X	X												2@38.5												2
																											50

Figure 3.7: Optimal Capacity Plan by Weeks for Assembly Plants Using Model Linearization

2016 CALENDAR YEAR JULY THROUGH DECEMBER

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	06/27	07/04	07/11	07/18	07/25	08/01	08/08	08/15	08/22	08/29	09/05	09/12	09/19	09/26	10/03	10/10	10/17	10/24	10/31	11/07	11/14	11/21	11/28	12/05	12/12	12/19	12/26	Total Down
Plant_3	0		1@32.6																										0
Plant_4	1@52.5		1@57.7																										0
Plant_5	1@30		2@36																										0
Plant_6	1@61.5		2@73.8													1@73.8													0
Plant_7	1@47.5	X	2@57													1@57											X		2
Plant_8	0		1@28																										0
Plant_9	1@52.5																												0
Plant_10	0		X	X	1@30.4																								2
Plant_11	2@30.3		2@36.3																										0
Plant_12	2@46.4																												0
Plant_13	0		1@40.8																										0
Plant_14	2@38		2@45.6																										0
Plant_15	2@61		2@73.2																										0
Plant_16	1@39.8		1@47.8																										0
Plant_17	2@38.5																												0
																													4

Legend:
- X — Down Week scheduled by the model
- X — Required Down Weeks
- 3@51.9 — Three shifts running at 51.9 production rate
- No shifts scheduled

Figure 3.8: Optimal Capacity Plan by Weeks for Assembly Plants Using Model Linearization

2017 CALENDAR YEAR JANUARY THROUGH JUNE

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	01/02	01/09	01/16	01/23	01/30	02/06	02/13	02/20	02/27	03/06	03/13	03/20	03/27	04/03	04/10	04/17	04/24	05/01	05/08	05/15	05/22	05/29	06/05	06/12	06/19	06/26	Total Down
Plant_1	2@62.2	1@62.2													2@62.2													0
Plant_2	1@57.4																											0
Plant_3	0	2@32.6	X																									1
Plant_4	1@57.7	X	X	X									X	X	2@57.7													5
Plant_5	2@36	1@36																										0
Plant_6	1@73.8														2@73.8													0
Plant_7	1@57																				X	X	X	X	X	X		6
Plant_8	1@28	X	X																						X			3
Plant_9	1@52.5																									X	X	2
Plant_10	1@30.4																											0
Plant_11	2@36.3														1@36.3													0
Plant_12	2@46.4														1@55.6													0
Plant_13	1@40.8	X	X																									2
Plant_14	2@45.6	X	1@45.6												2@45.6													1
Plant_15	2@73.2																											0
Plant_16	1@47.8																											0
Plant_17	2@38.5	2@64.1																										0
																												20

Legend:
- X — Down Week scheduled by the model
- X — Required Down Weeks
- 3@51.9 — Three shifts running at 51.9 production rate
- No shifts scheduled

Figure 3.9: Optimal Capacity Plan by Weeks for Assembly Plants Using Model Linearization

2017 CALENDAR YEAR JULY THROUGH DECEMBER

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	07/03	07/10	07/17	07/24	07/31	08/07	08/14	08/21	08/28	09/04	09/11	09/18	09/25	10/02	10/09	10/16	10/23	10/30	11/06	11/13	11/20	11/27	12/04	12/11	12/18	12/25	Total Down
Plant_1	2@62.2																											0
Plant_2	1@57.4																											0
Plant_3	0														1@32.6													0
Plant_4	2@57.7	1@57.7					X	X																				2
Plant_5	1@36	2@36																										0
Plant_6	2@73.8							X	X	X	X	X	X															6
Plant_7	1@57	2@57													1@57													0
Plant_8	1@28	X																								X		2
Plant_9	1@52.5	1@47.2																										0
Plant_10	1@30.4	X	X																									1
Plant_11	1@36.3	X	X	2@36.3																								1
Plant_12	1@55.6																											0
Plant_13	1@40.8						X	X	X																			3
Plant_14	0		1@30.4																									0
Plant_15	2@73.2																											0
Plant_16	1@47.8	X	2@47.8																									1
Plant_17	2@64.1	X																										1
																												17

Legend:
- X — Down Week scheduled by the model
- X — Required Down Weeks
- 3@51.9 — Three shifts running at 51.9 production rate
- No shifts scheduled

Capacity Plan by Weeks for Assembly Plants Using Model Linearization

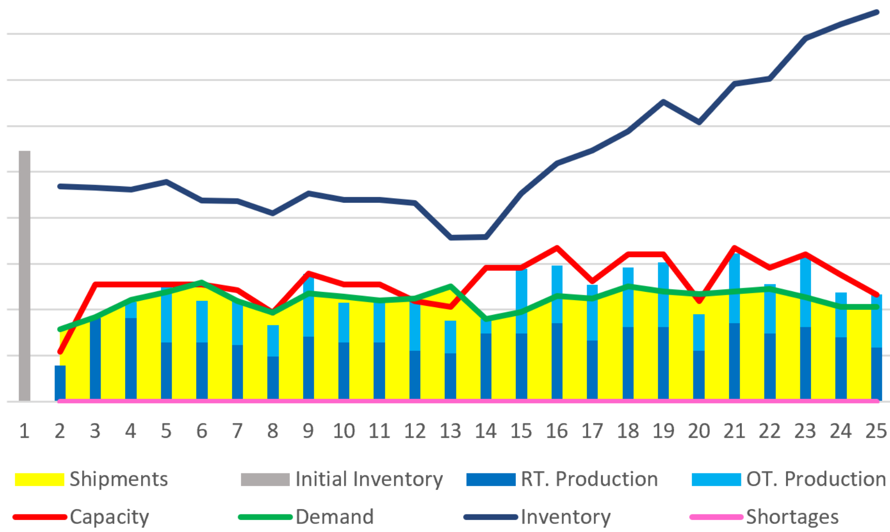


Figure 3.10: Optimal Solution for Sample Plant Using Model Linearization

3.6.2 Fix and Re-solve Iterative Heuristic Approach Results

Table 3.2 shows computational time of each sub-problem obtained by running the Fix and Re-solve algoritm using AMPL with CPLEX solver. For each stage the gap tolerance is as before set to 0.1% and time limit is set to three minutes. In the following section a comparison of the two approaches is provided. The breakdown of the optimal cost in using the iterative method is given in Figure 3.11 which shows that logistics, labor and inventory holding cost are three most significant costs of the total cost of production plants. The same conclusion is obtained using the global optimal approach. The optimal capacity plan for assembly plants including the optimal shift schedule, run rate and down periods is given in Figures 3.12, 3.13, 3.14, and 3.15. All other optimal decisions for a sample plant is shown in Figure 3.16, including monthly capacity, production during regular and over time, and shipments to markets.

	Size (Plants,Vehicles)	CPU
SUV	(1,6)	1.36(s)
Mid-size SUV	(1,5)	0.98(s)
Vans	(1,4)	1.15(s)
Trucks	(3,13)	7.87(s)
Cars	(11,31)	194.2(s)

Table 3.2: Fix and Re-Solve Heuristic Approach Results

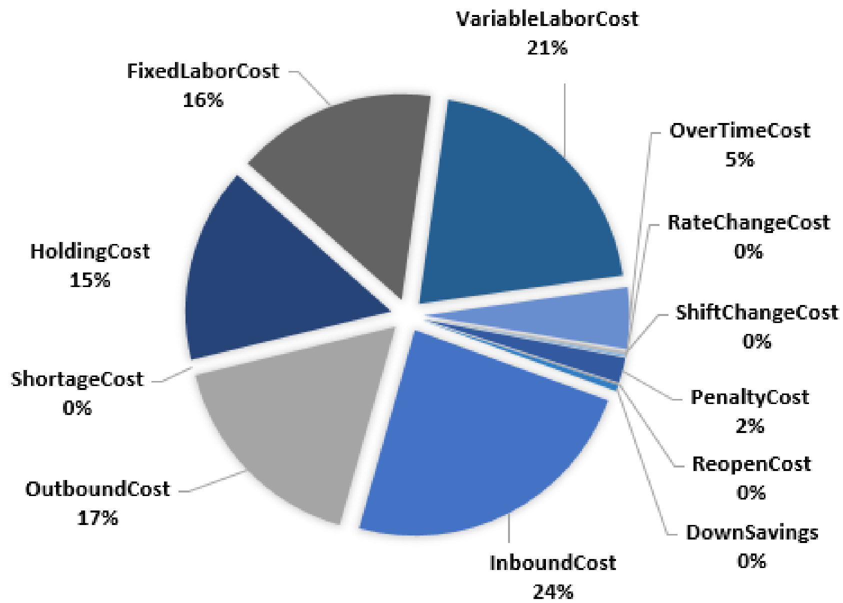


Figure 3.11: Breakdown of the Optimal Annual Cost Obtained by Fix and Re-Solve Heuristic

2016 CALENDAR YEAR JANUARY THROUGH JUNE

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	01/04	01/11	01/18	01/25	02/01	02/08	02/15	02/22	02/29	03/07	03/14	03/21	03/28	04/04	04/11	04/18	04/25	05/02	05/09	05/16	05/23	05/30	06/06	06/13	06/20	Total Down
Plant_1	3@51.9														2@51.9												0
Plant_2	3@47.9														1@47.9												0
Plant_3	1@40.8															X	X	X	X								4
Plant_4	3@52.5														2@52.5			X	X								2
Plant_5	3@30														2@30												0
Plant_6	3@61.5	X	X	X											1@61.5				X					X			5
Plant_7	3@47.5	X	X												2@47.5				X	X	X	X					6
Plant_8	2@25.9	X	X	X					X	X					1@25.9									X			6
Plant_9	3@28	X	X	X	X	X	X	X	X	X	X	X	X		1@28			X	X								14
Plant_10	2@52.5	X	X	X			X	X	X	X					1@52.5												7
Plant_11	2@30.3																	X	X								2
Plant_12	1@46.4														2@46.4												0
Plant_13	2@34					X	X	X	X																		4
Plant_14	2@38	X	X	X	X	X									1@38												5
Plant_15	2@61																										0
Plant_16	1@40	X	X	X	X													X	X								6
Plant_17	3@38.5	X	X																								2
																											63

Legend:
- X (green) Down Week scheduled by the model
- X (blue) Required Down Weeks
- 3@51.9 Three shifts running at 51.9 production rate
- (red) No shifts scheduled

Figure 3.12: Optimal Capacity Plan by Weeks for Assembly Plants Using Fix and Re-Solve Heuristic

2016 CALENDAR YEAR JULY THROUGH DECEMBER

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	06/27	07/04	07/11	07/18	07/25	08/01	08/08	08/15	08/22	08/29	09/05	09/12	09/19	09/26	10/03	10/10	10/17	10/24	10/31	11/07	11/14	11/21	11/28	12/05	12/12	12/19	12/26	Total Down
Plant_1	2@51.9		2@62.3																										0
Plant_2	1@47.9		1@57.5																										0
Plant_3	1@40.8																												0
Plant_4	'2@52.5		2@63	X	X											X													3
Plant_5	2@30		2@36																										0
Plant_6	1@61.5		2@73.8													1@73.8													0
Plant_7	2@47.5	X	2@57													1@57											X		2
Plant_8	1@25.9																												0
Plant_9	1@28		X	X								X	X			X	X				X	X						X	9
Plant_10	1@52.5		1@63																			X							1
Plant_11	2@30.3		2@36.3																										0
Plant_12	2@46.4		2@55.6																										0
Plant_13	2@34		2@40.8																										0
Plant_14	1@38		2@45.6																										0
Plant_15	2@61		2@73.2																										0
Plant_16	1@40		1@41.8													2@41.8													0
Plant_17	3@38.5		3@46.1																										0
																													15

Legend:
- X — Down Week scheduled by the model
- X — Required Down Weeks
- 3@51.9 — Three shifts running at 51.9 production rate
- No shifts scheduled

Figure 3.13: Optimal Capacity Plan by Weeks for Assembly Plants Using Fix and Re-Solve Heuristic

2017 CALENDAR YEAR JANUARY THROUGH JUNE

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	01/02	01/09	01/16	01/23	01/30	02/06	02/13	02/20	02/27	03/06	03/13	03/20	03/27	04/03	04/10	04/17	04/24	05/01	05/08	05/15	05/22	05/29	06/05	06/12	06/19	06/26	Total Down
Plant_1	2@62.3																											0
Plant_2	1@57.5																											0
Plant_3	1@40.8	X													X													2
Plant_4	2@63	X	X	X	X	1@63	X						X	X	2@63									X	X	X	X	11
Plant_5	2@36																											0
Plant_6	1@73.8	2@73.8																										0
Plant_7	1@57	2@57				X	X								1@57	X					X	X	X	X	X	X		9
Plant_8	1@25.9	X	X																	X	X				X	X	X	7
Plant_9	1@28						X	X	X	X													X	X	X	X	X	9
Plant_10	1@63	X	X				X	X	X							X	X	X										8
Plant_11	2@36.3																											0
Plant_12	2@55.6														1@55.6													0
Plant_13	2@40.8	X	X	1@40.8																								2
Plant_14	2@45.6	X																										1
Plant_15	2@73.2																											0
Plant_16	2@41.8	1@41.8													2@41.8													0
Plant_17	3@46.1																											0
																												49

Legend:
- X — Down Week scheduled by the model
- X — Required Down Weeks
- 3@51.9 — Three shifts running at 51.9 production rate
- No shifts scheduled

Figure 3.14: Optimal Capacity Plan by Weeks for Assembly Plants Using Fix and Re-solve Heuristic

2017 CALENDAR YEAR JULY THROUGH DECEMBER

ASSEMBLY PLANT DOWNTIME & RATE CHANGES

Plant	Rate	07/03	07/10	07/17	07/24	07/31	08/07	08/14	08/21	08/28	09/04	09/11	09/18	09/25	10/02	10/09	10/16	10/23	10/30	11/06	11/13	11/20	11/27	12/04	12/11	12/18	12/25	Total Down
Plant_1	2@62.3														3@62.3													0
Plant_2	1@57.5	2@57.5													1@57.5											X	X	2
Plant_3	1@40.8			X	X	X																						3
Plant_4	2@63	X					X	X							1@63					X	X							5
Plant_5	2@36																											0
Plant_6	2@73.8						X	X	X	X	X	X			3@73.8													6
Plant_7	1@57	2@57										X			1@57							X	X		X	X	X	6
Plant_8	1@25.9	X																									X	2
Plant_9	1@28	X	X									X	X			X	X	X	X			X						9
Plant_10	1@63	X	X				X	X				X				X	X					X						8
Plant_11	2@36.3	X	X																									2
Plant_12	1@55.6						X	X	X	X	X	X	X	X	X	X	X	X			X	X	X		X	X	X	18
Plant_13	1@40.8						X	X	X						2@40.8													3
Plant_14	2@45.6																											0
Plant_15	2@73.2																											0
Plant_16	2@41.8	X	1@41.8																									1
Plant_17	3@46.1	X																										1
																												66

Legend:
- X — Down Week scheduled by the model
- X — Required Down Weeks
- 3@51.9 — Three shifts running at 51.9 production rate
- No shifts scheduled

Figure 3.15: Optimal Capacity Plan by Weeks for Assembly Plants Using Fix and Re-solve Heuristic

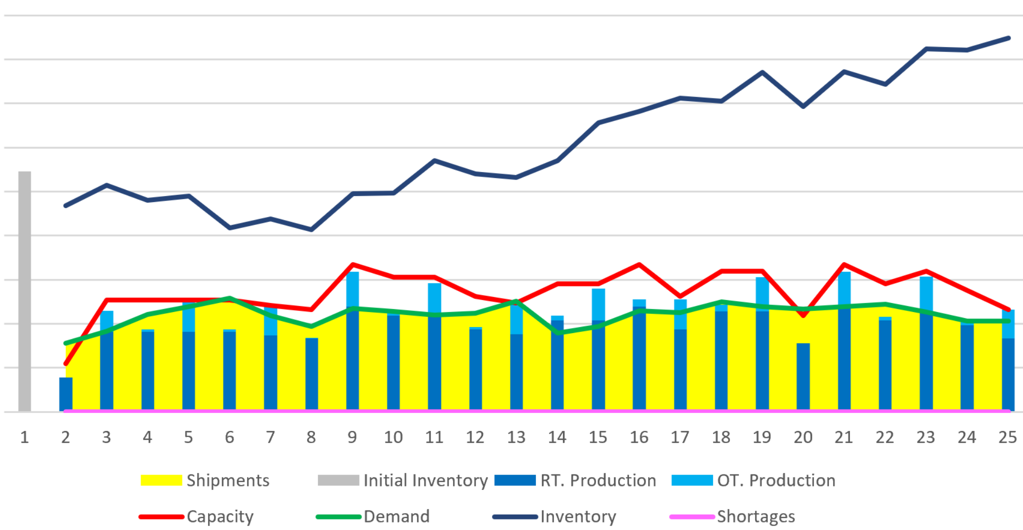


Figure 3.16: Optimal Solution for a Sample Plant Using Fix and Re-solve Heuristic

3.6.3 Comparison of Solution Approaches

Results suggest that the Fix and Re-solve heuristic algorithm is capable of producing near-optimal solutions in reasonable time. A comparison of the linearized and iterative results is summarized in Tables 3.3 and 3.4. The Fix and Re-solve approach results are very close to the results obtained from the global optimal approach. The gap between the optimal values using iterative approach and linearized approach is obtained to measure the quality of the iterative results. For the cars problem, the iterative approach objective value is compared to the best lower bound value, since the linearized solution is not optimal. The computational time improves significantly with the iterative approach. Cars problem has a 3.26% gap with the linearized formulation after running CPLEX for 24 hours. In approximately 3 minutes the iterative procedure produces a solution with 4% difference compared with best lower bound. Shift change occurs more often with 43 shift changes during the 24 months planning horizon in the global optimal approach as opposed to 35 shift changes the iterative approach . Run rate however changes more often in the iterative approach with 16 run rate changes during the 24 months planning horizon and 15 run rate changes in the global optimal approach. It is observed that the global optimal approach produces more plant closures (no shifts are scheduled) than in the iterative approach. The iterative approach produces 112 total down weeks as opposed to 91 down weeks using linearization approach during the 24 months planning horizon. Note that the total required down weeks is 76.

	CPU	Gap
SUVs		
Linearized	7(s)	$\leq 0.1\%$
Heuristic	1.36(s)	1.3%
Mid-size SUVs		
Linearized	33.5(s)	$\leq 0.1\%$
Heuristic	0.98(s)	1.11%
Vans		
Linearized	13.4(s)	$\leq 0.1\%$
Heuristic	1.15(s)	1.55%
Trucks		
Linearized	9.5(min)	$\leq 0.1\%$
Heuristic	7.78(s)	1.08%
Cars		
Linearized	24(hr)	3.26%
Heuristic	3.2(min)	4.25%

Table 3.3: Comparison of Solution Approaches

	Linearized	Heuristic
No. of shift changes	43	35
No. of rate changes	15	16
No. of down weeks	91	112

Table 3.4: Comparison of Solution Approaches

3.7 Rolling Schedule Scheme for the Evaluation of the Capacity Planning Model

Actual data is available for a two-year period (2016 & 2017) for comparison. In the rolling schedule scheme, we plan to solve the model each month (period) using forecasted demand. For decisions that cannot be implemented directly in practice, such as hiring workers or reconfiguring the manufacturing resources, a fixed, firm plan is to be used for the required implementation lead time. Given the two years of actual decisions for comparison, the model is to be run using a one-year planning horizon to allow performance evaluation over a one-year period. That is the model is solved monthly with a one year planning horizon and then one month's worth of decisions is fixed at each solution instance.

As actual demand for the company is known for the current month due to booked orders, actual demand is to be used for the current month. For the remaining months in the planning horizon, forecasted demand is to be used. Each month, production by product and plant is determined along with shipments to markets which are optimized by the model. This model's production decisions were constrained by the number of shifts and run rates set SLT (shift schedule change implementation lead time) and RRLT (run rate change implementation lead time) periods prior. These firm decisions are also to be used for determining shift change and run rate change costs for the period. The model's current month production and shif decisions determines inventory and logistics costs. Costs for each period over the year, based on the model's decisions, are to be added and compared to costs for the known decisions implemented by the company. In practice, forecasts are used in planning for months beyond the current month. The available data set contained actual demand but not the actual forecasts used. Thus, to better evaluate performance the known forecast error distribution is to be used to randomly generate five sample deviation streams from the actual demand. These are then used in five runs to simulate demand when running the optimization model, but then the actual demand is used to compute the estimated cost components for using the model. In this way a fair comparison to how the model would perform in practice is obtained. Rolling schedule scheme is described and summarized in the flow diagram shown in Figure 3.17.

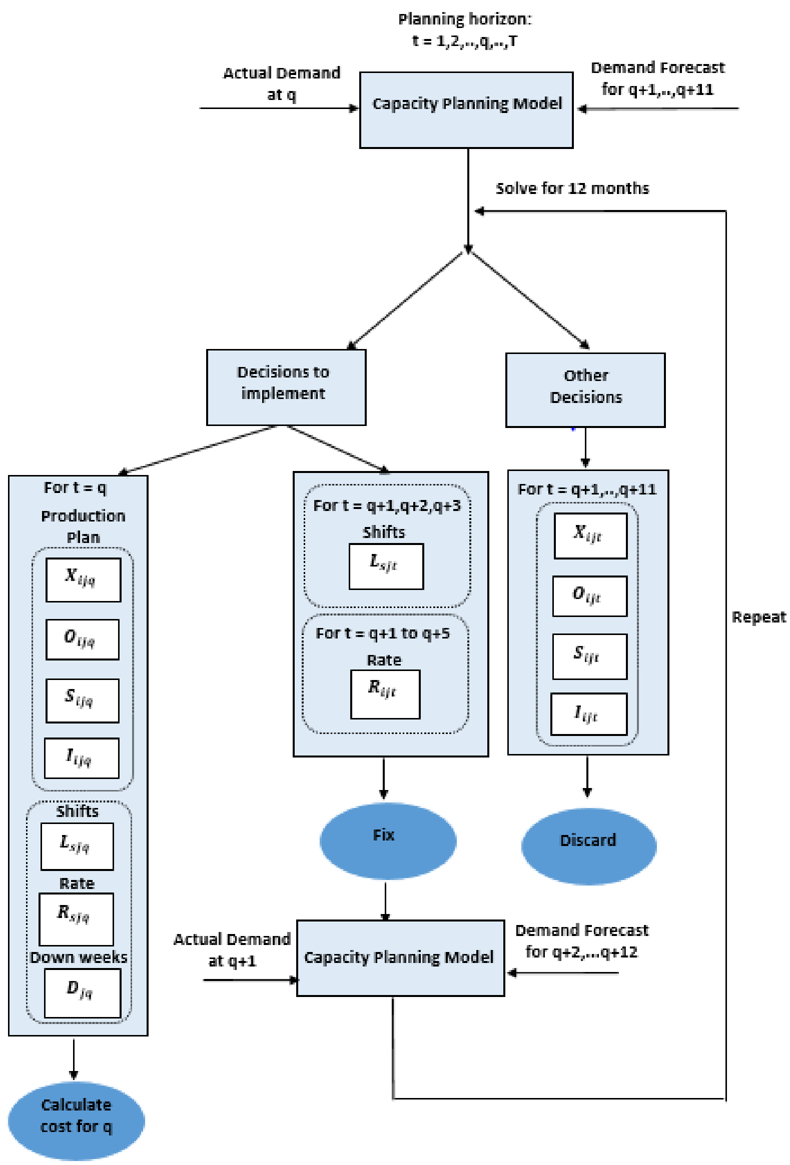


Figure 3.17: Rolling Schedule Scheme Flow Diagram

The capacity planning model is solved using the proposed Rolling schedule with the Fix and Re-solve heuristic algorithm. Results are summed over the five sub-problems and then compared to actual decisions. Table 3.5 shows a percentage improvement in cost compared to actual operations. Total cost has improved by 13%. This proves the value gained by employing shifts, run rate, down periods, and overtime jointly to set capacity level at production plants. The flexibility to adjust capacity with these levers is essential and prevents a huge loss.

	Logistics Cost	Production Cost	Inventory Cost
Savings	3.93%	14.59%	38.76%

Table 3.5: Savings Using Proposed Model With Rolling Schedule Compared to Actual Operations

3.8 Sensitivity Analysis

Supply chain networks are exposed to uncertainties therefore, sensitivity analysis is performed in order to identify the impact of uncertainties on the solution of the problem under study. The effect of model parameters which the most uncertainties come from is also investigated. The sensitivity analysis presented in the following sections is applied to the model with iterative approach.

3.8.1 The Value of Capacity Levers

It is informative to consider the relative value of each discrete capacity lever, namely shift, rate and down weeks. First consider the case where no shift changes are allowed throughout the 24 month planning horizon. In this case the initial shift schedule for each plant will be used for each month. The case where no run rate changes are allowed is also considered. For this case the initial run rate will be used for each month at each plant. Table 3.6 shows results from restricting planning options. Results are shown for both 18 and 24 months of the 24 month planning horizon. The 18 month values are included just to avoid any impact of end of horizon effects. Results are from using the iterative heuristic with a 3 minute limit at each iteration step for finding an improved solution. Total cost is increased by approximately 5% if shift schedule or run rate is not allowed to change with run rates having slightly more effect. Costs increase by 12% if neither lever is allowed. Down periods however have smaller effect, partly because labor rules prevent full cost recovery during those periods. The potential savings from allowing shifts and run rate to vary throughout the planning horizon is high for a capital-intensive industry such as automobile manufacturing with varying demand.

These potential savings from an integrated model are highly significant, and, in such a competitive market these options can determine financial sustainability.

Scenarios	18 Month Costs	24 Month Costs
Base Case	0%	0%
No shift or rate changes	12.27%	13.31%
No shift changes	4.60%	4.84%
No rate changes	5.17%	5.77%
No down periods	0.15 %	0.10%

Table 3.6: Cost Effect of Shift and Rate Changes

3.8.2 Parameter Sensitivity

Next the effect of changing certain cost parameters on the optimal total cost is studied. A variation in parameters of 100% or more from their original values is considered. Shift change cost $c_{sj}^{l+(-)}$ which includes planning, hiring and training for additions and severance/retraining for reductions is one of the most uncertain cost parameters in the model. The actual cost is difficult to estimate in practice thus it is useful to know its parameter sensitivity. Table 3.7 shows the percent change in the total cost compared to the base case when shift change cost is varied. Negative values mean that the cost decreased with the change. Doubling or eliminating shift change cost impacts total cost by +0.22% or -0.28% respectively the full horizon. Thus, while the cost is relevant, exact values are not critical for determining capacity plans when compared to the use of the model or its levers.

Scenarios	18 Months	24 Months
+300% S+R	1.40 %	0.99 %
+300% S	0.96 %	0.61 %
+100% S	0.55 %	0.22 %
Base Case	0.00 %	0.00%
-100% S	-0.03 %	-0.28%

Table 3.7: Shift (S) and Rate (R) Change Cost Variation Effect

The actual time to implement a change in shift schedule or a change in production rate are also uncertain. Therefore, we conduct sensitivity analysis on both shift and rate change lead time l^S, and l^R. The effect on the total cost is presented in Tables 3.8 and 3.9. Total cost has limited sensitivity to the exact value of cost to make a shift change, and is likewise only slightly sensitive to the lead time for implementing changes, relative to allowing consideration of such changes in operations planning. Changes in implementation time impacted total cost by less than 1%.

Lead Time(in months)	18 Months	24 Months
1	-0.60 %	-0.48%
2	-0.24%	-0.25%
Base Case (3)	0.00%	0.00%
4	0.64%	0.17%

Table 3.8: Shift Change Lead Time Variation Effect

Lead Time(in months)	18 Months	24 Months
1	-0.28%	-0.27%
3	-0.15%	-0.16%
Base Case (6)	0.00%	0.00%
12	0.39%	0.31%

Table 3.9: Rate Change Lead Time Variation Effect

3.9 Conclusion

A mathematical model is developed to make tactical decisions in the supply chain distribution network. The model proposed an optimal capacity plan for the capacitated manufacturing plants based on shift schedule, run rate selection and down periods. The model also proposed optimal production during regular and overtime, inventory levels, shortages and shipments by plants to demand markets. Two solution approaches are discussed, one provides optimal solution but with higher complexity and the other provides near optimal solution and is based on decomposition.

A case study is discussed to show the implementation of this model. This application used actual data from a major automotive producer in the U.S. Two solution approaches were discussed and compared. The model was solved using AMPL with CPLEX and solver for a 24-month planning horizon. Results showed that the production plant cost is primarily due to logistics, labor, and inventory holding costs. Therefore, the company can gain potential saving by further analyzing and studying these costs. Results also showed how capacity level changes for each production plant at each month. Capacity changes are facilitated by the flexibility of the model to add or eliminated shifts and the possibility to re-rate the production line. Increasing run rate is a general trend for all assembly plants. Increasing the run rate to the maximum value whenever it's possible is preferred over scheduling of shifts because of the high cost of labor needed to operate a shift compared to run rate increase cost. Essentially, use of overtime allows flexibility to mild monthly demand dynamics and avoids the fixed labor cost associated with adding shifts. It is observed that the model utilizes overtime as opposed to scheduling additional shifts to meet demand whenever possible. Overtime is kept at levels within the pre-defined organizational limits on over time usage.

Additionally, the model closes some manufacturing plants (schedules no shifts) for some period of time and re-open them again if needed. At the time of plant shut down demand is either satisfied from inventory or from other plants. It is also observed that the model utilizes down periods almost every month to save on variable labor cost. It was proven that the iterative approach gives a solution that is very close to the optimal solution. It's also more efficient in terms of computational time to solve the model especially for large problem. Sensitivity analysis of model parameters showed the effect on total cost when some model parameters are varied. This gives insight to company managers on potential savings.

This model has the advantage of integrating shift schedule, run rate, and down periods in one model which are the main component of the production capacity. This combination provides more flexibility to management, reflects reality, and presents a new model not previously discussed in the literature to my knowledge. This integration makes the proposed model unique since other models seen in the literature lack this integration. The proposed model in this research is considered a useful tool for firms seeking better capacity management. Using company defined operational rules on inventory and overtime the proposed model was demonstrated to provide solutions at significantly lower cost than those actually implemented.

Chapter 4

ON THE VALUE OF FLEXIBILITY

Manufacturing flexibility has become a very important aspect during the last decades especially in the competitive field of production firms. Rapid changes in customer expectations and technology creates an increasingly uncertain environment. Therefore, firms should be able to produce a variety of products in the quantities that customers demand while maintaining profitability. Companies face uncertainties of many types and thus try to introduce flexibility in their operations to cope with uncertainties. A major source of uncertainty is usually demand and the uncertainty is in both product-mix and volume. However, additional flexibility comes with cost. Many companies consider adding extra flexibility with out accurate quantitative evaluation of the value for various levels of flexibility, that is the relationship between the actual need of flexibility and the cost of acquiring it. Methods to increase flexibility and their value should be connected to the type of uncertainties that the companies face. Volume flexibility can be achieved by adjusting production capacity by shifts, rates and overtime. Product mix flexibility can be achieved by adding extra tooling or machine parts to be able to produce a wider range of products in facilities.

In this chapter, the value of adding extra tooling at manufacturing plants to increase the number of different products that can be produced at the production plant is investigated. The capacity planning model discussed in Chapter 3 is extended for this purpose by adding necessary terms to account for additional flexibility in tooling. Then results are compared with the model without additional flexibility.

4.1 Mathematical Model

Same notation from the model discussed in Chapter 3 is used for the model in this chapter with additional necessary sets, parameters, and variables. A binary variable is defined to decide whether a plant j is to be tooled to produced nameplate i. For the model with extra flexibility a new set $\mathcal{N}_j$ is added to represent the potential products that can be considered to be produced at production plants which are not initially in set $\mathcal{F}_j$. A capital fixed tooling cost per vehicle model is assumed to be known. (For the case study used in this dissertation values were obtained through discussions with the corporate planning staff). The required tooling fixed cost is set as a parameter. New terms for the flex model are as follows. The following assumptions are made:

- A one time per planning horizon fixed tooling cost for producing a new nameplate is assumed to be known.
- Outbound cost per nameplate is known for existing flexibility set and is assumed to be the same for the nameplates in the extra flexibility set.

4.1.1 Notation

Indices and sets:

- $\mathcal{N}_j$: additional flexibility set, representing additional products that can be made at plant j which are not initially in set $\mathcal{F}_j$

Parameters:

- c_j^{flex}: fixed cost to add tooling to production plant for for the full planning horizon

Decision Variables:

$$\boldsymbol{E_{ij}} = \begin{cases} 1, & \text{if plant } j \text{ is to be tooled to produce nameplate } i \\ 0, & \text{if otherwise} \end{cases}$$

The objective function as before minimizes total cost of logistics, production, and inventory cost as well as shortages, shift and rate changes. In the flexibility model, tooling cost is added whenever a new product is to be produced in a plant. Note that the summation over nameplate (i) is extended to include nameplates in the original feasibility set (feasible products to be produced at each plant) $\mathcal{F}_j$ as well as nameplates in the extra flexibility set $\mathcal{N}_j$.

$$\begin{aligned} \text{Minimize: } & \sum_{i\in(F_j\cup N_j)} \sum_{j=1}^{J} c_{ij}^{in}. \sum_{t=1}^{T}(X_{ijt}+O_{ijt}) + \sum_{i\in(F_j\cup N_j)} \sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{t=1}^{T}(c_{ij}^{out}.S_{ijmt}) \\ & + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T}(c_{im}^{u}.U_{imt}) + \sum_{i\in(F_j\cup N_j)} \sum_{j=1}^{J}\sum_{t=1}^{T}(c_i^I.I_{ijt}) + \sum_{i=1}^{I}\sum_{t=1}^{T}(c_i^{\Delta I}.\Delta I_{it}) \\ & + \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{sj}^{f}.L_{sjt}) + \sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=1}^{T}(c_{rj}^{v}.L_{sjt}.R_{rjt}) \\ & + \sum_{i\in(F_j\cup N_j)} \sum_{j=1}^{J}\sum_{t=1}^{T}((1+\alpha_j).c_j^o.O_{ijt}) + \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}((c_{sj}^{l+}.L_{sjt}^{+}) + (c_{sj}^{l-}.L_{sjt}^{-})) \\ & + \sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T}((c_j^{r+}.R_{jt}^{+}) + (c_j^{r-}.R_{jt}^{-})) + \sum_{j=1}^{J}\sum_{t=1}^{T}(c_j^{ro}.RO_{jt}) \\ & - \sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{rj}^{ds}.c_{rj}^{v}.R_{rjt}.L_{wjt}.Z_{jt}/w_{jt}) + \sum_{i\in N_j}\sum_{j=1}^{J}(c_j^{flex}.E_{ij}) \end{aligned} \tag{4.1}$$

Constraints (4.2) and (4.3) are defined to activate production during regular and over time at production plants if a product is selected from additional flexibility set. M needs to large enough if the constraint is violated.

M can be bounded by the maximum capacity of production plant j. This is shown in equations (4.2) and (4.3).

$$X_{ijt} \leq M_x.(E_{ij}) \quad \forall i \in N_j, j \in J, t \in T \tag{4.2}$$

$$O_{ijt} \leq M_o.(E_{ij}) \quad \forall i \in N_j, j \in J, t \in T \tag{4.3}$$

Constraints (4.4) to (4.31) are same as described in Chapter 3 except that the set of feasible product is adjust to include both the original flexibility $\mathcal{F}_j$ and extra flexibility $\mathcal{N}_j$ sets.

$$\sum_{j=1}^{J} S_{ijmt} + U_{imt} = d_{imt} \quad \forall i \in F_j \cup N_j, m \in M, t \in T \tag{4.4}$$

$$X_{ijt} + O_{ijt} + I_{ij}^{0} = \sum_{m=1}^{M} S_{ijmt} + I_{ijt} \quad \forall i \in F_j \cup N_j, j \in J, t = 1 \tag{4.5}$$

$$X_{ijt} + O_{ijt} + I_{ij(t-1)} = \sum_{m=1}^{M} S_{ijmt} + I_{ijt} \quad \forall i \in F_j \cup N_j, j \in J, t > 1 \tag{4.6}$$

$$\sum_{j=1}^{J} I_{ijt} \geq \underline{\eta_{it}}. \sum_{m=1}^{M} d_{imt} - \sum_{j=1}^{J} I_{ijt}^{-} \quad \forall i \in F_j \cup N_j, t \in T \tag{4.7}$$

$$\sum_{j=1}^{J} I_{ijt} \leq \overline{\eta_{it}}. \sum_{m=1}^{M} d_{imt} + \sum_{j=1}^{J} I_{ijt}^{+} \quad \forall i \in F_j \cup N_j, t \in T \tag{4.8}$$

$$\Delta I_{it} = \sum_{j=1}^{J} (I_{ijt}^{+} + I_{ijt}^{-}) \quad \forall i \in F_j \cup N_j, t \in T \tag{4.9}$$

$$\sum_{j=1}^{J} I_{ijt} = I_i^e \quad \forall i \in F_j \cup N_j, j \in J, t = T \tag{4.10}$$

$$\sum_{i \in F_j \cup N_j} X_{ijt} \leq \sum_{r=1}^{R} (\beta_r.r_j^0.R_{rjt}).(wd_{jt}.(1 - Z_{jt}/w_{jt})). \sum_{s=1}^{S}((h_{sj}.(L_{sjt} - \gamma_{sjt}.L_{sjt}^{+}))$$

$$\forall j \in J, t \in T \tag{4.11}$$

$$\sum_{i \in F_j \cup N_j} O_{ijt} \leq \sum_{r=1}^{R} (\beta_r . r_j^0 . R_{rjt}) . (wd_{jt} . (1 - Z_{jt}/w_{jt})) . \sum_{s=1}^{S} (\overline{o}_s . L_{sjt}) \quad \forall j \in J, t \in T \tag{4.12}$$

$$\sum_{i \in F_j \cup N_j} (O_{ijt} + O_{ij(t+1)}) \leq o'_{jt} . (O_{jt}^C + O_{j(t+1)}^C) \quad \forall j \in J, t \geq 1 \tag{4.13}$$

$$L_{sjt} = L_{sj}^0 \quad \forall s \in S, j \in J, t = 1 \tag{4.14}$$

$$L_{2jt} \leq L_{1jt} \quad \forall j \in J, t \in T \tag{4.15}$$

$$L_{3jt} \leq L_{2jt} \quad \forall j \in J, t \in T \tag{4.16}$$

$$L_{sjt} - L_{sj}^0 - L_{sjt}^+ + L_{sjt}^- = 0 \quad \forall s \in S, j \in J, t = 1 \tag{4.17}$$

$$L_{sjt} - L_{sj(t-1)} - L_{sjt}^+ + L_{sjt}^- = 0 \quad \forall s \in S, j \in J, t > 1 \tag{4.18}$$

$$\sum_{t' \in [t - l^s + 1, t]} (L_{sjt'}^+ + L_{sjt'}^-) \leq 1 \quad \forall j \in J, t \geq l^s \tag{4.19}$$

$$U_{imt} \leq \overline{u}_{im} \quad \forall i \in I, m \in M, t \in T \tag{4.20}$$

$$\sum_{r=1}^{R} R_{rjt} = 1 \quad \forall j \in J, t \in T \tag{4.21}$$

$$R_{rj1} = R_{rj}^0 \quad \forall r \in R, j \in J \tag{4.22}$$

$$\sum_{r=1}^{R} (\beta_r . R_{rjt}) - (\beta_r . R_{rj(t-1)}) \leq \max\{\beta_r\} . R_{jt}^+ \quad \forall j \in J, t \in T \tag{4.23}$$

$$\sum_{r=1}^{R} (\beta_r . R_{rj(t-1)}) - (\beta_r . R_{rjt}) \leq \max\{\beta_r\} . R_{jt}^- \quad \forall j \in J, t \in T \tag{4.24}$$

$$\sum_{t' \in [t - l^R + 1, t]} (R_{jt'}^+ + R_{jt'}^-) \leq 1 \quad \forall j \in J, t \geq l^R \tag{4.25}$$

$$Z_{jt}/w_{jt} \leq L_{1jt} \quad \forall j \in J, t \in T \tag{4.26}$$

$$Z_{jt} \geq \overline{Z}_{jt} \quad \forall j \in J, t \in T \tag{4.27}$$

$$L_{1jt} - L_{1j(t-1)} \leq RO_{jt} \quad \forall j \in J, t > 1 \tag{4.28}$$

$$X_{ijt}, O_{ijt}, I_{ijt}, \Delta I_{it}, I_{ijt}^+, I_{ijt}^-, S_{ijmt}, U_{imt} \geq 0 \quad \forall i \in F_j, j \in J, m \in M, t \in T \tag{4.29}$$

$$L_{sjt}, L^+_{sjt}, L^-_{sjt}, R_{rjt}, R^+_{jt}, R^-_{jt}, RO_{jt}, E_{ij} \in \{0,1\} \quad \forall w \in W, j \in J, t \in T, r \in R \quad (4.30)$$

$$Z_{jt} \in N \quad \forall j \in J, t \in T \quad (4.31)$$

4.2 Model Discussion and Results

The model with extra tooling flexibility is a MINLP. Therefore, solution methodologies applied in Chapter 3, namely model linearization and Fix and Re-Solve Heuristic are used to solve the model. Since there are no additional non-linear terms, no further linearization is needed. For the vehicle producer case study considered in this research the model determines if within each sub-problem it is economical to invest in tooling to increase nameplates mix a production plant can produce. Figure 4.1 shows the number of plants and vehicle models in each of the previously described five sub-problems. Solid arrows indicate that a plant is currently tooled to produce each vehicle model in the feasible set of vehicles. Doted arrows indicate that a plant is currently tooled to produce some vehicle models in the feasibility set. Out of the five sub-problems two of them are relevant for the model with flexibility, namely truck plants and car plants, since each of these two sub-problems plant sets are partially tooled. For the other three sub-problems there is a single plant per problem which is already tooled to produce all products in its feasibility set. Another problem of interest is the impact on production decisions of exploiting the use of the same architecture for trucks and SUV models. This potentially allows production of products from both sets in the same assembly plant with addition of the proper nameplate specific tooling (in practice, plants require both compatible architecture and nameplate tooling to product a product). Therefore, the combined TRK and SUV problem is also considered in the following analysis. Figure 4.2 shows the number of plants and vehicle models in the combined TRK and SUV problem.

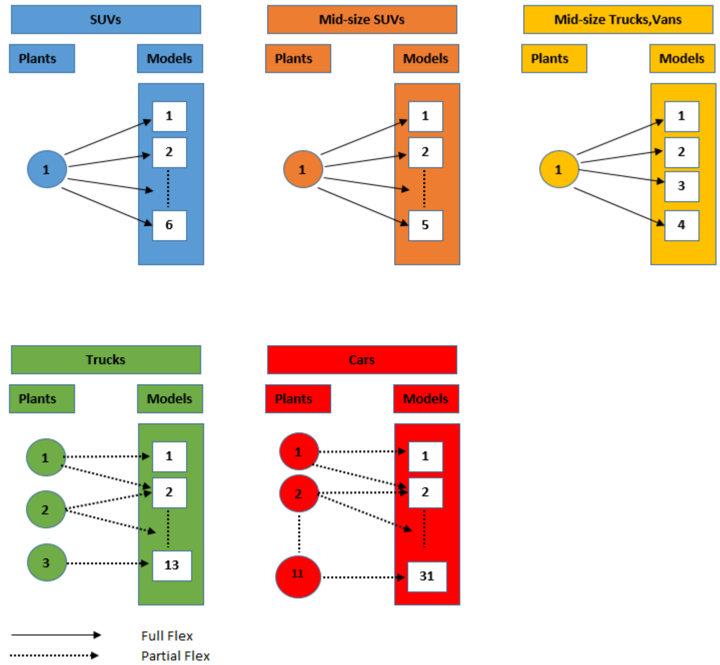


Figure 4.1: Current Product Mix Flexibility for the Case Study

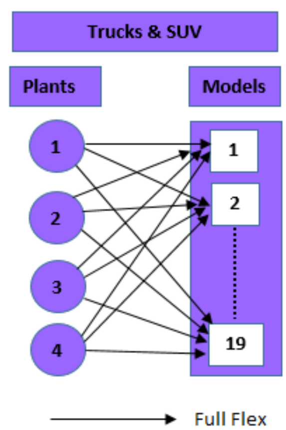


Figure 4.2: Combined TRK and SUV Problem

4.2.1 Full Flexibility Effect on Total Cost

The full flexibility model is solved for TRK, TRK combined with SUV, and Car problems. To evaluate the cost and benefits of product mix flexibility, two flexibility cases are considered and compared with no flexibility case. In the first case a positive fixed cost of tooling flexibility is considered. In the second case tooling cost is assumed to be zero. This second case was of interest just to explore the potential operating cost savings of flexibility. Flexibility model for each problem and for each flexibility case is solved using model linearization and also using the Fix and Re-solve heuristic discussed in Chapter 3. Table 4.1 present percentage changes in total cost when the full flexibility model with zero flexibility cost is compared with the model with no extra flexibility. For trucks only, reported savings from the linearized and the heuristic are 4.8% and 5.1% respectively over the less flex solution. Combining the SUV and TRK plants and assuming all 4 plants are tooled to produce all 19 models, the solution from both the linearized model and the heuristic saves 5.9% over the current SUV and TRK models solved separately. Car plants problem became more intractable with the addition of extra flexibility to the original capacity planning model. The linearized model can not be solved for CAR problem in reasonable time. The heuristic solves much faster than the linearized model. Reported savings for car models are 10.6% over the less flex solution. While the percent savings are significant in absolute terms, there would be extra tooling cost required to gain these savings that are not included in the model. Thus, the results simply indicate an upper bound on actual savings from a full flexibility configuration.

	Problem		
	TRK	SUV&TRK	CAR
Size	(3,13,39)	(4,19,76)	(11,31,341)
Linearized Model	4.8%	5.9 %	-
Time	10 mins	2 hours	-
Iterative Heuristic	5.1%	5.9%	10.6 %
Time	43 sec	3 mins	25 mins

Table 4.1: Percentage Change in Total Cost of Flexibility Model With Zero Flex Cost Compared With No Extra Flexibility

Table 4.2 shows the value of increased flexibility options when cost is included in the model for any additional tooling used. The cost of installing additional product tooling is amortized over its life expectancy. Results indicate that for TRK plants alone there is about 2% cost saving when the actual tooling cost is used. When SUV is combined with TRK and with actual tooling cost a cost savings of about 0.7% is achieved. For CAR plants the iterative heuristic is applied with a time limit of 1 hour per iteration. Cost savings of 2.5% is achieved.

	Problem		
	TRK	SUV&TRK	CAR
Size	(3,13,39)	(4,19,76)	(11,31,341)
Linearized Model	2.4%	0.68%	-
Time	30 mins	18 hours	-
Iterative Heuristic	2.1%	0.78%	2.5%
Time	10 mins	18 mins	24 hours

Table 4.2: Percentage Change in Total Cost of Flexibility Model With Additional Product Tooling Cost Compared With No Extra Flexibility

Table 4.3 shows the number of total flexibility options for each of the investigated flexibility cases. For TRK plants only, originally there are 20 flex options. With zero flex cost, the model uses the full flex capability to take advantage of plant cost structures and capacity, increasing the flex options to 39.

With new flex tooling cost included, only one flex option is added to the set of flex options making a 21 total flex options. When SUV is combined with TRK, the total number of flex options is 26 before adding extra tooling. Only one extra flex is added with positive flex cost. For CAR plants, originally there are a total of 41 flex options, which have increased to the maximum possible of 341 when zero flex cost is charged. Additional 7 flex options are utilized when the fixed tooling cost of added flex capability is included in the model.

Stats	Model		
	TRK	SUV&TRK	CAR
Size	(3,13,39)	(4,19,76)	(11,31,341)
Linearized Model with no flex	20	26	41
Linearized Model with zero flex cost	39	76	-
Iterative Heuristic with zero flex cost	39	76	341
Linearized Model with positive flex cost	21	27	-
Iterative Heuristic with positive flex cost	21	27	48

Table 4.3: Number of Total Flex Options Utilized

4.2.2 Full Flexibility Effect on Logistics Cost

In particular, there is interest in determining if logistics costs could be reduced by increasing flex capability. Tables 4.4 and 4.5 present percentage change in logistics cost (inbound plus outbound) and the interaction with other operational cost with and without tooling cost. Logistics and labor cost have improved the most with full flexibility in both TRK and SUV & TRK problems. The results reflect the different cost structures of the plants resulting from logistics costs and labor efficiencies.

Cases	TC	Logistics	Labor	Overtime
Zero flex cost	4.8%	6%	2.3%	3.8%
Positive flex cost	2.4%	1.7%	3%	0.6%

Table 4.4: % Savings in Cost Relative to the No Added Flex Model in Truck Plants

Cases	TC	Logistics	Labor	Overtime
Zero flex Cost	5.9%	9.3%	6.4%	-10%
Positive flex Cost	0.68%	2.2	%5%	-8.7%

Table 4.5: % Savings in Cost Relative to the No Added Flex Model in TRK-SUV Plants

In Table 4.6 results show an improvement in logistics and labor costs in car production plants. On the other hand, some other operational costs have increased. Re-opening cost has increased significantly when tooling cost is assumed to be zero. The drop in production in some plants creates plants shutdown for some periods of time. Therefore, a penalty to re-open the plant is counted. Some plants are temporarily closed, then shifts are later added to accommodate demand, incurring a penalty to re-open the plant. When actual tooling cost is included, plant closure has reduced. Overtime cost have increased in both cases.

Cases	TC	Logistics	Labor	Overtime	Re-opening
0_Flex_Cost	10.6%	18.5%	11.1%	-22%	-250%
+_Flex_Cost	2.5%	6.4%	4.4%	-20%	50%

Table 4.6: % Savings in Cost Relative to the No Added Flex Model in Car Plants

4.2.3 Full Flexibility Effect on Total Production

Figure 4.3 shows total production per plant for the entire planning horizon (24 months) at truck plants. The figure shows a modest shift in production between production plants as a result of full flexibility. Figure 4.4 shows the shift in production when SUV and TRk plants are combined.

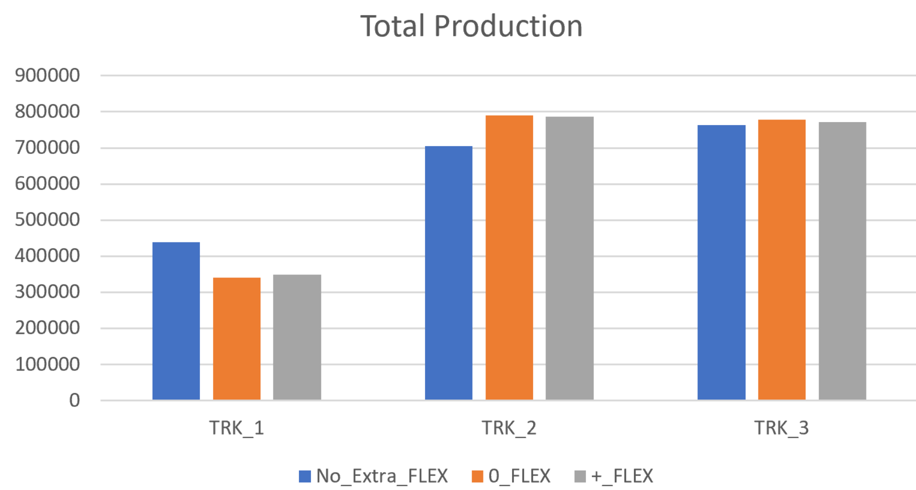


Figure 4.3: Total Production by Plant for Trucks

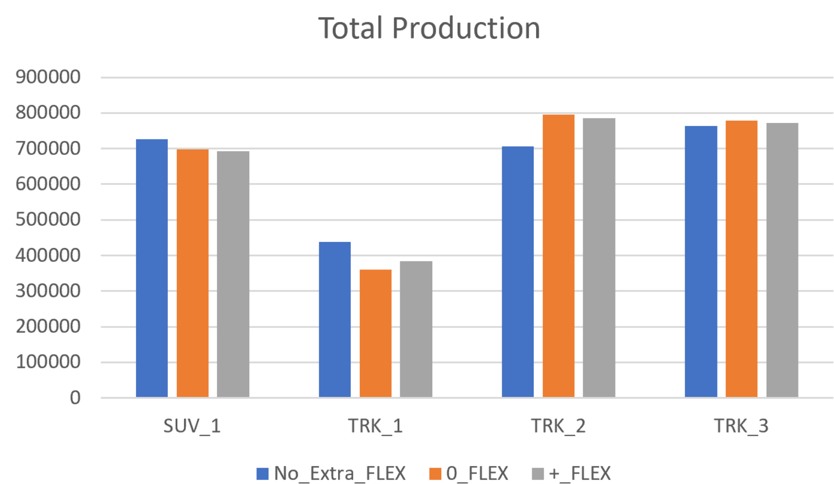
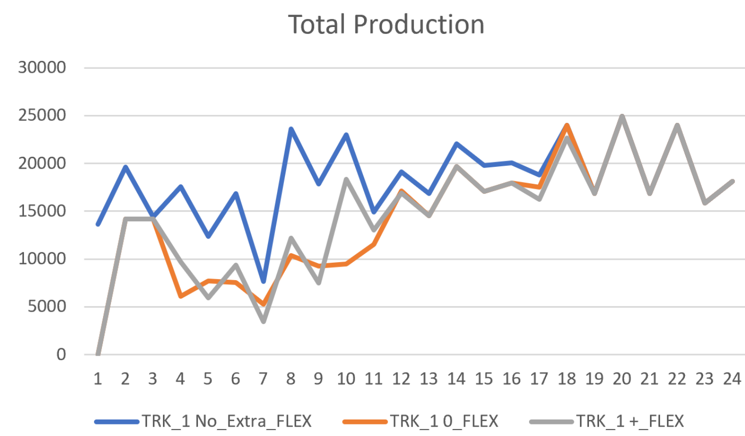
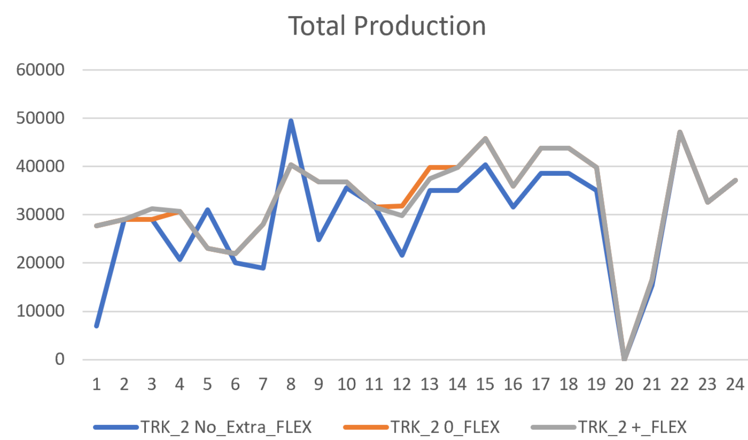


Figure 4.4: Total Production by Plant for SUV and TRK

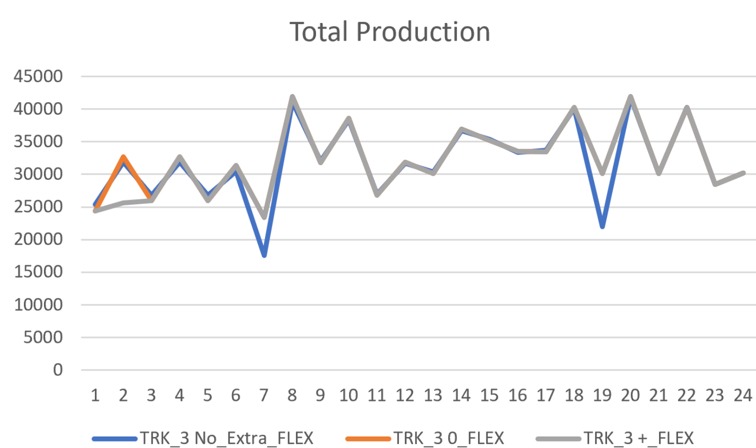
Graphs in Figure 4.5 show total production at each tucks plant by month for the model with no extra flexibility, zero tooling cost, and the model with positive tooling cost. Sub-figure (a), shows a drop in production as a result of an additional product flexibility. Production is shifted to other plants which explains the slight increase in production at plant-2 and plant-3 as shown in Sub-figure (b) and (c) respectively.



(a) Truck Plant-1



(b) Truck Plant-2



(c) Truck Plant-3

Figure 4.5: Flexibility Effect on Total Production at Truck Plants Over Time

From the results of the three problems, whenever it is possible the model satisfies demand from production plants where it is feasible and cheapest to be produced. Therefore, in some plants capacity plan is adjusted accordingly either by adding or removing shifts, increasing or decreasing the run rate or using overtime to meet the change in production and distribution plan.

4.3 Conclusion

In this chapter the value of product flexibility is investigated taking into account cost and benefits of this flexibility. Product flexibility allows production plants to produce various products to meet customer changing demand. However flexibility comes with a cost explained by the capital cost of tooling needed to enable production of a wider product mix. To achieve highest benefits from flexibility firms need to invest in it only when it is needed (i.e. results in cost savings). For this purpose the integrated capacity planning model described in Chapter 3 is extended to account for product flexibility. In this model it is investigated whether it is economical to increase product mix at production plants of an existing production and distribution supply chain network. The effect on total cost as well as production schedule are studied as well. A detailed mathematical formulation is first presented. Then, implementation of the proposed model for a real life case study is presented. To solve the proposed model for the case study, solution methodologies previously described in Chapter 3 are employed. Results showed the potential savings in total cost due to product flexibility in some production plants. To evaluate flexibility, the model with no flexibility is compared with two flexibility options, one using the actual capital cost of tooling and another assuming zero tooling cost.

Computational results show that it is advantageous to add some product flexibility at some production plant for the existing supply chain network. With partial product flexibility savings in total cost especially logistics cost is achieved.

Chapter 5

ALTERNATIVE CAPACITY PLANNING MODEL

In this chapter an alternative formulation for the capacity planning problem described in Chapter 3 is presented. Given the computational complexity of the problem as well as the large instance that needs to be solved, an alternative formulation is proposed to investigate its relative computational complexity and usefulness. There is no one unique model for a problem therefore, a problem can be formulated using different models. It useful to have another formulation for the same problem for comparison of solution quality and solution time. In this chapter the alternative mathematical model for the integrated capacity planning problem is presented. Solution from this model is compared to the model proposed in Chapter 3 including computational time and optimality gap. Weaknesses and strengths of each of the two models are also provided.

In the capacity planning problem described in Chapter 3, the optimal capacity plan that specifies shifts, run rate and down periods at each production plant at each time period is determined. Those three levers are represented by binary and integer variables that each grow linearly in the dimensions of the problem (periods, plants). Thus, there is a finite number of possible capacity plans for these three factors over the planning horizon. Each possible combination of shifts, run rate, and down periods can be considered as a schedule. Thus, one binary variable is used in the alternate formulation as an indicator if that schedule is used. Other continuous variable decisions such as production during regular and over time, shipments, and inventory remain the same.

Thus the model presented in this chapter selects a schedule including number of shifts, run rate and down period for each plant at each time period using one binary variable as opposed to using three separate variables for shift, rate and down weeks respectively each period. This formulation however, is complicated by the large number of possible schedules. Potentially 4 shifts x 7 run rates x 5 down weeks = 106 possible schedules per plant and month (plants without shifts do not need to specify run rate or down weeks). The alternative formulation is referred as the schedule generation model. In the next section results for the schedule generation model are presented for the case study. A comparison of the results and computational complexity of the schedule generation model with the results using the model described in Chapter 3 is presented.

5.1 Mathematical Model

The following observations are noted:

- Associated with each schedule there is a maximum regular time and a maximum overtime level of production that can be computed a priori.
- Associated with each schedule there is an associated fixed and regular time labor cost.

The following assumptions from the model described in Chapter 3 also hold for this model:

- The set of product allocations to production plants is given. Not all allowable allocations must be utilized but no additional allocations are considered in the model. Thus, investment cost is assumed fixed and exogenous to the model.

- Demand for each product type in each market each period is known deterministically over the planning horizon or through forecast. In practice only the first month's demand is known precisely.

- Cycle time is given for the plant based on an upper percentile of the slowest product and the product mix and sequencing are such that that cycle time can be met.

- Cycle time (inverse of run rate) limits are known by plant.

- Run rate can be varied at most twice a year and only be plus/minus 5%, 10% or 20% of current rate based on observed past practice.

- A lead time is required to change the run rate, nominally assumed to be six months. Similarly, adding or deleting a shift requires a known lead time, nominally set to three months.

- Inventory accumulation is to be generally avoided by matching production to demand but is allowable at production plants. Initial values are known.

- The minimum inventory and maximum inventory for each product at each time is limited by organizational policy.

- Inventory deviation penalty per unit is a percentage of product book value.

- The minimum period for assigning production volume is four-weeks/one month.

- Inbound logistics cost per product is constant over the time horizon and can be determined exogenous to the model by assuming existing suppliers will be used or suppliers will relocate.

- Outbound logistics cost per product-destination pair is constant over the time horizon.

- Production rate for a product at a plant may also be limited as a percentage of total production or a maximum rate for that product in addition to total plant capacity.

- Shifts one and two have eight productive hours per day but the third shift adds only an additional 6.5 productive hours.

- Overtime is limited to a maximum total activation of 22.5 hours per day with additional limits on length of full overtime usage.

- Rate change cost up or down is independent of number of shifts and is based on cost for required tooling adjustment

5.1.1 Notation

The same notation is used as described in Chapter 3. New terms for the alternative formulation are presented in this section.

Indices and sets:

- $\mathcal{D} : \{0, 1, 2, 3, 4\}$, set of down weeks;

Parameters:

- χ_{srdjt}: maximum allowable production during regular time of a schedule of s shifts, run rate r and down periods d in plant j in time period t;

- ν_{sjt}: maximum regular time production of shift s in plant j in time t;

- ϕ_{srdjt}: maximum allowable production during overtime of a schedule of s shifts run rate r and down period d in plant j in time period t;

- c_{srdjt}: fixed and variable labor cost minus down period saving from variable labor associated with a schedule of s shifts run rate r and down period d in plant j in time period t;

- θ^0_{srdj}: Binary indicator if initial schedule of s shifts, run rate r, and d down periods is used at plant j;

Decision Variables:

$$\boldsymbol{\Theta_{srdjt}} = \begin{cases} 1, & \text{if a schedule with } s \text{ shifts, run rate } r, \ d \text{ down periods is used} \\ & \text{at plant} j \text{ at period } t \\ 0, & \text{if otherwise} \end{cases}$$

The objective function in equation (5.1) as before minimizes total cost at production plants as well as logistics cost over the planning horizon. The first term is schedule cost minus possible savings from down periods. Schedule cost includes cost of shifts (i.e fixed and variable labor cost) which is proportional to run rate level. Other terms remain the same as described in Chapter 3.

$$\begin{aligned} Minimize: & \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{srdjt}.\Theta_{srdjt}) + \sum_{i\in F_j}\sum_{j=1}^{J} c^{in}_{ij}.\sum_{t=1}^{T}(X_{ijt}+O_{ijt}) \\ & + \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{t=1}^{T}(c^{out}_{ij}.S_{ijmt}) + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T}(c^{u}_{im}.U_{imt}) \\ & + \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{t=1}^{T}(c^{I}_{i}.I_{ijt}) + \sum_{i=1}^{I}\sum_{t=1}^{T}(c^{\Delta I}_{i}.\Delta I_{it}) \\ & + \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{t=1}^{T}((1+\alpha_j).c^{o}_{j}.O_{ijt}) + \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}((c^{l+}_{j}.L^{+}_{jt}) + (c^{l-}_{j}.L^{-}_{jt})) \end{aligned}$$

$$+\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T}((c_j^{r+}.R_{jt}^{+}) + (c_j^{r-}.R_{jt}^{-})) + \sum_{j=1}^{J}\sum_{t=1}^{T}(c_j^{ro}.RO_{jt}) \tag{5.1}$$

Constraint (5.2) defines limit on production during regular time determined by available regular time schedule capacity.

$$\sum_{i\in F_j} X_{ijt} \leq \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(\chi_{srdjt}.\Theta_{srdjt}) \quad \forall j \in J, t \in T \tag{5.2}$$

Constraint (5.3) defines limit on production during overtime determined by available overtime schedule capacity.

$$\sum_{i\in F_j} O_{ijt} \leq \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(\phi_{srdjt}.\Theta_{srdjt}) \quad \forall j \in J, t \in T \tag{5.3}$$

Overtime production limit is defined in constraint (5.4). Overtime production in every two consecutive time periods is at most as a percentage (i.e o'_{jt}) of the sum of available overtime capacity in these two time periods.

$$\sum_{i\in F_j}(O_{ijt} + O_{ij(t+1)}) \leq o'_{jt}.\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}((\phi_{srdjt}.\Theta_{srdjt}) + (\phi_{srdj(t+1)}.\Theta_{srdj(t+1)}))$$

$$\forall j \in J, t \leq T \tag{5.4}$$

Constraint (5.5) states that one schedule is to be selected at each plant at each time period.

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}\Theta_{srdjt} = 1 \quad \forall j \in J, t \in T \tag{5.5}$$

Constraint (5.6) fixes the schedule of the first time period to the initial schedule which is defined as a parameter in the model.

$$\Theta_{srdj1} = \theta^0_{srdj} \quad \forall s \in S, r \in R, d \in D, j \in J \tag{5.6}$$

As discussed in Chapter 3, to model shift changes as well as shift change frequency, a binary variable ($L^{(+)(-)}_{sjt}$) is used to indicate shift change at each plant at each time period. This variable is modified to fit in the alternative formulation. Let $L^{(+)(-)}_{jt}$ be the number of shift increase (+) or decrease(-) at plant j in time t. This continuous (but will take only integer values) variable is used to track number of shift changes at production plants and to compute shift change cost in the objective function. In order to implement shift change frequency, a binary variable is used to indicate a change in shifts (either increase or decrease).

$$\boldsymbol{LI^{(+)(-)}_{jt}} = \begin{cases} 1, & \text{if a shift is added(+) or removed(-) in plant } j \text{ at period } t \\ 0, & \text{otherwise} \end{cases}$$

Constraints (5.7) and (5.8) track number of shift increases or decreases at each plant at each time period through the continuous (but will take only integer values) variables $L^{(+)(-)}_{jt}$.

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdjt}) - \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdj(t-1)}) \leq L^{+}_{jt} \quad \forall j \in J, t \leq T \tag{5.7}$$

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdj(t-1)}) - \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdjt}) \leq L^{-}_{jt} \quad \forall j \in J, t \leq T \tag{5.8}$$

In constraints (5.9), (5.10), and (5.11) shift change frequency is limited to allow for shift change implementation. The constraints ensure that there is at most one shift change within the shift lead time window. Plant re-opening constraint is adjusted accordingly as shown in equation (5.12)

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdjt}) - \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdj(t-1)}) \leq 3.LI^{+}_{jt} \quad \forall j \in J, t \leq T \tag{5.9}$$

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdj(t-1)}) - \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(s.\Theta_{srdjt}) \leq 3.LI^{-}_{jt} \quad \forall j \in J, t \leq T \tag{5.10}$$

$$\sum_{t' \in [t-L^{LT}+1,t]} (LI^{+}_{jt'} + LI^{-}_{jt'}) \leq 1 \quad \forall j \in J, t \geq L^{LT} \tag{5.11}$$

$$\sum_{r=1}^{R}\sum_{d=0}^{D}(\Theta_{1rdjt}) - \sum_{r=1}^{R}\sum_{d=0}^{D}(\Theta_{0rdj(t-1)}) \leq RO_{jt} \quad \forall j \in J, t > 1 \tag{5.12}$$

Constraint (5.13) and (5.14) indicate if the run rate is increased or decreased between time periods t and $t-1$.

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(\beta_r.\Theta_{srdjt}) - \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(\beta_r.\Theta_{srdj(t-1)}) \leq (max\{\beta_r\}.R^{+}_{jt}) \quad \forall j \in J, t \leq T \tag{5.13}$$

$$\sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(\beta_r.\Theta_{srdj(t-1)}) - \sum_{s=0}^{S}\sum_{r=1}^{R}\sum_{d=0}^{D}(\beta_r.\Theta_{srdjt}) \leq (max\{\beta_r\}.R^{-}_{jt}) \quad \forall j \in J, t \leq T \tag{5.14}$$

Run rate change frequency (5.15),penetration constraints (5.16) and (5.17) ,demand balance (5.18), inventory balance (5.19),(5.20), inventory deviation (5.21),(5.22) ,(5.23),(5.24), shortage bound (5.25) constraints remain the same as in Chapter 3 as follows:

$$\sum_{t' \in [t-R^{LT}+1,t]} (R^+_{jt'} + R^-_{jt'}) \leq 1 \quad \forall j \in J, t \geq R^{LT} \tag{5.15}$$

$$X_{ijt} + O_{ijt} \leq \overline{\rho_{ij}}. \sum_{i \in F_j} (X_{ijt} + O_{ijt}) \quad \forall i \in I, j \in J, t \in T \tag{5.16}$$

$$X_{ijt} \leq \overline{r}_{ij}. \sum_{s=1}^{S} \nu_{sjt} \quad \forall i \in I, j \in J, t \in T \tag{5.17}$$

$$\sum_{j=1}^{J} S_{ijmt} + U_{imt} = d_{imt} \quad \forall i \in F_j, m \in M, t \in T \tag{5.18}$$

$$X_{ijt} + O_{ijt} + I^0_{ij} = \sum_{m=1}^{M} S_{ijmt} + I_{ijt} \quad \forall i \in F_j, j \in J, t = 1 \tag{5.19}$$

$$X_{ijt} + O_{ijt} + I_{ij(t-1)} = \sum_{m=1}^{M} S_{ijmt} + I_{ijt} \quad \forall i \in F_j, j \in J, t > 1 \tag{5.20}$$

$$\sum_{j=1}^{J} I_{ijt} \geq \underline{\eta_{it}}. \sum_{m=1}^{M} D_{imt} - \sum_{j=1}^{J} I^-_{ijt} \quad \forall i \in F_j, t \in T \tag{5.21}$$

$$\sum_{j=1}^{J} I_{ijt} \leq \overline{\eta_{it}}. \sum_{m=1}^{M} D_{imt} + \sum_{j=1}^{J} I^+_{ijt} \quad \forall i \in F_j, t \in T \tag{5.22}$$

$$\Delta I_{it} = \sum_{j=1}^{J} (I^+_{ijt} + I^-_{ijt}) \quad \forall i \in F_j, t \in T \tag{5.23}$$

$$\sum_{j=1}^{J} I_{ijt} = I^e_i \quad \forall i \in I, t = T \tag{5.24}$$

$$U_{imt} \leq \overline{u}_{im} \quad \forall i \in I, m \in M, t \in T \tag{5.25}$$

Continuous and integer variables are defined in the following constraints:

$$X_{ijt}, O_{ijt}, I_{ijt}, \Delta I_{it}, I^{+}_{ijt}, I^{-}_{ijt}, S_{ijmt}, U_{imt}, L^{+}_{jt}, L^{-}_{jt} \geq 0 \quad \forall i \in F_j, j \in J, m \in M, t \in T \tag{5.26}$$

$$\Theta_{srdjt}, LI^{+}_{jt}, LI^{-}_{jt}, R^{+}_{jt}, R^{-}_{jt}, RO_{jt} \in \{0,1\} \quad \forall s \in S, r \in R, d \in D, j \in J, t \in T \tag{5.27}$$

5.2 Model Discussion and Results

The schedule generation formulation is a linear MIP, which is considered as an advantage relative to the previous MINLP. A commercial solver can directly be used to solve it. However, in this formulation it is harder to define or adjust schedule rules since all factors: shifts, rate, and down variables are confounded in one single variable. Strengths and weaknesses of each formulation is discussed in the next section. $\mathbb{O}$ and $\mathbb{S}$ are used to refer to the original and schedule generation formulations respectively.

5.2.1 Results and Comparison

The schedule generation model ($\mathbb{S}$) is solved for the five sub-problems of the case study usign AMPL with CPLEX solver. A solution with at most 1% difference from $\mathbb{O}$ solution is obtained for each sub-problem. Table 5.1 shows the computational time and optimality gap of each formulation. Results indicate that the schedule generation formulation provides solution with approximately same gap and with in the same computational time compared with the original formulation.

	Size (Plants,Vehicles)	$\mathbb{O}$ CPU	$\mathbb{O}$ Gap	$\mathbb{S}$ CPU	$\mathbb{S}$ Gap
SUV	(1,6)	7(s)	0.01%	5.5(s)	0.08%
Mid SUV	(1,5)	33.5(s)	0.10%	43.1(s)	0.09%
Vans	(1,4)	13.4(s)	0.08%	10.3(s)	0.09%
Trucks	(3,13)	9.5(min)	0.10%	5.3(min)	0.10%
Cars	(11,31)	24(hr)	3.26%*	24(hr)	3.08*%

Table 5.1: Schedule Generation Results Compared with the Original Model

Table 5.2 shows a comparison of the two models' results in terms of scheduling of shifts, rate and down weeks. Number of shift and rate changes are also provided. While results are similar, they are not identical in all cases indicating the existence of nearly equivalent alternate solutions within the allowable solution gap specified.

	No.of shift changes		No.of Rate changes		Down Weeks	
	$\mathbb{O}$	$\mathbb{S}$	$\mathbb{O}$	$\mathbb{S}$	$\mathbb{O}$	$\mathbb{S}$
SUV	2	2	1	1	2	2
Mid SUV	6	5	1	1	12	12
Vans	3	3	1	1	3	3
Trucks	6	5	3	2	14	17
Cars	26	24	9	9	67	70

Table 5.2: Schedule Comparison

Table 5.3 presents a comparison between the two formulations including some features and limitations. The size of each formulation in terms of number of integer and binary variables are compared. Table suggest that ($\mathbb{O}$) has fewer number of binary variables because as opposed to formulation ($\mathbb{S}$), down periods are modeled using integer variable. In formulation ($\mathbb{S}$) the number of binary variables have exponentially increased due to the additional index D used for down periods.

	$\mathbb{O}$	$\mathbb{S}$
Schedule variables	R,L,D	Θ
Mathematical model	MINLP	MILP
Strengths	Easier to adjust constraints	All terms are linear
Weaknesses	Quadratic & cubic terms	Hard to construct data
No. of binary variables	$JT(3S+R+3) = 7,752$	$JT(SRD+5) = 59,160$
No. of integer variables	$JT = 408$	0

Table 5.3: Comparison of the Original and Schedule Formulations

5.3 Conclusion

In this chapter an alternative formulation of the capacity planning model in which a different way of modeling capacity configuration is presented. In this formulation the same material flow variables and constraints are used. However, the two formulation differ in the use of capacity planning variables and constraints. All quadratic and cubic terms no longer exist in the alternative formulation but binary variables have grown significantly. Although the performance of the two models are nearly the same, the alternative formulation provides a verification of model solution for the case study.

Chapter 6

PRICING MODEL

An important consideration in the analysis of supply chains is to express the relationship between the price and the demand of products. This relationship is modeled using the concept of price elasticity of demand. In this research, the effect of price elasticity of demand on the operational decisions of supply chains is investigated. Adding a pricing factor to the supply chain network model allows for a better match between supply and demand. The demands of each product in each market are affected by the product's prices in the market and by the price elasticity of demand. In some periods, the demand might be high for some products with insufficient capacity at the plants. Slightly increasing the price of that product can help shift the demand to other substitutable products (perhaps very close to the product needed) with excess capacity. On the other hand, a price cut could get buyers attention and increase the demand. Flexible prices is an option for manufacturers where increasing capacity is expensive or currently infeasible. In this research, the price-demand relationship is modeled using a price elasticity of demand coefficient (PED) which measures the percentage change in demand for a good in response to a percentage change in its price. In other words, it is a measure that captures the responsiveness of a good's quantity demanded to a change in its price. Specifically, PED is defined as the ratio of percentage change in quantity demanded to percentage change in its price. A product is elastic when a change in its price has a relatively larger effect on the quantity demanded. If the quantity demanded has a relatively smaller change in response to its price change the this product is inelastic.

The law of demand states that there is an inverse relationship between price and demand for a product, provided that all other factors are constant. Due to the inverse relation between price and demand PED is a negative value. The PED formula is as follows:

$$\text{PED} = \frac{\Delta Demand/Demand}{\Delta Price/Price} \tag{6.1}$$

In the proposed pricing model, optimal prices are to be obtained as well as all other operational decisions discussed in earlier chapters (i.e., production, distribution and inventory). Therefore, the Capacity Planning and Production Scheduling model described in Section 3.2.1 is extended to account for price decisions. In the following section, the mathematical formulation for the pricing model is presented. The model is built by defining and adding necessary terms to the original model. Next, solution methodologies to solve the pricing model are suggested. Results for the vehicle manufacturer case study is then presented and a comparison of solution approaches is also provided. At the end of this chapter, key findings and conclusion on the pricing model are presented.

6.1 Mathematical Models

Prices are modeled using two approaches. Prices can be discrete and modeled as the choice among several available product price levels. Prices can also be continuous. For discrete prices, a binary variable is used to indicate the price level for each product at each time period and a continuous variable is used for continuous prices. Each of the two approaches has its own complexity and therefore needs a specific solution methodology. In this section two mathematical models are presented, one using discrete prices and another model using continuous prices.

Solution methodologies and model complexity are also discussed. Since prices are considered in the model, revenues are included in the objective function. Therefore, the objective function is changed to total profit maximization instead of minimizing total cost. In doing so, the cost of materials is added to ensure the contribution to profit and administrative overhead is appropriately accounted for in setting production plans. This fixed cost per nameplate term was not necessary and thus was not included in the previous cost minimization model as it is independent of production source. The following assumptions are added to the list of assumptions of the original model:

- Product demand is a linear function of its price (i.e., the PED equation holds over the range of interest).
- Initial price of each product is known.
- Price elasticity of demand coefficients are known and assumed to be constant over the range of products and constant over time.
- Prices are determined at each period based on the price elasticity of demand (PED) of each product.
- Price change between time periods is based on change from initial price.
- At each period, initial demand is known from forecast of each product. After the price is determined, the actual demand is then obtained.
- Products may be modeled as being demand independent or related (substitutable) within classes.

The following Technological data are added to the original model:

- Initial price of each product.

- PED coefficients of each product.
- Product price levels and price at each level.
- Material cost per nameplate unit.

6.1.1 Notation

In this section, the same notation defined for the model described in Chapter 3 is used. Therefore, only additional terms needed for the pricing model are presented.

Indices and sets:

- $\mathcal{P}$: $\{1 \cdots, p, \cdots P\}$, set of possible prices measured as proportional changes from nominal price of a product (discrete price case);

Parameters:

- e_i: price elasticity of demand of product i, $e_i < 0$;
- p_i^0: initial price of product i;
- $\pi_{i,p}$: price of product i associated with price level p;
- c_i^{mat}: unit material cost of product i.

Decision Variables:

- D_{imt}: demand of product i in market m at time period t;
- Π_{it}: price of product i at time period t;

$$\boldsymbol{\Pi^I_{pit}} = \begin{cases} 1, & \text{if price level } p \text{ is selected for product } i \text{ at period } t \\ 0, & \text{if otherwise} \end{cases}$$

6.2 Discrete Pricing Model

Price is modeled as the choice among allowable product price levels determined by proportional changes from initial prices. In order to maintain a stable price level and to avoid large price fluctuations at the markets, three price levels are defined using index p as follows:

$$p = \begin{cases} 1, & \text{Initial price} \\ 2, & \text{Initial price -8\%} \\ 3, & \text{Initial price +8\%} \end{cases}$$

With this setting, product prices will either remain as initial prices, decrease by 8% or increase by 8%. The 8% change was selected after discussion with a representative from the industry. The objective function is formulated to maximize profits. To obtain profits, total revenue is first calculated then total cost. Total Revenue is calculated by Equation (6.2), which is formulated as the product of shipments (i.e., sales) and price for all product types produced from each plant and delivered to all demand markets over the planning horizon. Note that shipments cannot exceed demand since inventories are held at the plant level.

$$\text{Total Revenue} = \sum_{i \in F_j} \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{t=1}^{T} (S_{ijmt} . \Pi_{it}) \tag{6.2}$$

The total cost is as in the model described in Chapter 3. It accumulates the cost of inbound, outbound logistics, shortages, inventory holding and penalty for deviation from bounds. It also includes fixed and variable cost of labor, over time cost, shift and run rate change costs. The savings from down periods is also included.

For the pricing model row material cost is added to ensure that profit is appropriately calculated. The objective function is the difference between total revenue and total cost as shown in equation (6.4)

$$\begin{aligned}
\text{Total Cost} = & \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_i^{mat}.(X_{ijt}+O_{ijt})) + \sum_{i\in F_j}\sum_{j=1}^{J} c_{ij}^{in}.\sum_{t=1}^{T}(X_{ijt}+O_{ijt}) \\
& + \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{t=1}^{T}(c_{ij}^{out}.S_{ijmt}) + \sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T}(c_{im}^{u}.U_{imt}) + \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_i^{I}.I_{ijt}) \\
& + \sum_{i=1}^{I}\sum_{t=1}^{T}(c_i^{\Delta I}.\Delta I_{it}) \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{sj}^{f}.L_{sjt}) + \sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=1}^{T}(c_{rj}^{v}.L_{sjt}.R_{rjt}) \\
& + \sum_{i\in F_j}\sum_{j=1}^{J}\sum_{t=1}^{T}((1+\alpha_j).c_j^{o}.O_{ijt}) + \sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}((c_{sj}^{l+}.L_{sjt}^{+}) + (c_{sj}^{l-}.L_{sjt}^{-})) \\
& + \sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T}((c_j^{r+}.R_{jt}^{+}) + (c_j^{r-}.R_{jt}^{-})) + \sum_{j=1}^{J}\sum_{t=1}^{T}(c_j^{ro}.RO_{jt}) \\
& - \sum_{r=1}^{R}\sum_{s=1}^{S}\sum_{j=1}^{J}\sum_{t=1}^{T}(c_{rj}^{ds}.c_{rj}^{v}.R_{rjt}.L_{wjt}.Z_{jt}/w_{jt})
\end{aligned} \tag{6.3}$$

$$\text{Maximize Profit} = \text{Total Revenue - Total Cost} \tag{6.4}$$

The set of constraints of the model discussed in Chapter 3 is used for the pricing model with the following additional constraints. Equation (6.5) states that only one price is selected for each product type at each time among p levels of the price. Constraint (6.6) calculates the price of product i selected at time t as the product of price parameter and price indicator variable.

$$\sum_{p=1}^{P}\Pi_{pit}^{I} = 1 \quad \forall i \in I, t \in T \tag{6.5}$$

$$\Pi_{it} = \sum_{p=1}^{P}(\pi_{pi}.\Pi_{pit}^{I}) \quad \forall i \in I, t \in T \tag{6.6}$$

The price-demand equation can take different forms, it could be linear or in some cases it could be nonlinear. In this research, a linear function is used to express demand as a function of price. The equation is defined as follows:

$$\text{Demand} = \text{Initial Demand} + \text{PED.(Initial Demand).(\%change in price)} \tag{6.7}$$

In Constraint (6.8) elasticity coefficient is used to determine the demand of product i in market m at each time period t using product price, initial price and demand forecast information at time period t.

$$D_{imt} = d_{imt} + e_i.d_{imt}.((\Pi_{it} - p_i^0)/p_i^0) \quad \forall i \in I, m \in M, t \in T \tag{6.8}$$

Demand Constraint (5.18) discussed in Chapter 3 is updated using new demand variable as follows:

$$\sum_{j=1}^{J} S_{ijmt} + U_{imt} = D_{imt} \quad \forall i \in F_j, m \in M, t \in T \tag{6.9}$$

Constraints (6.10) and (6.11) define continuous and integer variables respectively

$$\Pi_{pit}^{I} \in \{0,1\} \quad \forall p \in P, i \in I, t \in T \tag{6.10}$$

$$\Pi_{it}, D_{imt} \geq 0 \quad \forall i \in I, m \in M, t \in T \tag{6.11}$$

6.2.1 Model Discussion and Solution Methodologies

The total revenue function defined in Equation (6.2) is non-linear since it is the product of continuous and binary variables. As discussed in Chapter 3, to obtain optimal solution non-linear terms need to be linearized. Linearizing the product of continuous and binary variables using McCormick envelopes is discussed in Section 3.2.3. Lower and upper bounds on shipments are set to zero and maximum demand respectively. Maximum demand is obtained at the lowest price and set as an upper bound on shipments and it is calculated as shown in equation (6.12). Note that price elasticity coefficient e_i is a negative value by definition. A new variable Q_{ijmpt} is defined, where $Q_{ijmpt} = S_{ijmt}.\Pi^{I}_{pit}$. The revenue function is reformulated as shown in equation (6.13).The set of constraints from (6.14) to (6.17) are added to the model.

$$D^{max}_{imt} = (1 - 0.08.e_i).d_{imt} \tag{6.12}$$

$$\text{Total Revenue} = \sum_{i \in F_j} \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{p=1}^{P} \sum_{t=1}^{T} (\pi_{pi}.Q_{ijmpt}) \tag{6.13}$$

$$Q_{ijmpt} \leq D^{max}_{imt}.\Pi^{I}_{pit} \quad \forall i \in F_j, j \in J, m \in M, p \in P, t \in T \tag{6.14}$$

$$Q_{ijmpt} \leq S_{ijmt} \quad \forall i \in F_j, j \in J, m \in M, p \in P, t \in T \tag{6.15}$$

$$Q_{ijmpt} \geq S_{ijmt} - D^{max}_{imt}.(1 - \Pi^{I}_{pit}) \quad \forall i \in F_j, j \in J, m \in M, p \in P, t \in T \tag{6.16}$$

$$Q_{ijmpt} \geq 0 \quad \forall i \in F_j, j \in J, m \in M, p \in P, t \in T \tag{6.17}$$

Given that the revenue function is linear and all other constraints are also linear, the pricing model can be solved using a commercial solver such as CPLEX. As described in Chapter 3, solving the linearized model is computationally difficult especially for a large size problem such as the vehicle manufacturer case study considered in this research.

Therefore, the linearized model is solved by dividing the problem into sub-problems and solving each independently. The problem separation discussed in Chapter 3 is used for solving the pricing model. An efficient algorithm is needed to solve the pricing model. Heuristics are usually used to obtain near optimal solutions in reasonable time. The proposed iterative Fix and Re-solve heuristic approach discussed in Chapter 3 can be modified and used to solve the pricing model. A fourth stage stage is added for prices. In the fourth stage rate, shift, down weeks variables are fixed and the model is solved for prices. A schematic of this approach is shown in Figure 6.1. As the pricing problem is slow to converge, it is allowed more time per iteration.

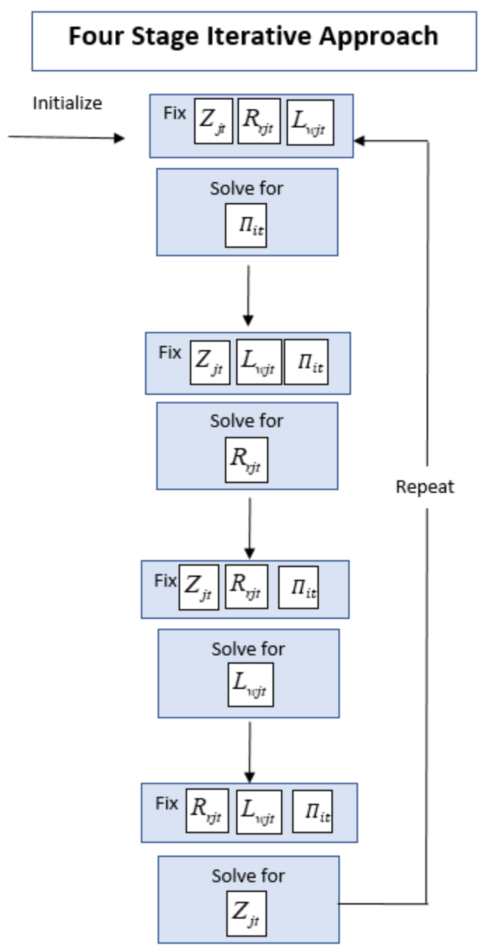


Figure 6.1: A Schematic of the Four-stage Fix and Re-solve Heuristic Approach

As shown in Algorithm 2, a fourth stage is added to solve for prices. In each of the four stages, three out of the four variables are fixed and the model is solved to optimality or until an allowed time limit. Then the new set of variables are fixed and the model is re-solved for the fourth. In the first step shifts, rate and down weeks are initialized by setting down weeks to the required down weeks (due to holidays, maintenance or upgrading) and all shifts and rates to the initial schedule. The first iteration starts with the first stage where the pricing model is solved for prices. Note that after fixing shifts, rate, and down weeks the objective function is still non linear because of the product of price and shipment variables. Therefore, linearization of the revenue function is also implemented when solving for prices. Gap tolerance is nominally set to 0.1% but since price convergence is slow, a longer time is allowed when solving for prices. This process continues until the gap between the objective value of two iterations is with in a pre-defined error. Note that the optimal solution is either increasing at each of the four stages iteration or unchanged. As there are a finite number of binary and integer assignments, the process will converge given that the algorithm is stopped when ever a solution is repeated.

Algorithm 2 Fix and Re-Solve Heuristic Algorithm for Pricing model

$z^* \leftarrow +\infty; \varepsilon \leftarrow 1e0;$ ▷ Ititialize optimal value bound and gap
$n \leftarrow 0;$ ▷ Setting number of times solution remain the same
$z \leftarrow z(R_0, L_0, D_0);$ ▷ initialalize shifts, rate, down
loop
  if $z - z^* < -\varepsilon$ **then** ▷ Checking gap tolerance
    $z^* \leftarrow z$; $n \leftarrow 0$; ▷ update objective
  else
    $n \leftarrow n + 1$ ▷
    If $n \geq 5$ **break;** ▷ break the loop
  end if
  fix Z, R, L; ▷ Fix down weeks, run rate and, shift
  solve; ▷ Solve the model for price
  $z \leftarrow$ Objective; ▷ set objective value
  if $z - z^* < -\varepsilon$ **then** ▷ Checking gap tolerance
    $z^* \leftarrow z$; $n \leftarrow 0$;
  else
    $n \leftarrow n + 1$
    If $n \geq 5$ **break;**
  end if
  fix Z, L, Π; ▷ Fix down weeks, shifts and, price
  solve; ▷ Solve the model for rate
  $z \leftarrow$ Objective;
  if $z - z^* < -\varepsilon$ **then**
    $z^* \leftarrow z$; $n \leftarrow 0$;
  else
    $n \leftarrow n + 1$
    If $n \geq 5$ **break;**
  end if
  fix Z, R, Π; ▷ Fix down weeks, run rate and price
  solve; ▷ Solve the model for shift
  $z \leftarrow$ Objective;
  if $z - z^* < -\varepsilon$ **then**
    $z^* \leftarrow z$; $n \leftarrow 0$;
  else
    $n \leftarrow n + 1$
    If $n \geq 5$ **break;**
  end if
  fix R, L, Π; ▷ Fix run rate, shifts and price
  solve; ▷ Solve the model for down weeks
  $z \leftarrow$ objective;
end loop

6.2.2 Demand Dual Variables Heuristic Algorithm

Another iterative heuristic algorithm is proposed for the pricing model in which, dual variables of the fixed prices model are employed. Dual variables are one of the most interesting values in the solution of LPs since they provide the improvement in objective function as a result of a unit change in the resource associated with the dual variable. For MIPs, dual variables can still be useful and provide estimates on marginal improvement in the objective function. In the pricing model, dual variables provide estimates of the marginal profit of a unit change in sales as a result of price change (either increase or decrease). From this value, profit change, either increase or decrease, can be estimated by knowing the new price and the new demand. In the proposed demand dual variables heuristic algorithm each price change option up or down is investigated for each product type and each month. In the pricing model only two price change options are allowed either +8%, or -8% of the current price. At each iteration a new price is selected and one price change is made at a time per month per product based on the the product that has most profit potential per unit. This change in price is implemented and the model is resolved. Since the price decision variable is fixed to some value at each iteration, the revenue function becomes linear. If the suggested new price actually increases the profit then this price is fixed and the procedure is repeated. If profit doesn't increase, the next best prospective price change is implemented and the model is resolved. The algorithm terminates when no more profitable price changes can be made. Figure 6.2 is a flow diagram describing the algorithm.

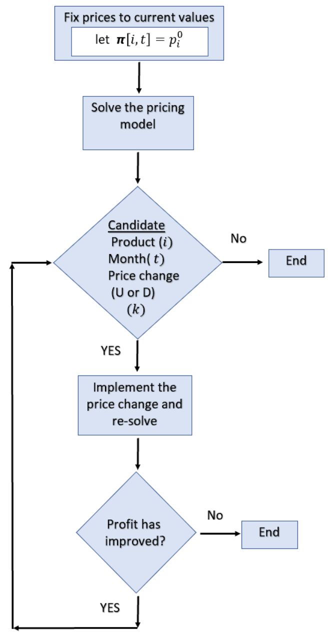


Figure 6.2: A Schematic of the Dual Variables Heuristic Algorithm

As described in the algorithm, at each iteration the algorithm finds a candidate set of product, month and a type of price change either up or down {product, month, up or down} for which the pricing model is re-solved. Let $k = u, d$ be the index for price change. Therefore, prospective sets are indexed by $\{i, t, k\}$. The algorithm iterates in the following steps:

1. Dual variables reported from the demand constraint defined in equation (6.9) are obtained after solving the pricing model with prices fixed to current values. Let $Demand\_dual_{imt}$ be the dual variable of product i of market m in time t.

2. Possible price changes are then calculated for each product in each time period. Let $New\_price\_up_{it\{k=u\}}$ be the new price of product i at time t after increasing initial price by 8%. Simillarly, let $New\_price\_down_{it\{k=d\}}$ be the new price of product i at time t after decreasing initial price by 8%.

3. Demand associated with new prices are calculated using price elasticity coefficient. As descried earlier, demand is inversely related to price by PED formula defined in equation (6.1). Let $New\_demand\_up_{it\{k=u\}}$ be the demand of product i in market m at time t associated with a price increase. Let $New\_demand\_down_{it\{k=d\}}$ be the demand of product i in market m at time t associated with a price decrease.

$$New\_demand\_up_{it\{k=u\}} = (1 - 8.e_i).\sum_{m=1}^{M} D_{i,m,t} \quad \forall i \in I, t \in T \tag{6.18}$$

$$New\_demand\_down_{it\{k=d\}} = (1 + 8.e_i).\sum_{m=1}^{M} D_{i,m,t} \quad \forall i \in I, t \in T \tag{6.19}$$

4. Revenue for each price option is then calculated as the product of the new price and the new demand descried in steps 3 and 4.

$$New\_Rev\_up_{it\{k=u\}} = New\_demand\_up_{it\{k=u\}}.New\_price\_up_{it\{k=u\}} \quad \forall i \in I, t \in T \tag{6.20}$$

$$New\_Rev\_down_{it\{k=d\}} = New\_demand\_down_{it\{k=d\}}.New\_price\_down_{it\{k=d\}} \tag{6.21}$$

$$\forall i \in I, t \in T$$

5. After calculating new revenues, revenue changes as a result of new prices are calculated as the difference between new revenue and current revenue.

$$Rev\_c\_up_{it\{k=u\}} = New\_Rev\_up_{it\{k=u\}} - (Pr_{it}.\sum_{m=1}^{M} D_{i,m,t}) \quad \forall i \in I, t \in T \tag{6.22}$$

$$Rev\_c\_down_{it\{k=d\}} = New\_Rev\_down_{it\{k=d\}} - (Pr_{it}.\sum_{m=1}^{M} D_{i,m,t}) \quad \forall i \in I, t \in T \tag{6.23}$$

6. Changes in total cost are estimated using dual variables. Dual variables provide information about the marginal profit of a unit change in sales (i.e. shipments). Note that shipments never exceed demand since inventory is stored at production plants. Demand changes are calculated first to estimate changes in total cost as follows:

$$Demand\_c\_up_{it\{k=u\}} = New\_demand\_up_{it\{k=u\}} - \sum_{m=1}^{M} D_{i,m,t} \quad \forall i \in I, t \in T \tag{6.24}$$

$$Demand\_c\_down_{it\{k=d\}} = New\_demand\_down_{it\{k=d\}} - \sum_{m=1}^{M} D_{i,m,t} \quad \forall i \in I, t \in T \tag{6.25}$$

Dual variables give profit per unit. Therefore, to obtain cost per unit demand dual is subtracted from unit price. Note that in previous calculations demand per product per month is summed over all markets. Thus, the average of dual variables over markets is used to estimate marginal costs. Change in total cost is then calculated as follows:

$$TC\_c\_up_{it\{k=u\}} = (Pr_{i,t} - (\sum_{m=1}^{M} Demand\_dual_{i,t,m}/M)).Demand\_c\_up_{it\{k=u\}} \tag{6.26}$$

$$TC\_c\_down_{it\{k=d\}} = (Pr_{i,t} - (\sum_{m=1}^{M} Demand\_dual_{i,t,m}/M)).Demand\_c\_down_{it\{k=d\}} \tag{6.27}$$

7. Changes in total profit associated with each price change option are estimated as follows:

$$TP\_c\_up_{it\{k=u\}} = Rev\_c\_up_{it\{k=u\}} - TC\_c\_up_{it\{k=u\}} \quad \forall i \in I, t \in T \tag{6.28}$$

$$TP\_c\_down_{it\{k=d\}} = Rev\_c\_down_{it\{k=d\}} - TC\_c\_down_{it\{k=d\}} \quad \forall i \in I, t \in T \tag{6.29}$$

8. After calculating all possible price changes for each product and time and the associated expected change in profit, the best prospective set $\{i, t, k\}$ is to be selected which is the one with maximum expected profit change.

$$\max\_TP\_change = \max_{it}(TP\_c\_up_{it\{k=u\}}, TP\_c\_down_{it\{k=d\}}) \tag{6.30}$$

9. Let $\{i_{max}, t_{max}, k_{max}\}$ be the index of the product, time, and price change respectively of $\max\_TP\_change$. The adjustment in price is implemented and the model is resolved with the new price. New profit is obtained and compared with initial profit.

   - If profit improves then the new price is fixed and a new prospective set is to be found.
   - If profit doesn't improve then the price is adjusted back to current prices and the next best prospective set is selected.

   Note that a unique set can appear once in the entire algorithm. Therefore, to prevent the algorithm from increasing (or decreasing) a price for the same product in the same period more than once, a new set $\mathcal{INV}_{itk}$, is defined to track the sets to be investigated. After each iteration the set which has been investigated is removed form $\mathcal{INV}_{itk}$.

Also, the solution from a run that doesn't improve the objective function is discarded and improvement in objective function if any is calculated based on the most recent iteration where the objective value has improved. let n^* be the iteration in which the last improvement in profit is achieved.

10. The algorithm iterates while there exist profitable price changes.

The algorithm is summarized in Algorithm 3.

Algorithm 3 Demand Dual Variables Heuristic Algorithm for Pricing model

$n \leftarrow 0$; ▷ Initialize iteration number
$n^* \leftarrow n$; ▷ Initialize reference iteration
fix $Pr_{i,t} := p_i^0$; ▷ fix price variable to current prices
Solve; ▷ Solve the pricing model
$max\_TP\_change[n] = \max_{it \in \mathcal{INV}_{itk}} (TP\_c\_up_{it\{k=u\}}[n], TP\_c\_down_{it\{k=d\}}[n])$;
while $max\_TP\_change[n] > 0$ **do**
  $\mathcal{INV}_{itk} \leftarrow \mathcal{INV}_{itk} diff \{i\_max[n], t\_max[n], k\_max[n]\}$
  fix $Pr[i\_max[n], t\_max[n]] := New\_price[n]$;
  $n \leftarrow n + 1$;
  Solve;
  $obj\_change = Profit[n] - Profit[n^*]$;
  if $obj\_change <= 0$ **then**
    fix $Pr[i\_max[n-1], t\_max[n-1]] := p_i^0$;
    $max\_TP\_change[n] = \max_{it \in \mathcal{INV}_{itk}} (TP\_c\_up_{it\{k=u\}}[n^*], TP\_c\_down_{it\{k=d\}}[n^*])$;
  else
    $n^* \leftarrow n$
    $max\_TP\_change[n] = \max_{it \in \mathcal{INV}_{itk}} (TP\_c\_up_{it\{k=u\}}[n], TP\_c\_down_{it\{k=d\}}[n])$;
  end if
end while

6.3 Continuous Pricing Model

In this section, a pricing mathematical model using continuous variable for prices is presented. In this model, prices can take any value from a defined range of possible prices. Prices are less restricted as opposed to discrete prices which provides more flexibility in practice. However, as shown next solving the model optimally with continuous prices is much harder. For the continuous pricing model, the objective function will be as discussed in Equation (**??**) except that the domain of the pricing variable is changed to to be continuous. Constraints (6.5) and (6.6) are eliminated and constraints (6.8) and (6.9) will be the same. Constraint (6.31) defines continuous variables.

$$\Pi_{it}, D_{imt} \geq 0 \quad \forall i \in I, m \in M, t \in T \tag{6.31}$$

6.3.1 Model Discussion and Solution Methodologies

The revenue function shown in the continuous pricing model is also non-linear since it's the product of continuous and another continuous variable. Exact linearization is not available but an approximation can be used. McCormick envelopes are commonly used as a linear relaxation in which linear under and overestimators of the function over its domain are to be found. The graph in Figure 6.3 shows a graphical representation of the relaxation. The envelopes of z are shown in the graph over the bounding box $[x^L, y^L]$x$[x^U, y^U]$ which are the actual bounds of each variable.

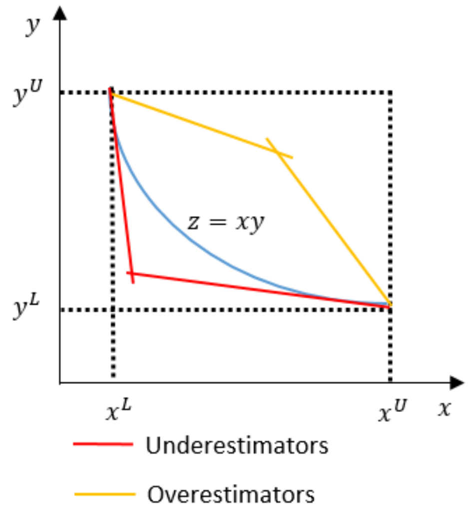


Figure 6.3: Piece-wise McCormick Envelopes Graphical Representation

McCormick (1976) proposed the following linear inequalities to replace the non-linear fuction z where $z = x.y$, x, $y \in \mathbb{R}$, $x \in [x^U, x^L]$ and $y \in [y^L, y^U]$.

$$z \geq x^L.y + x.y^L - x^L.y^L \tag{6.32}$$

$$z \geq x^U.y + x.y^U - x^U.y^U \tag{6.33}$$

$$z \leq x^U.y + x.y^L - x^U.y^L \tag{6.34}$$

$$z \leq x.y^U + x^L.y - x^L.y^U \tag{6.35}$$

The linear relaxation provides an an upper bound (for maximization) on the optimal solution of the original problem. Piece-wise McCormick envelopes is one of the linear relaxation techniques that provides tightest bounds (i.e., bounds that are closest to the optimal solution of the original problem) thus decreasing computational time. In the literature this approach is used in various applications. For example Al-Khayyal and Falk (1983) and Bergamini *et al.* (2005) applied it for the blending problem that arises in refinery processes in the petroleum industry. Commercial solvers such as CPLEX can be used to solve the problem relaxation linearization.

For the pricing model, shipments are bounded by a minimum of zero and a maximum of demand. Prices can also be bounded by a minimum of -8% and a maximum of +8% of the initial prices respectively. First the objective function is reformulated as shown in equations (6.36) and (6.37). Then it can be replaced by the linear constraints shown in equations (6.3.1) to (6.43).

$$\text{Total Profit} = \sum_{i \in F_j} \sum_{j \in J} \sum_{m \in M} \sum_{t \in T} (Revenue_{ijmt}) - \text{Total Cost} \tag{6.36}$$

$$Revenue_{ijmt} = S_{ijmt}.\Pi_{it} \quad \forall i \in F_j, j \in J, m \in M, t \in T \tag{6.37}$$

$$Revenue_{ijmt} \geq (0.\Pi_{it}) + S_{ijmt}.(0.92.P_i^0) - (0.(0.92.P_i^0)) \tag{6.38}$$

$$\forall i \in F_j, j \in J, m \in M, t \in T$$

$$Revenue_{ijmt} \geq (D_{imt}.\Pi_{it}) + (S_{ijmt}.1.08.P_i^0) - (D_{imt}.1.08.P_i^0) \tag{6.39}$$

$$\forall i \in F_j, j \in J, m \in M, t \in T$$

$$Revenue_{ijmt} \leq (x^U.S_{ijmt}) + (\Pi_{it}.y^L) - (x^U.y^L) \tag{6.40}$$

$$\forall i \in F_j, j \in J, m \in M, t \in T$$

$$Revenue_{ijmt} \leq (\Pi_{it}.y^U) + (x^L.y) - (x^L.y^U) \tag{6.41}$$

$$\forall i \in F_j, j \in J, m \in M, t \in T$$

$$0 \leq \sum_{j=1}^{J} S_{ijmt} \leq D_{imt} \quad \forall i \in F_j, m \in M, t \in T \tag{6.42}$$

$$(0.92.P_i^0) \leq \Pi_{it} \leq (1.08.P_i^0) \quad \forall i \in F_j, t \in T \tag{6.43}$$

6.4 Pricing Models Results and Discussion

To illustrate the applicability of the proposed pricing optimization models, pricing models discussed in previous sections have been implemented for the vehicle production supply chain network described in Chapter 3 . The vehicle producer is interested in knowing by how much prices can be adjusted to match supply with fluctuating demand within given elasticity coefficients for each nameplate. The effect of varying prices on the schedule of shifts, run rate and other operational decisions at production plants is also investigated. In this section, numerical results of the proposed pricing models are presented. Optimal prices of nameplates at each time period are presented as well as optimal capacity configuration (i.e., shifts, run rate and down periods). A comparison between fixed and variable price models in terms of profit is provided to prove the advantage of adding pricing decisions to the original capacity planning model. Sensitivity analysis is also presented to study the effect of change in model parameters on optimal solution.

6.4.1 Discrete Pricing Model Results

The linearized pricing model is solved using AMPL with CPLEX solver. All experiments were run on an Intel(R) core(TM) i7-8550U CPU @ 1.80 GHz and 8 GB RAM machine. The fixed prices model where prices are fixed to their current values is also solved for comparison purposes. Table 6.1 shows the results of fixed prices model for each sub-problem including problem size in terms of number of plants and number of different vehicles, computational time and optimality gap. Results indicate that the fixed prices model for SUV, Mid SUV, Vans, and Trucks sub-problems can be solved optimally in reasonable time.

As the problem gets larger as in the Cars problem, solution time increases considerably and the problem did not solve within the desired gap in 24 hours.

	Size (Plants,Vehicles)	CPU	Gap
SUV	(1,6)	6.26(s)	0.10%
Mid SUV	(1,5)	33.98(s)	0.10%
Vans	(1,4)	10.45(s)	0.10%
Trucks	(3,13)	11.38(min)	0.10%
Cars	(11,31)	24(hr)	3.38%*

Table 6.1: Fixed Prices Model Results

Table 6.2 summarizes the results of the discrete pricing model for the five sub-problems of the vehicle manufacturer case study. Computational time and optimality gap for each problem are shown. Optimality measures the quality of the current solution. The gap measures how far is the current best integer solution from the best possible upper bound on the objective function (Note that the objective is maximization as opposed to minimization). Initially, the LP relaxation is set to be the upper bound of the objective value, then as the solution improves a better bound is found. As discussed in Chapter 3, gap tolerance is set to 0.1% for the solution to be considered optimal and solution terminates after 24 hours if the desired gap is not reached. Percentage change in profit from the discrete pricing model compared with fixed prices is also presented.

	Size (Plants,Vehicles)	CPU	Gap	% Change in Profit
SUV	(1,6)	17.14(hr)	0.1%	4.50%
Mid SUV	(1,5)	76.30(s)	0.1%	4.90%
Vans	(1,4)	24(hr)	11.7%*	2.58%
Trucks	(3,13)	24(hr)	12.7%*	1.52%
Cars	(11,31)	24(hr)	21.2%*	2.63%

Table 6.2: Linearized Discrete Pricing Model Results

Both SUV and mid-size SUV problem can be solved optimally (gap is less than 1%). As problems get larger, the model cannot be solved optimally in reasonable time. For Vans, trucks, and cars the optimality gap after running for 24 hours is still high. The improvement of gap over time for car plants problem solved after linearization is plotted in Figure 6.4, it shows initially the gap drops drastically but then it is saturated and changes very slowly until the end of 24 hours.

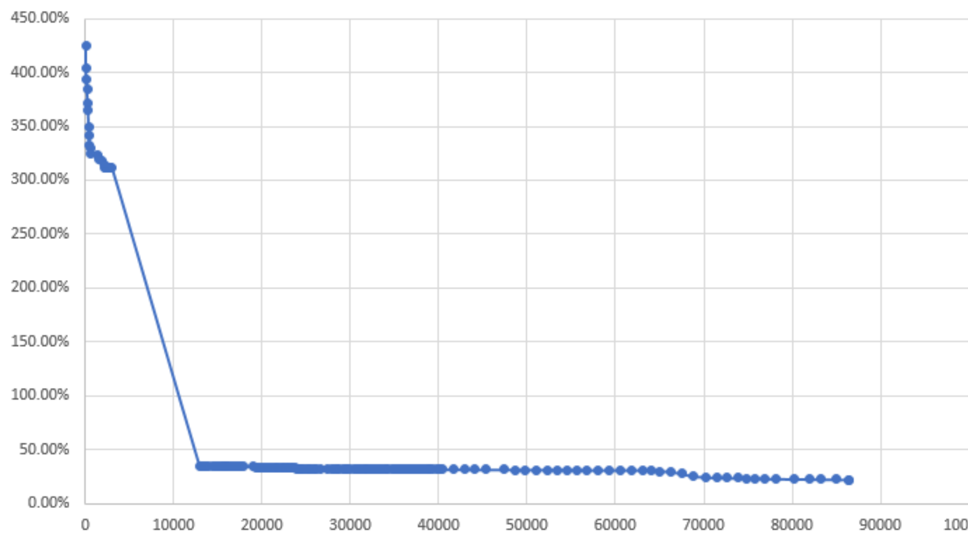
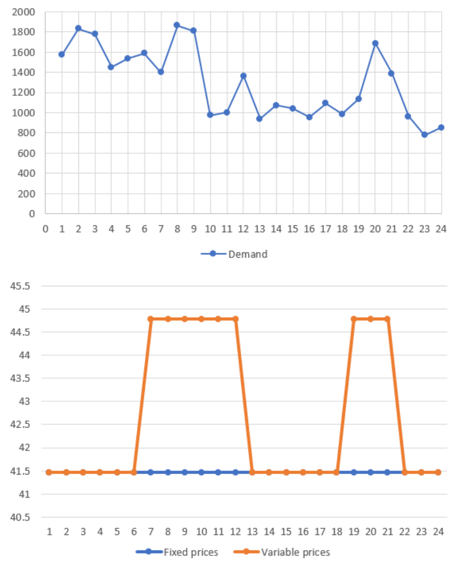


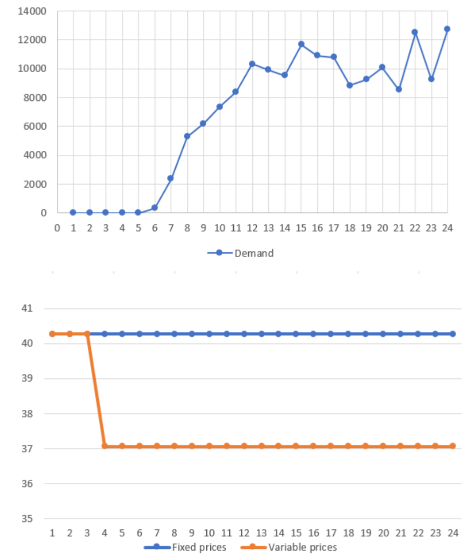
Figure 6.4: MIP Gap Improvement Over Time for Car Plants Problem Solved After Linearization

Figure 6.5 shows optimal prices obtained from the linearized model for sample vehicles over 24 months planning horizon compared to fixed prices. Demand is plotted as well to see the effect of price change on demand. In sub-figure (a) prices fluctuate between initial prices and maximum increase of 8% of initial prices. In sub-figure (b) prices are lowered by 8% until the end of planning horiaon. In sub-figure (c) prices are raised by 8% then lowered back to initial prices in the last quarter. The model suggests to decrease prices by 8% for some vehicles and increase sale volumes whenever there is an extra production capacity during regular time. If capacity is at the limit then prices are either increased by 8% or kept at nominal levels.

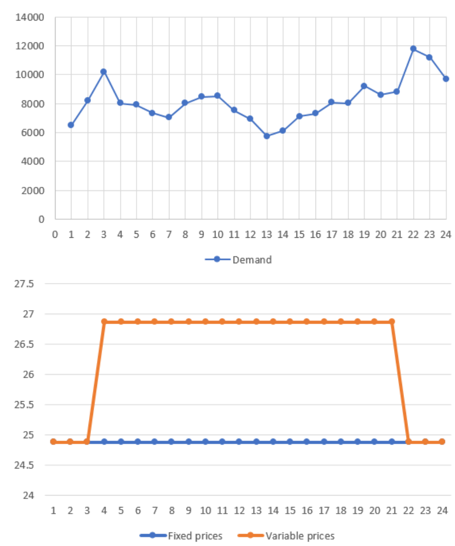
Total vehicle shipments have increased by 5.5% with variable price model. As a result, most plants have increased their shifts to meet the increase in shipments.The model also takes in to account the limitation on capacity adjustment determined by the shift, rate changes of three and six months respectively.



(a) Vehicle Model-14



(b) Vehicle Model-2



(c) Vehicle Model-9

Figure 6.5: Price and Demand Variation Over Time for Sample Vehicles

The Fix and Re-solve heuristic algorithm is applied to facilitate solving the problem. A restricted problem is solved at each stage in which the gap tolerance is set to 0.1% and solution time varies from 1 to 3 hours depending on problem size. Therefore, at each stage either an optimal solution is obtained or the time limit have been reached. Table 6.3 summarizes the results of the pricing model solved with the Fixed and Re-solve heuristic algorithm. Computational time represents the total time it takes the iterative scheme until it stops. The time limit represents the time allowed to solve each restricted problem at each stage. The table also shows the number of full iterations before stopping. In the gap column, the iterative objective value is not compared with linearized objective values since the solution obtained by solving the linearized model is not optimal. It is instead compared with the best upper bound found when solving the linearized model.

	Size (Plants,Vehicles)	CPU	Time Limit	Iterations	Gap
SUVs	(1,6)	2.00(hr)	2(hr)	1	4.22%
Mid SUVs	(1,5)	6.25(s)	3(min)	3	0.51%
Vans	(1,4)	2.01(hr)	2(hr)	3	12.24%
Trucks	(3,13)	2.00(hr)	2(hr)	2	12.95%
Cars	(11,31)	3.00(hr)	3(hr)	2	24.33%

Table 6.3: Discrete Pricing Model Results Using Fix and Re-solve Heuristic

Table 6.4 shows percentage change in objective function comparing the linearized and heuristic approach. Results indicate a loss in objective values of all sub-problems and that the Fix and Re-solve heuristic approach is not improving the objective function. This is because fixing the schedule consisting of shift, rate and down periods and solving for price limits the value of changing demand. Essentially, pricing without the ability to simultaneously adjust the key capacity levers, does not provide useful flexibility.

	%change in objective value
SUVs	-4.1%
Mid SUVs	-0.35%
Vans	-0.6%
Trucks	-1.9%
Cars	-2.2%

Table 6.4: Objective Function Value Comparison Between Linearized and Heuristic Approach

The Dual Variables driven heuristic algorithm is applied to solve the pricing model. Results in Table 6.5 shows percentage improvement in total profit compared with fix prices and linearized model prices respectively. The reported gap is calculated based on the upper bound obtained from the linearized model. Computational time for each sub problem together with the number of iterations are presented. Contrary to the Fix and Resolve heuristic results, the Demand Dual heuristic is able to find significantly better price-capacity configurations. Profit has improved for all five sub-problems when prices are allowed to change.

	Problem				
Stats	SUV	MID	VAN	TRK	CAR
Size (Plants,Vehicles)	(1,6)	(1,5)	(1,4)	(3,13)	(11,31)
Gap	0.1%	0.1%	11.67%	11.15%	17.22%
Time (min)	10.38	15.9	21.47	35.89	150
No. of iterations	120	81	48	160	409
Change in profit w.r.t fixed prices	4.5%	4.9%	2.6%	1.6%	2.9%
Change in profit w.r.t linearized	0.0%	0.0%	0.04%	0.11%	0.34%

Table 6.5: Results of the Demand Dual Variables Heuristic for Discrete Prices

Total production by month at a sample of two plants is plotted in Figures 6.6 and 6.7. In the graphs total production with fixed prices is also plotted for comparison. Figure 6.6 shows approximately same but smoother production levels over time as a result of variable prices. The graph in Figure 6.7 indicates that variable prices slightly reduced production as a result of price increase of vehicles produced at this plant.

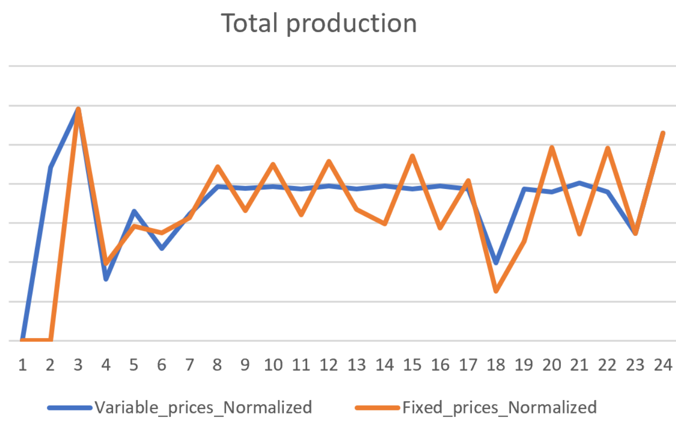


Figure 6.6: Total Production by Month at One of the Car Production Plants

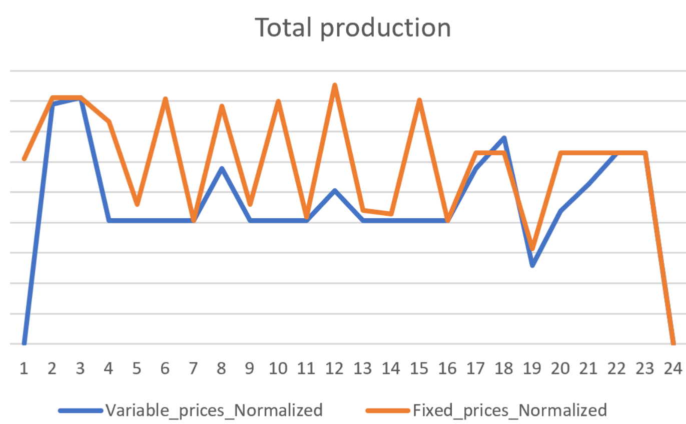


Figure 6.7: Total Production by Month at One of the Car Production Plants

To see the effect of varying prices on optimal capacity plan, regular time capacity at production plants is studied for both fixed and variable prices. Regular time capacity per month for a sample plant is plotted for both fixed and variable prices in Figure 6.8. The figure shows the increase in regular time capacity as a result of varying prices. For MID SUV, production volumes has been increased as a result of lower prices.

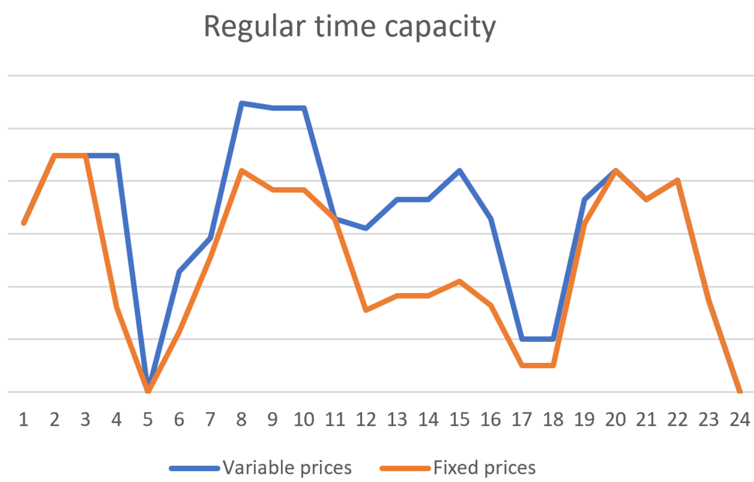


Figure 6.8: Regular Time Capacity by Month at MID SUV Production Plant

In this section several solution approaches have been presented to solve the discrete pricing model. Table 6.6 provides a comparison of these approaches. Computational time and optimality gap are presented. Optimalitiy gap is calculated based on the current best upper bound on the objective value which is obtained from solvig the linearized model. Results indicate that discrete pricing lineraized model can be solved optimally and in reasonable time for one sub-problem (MID) out of the five sub-problems. Results form the linearized discrete pricing model indicate that profit for all sub-problems have improved compared with fixed prices with a maximum improvement of 4.9%. Two heuristic algorithms are proposed to facilitate solving the pricing model. The Fix and Re-solve heuristic algorithm seems not to be improving the objective value as the gap reported for each sub-problem is higher than linearized model gaps. The Fix and Re-solve heuristic limits the interaction between prices and shifts, rate and down period. On the other hand, with the Demand Dual Variables heuristic algorithm the the computational time has improved for all sub-problems. The largest problem (CAR) is solved in about 2.5 hours with the Dual Variables heuristic as opposed to 24 hours using the linearized model.

Stats	Problem				
	SUV	MID	VAN	TRK	CAR
Size (Plants,Vehicles)	(1,6)	(1,5)	(1,4)	(3,13)	(11,31)
Linearized					
Gap	0.1%	0.1%	11.7%	12.7%	21.2%
Time	17.1(hr)	76.3(s)	24(hr)	24(hr)	24(hr)
Change in profit w.r.t fixed prices	4.5%	4.9%	2.5%	1.5%	2.6%
Fix and Re-solve Heurisitc					
Gap	4.22%	0.51%	12.24%	12.95%	24.33%
Time	2(hr)	6.5(s)	2(hr)	2(hr)	3(hr)
Change in profit w.r.t fixed prices	0.21%	4.5%	1.9%	0.4%	0.3%
Demand Dual Variables heuristic					
Gap	0.1%	0.1%	11.67%	11.15%	17.22%
Time(min)	10.38	15.9	21.47	35.89	150
Change in profit w.r.t fixed prices	4.5%	4.9%	2.6%	1.6%	2.9%

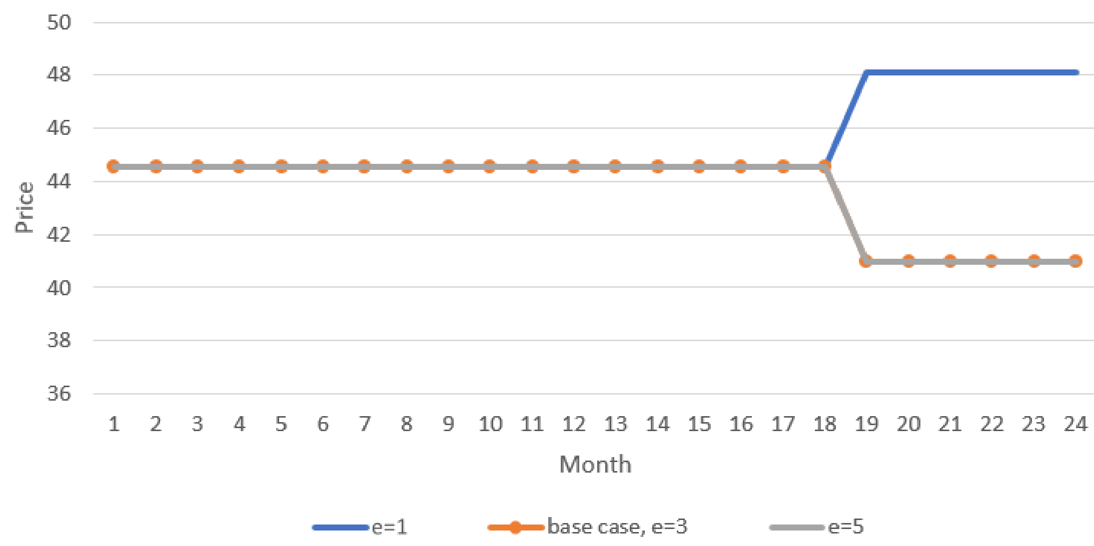
Table 6.6: Comparison of Solution Approaches for the Discrete Pricing Model

6.4.2 *Sensitivity Analysis*

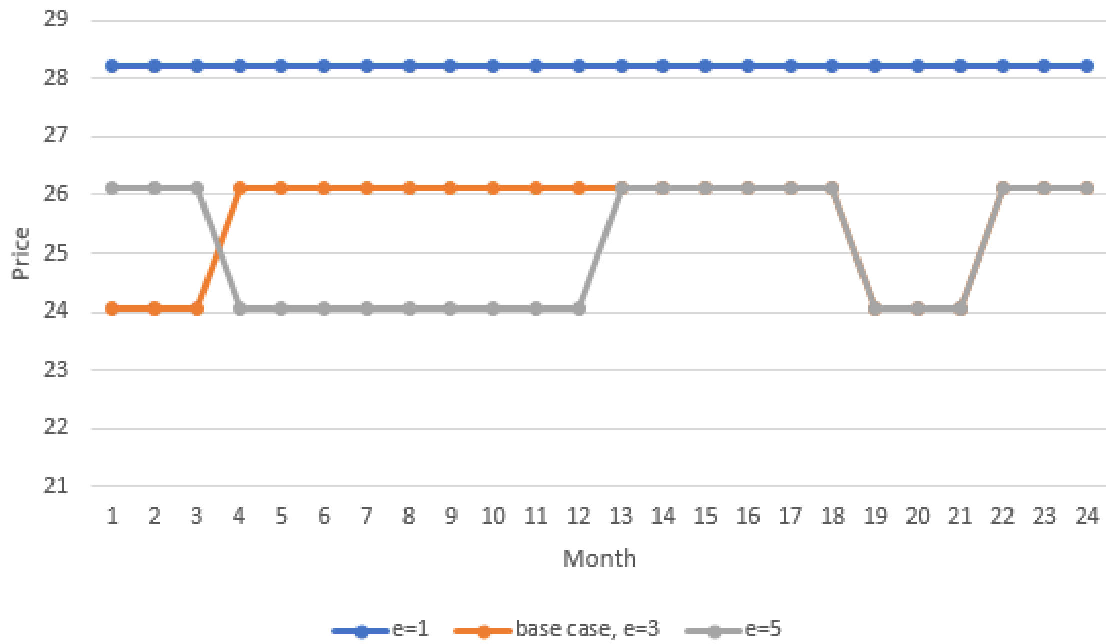
In this section different scenarios for the pricing model are investigated and the effect on optimal solution is studied. Sensitivity of some model parameters is tested. For sensitivity runs a sample of two problems out of the five problems are studied and solved using the linearizd model. A small and a medium size problem are selected namely, Mid SUV and Trucks. The following scenarios are investigated:

1. Price elasticity of demand effect

   Different factors may affect pricing decisions. One of the most important factors is price elasticity of demand (e_i). Different values of price elasticity coefficients may affect prices and profits. As per equation (6.7) elasticity is inversely related to prices. Therefore, smaller elasticity tend to increase prices while larger elasticity decrease prices. On the other hand, sales are affected. An increase in prices is accompanied by a decrease in sales which affects production volumes and optimal capacity levels. Total profits of the nominal elasticity value is compared with two values selected within the possible range of price elasticity of vehicles. In Figure 6.9 optimal prices are shown for a sample of two vehicles for a base case with $e_i = 3$, a case with $e_i = 1$, and a case with $e_i = 5$. In graph (a) prices remain unchanged for a period of 18 months, then are raised with a decrease in elasticity. In graph (b) prices increase with a decrease in elasticity, but with an increase in elasticity prices alternate between nominal case prices and minimal possible prices. This shows that prices for some vehicle models are sensitive to price elasticity change, especially with lower elasticity values.



(a) Sample Vehicle for Mid SUV Problem



(b) Sample Vehicle for TRK Problem

Figure 6.9: Optimal Prices Using Different Elasticity Coefficients for Sample Vehicles

Table 6.7 shows percentage changes in total profit corresponding to change in elasticity value compared to the base case for the mid-size SUV problem. Results show an improvement in total profits of about 9% in response to increase in elasticity. As discussed earlier, higher elasticity tends to reduce prices and as a result more sales are expected. The increase in sales increases total cost but total profit is improved.

% improvement in Total Profit		
	e_i	Linearized model
Case-1	1%	0.63%
Base case	3%	-
Case-2	5%	9.2%

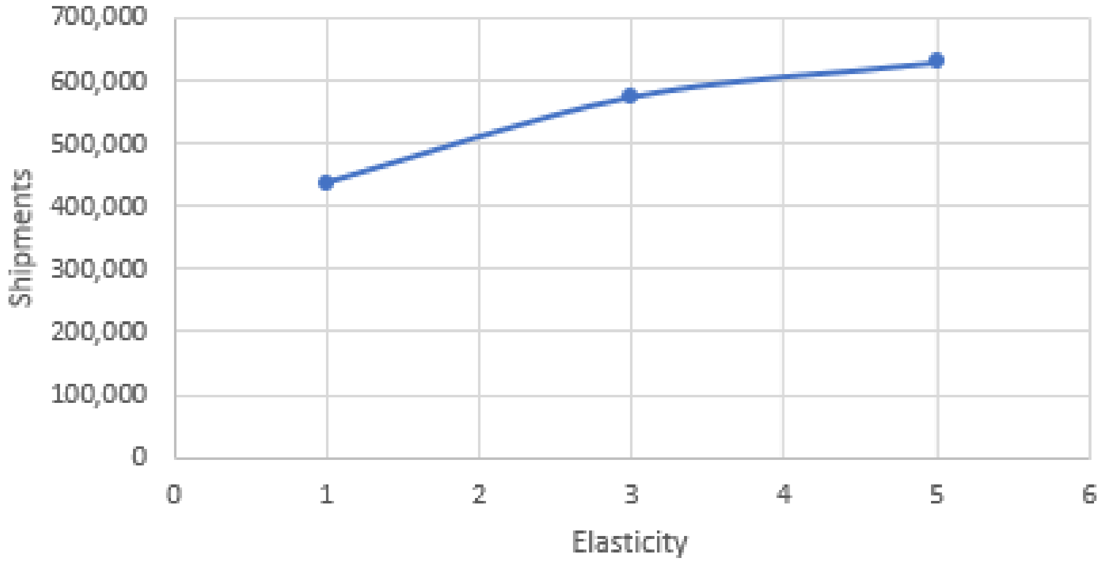
Table 6.7: Price Elasticity of Demand Effect for Mid-size SUV Problem

Results in Table 6.8 show an improvement in total profits. More improvement is obtained when elasticity is reduced. That's when prices are increased. For the trucks problem it's more economical to increase prices and sell fewer amounts of vehicles.

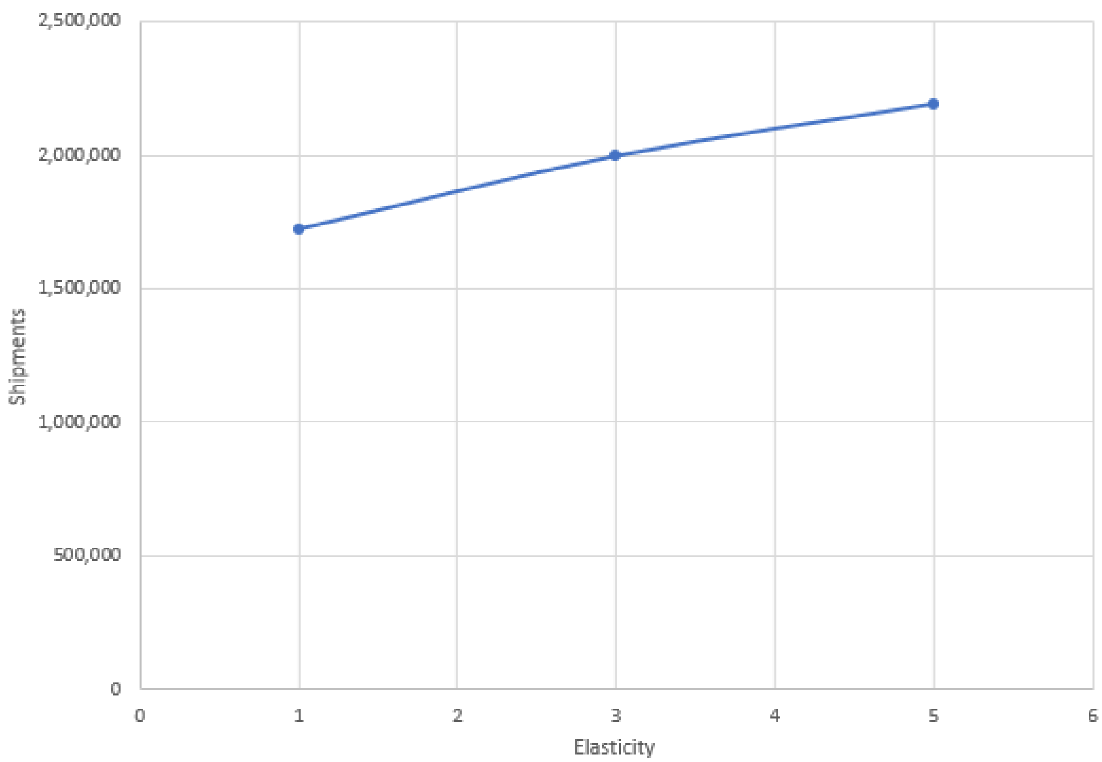
		% improvement in TP
	e_i	Linearized model
Case-1	1%	9.3%
Base case	3%	-
Case-2	5%	7.2%

Table 6.8: Price Elasticity of Demand Effect for Trucks Problem

Figure 6.10 shows shipments for different values of elasticity. As elasticity increases, shipments increase. Figures 6.11 show the change in profits, revenues and cost for three values of elasticity. Revenues and cost are directly related with elasticity and shipments as well. The increase in revenues is justified by the higher units sold caused by reduction in price.



(a) Mid-size SUVs



(b) Trucks

Figure 6.10: Elasticity Versus Shipments

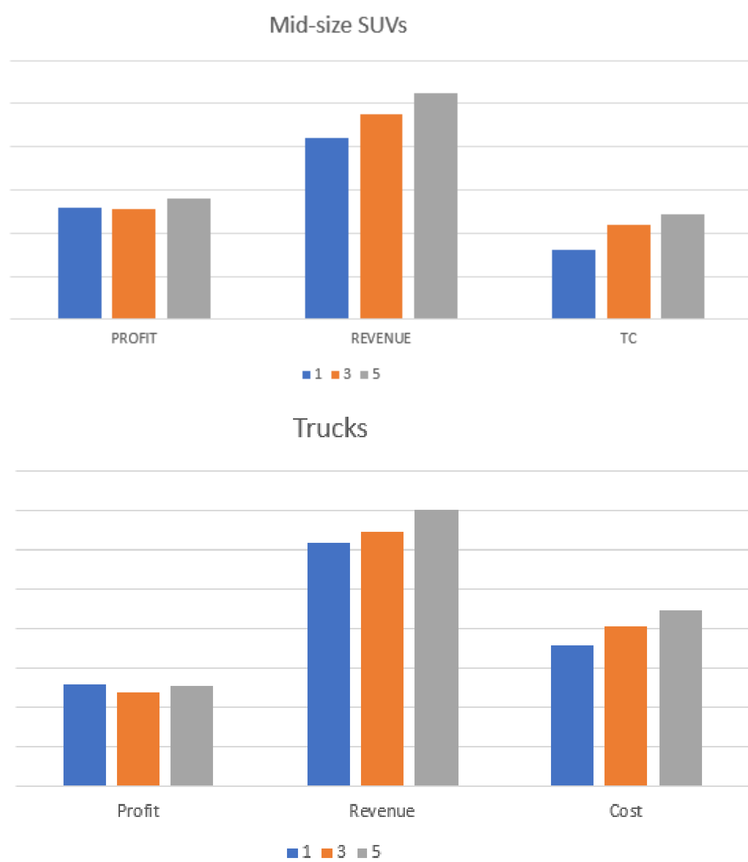


Figure 6.11: Profit, Revenue and Cost for Differed Elasticity Coefficients

Figure 6.12 shows the change in regular time capacity with the change in price elasticity demand. Note that an elasticity of zero is equivalent to fixing prices. Capacity decreases initially because with elasticity of one prices are increased then it starts increasing with the increase in elasticity.

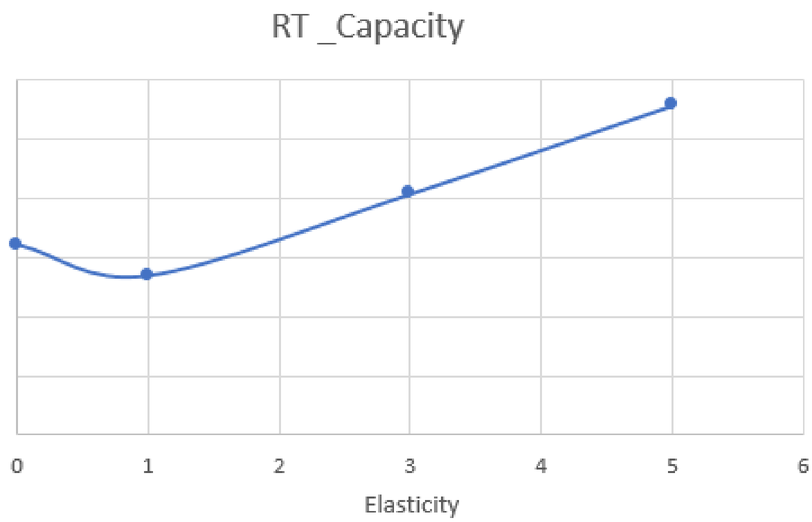


Figure 6.12: Regular Time Capacity Variation Over Different Elasticity Coefficients

2. Cross-price elasticity of demand effect

   Cross-price elasticity of demand (XPED) plays an important role for determining the prices of substitutable goods. The substitution effect is the decrease in a product's demand in response to consumers switching to cheaper similar products when its price rises. XPED measures the responsiveness in the demand of one good when the price of another good changes. XPED of products A and B is defined as follows:

$$XPED_{AB} = \frac{\Delta Demand_A / Demand_A}{\Delta Price_B / Price_B} \tag{6.44}$$

   In the pricing model a new parameter is defined for cross-elasticity as follows:

   - xe_{ii}: cross-price elasticity of demand of pair of product in set i

   The demand function in Equation (6.8) is adjusted as follows to account for cross-elasticity of each pair of substitutable products:

$$D_{imt} = d_{imt} + (xe_{ii}.d_{imt}).((PR_{it} - P_i^0)/P_i^0) + \sum_{i':i' \neq i} (xe_{ii'}.((PR_{i't} - P_{i'}^0)/P_{i'}^0)$$
$$\forall i \in I, m \in M, t \in T \tag{6.45}$$

   In constraint (6.45) the cross-elasticity coefficient is used to determine the demand of product i in market m at each time period t using product price, initial price and demand forecast information at time period t as well as the price change of substitutable products. To study the effect of substitutability for the vehicle manufacturer case study, vehicles are assumed to be partially substitutable.

Vehicles produced in the same plant are assumed to be substitutable since similar vehicles are usually produced in the same plant and that plant has the required tooling for producing same vehicle category (i.e. trucks, SUVs,...etc.). With the reformulated demand function the computational complexity is expected to increase, since the model will try to investigate the relation between each pair of products. Results in Table 6.9 shows percentage improvement in total profits as a result of substitutability. Computational time is also presented. From the table it is concluded that substitutability adds value to the model but it drastically increases computational time. The value of substitutability currently appears not to be significant, however, it is possible a larger effect would be discovered if solved to a smaller gap. And, the existence of cross-elasticity is a market property, not a decision option.

	Time	% improvement in TP
		Linearized model
Case-1	48(hr)	1.2%
Base case	101.6(s)	-

Table 6.9: Cross-elasticity Effect for Mid-size SUV Problem

3. Price change frequency effect

   How often prices change over the planning horizon might have an effect on total profits. This may also impact customer behavior, but data on that effect is not available and such consideration is not included in this study. Table 6.10 shows the effect of allowing monthly price changes as opposed to quarterly price changes which is the base case. Profit has improved by 2.6% when truck prices are allowed to changing more frequently however there is only a small effect on total profit for mid-size SUVs.

% improvement in Total Profit		
	SUVs	TRK
	Linearized model	Linearized model
Case-1 (monthly)	0.019%	2.6%
Base case (quarterly)	-	-

Table 6.10: Price Change Frequency Effect for Mid-size SUV and Trucks Problems

6.4.3 Continuous Pricing Model Results

The pricing model with continuous prices is solved using CPLEX solver for the vehicle manufacturer case study with the previously discussed problem separation. As discussed in Section 5.1.3 the model is linearized using an approximate procedure, and, therefore, solution is not guaranteed to be optimal. However, it provides an upper bound on the optimal solution. After analysing the results, it has been observed that the revenue value doesn't match the actual revenues (revenue calculated using shipments and prices from the solution). The gap between those values is due to the approximation of the over estimators of the objective function. The calculated actual revenue value is compared with the optimal revenues to see how significant is the gap. Revenues from the continuous prices model is also compared with the discrete prices revenue. Table 6.11 shows continuous pricing model results solved after linearization.

	Size (Plants,Vehicles)	CPU	Gap
SUV	(1,6)	31.2(min)	0.09%
Mid-size SUV	(1,5)	20.6(min)	0.09%
Vans	(1,4)	60.3(min)	0.09%
Trucks	(3,13)	24(hr)	0.2%
Cars	(11,31)	24(hr)	2.1%

Table 6.11: Linearized Continuous Pricing Model Results

In Figure 6.13 total profit of the continuous pricing model is compared with fixed prices model and discrete prices.

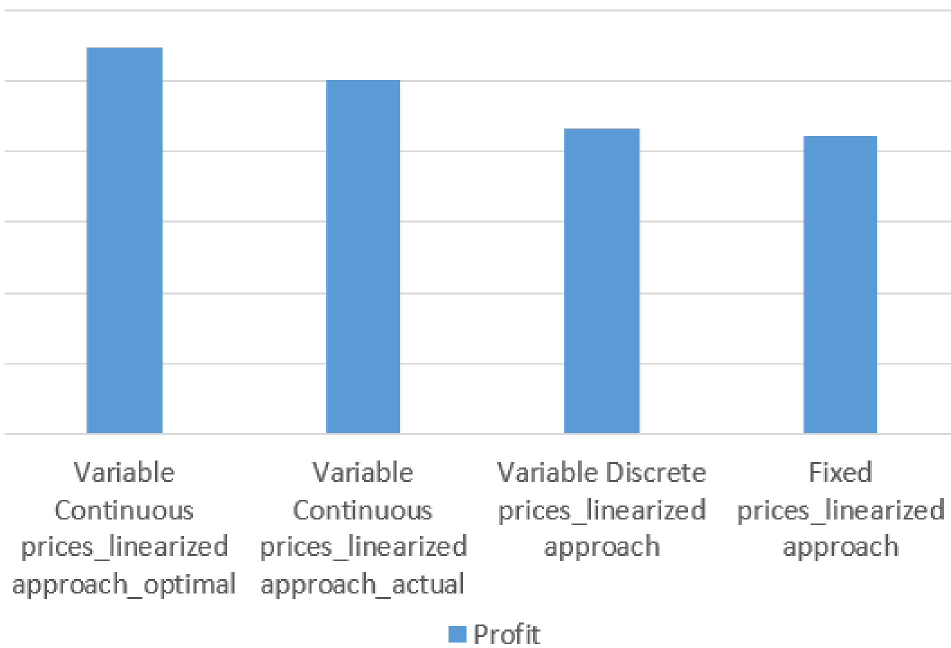


Figure 6.13: Total Profit Model Comparison for Continuous Pricing

In Figure 6.14 revenue of the continuous pricing model is compared with fixed prices model and discrete prices. Results indicate a 3.9% gap due to the approximate linearization. Results also indicate that with continuous pricing revenues have improved by 4.2% compared to discrete prices.

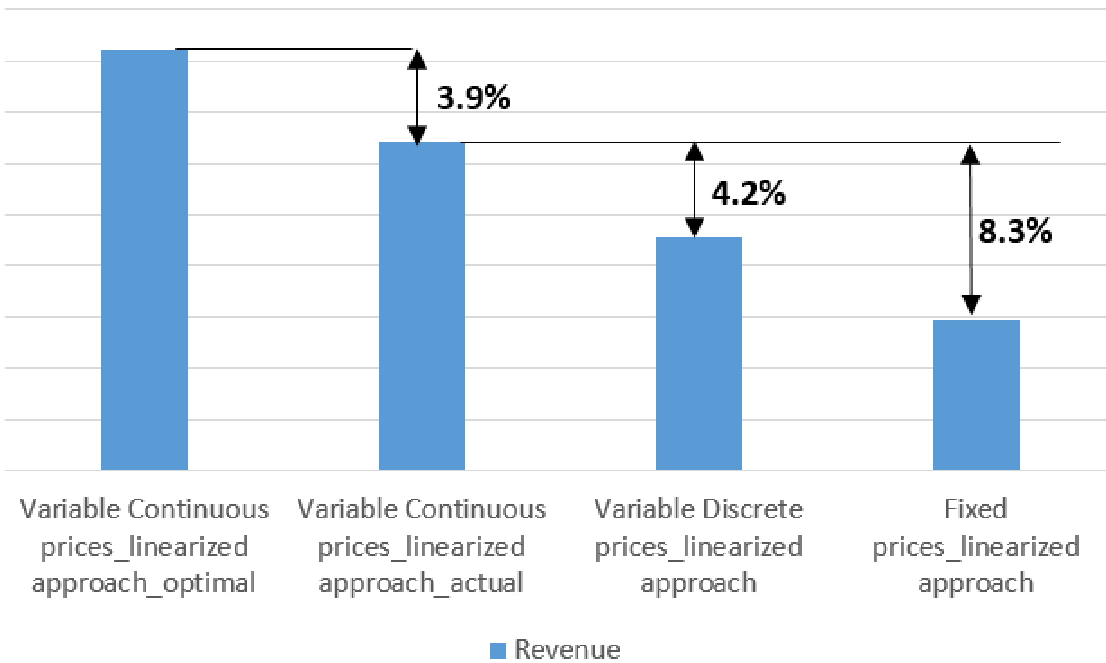


Figure 6.14: Total Profit Model Comparison for Continuous Pricing

To reduce the gap caused by the approximation, the approximate linearization can be improved by adding tighter bounds. Figure 6.15 shows the tighter bounds based on the tangent line as well as the line connecting the points (x^U, y^L) and (x^L, y^U). This approach needs further investigation and will be considered for future research.

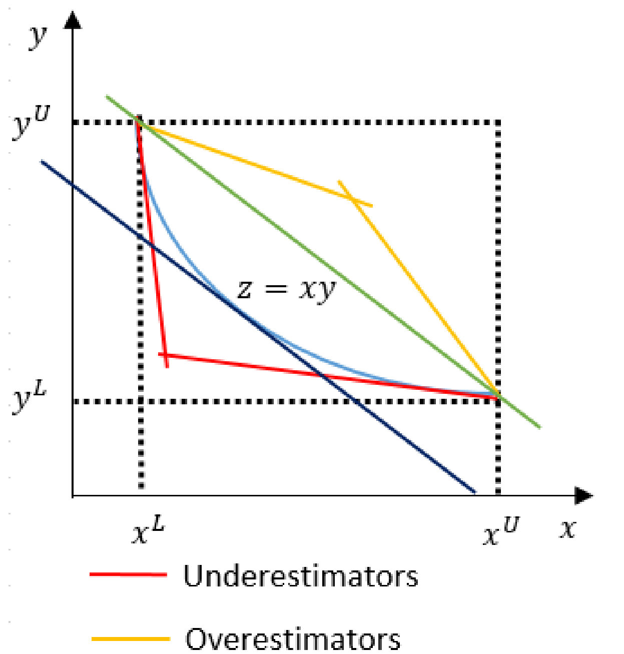


Figure 6.15: Improving the Linear Approximation

6.5 Conclusion

In this chapter pricing decisions are integrated in the capacity planning model proposed in Chapter 3. This model optimizes product prices by maximizing total profits. Solving the model also returns an optimal capacity plan as well as optimal operational decisions including production, inventory and distribution. The purpose of the proposed pricing model is to investigate the value of adding pricing factor to the original capacity planning model as well as the effect of varying prices on the optimal capacity plan (i.e. shifts, rate and down periods). Research has shown that demand triggers all supply chain operations. Therefore, it's important to study demand and factors that affect demand and try to find a better match between supply and demand. Price plays an important role in shaping demand. In economics the price elasticity of demand is used to represent the relation between demand changes and price changes. PED is used to define the relation between demand and prices in the proposed pricing model. The full problem formulation is presented as well as solution methodologies. Comparison of solution methodologies in terms of computational time and optimal objective gap is also provided. Sensitivity of model parameters is also conducted.

The vehicle manufacturer case study is considered for the implementation of the pricing model. Results show that it's economical to decrease prices within a given price elasticity and sell more products to maximize revenues. Although this strategy increases total cost since production volumes are increased, the marginal revenues exceed marginal costs resulting in positive marginal profits. Production plants adjust capacity to meet the increased production volume by scheduling of shifts and/or overtime. Since production volumes are increased, the model schedules more shifts with maximum run rates. In addition, increased sales increase market share, lower fixed cost per vehicle an enhance employee job security.

The pricing model is further evaluated by comparing the results to the fixed prices model. Results show in this case that varying prices improve total profits by 2.7% compared to fixed price profits. Additionally, results indicate that adjusting prices smoothed production at some production plants providing more production consistency over time.

In this chapter different solution methodologies for the pricing model are proposed. First, the MINLP pricing model is solved after linearizing non-linear constraints. However, due to the complexity of the model especially with large size problem the model doesn't solve in reasonable time. An efficient heuristic algorithm is proposed to overcome computational complexity. The Demand Dual Variables heuristic algorithm is implemented to solve the pricing model. Numerical results show that with the proposed heuristic algorithm solution time has significantly with better objective gap.

Chapter 7

CONCLUSION AND FUTURE RESEARCH

In this research tactical and operational planning of supply chain networks is investigated. The objective of this research is to develop efficient optimization models for the capacity planning and production and distribution scheduling problem of a multi-product, multi-plant supply chain network of durable consumer goods. The approach is based on mathematical programming techniques, especially MIP models, and their solution approaches. Limited options of capacity configuration may result in uneconomical production and distribution plans. Therefore, firms attempt to continuously improve their production plans to economically meet market demand by selecting optimal capacity plans. Integration of capacity planing and production scheduling is key to achieving this goal. In this research, an integrated capacity planning model with three key discrete decisions: choice of run rate, shifts and down periods as well as continuous production, overtime, inventory and distribution decisions is proposed. These key decisions are the most important factors of capacity determination at production plants. Based on the literature reviewed, none of the previous related research has considered these factors jointly in one model nor therefore, provided an integrated planning model at this level of tactical detail. Research and practice have shown that MIP can be a useful tool for supply chain network optimization. Solving such models can produce significant improvement in supply chain operations, and, as a result, better service level with minimal cost.

Although there has been a significant work done in this area in the literature, the models proposed in this research are intended to fill a gap by including detailed levers yielding flexibility to configure production capacity in a multi-stage, multi-product network and in a multi-period setting.

7.1 Research Summary

The integrated capacity planning, production and distribution scheduling model proposed in this research is a nonlinear mixed integer program. The model determines optimal capacity configuration and production at assembly plants to meet medium range demand forecasts at minimal cost. Production during regular and overtime and distribution decisions are included as well as inventory levels of finished goods. Flexible capacity is modeled by the choice among different possible run rate levels, adding or deleting shifts and scheduling of down periods at production plants in each time period. Changing capacity level however, requires cost and time, therefore cost and time to implement capacity changes is taken into account in the model. The proposed model accounts for cost of acquiring raw material, fixed and variable labor cost to operate shifts and cost of shipping final products to markets. Shortage and inventory holding costs are also considered as well as capacity limitations such as scheduling of overtime. Production limits by product and plant are included based on tooling and supplier capability.

An alternative formulation for the capacity planning, production and distribution scheduling problem is presented to allow comparison of solution quality and solution time. In the alternative formulation one variable is used to define a schedule of run rate, shift, and down periods as opposed to three separate variables. With this setting, the model is no longer non linear since all cubic and quadratic terms are substituted by linear terms.

The new formulation has the advantage of being linear which is more tractable than non linear models. However, adjusting model constraints becomes more difficult due to the aggregation of three key variables. Another limitation is constructing the data.

In this research a pricing model is also proposed which integrates product price decisions into the capacity planning model. In the pricing model, optimal prices of products are determined together with other operational decisions jointly. Product price and demand are inversely related by the law of demand, this relation is expressed by price elasticity of demand coefficients. The effect of price elasticity of demand on the operational decisions is studied. With this model, the value of adding pricing decisions in the supply chain optimization model is shown as well as potential profits gained by varying product prices as opposed to fixed prices.

Solution methodologies for the proposed mathematical models are discussed. Given the complexity of the integrated model, solving real life problems optimally is computationally difficult. Solving MINLPs is challenging and still under study in the area of operations research and optimization. Solution methodologies proposed in this research include linearization in an effort to gain potentially optimal solutions. This approach however, is computationally inefficient for large size problems. Therefore, an heuristic algorithm is proposed to solve the model to near optimally in reasonable time. A Fix and Re-solve heuristic algorithm is designed to solve the models. The idea behind this algorithm is to solve the full model iteratively by decomposition which is based on the key binary variables. Another efficient heuristic algorithm which is based on information from dual variables is proposed to solving the pricing model.

7.2 Key Findings

A real life problem is studied to implement the proposed models and obtain numerical results. A case study of a major vehicle manufacturer in North America is considered to prove the applicability of the proposed mathematical models. The vehicle production firm manufactures and sells a variety of vehicle models such as SUV, trucks, vans and cars. Their network consists of multiple assembly plants producing multiple vehicle types (nameplates). The proposed capacity planning model, the alternative model and pricing model of the vehicle manufacturer are solved using AMPL with CPLEX solver. Results of different solution methodologies are presented and discussed. Numerical results shows that with the proposed model, total cost has improved by 13% compared with current operations at production plants. This proves the value gained by employing shifts, run rate, down periods, and overtime jointly to set capacity level at production plants. The flexibility to adjust capacity with these levers is essential and prevents a huge loss. Results also show that the proposed iterative Fix and Re-solve heuristic is able to find significantly better capacity configurations in reasonable time when problem size makes solving the linearized model difficult.

On the other hand, the Fix and Re-solve heuristic approach is not improving profits in the pricing model because pricing without the ability to simultaneously adjust the key capacity levers, does not provide useful flexibility. Contrary to the Fix and Resolve heuristic results, the Dual Demand heuristic is able to find significantly better price-capacity configurations. Results of the pricing model reveals the advantage of varying prices over fixed prices. Results indicate smoother production. Moreover, with variable prices profits have improved by 2.7%.

The pricing model suggests to reduce prices of some vehicles within a defined elasticity and therefore, increasing production volumes. Increased sales on the other hand, increase market share, lower fixed cost per vehicle an enhance employee job security.

7.3 Future Research

A number of further research tasks can be done for this problem.

- One future work in this research could be modeling continuous prices. The model is specified but initial results indicate that existing solution techniques may not work well and thus new methodology will be needed.

- Another research task is to investigate solution approaches for the schedule generation formulation by column generation and branch and bound. Unfortunately the construction of such an algorithm will not be straightforward as the natural subproblem for generating columns is nonlinear. Efficient methods for defining the set of possible columns, computing their parameters and generating potential improving variables (columns) are needed.

- The alternative formulation defined variables as a shift, rate and down week solution for a plant in a period. This required complex constraints for linking periods due to implementation of changes. A different alternative formulation would define a schedule for a plant for the entire planning horizon in a single variable. The model would then need to only select one schedule for each plant. However, the number of possible variables would grow significantly. Column generation or other approaches could be explored for such a formulation.

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