Experimental Procedures and Data Analysis of Orthotropic Composites

by

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A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Science

Approved August 2016 by the
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December 2016

ABSTRACT

Composite materials are widely used in various structural applications, including within the automotive and aerospace industries. Unidirectional composite layups have replaced other materials such as metals due to composites' high strength-to-weight ratio and durability. Finite-element (FE) models are actively being developed to model response of composite systems subjected to a variety of loads including impact loads. These FE models rely on an array of measured material properties as input for accuracy. This work focuses on an orthotropic plasticity constitutive model that has three components – deformation, damage and failure. The model relies on the material properties of the composite such as Young's modulus, Poisson's ratio, stress-strain curves in the principal and off-axis material directions, etc. This thesis focuses on two areas important to the development of the FE model – tabbing of the test specimens and data processing of the tests used to generate the required stress-strain curves. A comparative study has been performed on three candidate adhesives using double lap-shear testing to determine their effectiveness in composite specimen tabbing. These tests determined the 3M DP460 epoxy performs best in shear. The Loctite Superglue with 80% the ultimate stress of the 3M DP460 epoxy is acceptable when test specimens have to be ready for testing within a few hours. JB KwikWeld is not suitable for tabbing. In addition, the Experimental Data Processing (EDP) program has been improved for use in post-processing raw data from composites test. EDP has improved to allow for complete processing with the implementation of new weighted least squares smoothing options, curve averaging techniques, and new functionality for data manipulation.

ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Rajan for his help and support during my time here. I would also like to thank my committee members Dr. Neithalath and Dr. Mobasher for their teaching. Finally, I would like to thank Canio Hoffarth and Bilal Khaled for their help in experimentation.

TABLE OF CONTENTS

CHAPTER Page

APPENDIX Page

LIST OF TABLES

Table Page

LIST OF FIGURES

Figure Page

Figure Page

1 Introduction

Composite materials are widely used in various structural applications, including within the automotive and aerospace industries. Unidirectional composite layups have replaced other materials such as metals due to composites' high strength-to-weight ratio and durability. Finite-element (FE) models are actively being developed to model response of composite systems subjected to a variety of loads including impact loads. These FE models rely on an array of measured material properties as input for accuracy. This work focuses on a constitutive model implemented in LS-DYNA® called MAT213, an orthotropic plasticity model that has three components – deformation, damage and failure (Livermore Software Technology Corporation (LSTC) 2015); (Goldberg, et al. 2015); (Harrington, et al. 2016). The model relies on the material properties of the composite such as Young's modulus, Poisson's ratio, stress-strain curves in the principal and off-axis material directions, etc.

1.1 Literature Review

This thesis has two major areas of focus related to the characterization of orthotropic composites: (1) sample preparation with focus on tabbing of the specimens to enable proper gripping and response of the specimen in the grips, and (2) development of a graphical-user interface (GUI)-based computer program for processing the experimental results from the material characterization tests.

Tabbing strategies for composite testing have been explored in the past with several tabbing guides available for use in composites testing (Adams and Adams 2002). Since tabs are attached to the test specimens via adhesives, studies of adhesives are often done as comparison studies between adhesives. Research in the area of adhesives has found that symmetric lap-shear testing is best for characterizing adhesives because the response from this test closely matches the expected true shear response (Renton 1976). In their analysis of several papers, Matthews et al. found primary characteristics necessary in adhesives in shear includes adherend type, overlap size, and low shear modulus, among others. Adhesives testing using isotropic materials such as metals simplifies the analysis by reducing the number of failure modes in test specimens. Anisotropy in composites introduces not only failure modes within the substrate but new interactions between the adhesive and substrate. Failure in the interface between substrate adhesives in any application have commonly been attributed to quality control problems, however (Matthews, Kilty and Godwin 1982). Yang and co-workers developed an analytical FE model of ASTM lap shear specimens using equivalent plastic strain criterion and the fracture mechanic approach for use in comparing adhesives (Yang, Tomblin and Guan 2003).

There is a variety of smoothing methods available for removing outliers that potentially show in raw data. Savitzky and Golay developed one of the early least-squares polynomial filters, using local nonlinear regression to smooth data (Savitzky and Golay 1964). Cleveland in 1979 developed the basic assumptions used in the Locally Weighted Regression and Smoothing Scatterplots (Lowess) method and its robust version, including parameter selection and weight function determination. An improvement from other least-squares smoothing techniques, Lowess and the related Local Regression (Loess) smoothing method use a local regression weighted based on the proximity and – for robustness and removing outliers – the residuals of each data point within the local region (Cleveland 1979). Other filters may include those based on power law, logarithmic, and exponential equations (Stroud 1999).

1.2 Thesis Objectives

This thesis focuses on two areas important to the development of the MAT213 FE model – tabbing of the test specimens and data processing of the tests used to generate the required stress-strain curves. The material chosen for validating MAT213 is the T800S/3900-2B unidirectional composite manufactured by Toray.

Tabbing: Some tabbing strategies for use in characterizing composites have been followed in characterizing the T800S/3900-2B composite at Arizona State University as previously described (Adams and Adams 2002). During this preliminary testing, G-10 glass fabric/epoxy (fiberglass) tabs sometimes de-bonded from the T800S/3900-2B substrate. This failure is in the adhesive rather than in either the fiberglass or the T800S/3900-2B substrate. Three candidate adhesives were selected for testing to determine which is best suited for use with composite tabbing: 3M DP460 – the original adhesive used, Loctite Super Glue, and J-B KwikWeld (3M 2015), (Henkel 2014), (J-B Weld It Australia 2012). All adhesives selected are paste adhesives that are easy to use during sample preparation. To test each adhesive, standard test procedures for double lap shear adhesive characterization tests is used (ASTM, 2008). Supplemental analysis using Digital Image Correlation (DIC) during these tests is also used to ensure consistent results between tests.

Data Processing: To simplify the process of reviewing experimental raw data and generating material constants and stress-strain curves, a program has been developed at

Arizona State University called Experimental Data Processing (EDP) program (Vokshi 2011). The current version of EDP has the following functionalities.

(1) Reading raw data from text file (*.txt) and comma separated file (*.csv) with several different field delimiters. Displaying the data as x-y plot.

(2) Displaying and editing raw data in the form of a spreadsheet.

(3) Identifying outliers using Chauvenet algorithm (Lin and Sherman 2007).

(4) Smoothing the raw data using the following options – median, simple moving average, polynomial moving average, Loess, robust Loess, Lowess, and robust Lowess. The smoothened data can then be used to create a stress-strain curve.

(5) Fitting curves to the smoothened data using the following fitting equations – linear polynomial, quadratic polynomial, cubic polynomail, quartic polynomial, qunitic polynomial, exponential function of the form ae^{bx}, logarithmic function of the form $a\log(bx)$ and power equation of the form ax^b.

(6) Creating a single stress-strain curve from the stress-strain curves processed from several replicates using a least-squares fit model.

(7) Creating the MAT213 constitutive model data for LS-DYNA analysis.

(8) Carrying out distribution analysis of experimental data using 2-parameter Weibull, 3-parameter Weibull, normal, log normal, Gamma and generalized exponential distribution models.

In this thesis, the focus is on items (1)-(6). Existing functionalities were enhanced and new functionalities were added.

2 Experimental Procedures

Building finite element models of composites requires material properties such as modulus of elasticity, Poisson's ratio, etc. or the entire stress-strain curve. These material property values are usually obtained from laboratory experiments. The typical pre-test steps are as follows.

Step 1. Identify the appropriate ASTM document governing the test for the selected material. This will help identify the test machine; the grips to be used; the test conditions such as loading rate, temperature etc.; how test data will be obtained in real time; and how the test data will be processed after the test to obtain the material properties.

Step 2. Generate the dimensions for a typical specimen. Machine the specimen.

Step 3. Identify how the specimen is inserted and held in the grips.

Step 4. Prepare the specimen for test – if required, bonding the tabs to the specimen, or bonding the specimen to the test fixture, and speckling the specimen.

In this chapter, the test methods for characterizing unidirectional composites are discussed and candidate adhesives that can be used in preparing the composite test specimens are investigated with the test results are discussed.

2.1 Experimental Procedures for Orthotropic Composites

This section covers the methods used to gather data to characterize the T800S/3900-2B composite for use with MAT 213.

2.1.1 Material Data

The T800S/3900-2B composite manufactured by Toray Carbon Fibers America, Inc. is a unidirectional fiber/epoxy resin composite material. Characterization testing is done with three different panel types of varying thickness to create specimens. The T800 composite has a tensile strength of 430 ksi (2950 MPa) (Toray Carbon Fibers America, Inc. n.d.)

2.1.2 Test Equipment

Data is collected using a MTS 810 Load Frame and Two Point Grey Grasshopper® 3 cameras. The details of the testing system are written in reference to testing at quasi-static and room temperature conditions.

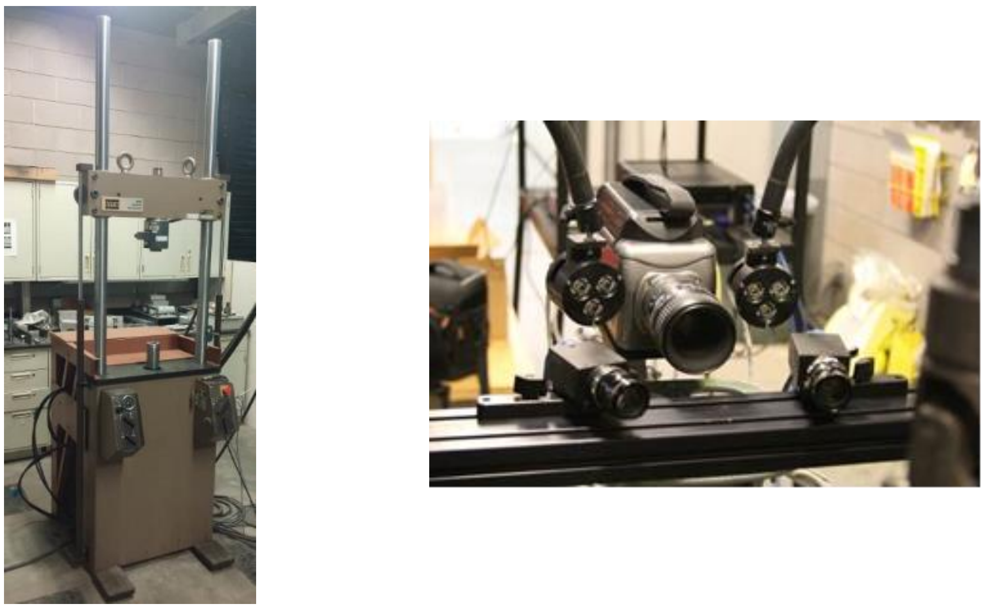


Figure 1. Photos of test equipment used in composites testing. MTS Load Frame (left) and Point Grey Grasshopper 3 camera set up (right)

Test Frame

The MTS 810 Load Frame has a 10 inch stroke and a 20 kip compression and tension capacity. The frame features interchangeable grips for testing different specimen types. The system is controlled by a MTS FlexTest® digital controller. The digital interface allows for load or stroke controlled load testing. Force and displacement data is collected every second.

Digital Image Correlation (DIC) Equipment

The Point Grey Grasshopper® 3 5.0 Megapixel cameras feature mono coloring, 15 max frames per second, and a digital interface. Cameras are calibrated using VIC3D 7© software (Correlated Solutions 2014). The resulting images from testing are then analyzed in VIC3D 7©.

2.1.3 Force Data Collection

Force data is collected from the MTS system once per second during testing and is stored in a .dat file. The test is actuator controlled, with the test speed defined as distance per minute. The test speed varies based on the test being done. Once collected, the force data is converted to stress by using the cross-sectional area of the testing region measured before testing. For each test, the thickness/width measurements are taken at 8-12 points in the area of interest.

2.1.4 Strain Data Collection

The DIC system relies on two cameras capturing images of the speckled specimen during quasi-static testing. DIC relies on smooth surfaces with few rapid changes in geometry. Speckling involves painting the entire surface of the testing region white, then applying a random pattern of black spots. While there are many methods to apply random speckling patterns, small specimens such as those used to characterize the T800S/3900-2B composite are painted using a fine mist of black paint.

Prior to each test, calibration must be done to ensure the VIC3D 7© software is able to accurately pick up on and quantify relative displacements. Pictures are taken over an interval varying from 3 to 10 second, depending on the quasi-static experiment being done. For example, a relatively quick test such as a 2-3 direction Iosipescu shear test takes one picture every 3 seconds; a longer test such as any off-axis compression test, which lasts approximately 15 minutes, involves one picture every 10 seconds.

The calibration and speckle images taken before and during testing are then analyzed using VIC3D 7© software. Within this program, the region of interest is selected for analysis. Once analyzed, various strain fields and displacements can be plotted over a 2-dimensional image of the specimen over time. Only index values and corresponding relevant strain values are necessary when exporting data in a Comma Separated Value (CSV) file. For example, in a 1-direction tension test, longitudinal (e_{yy}) strains are used. In a 2-3 shear test, shear (e_{xy}) strains are used. If the area of interest is large enough or stress concentrations are noticeable in certain regions, a new region may be defined to collect localized information that may be more indicative of the true behavior of the material.

2.1.5 Summary of Raw Data

Following testing, there are two files with unprocessed data that will be used to create a characteristic curve for a material in one test. The DAT file has information on the testing machine, basic testing information (including test name and length of time of testing), and the data itself: the raw time in seconds, the relative displacement of the loading head in inches, and the force value in pounds. The CSV file has just the raw data: the indices, the relevant strains, and any additional data sets selected for export.

Both raw data files may require modification in the following cases: if the test was stopped and resumed, the DAT file includes a new header at the stopping index; if either test has empty lines mixed with data for any reason, those lines must be deleted.

2.2 Experimental Procedures for Adhesives

During preliminary testing of the T800/F3900 unidirectional composites, it was found that parts of the test specimens in the grips were being crushed. Lowering the grip pressure did not help since then the specimens tended to slip. To mitigate crushing, fiberglass tabs are adhered to the specimens in the gripping regions. However, these adhesives can potentially fail in shear during testing. A test plan was created to evaluate candidate adhesives suitable for the composite tests described earlier. Specifically, double lap shear tests are performed based on standards and recommendations outlined in ASTM D3528-96 (ASTM, 2008).

2.2.1 Double Lap-Shear Test Plan

This subsection describes the test plan for the adhesive comparison study.

2.2.1.1 Adhesive Data

Three candidate adhesives were selected for evaluation based on prior experience with testing composites at Arizona State University and Ohio State University. The names and relevant data for the candidate adhesives are listed in Table 1.

Table 1. Adhesive Data

Adhesive Name	**Specimen Code**	**Mfr.**	**Type**	**Form**	**MFG Date**	**Code #**
3M DP460	3ME	3M	Epoxy	Paste	Resin: MAY-18-2015 Hardener: MAY-28-2015	7000000872
Loctite Super Glue	LSG	Henkel	Cyanoacrylate	Liquid	MAR-10-2015	43072
KwikWeld	JBQ	J-B Weld	Epoxy	Paste	APR-04-2015	8276F

Preparation instructions for each adhesive including substrate surface preparation procedure, mixing directions, application conditions, assembly conditions prior to pressure, curing conditions, and conditioning procedure are presented in Section 2.2.1.3.

2.2.1.2 Substrate Data

The substrate refers to the material on which the adhesive is applied. Most adhesives testing standards provide guidelines for using a metal substrate such as aluminum (ASTM 2010, ASTM 2008). For the tests described in this report, in order to best relate the results of these tests to application with unidirectional composites, fiberglass (Acculam® Epoxyglas™ G10-FR4 manufactured by Accurate Plastics Inc, NY) specimens are used. For these tests, the fiberglass is used as the only substrate whose tensile strength is 275.79 MPa (1-direction) and 241.32 MPa (2-direction) (Accurate Plastics, Inc. 2016). The tensile strength of this material exceeds that of the three adhesives in the test plan (see Table 2).

Table 2. Adhesive Shear Strength Summary

Adhesive	**Substrate**	**Maximum Shear Stress (MPa)**
3M DP460	Phenolic	9.653
	Aluminum	31.026
Loctite Super Glue	Not Provided (lower limit)	9.997
	Not Provided (upper limit)	19.995
JB Kwik Weld	Not Provided	7.171

2.2.1.3 Adhesive Preparation

The adhesive preparation information is available from the manufacturer's technical sheets (3M 2015), (J-B Weld It Australia 2012), (Henkel 2014). The details of the adhesive testing as performed are presented in Table 4 Table 5, and Table 6. All steps and recommendations were followed verbatim.

Materials Used and Terminology

This section details the tools and terminology referenced in the subsequent sections on adhesive-specific procedures. Preparation is done at room temperature for all adhesives. Each testing specimen is composed of several pieces adhered together. The pieces used in sample creation are outlined in Figure 2.

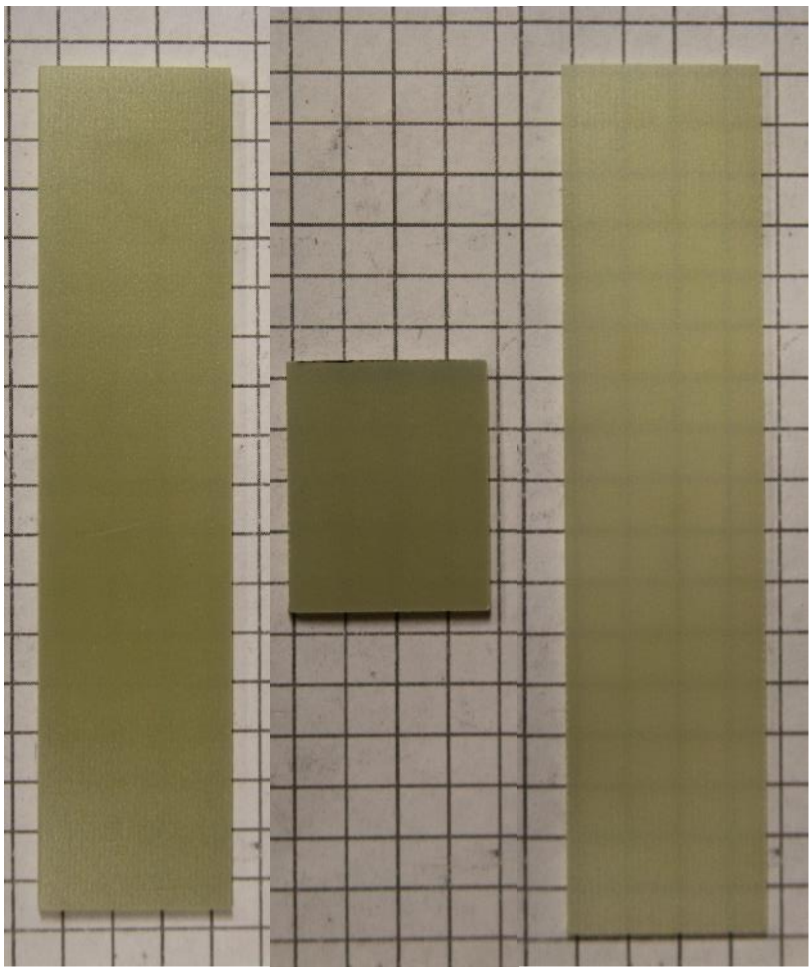


Figure 2. Photos of G10 Fiberglass specimen components. The top 2.44 mm piece (left). The bottom 2.44 mm spacer (center). One 1.52 mm piece for the bottom (right).

Step 1 is surface preparation. In all cases, this involves sanding the surface of the specimen with 1000-grit sandpaper and removing the dust with a water based cleanser (available from Micro Measurements, PA). The cleanser includes two parts: Conditioner

A, an acidic solution acting as both as etchant and cleanser, and Neutralizer 5A, an ammonia-based solution to neutralize the acidic conditioner. Following step 1, the surfaces of the bonding areas should appear as they do in Figure 3.

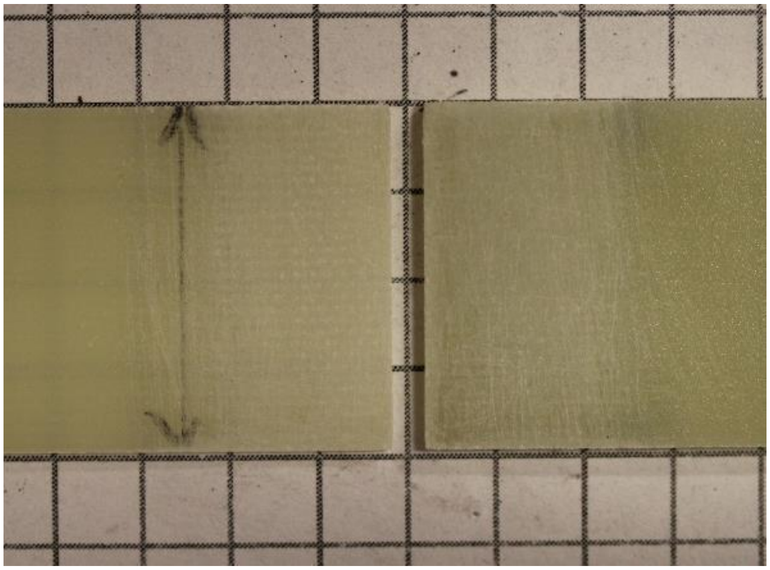


Figure 3. Complete substrate surface preparation example.

When applying the adhesive in step 3, special care should be taken to ensure the adhesive has spread evenly over the entire adhered surface. After placing the two surfaces together, the adhesive is visible beneath the fiberglass. As shown in Figure 4, the adhered gage region has a darker appearance where the adhesive has set. In addition, Figure 5 also shows the side view of an adhered specimen. While excess glue may be wiped away from the edges, a small amount of adhesive on the edge is acceptable.

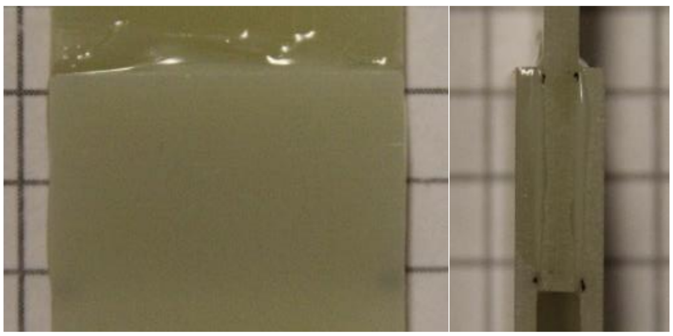


Figure 4. Adhered 3ME specimens with front view (left) and side view (right).

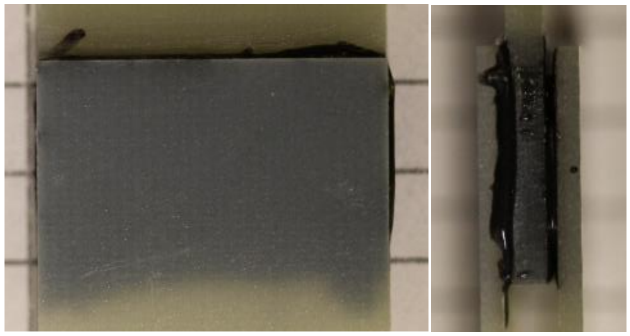


Figure 5. Adhered JBQ specimen with front view (left) and side view (right).

Step 4 involves assembly of the specimen. Photos of the assembly are shown in Figure 6.

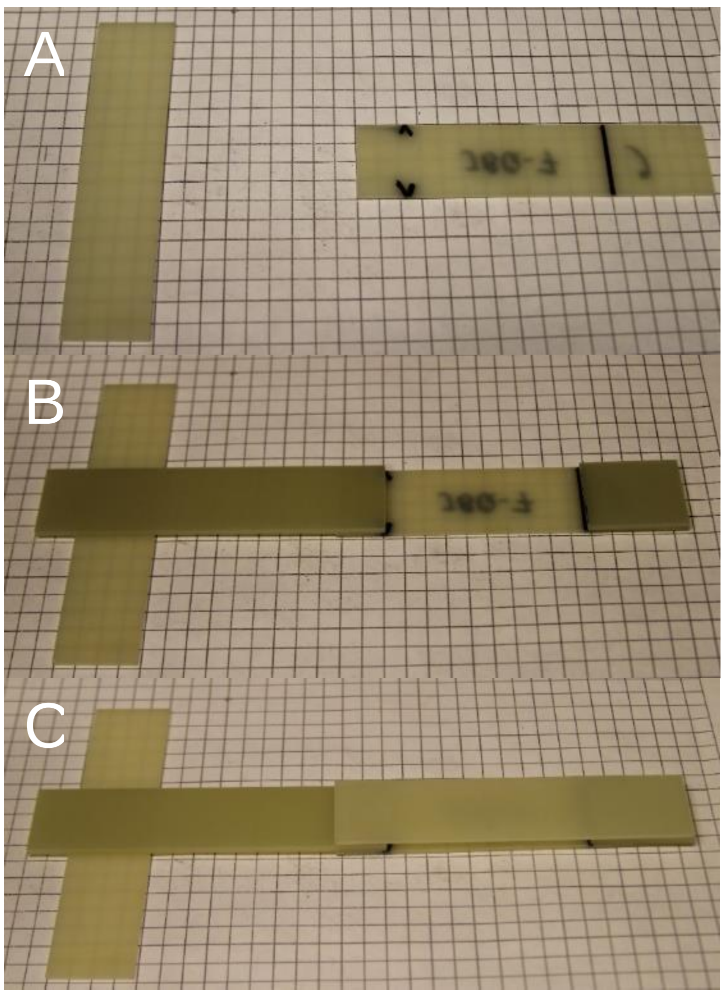


Figure 6. Assembly of adhesive test specimen. (A) Initial set up of first piece and 1.52 mm spacer. (B) Placing the 2.44 mm top piece and bottom spacer. (C) Placing the final 1.52 mm piece.

Complete bond information is summarized in Table 3.

Table 3. Bond Data

Adhesive	Application Method	Bonding Method	Thickness of Adhesive Layer (mm)	Length of Overlap (mm)	Conditioning Notes
3M DP460	Wooden Applicator	No-added pressure	~0.3	15	Store in dry place at room-temperature until testing
Loctite Super Glue	Long-neck Nozzle	Manual Pressure	~0.15		
JB Kwik Weld	Wooden Applicator	No-added pressure	~0.6		

Adhesive 1. 3M DP460

Table 4. Preparation instructions for adhesive 1: 3M DP460

Surface Preparation	Light sanding with 1000–grit sandpaper. All dust is then removed using a water based cleaning solution.
Mixing	DP460 is provided in a dual syringe plastic pack to be used with a manufacturer designed applicator system. Enough pressure is applied to the applicator on a surface designated for mixing to ensure even flow from the two packs. A small quantity may need to be discarded before the adhesive flows evenly. The two components are mixed by hand using a wooden applicator until adhesive is of consistent appearance.
Application Conditions	Adhesive is applied evenly in a thin layer over the bonding area using the wooden applicator. Excess adhesive is removed from edges using the applicator. Only one coat of adhesive is necessary.
Assembly Conditions	Immediately after the adhesive is applied, the second surface is placed on top with a 1.52 mm spacer on the opposing end to allow for even adhesion. The adhesive is then applied on the third and fourth surfaces with the second 1.52 mm piece being placed on top with the 2.44 mm spacer. Because adhesive does not cure immediately, the two sides are aligned as needed.
Curing Conditions	Once the surfaces are aligned, no external pressure is applied on the bond aside from the specimen's own weight. As the material sets, excess adhesive may be cleaned with tissue paper. The specimen is then left in a dry location at room temperature for 48 hours to allow for complete curing.
Conditioning Procedure	The specimens are stored at room temperature between bonding and testing. They are not to be exposed to heat or water.

Adhesive 2. Loctite Super Glue

Table 5. Preparation instructions for adhesive 2. Loctite Super Glue

Surface Preparation	Light sanding with 1000–grit sandpaper. All dust is then removed using a water based cleaning solution.
Mixing	This step is not needed for this adhesive.
Application Conditions	Application directions specify approximately one dot per square inch of surface to be bonded. To ensure a complete surface bond for tests, several drops of adhesive are placed in the center of the bonding area to allow for applied pressure to spread the adhesive to the edges. Only one coat is necessary.
Assembly Conditions	Immediately after the adhesive is applied, the second surface is placed on top.
Curing Conditions	Once the surfaces are aligned, they are held together by hand for 15-30 seconds at room temperature to allow the bond to set. As the material sets, excess adhesive may be cleaned with tissue paper. The procedure is repeated for the spacer and the second 1.52 mm piece.
Conditioning Procedure	The specimens are stored at room temperature between bonding and testing. They are not to be exposed to heat or water.

Adhesive 3. J-B KwikWeld

Table 6. Preparation instructions for adhesive 3. J-B Weld KwikWeld

Surface Preparation	Light sanding with 1000–grit sandpaper is performed first to clean bonding surfaces. Remove all dust with a water based cleaning solution.
Mixing	J-B Weld KwikWeld is provided two separate tubes: epoxy resin and epoxy hardener. Equal parts of both tubes are squeezed onto a disposable surface designated for mixing. The two components are mixed by hand using a wooden applicator until adhesive is consistent in appearance and color.
Application Conditions	Adhesive is applied evenly in a thin layer over the bonding area using the wooden applicator. Excess adhesive is removed from edges of the bonding area using applicator. Only one coat of adhesive is necessary.
Assembly Conditions	Immediately after the adhesive is applied, place the second surface on top with a 1.52 mm spacer on the opposing end to allow for even adhesion. The adhesive on the third and fourth surfaces are applied on the second 1.52 mm piece being placed on top with the 2.44 mm spacer. Because adhesive cures within 6 minutes, the two sides must be placed quickly with alignment occurring immediately.
Curing Conditions	Once the surfaces are aligned, no external pressure is applied on the bond aside from the specimen’s own weight. The specimen is left in a dry location at room temperature for 2 hours to allow for complete curing.
Conditioning Procedure	The specimens are stored at room temperature between bonding and testing. They are not to be exposed to heat or water.

2.2.1.4 Specimen Dimensions

ASTM D1002 suggests creating specimens five at a time from a single test panel from which individual 1 inch wide specimens are cut, though individually creating specimens are also acceptable (ASTM 2010). Specimen dimensions are shown in Figure 7. Non-metal substrates require joint overlaps to be chosen so that failure occurs in the joint. A bond width of 15 mm was used in the test to match the bond width used in 1-direction tension tests performed at Arizona State University for characterizing the T800S/3900-2B composite.

Shear characterization of adhesives is done using double-lap-shear tests. To maintain constant stress throughout the specimen while ensuring failure occurs in the bond, two different thicknesses of the substrate are used: 1.52 mm and 2.44 mm. The bottom of the specimen is made from two pieces bonded to the single top piece. The top piece is made from the single thicker 2.44 mm piece while the bottom is made from two 1.52 mm pieces, which together are closer to the thickness of the top at 3.04 mm, and an additional 2.44 mm thick spacer element adhered in the gripping region between the 1.52 mm pieces. Specimen dimensions are based on Type A specimens found in ASTM D 3528-96 (ASTM 2008).

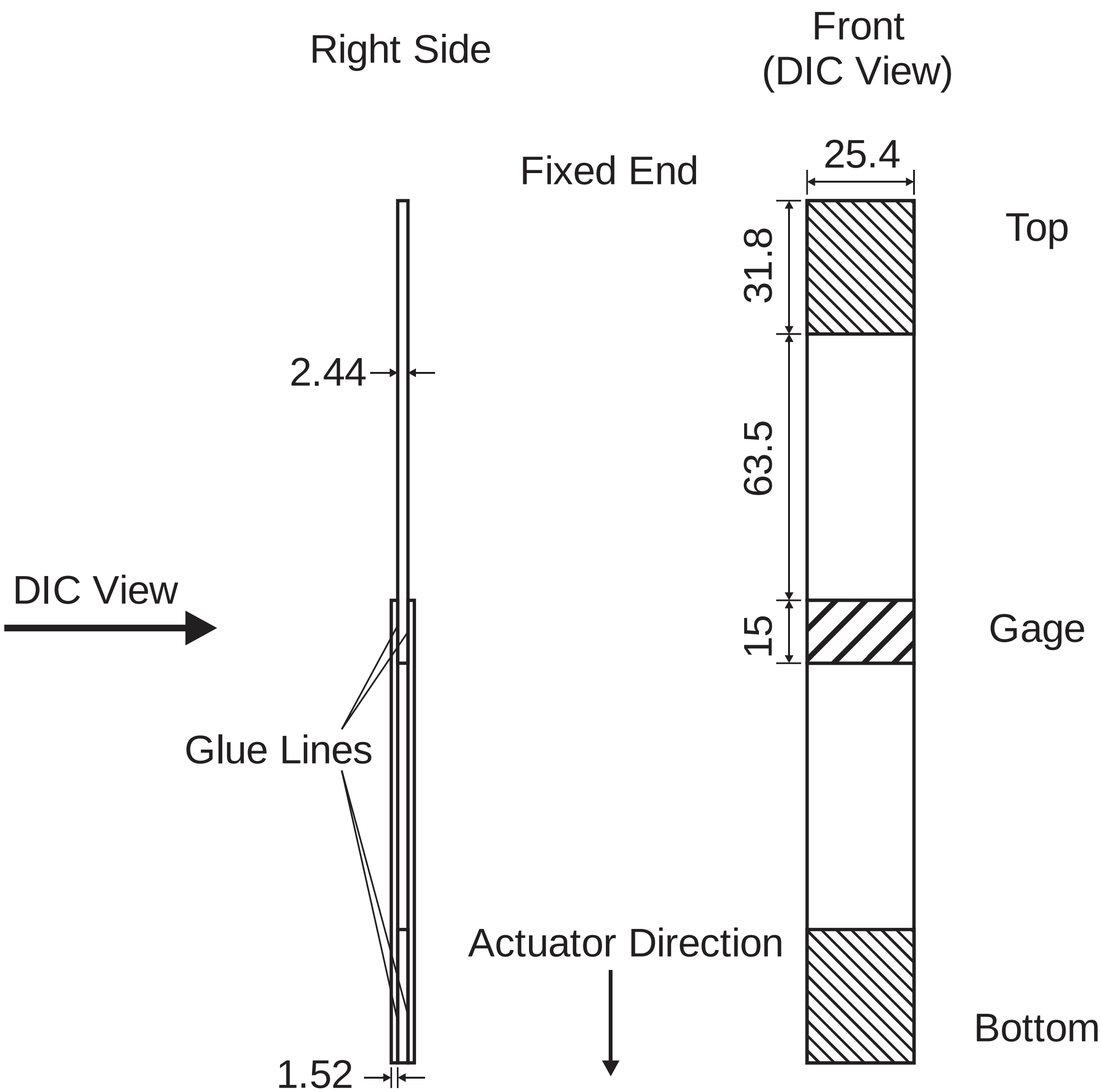


Figure 7. Form, dimension, and nomenclature of individual test specimen (all measurements in mm).

Each specimen was speckled for use with VIC3D 7© software for DIC analysis (Correlated Solutions 2014). A typical speckling pattern for a specimen is shown in Figure 8.

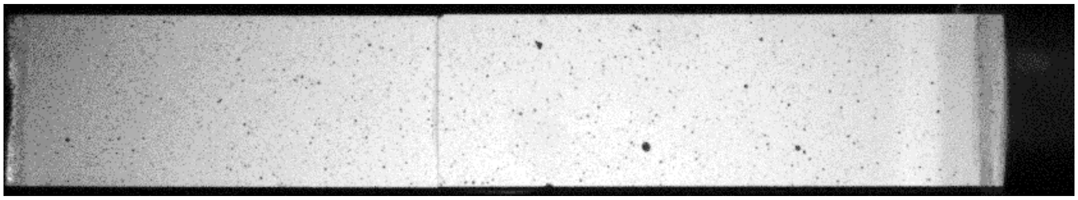


Figure 8. DIC image of the full 3ME-1 specimen during testing.

2.2.1.5 Test Procedure

The specimen is placed within the testing machine using vice grips according to the shaded region shown in Figure 7. The specimens are aligned vertically once the load is applied. A laser is used to verify that the specimens are perfectly aligned. The test is displacement controlled with the actuator moving at a constant rate of 0.001 in/s.

Digital image correlation (DIC) is run concurrently to verify no bending is taking place and to ensure strain fields are uniform. Strain is obtained from the adhesive length of the specimen (shaded region in Figure 9) plus 30 mm on either side.

Figure 9 shows the schematic diagram of the testing machine setup.

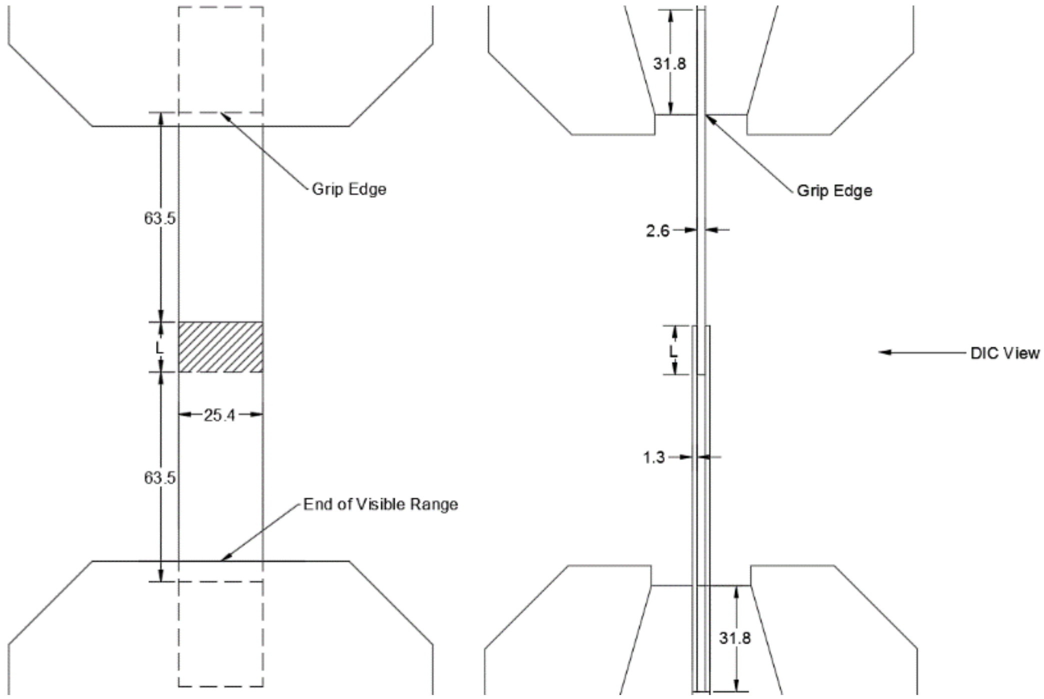


Figure 9. Front view (left) and side view (right) of shear testing setup.

2.2.2 Post-Processing

Five replicates for each adhesive were tested. During these tests, the maximum, minimum, and average failing force values were recorded as shown in Table 7. Additional notes concerning the nature of the failure were recorded following each test, e.g. brittle failures in the adhesive layer, failure patterns, etc. Images were also recorded before and after tests to compare specimen failure modes and patterns.

Strain data for the overlap area viewed from the front side of the specimen was analyzed to ensure there is no bending in the specimen. The DIC analysis provides 3 plots relevant to this analysis: normal strain, shear strain, and z (out-of-plane) displacement. Figure 10A shows a sample normal strain plot. The longitudinal strain field is uniform throughout the top, bottom, and gage sections and is directly proportional to the thickness of each section (the top section is 0.6 mm or 20% thinner than the bottom section, and the strain in the top section is also 20% less than that in the bottom section). Figure 10B shows a sample shear strain plot. The shear strains within the sample are close to zero indicating that the test conditions are correct to produce longitudinal tensile strain that dominate the test. Figure 10C shows the z-displacement plot along the specimen. The variation of the z-displacement along the specimen length is minimal indicating little or no bending in the specimen. Figure 11 shows another view of the z displacement plot. This side view of the specimen surface visually illustrates a small amount bending in the top section (small but acceptable). All tested specimens were checked for bending using these techniques.

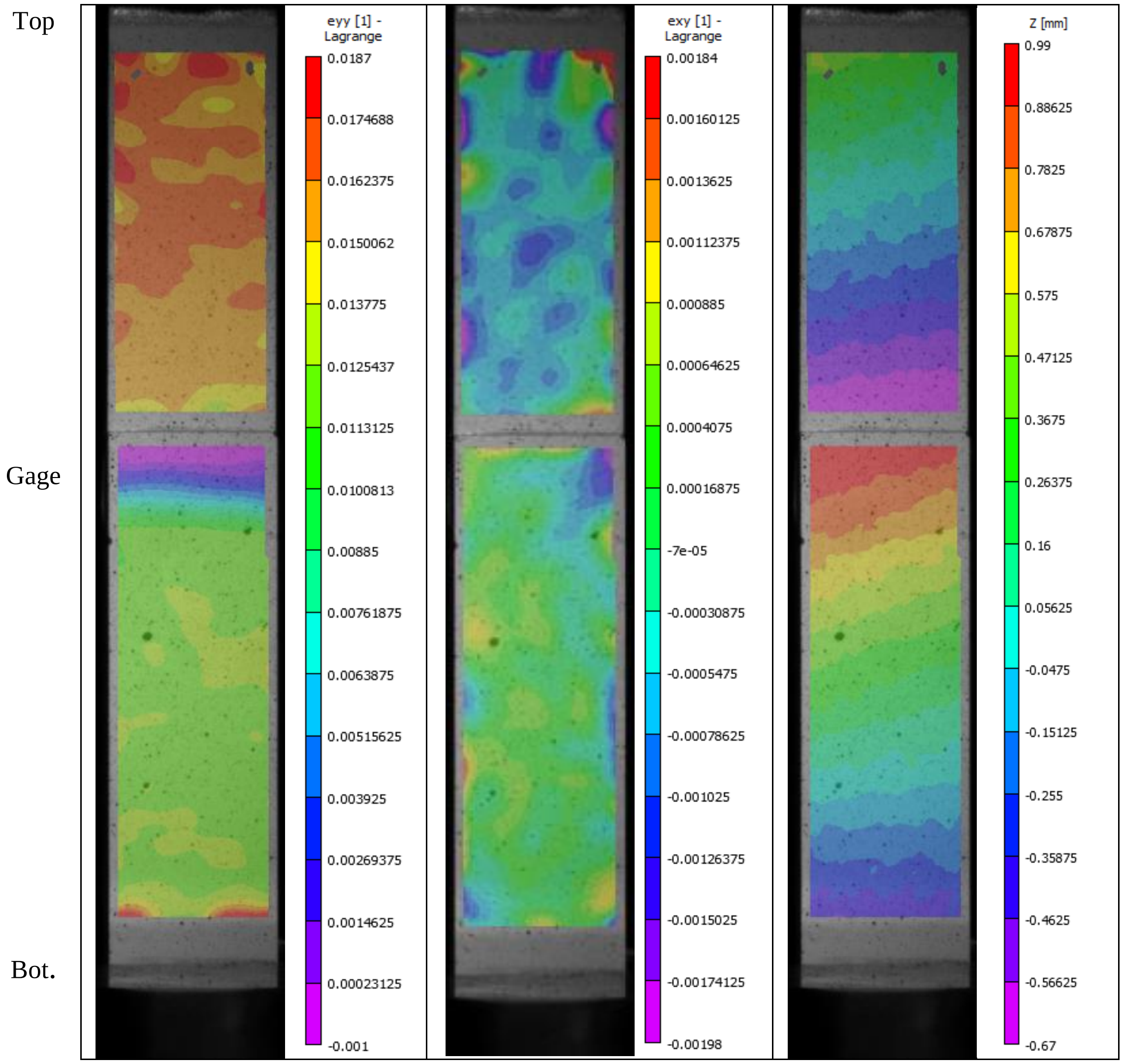


Figure 10. Normal (e_{yy}) strain field (left), shear (e_{xy}) strain field (center) and z position for specimen 3ME-1 just before failure.

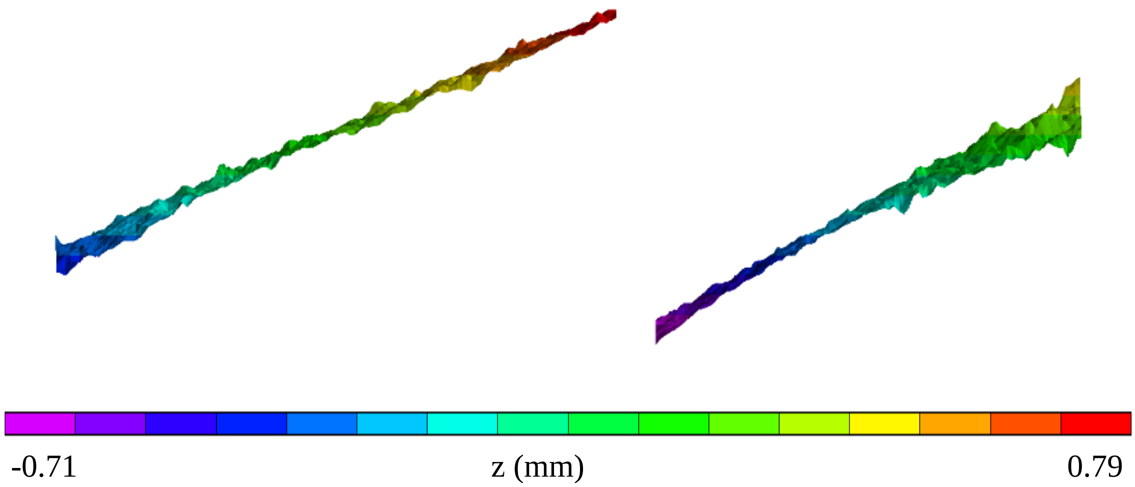


Figure 11. Displacement plot for specimen 3ME-1 just before failure.

In order to gage the quality of each experiment, shear stress were calculated from the applied force of the MTS using the following equation.

$$\tau = \frac{F}{2 \cdot b \cdot L} \tag{1}$$

where F is the applied tensile force, b is the width of the specimen (25.4 mm in this case), and L is the length of the adhered section (15 mm). The nominal measurements of *b* and *L* are obtained for each specimen after curing and prior to testing.

While stress-strain curves are not required for analyzing the results of the tests, they act as another way of gaging the quality of the tests. For example, when plotting stress-strain data for the 3ME tests, all had a similar response with close ultimate stress values, as shown in Figure 12.

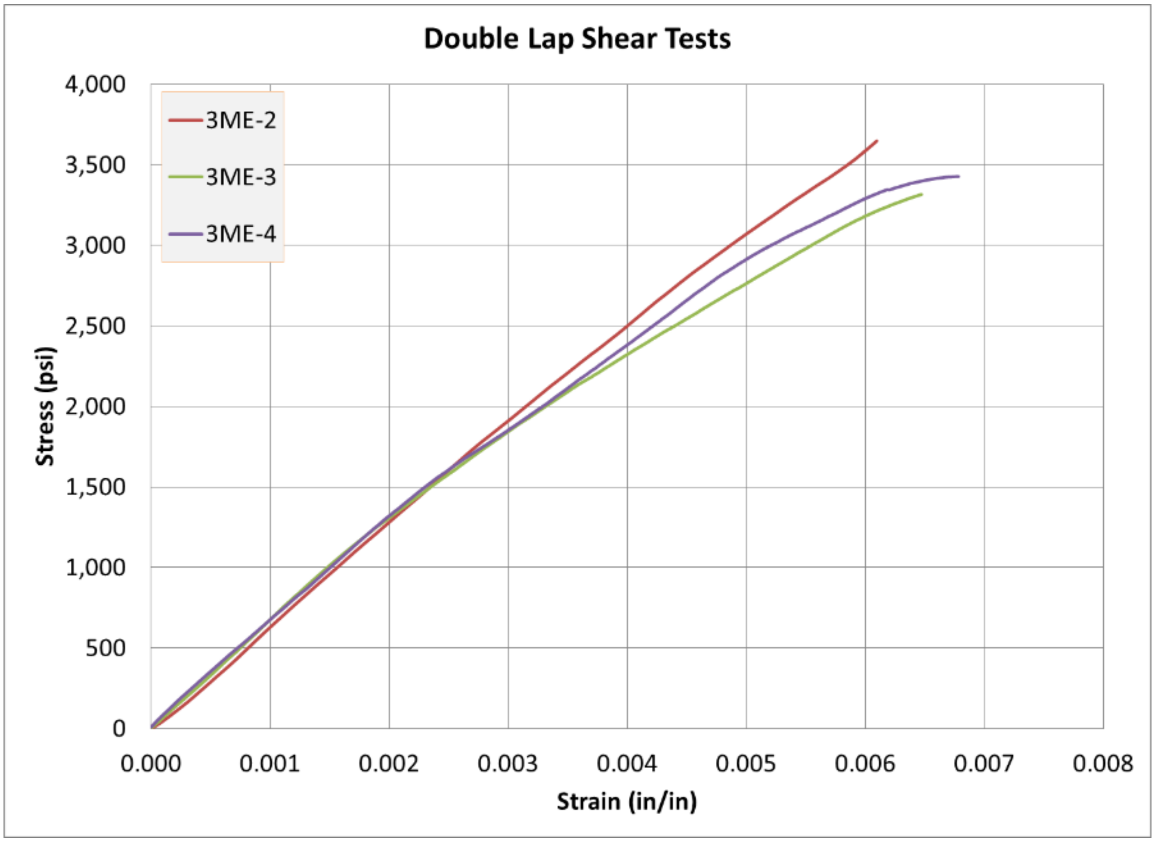


Figure 12. Stress-strain plot for 3M DP460 showing all 4 acceptable tests.

2.2.3 Results

Table 7 shows the numerical results of the fiberglass-on-fiberglass adhesive tests. Details of the results and analysis of each failure type for the three adhesives are detailed further in this section.

Table 7. Load Data summary table for Fiberglass-on-Fiberglass specimens

Adhesive	Number of Replicates	Maximum Failing Force (kN)	Minimum Failing Force (kN)	Average Failing Force (kN)	Coef. of Variation (%)
3M DP460	3	20.208	18.013	18.847	6.3
Loctite Super Glue	3	15.464	13.970	14.958	5.7
J-B Weld KwikWeld	2	8.350	5.785	7.068	26

3M DP460 Epoxy Results

The 3M DP460 specimens showed complete coverage of the adhered surface in the gage section. Figure 13A and B show the gage region of a 3ME specimen prior to testing. The consistent shading below the surface shows a complete bond. Figure 13C shows the gage region after testing. The fibers below the surface of the substrate have been exposed, indicating the top layer of the substrate failed at the same time as the adhesive, which failed along the center horizontal line of the gage region. The brittle failure also shows that the adhesive will not deform under load and thus is well suited for tabbing composite specimens.

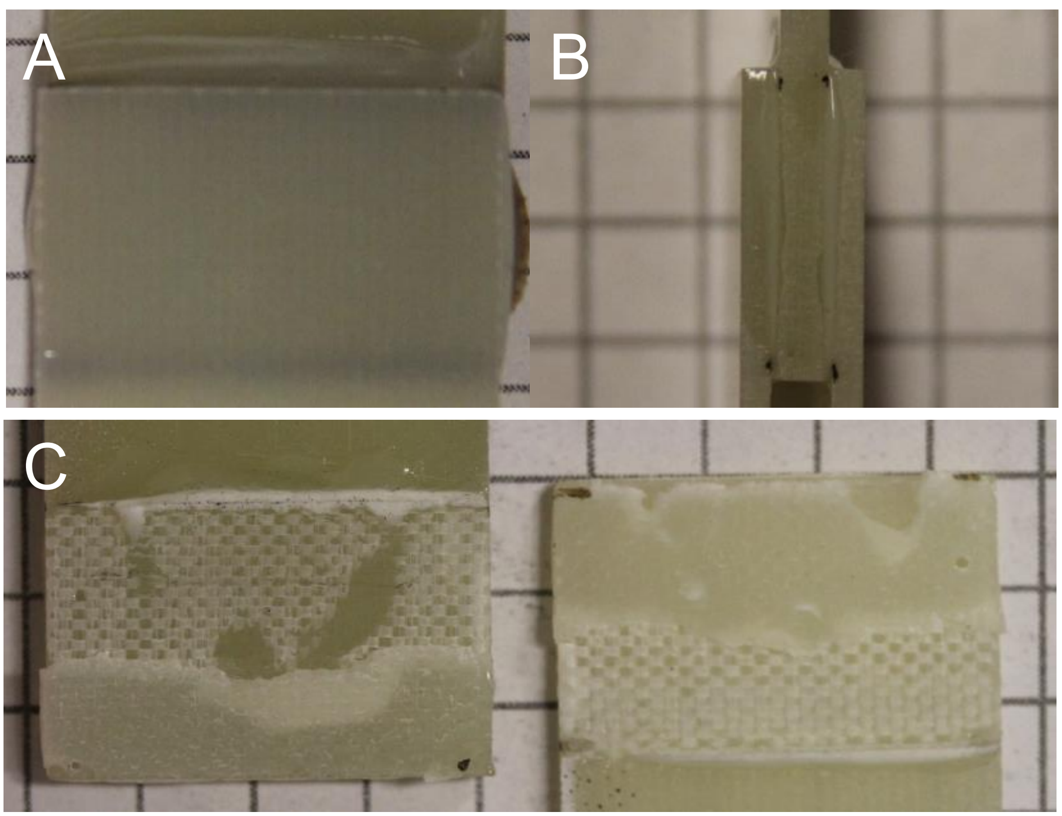


Figure 13. Specimen photos of gage region for 3M DP460 Epoxy (3ME). (A) Back side of 3ME-4 and (B) left side of 3ME-2 prior to testing. (C) Back side of 3ME-2 after testing.

The 3M DP460 epoxy had the highest failing force of the three tested adhesives. In addition, the 3M DP460 epoxy also was the most workable due to its long set time and relatively low viscosity after mixing. The only drawback is that the specimens require a 48 hour cure time.

Loctite Superglue Results

The Loctite Superglue features good workability allowing for complete coverage of the adhered surface as shown in Figure 14A where consistent shading is seen below the surface of the substrate. Figure 14B shows a consistent thickness along the edge of the specimen. Figure 14C shows the gage region after testing. The top layer of the substrate failed along with the adhesive during testing. The brittle failure shows that the adhesive will not deform under load and thus is well suited for tabbing composite specimens.

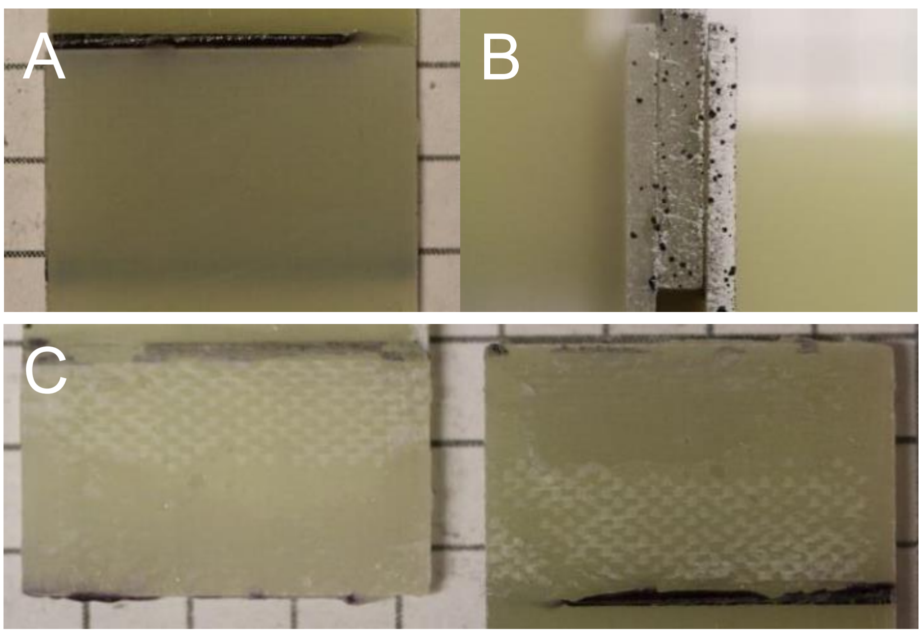


Figure 14. Specimen photos of gage region for Loctite Superglue (LSG). (A) Back side and (B) left side of LSG-4 before testing. (C) Back side surface of LSG-4 after testing.

The Loctite Superglue has 80% the ultimate force of the 3M DP460 epoxy. While the ultimate force is lower, the Loctite Superglue is an acceptable epoxy when test specimens have to be ready for testing within a few hours.

J-B Weld KwikWeld Adhesive Results

During testing, complications with consistent adhesive application with the J-B Weld adhesive led to having just 2 acceptable replicates. Figure 15A shows the back side of a specimen in which the adhesive is evenly applied. However, the entire region was not covered as seen in the gap in the top-right corner of the gage region. Figure 15B shows a sample with the adhesive applied with a consistent thickness. The results after testing show a variety of failure patterns in the adhesive. Figure 15C shows the adhesive flaking away from the substrate indicating the bond with the surface was incomplete. Figure 15D shows the adhesive experienced a brittle failure similar to those in the 3ME and LSG specimens. Figure 15E shows a ductile failure in which the adhesive did not completely fracture upon failure. The failure shown in Figure 15D is preferred, but the inconsistent performance even within the same specimen illustrate the difficulty of using the JB Kwik adhesive. The inconsistent failure patterns also correspond to the inconsistent results shown in Table 7.

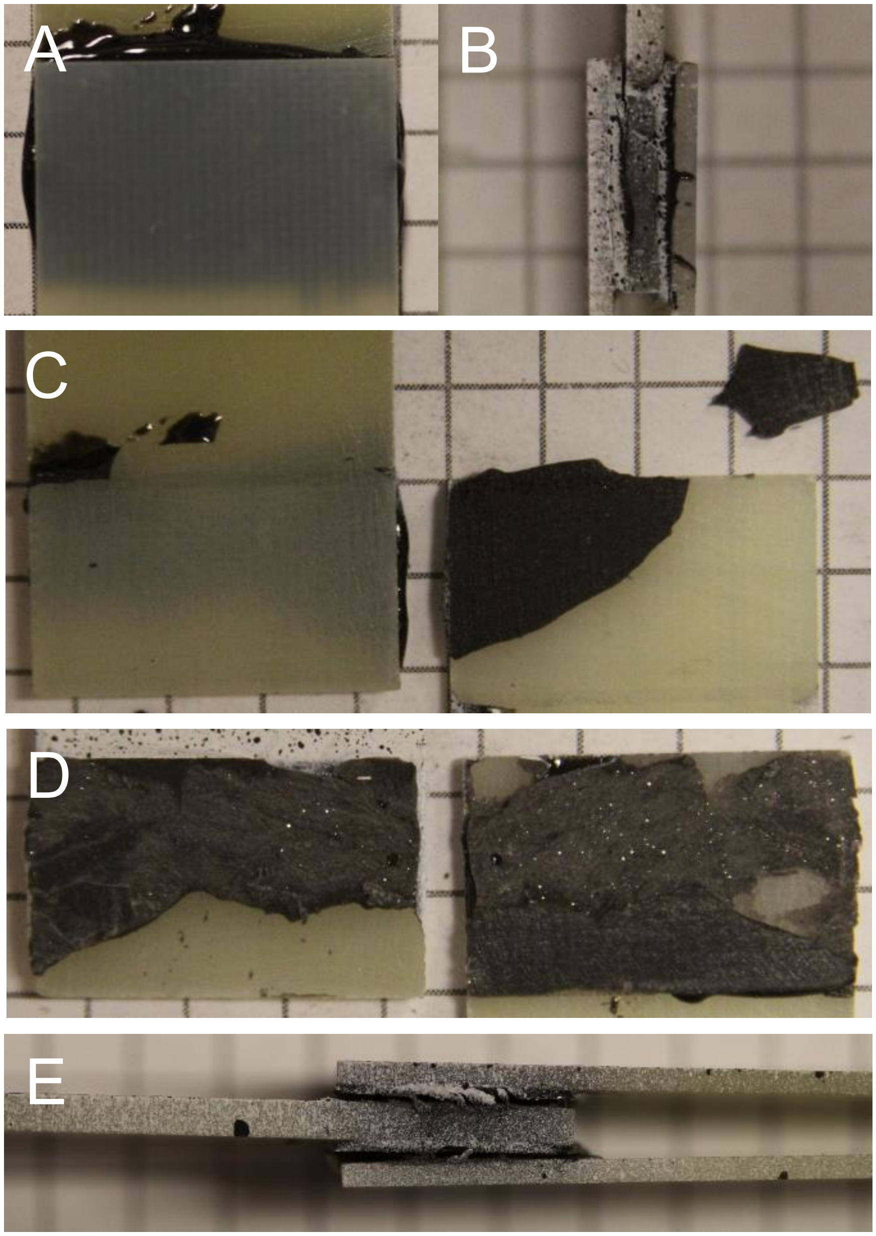


Figure 15. Specimen photos of gage region for JB Kwik KwikWeld (JBQ). (A) Back side of specimen JBQ-3 before testing. (B) Right side of specimen JBQ-2 before testing. (C) Back side of JBQ-3 after testing. (D) Front side of JBQ-3 after testing. (E) Left side of JBQ-2 after testing.

3 Experimental Data Processing Software Development

Determining the material properties from experimental data relies on processing. With the goal of improving the ease of use for data processing for unidirectional composites, the Experimental Data Processing (EDP) program is in ongoing development. EDP features functionality to allow for complete processing. Greater detail about this process is listed in 3.2. The general steps for post-processing are as follows.

Step 1. Read in the raw data. Raw data for composite testing is stored in two files with strain and force measurements taken over time.

Step 2. Process the raw data. Raw force data must be normalized by converting it to stresses. Raw data may also have inconsistent indexing, making unit conversions potentially important.

Step 3. Smooth the processed data. Raw data may include noise that is inherently a part of the testing process. Small irregularities during testing may also create outliers in the data. Smoothing removes these irregularities from the data.

Step 4. Plot the stress and strain data together.

Step 5. Generate the model curve from the stress-strain curves of each replicate.

3.1 Overview

EDP is an existing program developed at ASU since 2010. The program has evolved over time and is currently being developed for use with the MAT213 constitutive model. New features have concentrated on implementing the full data processing process. The most substantial improvement made to the program is the introduction of locally weighted least squares smoothing methods for both data smoothing and outlier removal. Smoothing functions already in place prior to improvement such as Moving Median and

Moving Average filters have also had bugs removed to improve accuracy and utility. New features such as data inversion and data set creation were added to increase efficiency. In addition to the processing improvements, an improved user interface has been implemented to help with viewing relevant information on the data being modified. The functions within EDP used in generating model curves from raw data are covered in greater detail in Appendix C.

3.2 Processing Steps

Processing data in EDP for use in MAT213 involves a simple series of steps. A full guide detailing all processing steps within EDP is found in chapter 4.

Prior to processing, the following information is required: raw DIC/MTS data, cross-sectional area, the test type, and the capture rate. As detailed in chapter 4, raw data for a single test is stored in two separate files: a CSV file with DIC data and a DAT file with force data. The test type is important to determine where to measure the cross-sectional area and the direction over which the strain is calculated. The capture rate for the DIC cameras is important to translate the raw strain indices to time.

3.2.1 Step 1. Read Raw Data

The raw data can be read without any prior modifications, though it is also important to note which columns store the relevant data (e.g. when reading raw DIC data, only the index and e_{xy} columns are necessary for a shear test even if e_{xx} and e_{yy} data are also exported. The user should be aware of other analysis regions in addition to the overall average if present). For a single test, multiple replicates are imported. To avoid confusion, all replicates should be named appropriately and have the correct delimiter selected. In total, six to ten files are read in depending on the number of replicates tested.

3.2.2 Step 2. Data Processing

Force and time data is used to determine the stress over time during the test. The stress in a specimen is considered the force applied over the cross section over which the load is applied. In a tension or compression specimen, the cross-section is in the direction of loading and is determined from the average of measurements taken in the region of failure prior to testing. Average stress in the specimen is calculated as follows.

$$\sigma = \frac{F}{A} \tag{2}$$

where F is the force applied at a single time point and A is the average cross-sectional area. In shear, the cross-section is parallel to the applied load. For Iosipescu tests, this area is between the notches. The average stress in the specimen is calculated as follows.

$$\tau = \frac{V}{A} \tag{3}$$

where V is the shear force applied at a single time point and A is the cross-sectional area. The strain data is calculated from the DIC images using Vic 3D 7® software. Depending on the test, different stain types are needed. In shear tests, Vic 3D 7® calculates engineering strain, requiring all strain values to be multiplied by two to convert to tensorial strain.

For both strain and force data, the time columns must start at zero and have consistent units. The strain indices must be converted to seconds based on the camera capture rate.

3.2.3 Step 3. Smoothing

In most cases, there is a level of noise within the raw data. To remove this, EDP has many different methods for smoothing data. The strengths and weaknesses for each method are outlined in section 3.3.6. For characterization tests at ASU, simple moving average (SMA) method is used.

3.2.4 Step 4. Plotting Stress-Strain Curves

After modifying the data, the stress and strain data must be reformatted so the time steps match. To do this, EDP has a reformat command that will rediscretize the data based on consistent time steps from a start time zero to a defined end time. Discretization is performed by interpolating between existing smoothed data points to generate new points. The end time is determined from matching time points in the force and strain data corresponding to the point of failure. Both strain and stress data must have the same start and end times and the same number of points.

Once all replicate data is reformatted over time, the stress data can now be plotted against the strain data for each replicate.

3.2.5 Step 5. Generating the Model Curve

The model curve is the final result that is used as input for MAT213 and is generated in four steps. First, the average ultimate strain is determined and is used as the end point of the model curve. Second, each replicate curve is either cut or extended to ensure all curves have the same end point determined from the average. If a replicate has a smaller ultimate strain than the average, the final point is extrapolated from the final points of the actual test. Similarly, if a replicate has a larger ultimate strain, the data is cut so it ends at the average. Third, each curve is rediscretized so all strain points match. From these points, the average stress is calculated and plotted to form the final model curve.

3.3 Implementation

The current version of the program has the following capabilities: editing raw data; Chauvenet's criterion for removing outliers; moving median, simple moving average (SMA), and polynomial moving average (PMA) filtering methods; curve reformatting; and curve averaging. These capabilities were enhanced with the addition of Lowess and Loess smoothing strategies and new implementation strategies for curve averaging techniques.

3.3.1 Curve Averaging

One new function creates the best fit of all curves currently being active (plotted) in the interface. The Generate Mean Curve function follows the post-processing steps detailed in section 3.2.

Step 1. Based on active flags for plotting, the function counts the number of active curves n and stores each corresponding replicate number j and data type (either raw, smoothed, or fitted). Knowing the usable curves, the program pulls the number of points P_j and the x and y column numbers for each active replicate.

Step 2. EDP next finds the average end strain based on the currently plotted data. The function allows for the largest, smallest, or average end strain value to be calculated and used.

Step 3. A new data set is created within the program that will contain the new x-column, the new average y-column, and the rediscretized y-columns for each input data set.

Step 4. Then, it discretizes the new x-axis (strain) array based on a starting point of zero and the new end strain. Each original data set is then cut or extended as needed based on the current end strain. To rediscretize the y values for each replicate, the function loops through each replicate then loops through each point within. When a rediscretized point x_k lies between two points in an original curve x_i and x_{i-1}, EDP interpolates between the original points to create the new stress value y_k.

$$m = \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \tag{4}$$

$$y_k = m(x_k - x_{i-1}) + y_{i-1} \tag{5}$$

Once a curve is reformatted, the program loops through each of the k points and begins to calculate the average as y_k/n.

3.3.2 Lowess/Loess Theory

Loess (Local regression) and Lowess (Locally weighted scatterplot smoothing) are similar methods for smoothing scattered data using a function based on weighted parameters to perform multiple local weighted regressions. Lowess uses a linear regression while Loess uses a higher degree polynomial for local fits (The MathWorks, Inc. 2016).

Lowess smoothing utilizes a function of the following form.

$$S(x_i) = \hat{w}_0 + \hat{w}_1 x_i \tag{6}$$

$\hat{w}_0$ and $\hat{w}_1$ are linear weight parameters generated within a local set of data defined by the user. Loess uses a similar form function for polynomial fitting.

The weight function has four properties.

1. $W(x) > 0$ for $|x| < 1$

2. $W(-x) = W(x)$ for $|x| < 1$
3. $W(x)$ is a nonincreasing function for $x \geq 0$
4. $W(x) = 0$ for $|x| \geq 1$

While any weight function with those four properties will work, EDP uses the normalized tri-cube weight function, which is of the following form.

$$w_i(x_i) = \left(1 - \left|\frac{x - x_i}{d(x)}\right|^3\right)^3 \tag{7}$$

where x is the focus point around which the local fitting function is formed, $d(x)$ is the distance between the focus point and the farthest point in the local array, and x_i is the i^{th} point in the local array. This function is utilized *i* times (the number of user defined points used to create the local fit) for all points *n*.

QR decomposition is used to calculate the least squares solution. Based on this solution, the slope and intercept of the local line are determined and used to calculate a new y-value based on each x-value. This process may be repeated for several iterations.

Note that when using the Loess algorithm, outliers have a more extreme effect on the surrounding smoothed data in certain cases; Loess is only recommended for sets of data with a large number of points and a high number of local points. Lowess is satisfactory in all other cases.

3.3.3 Lowess Algorithm

Input: Set of data (x,y), number of points to be used for local regression (*N*), number of repetitions (*R*), number of points within complete data set, *P*.

Output: Smoothed data

Step 1. Get the size of the data (*s*)

Step 2. Set $u=(N+1)/2$

Step 3. Loop through each point of focus i = 1 to i = P for R iterations

Step 3a. Establish local arrays of x and y data and determine the distance values, d, based on location within the global array.

- If $i<u$ (focus is on left)

$$d=x(N)-x(i) \tag{8}$$

- If $i>s-u$ (focus is on right)

$$d=x(i)-x(P-N+1) \tag{9}$$

- Else (focus is in the center range of data)

$$d=\max\left(x(i)-x(i-u+1),x(i+u-1)-x(i)\right) \tag{10}$$

Step 3b. Calculate the weight of each point in the local array j based on the focus point i.

- $$z_j=\frac{|x(j)-x(i)|}{d} \tag{11}$$
- $$w_j=\left(1-z_j^{\ 3}\right)^3 \tag{12}$$

Step 3c. Begin QR decomposition. Set up the following matrices.

$$\underline{x}=\begin{bmatrix}1 & x_j\\ 1 & x_{j+1}\\ \vdots & \vdots\\ 1 & x_N\end{bmatrix} \quad (13) \qquad \underset{\sim}{y}=\begin{bmatrix}y_j\\ y_{j+1}\\ \vdots\\ y_N\end{bmatrix} \quad (14) \qquad \underline{w}_{half}=\begin{bmatrix}w_j & 0 & \cdots & 0\\ 0 & w_{j+1} & \cdots & 0\\ \vdots & \vdots & \ddots & 0\\ 0 & 0 & 0 & w_N\end{bmatrix}^{0.5} \quad (15)$$

- $$\underline{x}_{half}=\underline{w}_{half}\,\underline{x} \tag{16}$$
- Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$
- $$\underline{y}_{half}=Q^{Á}\,\underline{w}_{half}\,\underline{y} \tag{17}$$

Step 3d. Calculate local slope and intercept from the following values of the $\underline{y}_{half}$ and R matrices.

- $$m=\frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} \tag{18}$$
- $$b=\frac{\left(\underline{y}_{half}\right)_{1,1}-R_{1,2}m}{R_{1,1}} \tag{19}$$

Step 3e. Calculate the new smoothed point based on equations (18) and (19).

- $$y_{i\ new}=mx_i+b \tag{20}$$

Step 3f. Calculate the residuals (necessary for robust Lowess, see section 3.3.5).

For an example showing how to implement Lowess for a sample set of data, see Appendix A.

3.3.4 Robust Local Regression Theory

Loess and Lowess methods for smoothing are both vulnerable to outliers. By implementing a robust procedure, the Loess/Lowess methods can become resistant to a small number of outliers. After following the Loess/Lowess procedure described in the previous section for one iteration, the residual for each point can be calculated.

$$r_i = y_i - \hat{y}_i \tag{21}$$

Where r_i is the residual, $\hat{y}_i$ is the smoothed value, and y_i is the recorded value.

From these residuals, a robust weight value can be calculated for each i^th point, which is based on a bi-square function with the following constraints.

$$w_{ri}(r_i) = \begin{cases} \left(1 - \dfrac{r_i}{6MAD}^2\right)^2 & |r_i| < 6MAD \\ 0 & |r_i| \geq 6MAD \end{cases} \tag{22}$$

Where MAD is the median absolute deviation of the residuals.

$$MAD = \text{median}(|r|) \tag{23}$$

The data is then smoothed again using the same tri-cube weights as before now multiplied by the robust weights. The residual values are also used to determine if a value should be used in the Loess/Lowess procedure. If the robust weight of a point is 0, then that point is not considered in determining the smoothed position of any point, including itself; instead, the closest neighboring values with non-zero robust weights are used.

The robust version of the process may be repeated for several iterations.

Note that when using the Robust Loess algorithm, outliers have a more extreme effect on the surrounding smoothed data in certain cases. As with Loess, Robust Loess is only recommended for sets of data with a large number of points and a high number of local points. Robust Lowess is satisfactory in all cases.

3.3.5 Robust Lowess Algorithm

This algorithm is continued from step 3 (performed for one iteration) of section 3.3.3.

Step 4. Calculate the residual weight based of each point *i*.

$$w_{ri}(r_i) = \begin{cases} \left(1 - \frac{r_i}{6MAD}^2\right)^2 & |r_i| < 6MAD \\ 0 & |r_i| \geq 6MAD \end{cases} \quad (22)$$

Step 5. Loop through each point of focus $i = 1$ to $i = P$ for R iterations

Step 5a. Establish local arrays of x and y data and determine the distance values, d, based on location within the global array. If a point has a residual weight of zero, do not include it in the local arrays for smoothing. The local coordinate vector only points with a residual weight. The x array no longer includes those points for calculating *d* and corresponding weights, but are still used as focus points.

- If $i < u$ (focus is on left)

$$d = x(N) - x(i) \quad (8)$$

- If $i > s - u$ (focus is on right)

$$d = x(i) - x(P - N + 1) \quad (9)$$

- Else (focus is in the center range of data)

$$d = \max\left(x(i) - x(i - u + 1), x(i + u - 1) - x(i)\right) \quad (10)$$

Step 5b. Calculate the weight of each point in the local array, *j*, based on the focus point, *i*. Note again that x values belonging to points with zero residual weight are excluded from this step and the next closest neighbor is used instead.

$$w_j = \left(1 - \frac{|x(j) - x(i)|^3}{d}\right)^3 w_{ri} \quad (22)$$

Step 5c. Begin QR decomposition for least squares. Set up the matrices in equations

$$\underline{x} = \begin{bmatrix} 1 & x_j \\ 1 & x_{j+1} \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix} \quad (13) \qquad \underset{\sim}{y} = \begin{bmatrix} y_j \\ y_{j+1} \\ \vdots \\ y_N \end{bmatrix} \quad (14) \qquad \underline{w}_{half} = \begin{bmatrix} w_j & 0 & \cdots & 0 \\ 0 & w_{j+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & w_N \end{bmatrix}^{0.5} \quad (15),$$

and (16). Perform QR decomposition on $\underline{x}_{half}$, calculate $Q^{Á}$, and calculate $\underline{y}_{half}$ (see equation (17)).

Step 5d. Calculate local slope and intercept from the following values of the $\underline{y}_{half}$ and R matrices using equations (18) and (19).

Step 5e. Calculate the new smoothed point using equation (20).

For an example showing how to implement Robust Lowess for a sample set of data, see Appendix B.

3.3.6 Comparison of Smoothing Strategies

EDP features many smoothing methods. moving median, simple moving average (SMA), polynomial moving average (PMA), Lowess, Loess, and the robust forms of Lowess and Loess. Each smoothing method has benefits and drawbacks for different data types. This section details scenarios for each smoothing type using actual data from quasi-static load tests on unidirectional composite specimens.

In general, increasing the number of points or the number of iterations will improve results. These variables should be selected based on the quality of the raw data. Changing the variables multiple times may be required to achieve desired results.

Four sample data sets are discussed for each method. Case 1 involves longitudinal strain data gathered from a 3-direction compression specimen using DIC shown in Figure 16. This data set features several minor plateaus that are common in raw compression test data. In addition, the data is constantly increasing, meaning there are no outliers. The original raw data has been modified to convert indices to units of time and to ensure the data starts at zero. With only 54 points, processing this data shows the effects of each method on a relatively small data set.

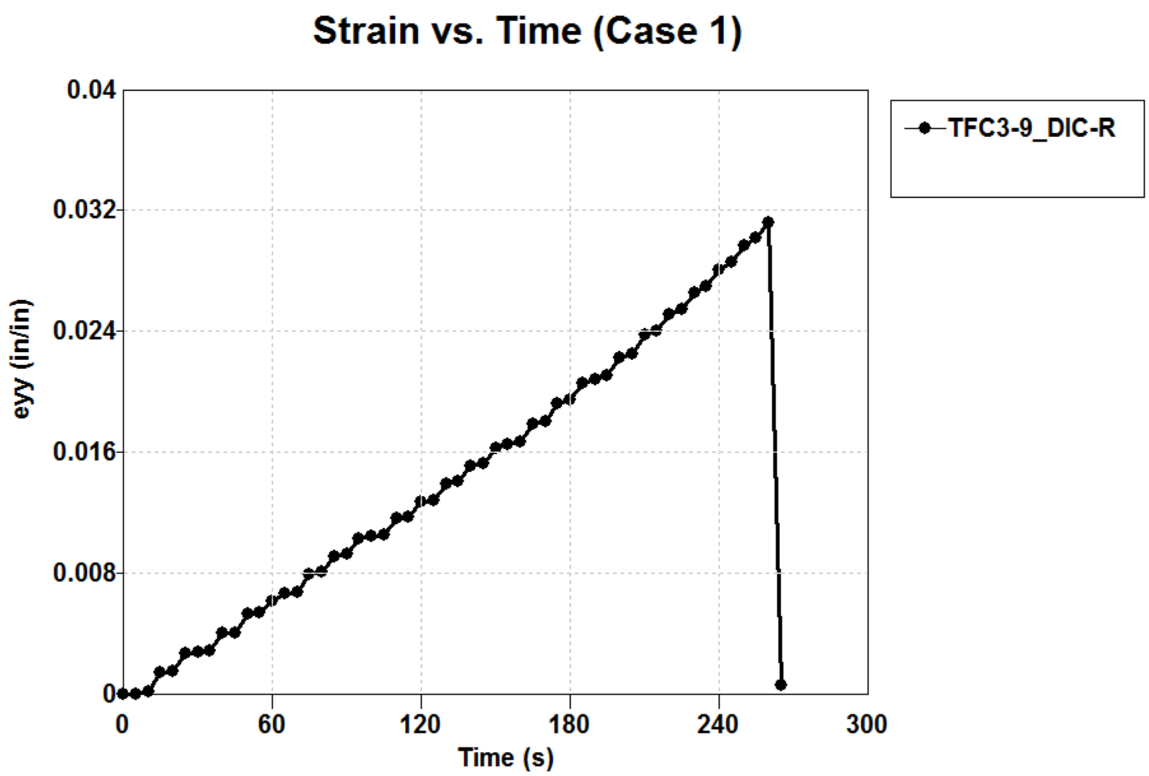


Figure 16. Raw strain data from 3-direction compression test (case 1).

Case 2 covers the stress data from the same 3-direction compression test, shown in Figure 17. Like case 1, this case also features plateaus common with compression tests with constantly increasing values. Unlike case 1, there are 266 points in this data set, opening up new possibilities in processing options.

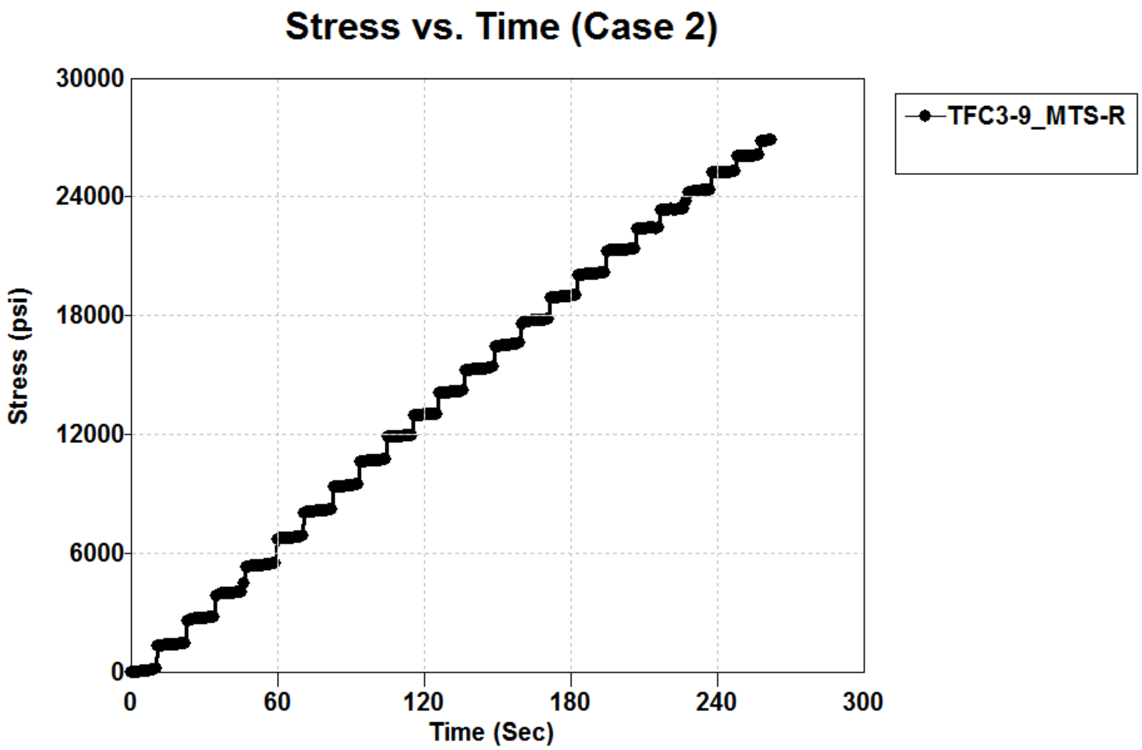


Figure 17. Raw stress data from 3-direction compression test (case 2).

Case 3 covers strain data from a 2-3 plane shear test featuring an anomalous reading in the center of the data, shown in Figure 18. The data was identified as anomalous based on a comparison with the raw force data for the same test, which did not include a corresponding jump. While there is no real outlier in this data set, certain smoothing options are preferred in this case. In processing, this data set will be cut at the point of failure at around 1050 seconds.

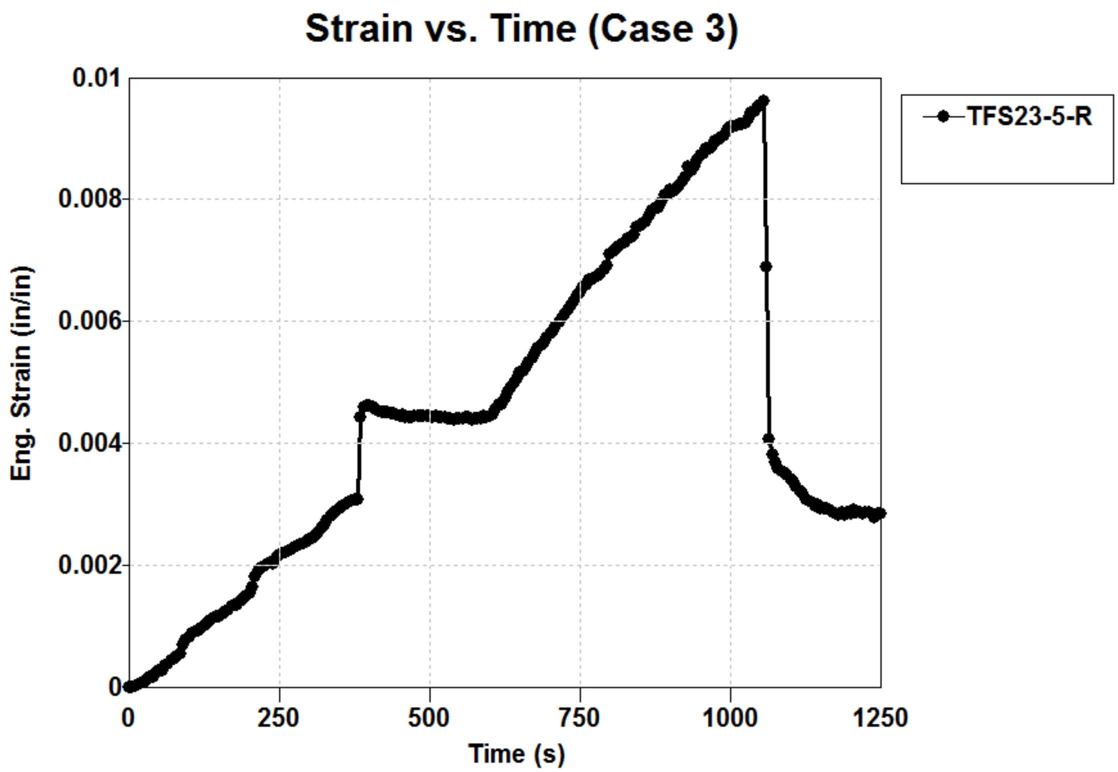


Figure 18. Raw engineering strain data from 2-3 plane shear test (case 3).

The final case shows 1-3 plane shear test data that includes several plateaus and possible outliers (see Figure 19). In processing, this data set will be cut at the point of failure at around 130 seconds.

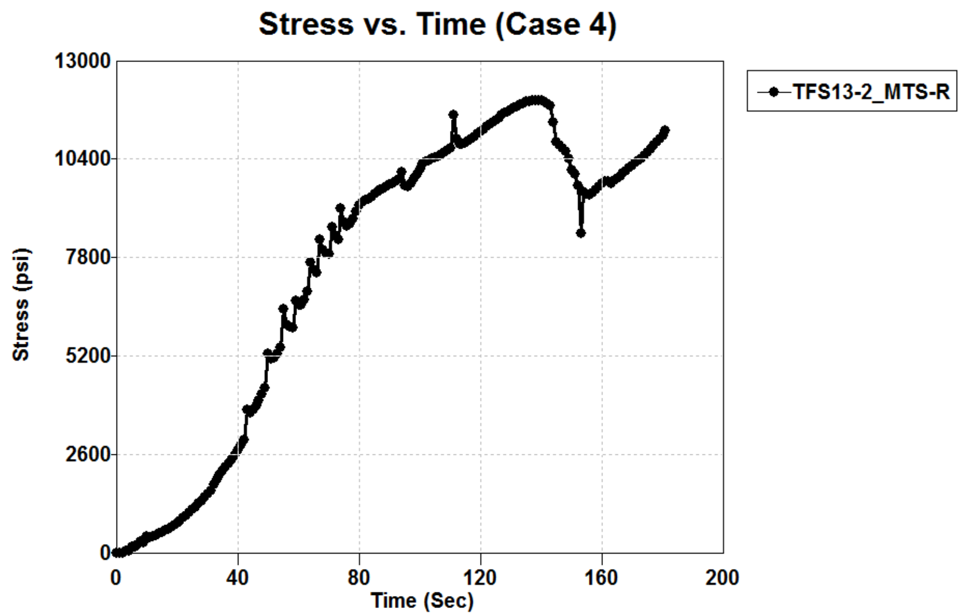


Figure 19. Raw stress data from 1-3 plane shear test (case 4).

3.3.6.1 Moving Median Filter

The Moving Median Filter is a simple filter that replaces each point with the median value of the local points (Vokshi 2011). Mathematically, this is expressed as.

$$m_i^{[N]} = med(y_{i-u},\ldots,y_i,\ldots,y_{i+u}) \tag{24}$$

where i is the current point ranging from 1 to P, P is the number of points within the data set, $u = \frac{N-1}{2}$, N is the local span size or number of points within the local range that the filter processes for each point, and $m_i^{[N]}$ is the new value of y_i. This filter can only process points from $i=u+1$ to $i=P-u$. The points at each end are unchanged.

Having relatively few points that are monotonically increasing will limit the effectiveness of the median filter. In case 1, using the median filter results in no change in the data (shown on the left in Figure 20) because any point value will always be in the center of the local region. Similarly, Median will have no effect on the constantly increasing data in case 2 (shown on the right in Figure 20).

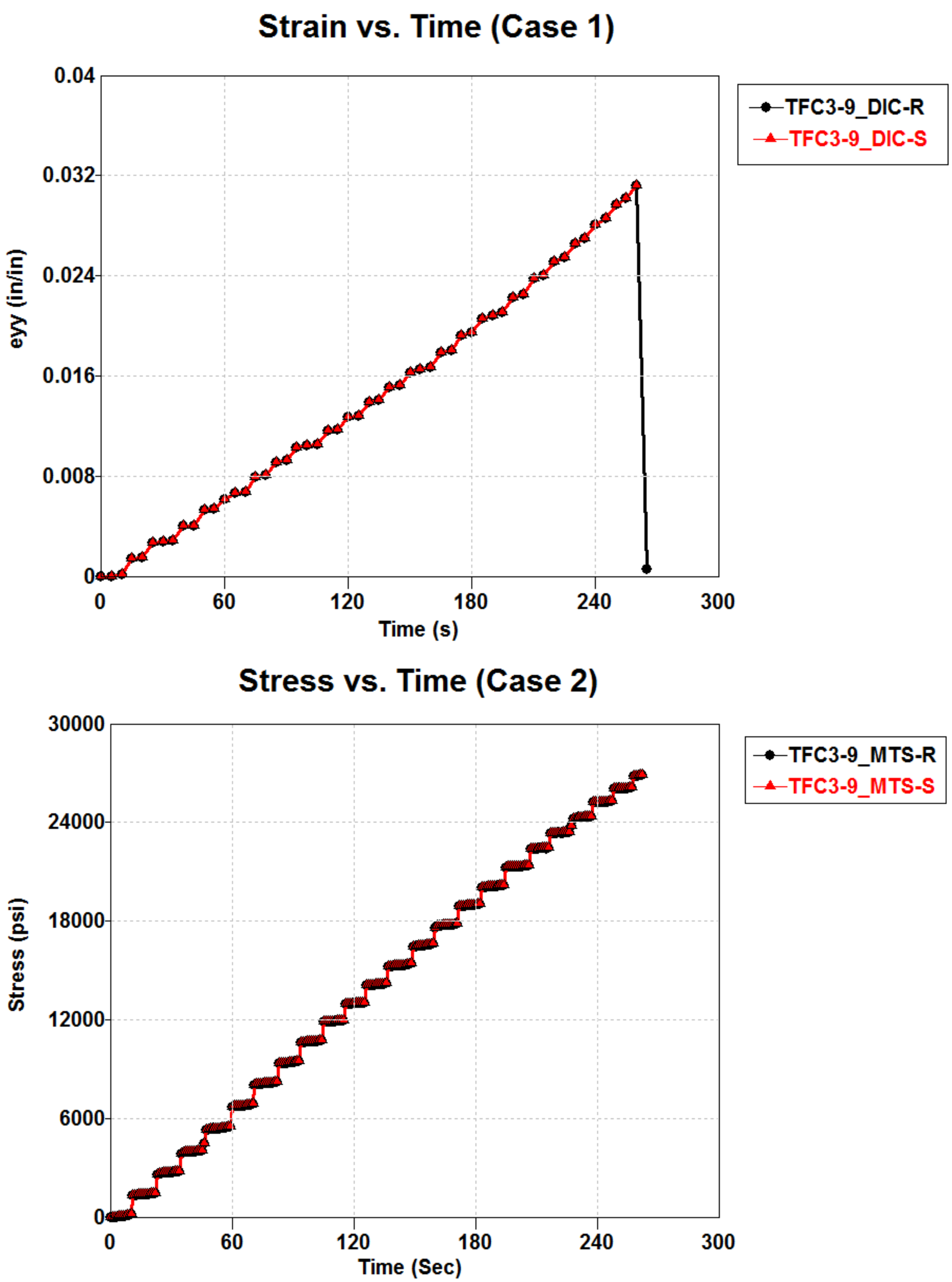


Figure 20. Moving Median Filter. Case 1 (top) and Case 2 (bottom)

In case 3, the anomaly in the data results in a single plateau. Similar to Cases 1 and 2, Median filter has a limited effect on the data, as shown on the left in Figure 21. The noise in this case, however, is somewhat mitigated by using the median filter, as shown in the zoomed part of the plot on the bottom in Figure 21.

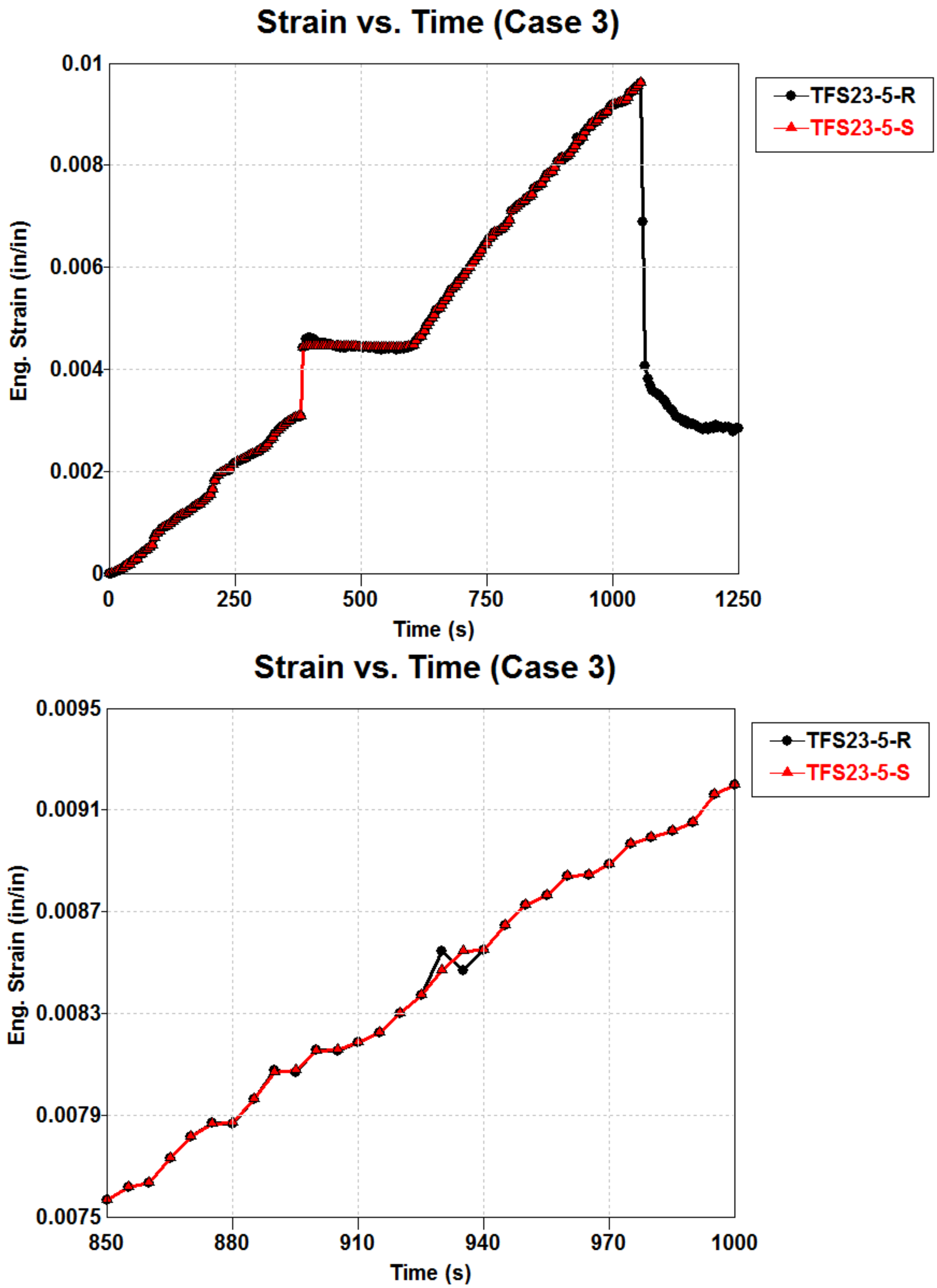
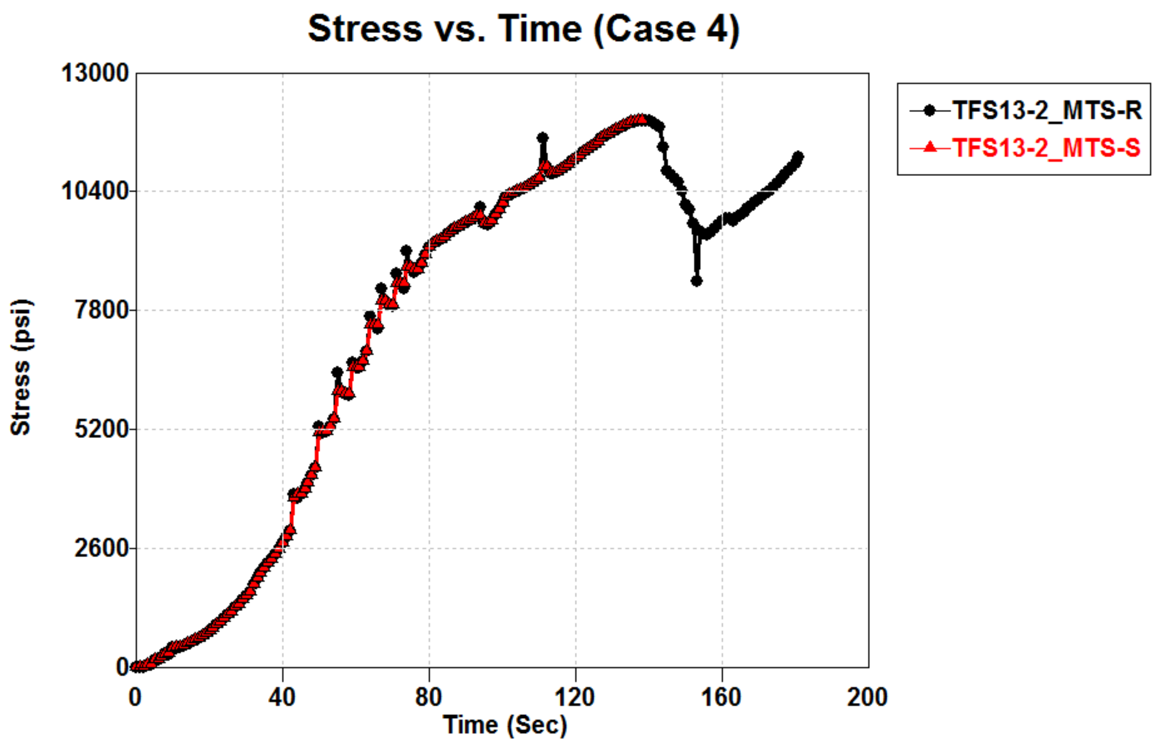


Figure 21. Case 3 smoothened with Moving Median Filter. Full plot (top) and zoomed plot (bottom).

Case 4, with several outliers, allows for a more effective use of the Median filter. When using more points, the median filter smooths outliers better than when using fewer points. Figure 22 shows how the number of points effects smoothing. In this case, using a wider range helps smooth regions with outliers. As with the previous cases, large jumps without outliers are not smoothened by this method.



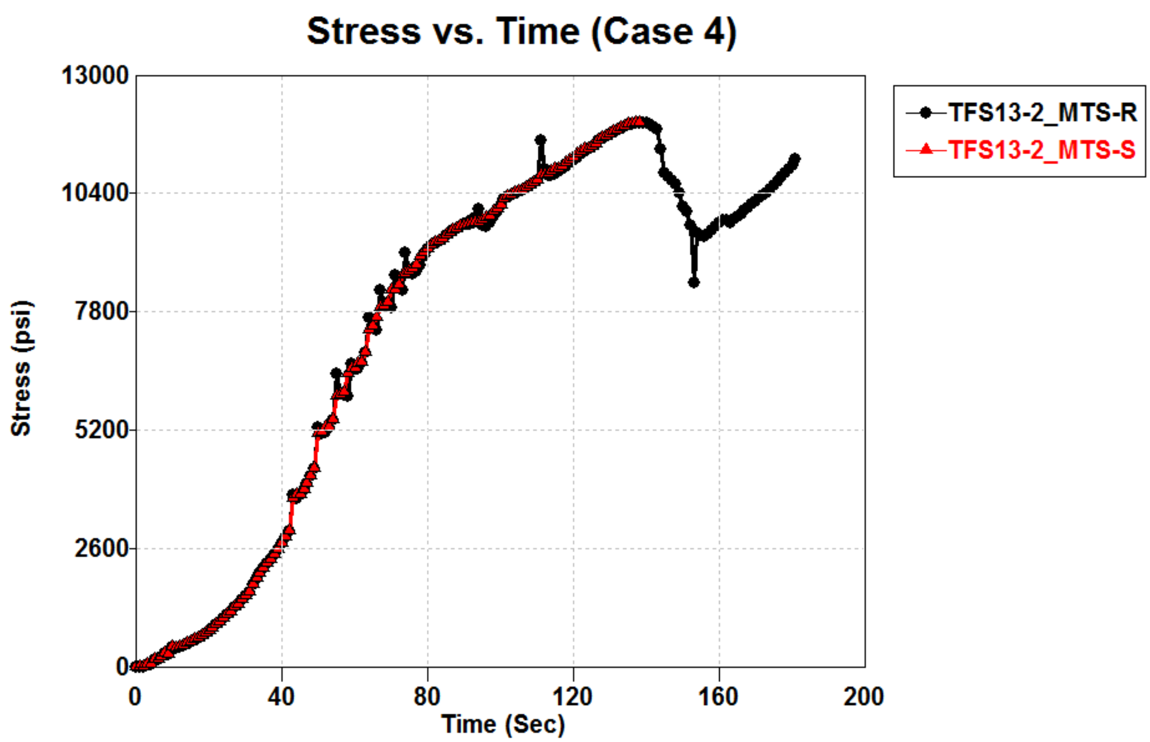


Figure 22. Case 4 smoothened with Moving Median Filter using 3 points (top) and 25 points (bottom).

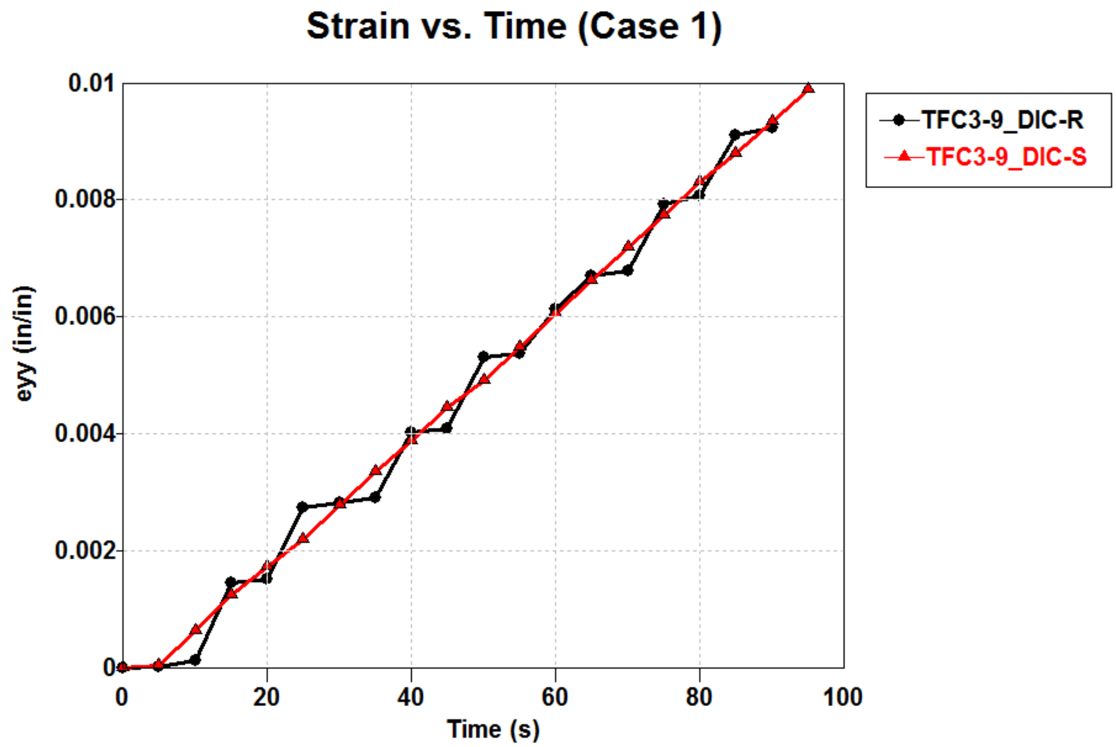
3.3.6.2 Moving Average Filters

The two types of moving average filters are Simple Moving Average (SMA) and Polynomial Moving Average (PMA). These are simple filters that replace each point with the non-weighted average value of the local points (Vokshi 2011). Mathematically, this is expressed as

$$y_i^{[N]} = \frac{\sum_{j=-u}^{u} y_j}{N} \tag{25}$$

where i is the current point ranging from 1 to P, P is the number of points within the data set, $u = \frac{N-1}{2}$, N is the local span size or number of points within the local range that the filter processes for each point, j is the local index for the running average, and $y_i^{[N]}$ is the new value of y_i. This filter can process all points except the start and end point for SMA and the first and final 2 points for PMA.

In case 1, with few points and no outliers, SMA and PMA are very effective at smoothing these plateaus. In this case, using 9 points necessitates only 1 iteration, though using 3 points requires 3 iterations to generate a comparable smoothed curve. Figure 23 shows the results of using SMA and PMA (third degree) both with 9 points and 1 iteration with similar results. Also note the ends of the smoothened data when using PMA remain unchanged, limiting the effectiveness of PMA on data sets with few points.



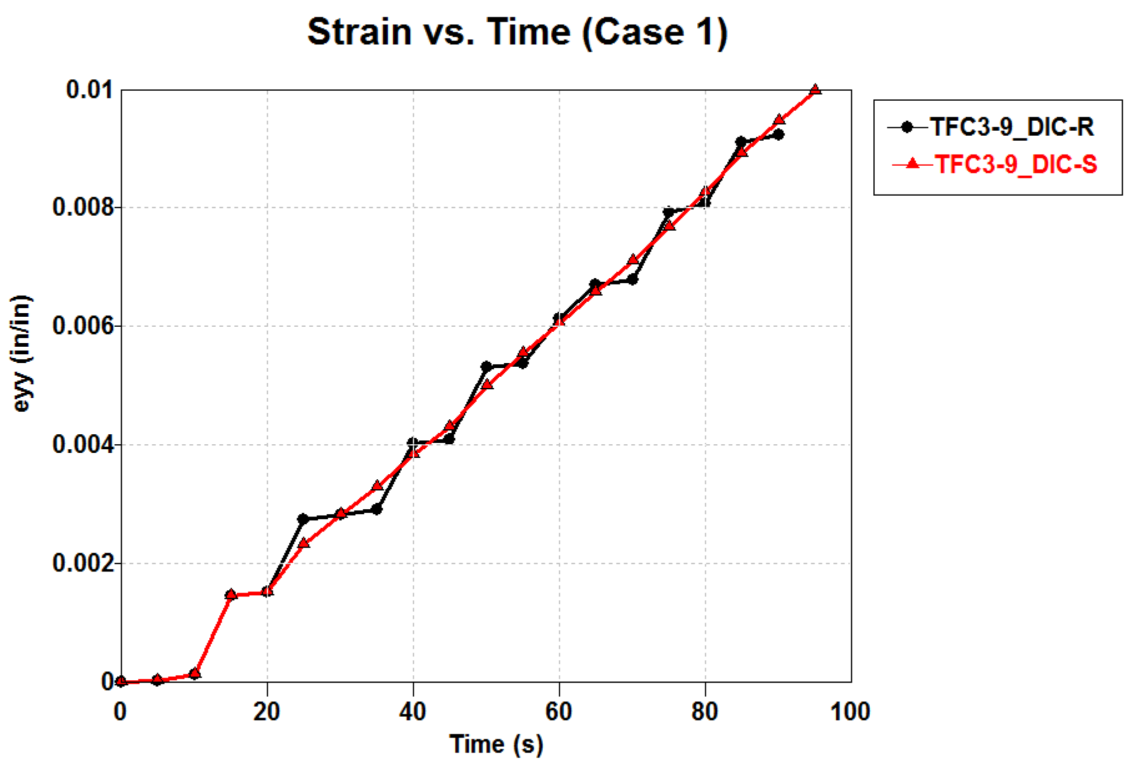
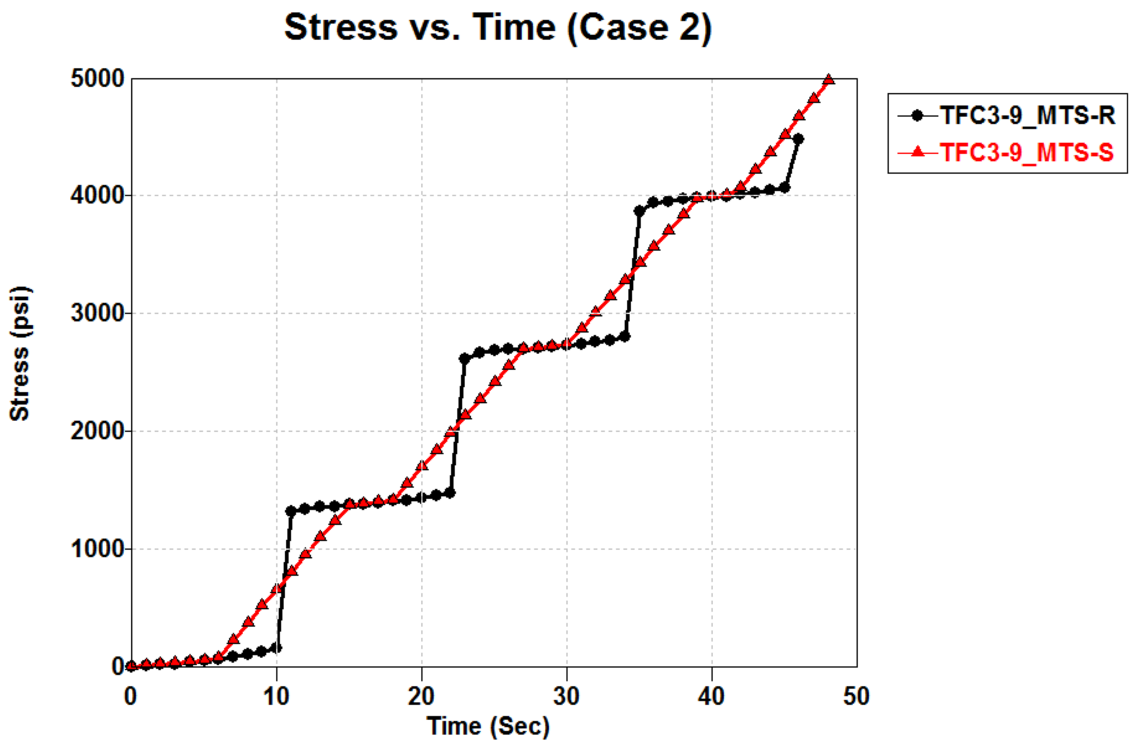


Figure 23. Case 1 with SMA (top) and PMA (bottom) zoomed to first 100 seconds.

With more points and large plateaus as in case 2, more local points are needed for SMA and PMA to achieve adequate smoothing. In this case, 13 is the best number of points needed for 1 iteration. Using fewer points with more iterations will be inadequate in this case while using more will introduce new issues with overcorrecting. This case also illustrates how PMA is susceptible to local patterns. With these plateaus, the curved smoothed with PMA will closely match the original curve. More points and iterations are needed to adequately smooth the data with PMA, though PMA’s issues with the ends of the data set are apparent. Figure 24 shows the results of using SMA and PMA both with 9 points and 1 iteration. Note the local patterns adhered to by the PMA filter.



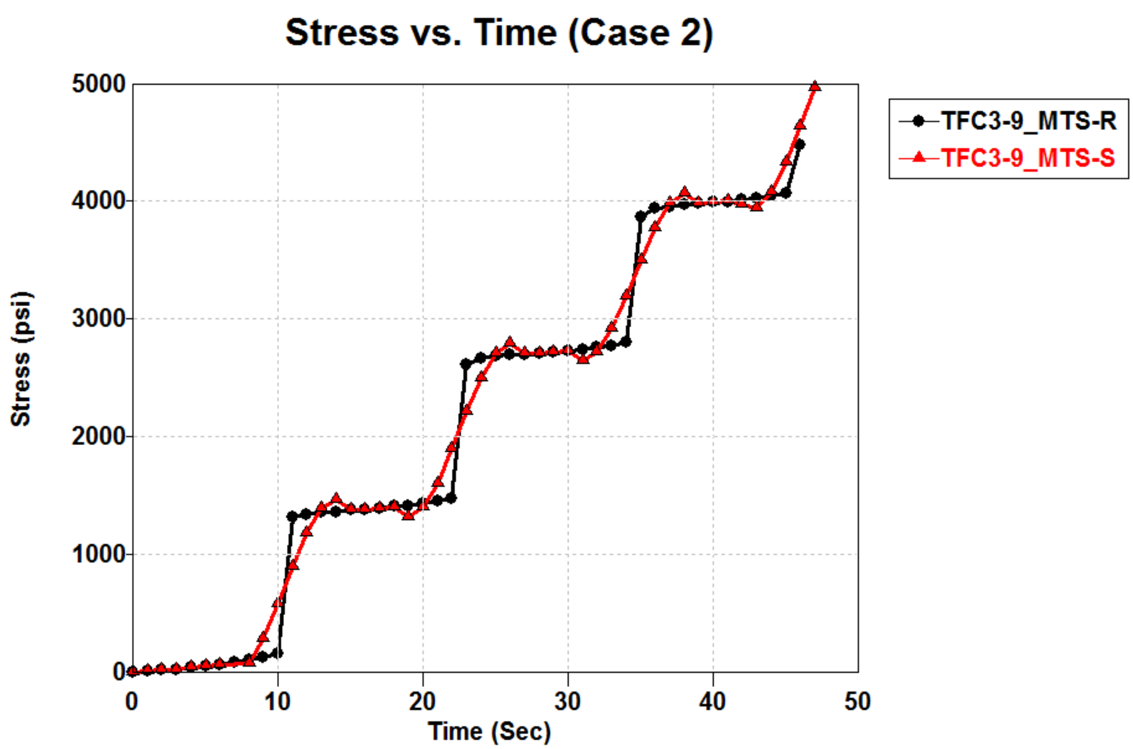


Figure 24. Case 2 smoothened with SMA (Top) and PMA (Bottom) zoomed to first 50 seconds.

Case 3 illustrates a case in which both a large number of iterations are necessary and PMA is not adequate. Using SMA with 25 points and 15 iterations is necessary to overcome the anomaly. Using PMA with an increasing number of iterations does not approach a smooth line. Figure 25 compares the SMA results with PMA results using 25 points and 30 iterations.

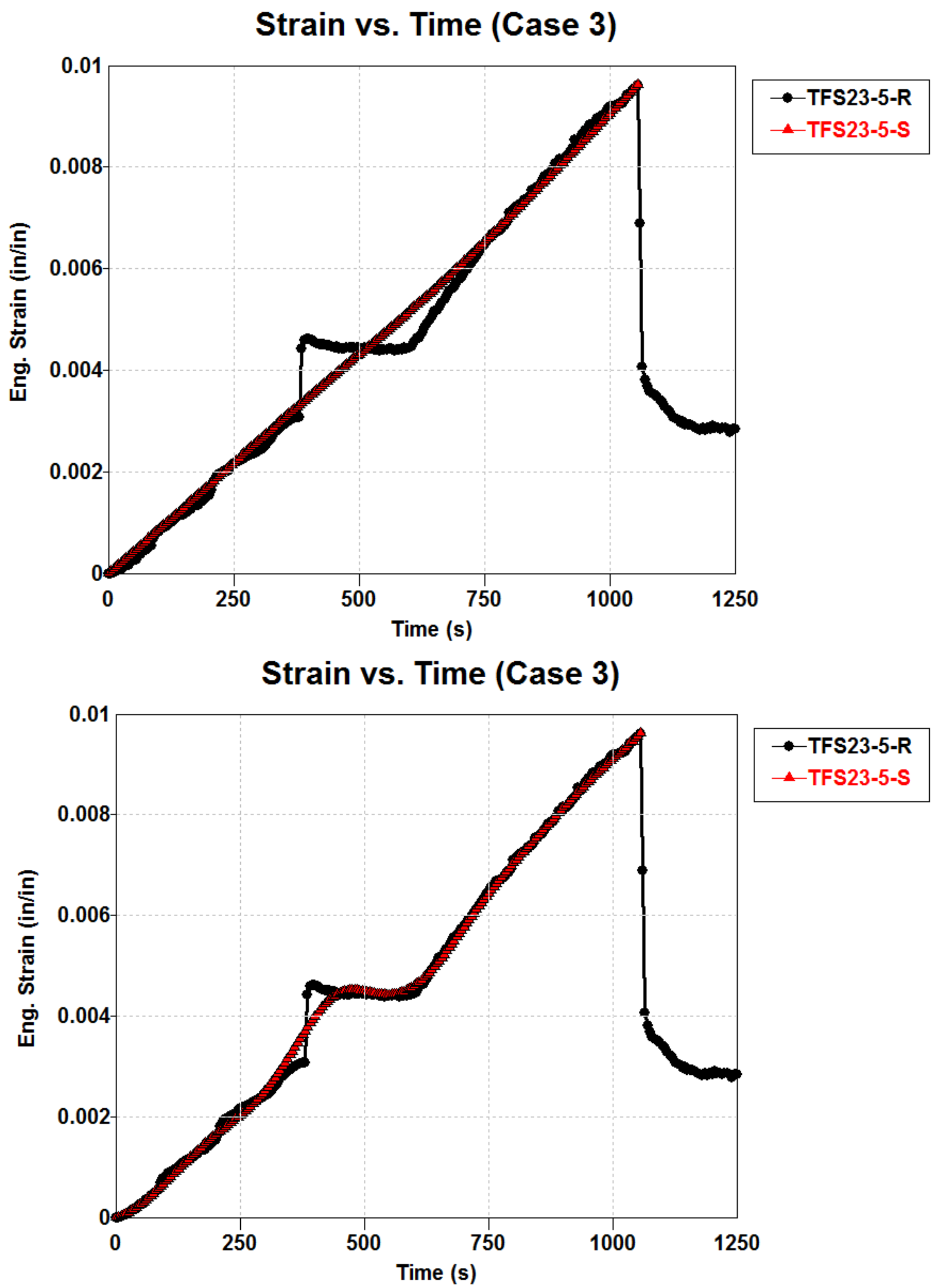
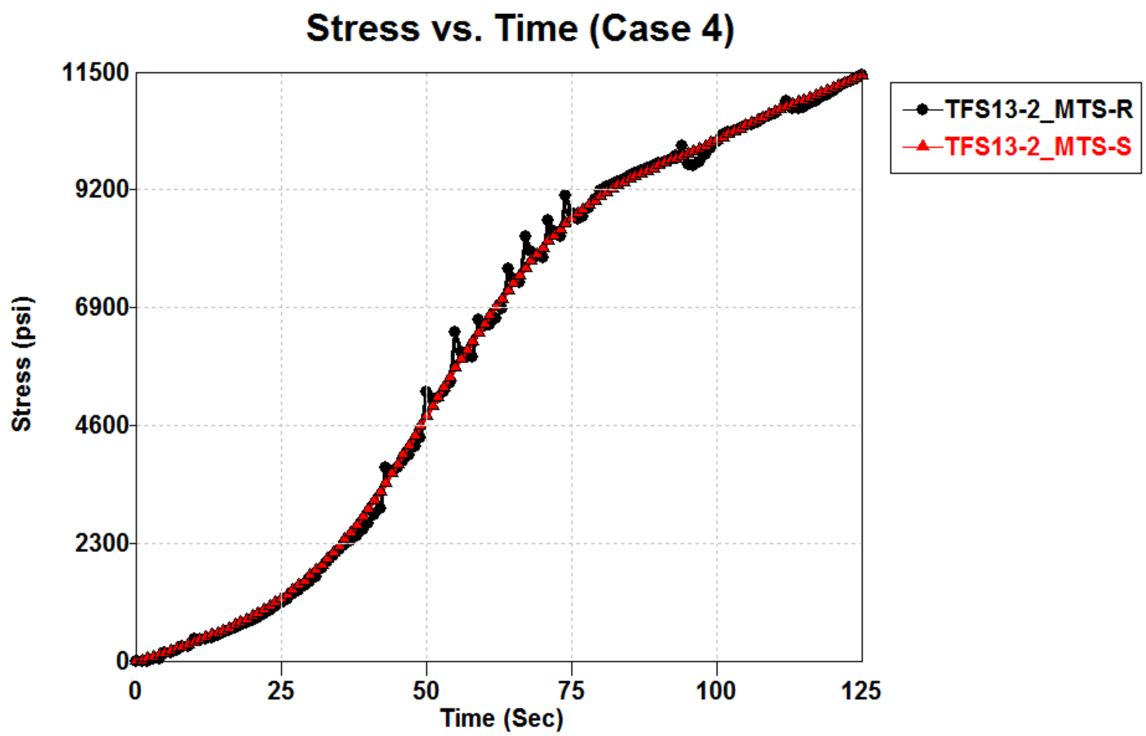


Figure 25. Case 3 smoothed with SMA (top) and PMA (bottom) methods.

SMA works well with data involving few noticeable outliers. PMA, however, shows susceptibility to local patterns, requiring more local points and more iterations. Figure 26 shows the results of using SMA and PMA in case 4 with the same inputs.



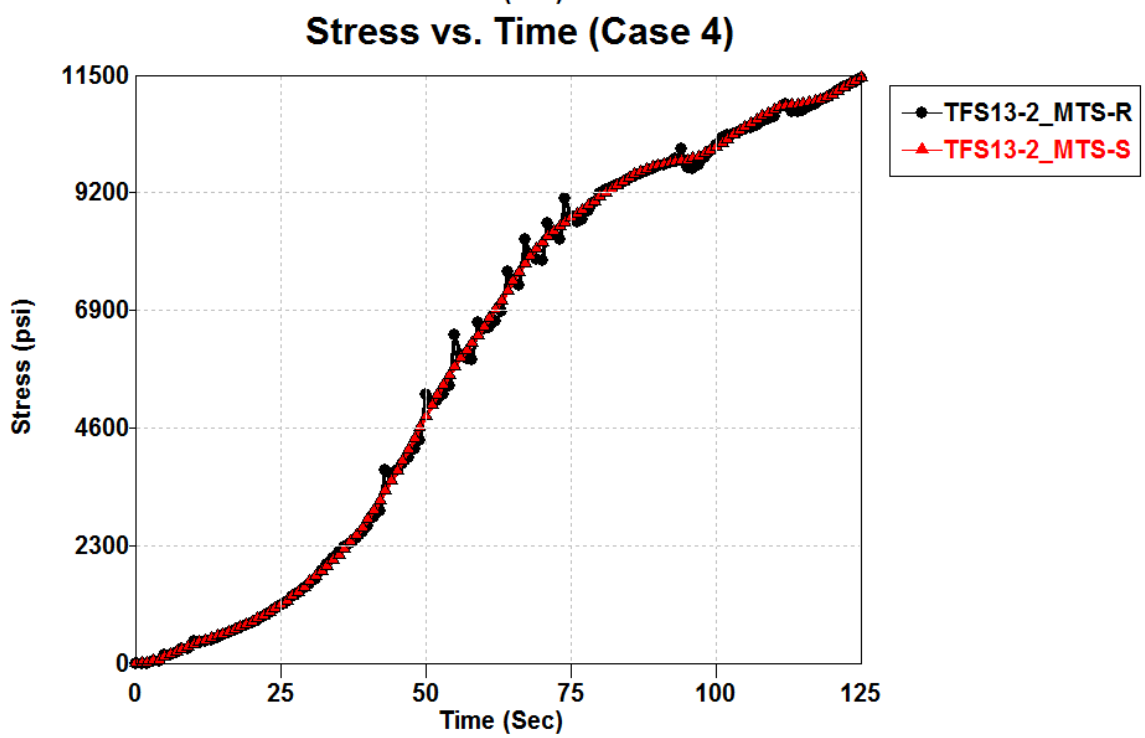


Figure 26. Case 4 smoothened with SMA (left) and PMA (right) using 15 points and 2 iterations zoomed to initial 125 seconds.

3.3.6.3 Lowess/Loess

Loess (Local regression) and Lowess (Locally weighted scatterplot smoothing) are similar methods for smoothing scattered data using a function based on weighted parameters to perform multiple local weighted regressions. Lowess uses a linear regression while Loess a higher degree polynomial for local fits. This filter allows for all points within a given set to be processed. For more information on these methods, see section 3.3.2.

This method is commonly used with high noise data, which is more common with high strain-rate testing than with quasi-static testing. The cases detailed here use data from quasi-static tests alone as high-strain rate test data for the same T800-F3900 composite is not currently available.

With case 1, Lowess and Loess methods offer similar functionality to SMA/PMA. With 9 points and 1 iteration, Lowess/Loess appear very similar to the results shown in Figure 23. While the two smoothed curves shown in Figure 26Figure 27 do not match, the overall trends are acceptable for a final smoothed curve. Note that these methods modify all points within the data set including the ends. Shifting the curve to ensure it starts at the origin may be necessary. Shifting should only be done if the shift is small relative to the maximum. In this case, the shift would be equal to about 1% of the maximum value.

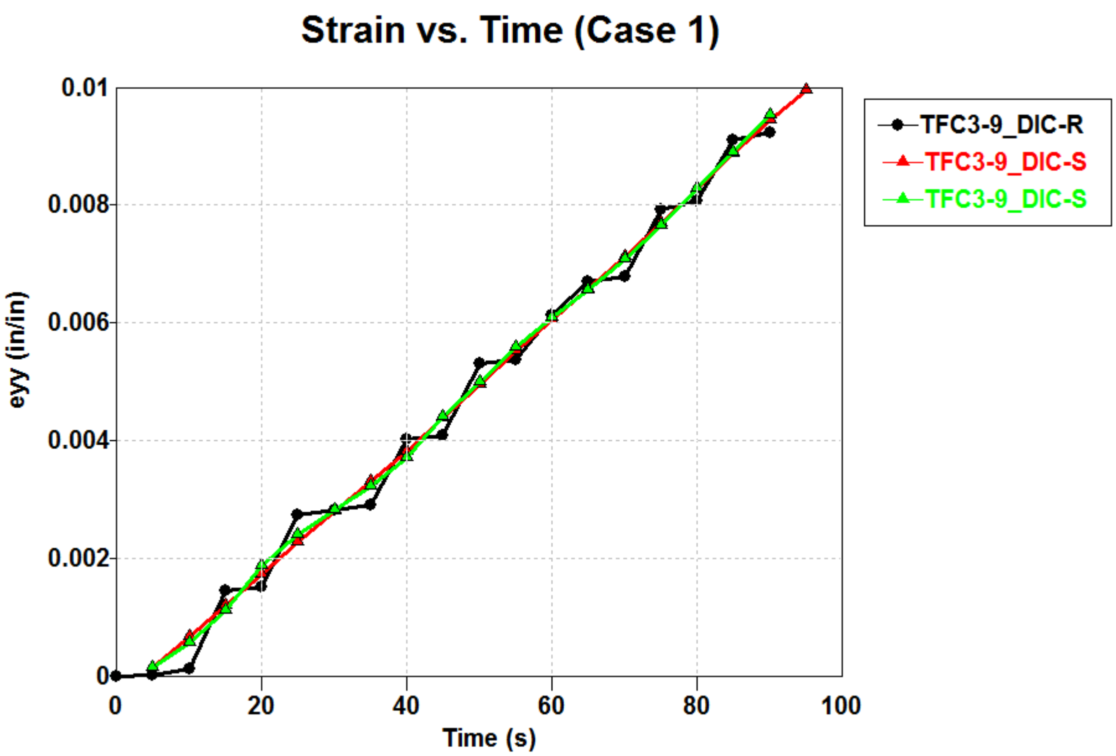


Figure 27. Case 1 smoothed with Lowess (red) and Loess (green) methods zoomed to the first 100 seconds

Lowess and Loess perform similarly to SMA and PMA in case 2. Loess, which smooths points based on a third degree polynomial regression, is also susceptible to local patterns like PMA. When using 11 points and 2 iterations, Lowess generates a straight line while Loess maintains some irregularities in the data while following the initial points closely. Increasing the number of local points leads to both methods matching closely as shown in Figure 28.

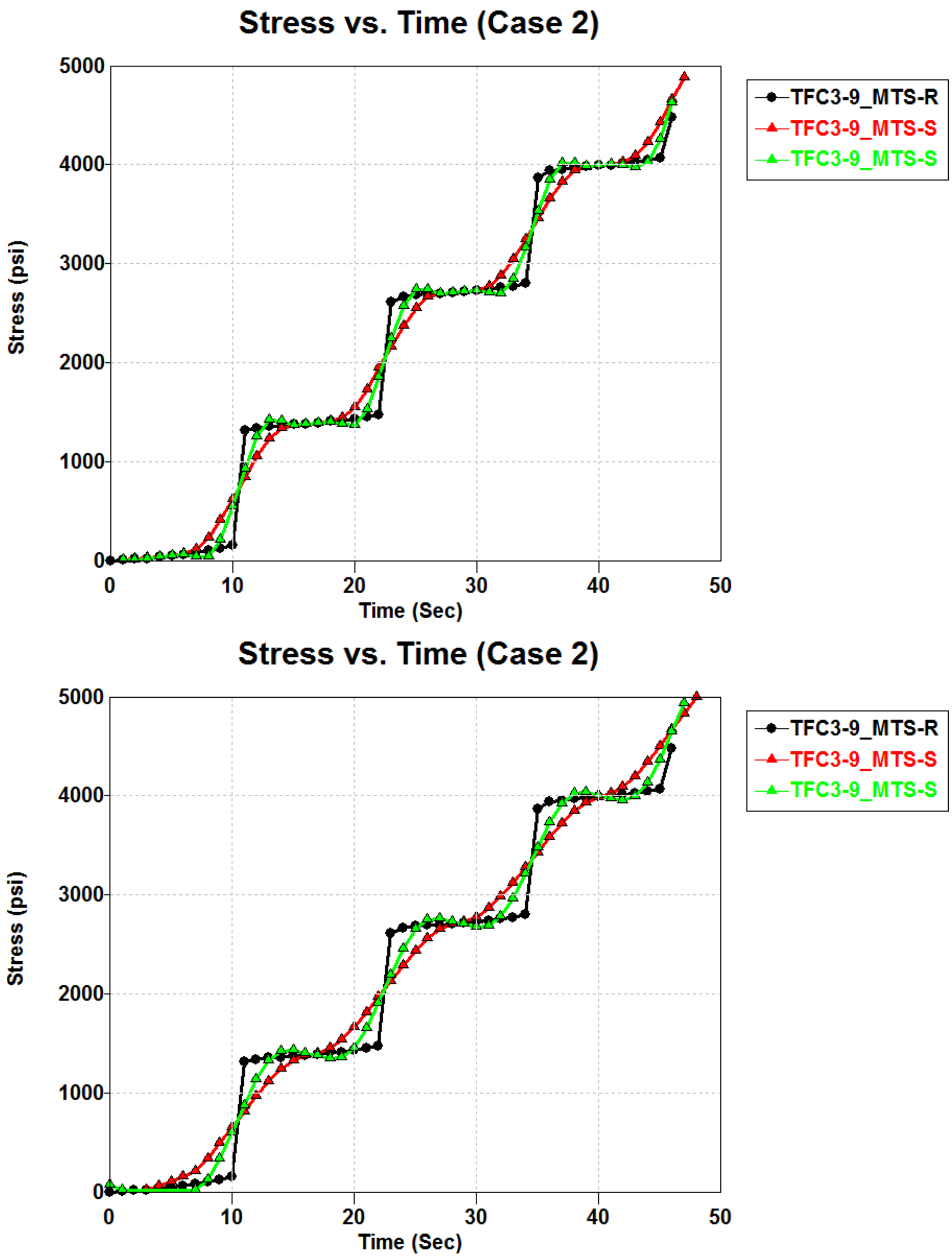


Figure 28. Case 2 smoothened with Lowess (red) and Loess (green) methods zoomed to the first 50 seconds with 11 points (top) and 15 points (bottom)

Lowess and Loess do have some weaknesses compared to SMA and PMA. When increasing the number of points and iterations as was done in Case 3 for SMA, the computation time increases dramatically for Lowess and Loess. For case 3, using 25 points and 15 iterations is inadequate for both Lowess as shown in Figure 29. Just like PMA, Loess smoothing does not converge on a smooth line in this case.

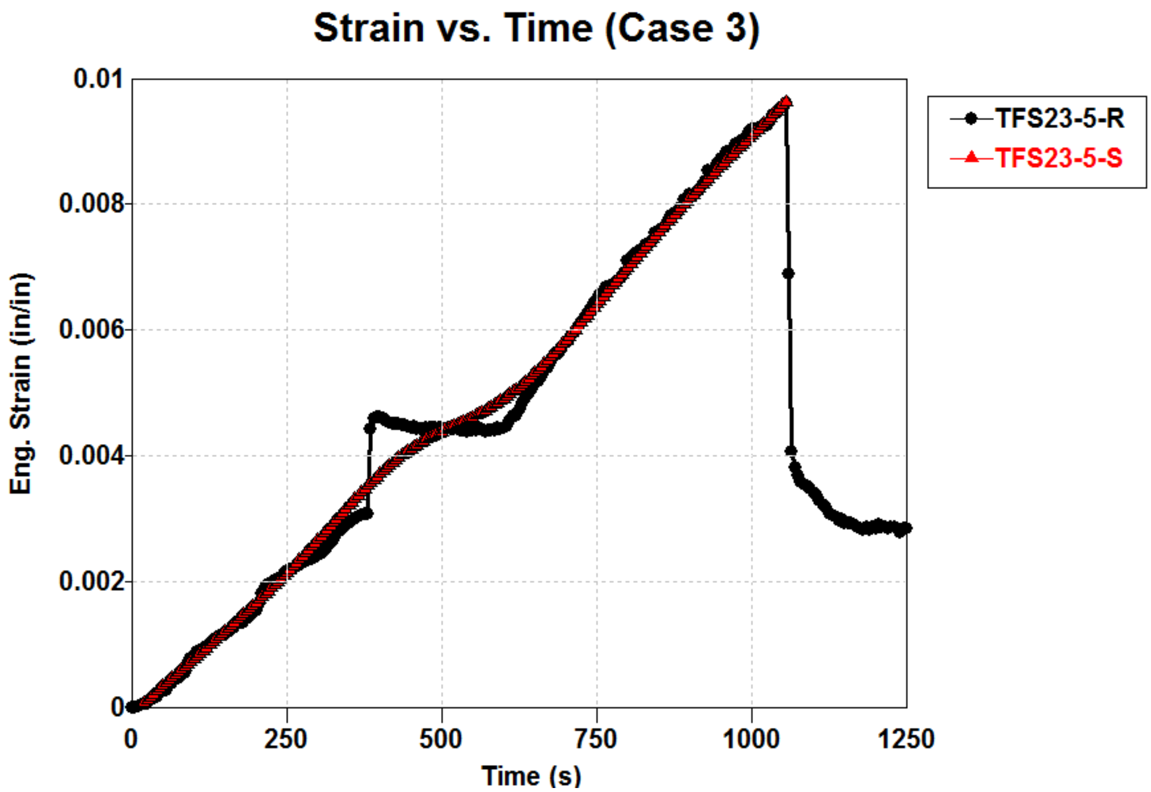


Figure 29. Case 3 smoothed with Lowess method.

In further contrast to SMA and PMA, Lowess and Loess are more susceptible to outliers. Looking at some of the outliers in case 4, neither Lowess nor Loess methods are able to smooth outliers as well as SMA. As shown in Figure 30, both methods result in inadequately smoothened data around large outliers without increasing the number of local points or iterations.

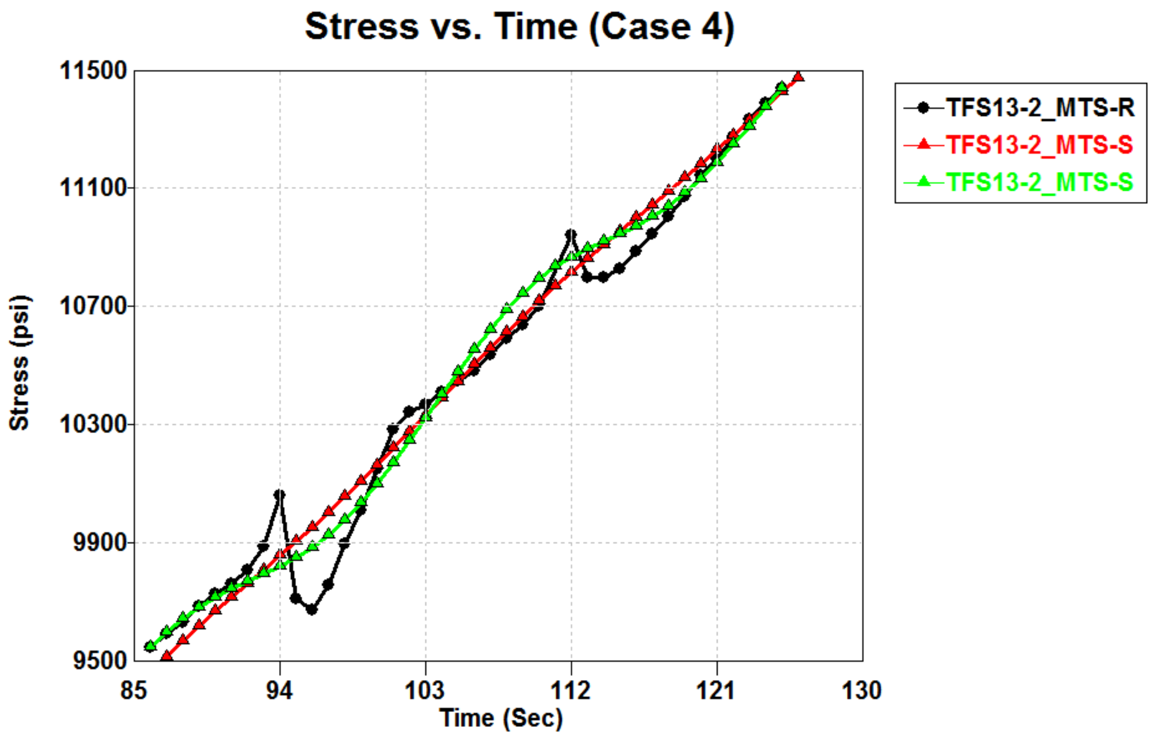


Figure 30. Case 4 smoothened with Lowess (red) and Loess (green) methods zoomed in to show local outliers

Overall, Lowess and Loess methods follow local trends very closely. In data with a high amount of noise, these local trends may be desirable. In materials testing, however, local trends may not be indicative of the material response. The choice between Lowess and Loess lies in the desired sensitivity to local phenomena in the data.

3.3.6.4 <u>Robust Lowess/Loess</u>

By implementing a robust procedure, the Loess/Lowess methods can become resistant to a small number of outliers. The robust procedure involves using Lowess/Loess method for one iteration, then determining the residual for each point based on that iteration. This residual is used as an additional weight to reduce the impact of relative outliers. This filter allows for all points within a set to be processed. For more information on these methods, see section 3.3.4.

Including the residual weights only helps with outliers, however. In cases 1, 2, and 3, without any outliers, Robust Lowess/Loess does not appear any different from their non-robust results. The results in case 3 are shown in Figure 31.

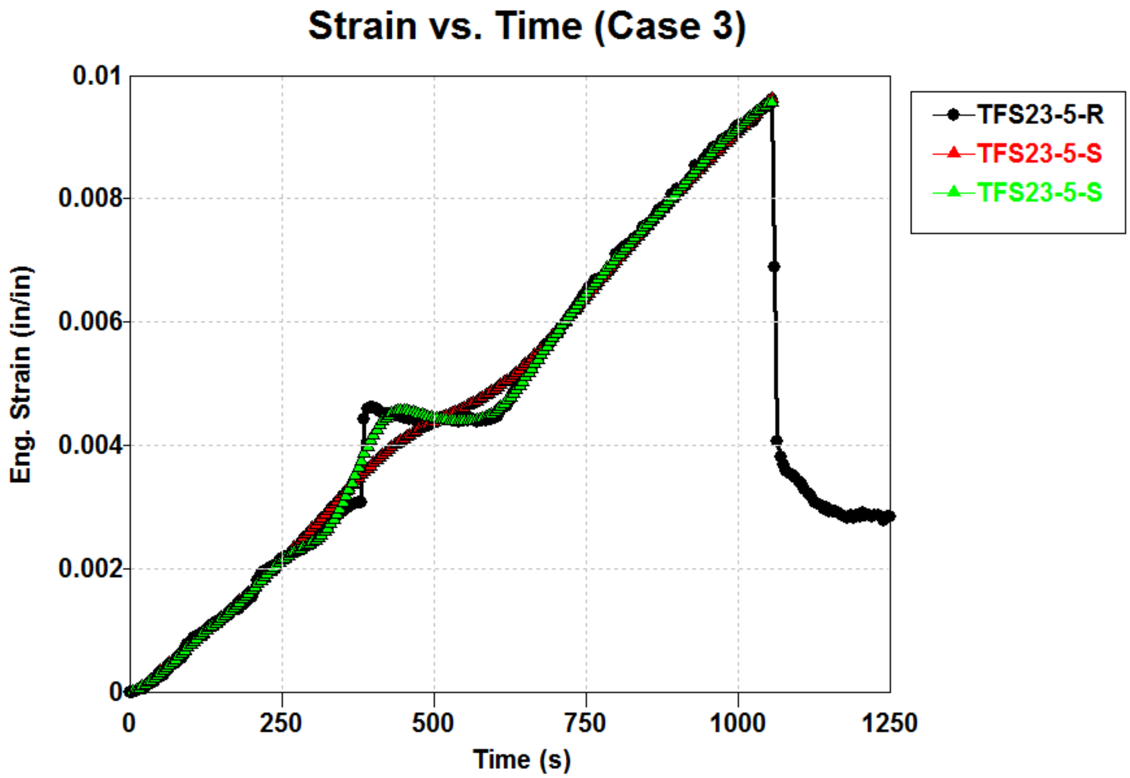


Figure 31. Case 3 smoothened with Robust Lowess (red) and Robust Loess (green) methods.

In case 4, the robust version of Lowess and Loess smoothen large outliers better than their basic counter parts as shown in Figure 32.

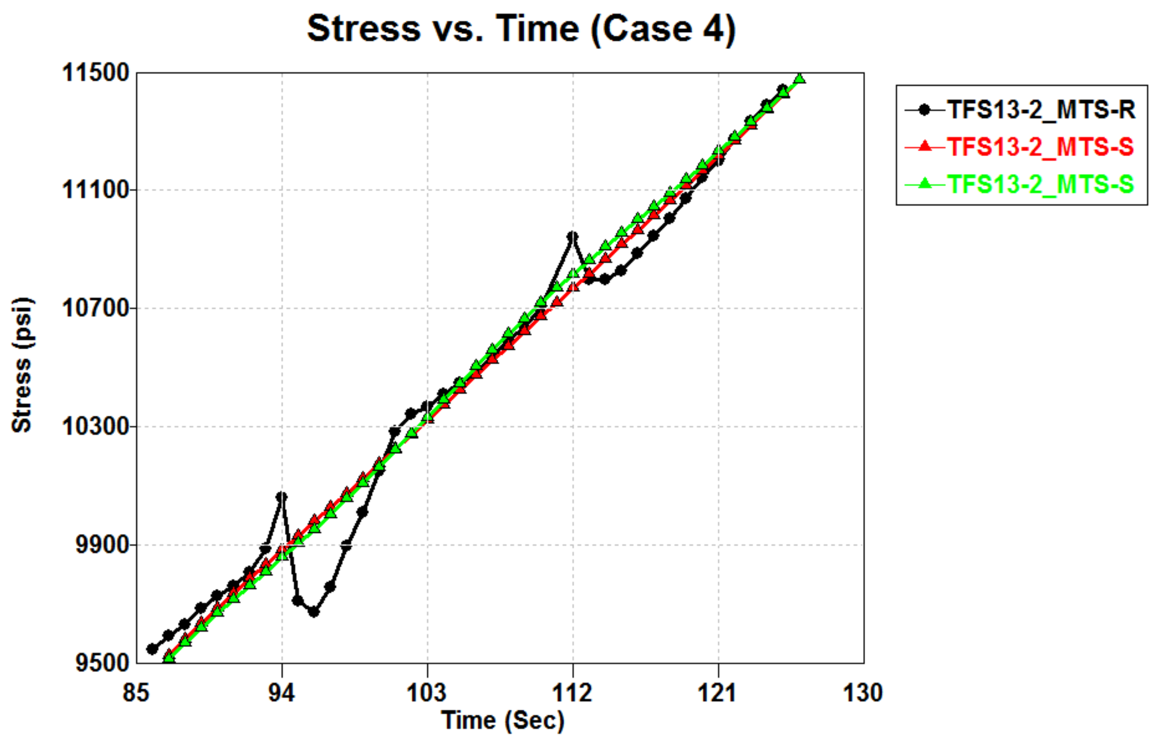


Figure 32. Case 4 smoothened with Robust Lowess (red) compared to Lowess (green).

4 Post-Processing to Generate Input for Constitutive Model

This chapter illustrates the complete process for processing raw data from multiple tests to generate a representative curve for the tension response of the 2-Direction of the T800S/3900-2B unidirectional composite. All model curves generated in EDP based on the ASU tests for the T800S/3900-2B composite are found in Appendix C.

Upon launching the program, note the basic commands available. The tool bar at the top includes several buttons. At the far left is the **New** button, which will clear all current data and present the screen as it is at startup. Next is the **Open** button, with which you may open a previously saved .edp file. The **Save** button is to the right of open and saves all work as currently stored. To get started, select **Read raw data** located in the toolbar as highlighted in red in Figure 33.

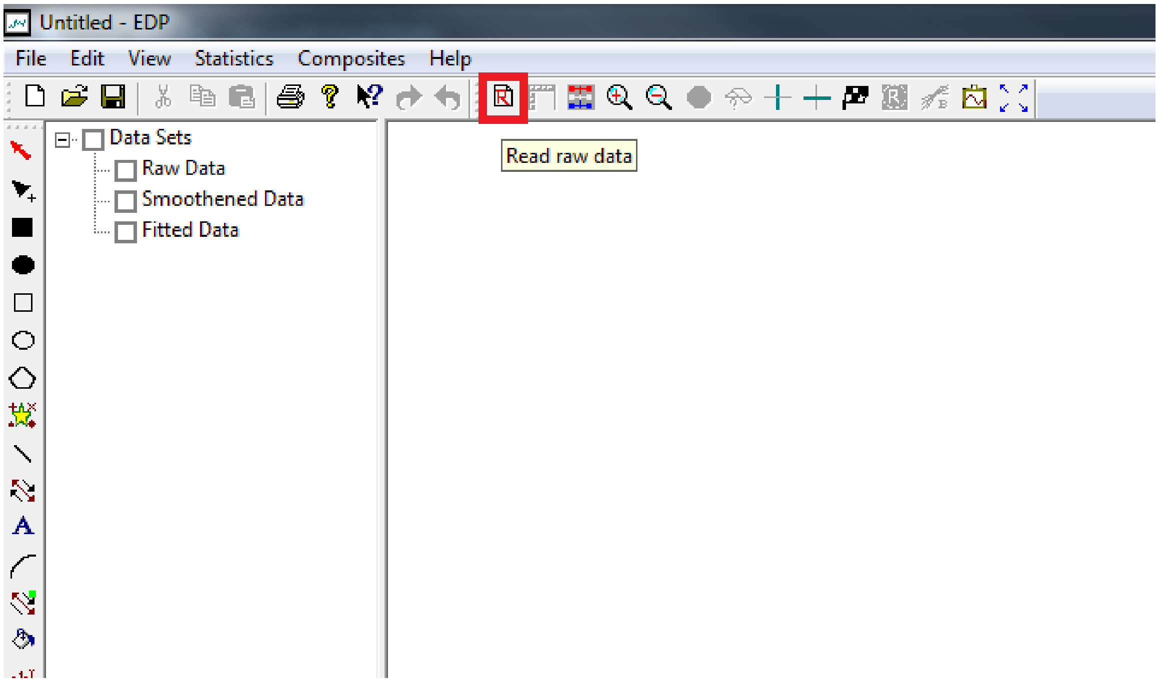


Figure 33. Step 1 of processing involves reading in the raw data. The Read raw data button (highlighted in red) is found at the top of the screen in the tool bar.

Once pressed, you will be presented with the Read Raw Data interface detailed in Figure 34. The first step is to **Browse** folders for one data file to be processed. Be sure to properly name each file prior to loading files.

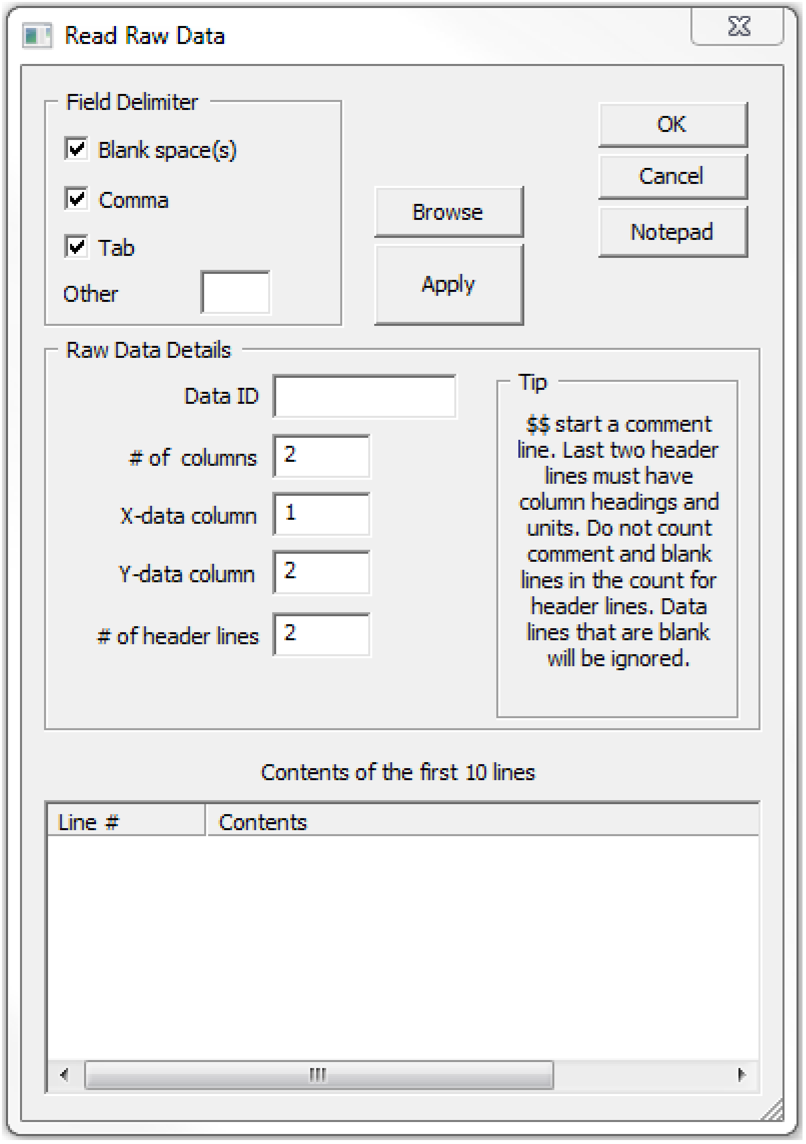


Figure 34. The Read Raw Data interface. In here, multiple sets of data may be read in at once before closing the interface.

After selecting the data file, a preview of the contents is shown at the bottom of the window. Using this, it is a simple matter to fill in the Data ID (the name of the data set or replicate), the number of columns, which columns are to be plotted as x and y-axes (this can be changed later), and the number of header lines. Note the tip on the right that specifies how to interpret the number of header lines.

It is important to differentiate between DIC and MTS data. In Figure 35, the DIC data for replicate 3 has been labelled "TFT2-3_DIC" to indicate the test, the replicate number, and the data being stored. In addition, since the DIC data is stored in a .csv file, only the comma delimiter should be selected in the upper left corner. The number of columns is determined for DIC by counting the number of commas in row one. Since there are 3 commas, there are 4 rows. It may also be necessary to count columns outside of EDP prior to reading raw data. For a tension test, the longitudinal strain is needed, which labelled as eyy in column 3. Vic 3D 7® exports data with 2 header rows with the analysis region in row 1 and the column title in row 2. Press apply to receive confirmation that the replicate successfully read.

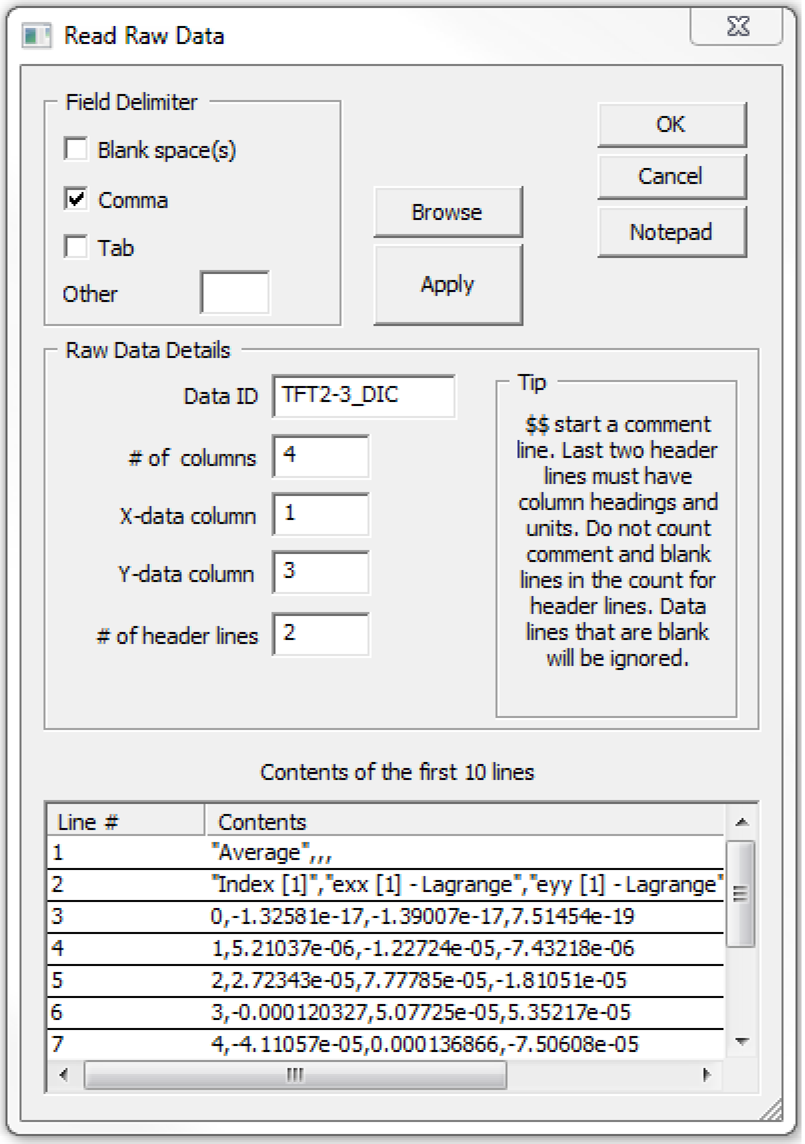


Figure 35. The Read Raw Data interface with DIC information entered.

Next, the MTS data is read. The MTS system exports force data using 5 header lines, but one is blank. As noted in the tip, 4 is entered as the number of header lines to accommodate. The MTS data is separated by tabs. The MTS data always has three columns: Time, Actuator Displacement, and Force. Time and force are the columns of interest. The Read Raw Data dialogue should appear as it does in Figure 36. Press **Okay** when finished reading all data sets.

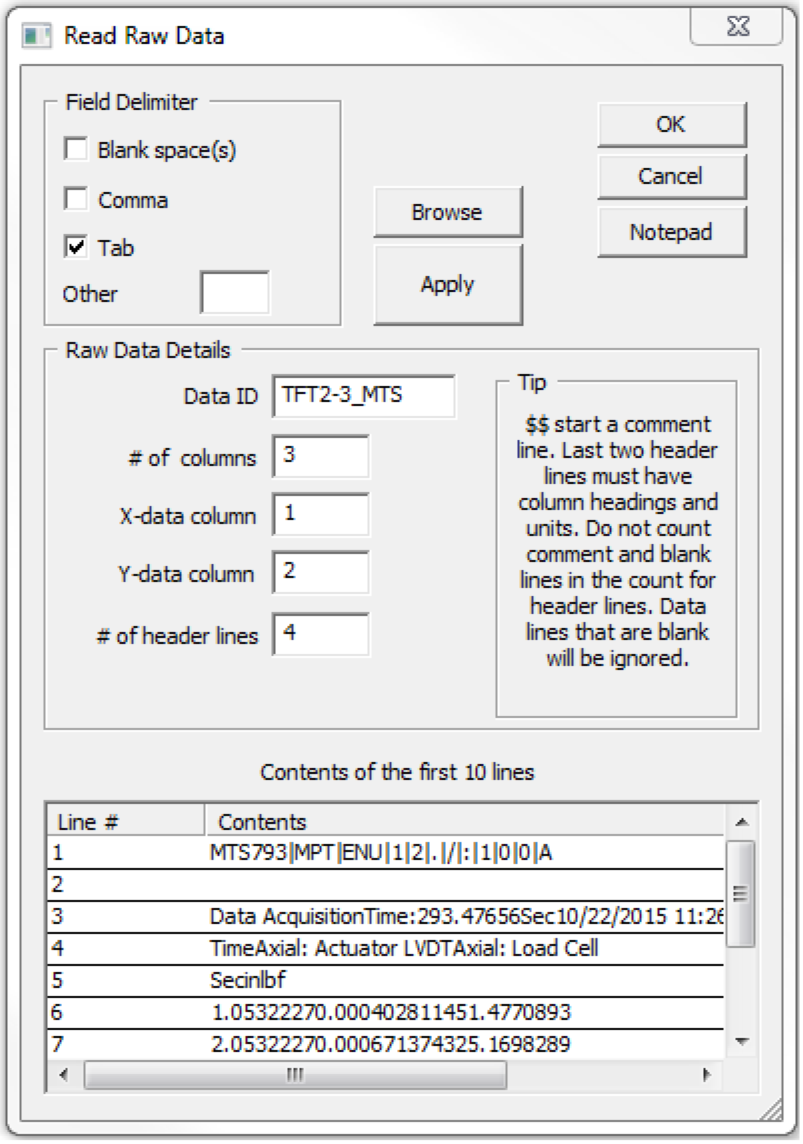


Figure 36. Read Raw Data dialogue with MTS data entered.

If a mistake is made when reading raw data, such as entering an incorrect number of header lines, the command **deleterep** is available for use in the Edit Raw Data dialogue box in the Action input box within the Cell Operations area (seen in Table 8). Take care when deleting data sets as it is not possible to rearrange data sets at this time.

The raw data is immediately plotted upon pressing okay. DIC and MTS data are on different scales, however, making visualization difficult at this point. Using the check boxes on the top left of the viewing area, it is possible to toggle each curve on and off (see Figure 37). When data is smoothened or fitted, toggling each smoothened or fitted set of data will also be possible. The raw DIC and MTS data can now be visualized as illustrated in Figure 38. Visualizing each data set helps to identify which smoothing methods would work best. In the 2-Direction Tension test, data is consistent with some noise. The drops at the end of each data set should be excluded from processing.

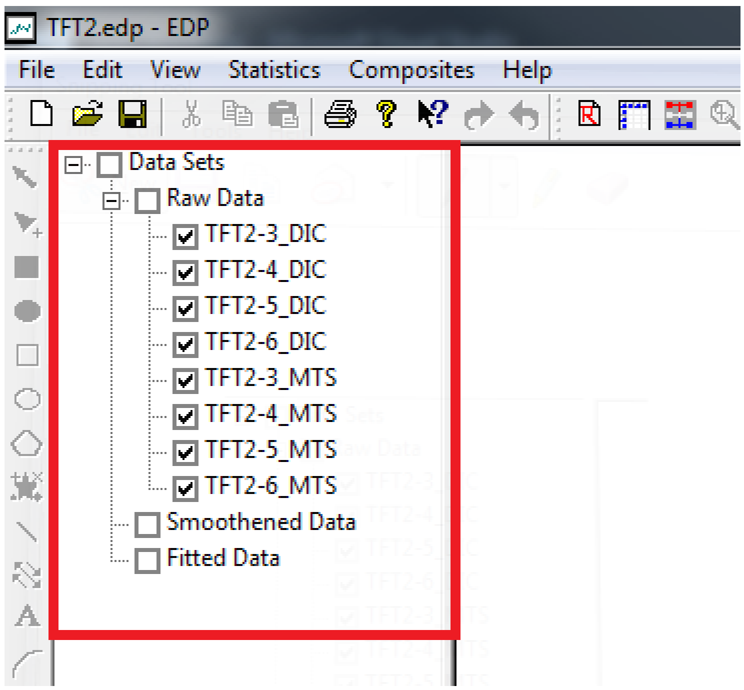


Figure 37. Plot toggle area in top left of view screen.

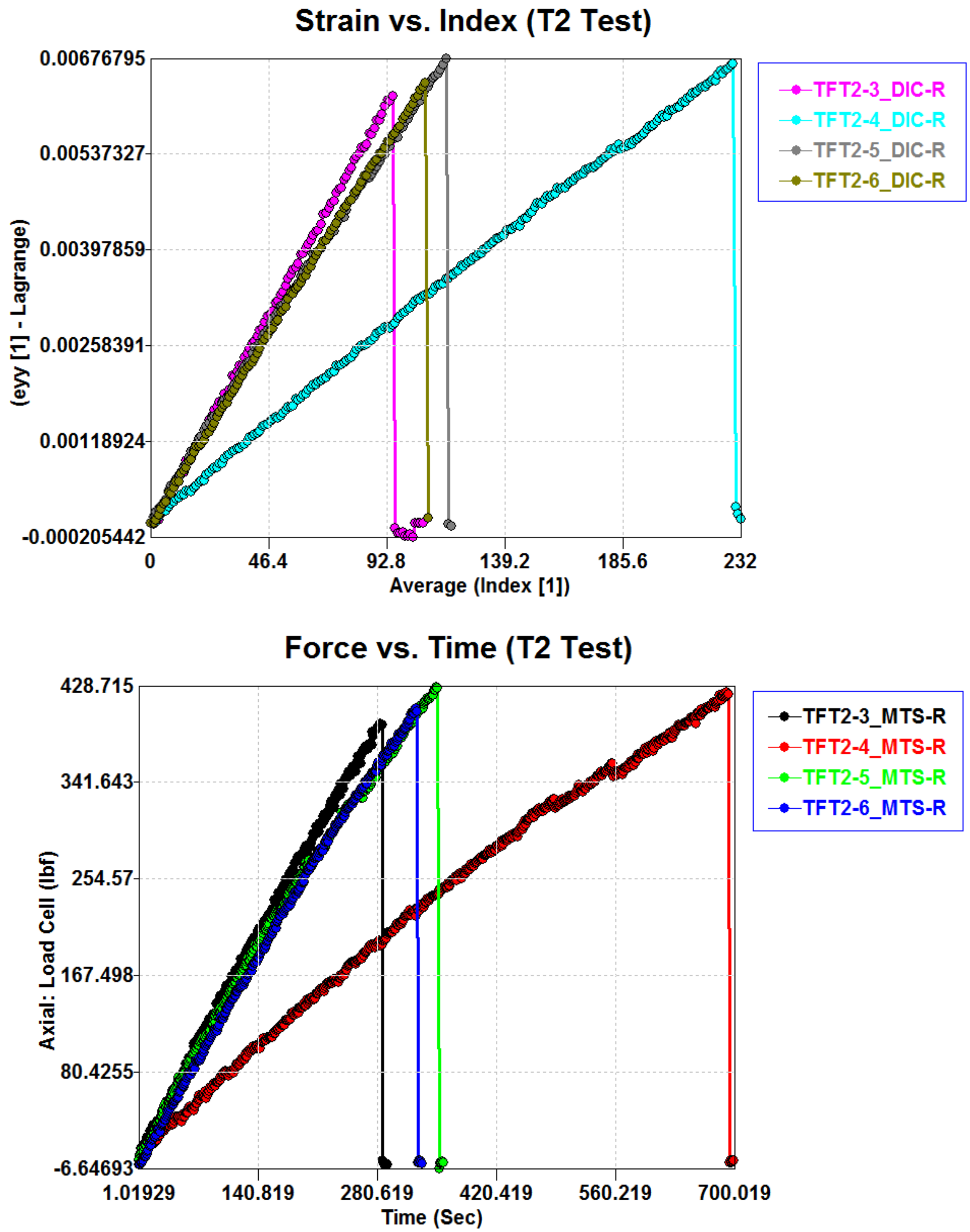


Figure 38. Raw data plotted in EDP.

To ensure all data set pairs match and are normalized, the raw data needs to be edited. Select **EDIT or view data** at the top of the screen next to the Read Raw Data button to view the data (see Figure 39).

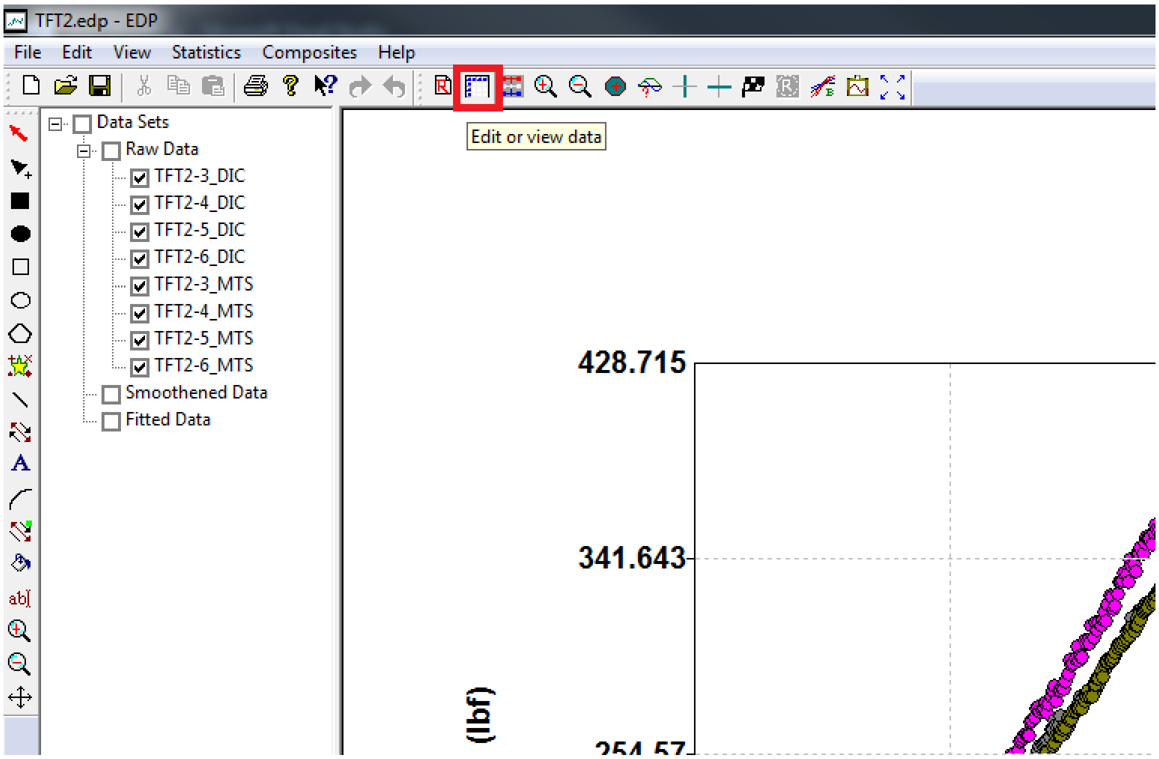


Figure 39. Location of Edit or view data button

Upon opening the Edit Raw Data & View Smoothened and Fitted Data, data from data set one is presented by default. Note the four major areas within this interface shown in Figure 40. At the top is list data showing relevant data for all replicates, including the set number, set name, the number of points, the number of columns, etc. The main feature is the data visualization in the center of the interface. Row ID's are shown on the left while column headers and units are shown above the column ID's. The currently selected data is viewable and editable in this area. Here, it is possible to insert rows/columns, delete rows/columns, and cut, copy, paste, and clear data using right click. To delete or insert, the header must be selected. On the bottom right of the window are the details of the data set currently on screen. Here, set name, column headers, column units, and the x and y-columns can be changed. This is also where the current data set and data type are selected for the viewing area. For a data change made in the details or viewing areas to be stored, the **Update** button must be pressed. Export will create a new text file with the x and y data separated by commas. Finally, in the upper right is the Cell Operations area. Here, Actions can be entered to modify raw and smoothened data. Current commands are shown in Table 8.

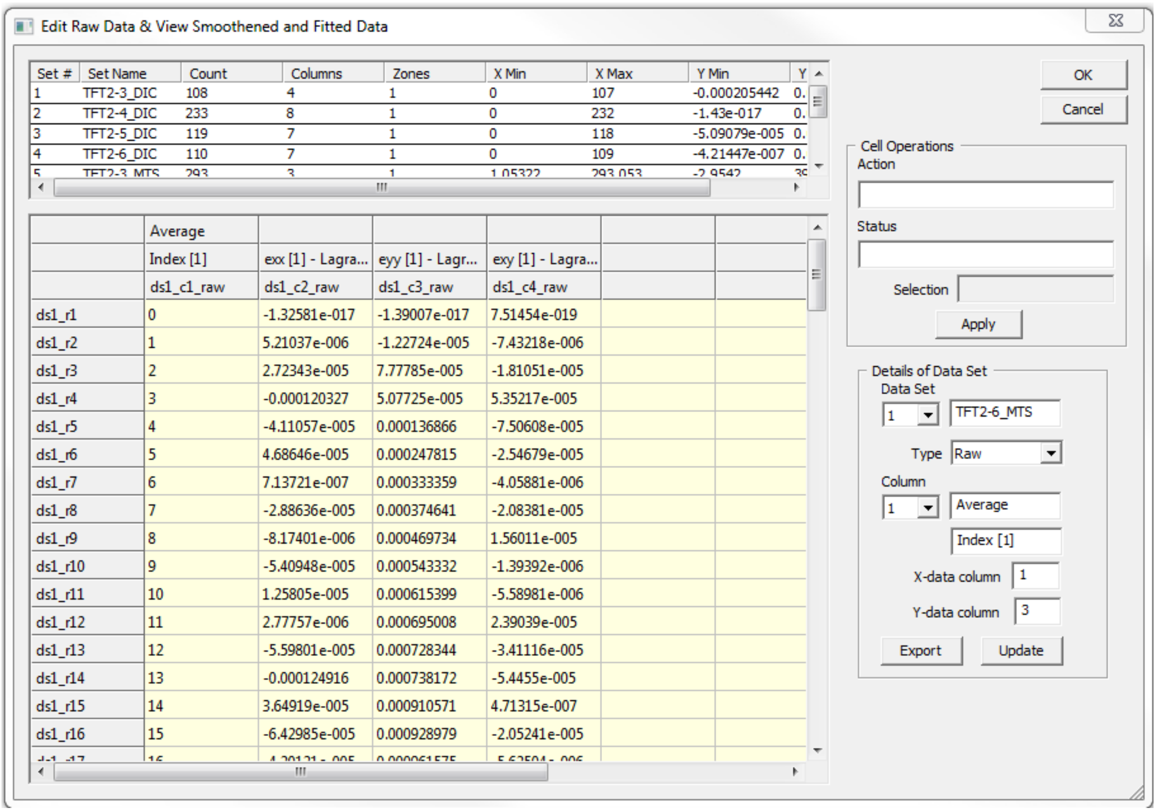


Figure 40. Edit Raw Data & View Smoothened and Fitted Data interface upon launch.

Table 8. Functions for editing data.

Function	**Description**	**Format**
Edit Raw Data	Edits an existing column or creates a new column within a data set based on an equation. Uses column ID available at the top of the viewing window.	ds?_c?_raw = expression involving other columns

Delete Replicate	Deletes a data set.	deleterep, replicate #
Reformat	Rediscretizes smoothened data within a zone of a data set based on an existing column.	reformat, replicate #, zone #, x column #, # of points
Create	Creates a new data set from existing data sets.	create, replicate name, type, x replicate #, x column #, y replicate #, y column #
Zero	Shifts a column to start at zero.	zero, replicate #, column #
Invert	Inverts a column.	inv, replicate #, column #
Zero and Invert	Performs both the Zero and Invert functions	zeroinv, replicate #, column #

The first step of editing data is normalizing values. The zero and invert commands are used here to ensure all sets of data start at zero time, zero strain, and zero stress as long as the initial values are small compared to the maximum values. For example, MTS time starts at 1.05322 seconds, despite representing the start of the test. Another example is the force column. For TFT2-3, the max force is 394.802 while the initial force is 1.47709, or about 0.4% of the maximum value. The zero command allows for multiple replicate and column pairs to be entered at once (e.g. zero, 1, 1, 1, 3, 5, 1, 5, 3 indicates the first and third columns of data sets 1 and 5 are to be zeroed). Since this is a tension test, there is no need to invert the data. Compression and shear tests may require inverting the data.

Once data is zeroed, the raw data must now be normalized. The DIC data is presented by index instead of time. The capture rate should be recorded during testing. For TFT2 tests, the capture rate was 3. By using the command "ds1_c1_raw=ds1_c1_raw*3", the indices are converted to units of time. MTS data also shows force instead of the normalized stress. Using the edit raw data function, the force can be converted to stress. To reflect these conversions, use the Details of Data Set area to update the column titles and units.

The changes made should be immediately apparent upon pressing Okay in the Edit Raw Data dialogue box.

Next, the data must be smoothed. Now that data is loaded, the **Process Data** button is available. Select the **Process Data** button in the top tool bar as shown in Figure 41.

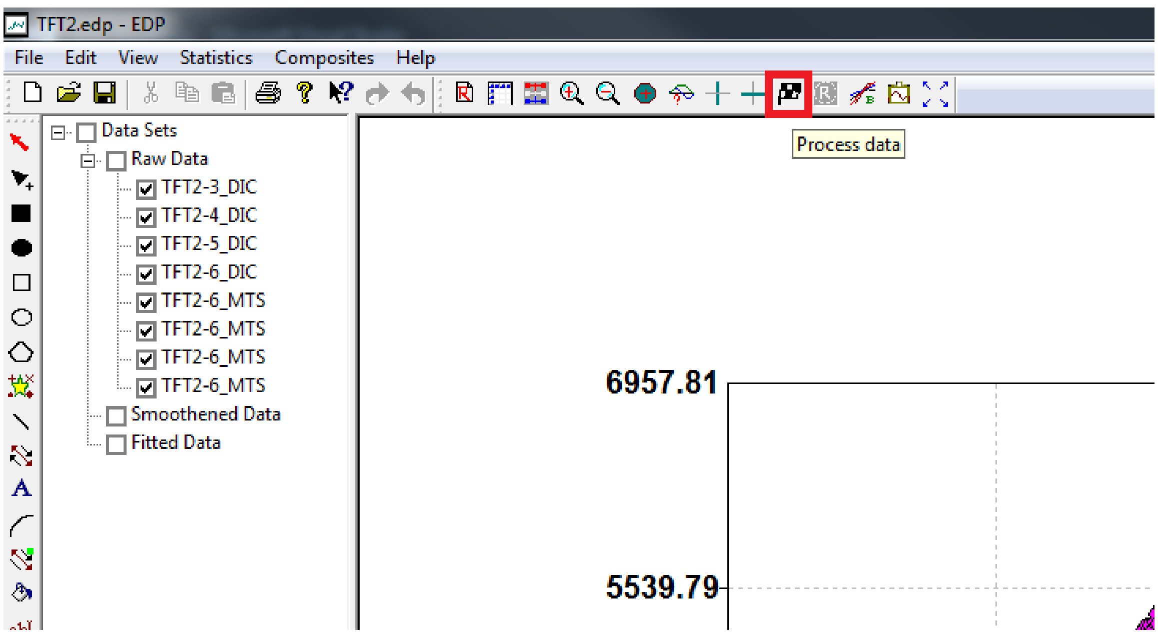


Figure 41. Interface with Process data button highlighted

The Data Set-Zone Data dialogue box is where smoothing and fitting options are selected, shown in Figure 42. Here, all changes to the current Data Set will only be stored upon pressing Update.

Smoothing and fitting is done by zone. If a data set has multiple regions of data (ex. linear and nonlinear regions), additional zones may be added with the Add Zone button on the right. Zones are defined by the start and x values. These values should correspond to those from the x-column defined upon reading or editing data. For the 2-

Direction tension tests, the start x should be 0 and the end x depends on the ultimate stress and strain values for each replicate. All end values should be multiples of 3 to ensure stress and strain points match. The current fit types and smoothing methods are outlined in Table 9 and Table 10. The number of points must be odd between 3 and 25 for most smoothing methods; PMA and Loess require a minimum of 5 points.

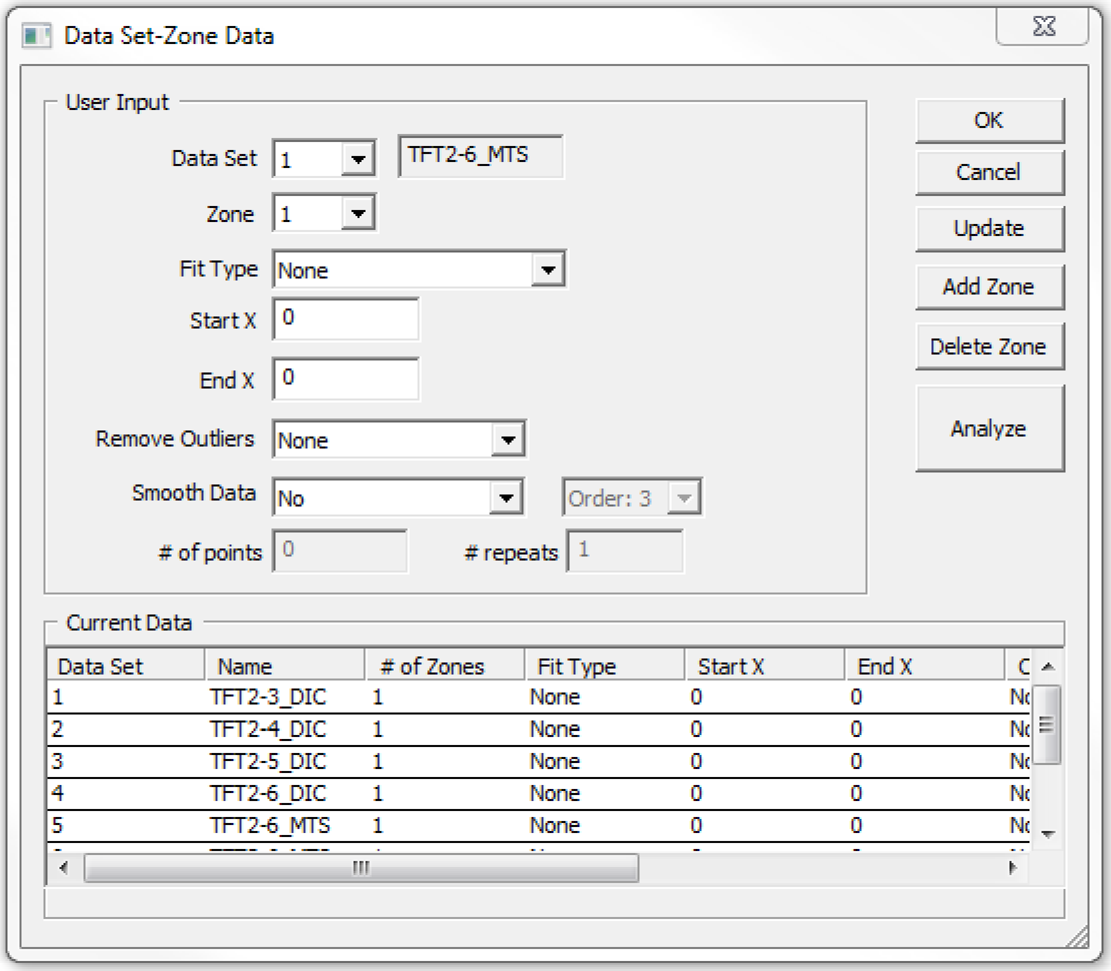


Figure 42. Data Set-Zone Data dialogue box

Since the data appears straight with a small amount of noise, an appropriate smoothing method must be selected. For this example, both SMA and Lowess smoothing are used. Additional information on smoothing methods is available in section 3.3.6. The results of both methods are shown in this guide. Select analyze once all zones are defined and press OK once the analysis is finished.

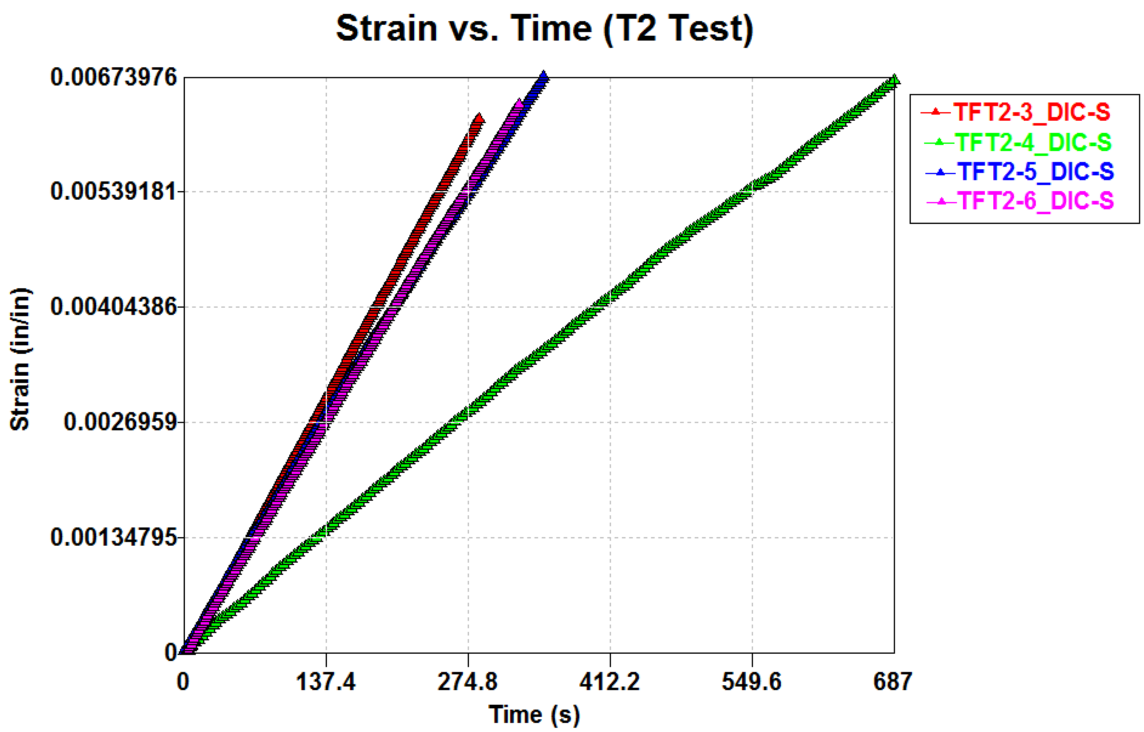
Table 9. Fitting Types and their descriptions

Fitting Type	**Description**
Polynomial	Fits the data within the zone to a first, second, third, fourth, or fifth degree polynomial.
Exponential	Fits the zone data to an exponential curve of the form $y = ae^{bx}$
Logarithmic	Fits the zone data to a logarithmic curve of the form $y = a\log(bx)$
Power	Fits the zone data to curve of the form $y = ax^{b}$

Table 10. Smoothing Methods and brief descriptions.

Smoothing Type	**Description**
Median	Replaces data points with the median value of a local range.
Simple Moving Average	Replaces data points with the unweighted average value of a local range.
Polynomial Moving Average	Replaces data points with the weighted average value of a local range.
Loess	Replaces data points with a weighted value calculated using local regression (using a second degree polynomial) within a local range giving more weight to proximate values.
Robust Loess	Loess with an additional weight considering outliers.
Lowess	Replaces data points with a weighted value calculated using local regression (using a first degree polynomial) within a local range giving more weight to proximate values.
Robust Lowess	Lowess with an additional weight considering outliers.

Once analyzed, the Smoothened Data is now able to be toggled on the left. Figure 43 shows the TFT2 smoothened data generated using Lowess with 15 points and 2 repeats.



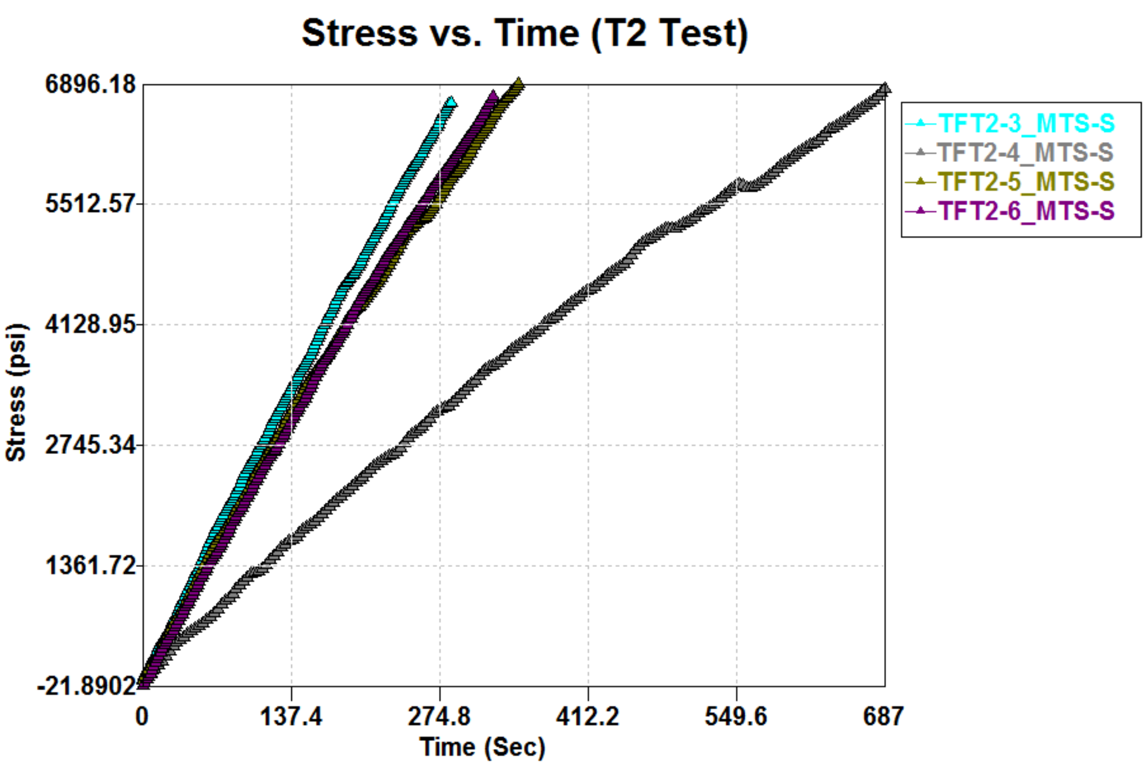


Figure 43. Smoothened data generated with Lowess smoothing plotted in EDP

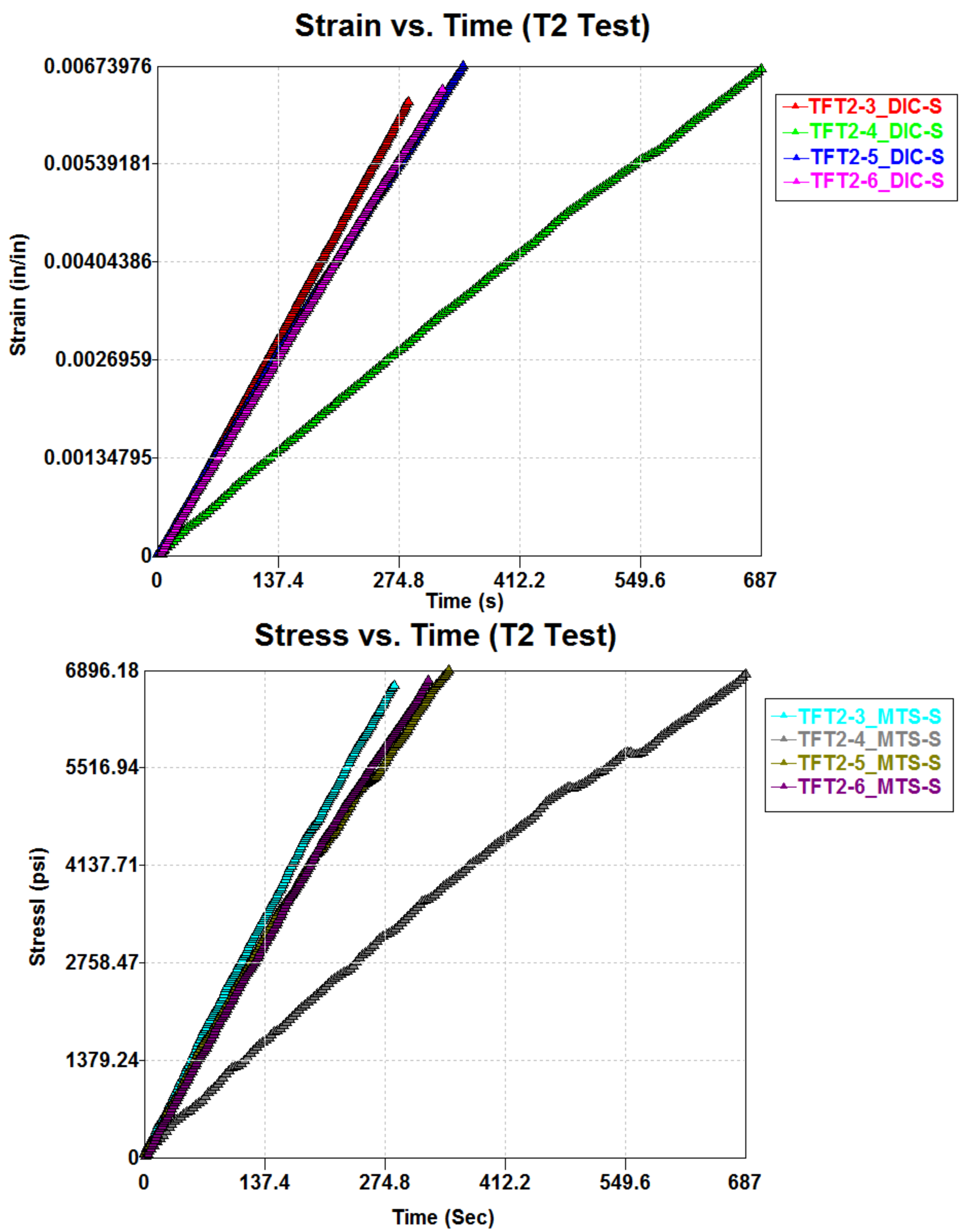


Figure 44. Smoothened data generated with SMA method plotted in EDP

The smoothened data must now be reformatted with the "Reformat" command (detailed in Table 8; ex. "reformat, 1, 1, 1, 200" rediscretizes data set 1, zone 1, based on column 1 into 200 points) using the Action prompt within the Edit Raw Data & View Smoothened and Fitted Data dialogue box.

Once all replicates are reformatted, the “create” command can be used (ex. “create, TFT2-3, smt, 1, 3, 5, 3” will create a new replicate titled TFT2-3 based on the smoothed data stored in column 3 of data set 1 and column 3 of data set 2). EDP alerts the user when the new data set is successfully created. The column titles and units are transferred from the source columns automatically. The results for creating the 4 stress-strain curves for the 2-Direction tension test are shown in Figure 45.

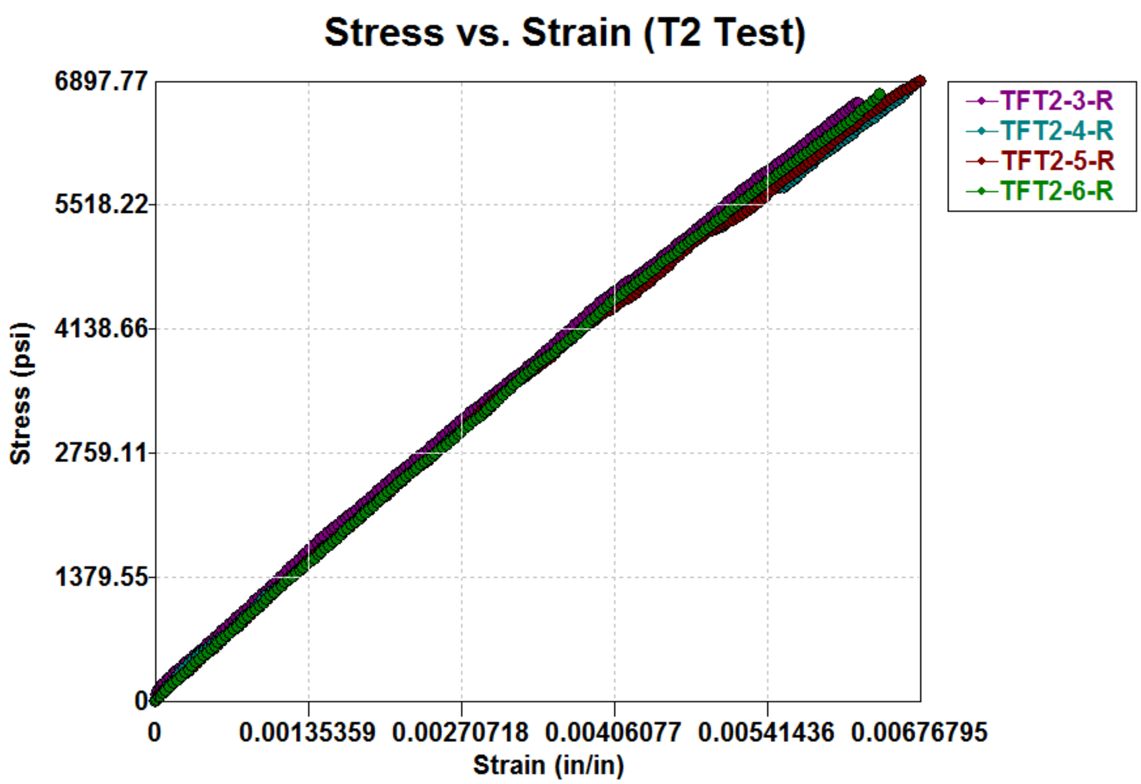


Figure 45. Stress-strain curves for the four replicates plotted in EDP

With all stress-strain curves generated, the final step is to generate the best fit or model curve. The **Best fit plot** button is in the top toolbar as shown in Figure 46.

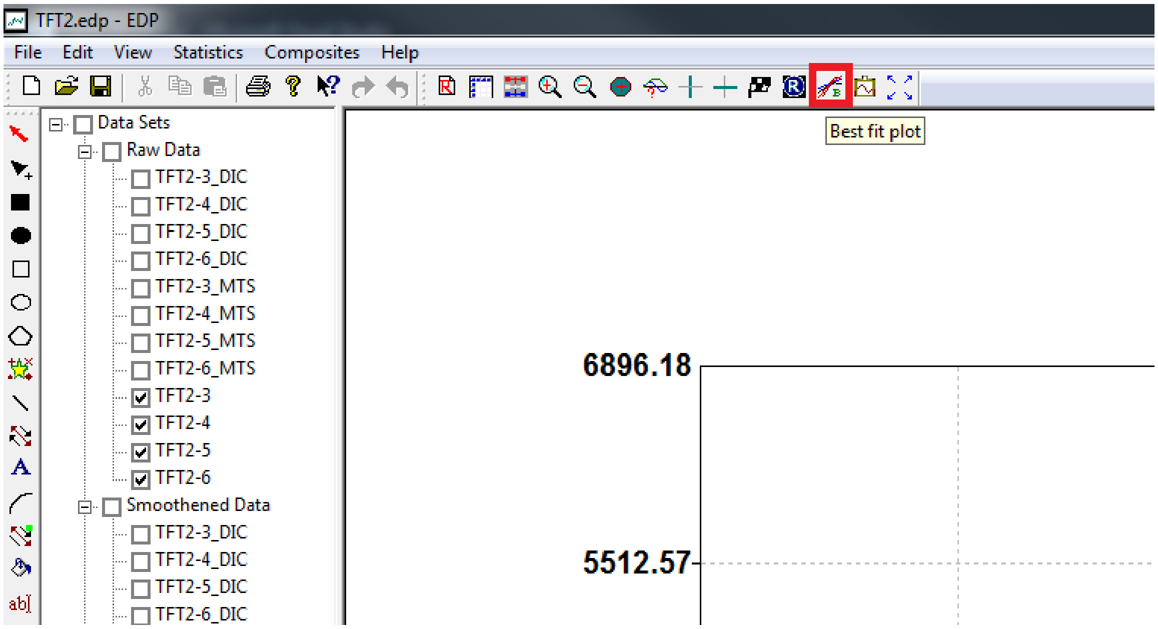


Figure 46. Interface with Best fit plot button highlighted.

The Best Fit plot will be generated based on all curves currently being plotted. When using Lowess or Loess smoothing, be sure all replicates start at 0 prior to creating the best fit (other methods will maintain the same start and end points). Upon pressing **Best fit plot**, the Best Fit Input dialogue box is presented, shown in Figure 47. There are three inputs for creating the best fit: the new data set name, the curve end type, and the number of data points. There are three curve end types: extend, average, and cut. Extend selects the largest end x value as the end point for the best fit and extrapolates shorter curves. Average takes the average of the end points and cuts/extends curves as needed. Cut selects the smallest end x value as the end point for the best fit curve and cuts longer curves. The number of points must be between 5 and 100. For this example, the name will be set to “TFT2”, the end type set to “Average”, and the number of points set to 100.

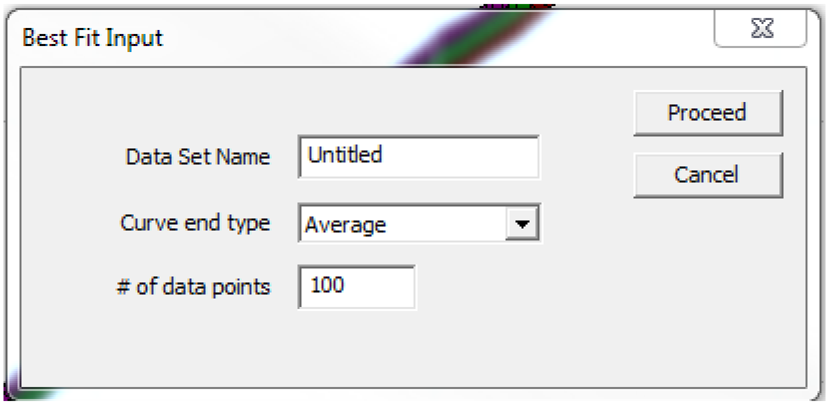


Figure 47. Best Fit Input dialogue box

The final curve can then be plotted alone. For better visualization, change the plot limits by pressing the **Edit graph settings** button in the top bar, as shown in Figure 48.

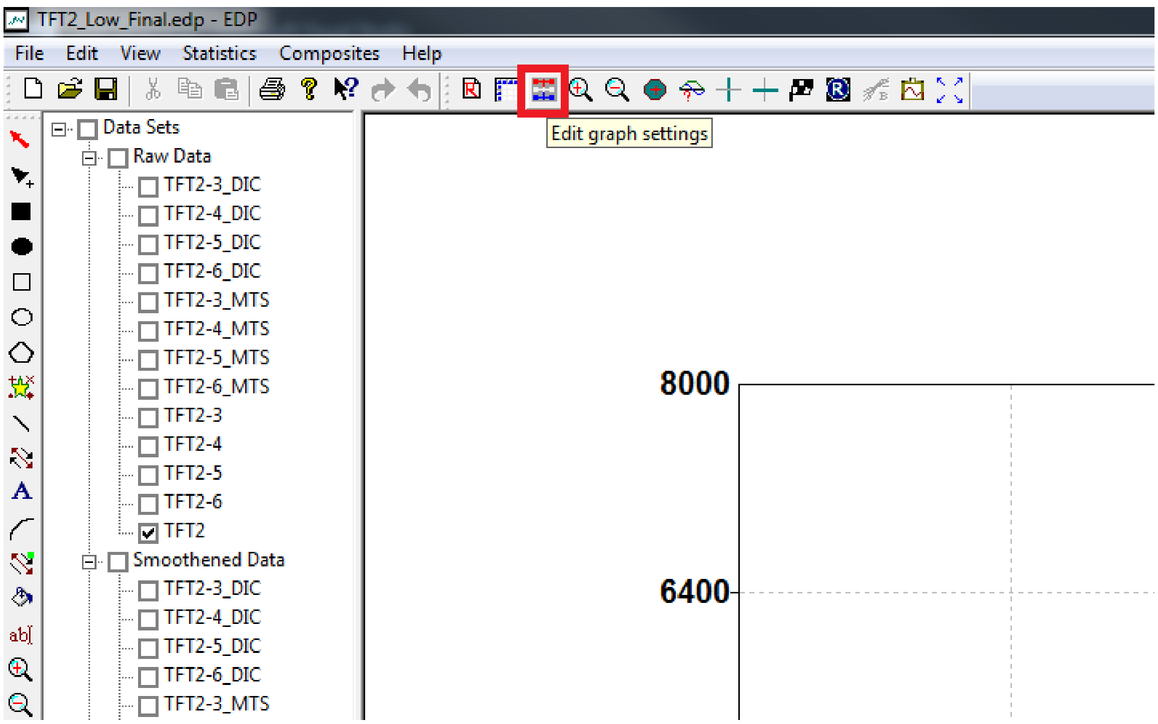


Figure 48. Interface with Edit graph settings button highlighted

The XY Graph Data dialogue box has two tabs. The first tab, Graph Attributes, allows for the axis labels and graph title to be edited. The curve colors, markers, and line types are also customized in this section. The Graph Layout tab allows for the plot limits to be modified. In this tab are miscellaneous options to further customize the plot appearance. Both tabs are shown in Figure 49.

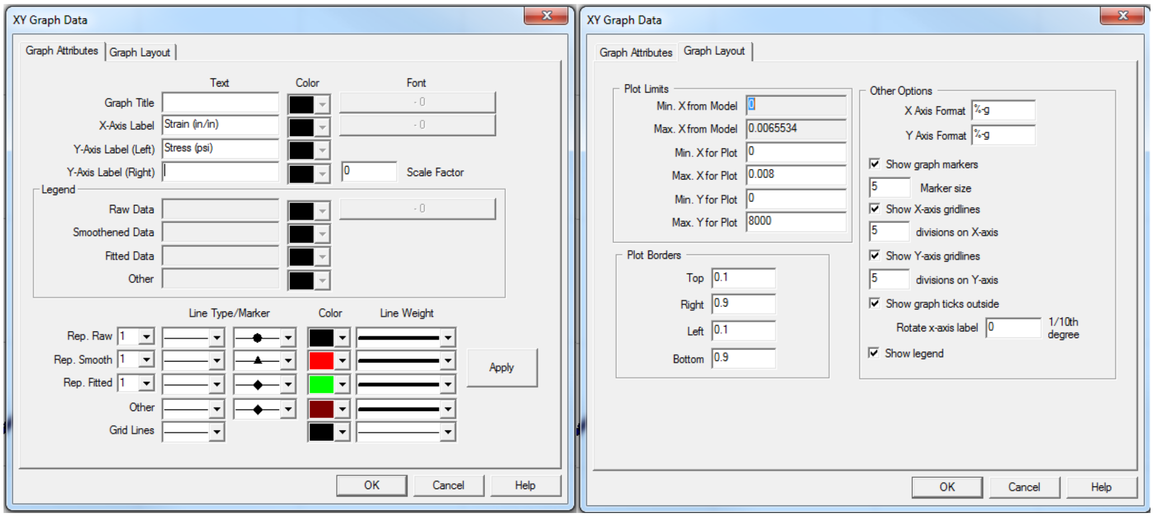


Figure 49. XY Graph Data interface with the Graph Attributes tab (left) and Graph Layout tab (right) shown

The final curve generated with Lowess and SMA smoothing methods with modified plot limits and alternate y-axis are shown in Figure 50. Both curves are very close together with the Lowess curve having a higher ultimate strain and the SMA curve having a higher ultimate stress. Both curves appropriately represent the response of the T800-F3900 composite in the 2-direction. When generating other curves, however, the best smoothing method depends on the quality of the raw data.

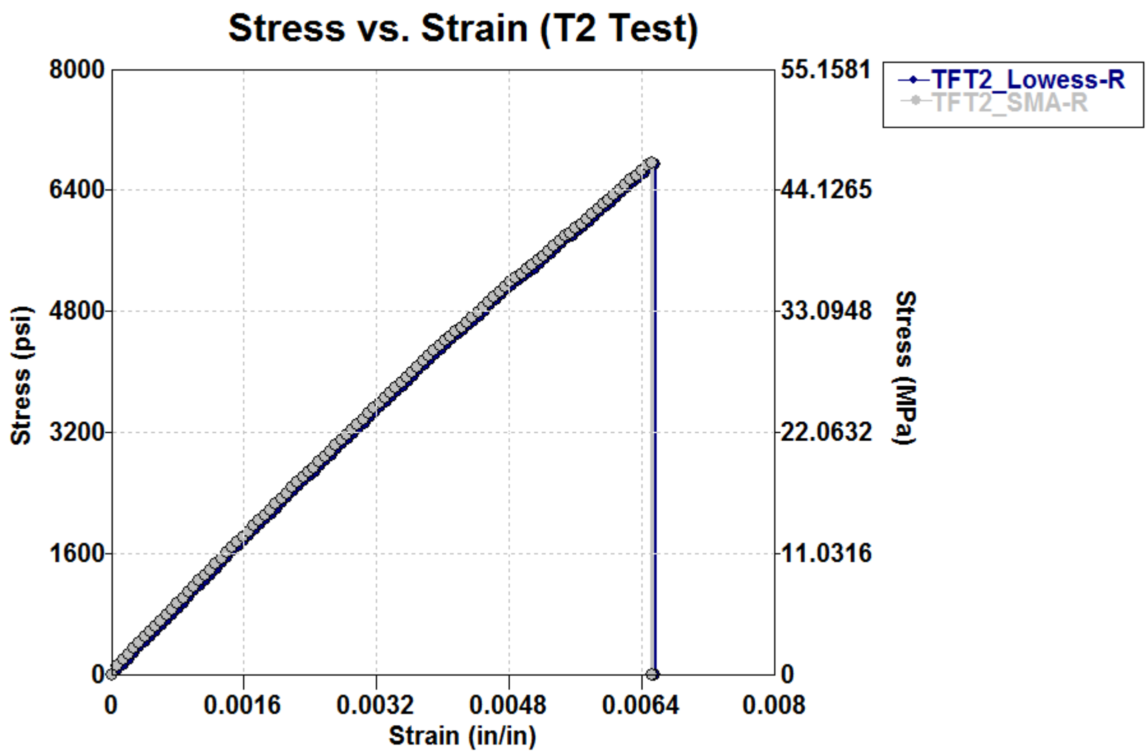


Figure 50. Final model curve for TFT2 generated with Lowess (blue) and SMA (grey) smoothing methods plotted in EDP

5 Conclusions

Composite materials are increasingly used in structural applications due to their high strength-to-weight ratio and durability. FE material models are actively being developed for modeling composites under load. FE models rely on characterization testing for input values including Young's modulus, Poisson's ratio, stress-strain curves, etc. Testing for unidirectional composites involves scenarios not seen in more commonly tested materials such as metals. Tabbing strategies have been developed though rely heavily on the characteristics of the adhesive used. Through the testing of three candidate adhesives, a comparison has made to determine the best of the candidate adhesives for use in the testing of composite materials. In addition, determining material characteristics depends on post-processing strategies. The development of the EDP software has allowed for a complete experience for generating some of the required inputs for the MAT213 constitutive model.

The results of the adhesives testing show that the 3M DP460 is the best performing adhesive of the candidate adhesives based on double-lap shear tests for tabbing unidirectional composites. The 3M epoxy achieved a shear strength of 3423.4 psi on the G10 fiberglass substrate, which is 1 ksi below the reported shear strength on an aluminum substrate. The Loctite Superglue has some useful functionality with 80% the strength of the 3M DP460 epoxy and a much quicker cure time. The JB KwikWeld proved inadequate for use with tabbing due to low workability and low apparent shear strength.

In the realm of processing, EDP has received a large number of features and improvements aimed at facilitating the post-processing of data for use with the MAT213 constitutive model. New locally weighted regression techniques have been implemented to both improve upon the smoothing capabilities of the software and help remove outliers from raw data. Additional supplementary functions have been implemented to allow for improved ease of use while processing. Using the new processes implemented in EDP, 11 of the characteristic curves for the T800 composite have been generated for eventual use in MAT213.

5.1 Future Work

The EDP currently lacks some supplementary functions for current functionality. A foundation for the student's t-test had been introduced previously but not fully implemented. The ability to manipulate raw data is currently limited without the introduction of potentially useful functions such as backward extrapolation. The program will require the ability to easily determine the slope of a line as well as the area under a generated curve.

The program also lacks major functionality for the post-processing of data for use in MAT213. A function to generate flow rule coefficients and create the material card based on data read into EDP would mean only one program is necessary for the processing of raw data for use in MAT 213.

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APPENDIX A

LOWESS EXAMPLE

Consider the set of data shown in Table 11.

Table 11. Lowess Example Data

x	**y**
0	5
5	9
10	6
15	25
20	300
25	15
30	27

Step 1. The size of the data set is s = 7.

Step 2. Applying a span of five (N = 5, u = 3).

$$d_i^{[5]} = \begin{cases} x(N)\text{-x}(i) & \text{if } i < u \\ \max\left(x(i) - x(i-u+1), x(i+u-1) - x(i)\right) & \text{if } u \le i \le (P-u+1) \\ x(i) - x(P-N+1) & \text{if } i > (P-u+1) \end{cases} \quad \begin{pmatrix} i < 3 \\ 3 \le i \le 5 \\ i > 5 \end{pmatrix}$$

Step 3. For i = 1 (x = 0)

Step 3a. $d_1^{[5]} = x(5) - x(1) = 20 - 0 = 20$

Step 3b. $x_{local}' = \begin{bmatrix} 0 & 5 & 10 & 15 & 20 \end{bmatrix}$

$$z_1 = \frac{\left|x_{local}(1) - x(1)\right|}{d_1} = 0,\ z_2 = \frac{\left|x_{local}(2) - x(1)\right|}{d_1} = 0.25,\ z_3 = 0.5,\ z_4 = 0.75,\ z_5 = 1$$

$$w_1 = \left(1 - z_1^{\ 3}\right)^3 = 1,\ w_2 = \left(1 - z_2^{\ 3}\right)^3 = 0.954,\ w_3 = 0.670,\ w_4 = 0.193,\ w_5 = 0$$

Step 3c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 0 \\ 1 & 5 \\ 1 & 10 \\ 1 & 15 \\ 1 & 20 \end{bmatrix},\ \underset{\sim}{y} = \begin{bmatrix} 5 \\ 9 \\ 6 \\ 25 \\ 300 \end{bmatrix},\ \underline{w}_{half} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 0.954 & 0 & 0 & 0 \\ & & 0.670 & 0 & 0 \\ & sym & & 0.193 & 0 \\ & & & & 0 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\,\underline{x} = \begin{bmatrix} 1 & 0 \\ 0.977 & 4.883 \\ 0.819 & 8.185 \\ 0.440 & 6.594 \\ 0 & 0 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{\text{Á}}$.

$$Q^{\text{Á}} = \begin{bmatrix} -0.596 & -0.582 & -0.488 & -0.262 & 0 \\ 0.653 & 0.013 & -0.513 & -0.557 & 0 \\ 0.231 & -0.629 & 0.656 & -0.349 & 0 \\ 0.407 & -0.516 & -0.262 & 0.707 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{y}_{half} = Q^{\text{Á}} \underline{w}_{half} \underline{y} = \begin{bmatrix} -13.367 \\ -5.268 \\ -4.983 \\ 3.986 \\ 0 \end{bmatrix}$$

Step 3d. Calculate the local slope and intercept.

$$m = \frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} = \frac{-5.268}{-7.813} = 0.674 \qquad b = \frac{\left(\underline{y}_{half}\right)_{1,1} - R_{1,2} m}{R_{1,1}} = 4.525$$

Step 3e. $y_{1\,new} = 0.674(0) + 4.525 = 4.525$

Step 3f. $r_1 = 5 - 4.525 = 0.475$

Step 3. For i = 2 (x = 5)

Step 3a. $d_2^{[5]} = x(5) - x(2) = 20 - 5 = 15$

Step 3b. $x_{local}\,' = \begin{bmatrix} 0 & 5 & 10 & 15 & 20 \end{bmatrix}$

$$z_1 = \frac{\left|x_{local}(1) - x(2)\right|}{d_2} = 0.333,\; z_2 = \frac{\left|x_{local}(2) - x(2)\right|}{d_2} = 0,\; z_3 = 0.333,\; z_4 = 0.667,\; z_5 = 1$$

$$w_1 = \left(1 - z_1^{\;3}\right)^3 = 0.893,\; w_2 = \left(1 - z_2^{\;3}\right)^3 = 1,\; w_3 = 0.893,\; w_4 = 0.348,\; w_5 = 0$$

Step 3c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 0 \\ 1 & 5 \\ 1 & 10 \\ 1 & 15 \\ 1 & 20 \end{bmatrix},\; \underset{\sim}{\underline{y}} = \begin{bmatrix} 5 \\ 9 \\ 6 \\ 25 \\ 300 \end{bmatrix},\; \underline{w}_{half} = \begin{bmatrix} 0.893 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 0.893 & 0 & 0 \\ & sym & & 0.348 & 0 \\ & & & & 0 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half} \underline{x} = \begin{bmatrix} 0.945 & 0 \\ 1 & 5 \\ 0.945 & 9.450 \\ 0.590 & 8.855 \\ 0 & 0 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{\text{Á}}$

$$\underline{y}_{half} = Q^{Á}\underline{w}_{half}\,\underline{y} = \begin{bmatrix} -15.552 \\ -7.011 \\ -6.388 \\ 4.629 \\ 0 \end{bmatrix}$$

Step 3d. Calculate the local slope and intercept.

$$m = \frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} = \frac{-7.011}{-8.696} = 0.8063 \qquad b = \frac{\left(\underline{y}_{half}\right)_{1,1} - R_{1,2}m}{R_{1,1}} = 3.857$$

Step 3e. $y_{2\ new} = 0.806(5) + 3.857 = 7.888$

Step 3f. $r_2 = 9 - 7.888 = 1.112$

Step 3. For i = 3 (x = 10)

Step 3a. $d_3^{[3]} = \max\left(x(5) - x(3), x(3) - x(1)\right) = \max\left(20 - 10, 10 - 0\right) = 10$

Step 3b. $x_{local}\,' = \begin{bmatrix} 0 & 5 & 10 & 15 & 20 \end{bmatrix}$

$$z_1 = \frac{\left|x_{local}(1) - x(3)\right|}{d_3} = 1,\ z_2 = \frac{\left|x_{local}(2) - x(3)\right|}{d_1} = 0.5,\ z_3 = 0,\ z_4 = 0.5,\ z_5 = 1$$

$$w_1 = \left(1 - z_1^{\,3}\right)^3 = 0,\ w_2 = \left(1 - z_2^{\,3}\right)^3 = 0.670,\ w_3 = 1,\ w_4 = 0.670,\ w_5 = 0$$

Step 3c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 0 \\ 1 & 5 \\ 1 & 10 \\ 1 & 15 \\ 1 & 20 \end{bmatrix},\ \underset{\sim}{y} = \begin{bmatrix} 5 \\ 9 \\ 6 \\ 25 \\ 300 \end{bmatrix},\ \underline{w}_{half} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ & 0.670 & 0 & 0 & 0 \\ & & 1 & 0 & 0 \\ & sym & & 0.670 & 0 \\ & & & & 0 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\,\underline{x} = \begin{bmatrix} 0 & 0 \\ 0.819 & 4.092 \\ 1 & 10 \\ 0.819 & 12.277 \\ 0 & 0 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$ and R

$$\underline{y}_{half} = Q^{Á}\underline{w}_{half}\,\underline{y} = \begin{bmatrix} -18.813 \\ 9.260 \\ -6.299 \\ 5.442 \\ 0 \end{bmatrix}$$

Step 3d. Calculate the local slope and intercept.

$$m = \frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} = \frac{9.260}{5.788} = 1.600 \qquad b = \frac{\left(\underline{y}_{half}\right)_{1,1} - R_{1,2}m}{R_{1,1}} = -3.701$$

Step 3e. $y_{3\,new} = 1.600(10) - 3.701 = 12.299$

Step 3f. $r_3 = 6 - 12.299 = -6.299$

⋮

Step 3. For i = 7 (x = 30)

Step 3a. $d_7^{[5]} = x(7) - x(7-5+1) = 30 - 10 = 20$

Step 3b. $x_{local}' = [10 \quad 15 \quad 20 \quad 25 \quad 30]$

$$z_1 = \frac{|x_{local}(1) - x(7)|}{d_7} = 1,\ z_2 = \frac{|x(2) - x(7)|}{d_7} = 0.75,\ z_3 = 0.5,\ z_4 = 0.25,\ z_5 = 0$$

$$w_1 = \left(1 - z_1^{\ 3}\right)^3 = 0,\ w_2 = \left(1 - z_2^{\ 3}\right)^3 = 0.193,\ w_3 = 0.670,\ w_4 = 0.954,\ w_5 = 1$$

Step 3c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 10 \\ 1 & 15 \\ 1 & 20 \\ 1 & 25 \\ 1 & 30 \end{bmatrix},\ \underset{\sim}{y} = \begin{bmatrix} 6 \\ 25 \\ 300 \\ 15 \\ 27 \end{bmatrix},\ \underline{w}_{half} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ & 0.193 & 0 & 0 & 0 \\ & & 0.670 & 0 & 0 \\ & sym & & 0.954 & 0 \\ & & & & 1 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\,\underline{x} = \begin{bmatrix} 0 & 0 \\ 0.440 & 6.594 \\ 0.819 & 16.370 \\ 0.977 & 24.416 \\ 1 & 30 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$ and R

$$\underline{y}_{half} = Q^{Á}\,\underline{w}_{half}\,\underline{y} = \begin{bmatrix} -147.233 \\ -114.357 \\ 145.135 \\ -70.328 \\ -24.338 \end{bmatrix}$$

Step 3d. Calculate the local slope and intercept.

$$m = \frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} = \frac{-114.357}{7.813} = -14.637 \quad b = \frac{\left(\underline{y}_{half}\right)_{1,1} - R_{1,2}m}{R_{1,1}} = 452.177$$

Step 3e. $y_{7\,new} = -14.637(30) + 452.177 = 13.074$

Step 3f. $r_7 = 27 - 13.074 = 13.926$

The smoothed data with each residual is shown in Table 12.

Table 12. Smoothed Lowess example data and residual data.

x	y
0	4.525
5	7.888
10	12.299
15	98.296
20	139.666
25	86.6836
30	13.074

r
0.475
1.112
-6.299
-73.295
160.334
-71.684
13.926

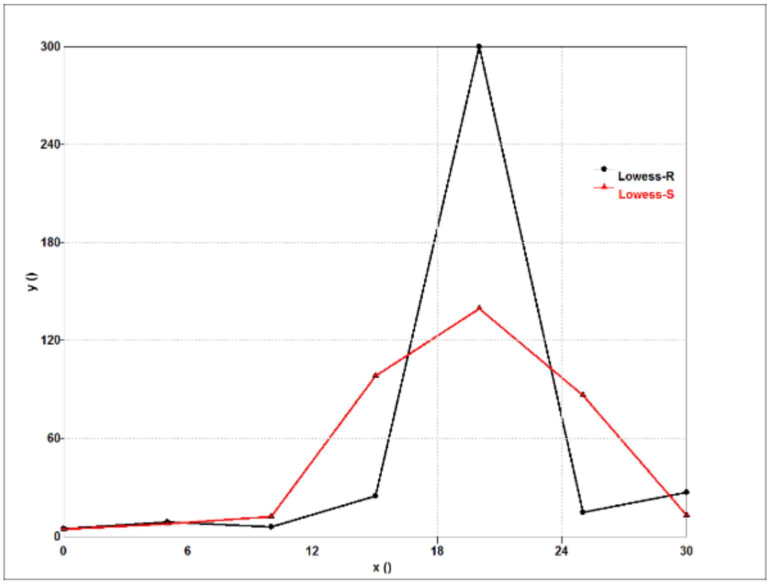

Figure 51. Lowess example data and smoothed data plots.

When visualizing the data in Table 12 as shown in Figure 53, it should be noted that the outlier corresponding to y=300 has been reduced but not been eliminated, and the surrounding data points have been skewed towards the outlier. Also note that following the Loess algorithm, this outlier would have a more extreme effect on the surrounding data, as shown in Figure 52. Therefore, Loess is only recommended for sets of data with more points and a higher number of local points.

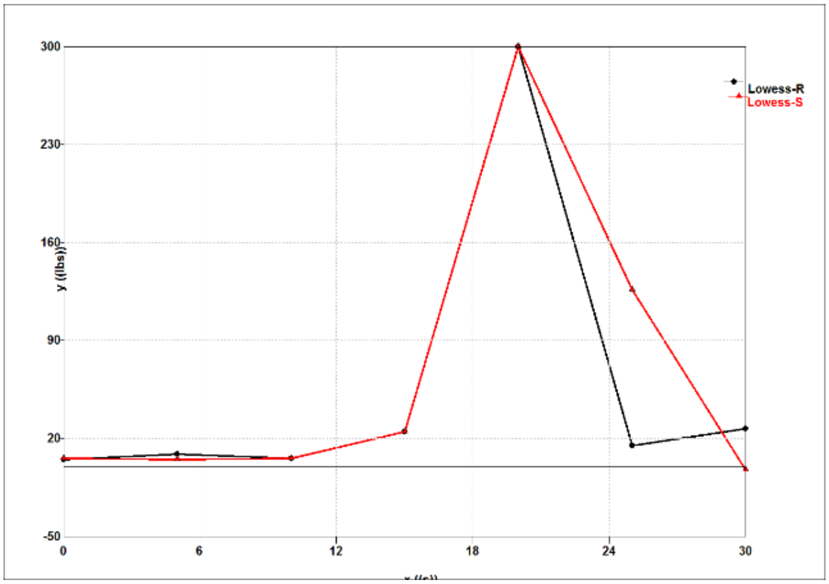

Figure 52. Example data with smoothed data generated with Loess method.

APPENDIX B

ROBUST LOWESS EXAMPLE

Consider the set of data shown in Table 13.

Table 13. Robust Lowess example data. The original data and corresponding residuals from a single iteration of Lowess smoothing are shown.

x	**y**
0	5
5	9
10	6
15	25
20	300
25	15
30	27

r
0.475
1.112
-6.299
-73.295
160.334
-71.684
13.926

These steps continue from the process detailed in Appendix A.

Step 4. Determining *MAD* = 13.9256
Calculate weights based on residuals.

$$w_{ri}\left(r_i\right)=\begin{cases}\left(1-\dfrac{r_i}{83.5536}^{2}\right)^{2} & \left|r_i\right|<83.5536\\ 0 & \left|r_i\right|\geq 83.5536\end{cases}$$

$$w_{ri}=\begin{bmatrix}0.9999 & 0.9996 & 0.9887 & 0.0531 & 0 & 0.0697 & 0.9452\end{bmatrix}$$

Step 5. For *i* = 1 (*x* = 0)
Step 5a. Apply a span of three (*N* = 5, *u* = 3).
Because point 5 is within the local range and $w_{r5} = 0$, this point must be ignored when calculating weights for this section. Point 6 is used instead.

$$d_1^{[5]}=x\left(6\right)-x\left(1\right)=25-0=25$$

Step 5b. $x_{local}'=\begin{bmatrix}0 & 5 & 10 & 15 & 25\end{bmatrix}$

$$z_1=\frac{\left|x_{local}(1)-x(1)\right|}{d_1}=0\,,\; z_2=\frac{\left|x_{local}(2)-x(1)\right|}{d_1}=0.2\,,\; z_3=0.4\,,\; z_4=0.6\,,\; z_5=1$$

$$w_1=\left(1-{z_1}^3\right)^3 w_{r1}=0.9999\,,\; w_2=\left(1-{z_2}^3\right)^3 w_{r2}=0.9758\,,\; w_3=0.8107\,,\; w_4=0.0256\,,\; w_5=0$$

Step 5c. Begin QR decomposition.

$$\underline{x}=\begin{bmatrix}1 & 0\\ 1 & 5\\ 1 & 10\\ 1 & 15\\ 1 & 20\end{bmatrix},\; \underset{\sim}{y}=\begin{bmatrix}5\\ 9\\ 6\\ 25\\ 15\end{bmatrix},\; \underline{w}_{half}=\begin{bmatrix}0.9999 & 0 & 0 & 0 & 0\\ & 0.9758 & 0 & 0 & 0\\ & & 0.8107 & 0 & 0\\ & sym & & 0.0256 & 0\\ & & & & 0\end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\,\underline{x} = \begin{bmatrix} 1 & 0 \\ 0.988 & 4.939 \\ 0.900 & 9.004 \\ 0.160 & 2.400 \\ 0 & 0 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$.

$$Q^{Á} = \begin{bmatrix} -0.596 & -0.589 & -0.537 & -0.095 & 0 \\ 0.689 & 0.035 & -0.684 & -0.237 & 0 \\ 0.371 & -0.779 & 0.474 & -0.176 & 0 \\ 0.1807 & -0.212 & -0.137 & 0.951 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{y}_{half} = Q^{Á}\,\underline{w}_{half}\,\underline{y} = \begin{bmatrix} -11.501 \\ -1.515 \\ -3.214 \\ 2.081 \\ 0 \end{bmatrix}$$

Step 5d. Calculate the slope and intercept for local regression.

$$m = \frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} = \frac{-1.515}{-6.903} = 0.219 \qquad b = \frac{\left(\underline{y}_{half}\right)_{1,1} - R_{1,2}m}{R_{1,1}} = 5.815$$

Step 5e. $y_{1\,new} = 0.219(0) + 5.815 = 5.815$

Step 5. For $i = 2$ ($x = 5$)

Step 5a. $d_2^{[5]} = x(5) - x(2) = 20 - 5 = 15$

Step 5b. $x_{local}{}' = \begin{bmatrix} 0 & 5 & 10 & 15 & 25 \end{bmatrix}$

$$z_1 = \frac{\left|x_{local}(1) - x(2)\right|}{d_2} = 0.25\,,\ z_2 = \frac{\left|x_{local}(2) - x(2)\right|}{d_2} = 0\,,\ z_3 = 0.25\,,\ z_4 = 0.5\,,\ z_5 = 1$$

$$w_1 = \left(1 - z_1^{\,3}\right)^3 w_{r1} = 0.9538\,,\ w_2 = \left(1 - z_2^{\,3}\right)^3 w_{r2} = 0.9996\,,\ w_3 = 0.9430\,,\ w_4 = 0.0356\,,\ w_5 = 0$$

Step 5c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 0 \\ 1 & 5 \\ 1 & 10 \\ 1 & 15 \\ 1 & 25 \end{bmatrix},\ \underset{\sim}{y} = \begin{bmatrix} 5 \\ 9 \\ 6 \\ 25 \\ 15 \end{bmatrix},\ \underline{w}_{half} = \begin{bmatrix} 0.9538 & 0 & 0 & 0 & 0 \\ & 0.9996 & 0 & 0 & 0 \\ & & 0.9430 & 0 & 0 \\ & sym & & 0.0356 & 0 \\ & & & & 0 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\,\underline{x} = \begin{bmatrix} 0.977 & 0 \\ 0.9998 & 4.999 \\ 0.971 & 9.711 \\ 0.189 & 2.830 \\ 0 & 0 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$

$$\underline{y}_{half} = Q^{Á}\underline{w}_{half}\underline{y} = \begin{bmatrix} -11.863 \\ -1.576 \\ -3.428 \\ 2.457 \\ 0 \end{bmatrix}$$

Step 5d. Calculate the slope and intercept for local regression.

$$m = \frac{\left(\underline{y}_{half}\right)_{2,1}}{R_{2,2}} = \frac{-1.576}{-7.138} = 0.221 \qquad b = \frac{\left(\underline{y}_{half}\right)_{1,1} - R_{1,2}m}{R_{1,1}} = 5.801$$

Step 5e. $y_{2\ new} = 0.221(5) + 5.801 = 6.905$

Step 5. For i = 3 (x = 10)

Step 5a. $d_3^{[5]} = \max\left(x(6) - x(3), x(3) - x(1)\right) = \max\left(25 - 10, 10 - 0\right) = 15$

Step 5b. $x_{local}{}' = \begin{bmatrix} 0 & 5 & 10 & 15 & 25 \end{bmatrix}$

$$z_1 = \frac{|x_{local}(1) - x(3)|}{d_3} = 0.667,\ z_2 = \frac{|x_{local}(2) - x(3)|}{d_1} = 0.333,\ z_3 = 0,\ z_4 = 0.333,\ z_5 = 1$$

$$w_1 = \left(1 - {z_1}^3\right)^3 w_{r1} = 0.349,\ w_2 = \left(1 - {z_2}^3\right)^3 w_{r2} = 0.893,\ w_3 = 0.989,\ w_4 = 0.047,\ w_5 = 0$$

Step 5c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 0 \\ 1 & 5 \\ 1 & 10 \\ 1 & 15 \\ 1 & 25 \end{bmatrix},\ \underset{\sim}{y} = \begin{bmatrix} 5 \\ 9 \\ 6 \\ 25 \\ 15 \end{bmatrix},\ \underline{w}_{half} = \begin{bmatrix} 0.349 & 0 & 0 & 0 & 0 \\ & 0.893 & 0 & 0 & 0 \\ & & 0.989 & 0 & 0 \\ & sym & & 0.047 & 0 \\ & & & & 0 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\underline{x} = \begin{bmatrix} 0.590 & 0 \\ 0.945 & 4.724 \\ 0.994 & 9.943 \\ 0.218 & 3.267 \\ 0 & 0 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$ and R

$$\underline{y}_{half} = Q^{Á}\underline{w}_{half}\underline{y} = \begin{bmatrix} -11.195 \\ -0.976 \\ -3.481 \\ 2.802 \\ 0 \end{bmatrix}$$

Step 5d. Calculate the slope and intercept for local regression.

$$m = \frac{(\underline{y}_{half})_{2,1}}{R_{2,2}} = \frac{-0.976}{-5.678} = 0.172 \qquad b = \frac{(\underline{y}_{half})_{1,1} - R_{1,2}m}{R_{1,1}} = 6.282$$

Step 5e. $y_{3\,new} = 0.172(10) + 6.282 = 8.001$

⋮

Step 5. For i = 7 (x = 30)

Step 5a. $d_7^{[5]} = x(7) - x(2) = 30 - 5 = 25$

$$x_{local}' = [5 \quad 10 \quad 15 \quad 25 \quad 30]$$

$$z_1 = \frac{|x_{local}(1) - x(7)|}{d_7} = 1,\ z_2 = \frac{|x(2) - x(7)|}{d_7} = 0.8,\ z_3 = 0.6,\ z_4 = 0.2,\ z_5 = 0$$

$$w_1 = (1 - z_1^{\,3})^3 = 0,\ w_2 = (1 - z_2^{\,3})^3 = 0.193,\ w_3 = 0.670,\ w_4 = 0.954,\ w_5 = 1$$

Step 5c. Begin QR decomposition.

$$\underline{x} = \begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 15 \\ 1 & 25 \\ 1 & 30 \end{bmatrix},\ \underset{\sim}{y} = \begin{bmatrix} 9 \\ 6 \\ 25 \\ 15 \\ 27 \end{bmatrix},\ \underline{w}_{half} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ & 0.115 & 0 & 0 & 0 \\ & & 0.026 & 0 & 0 \\ & sym & & 0.0658 & 0 \\ & & & & 0.945 \end{bmatrix}^{0.5}$$

$$\underline{x}_{half} = \underline{w}_{half}\,\underline{x} = \begin{bmatrix} 0 & 0 \\ 0.339 & 3.390 \\ 0.160 & 2.400 \\ 0.261 & 6.520 \\ 0.972 & 29.167 \end{bmatrix}$$

Perform QR decomposition on $\underline{x}_{half}$ and calculate $Q^{Á}$ and R

$$\underline{y}_{half} = Q^{Á}\,\underline{w}_{half}\,\underline{y} = \begin{bmatrix} -25.947 \\ 6.599 \\ 2.134 \\ -1.761 \\ 0.194 \end{bmatrix}$$

Step 5d. Calculate the slope and intercept for local regression.

$$m = \frac{(\underline{y}_{half})_{2,1}}{R_{2,2}} = \frac{6.5989}{6.746} = 0.978 \qquad b = \frac{(\underline{y}_{half})_{1,1} - R_{1,2}m}{R_{1,1}} = -2.628$$

Step 5e. $y_{7\,new} = 0.978(30) - 2.628 = 26.719$

The smoothed data is shown in Table 14.

Table 14. Smoothened Lowess example data.

x	y

0	5.8153
5	6.90535
10	8.00056
15	9.20848
20	16.7889
25	21.8438
30	26.7192

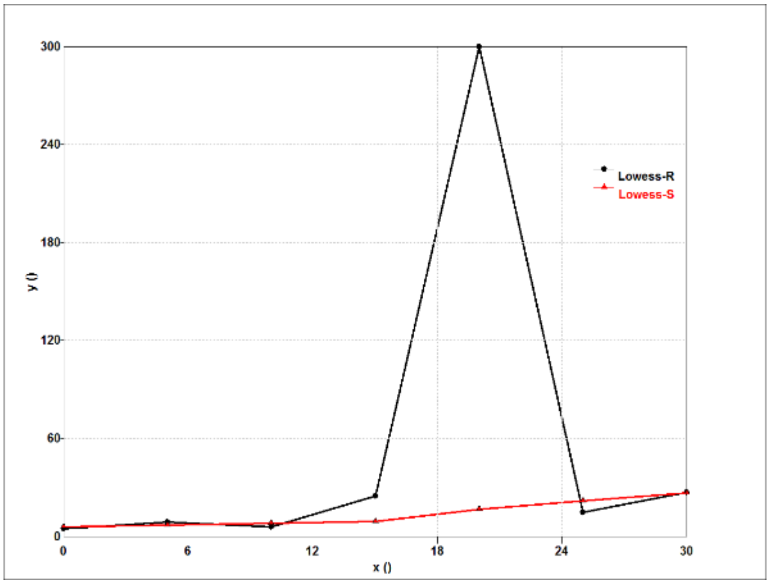

Figure 53. Robust Lowess example raw data and smoothened data plots

When visualizing the smoothed data in Table 14, as shown in Figure 51, note the outlier corresponding to y=300 has been smoothened. Also note that following the Robust Loess algorithm, this outlier would have a more extreme effect on the surrounding data, as shown in Figure 54. Therefore Robust Loess is only recommended for sets of data with more points and a higher number of local points.

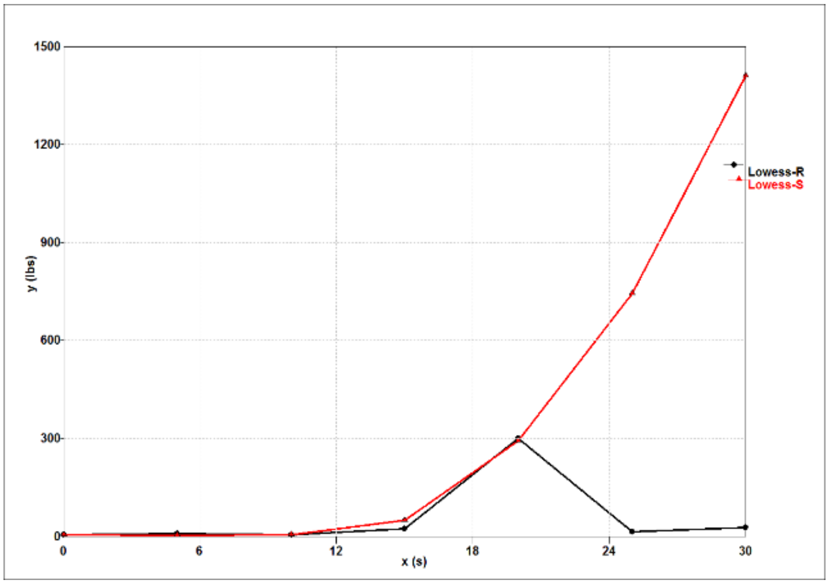

Figure 54. Smoothing example data with smoothed data from Robust Loess method.

APPENDIX C

EDP GENERATED MODEL CURVES FOR T800/F3900 COMPOSITE

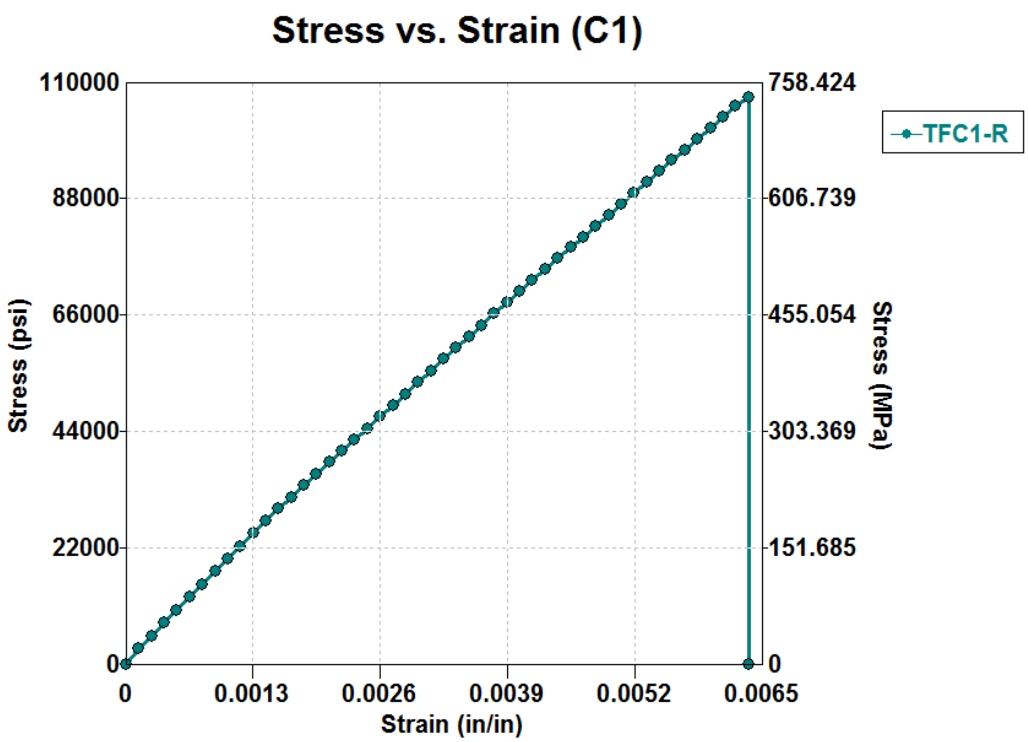

Figure 55. 1-Direction Compression Model Curve

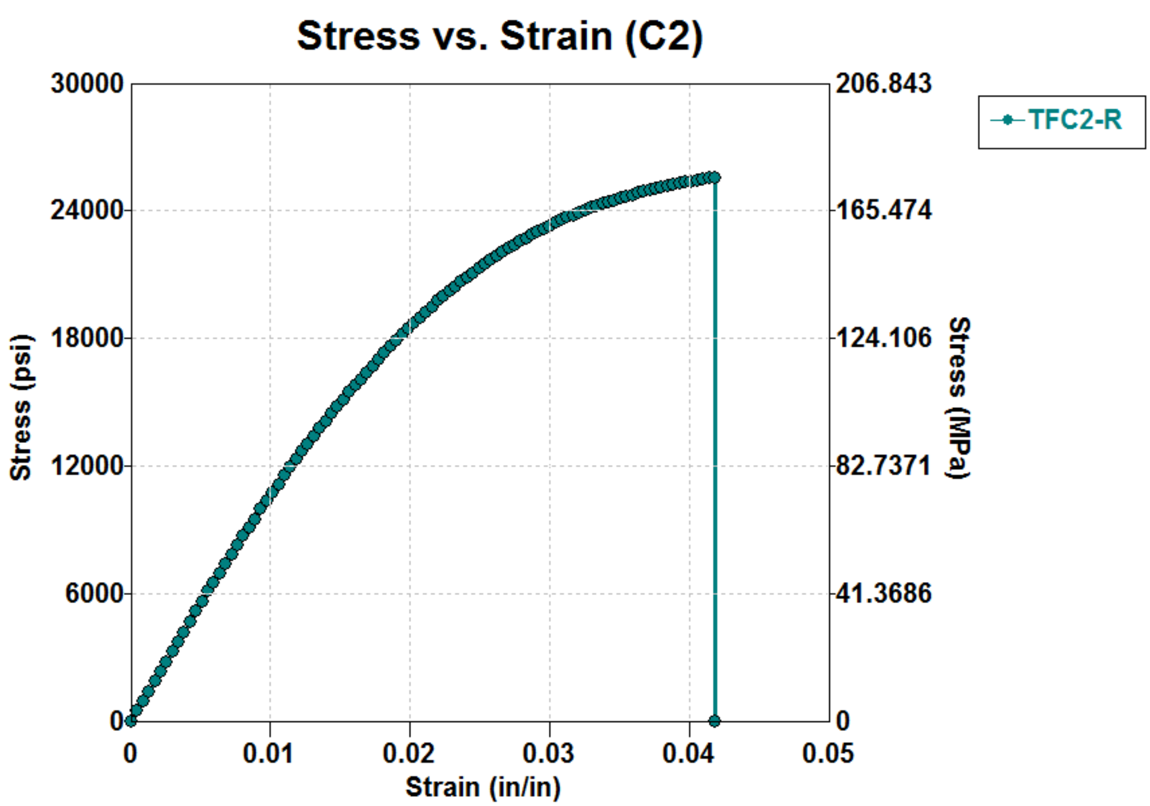

Figure 56. 2-Direction Compression Model Curve

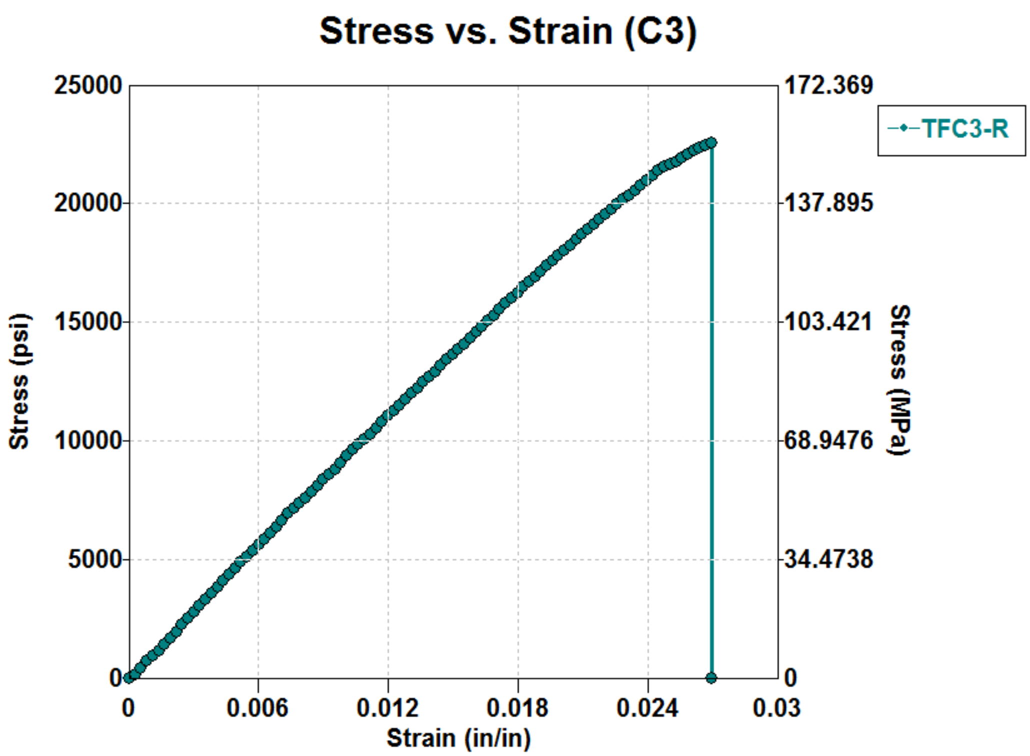

Figure 57. 3-Direction Compression Model Curve

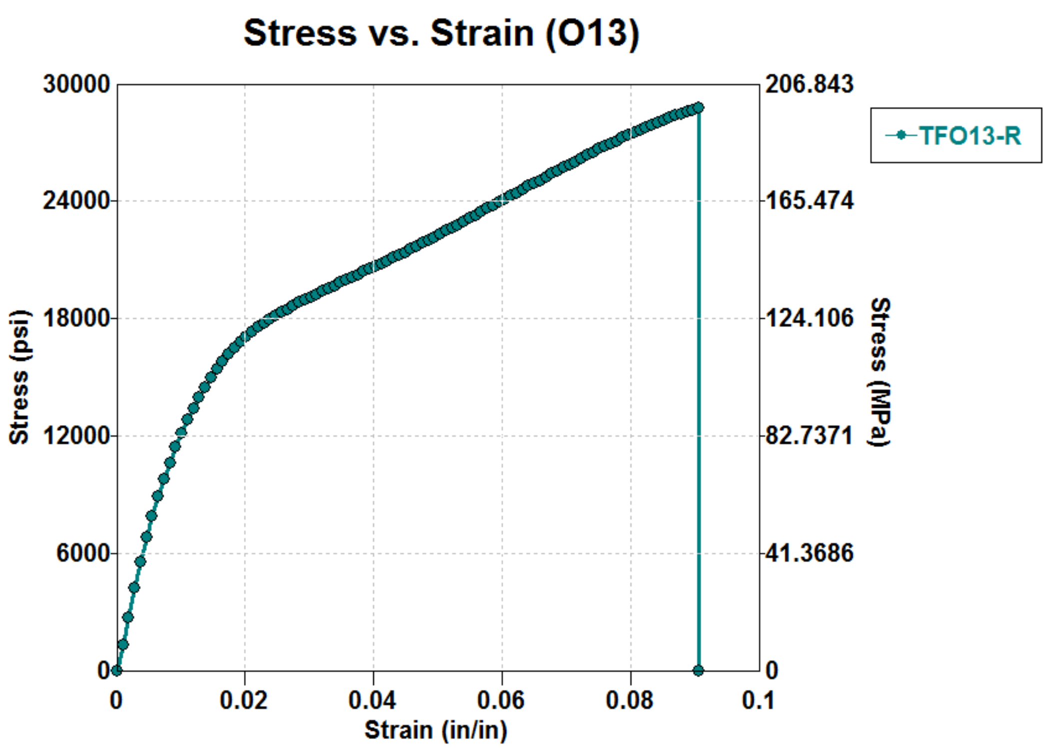

Figure 58. 1-3 Off-Axis Compression Model Curve

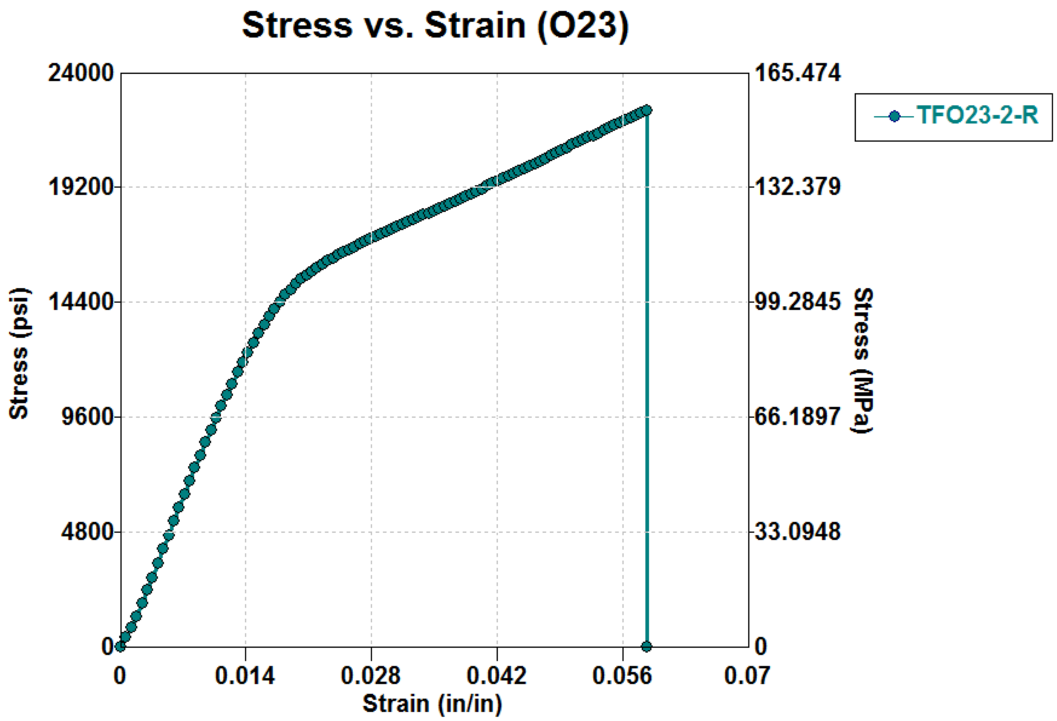

Figure 59: 2-3 Off-Axis Compression Model Curve

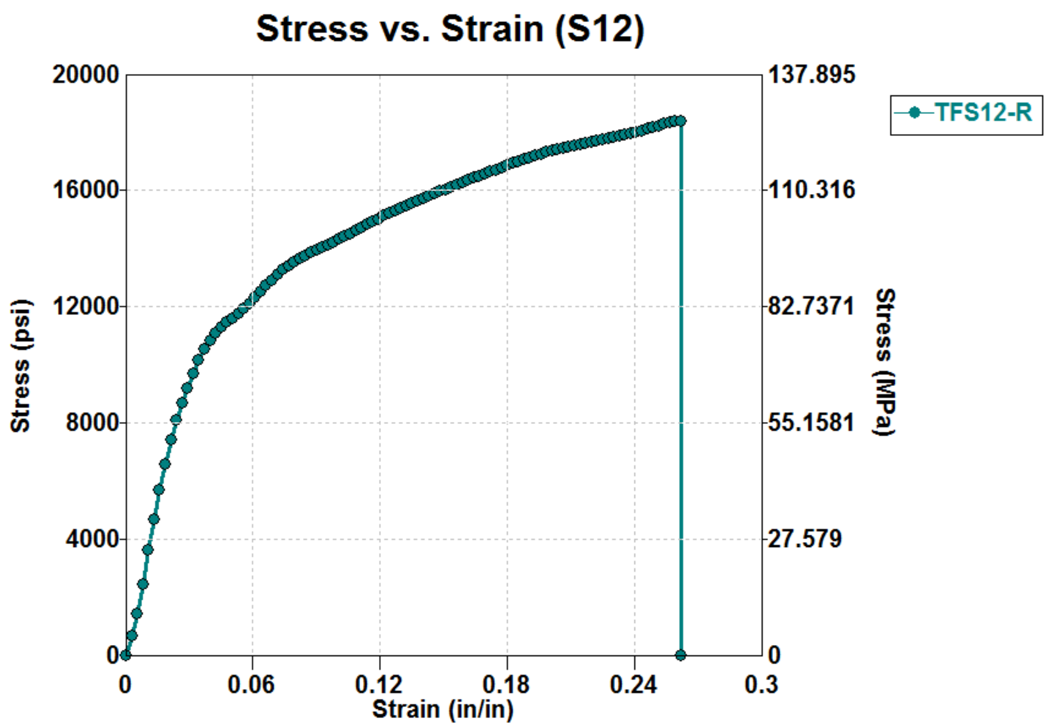

Figure 60. 1-2 Shear Model Curve

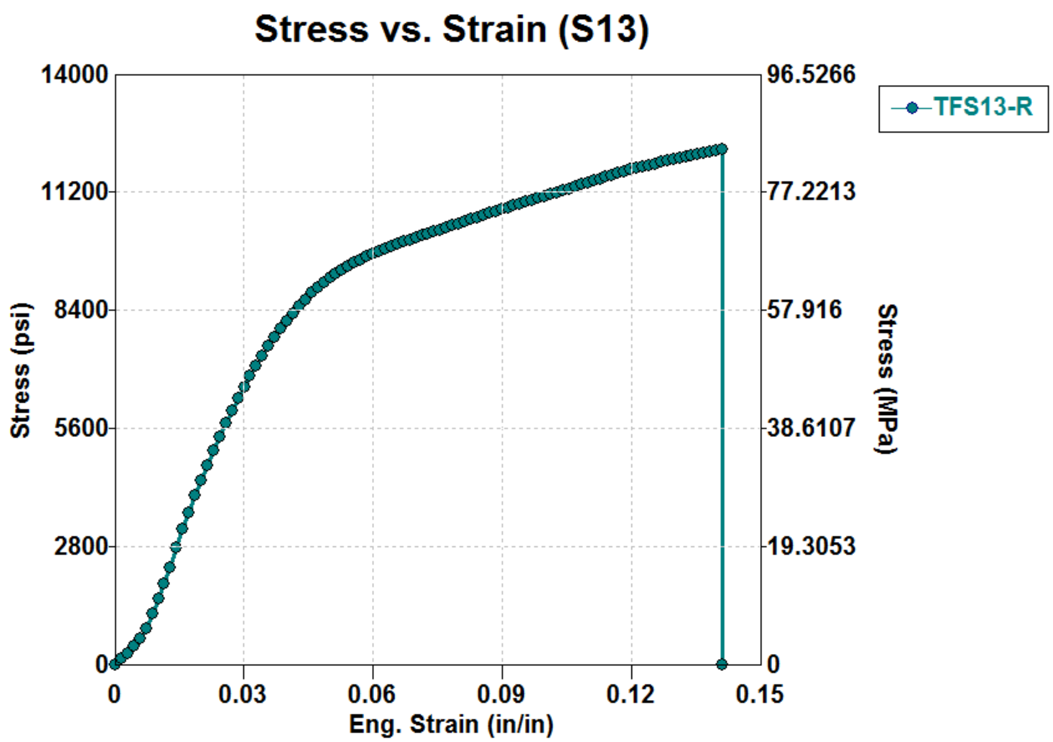

Figure 61. 1-3 Shear Model Curve

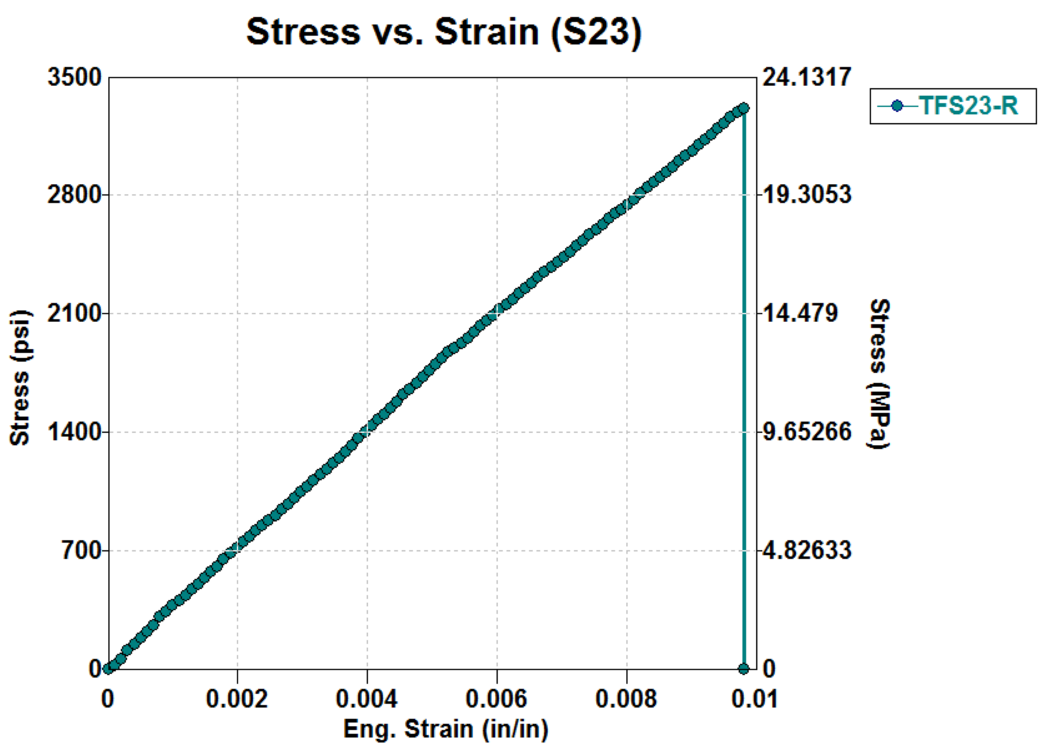

Figure 62. 2-3 Shear Model Curve

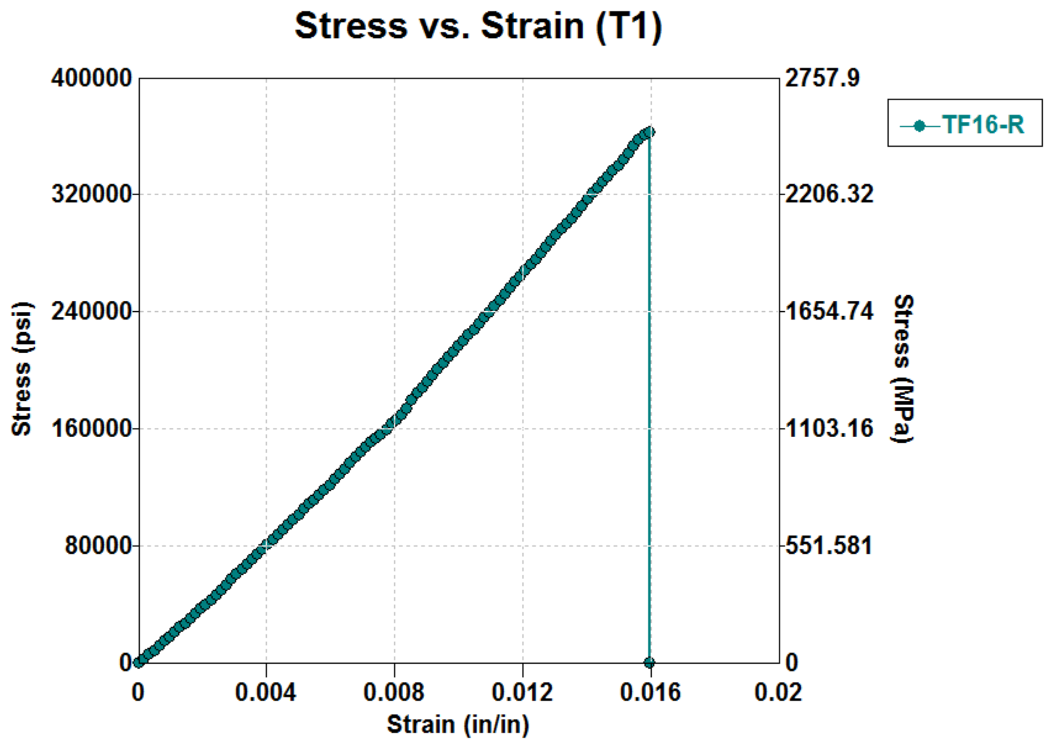

Figure 63. 1-Direction Tension Model Curve

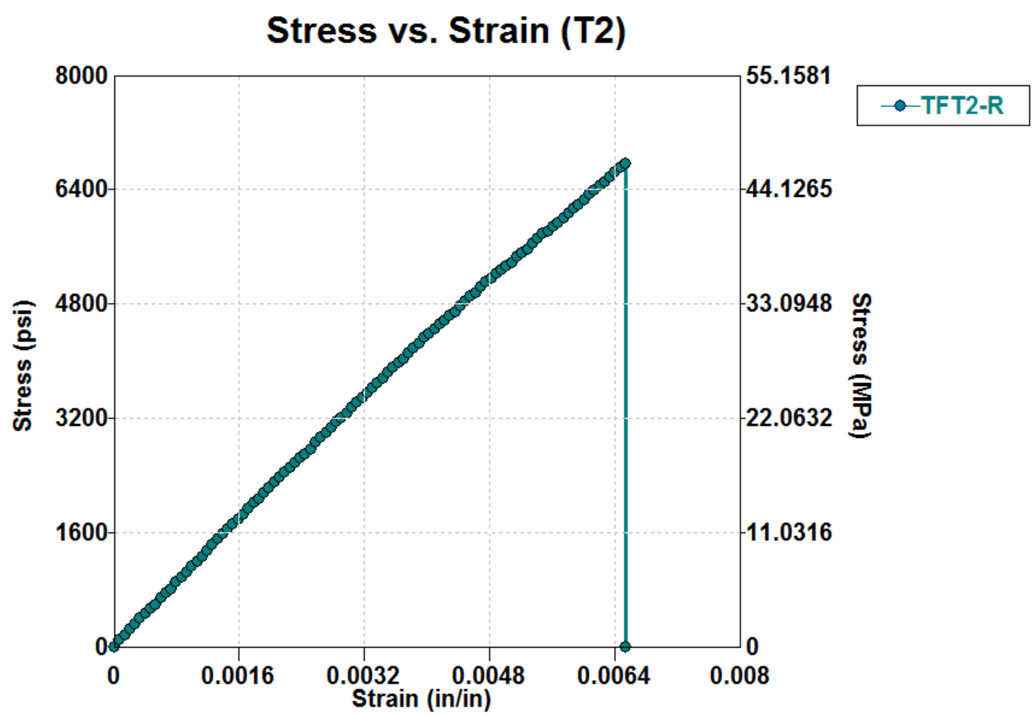

Figure 64. 2-Direction Tension Model Curve

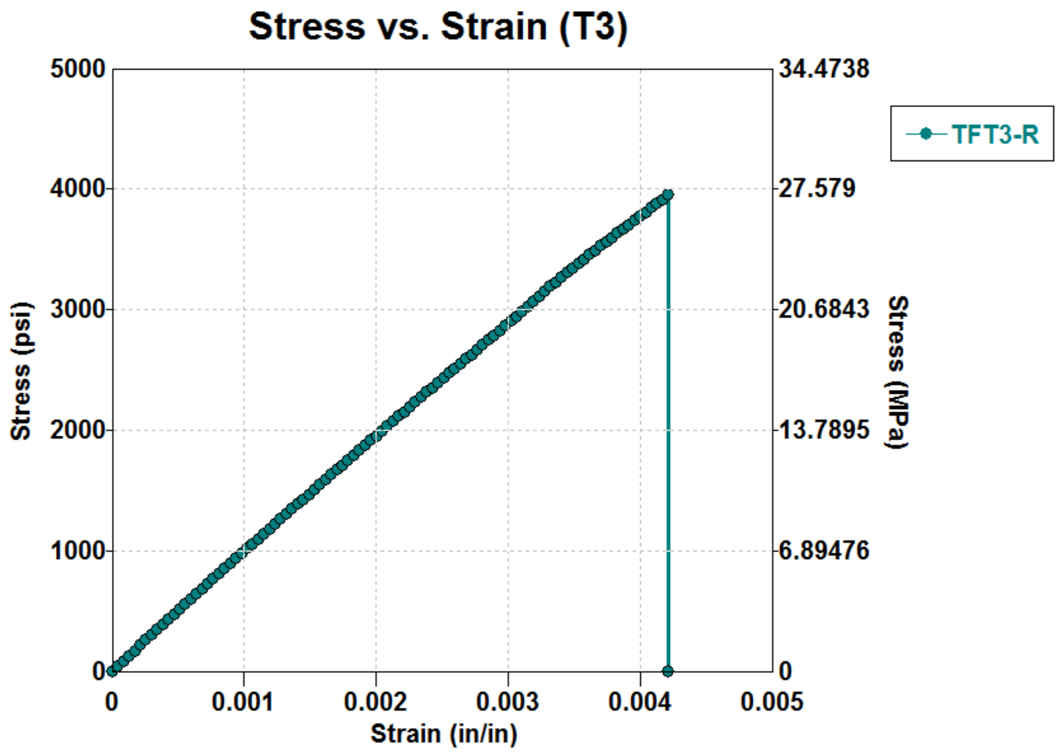

Figure 65. 3-Direction Tension Model Curve

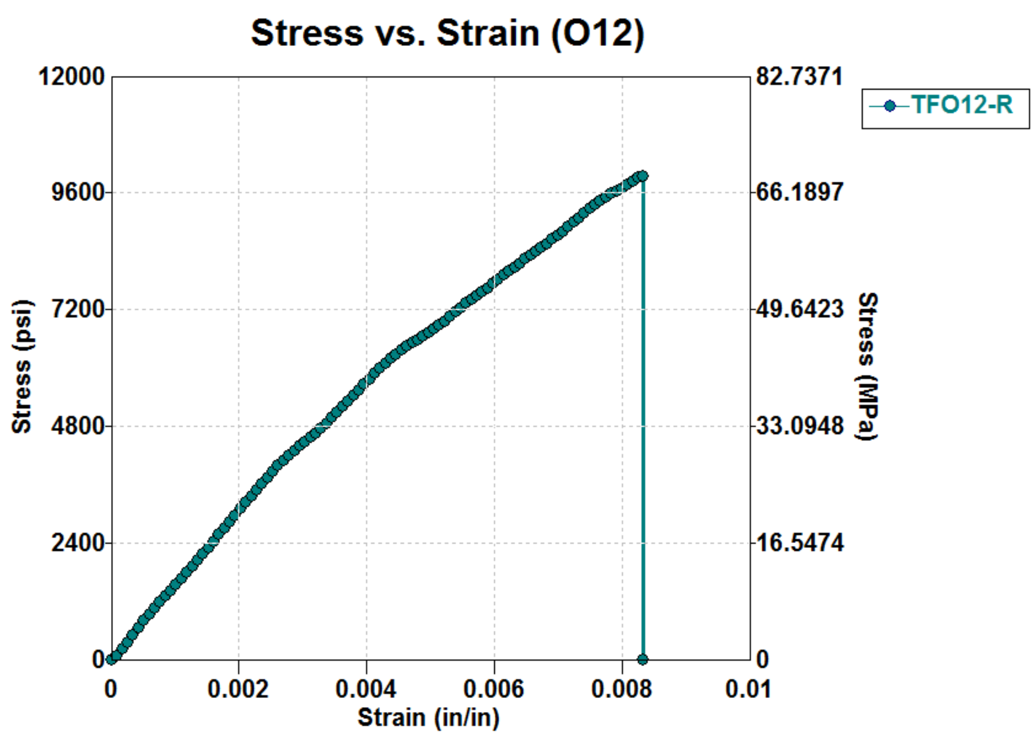

Figure 66. Off-Axis Tension Model Curve