A Probabilistic Cost to Benefit Assessment

of a

Next Generation Electric Power Distribution System

by

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ABSTRACT

This thesis provides a cost to benefit assessment of the proposed next generation distribution system, the Future Renewable Electric Energy Distribution Management (FREEDM) system. In this thesis, a probabilistic study is conducted to determine the payback period for an investment made in the FREEDM distribution system. The stochastic study will help in performing a detailed analysis in estimating the probability density function and statistics associated with the payback period.

This thesis also identifies several parameters associated with the FREEDM system, which are used in the cost benefit study to evaluate the investment and several direct and indirect benefits. Different topologies are selected to represent the FREEDM test bed. Considering the cost of high speed fault isolation devices, the topology design is selected based on the minimum number of fault isolation devices constrained by enhanced reliability. A case study is also performed to assess the economic impact of energy storage devices in the solid state transformers so that the fault isolation devices may be replaced by conventional circuit breakers.

A reliability study is conducted on the FREEDM distribution system to examine the customer centric reliability index, System Average Interruption Frequency Index (SAIFI). It is observed that the SAIFI was close to 0.125 for the FREEDM distribution system. In addition, a comparison study is performed based on the SAIFI for a representative U.S. distribution system and the FREEDM distribution system.

The payback period is also determined by adopting a theoretical approach and the results are compared with the Monte Carlo simulation outcomes to understand the variation

in the payback period. It is observed that the payback period is close to 60 years but if an annual rebate is considered, the payback period reduces to 20 years. This shows that the FREEDM system has a significant potential which cannot be overlooked. Several direct and indirect benefits arising from the FREEDM system have also been discussed in this thesis.

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NOMENCLATURE

AC	Alternating current
ASAI	Average Service Reliability Index
B	Annual benefit
$B_{n\text{-}years}$	Benefit after *n* years due to rate of interest
B_o	Benefit factor
B_r	Benefit due to reliability
CAIDI	Customer Average Interruption Duration Index
CBEMA	Computer and Business Equipment Manufacturer's Association
C_{cap}	Capacitance of the energy storage capacitor in SST
C_f	Annual cost of the FREEDM system
C_{fid}	Annual cost of a fault isolation device
C_{fidnet}	Annual cost of a group of FIDs in the FREEDM system
C_{sst}	Annual cost of the solid state transformer
DC	Direct Current
DGI	Distribution Grid Intelligence
DRER	Distributed Renewable Energy Resource
DSED	Distributed Energy Storage Device
E	Total energy required by the load over a year
E_{cap}	Energy stored in the capacitor
ECOT	Expected customer outage cost
EENS	Expected Energy Not Supplied

EIA	Energy Information Administration
$E_{not\text{-}ser}$	Energy not served by the FREEDM system
$E_{not-ser}^{nfid=0}$	Energy unserved with 0 FID on the system
E_{ser}	Energy served by the FREEDM system
EUE	Expected Unserved Energy
Ex	Expectation or mean of the samples
f(n)	Function of number of FIDs based upon the topology
FAIDI	Feeder Average Interruption Duration Index
FAIFI	Feeder Average Interruption Frequency Index
FID	Fault isolation device
FREEDM	Future Renewable Electric Energy Delivery and Management
$f_Y(Y)$	*PDF* of payback period
I	Investment
IEAR	Interrupted Energy Assessment Rate
IEEE	Institute of Electrical and Electronics Engineers
IEM	Intelligent Energy Management
IFM	Intelligent Fault Management
ITIC	Information Technology Industry Council
$I_{n\text{-}years}$	Investment after n years due to rate of interest
kVA	Kilo Volt Amperes
$Load_{system\text{-}peak}$	Peak system load in the FREEDM system
LOLE	Loss Of Load Expectation

LPCDF	Load Point Customer Damage Function
M	Number of customers on the FREEDM system
M_o	Annual maintenance
MP_{fid}	Market price of an FID
MP_{sst}	Market price of an SST
MWh	Megawatt hour
N	Normal distribution
$N_{customers}$	Number of customers lost due to an electrical outage in the FREEDM system
NERC	North American Electric Reliability Corporation
n_{fid}, N_{fid}	Number of FIDs
NSF	National Science Foundation
n_{sst}, N_{sst}	Number of SSTs
N_x	Optimal number of FIDs in the FREEDM system
N_y	Optimal number of SSTs in the FREEDM system
P_r	Probability of samples
PDF	Probability Density Function
ROI	Rate of Interest
RPS	Renewable Portfolio Standard
SAIDI	System Average Interruption Duration Index
SAIFI	System Average Interruption Frequency Index
SIC	Standardized Industrial Classification

SRA	Substation reliability models
SST	Solid State Transformer
SST_{rating}	kVA rating of a solid state transformer
T	Triangular distribution
T_{fid}	Service life of FID
T_{sst}	Service life of SST
U	Uniform distribution
V_1	Operating voltage of the SST
V_2	Residual voltage across the capacitor
Y	Payback period
μ	Mean
μ_B	Mean of annual benefit
μ_I	Mean of investment
μ_Y	Mean of payback period
¥	Ratio of standard deviation to mean
σ	Standard deviation
σ_B	Standard deviation of annual benefit
σ_I	Standard deviation of investment
σ_Y	Standard deviation of payback period

CHAPTER 1 THE FREEDM SYSTEM

1.1 Introduction

The Future Renewable Electric Energy Delivery and Management (FREEDM) center is a U.S. National Science Foundation (NSF) supported engineering research center. The level of funding is about 40 million dollars for 2008-2018. Reference [1] describes the FREEDM center. The FREEDM distribution system has been proposed as an innovative technology for the electric power industry. The aim of the FREEDM distribution system is to transform the present grid into a more reliable, secure and smart grid. The technology unites power electronics with information technology to offer a robust distribution system, which would have the capability to allow 100% autonomous operation on renewable technology [2] [3] [5]. By implementing this technology, efficient microgrid operation would be possible [2] [5]. Other features of the FREEDM system include perfect power quality, system operation at unity power factor, plug and play of energy storage devices and efficient communication infrastructure [2] [3].

The primary objective of this thesis is to provide a detailed assessment on the cost to benefit analysis for the FREEDM distribution system. The analysis would help in understanding the capital investment and the payback period for deployment of such a system in near future. In addition, the thesis aims at providing a probabilistic model to gauge the payback period for changes in parameters related to investment/capital cost.

1.2 The FREEDM system

According to the U.S Energy Information Administration (EIA) report for the year 2014, the energy produced from the renewable resources is close to 10% of total energy requirement in the country [4]. The energy production in the U.S is primarily based on non-renewable resources. The FREEDM center envisions a power system that emphasizes wide scale distributed renewable technology. Figure 1.1 depicts the concept of the FREEDM system where each residential load may act as an independent power producer and send the excess power back to the grid at competitive prices [5]. This figure is inspired from [5].

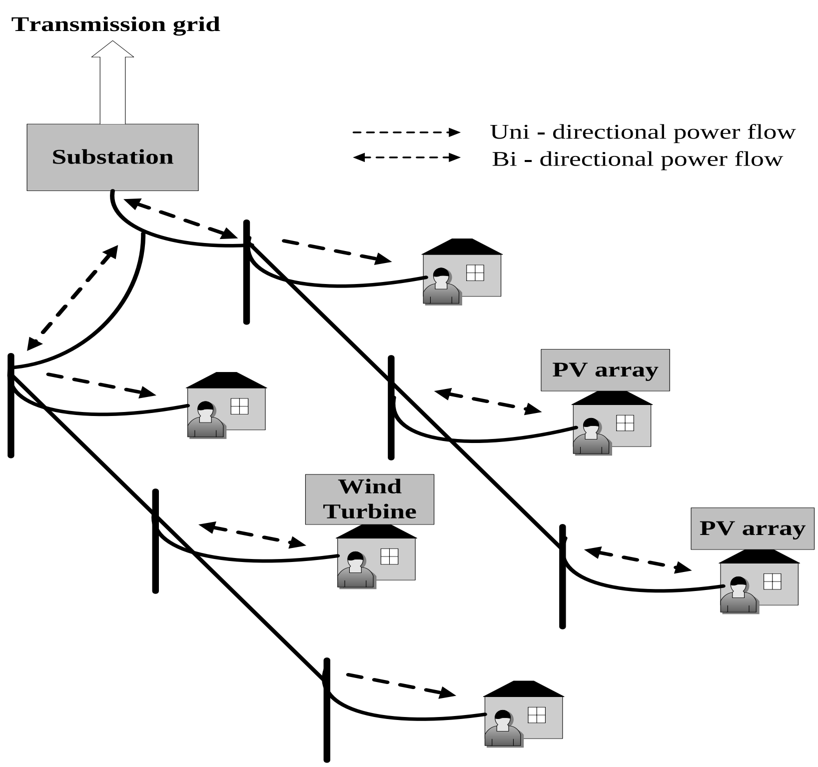

Figure 1.1 Envisioned FREEDM system

The FREEDM system primary components are the solid state transformers (SSTs) and the fault isolation devices (FIDs). These two components account for majority of the capital investment in the FREEDM distribution system [6]. The current power system is designed for delivering energy from larger centralized power stations to the load. For realizing the vision of the FREEDM system, a new infrastructure must be developed suited to two way power distribution and energy management [2] [3] [5]. The key aspects conceptualizing the idea of the future FREEDM distribution system is shown in Figure 1.2, which is inspired from [2] [5].

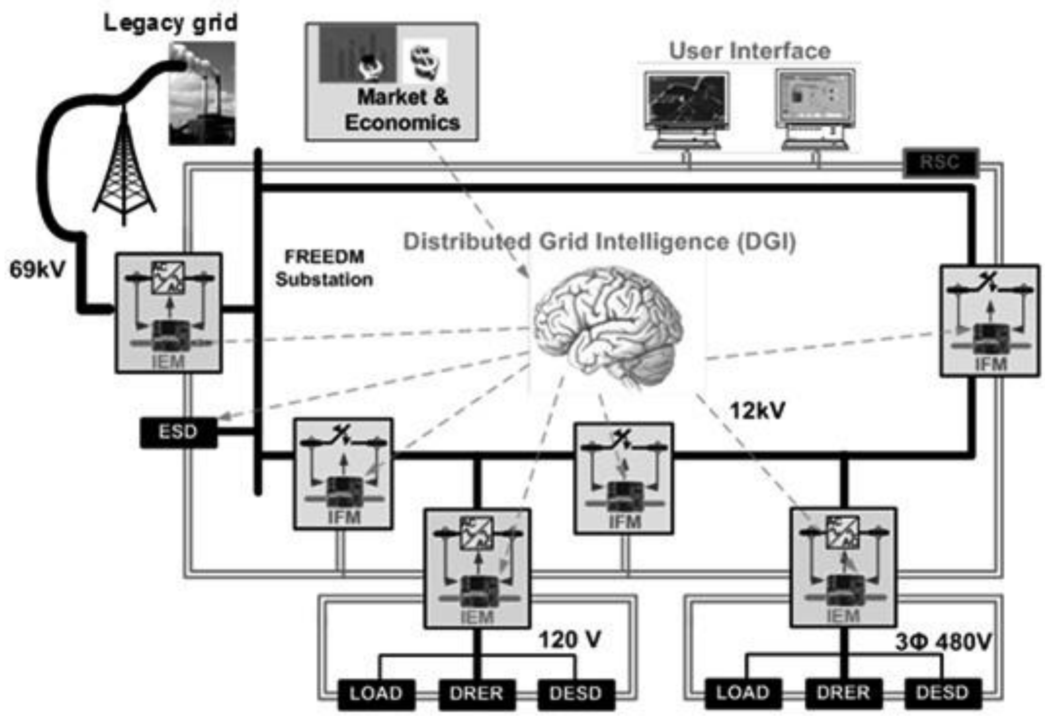

Figure 1.2 Conceptual FREEDM system [2, 5]

The Distributed Grid Intelligence (DGI) connects the entire system together. The primary function of the DGI is to provide a platform for efficient communication between multiple FREEDM hubs. In addition, the DGI must efficiently manage power flow within a certain FREEDM hub [7]. The Intelligent Energy Management (IEM) system is used to interface the solid state transformers with the DGI.

The principal objective of IEM is to ensure normal operation at the microgrid. If there is an excess production of power at the microgrid, the IEM system must allow bi directional power flow so that the excess energy may be delivered back to the grid [3] [5] [7]. On the other hand, the Intelligent Fault Management (IFM) system ensures that the system is safe during a fault. The FREEDM protection strategy is based on separation of the network into individual zones and monitoring of these individual zones for faults. Each zone sends digital signals to the IFM unit and the IFM unit takes decisions to trip the faulty sections from the healthy circuit based upon the signals [8].

1.3 The FREEDM distribution feeder

The basic configuration of the FREEDM system is shown in Figure 1.3, inspired from [9]. Two single-phase distribution level SSTs are connected to one selected phase of the 12.45 kV, three phase bus. The SST is a three-port network, which allows bi directional power flow control [2]. The FREEDM system is powered from a 69 kV sub transmission grid. The substation solid state transformer steps down the voltage to 12.45 kV, AC. The voltage at the primary distribution level is transformed to the secondary distribution voltage, which are 120 V, single phase AC and 380 V DC [9].

The primary feature of the FREEDM system is the plug and play interface at the AC and DC microgrid. Any load connected to the hub can be instantly recognized by the control center due to the open standard communication between the FREEDM hub and the control center which makes the FREEDM system more secure and efficient [2] [3] [5].

The communication network must promptly describe the load connected to the system and must even distinguish between the Distributed Energy Storage Device (DSED), load and Distributed Renewable Energy Resource (DRER) which will allow seamless plug and play of device in the FREEDM system [5].

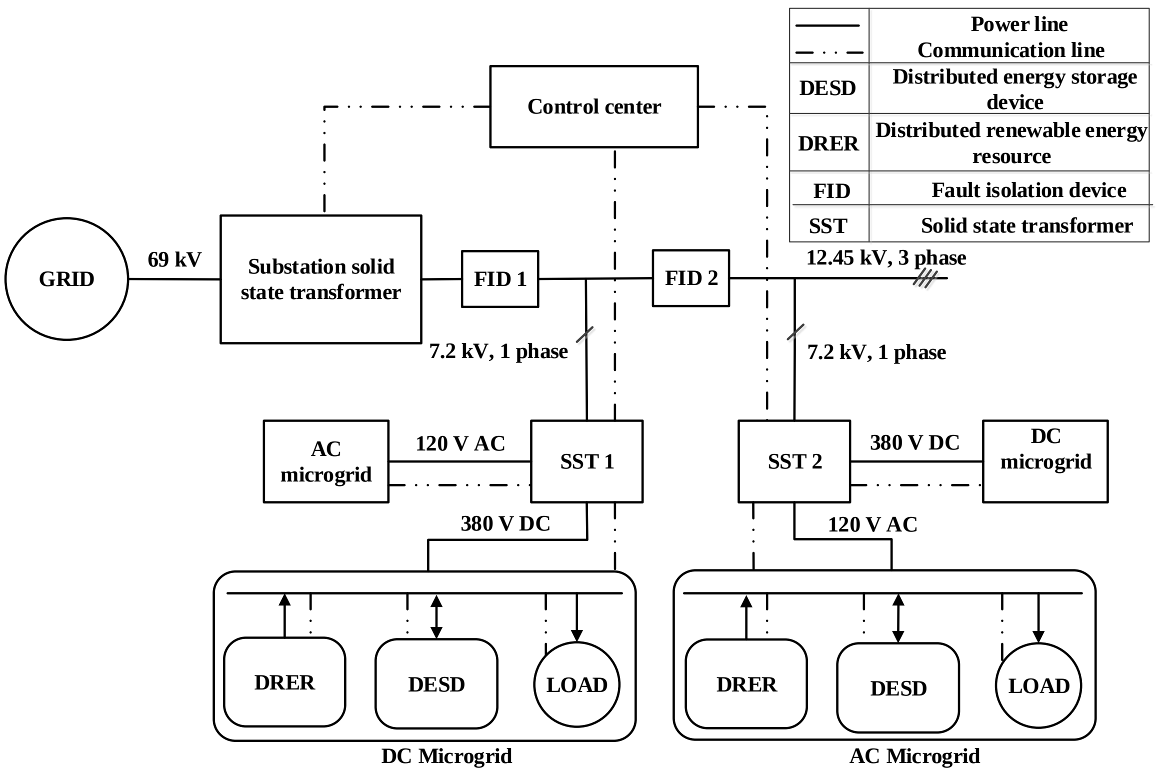

Figure 1.3 A basic FREEDM system [9]

1.4 FREEDM system components

The FREEDM system is a futuristic model encompassing several features, which the present distribution system lacks. These include improved security, better communication/control in the grid and higher renewable penetration capability [2] [3] [9]. The main components of the FREEDM system are the solid state transformers (SSTs) and the fault isolation devices (FIDs) [6] [9]. The solid state transformer is a digitally controlled converter, which has the prospect to replace the conventional magnetic transformer. It works on the principle of AC – AC conversion. The switching is performed at high frequency to reduce the size of the transformer [10]. The solid state transformer offers several significant advantages over the traditional transformer. Some of them are listed below,

- The solid state transformer allows bi directional AC or DC power flow.
- The size and weight of the solid state transformer is less in comparison to the magnetic transformers.
- It encourages integration of renewable technology and distributed energy storage devices.
- The power quality in the system is improved and operation is carried out at unity power factor [6].
- The solid state transformer has the capability to control the power flow by controlling the voltage magnitude and phase relationship between current and voltage [10] [11].

If a fault is detected in the system, the solid state transformers have the capability to disconnect the secondary end. This is impossible in case of conventional transformers. In addition, the solid state transformers have the control capability to limit the fault current [11].

During faults in the system, the circuit must be interrupted quickly so that the healthy system is isolated from the faulty section and normal operation is restored [10] [11]. The whole operation should comply with Information Technology Industry Council (ITIC) curve [12] [14]. According to the ITIC curve, an outage event of 100% low voltage is acceptable if the event does not last for more than 20 milliseconds in a 60 Hertz system as explained in Figure 1.4 [6] [12]. The figure is inspired from [14]. The ITIC curve is a successor of Computer and Business Equipment Manufacturer's Association (CBEMA).

Fault isolation devices (FIDs) or semiconductor based isolation devices interrupt the fault quickly (within a few microseconds) and the load is not be lost as per the CBEMA curve. The simulation results demonstrated in [12] [13] [14] show that for a three-phase short circuit, the impact lasts only for 100 microseconds and the circuit is tripped by the fault isolation device, thereby complying with the ITIC curve.

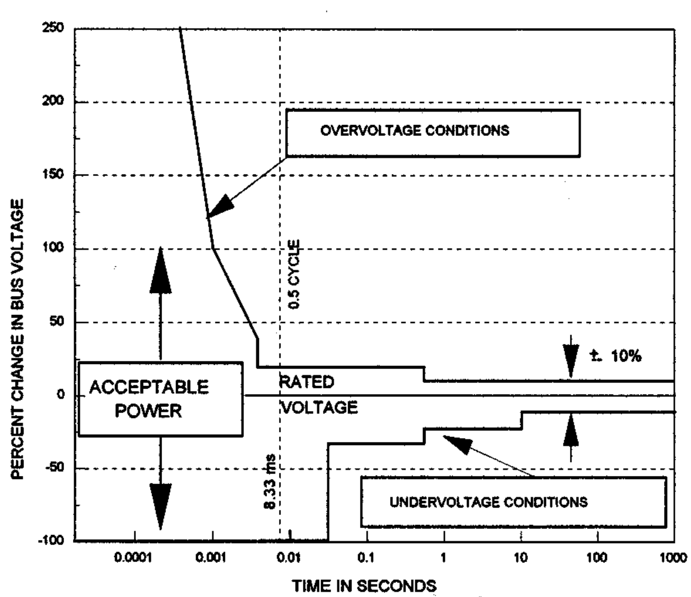

Figure 1.4 The ITIC curve [12] [14]

1.5 Cost to benefit analysis

According to [15], “The cost to benefit analysis is a systematic study to estimate the strengths and weaknesses of an alternative that satisfy the functional requirements of any business.” In other words, the cost to benefit analysis is a process by which business decisions of a firm can be influenced. A cost to benefit study is significant in understanding the capital investment and the payback period. From this study, the numerous benefits that can be obtained from the FREEDM distribution system are identified. The known benefits are quantified and annual profit is estimated to determine the time required to break even. Figure 1.5 explains the systematic process involved in the cost to benefit analysis.

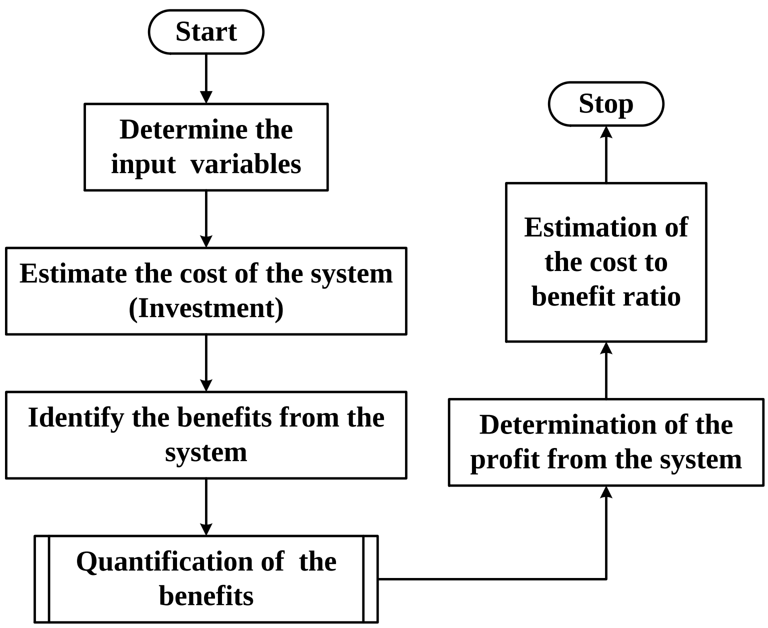

Figure 1.5 Flowchart for cost to benefit assessment

A cost benefit analysis provides a foundation in prioritizing decisions and understanding the trade-off between the desired results and investment made towards achieving those results. For example, after achieving a desired level of reliability in a distribution system, further investment might not guarantee significant improvement in the system reliability. Although, the system reliability does not significantly improve, the capital cost may increase substantially. The excess investment is unnecessary and thus it becomes obligatory to estimate the trade-off between the two [16]. One way of performing the cost to benefit analysis is by assessing the cost to benefit ratio as mentioned in [16]. In this method, the benefit is determined by considering the change in unserved energy. The product of unserved energy and the customer perceived cost of that unserved energy gives the net benefit of the system.

In [17] [18], another approach has been presented to evaluate the cost to benefit ratio. Here, the author has defined reliability based upon the number of customers interrupted in a distribution system during a fault. The cost to benefit ratio is estimated by gauging the improvement in the reliability of the system. The number of customers interrupted due to the failure of a component in the distribution feeder is estimated which provides the reliability indices (FAIDI and FAIFI). These indices are used to perform the cost to benefit study. By changing the feeder specifications such as addition of interrupters, fuses and fault indicators, the improved reliability index is calculated. The change in the index per unit cost incurred provides the cost to benefit ratio for the system.

Generally, when a cost to benefit study is performed, equilibrium is attained where the marginal benefit is equal to the marginal cost of producing the good. Figure 1.6 represents the equilibrium point where the marginal cost is equal to the marginal benefit for a system. This point represents a Pareto-optimal solution for the system. Figure 1.6 is inspired from [19].

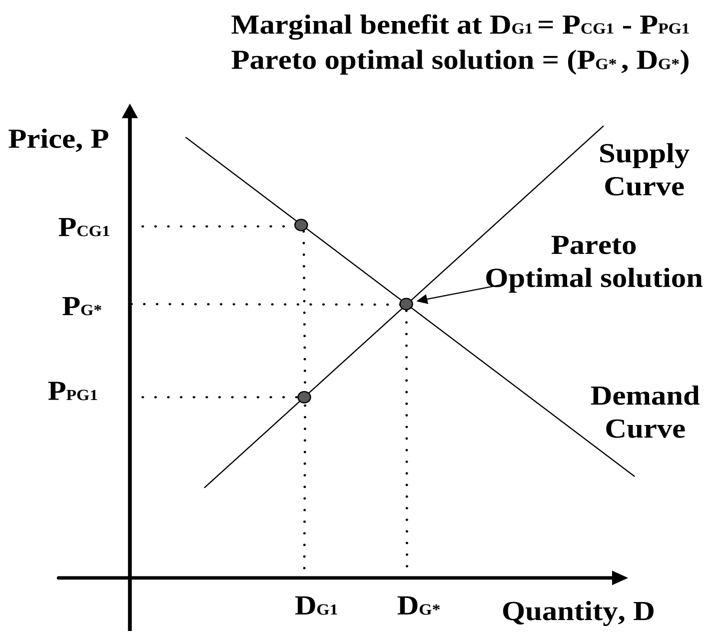

Figure 1.6 Market equilibrium model [19]

In [20], the cost to benefit ratio is estimated by quantifying the benefits. In this method, the benefits (increased expected energy served, reduction in network losses) are quantified and calculated. The investment cost associated to the distribution system (cost of network losses, penalty costs of unserved load points, and cost of service restoration) is also determined. The difference in the cost to benefit ratio between the old distribution system and the reconfigured distribution system is estimated. The difference in the cost to benefit ratio provides information regarding the improved distribution system performance and reliability.

All the methods discussed above fall under the category of 'comparison against a baseline'. In this approach, an alternative solution is suggested for a pre-existing problem. The investment cost and benefits are determined for an alternative solution, which are compared with the existing system. The difference in the investment cost and benefits reflect the economic impact of the alternative solution [21].

1.6 Probabilistic analysis

Probabilistic analysis is a discipline in engineering design, which is primarily used in the areas of quality assessment and reliability. The probabilistic approach models the effect of random variables on an engineering design. The variables involved in the problem are no longer thought as a single number; instead, a probability distribution is assumed [22]. In a cost to benefit assessment, the benefits are quantified and monetary values are identified for the benefits. Traditionally, the investment cost of an alternative solution and benefits accumulated from such a system is purely deterministic [16] [17] [18] [20]. A probabilistic model, on the other hand, assumes the system parameters to be stochastic. Randomness is modelled by a probability distribution function.

In [23], the author discusses a few probabilistic methods and tools for transmission planning, which can be used to perform reliability analysis and cost to benefit assessment. The literature lays emphasis on the following methods,

- Contingency enumeration
- Monte Carlo simulation.

Contingency enumeration follows a deterministic approach where the algorithm checks for the number of undesirable violations in the system (voltage violations, voltage collapse, and frequency violation) due to a fault. Outage statistics and impact on customers are accounted which gives the probability of occurrence of such events. Corrective measures are taken and the reliability index is calculated for understanding the cost of unreliability and customer damage function [23].

In [23], the author also uses the Monte Carlo simulation technique to compute power system indices such as Loss of Load Expectation (LOLE) and Expected Unserved Energy (EUE) for the system. The study is carried out by assuming probability distribution functions for the input parameters. The reliability indices are calculated based on the input functions. The process is carried out for several input samples to attain an acceptable amount of confidence over the means and variances of the statistical parameters involved in determining the reliability indices [23].

In [24], a method for reliability analysis in industrial distribution systems is presented. The study was performed using Monte Carlo simulation by analyzing the system behavior on occurrence of a fault followed by intervention of a circuit breaker. The Monte Carlo simulation generated failure events (samples) for the defined power system. The stochastic event (failure event) resulted in a change in the electrical topology. The voltages at the node points were captured subsequent to the event. After a fault was initiated, the circuit breaker would trip and a certain part of the network is lost. Depending upon the isolated section, the loss of load and the number of interruptions were estimated. The node voltages would be calculated for this new network and the customer interruptions would be assessed. The time of each interruption was approximated using Weibull, normal and uniform distribution through which the reliability index was estimated [24].

Another approach for reliability assessment using probabilistic model is stated in [25]. In this model, ‘contingency enumeration’ was implemented. The aim behind this literature is to develop reliability measures for an urban network. In [25], a reliability study is performed by estimating outages in substation and transmission facilities. Substation reliability models (SRA) were developed to study the power transfer capability for various usual/unusual cases. A contingency such as outage of any power system equipment is selected. A power flow study is run to evaluate the steady state conditions and abnormal situations (such as thermal overloads, over and under voltages). The model has an optimal power flow implementation through which it takes necessary corrective measures such as load shedding to restore the system. Additional contingencies are added sequentially and the reliability index is evaluated to obtain the Expected Unserved Energy (EUE) and Loss of Load Expectation (LOLE) [25].

The probabilistic method can also be adopted to determine certain parameters, which govern the economic aspects of an engineering problem. In general, the payback period for an investment may be defined as the ratio of the total investment made towards a project to the annual benefit incurred from the project [38]. If a probabilistic study is adopted, the PDF of the payback period is the ratio of two random variables, investment cost and annual benefit.

In [39 - 42], the author has presented a method to determine the PDF of a random variable, which is the ratio of two independent random variables where the two variables are normally distributed. In this study, it is assumed that the mean of the input random variables is sufficiently large than their respective standard deviation. In [41], the PDF of a slash distribution is also estimated. A slash distribution is the ratio of a normally distributed random variable to uniformly distributed random variable. By adopting the methods presented in [39 - 42], the PDF of the resultant output variable can be evaluated.

Reference [46] is a recently submitted paper on the subject of cost/benefit assessment for distribution systems with an application to the FREEDM distribution system. The literature discusses about the Monte Carlo simulation method and theoretic approach to determine the payback period probabilistically for the given FREEDM system

1.7 Cost of customer interruptions

Customer interruption cost may be defined as the average cost incurred by a customer due to an electrical outage [26]. The cost of an outage, in general, can be direct or indirect. Direct outage costs reflect towards the loss of production in a manufacturing firm, casualties in healthcare sectors and many other factors, which are directly influenced by the loss of electrical supply. On the other hand, an indirect cost may be associated to business relocation due to frequent outages [26] [27]. In [26] [27], the author presents several methods to quantify these impacts in monetary terms so that the true value of customer interruption cost may be estimated. A survey on the various methods used to determine the cost of an outage or customer interruption cost is presented in [27].

In direct costing method [27], a survey is performed to determine the net monetary loss due to an electrical interruption. It is relatively a straightforward approach where the customer outage cost is estimated based on the monetary loss incurred by the customer during the loss of supply. This method is valid and effective only when the loss is tangible, quantifiable and easily identifiable.

In case of a residential sector, this method may not be practical since it is difficult to discern the losses due to an outage [27]. An indirect costing method is more inclined towards the residential sector outages. The method is based on the principle of replacement of a good with an equivalent good, which is easily quantifiable and measure the worth of the original good. For example, the cost of an insurance policy which would alleviate the interruptions/outages at the customer end would give a measure of the amount of money the customer is willing to pay to obviate the interruption [27]. This amount would be equivalent to the loss incurred by the customers during an outage.

1.8 Indices to assess the reliability of a distribution system

The term reliability differs for different domains in the area of power systems. The definition of reliability may not be the same for a customer and a producer. According to NERC, reliability is defined based on two fundamental concepts, which are ‘adequacy’ and ‘operational reliability’ [36]. Reliability indices on the other hand, provide quantitative measure of the reliability of a system. These indices have been defined to monitor the duration and frequency of outages in a system. The reliability indices have been categorized in two domains namely, customer based indices and load based indices [34]. The definition of the various different reliability indices can be found from [33] [34].

In [34], different indices have been presented to determine the reliability of a distribution system. The paper presents a detailed information about the various terms associated with the reliability indices. The author has presented a survey report, which provides statistical information on the different reliability indices for the year 1990 and 1995. The author has also pointed out several pitfalls and shortcomings associated with the comparison of reliability indices between different utilities.

1.9 Organization of the thesis

Chapter 1 introduced the present challenges in the power industry. The chapter also explained the need to adopt a new infrastructure for better integration of renewable technology. The features and characteristics of an envisioned FREEDM distribution system were also discussed. This chapter also presented the relevant literature review related to the cost to benefit analysis, probabilistic study, reliability study and customer interruption cost.

Chapter 2 discusses the FREEDM distribution test bed and components used in the FREEDM system. In the chapter, the architecture of the three topologies is discussed. The FREEDM system parameters are defined which form the basis of the cost benefit assessment study. In addition, the chapter also presents a method to determine the optimal number of FIDs in a distribution system.

Chapter 3 discusses the reliability studies associated with the FREEDM system. The customer based reliability index called as the SAIFI is studied to understand the reliability of the FREEDM distribution system. A general expression is determined to evaluate the SAIFI for different topologies considered in the FREEDM cost benefit assessment study. In addition, the various direct and indirect benefits are discussed which may contribute towards making the FREEDM system more acceptable and advantageous in future.

Chapter 4 presents a probabilistic model for determination of the payback period and other associated statistical parameters. The cost of FIDs and SSTs cannot be precisely estimated since these components are still in the laboratory testing phase. A major portion of the FREEDM system investment is based on these two components. Due to this reason, any uncertainty in the cost of these components will result in an inaccurate assessment of the payback period. In other words, the cost to benefit analysis will be incorrect without proper knowledge of the cost of these components. This chapter presents a method to account for this problem by considering a stochastic model for the cost to benefit assessment of the FREEDM system.

Chapter 5 presents a theoretic approach to determine the probability density function for a random variable, which is a function of two variables. In this chapter, the expression of the probability density function for the payback period has been evaluated theoretically. In addition, the results from the system theoretic approach are compared with the Monte Carlo simulation results.

Chapter 6 summarizes the main conclusions of the research done on the cost to benefit assessment for the FREEDM distribution system. In addition, this chapter also discusses some of the future work for further advancement of the project. Appendices A through I support the analysis done in Chapter 2 through 5.

CHAPTER 2 FREEDM SYSTEM ARCHITECTURE AND INCLUSION OF ENERGY STORAGE

2.1 The FREEDM test bed

A cost to benefit analysis is required in the progression of a power engineering project to analyze the economic advantages and viability of the FREEDM system. The vision of the FREEDM center is to revamp the existing distribution system by introducing solid state components for better reliability and communication [2] [3]. The FREEDM approach replaces conventional distribution transformers with solid-state transformers and introduces fault isolation devices for quick interruption. Addition of such expensive components at the distribution level will significantly increase the cost of the distribution feeder. A cost to benefit study will help in determining the benefits amassed from deployment of such a system. In addition, this study would confirm whether the investment could be recovered within a reasonable period.

For the analysis of the FREEDM distribution system, a test distribution feeder is assumed with a rating of 1 MVA. In general, the distribution feeders are radial in nature and thus a fault on the system disconnects the entire load present on the feeder. However, noting the configuration in Figure 2.1, the FREEDM distribution feeder is energized at the two ends, i.e. the load may be served from either end. In a sense, Figure 2.1 is networked: Note, however that the concept of networked circuit refers to a set of buses that are joined by several interconnections that permit electrical service from several paths.

The conventional feeder can be transformed into the FREEDM feeder by incorporating the necessary components such as the solid-state transformers and the fault isolation devices. The FREEDM system uses a three phase 4 wire multi-grounded feeder, rated for 15 kV and operated at 12.47 kV [30]. The solid-state transformer used in the FREEDM system is a single phase 7.2/0.120 kV, 60 Hz transformer rated at 20 kVA [31]. The fault isolation is a single phase device rated at 15 kV, 200 A [28] [29].

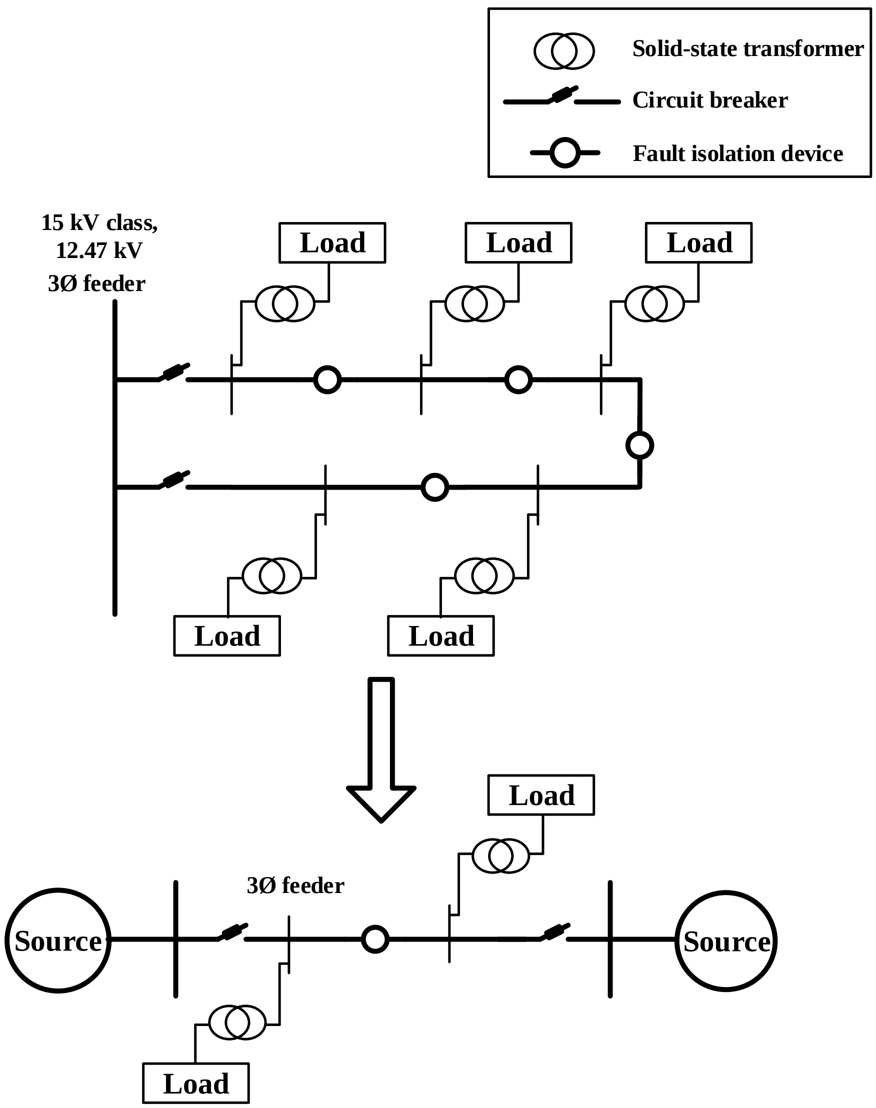

Figure 2.1 FREEDM test bed

2.2 Assumptions in the cost benefit study

The thesis aims at deriving the payback period of the FREEDM system. In the report, several assumptions are made due to lack of data and current technological limitations. The data presented in the thesis has been discussed with the FREEDM research team and other participating universities. The assumptions made in the thesis are discussed below,

Relating to FREEDM components

- The production cost of a single phase, 20 kVA, 7.2 kV solid-state transformer is assumed to be in the range of $15000 - 30000.
- The production cost of a single phase, 15kV, 200A fault isolation device is assumed to be in the range of $15000-$25000.
- The service life of a solid-state transformer is taken to be in the range of 15 – 20 years.
- The service life of a fault isolation device is assumed to be in the range of 15 – 20 years.

Relating to FREEDM feeder

- The FREEDM feeder is a 12.45 kV, 1 MVA feeder, which is powered from a 69 kV sub transmission grid.
- The FREEDM feeder is equivalent to a radial feeder with sources at both the ends of the feeder.

- The load is evenly distributed along the line unless specified otherwise.

Relating to FREEDM feeder load

- The load on the FREEDM feeder is purely residential for the study.
- The load is assumed non-diverse. This means that the residential loads connected to the FREEDM feeder have similar demand versus time characteristics.

2.3 FREEDM topology

The FREEDM test distribution feeder is a networked feeder with the same source at both ends. For the cost benefit study, different topologies were examined to understand the impact of the topological construction on the investment and accrued benefits. The goal is to select a topology which best reflects the FREEDM distribution test feeder and meets a high reliability standard at low investment cost. A detailed study was performed on different topologies out of which the following three topologies were selected for illustration:

i) Topology 1: One line feeder with same source at both ends

Topology 1 is the basic topology where a single feeder is assumed with the same source at both ends of the feeder. Circuit breakers are connected at both the ends of the feeder, which would isolate the FREEDM feeder from the external sub transmission grid (69 kV) in case of a fault. Figure 2.2 represents the arrangement of topology 1. During a fault in the feeder, the fault isolation devices are placed in between the load points to isolate a particular feeder section. A solid-state transformer would step down the voltage level from 7.2 kV (phase to ground) to 120 V (phase to ground), 60 Hz for residential loads at every load point. The feeder is assumed to be a three phase feeder.

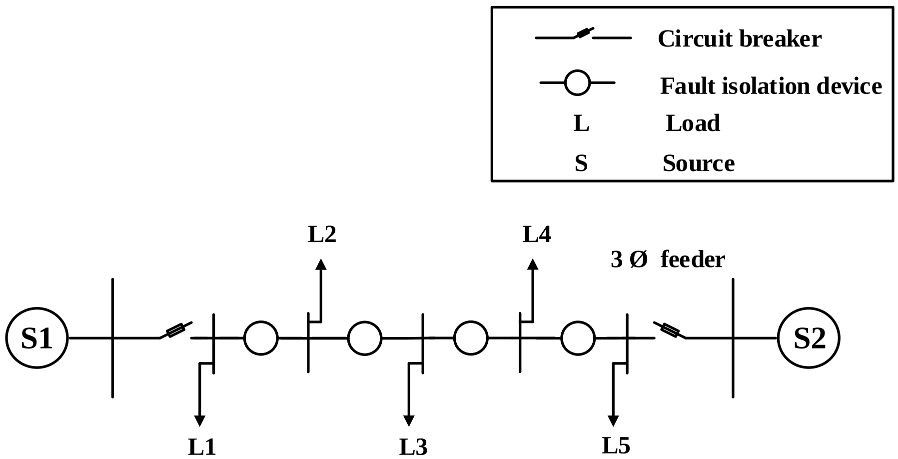

Figure 2.2 Architecture of topology 1

ii) Topology 2: Two line feeder with same source at both ends

Topology 2, with two parallel feeders connected to the same source at both the ends, is a more reliable topology when compared to topology 1. The reliability of the system has improved by introduction of two parallel feeders. Due to the presence of two parallel feeders, a fault on the line would interrupt only 50% of the customers when compared to topology 1. Figure 2.3 presents the arrangement of topology 2, where all the individual feeders are three phase feeders.

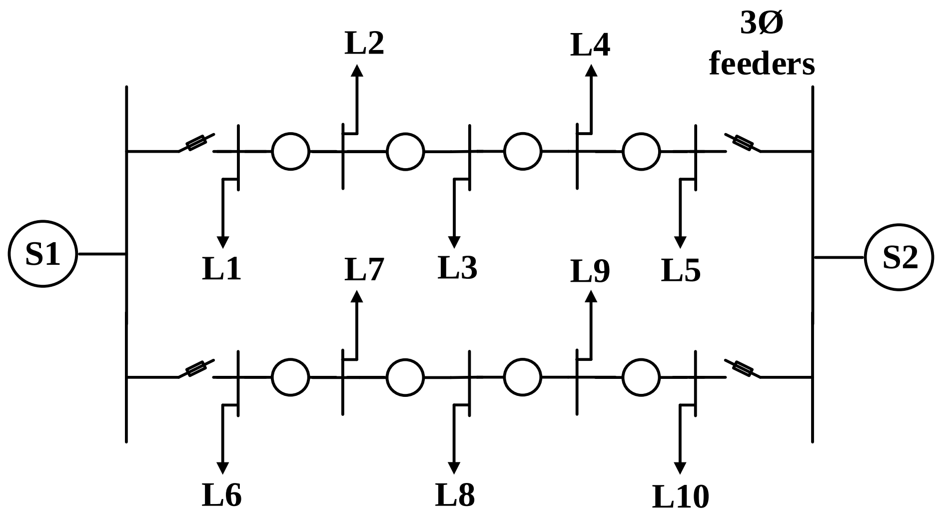

Figure 2.3 Architecture of topology 2

iii) Topology 3: Four line feeder with identical sources at the end

Topology 3 is the most reliable architecture with four parallel feeders connected to the same source at both the ends. The improved reliability is accounted by introduction of four parallel lines, which reduces the probability of a customer interruption by factor of four. Increase in the number of parallel paths reduces the load distribution on a particular feeder, thereby reducing the number of customer interruptions during a fault. As a result, the reliability index improves by a factor of four in this architecture. Figure 2.4 represents the architecture for topology 3 where all the individual feeders are three phase feeders.

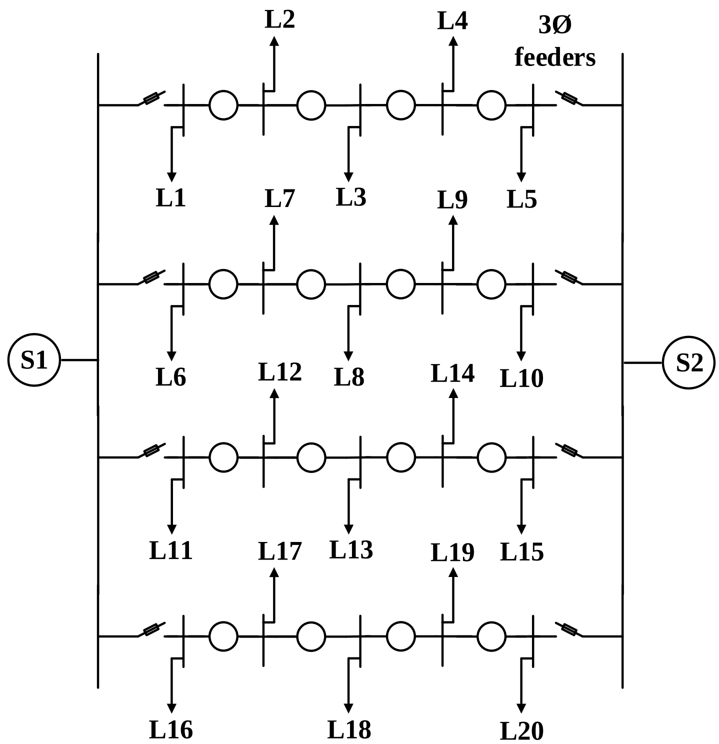

Figure 2.4 Architecture of topology 3

2.4 FREEDM system parameters

While performing the cost to benefit study, it becomes important to define several parameters mathematically, which directly or indirectly affect the investment cost and benefits of the FREEDM system. The aim is to understand the variation in cost and benefits with respect to these system parameters and adjust the parameters in such a way so that an optimal cost to benefit ratio can be achieved. The parameters are defined as listed below:

i) Annual cost

The *annual cost* of the FREEDM distribution system is defined as the combined cost of the solid state transformers and the fault interruption devices per unit operating life of these devices. The annual cost is denoted by C_f. The unit of annual cost is given in *\$/year* and mathematically represented as:

$$C_f = C_{fid} n_{fid} + C_{sst} n_{sst} ,$$

and,

$$C_{fid} = MP_{fid} / T_{fid} ,$$
$$C_{sst} = MP_{sst} / T_{sst} ,$$

where,

C_f is the annual cost of the FREEDM system,

C_{fid} is the annual cost of an FID,

C_{sst} is the annual cost an SST,

n_{fid} is the number of FIDs in the FREEDM distribution system,

n_{sst} is the number of SSTs in the FREEDM distribution system,

MP_{fid} is the market price of an FID,

MP_{sst} is the market price of an SST,

T_{fid} is the service life of an FID,

T_{sst} is the service life of an SST.

ii) Energy served

Energy served by the FREEDM distribution feeder is defined as the total energy supplied by the feeder to the residential units on an annual scale. The parameter is dependent on the number of FIDs, topology under consideration, number of interrupted customers and the fault location. With the change in the location of the fault, the number of customers interrupted may vary, which affects the *energy served* by the distribution feeder. Energy served by the distribution feeder is denoted by E_{ser} and the unit is given by *MWh/year*. The mathematical representation is given by:

$$E_{ser} = E\ f(n_{fid}),$$

where,

E is the total energy required by the load annually,

$f(n_{fid})$ is a function of number of FIDs in the distribution system depending upon the topology.

iii) Energy not served

Energy not served by the FREEDM distribution feeder is defined as the net energy that could not be served by the feeder to the residential units on an annual scale due to an outage in the system. With lower number of FIDs in the system, the reliability reduces which increases the number of customer interruptions during an outage. If the energy not served in a system is large, the overall system benefits reduce due to low reliability.

Attempts must be made to minimize *energy not served* in a distribution feeder. It is denoted by $E_{not\text{-}ser}$ and the unit is *MWh/year*. The mathematical expression is given by:

$$E_{not-ser} = E(1 - f(n_{fid})),$$

where,

E is the total energy required by the load annually,

$f(n_{fid})$ is a function of number of FIDs in the distribution system depending upon the topology.

iv) Benefit factor

The *benefit factor* represents the amount of money the customer is willing to pay to the utility to avoid one MWh of interruption. The term benefit factor is identified from [27] [29] as an indirect costing method. The benefit factor varies from one sector to another. For example, the benefit factor in a residential sector would be significantly lower when compared to a commercial sector. An outage for a small time period in a commercial sector might result in a huge economic loss. This loss is incomparable to a similar event in a residential sector. The benefit factor is denoted by B_o and the unit is *$/MWh*.

v) Benefit due to reliability

The term *benefit due to reliability* is defined as the annual profit accumulated due to an improvement in the reliability of the FREEDM distribution system. This term is coined specifically for determination of the number of FIDs that must be installed in the FREEDM system to meet the optimal investment cost and reliability requirements.

Benefit due to reliability must not be confused with the annual benefit accrued from the FREEDM system. Benefit due to reliability is a segment of the net annual benefit. There may be various other factors such as improved power quality and better integration with renewable technology, which can add towards the net benefit from the FREEDM system along with benefit due to reliability. Benefit due to reliability is denoted by B_r and the unit is *$/year*. The mathematical formulation is given by:

$$B_r = B_o(1 - f(n_{fid}))E_{not-ser}^{n_{fid}=0},$$

where,

B_o is the benefit factor,

$f(n_{fid})$ is a function of number of FIDs in the distribution system depending upon the topology,

$E_{not-ser}^{n_{fid}=0}$ is the energy not served by the distribution feeder when the number of FIDs in the distribution system is 0.

vi) Annual benefit

The *annual benefit* is defined as the sum of individual benefits that is obtained due to implementation of the FREEDM distribution system. These benefits may arise due to reduced customer interruptions, benefit due to improved reliability (B_r), improved power quality, reduced carbon emissions and enhanced penetration of renewable technology. The unit of annual benefit is *$/year* and is represented by *B*. The mathematical formulation is given by:

$$B = B_r + \textit{other benefits due to the FREEDM system}$$

vii) *Optimal number of FIDs*

Optimal number of FIDs represents the number of FIDs required to achieve a condition where the annual FID cost is equal to the benefit due to improved reliability. The intersection of the annual FID cost curve and the benefit due to reliability (*Br*) curve gives the optimal value of FIDs for the FREEDM distribution system.

The optimal number of FIDs is an indicator of the benefit factor. If the number of FIDs in a system is high, it infers that the benefit factor is high and vice versa. The optimal number of FIDs is denoted by N_x.

2.5 Determination of optimal number of FIDs

The determination of optimal number of FIDs is necessary in the cost to benefit assessment of the FREEDM system due to the high cost of an FID. The optimal number of FIDs is estimated from the FREEDM system parameters. This study was performed using MATLAB software and the code has been outlined in Appendix A. For determination of the optimal number of FIDs, the value of benefit factor, cost of an FID, service life of an FID and information regarding the topology must be available.

Table 2.1 gives the expression of optimal number of FIDs for three different topologies. The derivation of optimal number of FIDs has been outlined in Appendix B. Table 2.2 presents the details of the several input parameters used to perform the study. Figure 2.5 presents the simulation result for the three topologies. From Figure 2.5, it can be observed that the net FID annual cost increases linearly with the increase in the number of FIDs. The benefit due to reliability curve increases initially and then saturates with further increase in the number of FIDs for all the three topologies. As the number of FIDs in the distribution feeder increase, the reliability increases. After a certain point, a further increase in the number of FIDs does not improve the reliability of the system.

Table 2.1 Expression for optimal number of FIDs

Topology	Optimal number of FIDs (N_x)
Topology 1	$N_x = (B_o / C_{fid}) - 1$
Topology 2	$N_x = (0.5B_o - 2C_{fid} + \sqrt{(4C_{fid}^2 + 0.25B_o^2})) / 2C_{fid}$
Topology 3	$N_x = (0.25B_o - 4C_{fid} + \sqrt{(16C_{fid}^2 + 0.0625B_o^2 + B_oC_{fid}})) / 2C_{fid}$

Table 2.2 Input parameters for determination of N_x

Description	Value
Benefit factor (B_o)	$ 4000 / MWh
Cost of an FID (MP_{fid})	$ 15000
Service life of an FID(T_{fid})	15 years

The simulation shows that topology 3 requires the minimum number of FIDs amongst the three topologies under consideration. Topology 3 requires 1 FID for maintaining the same level of reliability which topology 1 and topology 2 would achieve by installing 3 and 2 FIDs respectively in the distribution system. This shows that topology

3 is the most cost effective topology. In addition, topology 3 can also be considered as the most reliable topology since the number of FIDs required is lower in comparison to topology 1 and topology 2.

This study is helpful in understanding the tradeoff between investment cost and reliability, which cannot be overlooked in a cost benefit assessment study. The results display that topology 2 and topology 3 exhibit a higher level of reliability at a lower investment cost in comparison to topology 1.

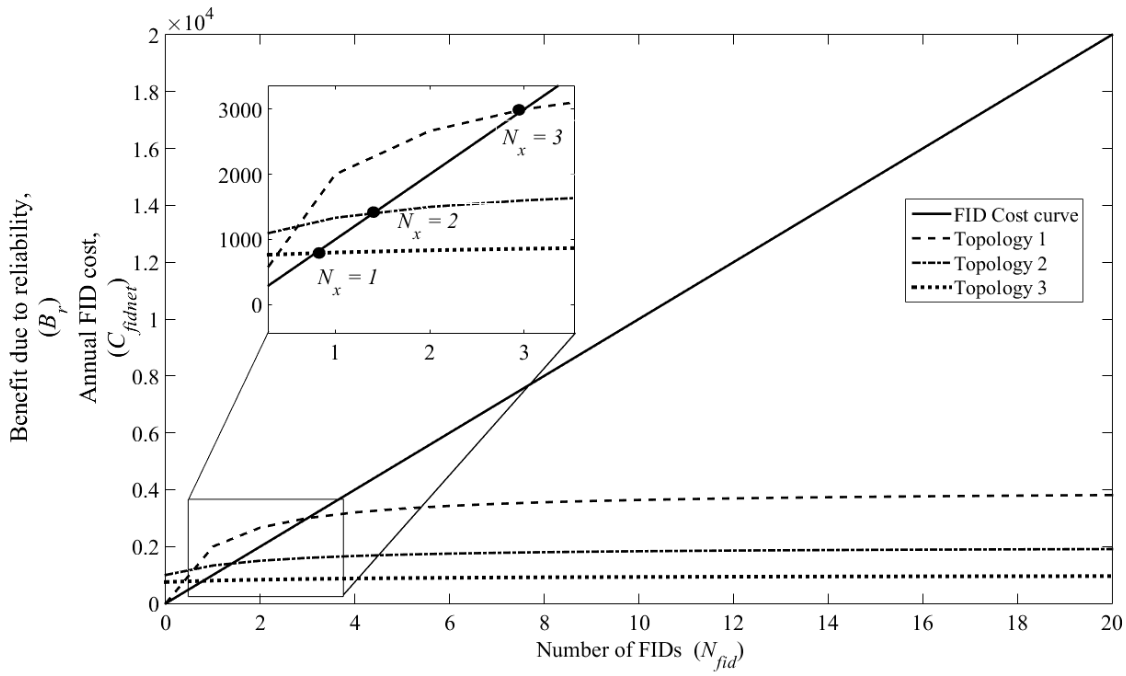

Figure 2.5 Determination of N_x for the three topologies

2.6 Variation of optimal number of FIDs with FREEDM system parameters

In Section 2.5, the optimal number of FIDs was estimated for the FREEDM system with the input parameters being fixed at specific values as shown in Table 2.2. The variation in the optimal number of FIDs with the FREEDM system parameters must be studied due to lack of information regarding the actual price of an FID. The cost of an FID is liable to change in future. In addition, the value of benefit factor may also vary depending upon the type of load under consideration. These changes must be accounted in deciding the optimal number of FIDs for the FREEDM distribution system.

A study was performed to understand the variation in the optimal number of FIDs with changes in benefit factor, annual cost of an FID, service life of an FID and topology. The test case used to perform the study was built on MS-Excel. Figure 2.6 presents the result for the case study in which the variation of N_x with the FREEDM system parameters is studied individually. It can be seen that as the topology changes from 1 to 3, the optimal number of FIDs reduce.

In addition, with the increase in the annual cost of an FID, the optimal number of FIDs reduces. As the cost of an FID increases, the intersection of the two curves (FID cost curve and benefit due to reliability curve) would occur for a lower number of FIDs. This has been shown in Figure 2.7. It can be seen that for an increased annual cost of an FID, the optimal number of FIDs reduces for the three topologies.

The optimal number of FIDs increases with the increasing value of benefit factor. The benefit factor represents the amount of money the customer is ready to pay to the utility for avoiding 1 MWh of interruption. If the customer is ready to pay more, then the reliability of the system must be high in order to avoid outages. In such a case, the optimal number of FIDs will rise.

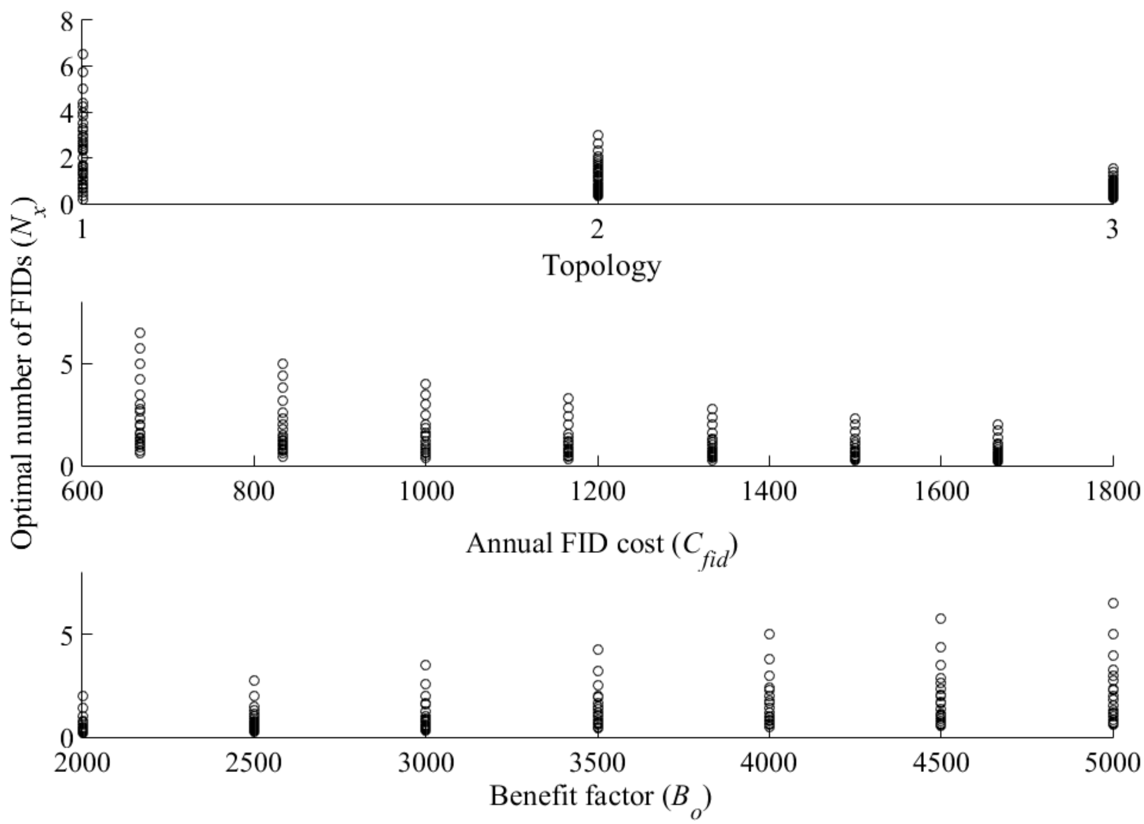

Figure 2.6 Variation of N_x with FREEDM system parameters

Figure 2.8 presents the simulation result when the variation in the optimal number of FIDs is influenced by the combined effect due to all the FREEDM system parameters. It can be observed that the optimal number of FIDs decreases with an increase in the annual cost of an FID and increases with the increase in the value of benefit factor. In addition, as the topology changes from 1 to 3, N_x reduces which shows that the topology 3 is the most reliable topology.

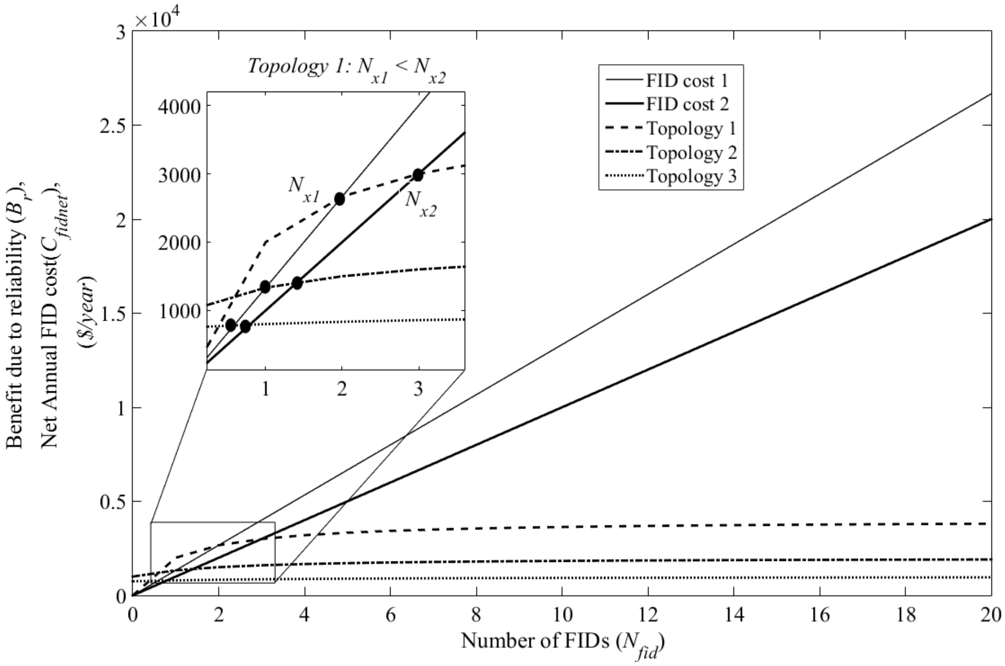

Figure 2.7 Variation of N_x with change in FID annual cost

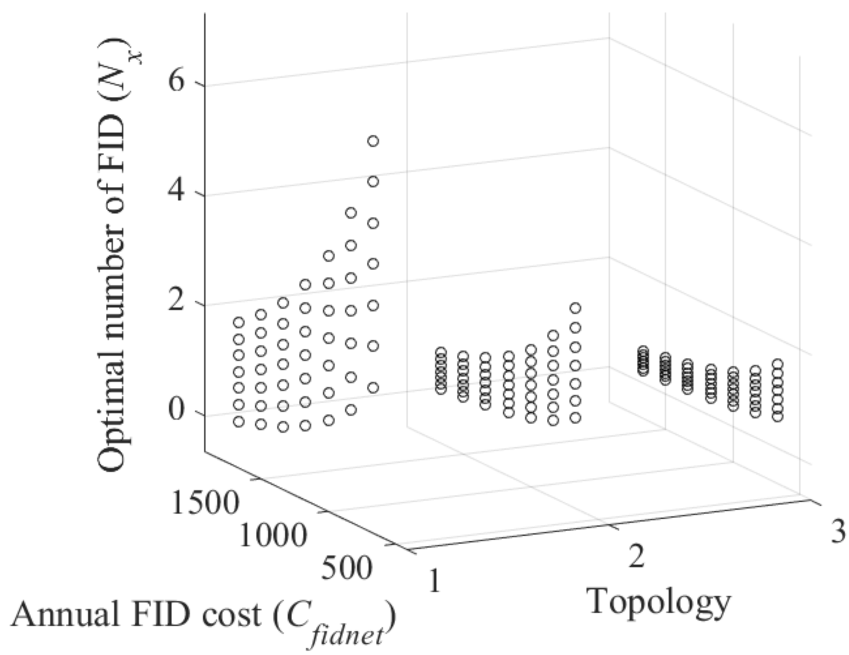

Figure 2.8 Variation of N_x with change in FREEDM parameters

2.7 Inclusion of energy storage elements in the FREEDM system

The FIDs are used in the FREEDM distribution system for clearing the faults quickly so that the load is not lost as per the ITIC/CBEMA curve. According to the ITIC curve, an outage of 20 ms is permissible in a 60 Hz system [13] [14]. Figure 2.9 pictorially represents the advantage of using an FID over a circuit breaker in a distribution system. In general, a circuit breaker requires about 3 – 6 cycles to clear a fault. Consequently, a low voltage would be observed on the system for 3 – 6 cycles, which does not comply with the ITIC/CBEMA curve and the load would be lost. An FID clears the fault within ½ cycle (8.33 ms) but the cost associated with an FID is high in comparison with a circuit breaker.

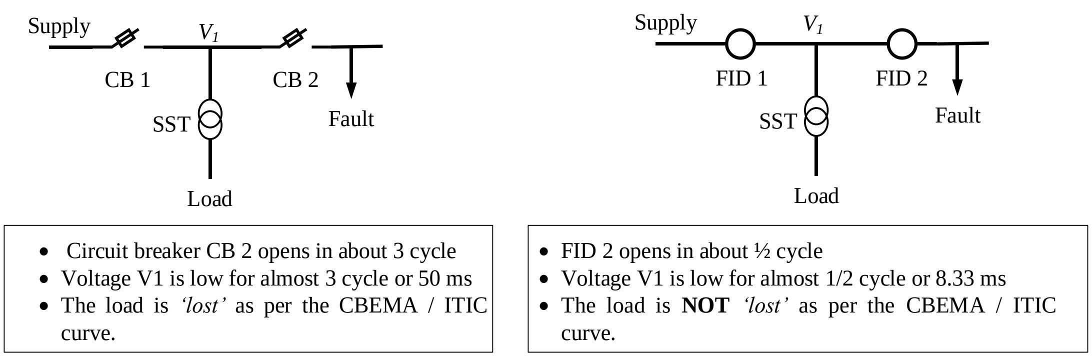

Figure 2.9 Impact of the use of FID for load retention

An energy storage device such as a capacitor in the SST might allow the replacement of FIDs with circuit breakers. The energy storage devices will support the load until the circuit breaker opens and isolates the fault. As the fault is isolated, normal supply to the SSTs would be restored due to the networked nature of the FREEDM distributed system. The inclusion of energy storage devices in the SSTs might result in significant cost savings since it would eliminate the requirement of FIDs from the FREEDM distribution system. Figure 2.10 describes the use of an energy storage device in the solid state transformer.

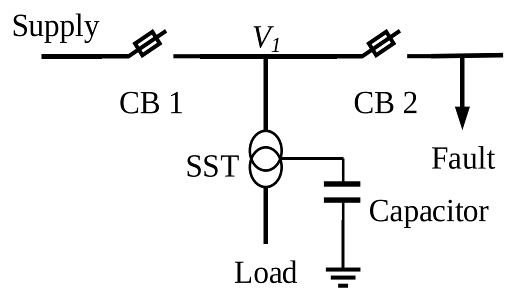

Figure 2.10 Energy storage in solid state transformer

The primary objective of this study is to assess the economic advantages of eliminating the use of FIDs. A simulation study is presented in Chapter 4 to understand the payback period of the FREEDM distribution system where the FIDs are replaced with circuit breakers. The study is presented in Case E where the payback period is compared with other cases to understand the benefit of energy storage devices in the FREEDM distribution system. It was predicted that the payback period would reduce drastically if the FIDs were replaced with circuit breakers due to a large difference in the cost of the two components. In contrast, it is observed that the payback period is higher. A detailed discussion is present in Chapter 4, Case E.

In this section, the value of capacitance used in the energy storage devices is also determined. For the analysis, it is assumed that the peak demand of each house is around 10 kW [32]. An SST can support two residential loads and the peak demand is 20 kW at any given time. It is estimated that for supporting 20 kW load for a period of 10 cycles, a capacitor of 46.25 μF is required, if it is fully discharged or a capacitor with a rating of

61.67 μF would be required if the capacitor is discharged partially to 6 kV. The calculation for determination of the capacitance is given in Appendix C.

2.8 Summary

In this chapter, the FREEDM distribution system architecture was exemplified by three different topologies. The expressions for determination of optimal number of FIDs were derived based on the three topologies using the FREEDM system parameters. The system parameters were defined and a study was conducted to understand the variation in the optimal number of FIDs. In addition, the inclusion of energy storage devices was also discussed.

CHAPTER 3 RELIABILITY STUDIES AND ESTIMATION OF BENEFITS IN FREEDM SYSTEM

3.1 Introduction

The primary objective of the power system is to provide adequate electric supply to the customers with a reasonable level of reliability. At a distribution level, reliability refers to the ability of the power system to avoid outages. These outages may be forced due to maintenance issues or unanticipated due to natural causes or failure of an equipment. Various definitions exist in the power system industry describing reliability, which varies from one type of sector to another [33] [34].

The definition of reliability is different for a customer in comparison to a generation station or a network operator. From a customer's perspective, 100% reliability would refer to zero interruption in the power supply. In comparison, from a network operator or producer's standpoint, reliability has a deeper meaning. Reliability, in a broader sense refers to the transportation of power across the grid to supply the loads while maintaining adequate ancillary services even upon malfunction of any electrical component in the system [33] [34] [36].

The power system industry has coined various reliability indices, approved by IEEE. The two basic categories of reliability indices are *customer based indices* and *load based indices*. *Customer based indices* record the frequency and duration of outages for individual customers. This index is helpful in residential sector. On the other hand, *load based indices* monitor duration and frequency of interruption for commercial loads. The most commonly used reliability indices are SAIDI, SAIFI, CAIDI and ASAI. These terms are discussed in [34].

In this chapter, the reliability of the FREEDM test bed is analyzed based on the customer based reliability index, SAIFI. The study is performed on the three different topologies discussed in Chapter 2. The aim of this study is to compare the results of the evaluated reliability index from the FREEDM system with the present available data on distribution systems. The results would help in understanding the extent of benefit that could be obtained due to improvement in reliability.

3.2 Determination of System Average Interruption Frequency Index (SAIFI)

The SAIFI index is a useful reliability index to understand the percentage of customers interrupted in a system during a fault. For the FREEDM distribution system, the SAIFI is evaluated for the three topologies as discussed in Chapter 2. A generalized expression for SAIFI is determined based on the number of FIDs in the system and type of topology under consideration. An improvement in the reliability of the system would be reflected as a reduction in the value of SAIFI. A lower SAIFI would mean that lesser number of customers have been affected due to an outage.

Determination of SAIFI: Topology 1

Consider the architecture of topology 1 as shown in Figure 3.1. It can be observed that topology 1 is a three phase, single line feeder. It is assumed that the FIDs are placed on the feeder with the load being distributed uniformly across the length of the feeder. From Figure 3.1 (a), it can be observed a fault in the feeder at any point would interrupt

'M' customers. Similarly from Figure 3.1 (b) to Figure 3.1 (d), a fault would interrupt *'M/2', 'M/3'* and *'M/(N_x +1)'* customers respectively. Table 3.1 summarizes the relationship between the number of customers interrupted and the change in the number of FIDs during a fault.

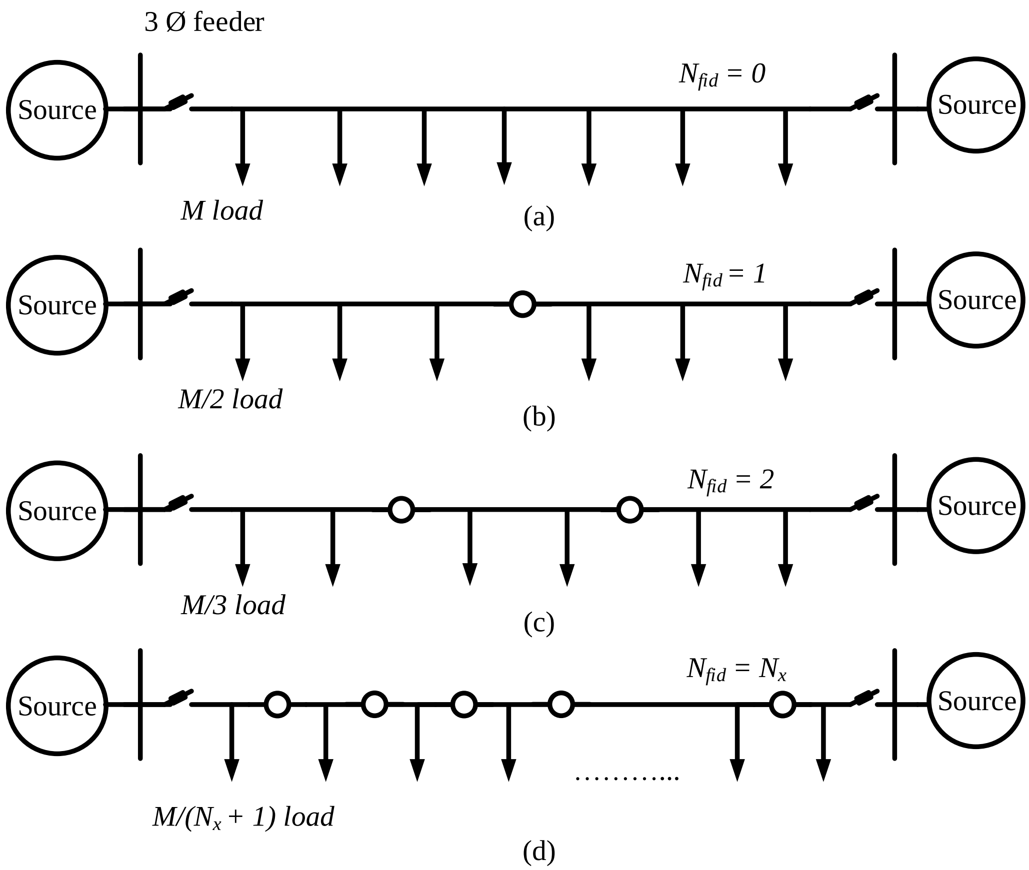

Figure 3.1 A pictorial for calculation of SAIFI for topology 1

Table 3.1 Affected customers for an outage in topology 1

Number of FIDs (N_{fid})	Numbers of customers interrupted ($N_{customers}$)
0	*M*
1	*M/2*
2	*M/3*
.	.
.	.
N_x	$M/(N_x+1)$

From Table 3.1 it can be seen that the expression for the number of customers affected due to an outage event is given by:

$$N_{customers} = M/(N_x+1).$$

And, the expression for SAIFI is given by:

$$\text{SAIFI} = N_{customers}/M\ ,$$

$$\text{SAIFI} = 1/(N_x+1).$$

If the value of N_x is assumed to be sufficiently large in comparison to 1, then by approximation,

$$\text{SAIFI} = (1/N_x)(1-1/N_x), \tag{3.1}$$

where,

$N_{customers}$ is the number of affected customers due to an outage,

M is the total number of customers present on the distribution system,

N_x is the optimal number of FIDs required.

Determination of SAIFI: Topology 2

Consider the architecture of topology 2 as shown in Figure 3.2. It can be observed that topology 2 is a three phase, two line feeder. It is assumed that the FIDs are placed on the feeder with the load being distributed uniformly across the length of the feeder. In addition, it is assumed that the odd numbered FIDs are placed on one line whereas the even numbered FIDs are placed on the other line as shown in Figure 3.2. An outage is assumed on either line for half of the total fault time period.

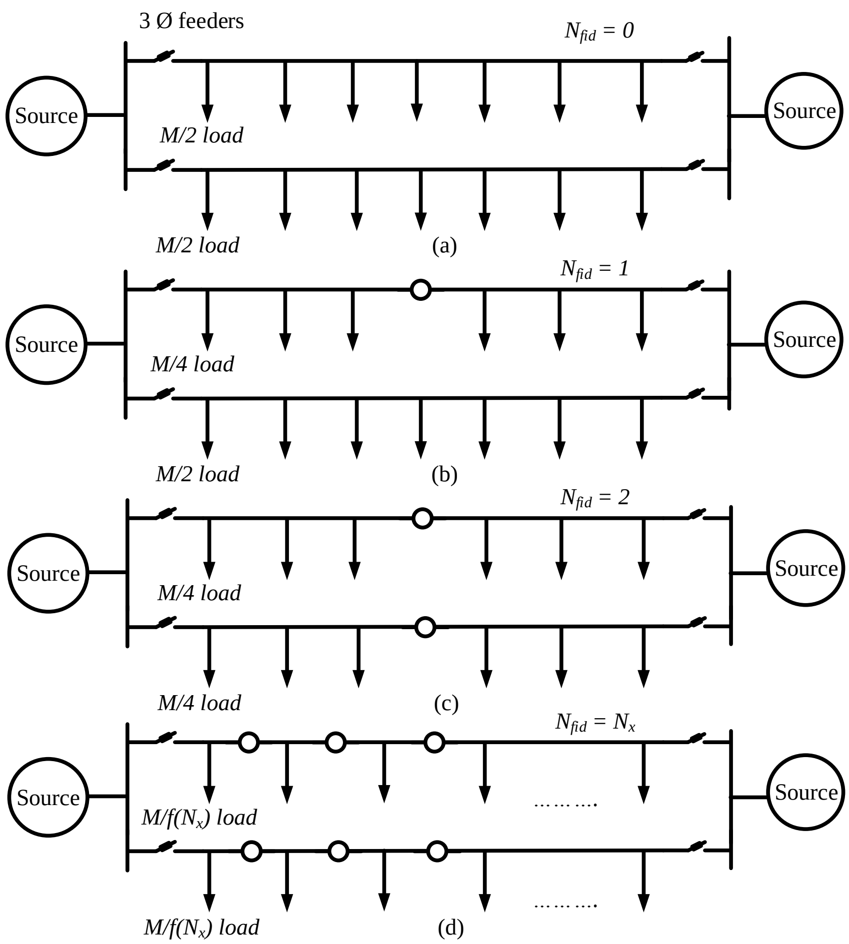

Figure 3.2 A pictorial for calculation of SAIFI for topology 2

From Figure 3.2 it can be observed that the number of customers interrupted would depend upon the number of FIDs in the system. Table 3.2 summarizes the relationship between the number of customers interrupted and the change in the number of FIDs during a fault.

Table 3.2 Affected customers for an outage in topology 2

Number of FIDs (N_{fid})	Numbers of customers interrupted ($N_{customers}$)
0	$M/2$
1	$\{(1/2)\ (M/2)\} + (1/2)\{(1/2)\ (M/2)\} = 3M/8$
2	$(1/2)\{(1/2)\ (M/2)\} + (1/2)\{(1/2)\ (M/2)\} = M/4$
3	$(1/2)\{(1/3)\ (M/2)\} + (1/2)\{(1/2)\ (M/2)\} = 5M/24$
4	$(1/2)\{(1/3)\ (M/2)\} + (1/2)\{(1/3)\ (M/2)\} = M/6$
.	.
.	.
N_x	$(1/2)\ M\{1/(N_x +1) + 1/(N_x +3)\}$, *if* N_x *is odd*
	$M\{(1/N_x + 2)\}$, *if Nx is even*

From Table 3.2 it can be seen that the expression for the number of customers affected due to an outage event is given by:

$$N_{customers} = (M/2)(1/(N_x+1)+1/(N_x+3)),\ \text{if } N_x \text{ is odd}$$

and,

$$N_{customers} = M/(N_x+2)),\ \text{if } N_x \text{ is even.}$$

The SAIFI for topology 2 is given by:

$$\text{SAIFI} = N_{customers}/M\ ,$$

$$\text{SAIFI} = (1/2)(1/(N_x+2)+(1/4)(1/(N_x+1)+1/(N_x+3)))\ .$$

If the value of N_x is assumed to be sufficiently large in comparison to 1, then by approximation,

$$\text{SAIFI} = (1/N_x)(1-2/N_x) \tag{3.2}$$

where,

$f(N_x)$ is the function of optimal number of FIDs in the system,

$N_{customers}$ is the number of affected customers due to an outage,

M is the total number of customers present on the distribution system,

N_x is the optimal number of FIDs required.

Determination of SAIFI: Topology 3

The architecture of topology 3 is similar to topology 2 where all the feeders are three phase feeders. There are four feeders in parallel. An outage is assumed on all the four feeders for one-fourth of the total fault time period. Table 3.3 summarizes the relationship between the number of customers interrupted and the change in the number of FIDs during a fault.

Table 3.3 Affected customers for an outage in topology 3

Number of FIDs (N_{fid})	Numbers of customers interrupted ($N_{customers}$)
0	$M/4$
1	$(1/4)\{(1/2)(M/4) + M/4 + M/4 + M/4\} = 7M/32$
2	$(1/4)\{(1/2)(M/4) + (1/2)(M/4) + M/4 + M/4\} = 6M/32$
3	$(1/4)\{(1/2)(M/4) + (1/2)(M/4) + (1/2)(M/4) + M/4\} = 5M/32$
4	$(1/4)\{(1/2)(M/4) + (1/2)(M/4) + (1/2)(M/4) + (1/2)(M/4)\} = M/8$
.	.
.	.
N_x	$(M/4)/(N_x+4)$, if N_x is (0, 4, 8, 12….)
	$(M/4)((1/(N_x+7) + 3/(N_x+3))$, if N_x is (1, 5, 9, 13….)
	$(M/4)((2/(N_x+6) + 2/(N_x+2))$, if N_x is (2, 6, 10, 14….)
	$(M/4)((3/(N_x+5) + 1/(N_x+1))$, if N_x is (3, 7, 11, 15….)

From Table 3.3 it can be seen that the expression for the number of customers affected due to an outage event is given by:

$$N_{customers} = (M/4)(4/(N_x+4)),$$ if N_x is (0, 4, 8, 12….),

$$N_{customers} = (M/4)(1/(N_x+7)+3/(N_x+3)),$$ if N_x is (1, 5, 9, 13….),

$$N_{customers} = (M/4)(2/(N_x+6)+2/(N_x+2)),$$ if N_x is (2, 6, 10, 14….),

$$N_{customers} = (M/4)(3/(N_x+5)+1/(N_x+1)),$$ if N_x is (3, 7, 11, 15….),

The SAIFI for topology 3 is given by:

$$\text{SAIFI} = N_{customers}/M,$$

$$\text{SAIFI} = \left(\frac{1}{16}\right)\left(\frac{4}{(N_x+4)}+\frac{1}{(N_x+7)}+\frac{3}{(N_x+3)}+\frac{2}{(N_x+6)}+\frac{2}{(N_x+2)}+\frac{3}{(N_x+5)}+\frac{1}{(N_x+1)}\right).$$

If the value of N_x is assumed to be sufficiently large in comparison to 1, then by approximation,

$$\text{SAIFI} = (1/N_x)(1-4/N_x), \tag{3.3}$$

where,

$N_{customers}$ is the number of affected customers due to an outage,

M is the total number of customers present on the distribution system,

N_x is the optimal number of FIDs required.

From the expression for SAIFI for the three topologies stated above in (3.1), (3.2) and (3.3), a generalized expression can be formed for an '*X*' line feeder. Table 3.4 summarizes the results for the SAIFI for an *'X'* line feeder in the FREEDM distribution system.

Table 3.4 Summary of results

Number of Feeders	SAIFI index
1	$(1/N_x)(1-1/N_x)$
2	$(1/N_x)(1-2/N_x)$
3	$(1/N_x)(1-3/N_x)$
4	$(1/N_x)(1-4/N_x)$
X	$(1/N_x)(1-X/N_x)$

It can be observed that the SAIFI index depends on the number of feeders present in distribution system. If the value of N_x is very large, then the SAIFI becomes independent of the number of feeders present in the topology. The SAIFI for a FREEDM distribution system with '*X'* feeder is given by:

$$\text{SAIFI} = (1/N_x)(1-X/N_x),$$

$$If\ N_x >> 1,$$

$$1/N_x^2 << 1,$$

$$\text{SAIFI} \approx 1/N_x .$$

3.3 Determination of SAIFI using Monte Carlo simulations

In Section 3.2, a generalized expression was obtained to evaluate the SAIFI for different topologies. While determining the value of SAIFI, it was assumed that the load distribution is uniform. This assumption obviated all the complexities that may arise due to non-uniformities in the load distribution along the length of the feeder. In this section, the value of SAIFI is determined by considering random distribution of load across the load points.

The study is performed by using Monte Carlo simulation in which random faults are applied at different points along the length of the feeder. During the event of an outage, the respective FIDs governing a particular zone would open, thereby isolating the fault from the rest of the network. Due to the outage, few residential customers would be isolated from the FREEDM distribution system.

The FREEDM test feeder explained in Chapter 2 is considered for performing the case study. A MATLAB code was written for simulating random faults on the system and estimating the FREEDM system SAIFI. The MATLAB code used in the simulation appears in Appendix C.

Table 3.5 presents the simulation result for a 10 FID distribution system. It is assumed that 500 residential customers are being served by the FREEDM distribution feeder. From Table 3.5, it can be observed that the evaluated SAIFI is very close to the result obtained from system theoretic study performed in Section 3.2.

Table 3.5 SAIFI results for a 10 FID system

SAIFI (System theoretic study)	SAIFI (Monte Carlo simulation)	SAIFI (Present U.S. distribution system) *
0.1	0.125	0.85 – 1.1

- * A 10 FID distribution feeder does not apply to the present US distribution system.
- * SAIFI index for the present U.S. distribution system is obtained from [35].

In addition, the evaluated SAIFI is compared with the SAIFI of the present U.S. distribution system. It is observed that the FREEDM system is almost nine times more reliable than the present distribution system. Figure 3.3 shows the result for the Monte Carlo simulation. The result presents the density of the SAIFI index for one million cases of random fault on the FREEDM distribution feeder.

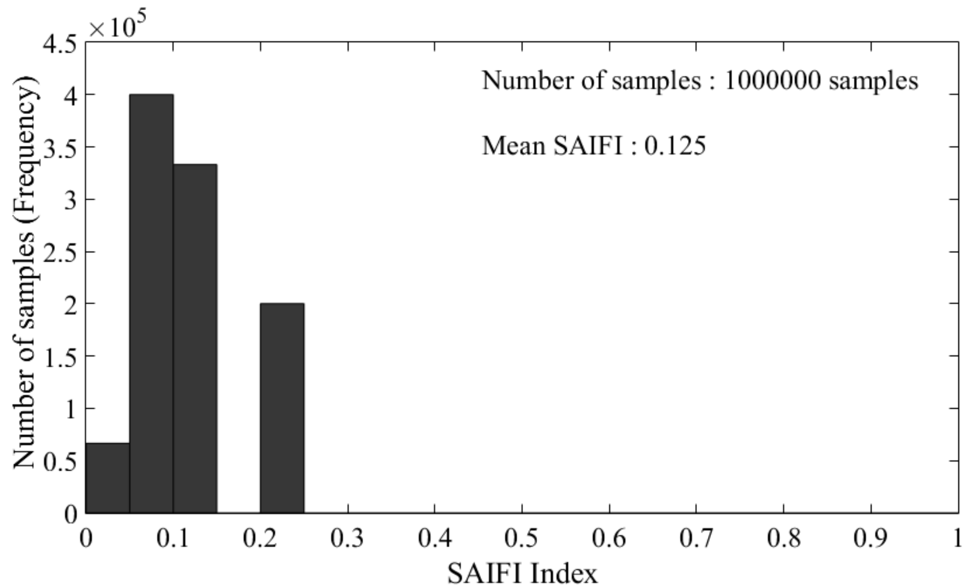

Figure 3.3 Monte Carlo simulation result, SAIFI determination for a 10 FID system

3.4 Direct and indirect benefits from the FREEDM system

Adoption of the FREEDM system over the traditional system, can lead to various benefits. It can be categorized as 'Direct' and 'Indirect'. It is unknown whether these terms are accurate, but they are useful to categorize benefits as per discussions over the last many years in the IEEE Power and Energy Society meetings. The categories are:

- Direct benefits

  i) Reduction of primary conductor active power losses

  ii) Reduction or elimination of the need for voltage regulation assets in the distribution system (e.g., shunt capacitors)

  iii) Implementation of power flow control through the use of SSTs (actually, secondary voltage control)

  iv) Implementation of energy storage systems and concomitant increased reliability (e.g., reduction of SAIDI, SAIFI)

v) Reduction of energy and demand charges for systems with distributed energy installed (this benefit is actually the difference between the reduction of energy and demand charges via FREEDM minus that attainable via conventional, commercially available technologies)

vi) The value of improved SAIDI due to high speed fault interruption

vii) High renewable penetration

viii) Subsidy provided by government over the use of renewable technology.

- Indirect benefits

i) Reduction of CO_2 due to the encouragement of the use of photovoltaic and similar distributed generation

ii) The reduction of greenhouse gas emissions, global warming, and other large scale phenomena

iii) Reduction of fossil fuel use for generation

iv) Modernizing the distribution system with new, electronic components

v) High speed switching capability in the distribution system (e.g., at electronic sub-cycle speeds).

3.5 Inclusion of the dollar benefits of indirect benefits

The inclusion of the dollar value of indirect benefits has long been discussed in *IEEE* PES venues and elsewhere. It is evident that including the actual dollar values of these indirect benefits is not quite as straightforward as in the case of direct benefits. For example, if one conventional 10 MVA distribution feeder were replaced by a 10 MVA FREEDM feeder, there would probably be no measurable benefit of CO_2 reduction or reduction of fossil fuel use. It is reasonable to say that if a given distribution company made a large scale effort to replace existing (or new) services with FREEDM technology, it would be nonetheless difficult to measure the indirect benefits. There are confounding issues such as the impact of the energy required to manufacture photovoltaic cells and the electronic components of the FREEDM system. The issue that is often raised is the calculation of how long it would take to recover the energy required for the manufacture of the PV and other electronic components.

A common solution to the foregoing issues is to address the value of indirect benefits through an attainment approach. That is, in a service territory in which a Renewable Portfolio Standard (RPS) is in place, calculate the cost to attain (comply) with the RPS by the FREEDM systems versus alternatives. In this attainment approach, the target RPS is fixed, and the question becomes the calculation of the cost to attain that target. As applicable to the FREEDM cost/ benefit project, this would mean that a target RPS is assumed (e.g., for the use of photovoltaic energy, perhaps 10 – 20% of residential energy use generated by PV, which would translate into about 30 – 60% of solar penetration by nameplate power rating), and the cost is calculated to attain that target using FREEDM versus conventional technology. The most logical conventional technology would be through the use of PWM inverters and no local energy storage (this is the technology presently commercially available and in widespread use in the United States).

3.6 Summary of the reliability study

In this chapter, the reliability of FREEDM distribution system was studied based on a customer centered reliability index called SAIFI. A general expression for the SAIFI was obtained for a three phase multi feeder FREEDM distribution system. In this study, the load was assumed to be uniformly distributed across the load points. Another study involved the use of Monte Carlo simulation method to evaluate the SAIFI for a non-uniform loading across the feeder. It was observed that the results from the two studies were close to each other. In addition, the FREEDM distribution feeder was found to be more reliable than the present distribution network. In addition, the chapter also discussed about the potential direct and indirect benefits from the FREEDM system.

CHAPTER 4 THE FREEDM COST – BENEFIT ANALYSIS PROBABILISTIC MODEL

4.1 Introduction

The deterministic approach is the traditional method for determination of the payback period of a given engineering project. This deterministic approach does not capture the statistics of the study input parameters. The results obtained by such a process may not reflect the true value of the payback period since the uncertainties in the cost and benefit data are not included in the analysis.

The two important components in the FREEDM system (FIDs and SSTs) are still in the test phase. The true cost of these components cannot be precisely determined. A better approach to determine the payback period and other performance indicators would be by adopting a probabilistic approach by assuming randomness in the input data. In this chapter, a probabilistic model is used to evaluate the PDF and statistics associated with the payback period. The study helps in analyzing the cost effectiveness of implementing the FREEDM system.

4.2 A probabilistic approach for cost / benefit analysis of the FREEDM system

The probabilistic study for the FREEDM system is done by assuming a model to represent the input and output parameters of a cost / benefit study. Figure 4.1 presents the generalized system model, which is used in determining the payback period and associated statistics. The input parameters include annual cost of the FIDs and SSTs, benefit factor and annual benefit from the system. Several approximations are assumed to simplify the model. The approximations in the model presented in Figure 4.1 are:

- The annual maintenance is ignored in the study
- Rate of interest is ignored
- Annual cost, benefit factor and annual benefit are univariate.

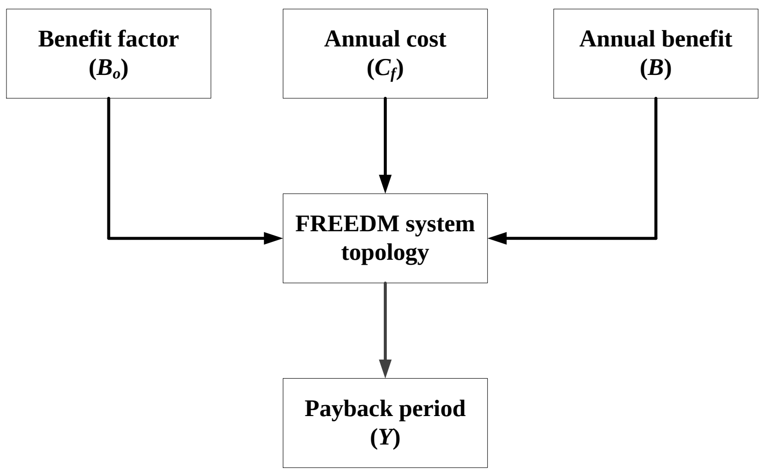

Figure 4.1 FREEDM system simplified probabilistic model

Figure 4.2 presents a detailed model for the FREEDM system where the approximations stated above are considered and a more sophisticated study is performed. This model is similar to the model explained in Figure 4.1 with the same input parameters. The net investment in the FREEDM system is evaluated by determining the optimal number of FIDs and SSTs. This is achieved by selecting a particular topology and calculating the optimal number of FIDs using the expression presented in Table 2.2. In addition, the number of SSTs can be evaluated based on the peak energy consumption by a residential load on the FREEDM system.

In Chapter 2, the FREEDM test distribution feeder is assumed to be rated 1 MVA with 40 residential loads. It is assumed that the peak load requirement by a residential load at any hour of the day is equal to 15 kW [65]. The SST is rated 20 kVA and the number of SST required in the FREEDM system is calculated based upon the peak demand at any hour of the day. In actuality, there would likely be about four customers (points of residential service) for each SST. With these data, the payback period can be evaluated if the annual benefit from the FREEDM system is known. The annual benefit accounts for the net capital generated from the FREEDM distribution system.

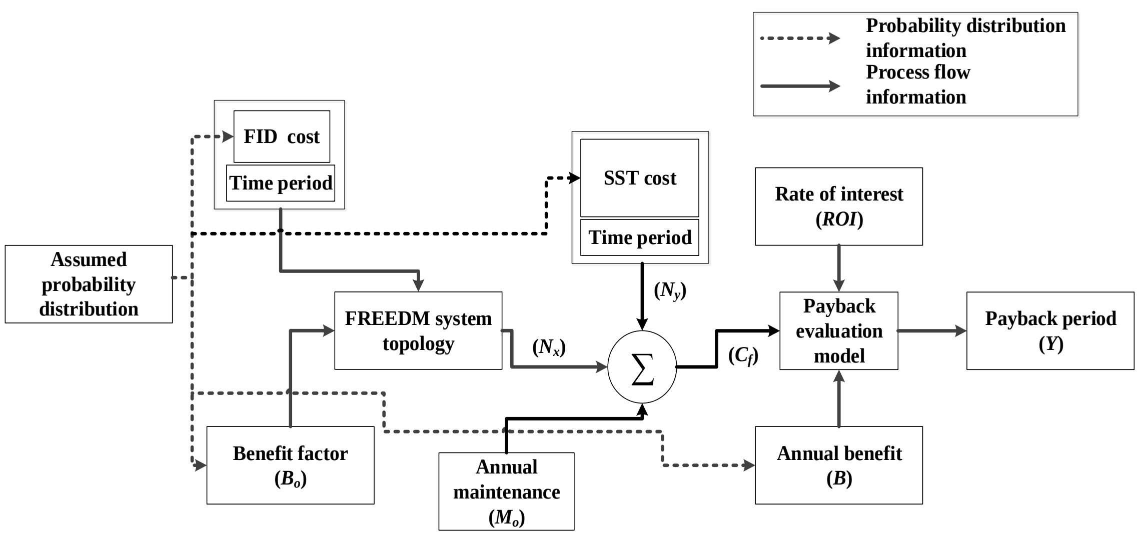

Figure 4.2 Detailed probabilistic model for FREEDM system

The annual benefit is different from the benefit factor since benefit factor. The benefit factor is an indicator, which determines how much amount of money the load is ready to pay to the utility for avoiding an interruption. In comparison, the annual benefit is a cumulative sum of individual benefits obtained because of adopting the FREEDM system. These individual benefits may include reduced carbon emissions, improved power quality, reduced customer interruptions and enhanced use of renewable energy.

A more formal definition of benefit factor and annual benefit is presented in Chapter 2. The net investment, annual benefit and rate of interest would determine the payback period and associated statistics.

4.3 Payback period estimation algorithm

Payback period may be defined as the time period in which the investment made in a system is recovered. The payback period estimation is computed based upon the intersection of the investment curve and the annual benefit curve as shown in Figure 4.3. The payback period is dependent on the rate of interest, which can also be noticed from Figure 4.3.

The payback period and its statistics can be estimated using the two models presented in Figures 4.1 and 4.2. One method used to determine the payback period is the Monte Carlo simulation technique. The algorithm used in the Monte Carlo simulation method is shown in Figure 4.4. The calculation of the payback period involves the following steps:

Step 1: Annual cost of the SST and FID is estimated in a range with a definite mean and standard deviation

Step 2: The benefit factor and annual benefit from the FREEDM system is defined in a range with a definite mean and standard deviation

Step 3: The type of distribution is decided for the different parameters discussed in i) and ii)

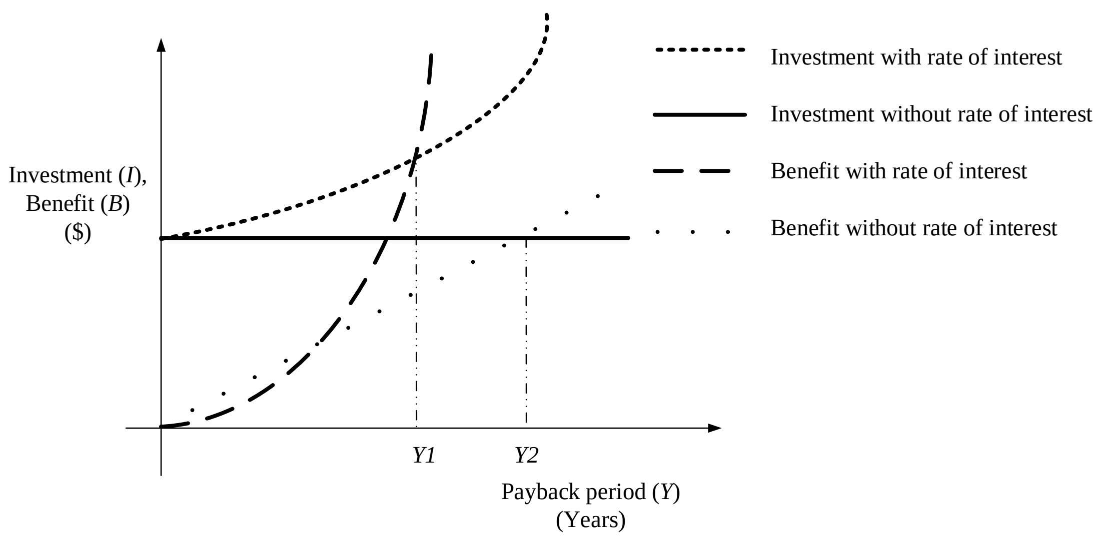

Figure 4.3 Graphical representation of payback period

Step 4: The topology is chosen and the optimal number of FIDs is calculated from the benefit factor and annual system cost

Step 5: The optimal number of FIDs in addition with the desired number of SSTs provide the net investment in the FREEDM system

Step 6: The investment along with the annual benefit and rate of interest provides the payback period and other statistical information related to the payback period.

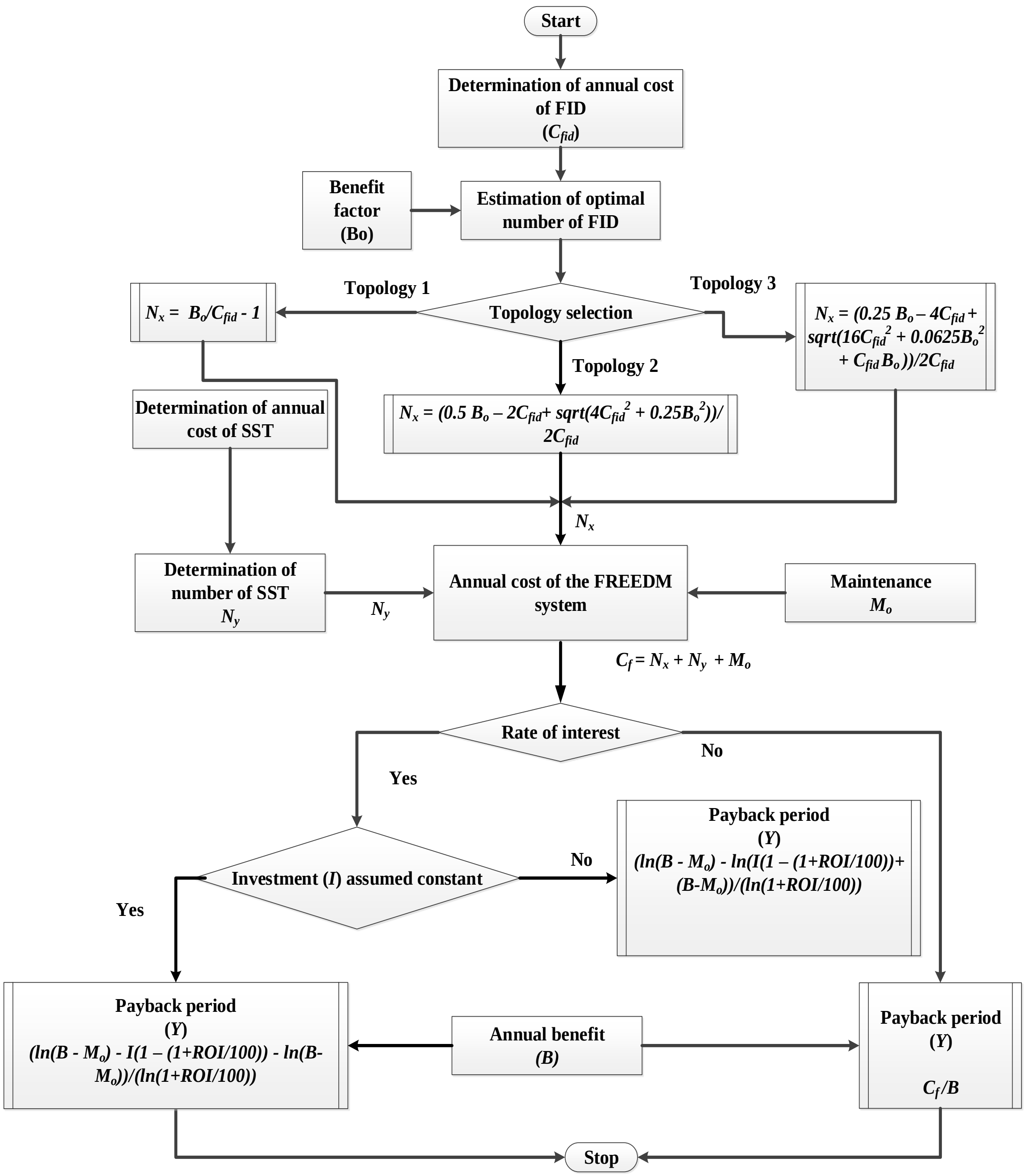

Figure 4.4 Algorithm for payback period estimation

The whole process is made probabilistic by specifying a PDF for the input parameters. The specified probability density functions should exhibit the desired mean and standard deviation of their respective variates. The PDF of the payback period is obtained from the Monte Carlo simulation process. In addition, the Monte Carlo simulation also provides the statistical information related to the payback period. The calculation of the payback period is performed for the three topologies using the Monte Carlo simulation technique for the following different cases:

- In Case A, the payback period and associated statistical parameters are determined by ignoring the rate of interest and annual maintenance of the FREEDM components (SSTs and FIDs). In addition, the input parameters are univariate.

- In Case B, the rate of interest is considered in determination of the payback period. In addition, the annual maintenance of the FREEDM components is also considered. The investment is constant, i.e. it does not increase as per the rate of interest. The annual benefit is assumed to increase as per the interest rate.

- In Case C, the investment and annual benefit grow as per the rate of interest. This case is similar to Case 2. The only difference between the two cases is that in Case 2, the investment is constant whereas in Case 3, the investment grows as per the rate of interest.

- In Case D, the input parameters are a function of several random variables. In addition, the investment is assumed to grow constantly depending upon the rate of interest. The input parameters are a combination of several random variables with the same or different distribution.
- In Case E, the economic impact of energy storage devices on the FREEDM system is studied. the FIDs are replaced with circuit breakers and energy storage devices are considered in the SSTs. The payback period is estimated for replacement of FIDs with circuit breakers and inclusion of energy storage devices in the FREEDM system.

4.4 Probabilistic cost / benefit analysis results

The payback period is computed using the algorithm mentioned in Figure 4.4. The expressions and results of the payback period are presented for different cases and are given by:

Case A: Simplified consideration

In Case A, the payback period is estimated using the model presented in Figure 4.1. The payback period is calculated by considering the overall investment cost and net annual benefit from the system as shown by the following formula,

$$Y = I / B, \tag{4.1}$$

where,

B is the annual benefit,

I is the net investment in the FREEDM system,

Y is the payback period.

This equation is further resolved using the relationship between optimal number of FIDs, optimal number of SSTs and net investment in the FREEDM system. The relationship between optimal number of FIDs and investment varies with different topologies. A generalized payback period expression is determined for the three topologies using the relationship presented in (4.1) and Table 2.1. The same has been presented below:

Derivation of payback period for topology 1

$$N_x = (B_o / C_{fid}) - 1, \tag{4.2}$$

$$N_y = Load_{system-peak} / SST_{rating}, \tag{4.3}$$

$$I = N_x C_{fid} T_{fid} + N_y C_{sst} T_{sst}, \tag{4.4}$$

$$Y = ((B_o - C_{fid}) T_{fid} + N_y C_{sst} T_{sst}) / B. \tag{4.5}$$

Derivation of payback period for topology 2

$$N_x = (0.5 B_o - 2 C_{fid} + \sqrt{4 C_{fid}^2 + 0.25 B_o^2}) / 2 C_{fid}, \tag{4.6}$$

$$N_y = Load_{system-peak} / SST_{rating}, \tag{4.7}$$

$$I = N_x C_{fid} T_{fid} + N_y C_{sst} T_{sst}, \tag{4.8}$$

Using the approximation,

$$\sqrt{A+B} = \sqrt{A}(1 + 0.5(B/A)),\ if\,(B/A) << 1,$$

$$Y = (((0.5B_o + B_o^2/16C_{fid})/2)T_{fid} + N_y C_{sst} T_{sst})/B. \qquad (4.9)$$

Derivation of payback period for topology 3

$$N_x = (0.25B_o - 4C_{fid} + \sqrt{16C_{fid}^2 + 0.0625B_o^2 + C_{fid}B_o})/2C_{fid}, \qquad (4.10)$$

$$N_y = Load_{system-peak}/SST_{rating}, \qquad (4.11)$$

$$I = N_x C_{fid} T_{fid} + N_y C_{sst} T_{sst}, \qquad (4.12)$$

Using the approximation,

$$\sqrt{A+B} = \sqrt{A}(1+0.5(B/A)),\ if\,(B/A) << 1,$$

$$Y = (((0.375B_o + B_o^2/128C_{fid})/2)T_{fid} + N_y C_{sst} T_{sst})/B, \qquad (4.13)$$

where,

B is the annual benefit generated from the FREEDM system,

C_{fid} is the annual cost of an FID in the FREEDM system,

C_{sst} is the annual cost of an SST,

I is the net investment made in the FREEDM system,

$Load_{system-peak}$ is the peak load on the FREEDM distribution feeder at any given time,

N_x is the optimal number of FIDs in the FREEDM system,

N_y is the optimal number of SSTs in the FREEDM system,

SST_{rating} is the kVA rating of an SST,

T_{fid} is the service life of an FID,

T_{sst} is the service life of an SST,

Y is the payback period for an investment made in the FREEDM system.

From these relationships, the payback period for the three different topologies is computed. It must be noted that the rate of interest and annual maintenance cost are not considered in determination of the payback period. The overall investment made in the FREEDM system and the annual benefit from the system are considered as random variables, distributed in a range with a given mean and standard deviation. Using the Monte Carlo simulation, the PDF of the payback period is generated which helps in estimating the other statistical parameters associated with the payback period. For purposes of performing the FREEDM cost to benefit assessment, the data shown in Table 4.1 are used.

Table 4.1 Input parameters for determination of payback period

Annual maintenance (M_o) : $ 1000				Rate of interest (*ROI*) : 1.5 %	
Service life of SST (T_{sst}) : 15 years				Service life of FID (T_{fid}) : 15 years	
Description		Symbol	Range	Mean**	Standard deviation**
	(Unit)			(μ)	(σ)
FID cost*	($)	C_{fid}	(10000 – 25000)	17500	2500
SST cost*	($)	C_{sst}	(15000 – 30000)	22500	2500
Benefit factor	($/MWh)	B_o	(2000 – 5000)	3500	500
Annual benefit	($/year)	B	(8000 – 14000)	11000	1000

- * The estimated cost for the SST and FID is based on the three phase design
- ** Data to define a random variable if normally distributed

Figure 4.5 presents the PDF of the input parameters for test case A3, where all the input parameters are triangular distributed. Figure 4.6 presents the density of input parameters. The simulation results for Case A are presented in Table 4.2 and Table 4.3 and the respective MATLAB code for Case A appears in Appendix E.

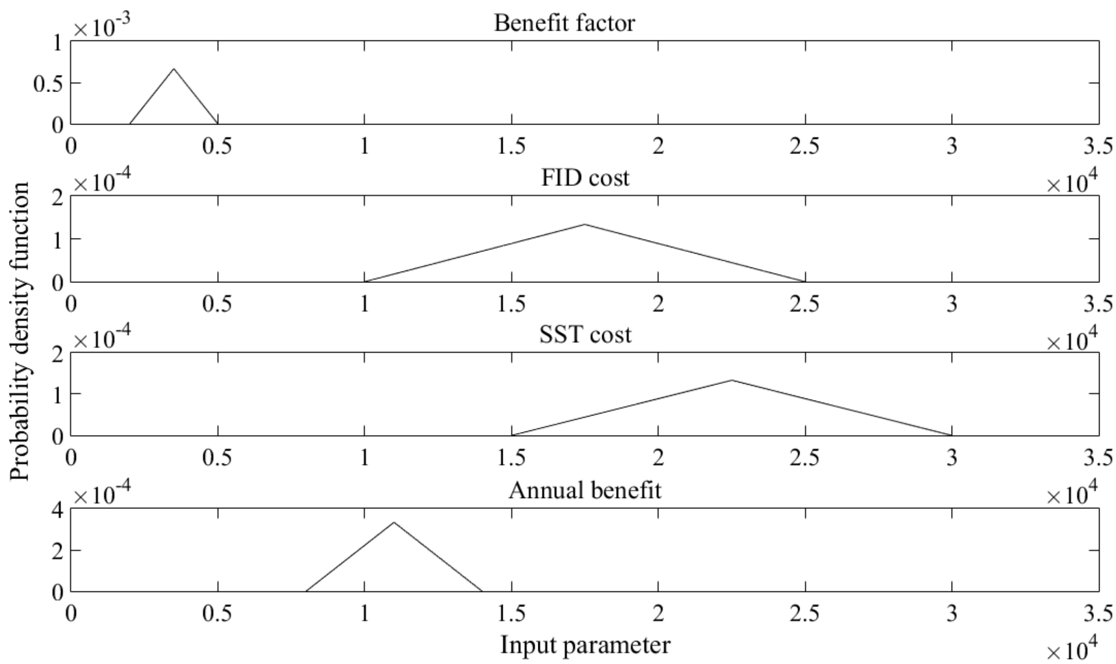

Figure 4.5 Probability density function for the input parameters

Figure 4.7 presents the density of payback period in different time intervals for test case A3. In test case A3, it can be observed that the probability of payback within 30 years is insignificant (nearly zero), which infers that the investment cannot be recouped within 30 years. The probability of investment recovery within 75 years is around 90 – 99% in all the test cases, which reflects a very high possibility of investment recovery.

Figure 4.8 presents the cumulative distribution plot for the three topologies. From the result, it can be observed that the simulation results for topology 2 and topology 3 are very close to each other. In other words, the probabilities of payback period for these two topologies are nearly the same. For any particular year, the probability of payback period for topology 1 is significantly lower than the other two topologies.

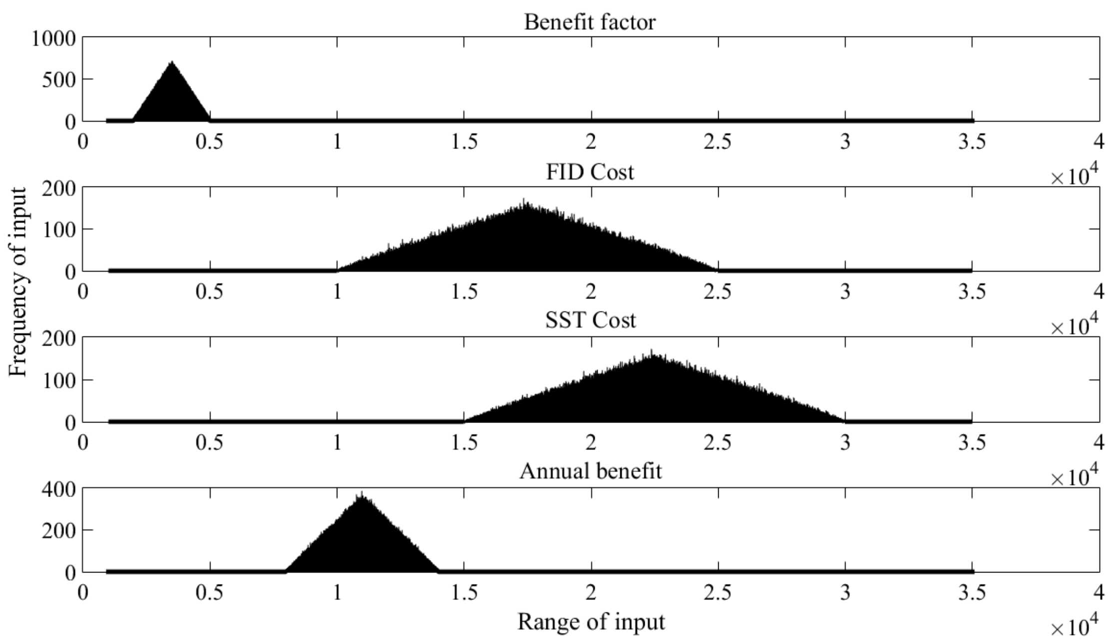

Figure 4.6 Density of the input parameters

Figure 4.9 presents a comparison between different test cases based on the payback period for each topology. It can be seen that the probability of payback is slightly affected due to change in distribution of the input parameters for the three topologies. In test case A1, where the random variables are distributed normally, there is a steep transition before and after 50 years. The transition is caused due to the small standard deviation, which results in a steep increase in the probability of the payback period. In contrast, the results obtained from the other test cases are very close to each other and increase gradually.

Table 4.2 Determination of probability of payback period for Case A

Test case	Topology	Probability distribution	Payback period probability (Y in years)					
			$P_r (Y \leq 25)$	$P_r (Y \leq 35)$	$P_r (Y \leq 45)$	$P_r (Y \leq 55)$	$P_r (Y \leq 65)$	$P_r (Y \leq 75)$
A1	1	C_{fid}: Normal	0	0.003	0.144	0.643	0.942	0.995
	2	C_{sst}: Normal	0	0.006	0.199	0.716	0.962	0.997
	3	B_o: Normal B: Normal	0	0.009	0.229	0.747	0.968	0.998
A2	1	C_{fid}: Uniform	0	0.060	0.284	0.581	0.803	0.925
	2	C_{sst}: Uniform	0	0.083	0.325	0.625	0.830	0.941
	3	B_o: Uniform B: Uniform	0	0.094	0.345	0.644	0.842	0.947
A3	1	C_{fid}: Triangular	0	0.010	0.200	0.613	0.897	0.985
	2	C_{sst}: Triangular	0	0.020	0.254	0.674	0.923	0.991
	3	B_o: Triangular B: Triangular	0	0.024	0.280	0.700	0.934	0.993
A4	1	C_{fid}: Uniform	0	0.037	0.279	0.584	0.843	0.965
	2	C_{sst}: Uniform	0	0.058	0.324	0.628	0.874	0.976
	3	B_o: Normal B: Triangular	0	0.071	0.346	0.650	0.887	0.980
A5	1	C_{fid}: Triangular	0	0.021	0.241	0.589	0.832	0.956
	2	C_{sst}: Triangular	0	0.034	0.290	0.637	0.864	0.969
	3	B_o: Normal B: Uniform	0	0.042	0.314	0.660	0.877	0.974

Table 4.3 Determination of statistics of payback period for Case A

Test case	Topology	Probability distribution	Expectation*(E) (years)	Standard Deviation*(σ) (years)	Conditional expectation $(Ex \leq 25)$ (years)	Conditional expectation $(Ex \leq 35)$ (years)	Conditional expectation $(Ex \leq 45)$ (years)	Conditional expectation $(Ex \leq 55)$ (years)	Conditional expectation $(Ex \leq 65)$ (years)	Conditional expectation $(Ex \leq 75)$ (years)
A1	1	C_{fid}: Normal	52.71	7.43	-	33.34	41.81	48.35	51.71	52.59
	2	C_{sst}: Normal	51.19	7.30	-	33.16	41.51	47.65	50.48	51.11
	3	B_o: Normal B: Normal	50.48	7.26	-	23.94	33.14	41.35	47.28	49.88
A2	1	C_{fid}: Uniform	53.66	13.18	-	32.50	38.73	44.53	48.68	51.43
	2	C_{sst}: Uniform	52.08	12.97	-	32.16	38.27	43.88	47.73	50.29
	3	B_o: Uniform B: Uniform	51.37	12.88	-	31.96	38.05	43.55	47.31	49.77
A3	1	C_{fid}: Triangular	52.95	9.15	-	33.47	40.94	47.11	50.96	52.56
	2	C_{sst}: Triangular	51.40	9.01	-	33.27	40.59	46.42	49.87	51.15
	3	B_o: Triangular B: Triangular	50.70	8.94	-	33.14	40.39	46.08	49.35	50.50
A4	1	C_{fid}: Uniform	52.92	11.39	-	33.09	39.54	45.01	49.50	51.98
	2	C_{sst}: Uniform	51.42	11.29	-	32.78	38.98	44.32	48.60	50.73
	3	B_o: Normal B: Triangular	50.68	11.24	-	32.63	38.70	43.98	48.12	50.10
A5	1	C_{fid}: Triangular	53.63	11.18	-	33.18	40.32	45.99	49.95	52.45
	2	C_{sst}: Triangular	52.06	10.94	-	32.90	39.89	45.31	49.02	51.22
	3	B_o: Normal B: Uniform	51.35	10.85	-	32.75	39.69	44.99	48.59	51.35

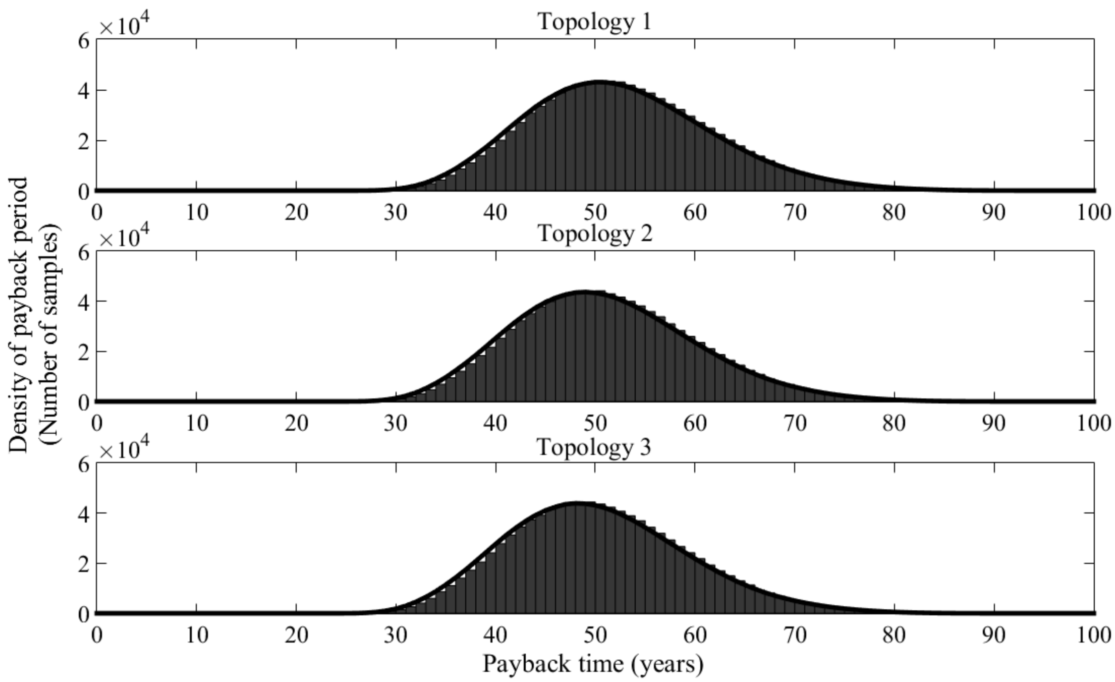

Figure 4.7 Density of probability of payback period for test-case A3, Case A

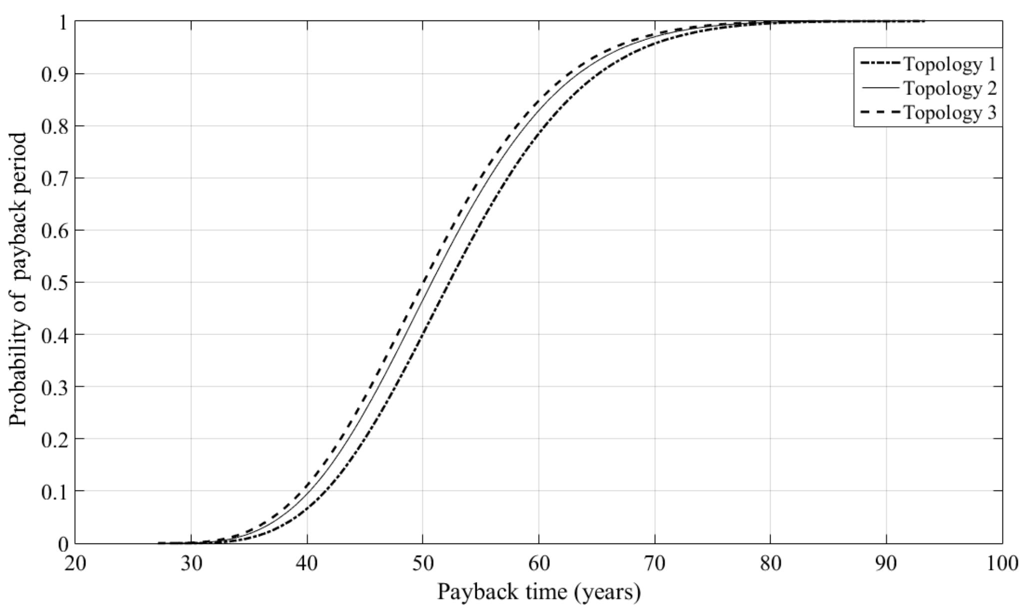

Figure 4.8 Cumulative distribution plot of payback period for test case A3, Case A

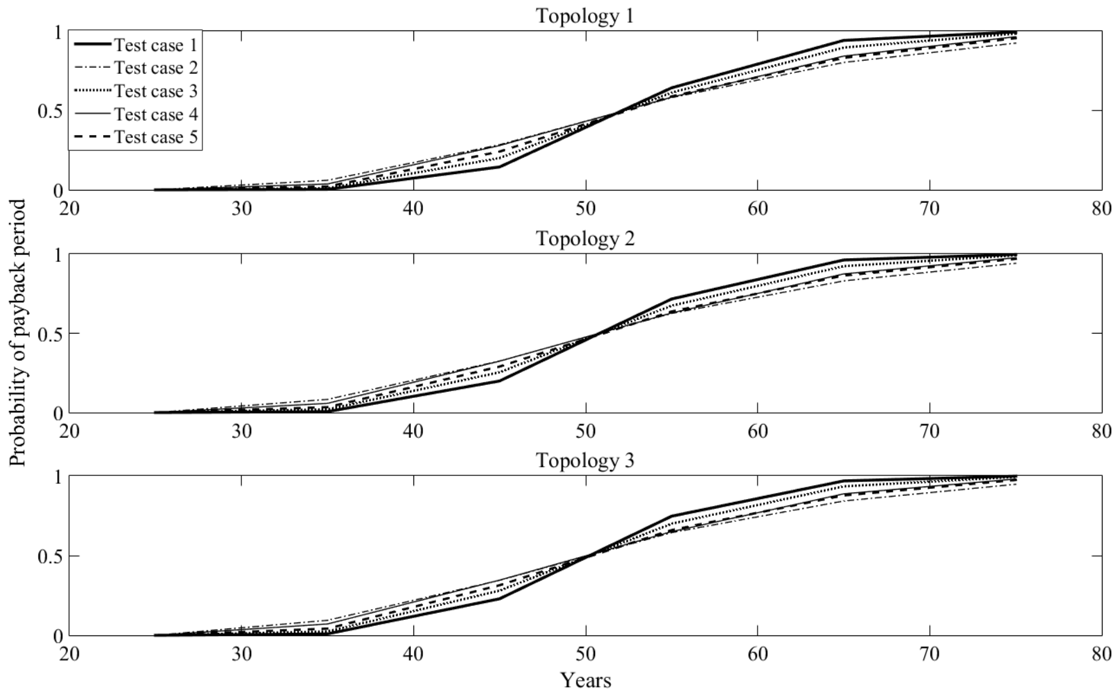

Figure 4.9 Comparison of payback period for different topologies, Case A

Case B: Investment assumed constant, annual benefit increases as per ROI

The approximations made in Case A are relaxed in Case B. The rate of interest and annual maintenance are considered to deduce the payback period and associated statistical parameters. The model presented in Figure 4.2 is used for determining the payback period. The investment is constant with time. The relationship between the payback period and input parameters is determined based on the relationships between investment, annual benefit and rate of return. The intermediate steps can be referred from Appendix F. The expressions are presented below:

$$I = N_x C_{fid} T_{fid} + N_y C_{sst} T_{sst}, \tag{4.14}$$

$$I_{n,years} = I(1+ROI/100)^N, \qquad (4.15)$$

$$B_{n,years} = (B - M_o)(1-(1+ROI/100)^N)/(1-(1+ROI/100)), \qquad (4.16)$$

If Y years is assumed to be the payback period, then for Case B,

$$I = (B - M_o)(1-(1+ROI/100)^Y)/(1-(1+ROI/100)), \qquad (4.17)$$

$$Y = \frac{(ln((B - M_o) - I(1-(1+ROI/100))) - ln(B - M_o))}{ln(1+ROI/100)}, \qquad (4.18)$$

where,

B is the annual benefit generated from the FREEDM system,

C_{fid} is the annual cost of an FID in the FREEDM system,

C_{sst} is the annual cost of an SST,

I is the net investment made in the FREEDM system,

$Load_{system\text{-}peak}$ is the peak load on the FREEDM distribution feeder at any given time,

N_x is the optimal number of FIDs in the FREEDM system,

N_y is the optimal number of SSTs in the FREEDM system,

ROI is the rate of interest,

SST_{rating} is the kVA rating of an SST,

T_{fid} is the service life of an FID,

T_{sst} is the service life of an SST,

Y is the payback period for an investment made in the FREEDM system.

The expressions for optimal number of FIDs is obtained from (4.2), (4.6) and (4.10) for topologies 1, 2 and 3 respectively. Similarly, the expression for the number of SSTs is obtained from (4.11). The sum of the two would give an estimate of the overall investment made towards the FREEDM system.

In (4.17), it is assumed that at the end of Y years, the net investment would be equal to the net benefit from the system. This equation is further solved to determine the payback period as mentioned in (4.18). It can be seen that the annual profit is from the system is the difference between the annual benefit and the annual maintenance given by ($B - M_o$). Table 4.1 presents the input parameters used for determining the payback period for Case B. The simulation is performed using MATLAB to evaluate the payback period and the respective code is presented in Appendix G. The results are tabulated in Table 4.4 and Table 4.5, which presents the probability of the payback period for different topologies and related statistical information under different test cases.

The PDF of the input parameters is shown in Figure 4.5 where all the system parameters are triangular distributed. Figure 4.10 presents simulation results for the determination of the payback period for test case B3. From the results, it is observed that the probability of payback period under 30 years is significantly low, almost zero. In addition, the probability of payback is high in the range of 50 – 55 years with the probability ranging from 0.90 – 0.98.

Table 4.4 Determination of probability of payback period for Case B

Test case	Topology	Probability distribution	Payback period probability (Y in years)					
			$P_r\,(Y \leq 30)$	$P_r\,(Y \leq 35)$	$P_r\,(Y \leq 40)$	$P_r\,(Y \leq 45)$	$P_r\,(Y \leq 50)$	$P_r\,(Y \leq 55)$
B1	1	C_{fid}: Normal C_{sst}: Normal B_o: Normal B: Normal	0.002	0.046	0.307	0.724	0.945	0.994
	2		0.004	0.074	0.388	0.789	0.964	0.996
	3		0.005	0.090	0.427	0.816	0.970	0.997
B2	1	C_{fid}: Uniform C_{sst}: Uniform B_o: Uniform B: Uniform	0.046	0.185	0.392	0.633	0.808	0.920
	2		0.065	0.217	0.434	0.669	0.833	0.934
	3		0.072	0.229	0.452	0.684	0.842	0.940
B3	1	C_{fid}: Triangular C_{sst}: Triangular B_o: Triangular B: Triangular	0.006	0.090	0.350	0.685	0.902	0.983
	2		0.012	0.125	0.415	0.739	0.928	0.989
	3		0.014	0.136	0.433	0.754	0.934	0.990
B4	1	C_{fid}: Uniform C_{sst}: Uniform B_o: Normal B: Triangular	0.024	0.164	0.389	0.644	0.857	0.964
	2		0.041	0.205	0.438	0.690	0.886	0.974
	3		0.046	0.216	0.452	0.703	0.893	0.976
B5	1	C_{fid}: Triangular C_{sst}: Triangular B_o: Normal B: Uniform	0.015	0.130	0.381	0.640	0.832	0.950
	2		0.026	0.170	0.435	0.686	0.863	0.964
	3		0.029	0.180	0.447	0.695	0.870	0.967

Table 4.5 Determination of statistics of payback period for Case B

Test case	Topology	Probability distribution	Expectation* *(E) (years)*	Standard deviation**(σ) (years)*	Conditional expectation *(Ex ≤ 30) (years)*	Conditional expectation *(Ex ≤ 35) (years)*	Conditional expectation *(Ex ≤ 40) (years)*	Conditional expectation *(Ex ≤ 45) (years)*	Conditional expectation *(Ex ≤ 50) (years)*	Conditional expectation *(Ex ≤ 55) (years)*
B1	1	C_{fid}: Normal	42.41	4.59	28.82	33.31	37.27	40.25	41.83	42.31
	2	C_{sst}: Normal	41.42	4.58	28.77	33.15	36.98	39.70	40.01	41.36
	3	B_o: Normal B: Normal	40.98	4.56	28.77	33.09	36.83	39.44	40.63	40.93
B2	1	C_{fid}: Uniform	42.67	8.03	28.39	31.64	34.80	37.71	39.79	41.31
	2	C_{sst}: Uniform	41.79	7.99	28.16	31.35	34.49	37.26	39.23	40.65
	3	B_o: Uniform B: Uniform	41.43	7.97	28.07	31.24	34.37	37.08	39.01	40.38
B3	1	C_{fid}: Triangular	42.42	5.64	29.01	32.93	36.52	39.41	41.29	42.16
	2	C_{sst}: Triangular	41.45	5.61	28.84	32.70	36.20	38.91	40.58	41.28
	3	B_o: Triangular B: Triangular	41.19	5.60	28.82	32.64	36.10	38.77	40.38	41.04
B4	1	C_{fid}: Uniform	42.31	6.89	28.80	32.29	35.36	38.19	40.45	41.74
	2	C_{sst}: Uniform	41.33	6.92	28.55	31.93	34.94	37.68	39.81	40.91
	3	B_o: Normal B: Triangular	41.07	6.92	28.50	31.84	34.82	37.56	39.62	40.68
B5	1	C_{fid}: Triangular	42.80	6.93	28.84	32.56	35.90	38.54	40.57	42.01
	2	C_{sst}: Triangular	41.78	6.88	28.61	32.30	35.51	38.02	39.93	41.20
	3	B_o: Normal B: Uniform	41.57	6.87	28.57	32.24	35.42	37.90	39.80	41.03

Figure 4.11 presents the cumulative distribution function plot for test case B3. It can be observed that the probability of payback period is nearly 90 – 95 % within 50 years. In addition, the cumulative distribution plot for topology 2 and topology 3 nearly overlap each other, which shows that the probability for payback period in any interval is nearly the same for both the topologies.

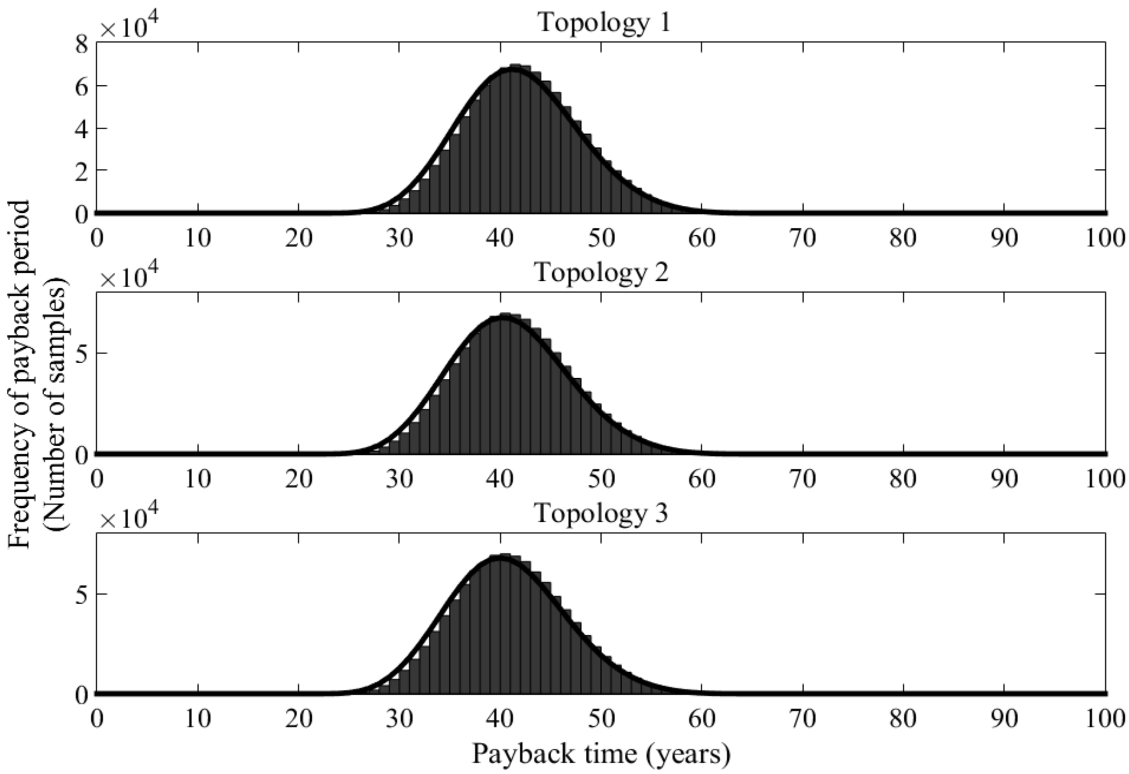

Figure 4.10 Frequency of payback period for three topologies for test-case B3, Case B

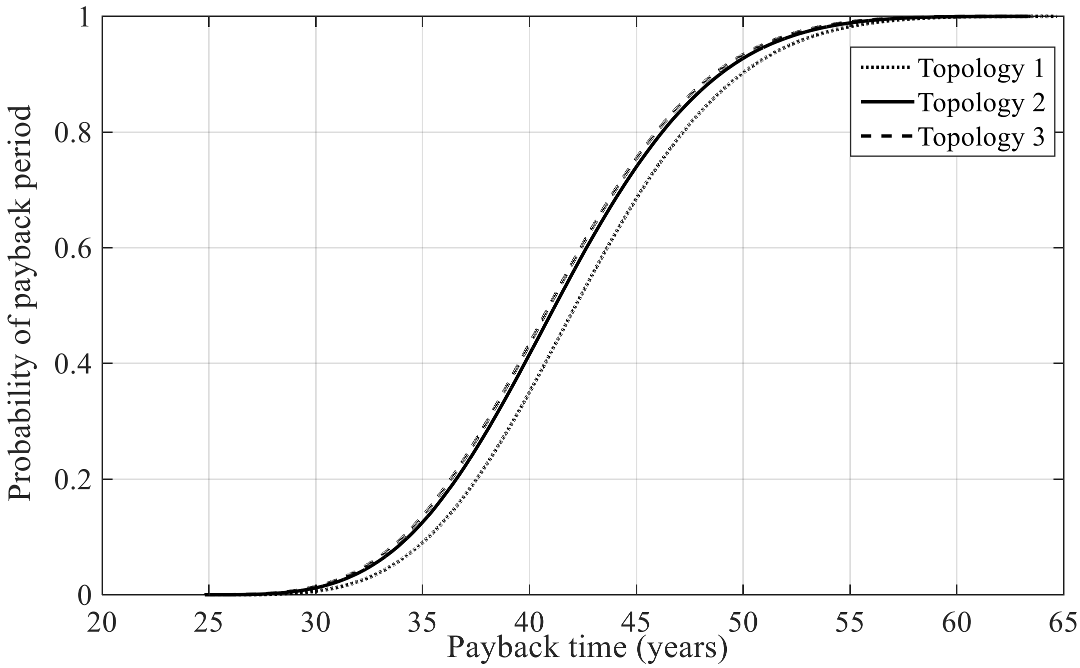

Figure 4.11 Cumulative distribution plot for test-case B3, Case B

From the results, it can be inferred that for the same measure of reliability, the probabilities of payback period obtained by considering topology 2 and topology 3 under different test cases are nearly the same. In such a scenario, topology 2 is more preferred due to its lesser structural complexity in comparison to topology 3.

Figure 4.12 presents the comparison for different topologies based on the test cases presented in Table 4.5. In test case B1 where the random variables are normally distributed, there is a steep transition before and after 43 years due to a small standard deviation. It can be observed that in all other test cases, the probabilities of payback period in a particular time interval are close to each other and increases gradually. There is a 50% payback probability in all the test cases at 43 years.

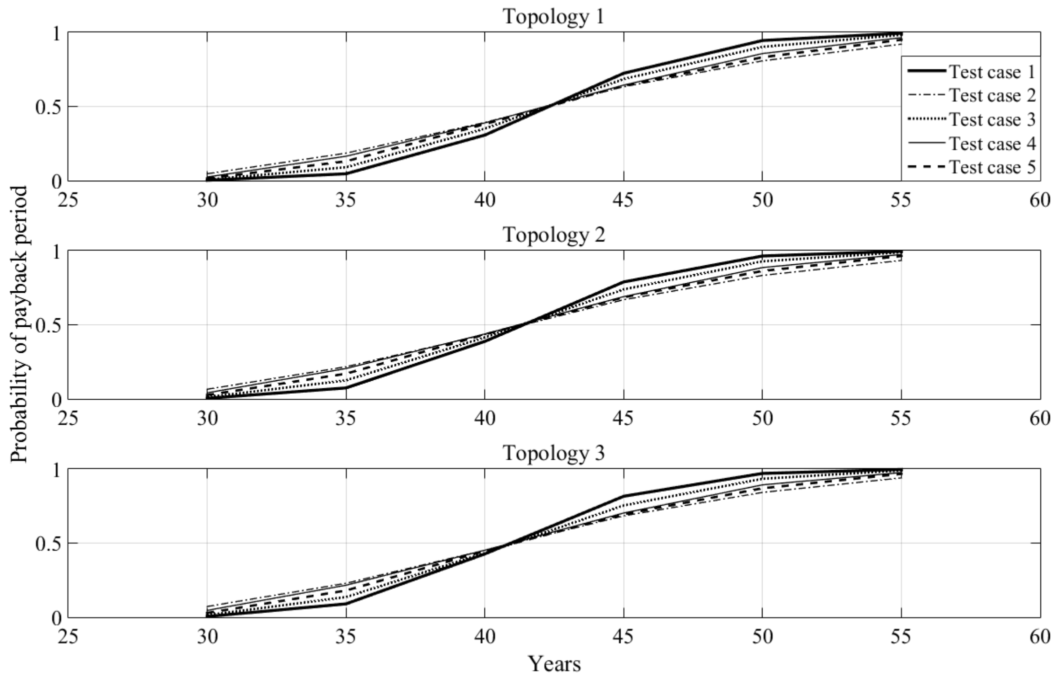

Figure 4.12 Comparison of payback period for different topologies, Case B

Case C: Investment and annual benefit increase as per ROI

In Case C, the investment is increasing with time depending upon the rate of interest. The payback period is then defined as:

$$I(1+ROI/100)^{Y} = (B-M_o)(1-(1+ROI/100)^{Y})/(1-(1+ROI/100)) \tag{4.19}$$

$$Y = (\ln(B-M_o) - \ln(I(1-(1+ROI/100)) + (B-M_o)))/\ln(1+ROI/100) \tag{4.20}$$

where,

B is the annual benefit generated from the FREEDM system,

C_{fid} is the annual cost of an FID in the FREEDM system,

C_{sst} is the annual cost of an SST,

I is the net investment made in the FREEDM system,

$Load_{system\text{-}peak}$ is the peak load on the FREEDM distribution feeder at any given time,

N_x is the optimal number of FIDs in the FREEDM system,

N_y is the optimal number of SSTs in the FREEDM system,

ROI is the rate of interest,

SST_{rating} is the kVA rating of an SST,

T_{fid} is the service life of an FID,

T_{sst} is the service life of an SST,

Y is the payback period for an investment made in the FREEDM system.

Table 4.6 and Table 4.7 presents the simulation results for the determination of payback period and other associated parameters for Case C. Figure 4.13 shows the simulation result for the test case C3 when the random variables representing the input parameters are triangular distributed. It can be seen that the probability of payback period within 50 years is highly unlikely. In addition, it can be observed that the probability of payback within 185 – 200 years is around 85 – 95%. The MATLAB file associated with the simulation can be referred from Appendix G.

Figure 4.14 shows the cumulative distribution plot for test case C3. The probability of the payback within 150 years in case of topology 3 is close to 80%. In addition, the results for topology 2 and topology 3 are very close to each other. In both the cases, the probabilities of payback period are almost the same.

In addition, Figure 4.15 presents a comparison between different test cases by considering each topology. Test case C1, in which the random variables are normally distributed, provides highly pessimistic results whereas test case C2, where the random variables are triangular distributed provides the most optimistic results. Moreover, it can be observed that recovery of 100% investment is loosely dependent on the type of distribution.

Table 4.6 Determination of probability of payback period for Case C

Test case	Topology	Probability distribution	Payback period probability (Y in years)					
			$P_r\ (Y \leq 35)$	$P_r\ (Y \leq 70)$	$P_r\ (Y \leq 105)$	$P_r\ (Y \leq 140)$	$P_r\ (Y \leq 175)$	$P_r\ (Y \leq 210)$
C1	1	C_{fid}: Normal	0	0.025	0.292	0.610	0.793	0.888
	2	C_{sst}: Normal	0	0.041	0.363	0.674	0.835	0.912
	3	B_o: Normal B: Normal	0	0.050	0.396	0.701	0.852	0.922
C2	1	C_{fid}: Uniform	0	0.196	0.508	0.725	0.849	0.915
	2	C_{sst}: Uniform	0	0.225	0.539	0.751	0.865	0.923
	3	B_o: Uniform B: Uniform	0	0.238	0.553	0.762	0.871	0.927
C3	1	C_{fid}: Triangular	0	0.065	0.382	0.655	0.811	0.895
	2	C_{sst}: Triangular	0	0.091	0.435	0.699	0.839	0.911
	3	B_o: Triangular B: Triangular	0	0.099	0.451	0.711	0.847	0.916
C4	1	C_{fid}: Uniform	0	0.152	0.480	0.698	0.828	0.901
	2	C_{sst}: Uniform	0	0.192	0.519	0.725	0.845	0.911
	3	B_o: Normal B: Triangular	0	0.202	0.529	0.733	0.849	0.914
C5	1	C_{fid}: Triangular	0	0.113	0.470	0.712	0.840	0.908
	2	C_{sst}: Triangular	0	0.147	0.516	0.742	0.857	0.912
	3	B_o: Normal B: Uniform	0	0.155	0.524	0.747	0.861	0.921

Table 4.7 Determination of statistics of payback period for Case C

Test case	Topology	Probability distribution	Expectation* (E) *(years)* (years)	Standard deviation* (σ) *(years)* (years)	Conditional expectation $(Ex \leq 35)$ *(years)*	Conditional expectation $(Ex \leq 70)$ *(years)*	Conditional expectation $(Ex \leq 105)$ *(years)*	Conditional expectation $(Ex \leq 140)$ *(years)*	Conditional expectation $(Ex \leq 175)$ *(years)*	Conditional expectation $(Ex \leq 210)$ *(years)*
C1	1	C_{fid}: Normal	141.4	61.0	-	63.7	88.0	105.4	117.1	124.9
	2	C_{sst}: Normal	132.6	57.4	-	63.2	86.5	102.4	112.6	119.2
	3	B_o: Normal B: Normal	128.7	55.7	-	63.1	85.8	101.1	110.6	116.7
C2	1	C_{fid}: Uniform	120.1	63.6	-	58.6	75.9	89.5	99.1	105.7
	2	C_{sst}: Uniform	115.7	62.3	-	57.6	74.5	87.7	96.5	102.5
	3	B_o: Uniform B: Uniform	113.9	61.7	-	57.3	74.0	86.9	95.4	101.2
C3	1	C_{fid}: Triangular	134.3	63.1	-	62.7	84.1	99.6	110.3	117.8
	2	C_{sst}: Triangular	127.5	60.6	-	61.9	82.5	97.0	106.6	113.9
	3	B_o: Triangular B: Triangular	125.6	59.8	-	61.7	82.1	96.2	105.7	112.1
C4	1	C_{fid}: Uniform	125.6	65.7	-	60.9	78.5	91.8	101.8	109.1
	2	C_{sst}: Uniform	120.3	64.3	-	59.7	76.5	89.1	98.6	105.3
	3	B_o: Normal B: Triangular	119.1	64.1	-	59.4	76.0	88.5	97.7	104.3
C5	1	C_{fid}: Triangular	125.5	62.6	-	61.5	81.2	94.6	103.9	110.4
	2	C_{sst}: Triangular	120.2	61.2	-	60.6	79.3	91.9	100.5	106.5
	3	B_o: Normal B: Uniform	119.1	61.0	-	60.4	79.0	91.4	99.8	105.7

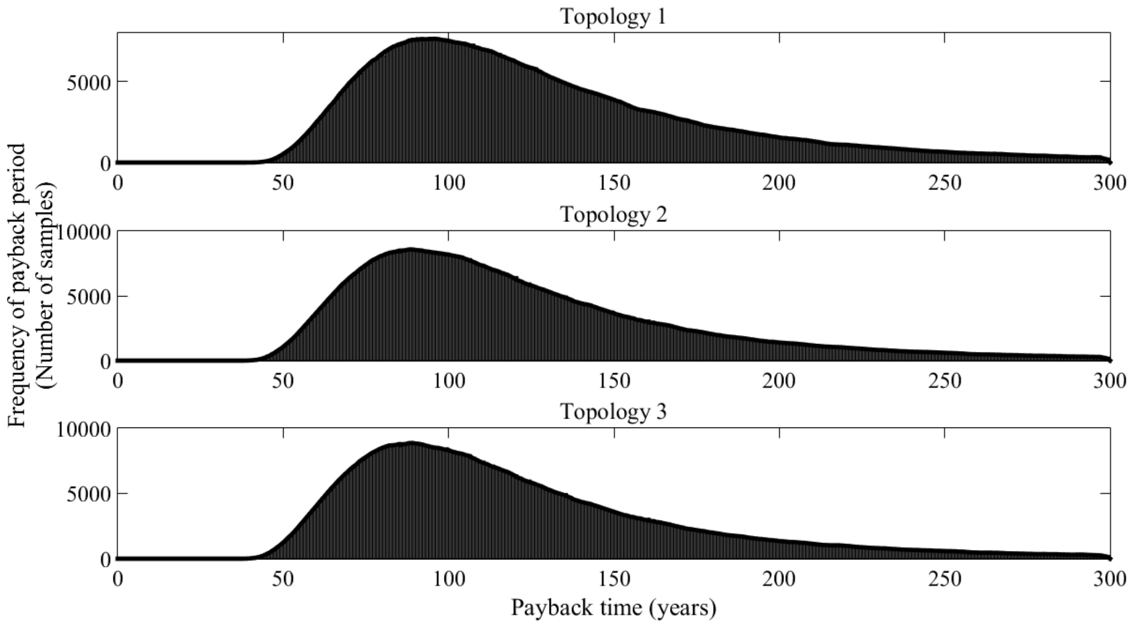

Figure 4.13 Frequency of payback period for three topologies for test-case C3

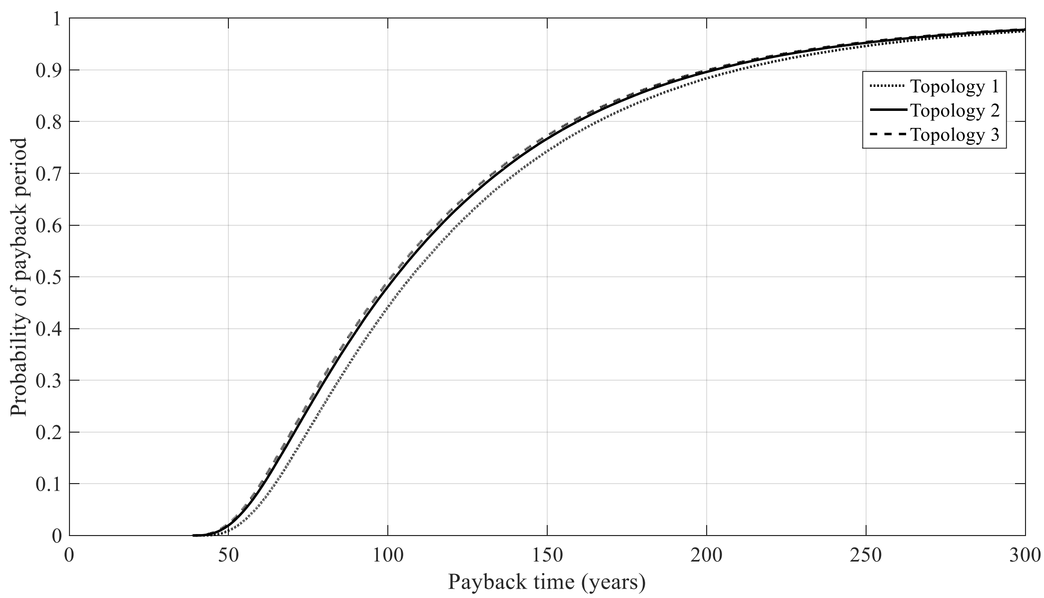

Figure 4.14 Cumulative distribution function plot for test-case C3

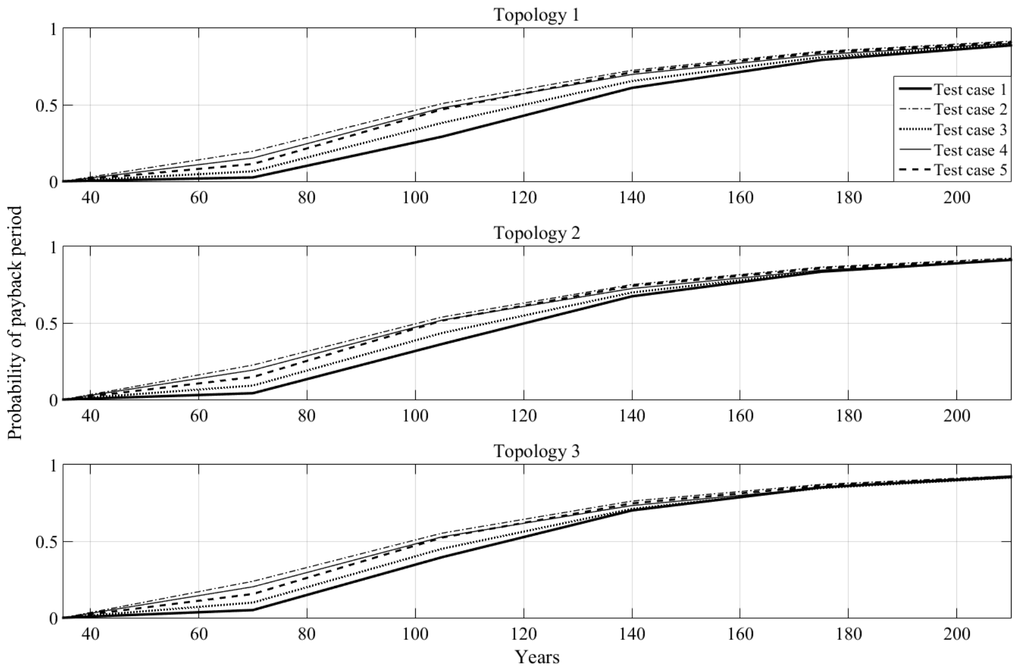

Figure 4.15 Comparison of payback period for different topologies, Case C

4.5 Normally distributed random variables as input data

In the simulation results for the different cases shown in Figures 4.8, 4.12 and 4.15, it is observed that the test cases where the random variables of the input parameters are normally distributed, the probability of the payback period is either highly optimistic or highly pessimistic when compared with the other test cases. One important observation is that the results with normal distribution cannot be trusted since the standard deviation considered in the study is small which has an impact on the determination of the payback period.

Representation of the input parameters as random variables with a normal distribution is highly unlikely since the input parameters are cost terms and cost cannot be negative. In the samples, there is a possibility that the cost may be negative depending upon the standard deviation of the normally distributed curve. It is observed that if the standard deviation is large, there is a high probability of input parameters being negative. In such a case, the payback period may not be a true representation of the actual value and study would be insignificant. This forces the random variable to be in a very narrow range so that the cost may not become negative. This degrades the probabilistic nature of the problem. Thus, it can be said that normal distribution may not be a good choice for representing the input parameters.

In statistical study, the three-sigma rule of thumb provides a conventional heuristic that 99.73% of the values in a normal distribution would be covered within three standard deviations [37]. If the standard deviation is very small, there is a possibility that the random samples would be repetitive. In contrast, if the standard deviation is large, the values may turn negative. Thus, the calculated payback period could not be considered as an indicator for the return on investment since the input parameter is not justifiable, if normally distributed.

Case D: Multivariate case

In the previous cases, the probability of the payback period was estimated based upon the assumption that the input parameters are a function of one random variable. In this section, it has been assumed that the input parameters are a function of several random variables. These random variables may have the same or different distributions. The input parameters are the same as shown in Table 4.1 and the assumed distribution of these parameters are a combination of two or more random variables. The different test cases are shown in Table 4.8. It must be noted that the investment and annual benefit grow as per the rate of interest.

Table 4.8 Assumed distribution for input parameters for Case D

Test case	Cost of SST	Cost of FID	Benefit factor	Annual benefit
D1	T + U	T + U	T + U	T + U
D2	T + N	T + N	T + N	T + N
D3	U + N	U + N	U + N	U + N
D4	T + U + N	T + U + N	T + U + N	T + U + N

- T: Triangular distribution, N: Normal distribution and U: Uniform distribution

A simulation was performed to understand the impact of this change on the payback period. The simulation was performed on MATLAB and the corresponding code is present in Appendix H. Table 4.9 presents the simulation result for determination of payback period if the input parameters are considered as a function of several random variables. In addition, Table 4.10 presents the statistics associated with the payback period.

Figure 4.16 shows the density of the input parameters for test case D4. Figure 4.17 presents the simulation results for test case D4 in which the probability of payback period at different intervals are determined for the three topologies. It can be observed that the probability of the payback period within 50 years is highly unlikely. The payback period is most likely to be within the range of 180 - 210 years.

Figure 4.18 presents the cumulative distribution function plot for the test case D4. The plot shows a similar result as observed earlier in different cases. This study shows that the simulation results from topology 2 and topology 3 are identical and thus topology 2 can be given preference over topology 3 in the FREEDM distribution system due to its structural simplicity.

Figure 4.19 presents a comparison between different test cases. It can be observed that in all the test cases, the probability of payback period is nearly the same. In addition, the probability of payback within 50 years is nearly equal to zero whereas the probability of payback within 125-150 years is close to 80%.

Table 4.9 Determination of probability of payback period for Case D

Test case	Topology	Payback period probability (Y in years)					
		$P_r\ (Y \leq 35)$	$P_r\ (Y \leq 70)$	$P_r\ (Y \leq 105)$	$P_r\ (Y \leq 140)$	$P_r\ (Y \leq 175)$	$P_r\ (Y \leq 210)$
D1	1	0	0.032	0.329	0.629	0.800	0.889
	2	0	0.053	0.395	0.685	0.836	0.911
	3	0	0.059	0.411	0.696	0.842	0.915
D2	1	0	0.004	0.206	0.573	0.79	0.892
	2	0	0.009	0.299	0.674	0.852	0.927
	3	0	0.010	0.312	0.685	0.859	0.932
D3	1	0	0.012	0.275	0.606	0.792	0.888
	2	0	0.024	0.353	0.675	0.836	0.913
	3	0	0.027	0.369	0.688	0.847	0.918
D4	1	0	0.005	0.227	0.579	0.787	0.889
	2	0	0.014	0.318	0.671	0.845	0.923
	3	0	0.015	0.331	0.682	0.852	0.926

Table 4.10 Determination of statistics of payback period for Case D

Test case	Topology	Expectation* *(E) (years)*	Standard Deviation* *(σ) (years)*	Conditional expectation *(Ex ≤ 35) (years)*	Conditional expectation *(Ex ≤ 70) (years)*	Conditional expectation *(Ex≤ 105) (years)*	Conditional expectation *(Ex ≤ 140) (years)*	Conditional expectation *(Ex ≤ 175) (years)*	Conditional expectation *(Ex ≤ 210) (years)*
D1	1	138.87	62.04	-	64.42	87.03	103.30	114.53	122.18
	2	130.77	58.97	-	63.69	85.33	100.40	110.34	116.97
	3	129.05	58.29	-	63.57	84.99	99.69	109.37	115.81
D2	1	145.92	56.27	-	66.23	92.58	111.27	123.35	131.04
	2	133.92	51.13	-	65.77	90.90	107.61	117.53	123.42
	3	132.43	50.23	-	65.77	90.76	107.14	116.80	122.50
D3	1	142.62	59.91	-	65.52	90.00	107.09	118.51	126.27
	2	133.27	56.53	-	64.97	88.23	103.76	113.72	120.21
	3	131.41	55.65	-	64.93	87.94	103.14	112.81	119.05
D4	1	145.37	57.99	-	66.23	91.71	109.91	121.91	129.70
	2	133.84	52.91	-	65.70	89.92	106.23	116.33	122.52
	3	132.36	52.22	-	65.66	89.72	105.73	115.58	121.55

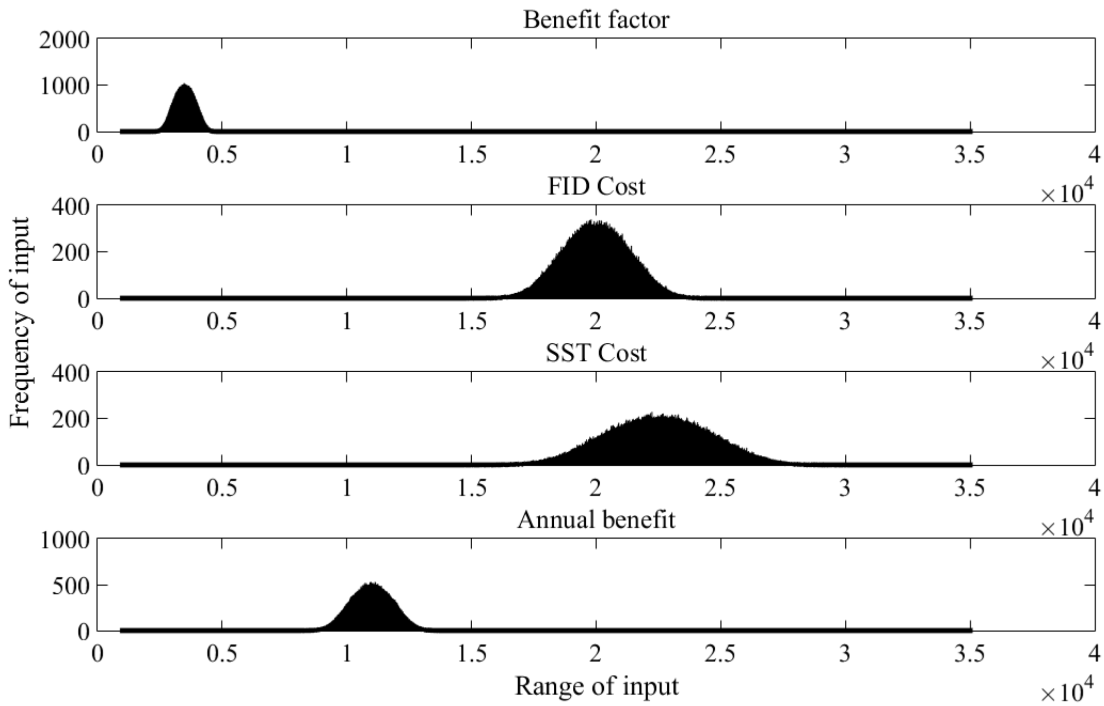

Figure 4.16 Input parameters for test-case D4

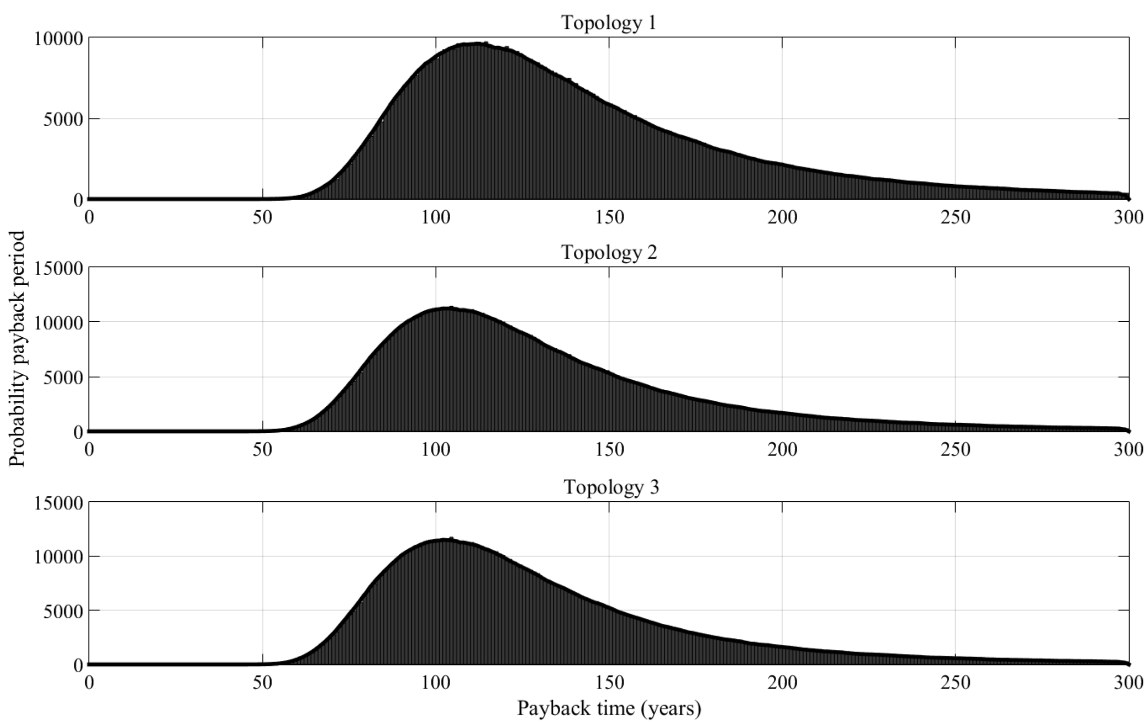

Figure 4.17 Determination of payback period for test-case D4

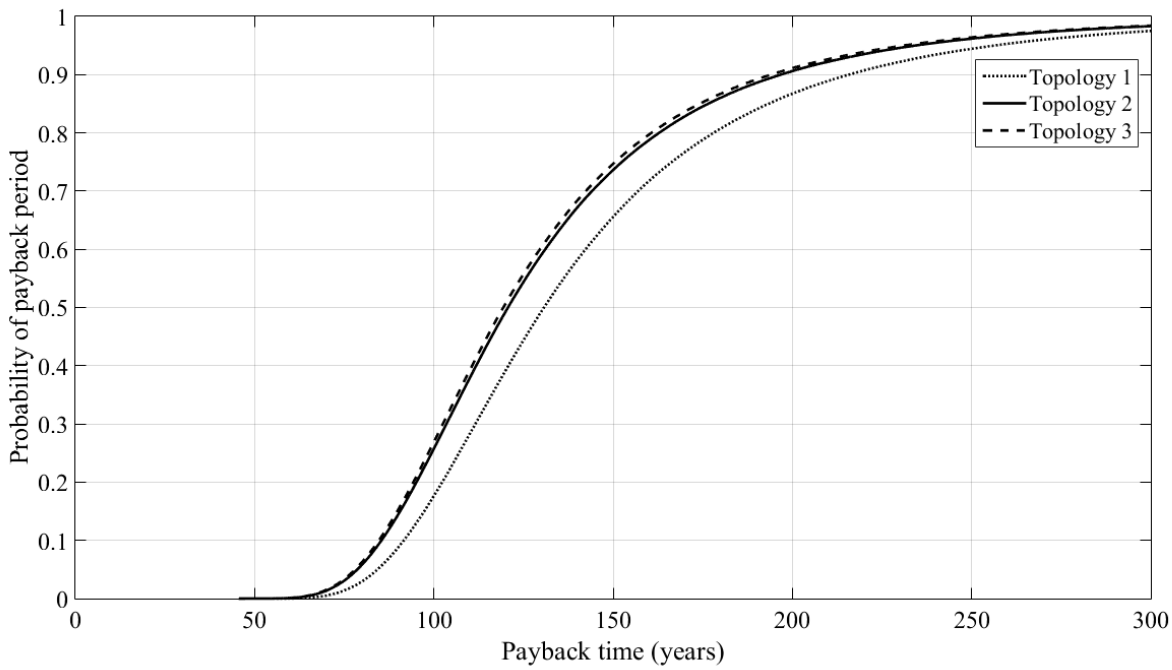

Figure 4.18 Cumulative distribution function plot for test-case D4

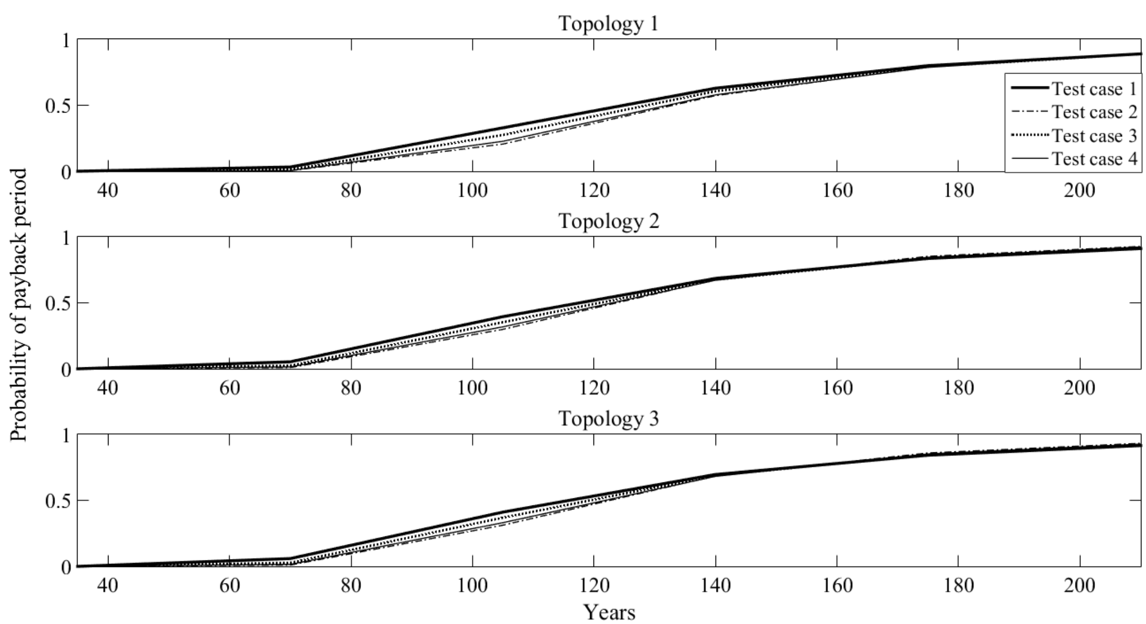

Figure 4.19 Comparison of payback period based on topology, Case D

Case E: Energy storage – FIDs replaced with circuit breakers

In Chapter 3, it was proposed that energy storage devices might result in significant cost savings due to replacement of FIDs with circuit breakers. In addition, the payback period was expected to reduce. In this section, a study is conducted to understand the economic impacts of inclusion of energy storage components in the FREEDM system.

A simulation study is done in order to estimate the probability of the payback period where the investment is constant with time. The associated MATLAB code is presented in Appendix I. Table 4.11 presents the additional input data used in the simulation study of Case E along with the data presented in Table 4.1. Figure 4.20 presents the simulation result, which represents the probability of payback period at different time intervals. It must be noted that the simulation results are compared with the result obtained from Case B.

Table 4.11 Input parameters for Case E

Description	Range of value ($)	Type of distribution assumed
Cost of capacitor	2000 – 2500	Triangular
Cost of circuit breaker	1500 – 3000	Triangular

From the results, it can be observed that there is no significant difference in the payback period when compared with test case B3 of Case B. The probability of payback within 50-60 years is close to 0.95, which is more or less in agreement with the results obtained from Case B.

The reason can be explained based upon the net investment cost in both the cases. In Case B, the investment is primarily due to the cost of FIDs and SSTs whereas in Case E the investment is due to the circuit breakers, SSTs and the energy storage capacitors. In both the cases, the investment is more or less the same. Due to this reason, the impact of both the cases on the payback period is similar.

In the simulation study for Case E, the investment was expected to reduce due to replacement of FIDs with circuit breakers. In contrast, it is observed that the investment remained the same. This is due to an added cost of the energy storage devices (capacitors) in the SSTs. The net investment in Case E is similar to the investment in Case B due to which the simulation results related to the payback period are very close to each other.

In addition, the probability of payback within a certain interval for all the three topologies is nearly the same in Case E unlike any other cases. It was seen that topology 3 is the most effective topology due to its high reliability. As shown in Chapter 2, topology 1 requires more number of FIDs in comparison to topology 3 to achieve a similar level of reliability. This shows that the investment in topology 1 is more in comparison to topology 3. However, the prices of circuit breakers are very less in comparison to FIDs due to which the difference in the investment cost between the 3 topologies is diminished. On the other hand, the number of SSTs required in all the three topologies is the same. This ensures that the investment in the three topologies is very close to each other. Similar implications can be inferred from Figure 4.21, which presents the cumulative distribution plot for the three topologies. It is observed that the probability of payback period during any interval is nearly the same for all three topologies.

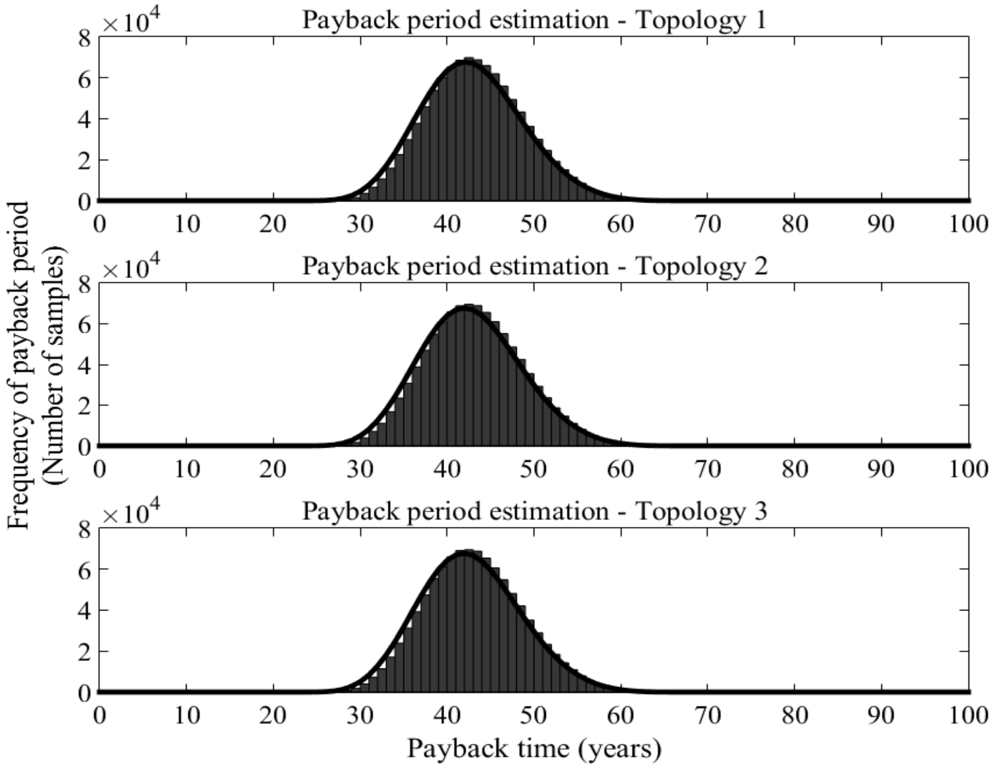

Figure 4.20 Determination of probability of payback period for Case E

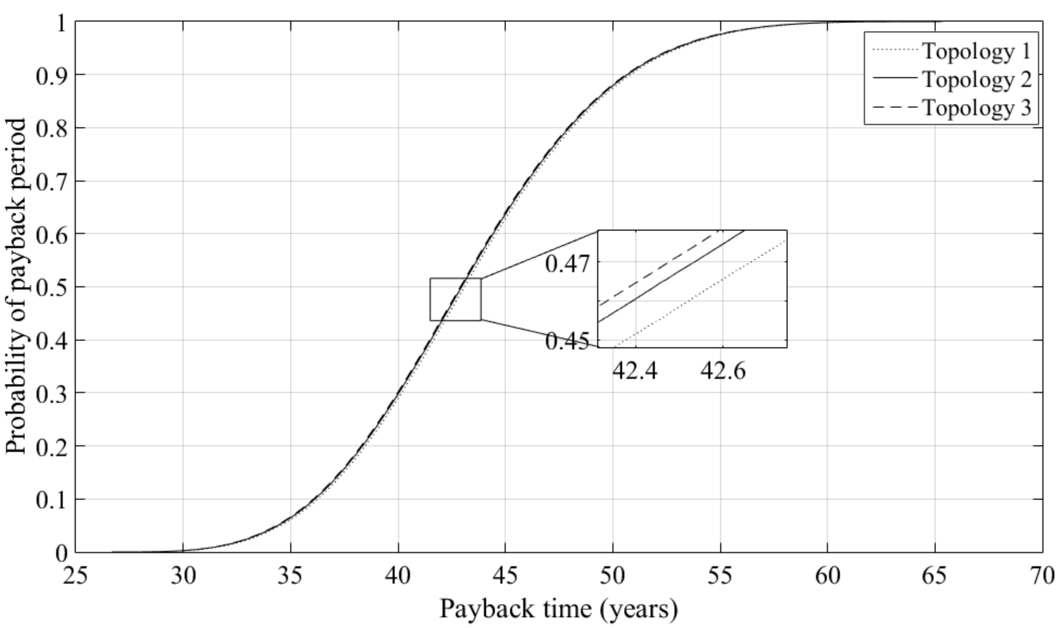

Figure 4.21 Cumulative distribution function plot for Case E

This study concludes that introduction of energy storage elements in the distribution system obviates the necessity of considering different topologies for the FREEDM distribution system. The payback period simulation results for all the three topologies closely match each other. In addition, the probability of payback period for the FREEDM system within 50-60 years is around 0.95, which is very close to results obtained from Case B. The results are subjected to change with changes in the cost of the energy storage devices.

4.6 Summary of test results

The payback period and associated statistical parameters are evaluated for different test cases. From the simulation results, it is observed that the probability of payback period within a specific year range varies vastly for the considered cases. It can also be seen that topology 3 has the lowest payback period in all the cases whereas topology 1 has the highest payback period. The simulation results also indicate that the results obtained by using topology 2 closely matches with topology 3. Figure 4.22 presents the density of the payback period for the different cases based on the simulation results obtained for topology 3. It can be seen that in Case B, the probability of payback within 50 years is close to 1 whereas in Case A and Case C the probability nears 0.5 and 0.05 respectively. Figure 4.23 shows the cumulative distribution function plot for the three cases with an assumed configuration of topology 3.

Case B presents a more realistic scenario since it considers the investment to be a sunk. The investment is constant with time whereas the annual benefit increases as per the rate of interest. From the simulation results for Case B, the probability for the payback within 50 years is nearly 0.95 - 1.

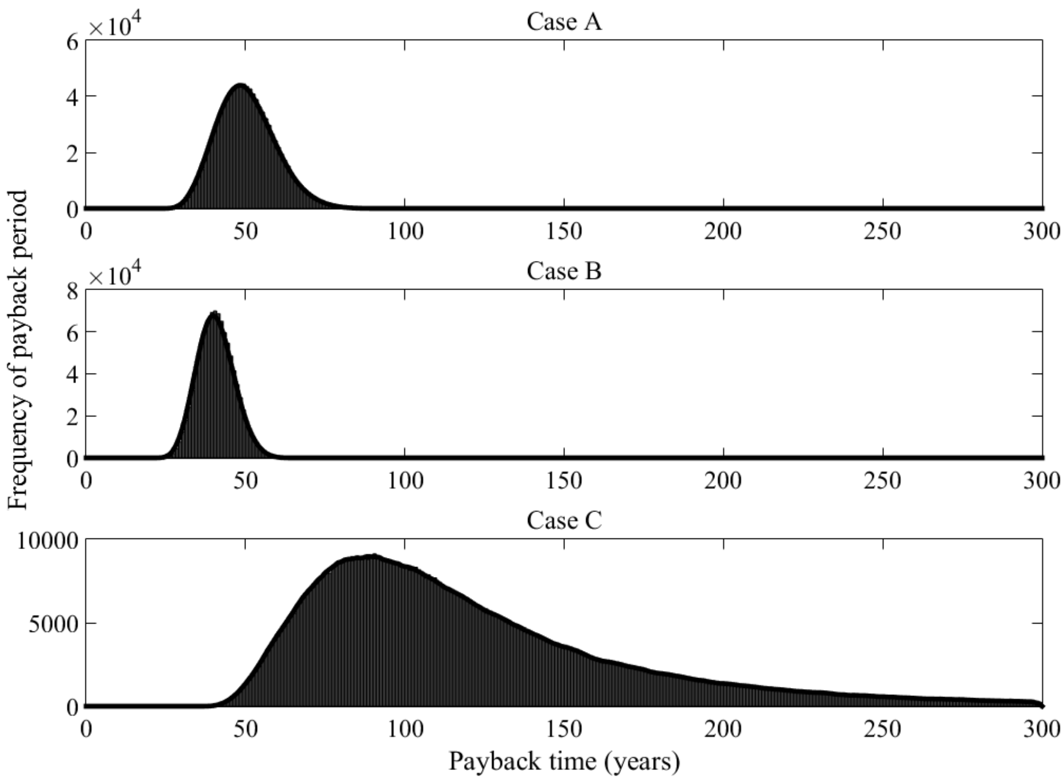

Figure 4.22 Comparison based on topology 3 simulation result for Case A, B and C

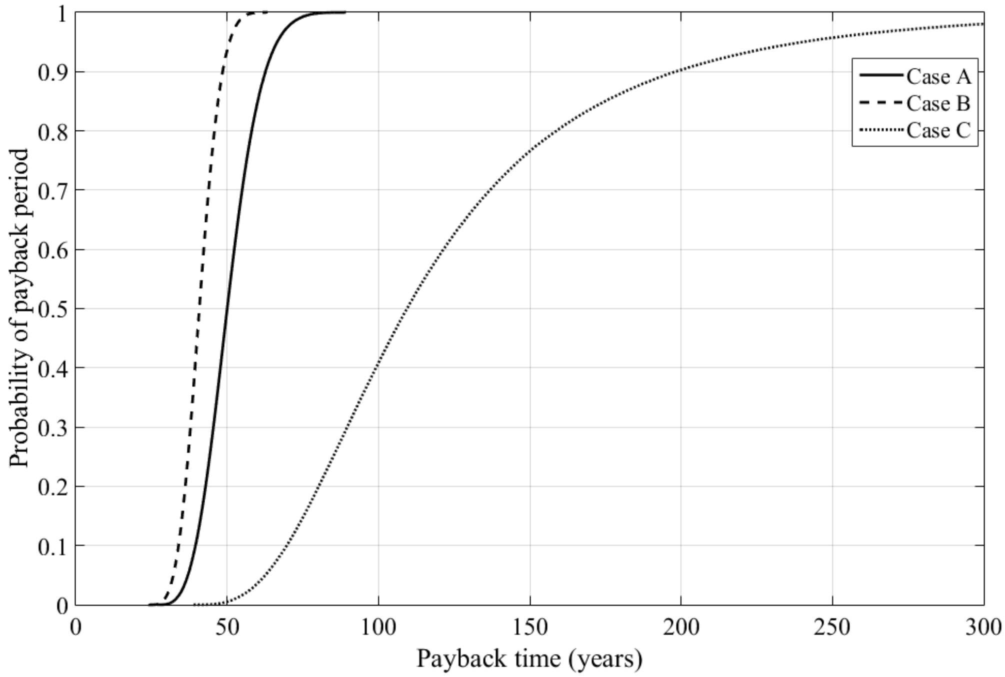

Figure 4.23 Cumulative distribution plot for topology 3, Case A, B and C

4.7 Development of a case study with a shorter payback period

In the previous cases (Case A, Case B, Case D and Case E), it was observed that the payback period lies in the range of 50-60 years. In this section, the effect of increased benefit in terms of state rebate due to the adoption of the FREEDM system is studied on the payback period. A state rebate of 20% on the customer's annual bill is assumed due to the adoption of FREEDM system. The state rebate adds towards the annual benefit. In addition, the investment cost is expected to reduce by 20-40%. The SSTs and FIDs are presently in the laboratory testing stage and the prices of these components are expected to reduce upon commercialization. The number of customers present on the FREEDM feeder is 40. Table 4.12 presents the net improvement in the annual benefit due to the rebate in the customer's annual bill.

Table 4.12 Estimation of improvement in annual benefit due to rebate on annual electric bill

Average monthly electric bill ($)	Annual rebate for adoption of FREEDM system ($)
138	13000

- The data for average monthly electric bill is considered for a 1000 sq. ft. apartment and the relevant information can be found in [47]
- The number of customers assumed in the FREEDM distribution system is 40
- The annual rebate is an added benefit in the FREEDM system

A simulation study is performed to estimate the payback period for this added benefit. Figure 4.24 presents the simulation result showing the density of payback period at different time intervals. From the simulation results, it was observed that the probability of payback within 20 years is close to 1. In addition, Figure 4.25 presents the cumulative distribution function plot, which shows that the payback period lies within 20-22 years.

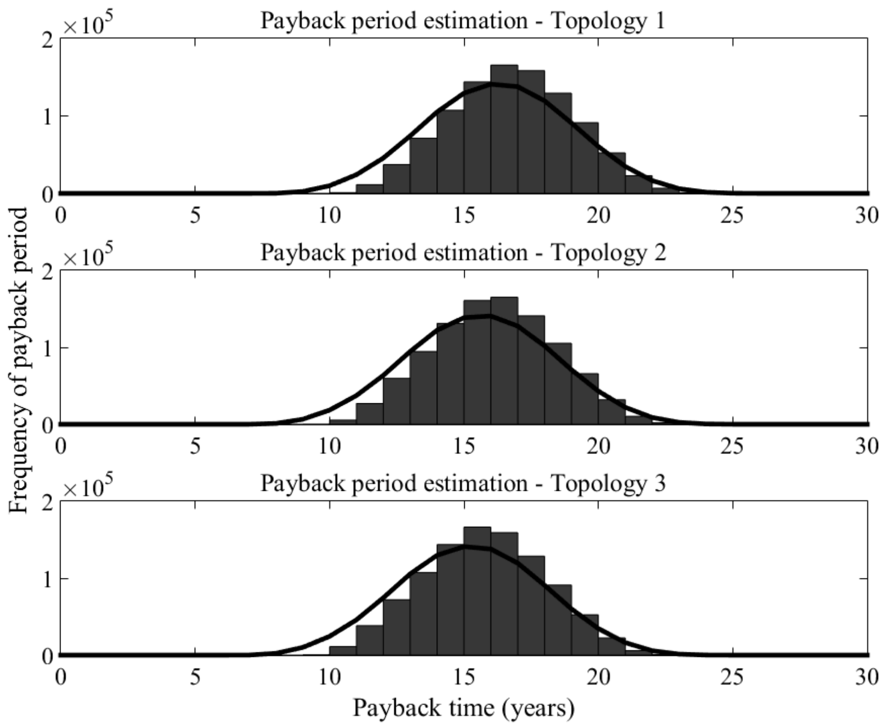

Figure 4.24 Density of probability of payback period: Improved benefit due to annual rebate

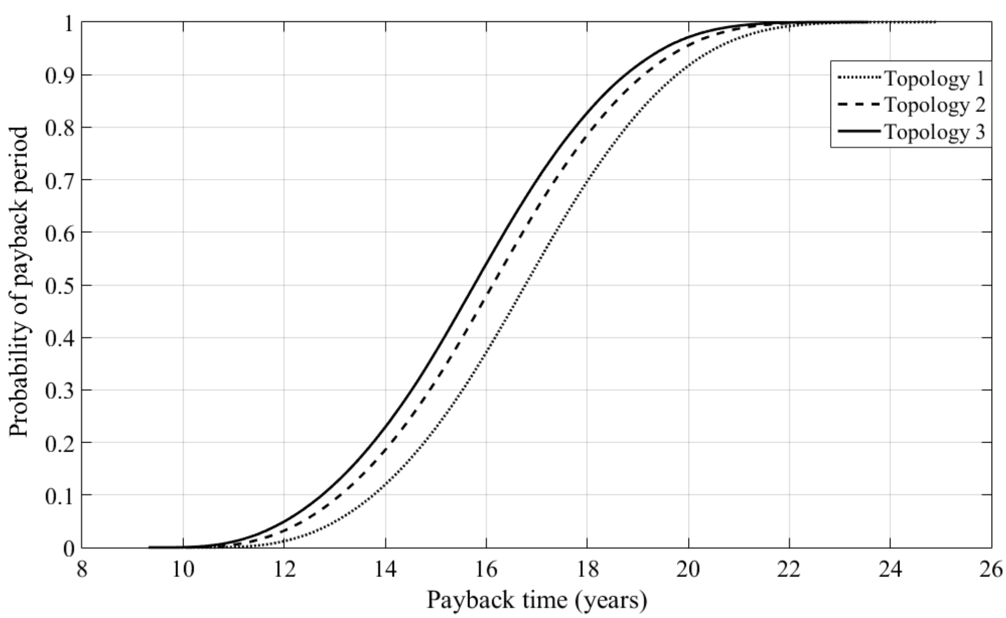

Figure 4.25 Cumulative distribution function: Improved benefit due to annual rebate

4.8 Conclusions drawn from the Monte Carlo simulation tests

The probabilistic model is developed in an attempt to mitigate the uncertainties in the determination of the payback period due to lack of information in the pricing structure of the SSTs and the FIDs. The true cost of an FID and SST is unknown presently, making the determination of payback period complex and uncertain. However, the probabilistic model is successful in estimating the probability of payback period within a specific time frame, making the model a respectable indicator to understand the advantages/disadvantages of investing in the FREEDM system. The results reflect that the investment in the FREEDM system is not advantageous at present since the payback time is very large, as seen from Case B. This figure is likely to improve with developments in the field of solid state devices which would bring down the cost of the SSTs and FIDs in future.

The FREEDM distribution system is studied for a residential sector in this thesis. If an industrial sector is considered, the net annual benefits from such a system will increase. Collectively, it will bring down the payback period and the investment made in the FREEDM distribution system can be recovered much faster than the stipulated time frame obtained from the simulation results.

In addition, the results indicate that topology 2 and topology 3 exhibit similar results in all the cases. Topology 2 has a lower infrastructural cost when compared to topology 3 due to reduced number of circuit breakers and feeder sections. This makes adoption of topology 2 more desirable.

CHAPTER 5 DETERMINATION OF PROBABILITY DENSITY FUNCTION USING A THEORETIC APPROACH

5.1 Introduction

In Chapter 4, a detailed study was performed to determine the payback period. The payback period was estimated for the FREEDM system by adopting the Monte Carlo simulation technique. In Chapter 5, the PDF of payback period is evaluated theoretically. This study will provide a foundation to conduct probabilistic studies to determine the payback period for the FREEDM system theoretically.

This chapter will provide a comparison between the Monte Carlo simulation results obtained in Chapter 4 with the system theoretic results obtained from [37 - 42]. The notion behind this study is to ensure that the Monte Carlo simulation results agree with the system theoretic results for the cost benefit assessment of the FREEDM system.

5.2 PDF of independent random variables

Consider that there are two random variables X and Y with a given PDF. For the purposes of discussion related to FREEDM system, it is assumed that the two variables are independent of each other. Consider an arithmetic process is performed on the two variables. The resultant random variable is denoted as Z. Table 5.1 presents the generalized PDF for resultant random variable Z for different arithmetic operation on two random variable X and Y [43] [44].

Table 5.1 Generalized PDF expressions for different arithmetic operations

Arithmetic operation	PDF	Range of κ
Addition	$f_z(\kappa)= f_x(\kappa)* f_y(\kappa)$	$\kappa \in [0,\infty]$
Subtraction	$f_z(\kappa)= f_x(-\kappa)* f_y(\kappa)$	$\kappa \in [0,\infty]$
Product	$f_z(\kappa)=\int_{-\infty}^{\infty}(1/\mid y\mid)f_{xy}(\kappa/y,y)dy$	$\kappa \in [0,\infty]$
Ratio	$f_z(\kappa)=\int_{-\infty}^{\infty}(\mid y\mid)f_{xy}(\kappa/y,y)dy$	$\kappa \in [0,\infty]$

In Chapter 4, the payback period was estimated for a simplified case. In the simplified case, the investment and annual benefit were represented by random variables distributed over a defined range. The PDF of these two variables were known and the payback period was estimated using the Monte Carlo simulation technique. The relationship between the payback period and two random variables is given by:

$$Y = I / B,$$

where,

B is the annual benefit from the FREEDM system,

I is the investment made in the FREEDM system,

Y is the payback period.

It can be observed that the payback period can be estimated theoretically by evaluating the ratio of the two random variables. This can be done by using the relationship defined in Table 5.1. In the discussion related to cost benefit assessment, it is assumed that the two random variables are normally distributed for a sufficiently large sample. This is in accordance with the central limit theorem mentioned in [45]. In addition, it is also assumed that the mean of the two random variable are sufficiently larger than their respective standard deviations. Therefore, the PDF of the payback period can be determined by using the frequency function mentioned in [39] [40] [42] [46]. It must be noted that the two input random variables (I and B) are normally distributed. The expression used to determine the PDF of the payback period is strictly valid for normally distributed input variables. The frequency function is given by:

$$f_Y(Y) = (1/\sqrt{2\pi})\frac{(\mu_B\sigma_I^2 + \mu_I\sigma_B^2 Y)}{\sqrt{(\sigma_I^2 + \sigma_B^2 Y^2)}} exp((-0.5)\frac{(\mu_I - \mu_B Y)^2}{(\sigma_I^2 + \sigma_B^2 Y^2)}), \tag{5.1}$$

where,

μ_I is the mean of investment,

μ_B is the mean of benefit,

σ_I is the standard deviation of investment,

σ_B is the standard deviation of annual benefit,

f_Y is the frequency function of the payback period, *Y* is the payback period.

It must be noted that the expression for PDF of payback period in (5.1) is bounded by certain assumptions, which have been discussed above. A more generalized expression for determination of the PDF of the payback period is given in (5.2) [41],

$$f_Y(Y)=\frac{b(Y)d(Y)}{a^3(Y)}\frac{1}{\sqrt{2\pi}\sigma_I\sigma_B}[\phi(\frac{b(Y)}{a(Y)})-\phi(-\frac{b(Y)}{a(Y)})]+\frac{1}{a^2(Y)\pi\sigma_I\sigma_B}e^{-c/2}, \qquad (5.2)$$

where,

$$a(Y)=\sqrt{\frac{1}{\sigma_I^2}Y^2+\frac{1}{\sigma_B^2}}, \qquad (5.3)$$

$$b(Y)=\frac{\mu_I}{\sigma_I^2}Y+\frac{\mu_B}{\sigma_B^2}, \qquad (5.4)$$

$$c=\frac{\mu_I^2}{\sigma_I^2}+\frac{\mu_B^2}{\sigma_B^2}, \qquad (5.5)$$

$$d(Y)=exp\frac{(b^2(Y)-ca^2(Y))}{2a^2(Y)}, \qquad (5.6)$$

$$\phi(t)=\int_{-\infty}^{t}\frac{1}{\sqrt{2\pi}}exp(-\frac{1}{2}u^2)du. \qquad (5.7)$$

5.3 FREEDM system theoretic study results

A simulation study is performed to determine the PDF of the payback period using a system theoretic approach. In this method, the PDF of the payback period is estimated using (5.1). It is assumed that the two random variables (investment and annual benefit) are normally distributed. The total investment required in the FREEDM system is calculated based upon the number of FIDs and SSTs required for a particular topology. Table 5.2 presents the input parameters used in the simulation study. The associated MATLAB code is present in Appendix I.

Table 5.2 Input parameters for system theoretic study

Description	Value	
	Investment (I)	Annual benefit (B)
Mean ($)	μ_I = 650000	μ_B = 10000
Standard deviation ($)	σ_I = 10000	σ_B = 500

The PDF of payback period obtained from the theoretical study is compared with the existing results from Chapter 4. This is done in order to understand the variation between the theoretic results and the results obtained from Monte Carlo simulation. If the two results match each other, it can be confirmed that the Monte Carlo simulation technique is an effective algorithm to determine the payback period for the FREEDM system.

From Figure 5.1, it is observed that the two results (one from Monte Carlo simulation and other from the system theoretic approach) closely trail each other. In other words, the PDF of the payback period evaluated in both the cases are almost the same, which ratifies that the Monte Carlo simulation method conforms the system theoretic results. Table 5.3 presents the simulation results, which gives an insight towards the statistics associated with the two simulation results. On comparing the results from both the methods, it is seen that the mean and standard deviation of the payback period from both the methods closely match each other. In other words, the probability of the payback period in any interval is approximately the same in both the methods.

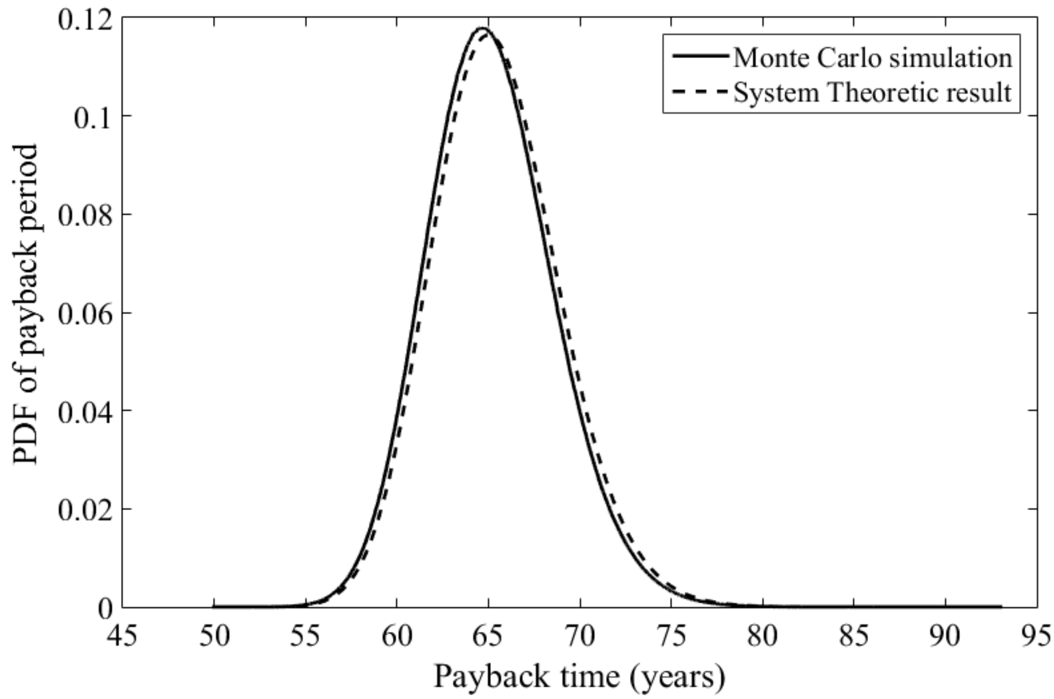

Figure 5.1 System theoretic simulation result

Table 5.3 Comparison of results from the two methods

Description	Probability of payback period (Y in years)				Payback period mean ($)	Payback period standard deviation ($)
	$P_r(Y \leq 60)$	$P_r(Y \leq 65)$	$P_r(Y \leq 70)$	$P_r(Y \leq 75)$	(μ_Y)	(σ_Y)
Monte Carlo simulation	0.057	0.499	0.915	0.995	65.16	3.43
System theoretic method	0.047	0.462	0.899	0.993	65.49	3.47

5.4 Sensitivity study to determine the operating limits

The expression used to determine the PDF of the payback period in (5.1) is based on certain assumptions, which must be followed. The assumptions are,

- The random variables are normally distributed along their mean.
- The mean of the random variables is sufficiently large when compared with the standard deviation.

If these assumptions are violated, the system theoretic results may start to differ from the Monte Carlo simulation results. A study is performed to determine the permissible difference between the two methods used in determination of the payback period for the FREEDM system. The ratio of standard deviation to mean (¥) of investment and annual benefit will reflect the limiting factor for the assumption to be valid. In addition, this study will help in understanding whether the expression used for determination of PDF of the payback period in (5.1) is valid for the cost benefit assessment of the FREEDM system. The results are compared based on the difference between the mean and standard deviation obtained from the two methods. A MATLAB code has been presented in Appendix I representing the same.

From Figure 5.2 and Figure 5.3, it can be seen that as the value of ¥ increases beyond 0.15, there is a significant difference in the results obtained by the two methods (system theoretic and Monte Carlo simulation). There are two different reasons associated to this aberration. They are:

- As the value of ¥ increases, the standard deviation increases depending upon the value of ¥. Due to the increase in the standard deviation of investment and annual benefit, the expression to determine the PDF of the payback period mentioned in (5.1) does not hold and generates erroneous data. This can be seen as a deviation of the system theoretic data from the Monte Carlo simulation data in Figure 5.2 and Figure 5.3 when ¥ increases beyond 0.13.
- In addition, an increase in ¥ beyond 0.13 would mean that the standard deviation is beyond 13% of the mean which will result in generation of erroneous data from Monte Carlo simulation. The Monte Carlo simulation will generate certain samples of payback period, which are at extremities that will be incoherent with the system theoretic result. A further increase in the standard deviation might also result in generation of certain samples where the payback period is negative, which is not possible.

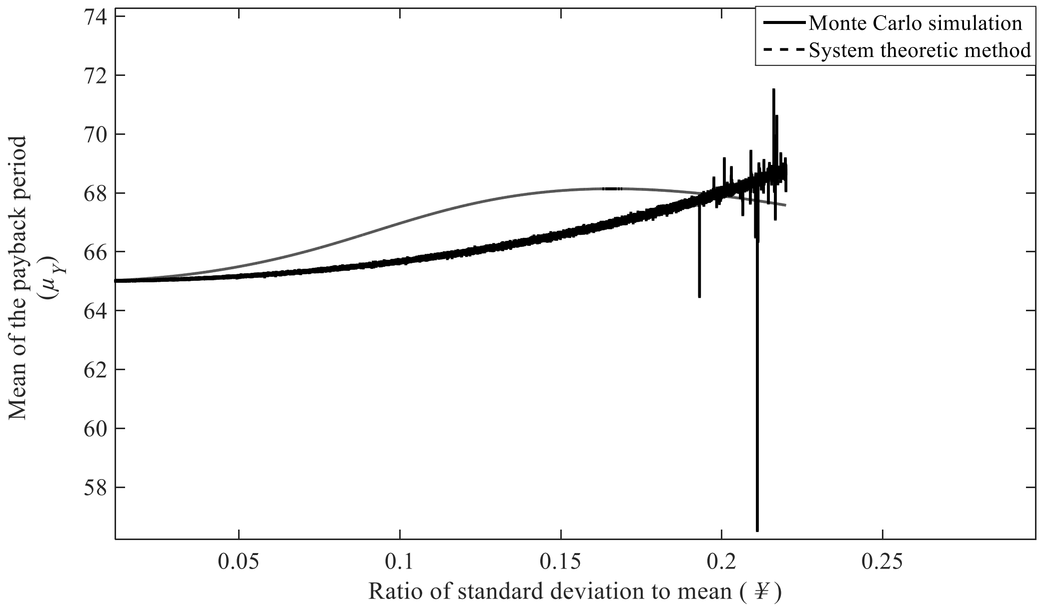

Figure 5.2 Sensitivity study based on the mean of the payback period

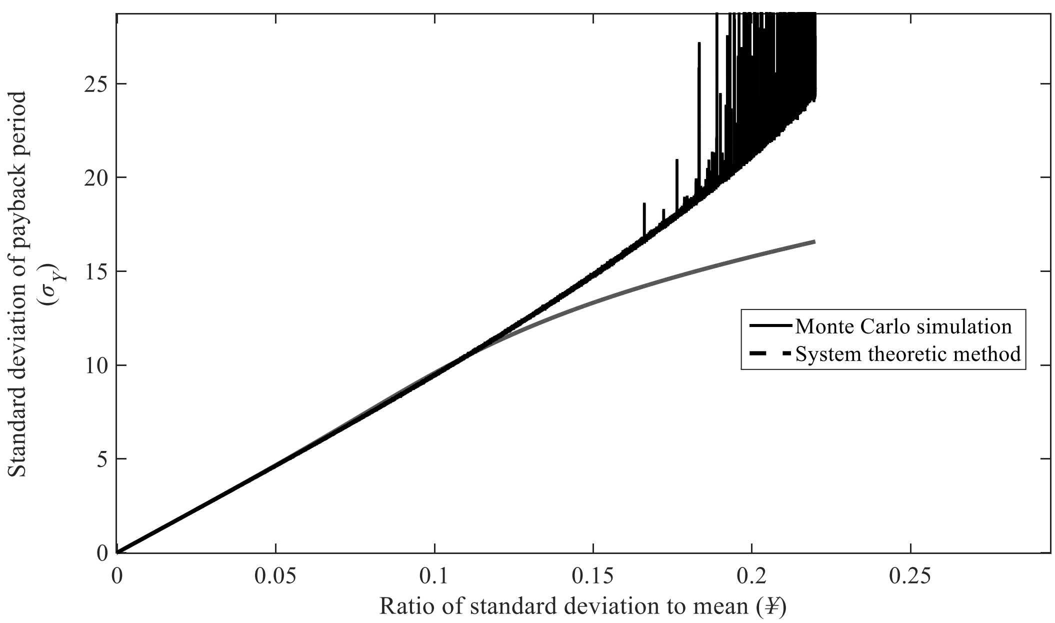

Figure 5.3 Sensitivity study based on the standard deviation of the payback period

From the study it can be confirmed that the value of ¥ cannot be greater than 0.13. In other words, the value of standard deviation of the two input variables cannot be more than 13% of their respective mean. If the probabilistic estimation of the payback period is performed within this range of ¥, the results from the two methods will be accurate and coherent with each other. Table 5.4 summarizes the study and presents the result for determination of the operating limit of ¥. It is observed that the percentage difference in the mean and standard deviation of the payback period from the two methods, namely the system theoretic method and Monte Carlo simulation is around 2% and 4% respectively.

Table 5.4 Summary of the sensitivity study

¥	Description	Simulation result	Operating limits
¥ ≤ *0.13*	$\sigma_I \leq 0.13\ \mu_I$ and $\sigma_B \leq 0.13\ \mu_B$	Monte Carlo result and system theoretic result match	Acceptable At ¥ = 0.13 Percentage difference in mean of payback period: 2.1% Percentage difference in standard deviation of payback period: 4.2%
¥ > *0.13*	$\sigma_I > 0.13\ \mu_I$ and $\sigma_B > 0.13\ \mu_B$	Monte Carlo result and system theoretic result do not match.	Unacceptable

5.5 Summary

This chapter introduced the theoretical approach to determine the PDF of a random variable, which is a function of two random variables. The PDF of the payback period for the FREEDM system was computed theoretically and the results were confirmed with the Monte Carlo simulation results obtained in Chapter 4. It was observed that the results from both the methods were identical and coherent with each other up to a certain value of ¥.

A sensitivity study was performed to determine the value of ¥ which provided the operating limit. It was observed that the standard deviation of the investment and annual benefit should not be above 13% of their respective mean values. The assumptions were valid for the FREEDM cost benefit assessment, thereby conforming the operating limit and assumptions discussed above.

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

This thesis focuses on the cost benefit assessment of the FREEDM system. The primary goal of this thesis was to set up a method to perform cost benefit assessment under uncertainty and to determine the number of years required to recoup the investment made in the FREEDM distribution system. The FREEDM test bed was studied and three distinct topologies were suggested which closely represented the distribution system.

A detailed study was done in order to determine the number of fault isolation devices required to meet reliability requirements. The optimal number of FIDs was determined because of the tradeoff study between benefit due to reliability and annual cost the FID. A better solution to the existing problem related to high cost of FIDs was also suggested in terms of energy storage devices. Studies indicated that a capacitor with a capacitance close to 93μF is required to support a load of 30 kW for a period of 10 cycles in a 60 Hz system. The inclusion of energy storage devices in the SSTs will replace the FIDs with conventional circuit breakers.

The reliability study results indicated that the FREEDM system has a great potential since the evaluated SAIFI was close to 0.125, which is almost 9 times better than the current U.S. SAIFI of 0.95, as per the PG&E reliability report for the year 2015. A detailed system theoretic study was also performed for the three topologies and a general expression was derived for the SAIFI, which was in accordance with the evaluated results.

In addition, the payback period was estimated for the investment made towards the FREEDM distribution system by conducting a probabilistic study. Due to uncertainties in the cost of the FREEDM components, a stochastic study was performed to obviate any irregularities in the estimation of the payback period. From the results, it became clear that the payback period lies in the range of 50 – 60 years and the probability of payback within this period is around 95 - 100%.

The payback period was also estimated for improved benefit in the FREEDM system due to an annual rebate on customer's electric bills. It was observed that the payback period is in the range of 18-20 years. To achieve this, the annual benefit must be at least $ 22000 whereas the investment must be close to $ 450000.

Presently, the obtained results may seem to be pessimistic since the payback period is high and there are limited benefits of adopting the FREEDM system. However, there are prospective outcomes from the thesis, which points towards the fact that with time the FREEDM distribution system may seem to be a lucrative option.

6.2 Recommendations for future work

The main recommendations for future work relating to the cost benefit assessment of the FREEDM system are:

- Develop a model to study the impact of energy storage devices on the SSTs. The model should include the battery itself, the solid state interface, constraints such as the energy storage and charge / discharge power limits.

- Perform a detailed study to determine other potential benefits from the FREEDM system. This should include: CO_2 reduction, reduction of fossil fuel use, reduction in transmission and sub transmission system size, maximization of renewable resources and their concomitant sustainability aspects.

- A study of the sociopolitical issues of rebates for the purpose of implementing a FREEDM-like system.

- A realistic study of the impact of fossil fuels, especially natural gas, on the entire cost / benefit study. Further, the question of the limits and acceptability of hydraulic fracturing (fracking).

- The inclusion of inflation in equipment and energy prices (this is partially done in [46]).

- Put the FREEDM system in the real time setting and determine the potential benefits and investment costs associated to it. This includes the realistic development of the infrastructure to implement FREEDM.

- Develop detailed system theoretic models to study the payback period based on different probability distributions.

- Life estimation study of the FREEDM components to determine the failure rates of FIDs and SSTs. The relationship to reliability should be investigated as well.

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[47] Average electric bill for customers in the U.S can be found online: http://eyeonhousing.org/2015/03/average-monthly-electrical-bill-by-state-2013/

APPENDIX A

MATLAB CODE FOR DETERMINATION OF OPTIMAL NUMBER OF FIDs

```
% Matlab code for determination of Nx - Optimal number of FIDs

clear all;
close all;
clc;

%input parameters
cfid = 10000; %FID cost
ccb = 0; % Circuit breaker cost - not considered
time = 15; % service life of FID
ben_fact = 4000; % benefit factor

no=0:1:20;  % Considering for 20 FID counts in the system

% selection of topology

top1 = 0;
top2 = 0;
top3 = 1;

if(top1 == 1)
    cf=(cfid+ccb*2)/time;
    for i=1:1:21;
    cost(i)=(no(i)*cf);
    eser(i)=no(i)/(no(i)+1);
    benefit=ben_fact*eser;
    end
    nx = (ben_fact/cf)-1;

    plot(no,cost,no,benefit,'linewidth',4);
    hold on;
  xlabel('Number of FIDs (N)');
 ylabel('Benefit (B), Investment(I)');
 title('Determination of optimal number of FIDs (Nx)');
 set(gca,'fontname', 'Times New Roman', 'fontsize', 18);

    display(nx);
    display(benefit);
    display(eser);
    display(cost);

end

if(top2 == 1)

    cf=(cfid+ccb*4)/time;
    for i=1:1:21;
    cost(i)=(no(i)*cf);
    eser(i)=(no(i)+1)/(no(i)+2);
    benefit=0.5*ben_fact*eser;
    end
    nx = ((0.5*ben_fact-2*cf)+sqrt((0.5*ben_fact)^2+4*cf^2))/(2*cf);
    plot(no,cost,no,benefit,'linewidth',4);
    hold on;
```

```
xlabel('Number of FIDs (N)');
 ylabel('Benefit (B), Investment(I)');
 title('Determination of optimal number of FIDs (Nx)');
 set(gca,'fontname', 'Times New Roman', 'fontsize', 18);

    display(nx);
    display(benefit);
    display(eser);
    display(cost);
end

if(top3 == 1)
    cf=(cfid+ccb*8)/time;
    for i=1:1:21;
    cost(i)=(no(i)*cf);
    eser(i)=(no(i)+3)/(no(i)+4);
    benefit=0.25*ben_fact*eser;
    end
    nx=((0.25*ben_fact-
4*cf)+sqrt((0.25*ben_fact)^2+16*cf^2+4*cf*(0.25*ben_fact)))/(2*cf);
    plot(no,cost,no,benefit,'linewidth',4);
    hold on;

xlabel('Number of FIDs (N)');
 ylabel('Benefit (B), Investment(I)');
 title('Determination of optimal number of FIDs (Nx)');
 set(gca,'fontname', 'Times New Roman', 'fontsize', 18);

    display(nx);
    display(benefit);
    display(eser);
    display(cost);
end
```

APPENDIX B

DETERMINATION OF OPTIMAL NUMBER OF FIDs FOR DIFFERENT TOPOLOGIES

The expression for N_x is derived by considering that the annual FID cost (C_{fid}) to be equal to the benefit due to reliability (B_r). The expression is determined separately for the three different topologies and is presented below,

Topology 1:

Figure 2.2 presents the architecture for topology 1. It can be seen that if a fault occurs in the FREEDM distribution system, the energy served by the feeder to the load would vary depending upon the number of FIDs in the system. Table B.1 gives the amount of the energy served by the feeder to the loads for different number of FIDs in the line during a fault for topology 1.

Table B.1 Variation of modeled parameters with FID count for topology 1

Number of FID	Energy served (E_{ser})	Energy not served ($E_{not\text{-}ser}$)	Annual cost of FID (C_{fid})	Benefit due to reliability (B_r)
	(MWh/year)	(MWh/year)	($/year)	($/year)
0	*0*	*E*	*0*	*0*
1	*1/2E*	*1/2E*	$1C_f$	$1/2\ E\ B_o$
2	*2/3E*	*1/3E*	$2C_f$	$2/3\ E\ B_o$
3	*3/4E*	*1/4E*	$3C_f$	$3/4\ E\ B_o$
4	*4/5E*	*1/5E*	$4C_f$	$4/5\ E\ B_o$
5	*5/6E*	*1/6E*	$5C_f$	$5/6\ E\ B_o$

From Table B.1, a general mathematical formulation can be derived for all the system parameters,

i) Energy served by the feeder

$$E_{ser} = (n/(n+1))E,$$

ii) Energy not served by the feeder

$$E_{not-ser} = E - E_{ser},$$

$$E_{not-ser} = (1/(n+1))E,$$

iii) Annual cost of FIDs

$$Annual\,\mathrm{cost\,of}\ FIDs = nC_{fid},$$

iv) Benefit due to reliability

$$B_r = B_o(n/(n+1))E_{not-ser}^{n=0},$$

For determination of the optimal number of FIDs,

$$Annual\,cost\ of\ FIDs = Benefit\,due\,to\,reliability$$

$$N_x C_{fid} = B_o(N_x/(N_x+1))E_{not-ser}^{n=0},$$

$$N_x = (B_o/C_f)E_{not-ser}^{n=0} - 1,$$

For topology 1,

$$E_{not-ser}^{n=0} = 1\ \text{MWh /year},$$

The optimal number of FIDs is given by,

$$N_x = (B_o / C_f) - 1.$$

Topology 2:

Figure 2.3 presents the architecture for topology 2. Table B.2 gives the amount of the energy served by the feeder to the loads for different number of FIDs in the line during a fault for topology 1.

Table B.2 Variation of modeled parameters with FID count for topology 2

Number of FID	Energy served (E_{ser})	Energy not served ($E_{not\text{-}ser}$)	Annual cost of FID (C_{fid})	Benefit due to reliability (B_r)
	(MWh/year)	(MWh/year)	($/year)	($/year)
0	*1/2E*	*1/2E*	*0*	$(1/4)\ (1/2E)\ B_o$
2	*3/4E*	*1/4E*	$2C_f$	$(3/8\ )\ (1/2E)\ B_o$
4	*5/6E*	*1/6E*	$4C_f$	$(5/12\ )\ (1/2E)\ B_o$
6	*7/8E*	*1/8E*	$6C_f$	$(7/16\ )\ (1/2E)B_o$
8	*9/10E*	*1/10E*	$8C_f$	$(9/20\ )\ (1/2E)\ B_o$
10	*11/12E*	*1/12E*	$10C_f$	$(11/24)\ (1/2E)\ B_o$

From Table B.2, a general mathematical formulation can be derived for all the system parameters,

i) Energy served by the feeder

$$E_{ser} = ((n+1)/(n+2))E\ ,$$

ii) Energy not served by the feeder

$$E_{not-ser} = E - E_{ser} \ ,$$

$$E_{not-ser} = (1/(n+2))E \ ,$$

iii) Annual cost of FIDs

$$Annual \,\mathrm{cost\,of}\; FIDs = nC_{fid} \ ,$$

iv) Benefit due to reliability

$$B_r = B_o((n+1)/(n+2))E_{not-ser}^{n=0} \ ,$$

For determination of the optimal number of FIDs,

$$Annual\,cost\;of\;FIDs = Benefit\,due\,to\,reliability$$

$$N_x C_{fid} = B_o((N_x+1)/(N_x+2))E_{not-ser}^{n=0} \ ,$$

For topology 2,

$$E_{not-ser}^{n=0} = 0.5\,\mathrm{MWh/year} \ ,$$

$$N_x = (0.5B_o - 2C_{fid} + \sqrt{(4C_f^2 + 0.25B_o^2}))/2C_{fid} \ ,$$

The optimal number of FIDs is given by,

$$N_x = (0.5B_o - 2C_{fid} + \sqrt{(4C_f^2 + 0.25B_o^2}))/2C_{fid} \ .$$

Topology 3:

Figure 2.4 presents the architecture for topology 3. Table B.3 gives the amount of the energy served by the feeder to the loads for different number of FIDs in the line during a fault for topology 1.

Table B.3 Variation of modeled parameters with FID count for topology 3

Number of FID	Energy served (E_{ser})	Energy not served $(E_{not\text{-}ser})$	Annual cost of FID (C_{fid})	Benefit due to reliability (B_r)
	(MWh/year)	(MWh/year)	($/year)	($/year)
0	*3/4E*	*1/4E*	*0*	*(3/16) (1/4E)* B_o
4	*7/8E*	*1/8E*	$4C_f$	*(7/32) (1/4E)* B_o
8	*11/12E*	*1/12E*	$8C_f$	*(11/48) (1/4E)* B_o
12	*15/16E*	*1/16E*	$12C_f$	*(15/64) (1/4E)* B_o
16	*19/20E*	*1/20E*	$16C_f$	*(19/80) (1/4E)*B_o
20	*23/24E*	*1/24E*	$20C_f$	*(23/96) (1/4E)* B_o

From Table B.3, a general mathematical formulation can be derived for all the system parameters,

i) Energy served by the feeder

$$E_{ser} = ((n+3)/(n+4))E \,,$$

ii) Energy not served by the feeder

$$E_{not-ser} = E - E_{ser} \,,$$

$$E_{not-ser} = (1/(n+4))E \,,$$

iii) Annual cost of FIDs

$$Annual\,\mathrm{cost\,of}\; FIDs = nC_{fid} \,,$$

iv) Benefit due to reliability

$$B_r = B_o((n+3)/(n+4))E_{not-ser}^{n=0} \,,$$

For determination of the optimal number of FIDs,

$$Annual\,\mathrm{cost}\;\mathrm{of}\;\; FIDs = Benefit\,due\,to\,reliability \,,$$

$$N_x C_{fid} = B_o((N_x+3)/(N_x+4))E_{not-ser}^{n=0} \,,$$

For topology 3,

$$E_{not-ser}^{n=0} = 0.25\,\mathrm{MWh/year} \,,$$

$$N_x = (0.25B_o - 4C_{fid} + \sqrt{(16C_f^2 + 0.0625B_o^2 + C_f B_o})) / 2C_{fid} \,,$$

The optimal number of FIDs is given by,

$$N_x = (0.25B_o - 4C_{fid} + \sqrt{(16C_f^2 + 0.0625B_o^2 + C_f B_o})) / 2C_{fid} \,.$$

APPENDIX C

DETERMINATION OF THE CAPACITOR VALUE FOR ENERGY STORAGE IN SST

The appendix illustrates the steps for determination of the capacitance to be used as an energy storage device for replacement of FID with circuit breakers. The following data must be considered for the determination of the capacitance:

i) The operating voltage *(V_1)* which must be maintained during an outage is 12kV

ii) Residual voltage across the capacitor *(V_2)* to be considered is 6kV

iii) The capacitor must maintain the system voltage for a minimum of 10 cycles

iv) 20 kW of load is considered for one solid state transformer

v) Frequency of the system is 60 Hz.

The energy stored in a capacitor is given by,

$$E_{cap} = 0.5C_{cap}(V_1^2 - V_2^2),$$

The energy stored in the capacitor for the desired load of 20 kW is given by,

$$E_{cap} = 20\,kW(10/60)s$$

$$E_{cap} = 3.33\,kJ,$$

For fully discharged condition, $V_2 = 0$,

$$C_{cap} = 46.25\ \mu F.$$

For a partial discharge condition, $V_2 = 6\ kV$,

$$C_{cap} = 61.67\ \mu F.$$

APPENDIX D

DETERMINATION OF SAIFI USING MONTE CARLO STUDY FOR A 10 FID DISTRIBUTION SYSTEM

```
% Matlab code

%Monte carlo simulation for Single line equivalent
% understanding change in SAIFI index by varying the FID
clear all;
close all;
%line length
A=10;
B=10;
C=10;

FID_pos = sort(randi((A+B+C),10,1));

%Assuming  one SST FEEDS 4 customers at the max
%assuming 1000 customers in the line, SST=250
SST_pos = sort(randi((A+B+C),250,1));

% n=100000;
% p=1;
% while (p<n)
%Initialization

Region_01=0;
Region_12=0;
Region_23=0;
Region_34=0;
Region_45=0;
Region_56=0;
Region_67=0;
Region_78=0;
Region_89=0;
Region_910=0;
Region_X=0;

for i=1:1:250

    % defining SST regions and No of customers in each region

        if(SST_pos(i)<=FID_pos(1))
        Region_01 = Region_01+1;

        elseif(SST_pos(i)>FID_pos(1) && SST_pos(i)<= FID_pos(2))
        Region_12 = Region_12 + 1;

        elseif(SST_pos(i)>FID_pos(2) && SST_pos(i)<=FID_pos(3))
         Region_23 = Region_23 +1;

        elseif(SST_pos(i)>FID_pos(3) && SST_pos(i)<=FID_pos(4))
         Region_34 = Region_34 +1;

          elseif(SST_pos(i)>FID_pos(4) && SST_pos(i)<=FID_pos(5))
         Region_45 = Region_45 +1;

          elseif(SST_pos(i)>FID_pos(5) && SST_pos(i)<=FID_pos(6))
         Region_56 = Region_56 +1;
```

```
         elseif(SST_pos(i)>FID_pos(6) && SST_pos(i)<=FID_pos(7))
        Region_67 = Region_67 +1;

         elseif(SST_pos(i)>FID_pos(7) && SST_pos(i)<=FID_pos(8))
        Region_78 = Region_78 +1;

        elseif(SST_pos(i)>FID_pos(8) && SST_pos(i)<=FID_pos(9))
        Region_89 = Region_89 +1;

         elseif(SST_pos(i)>FID_pos(9) && SST_pos(i)<=FID_pos(10))
        Region_910 = Region_910 +1;

        elseif(SST_pos(i)>FID_pos(10))
           Region_X = Region_X+1;
        end

end


    F = 1000000;

         for j=1:1:F

 % Random fault positions

             fault=(A+B+C)*rand(1,1);

%               count =0;

% estimating the number of customers affected due to fault at a
location
             if(fault<=FID_pos(1))
                count = Region_01;

            elseif(fault > FID_pos(1) && fault <= FID_pos(2))
            count = Region_12;

            elseif(fault > FID_pos(2) && fault <= FID_pos(3))
            count = Region_23;

            elseif(fault > FID_pos(3) && fault <= FID_pos(4))
            count = Region_34;

            elseif(fault > FID_pos(4) && fault <= FID_pos(5))
            count = Region_45;

            elseif(fault > FID_pos(5) && fault <= FID_pos(6))
            count = Region_56;

            elseif(fault > FID_pos(6) && fault <= FID_pos(7))
            count = Region_67;

            elseif(fault > FID_pos(7) && fault <= FID_pos(8))
            count = Region_78;
```

```
            elseif(fault > FID_pos(8) && fault <= FID_pos(9))
            count = Region_89;

            elseif(fault > FID_pos(9) && fault <= FID_pos(10))
            count = Region_910;

            elseif(fault > FID_pos(10))
            count = Region_X;
            end

%Evaluating SAIFI index
            %SAIFI index
            Num_cust_interrupted = count*4;
            SAIFI=Num_cust_interrupted/1000;
            Y(j) = SAIFI;
         end


            A1 = Y;
            binranges= 0:0.05:1;
            B1 = histc(A1,binranges);
            bar(binranges,B1,'histc');
            set(gca,'fontname', 'Times New Roman', 'fontsize', 18);
            xlabel('SAIFI Index');
            ylabel('Number of samples (Frequency)');
            xlim([0,1]);
           Saifi_index = sum(Y)/F;
           display(Saifi_index);
```

APPENDIX E

DETERMINATION OF THE PAYBACK PERIOD FOR THE SIMPLIFIED CASE

Matlab code:

```
% Case A Simplified Consideration
% Calculation of payback period for all the three topologies
% Ranges assumed for the parameters
% Benefit_factor = 2000-5000
% Annual benefit = 8000-14000
% Cost of an FID = 10000-25000
% Cost of SST = 15000-30000

% Fixed for all the distribution

% Other parameters assumed to constant such as
% Life of FID = 15 years
% Life of CB = 15 years

clc;
clear;
close all;
%Number of samples
n=1000000;


%Service life of FID and SST
life_FID=15;
life_sst=15;

% Feeder rating = 1 MVA
% Assumed peak demand per residential load = 15kVA
% Three phase SST rating = 25 kVA
% Assumed customers on the line = 40

SST_rating = 25e03;
Resid_peak_demand = 15e03;

%Customers on line and SST count
Cust_line = 40;
SST_count = Cust_line*Resid_peak_demand/SST_rating;

% distribution of input variables - Triangular

Benefit_factor = 2000+(3000)*0.5*(rand(n,1)+rand(n,1));

Annual_cost_fid =(10000+15000*0.5*(rand(n,1)+rand(n,1)))/life_FID;

Annual_cost_sst =(15000+15000*0.5*(rand(n,1)+rand(n,1)))/life_sst;

Benefit_per_year = 8000+6000*0.5*(rand(n,1)+rand(n,1));

% for pp = 1:1:3

%Topology selection
top1=1;
top2=0;
top3=0;
```

```
% Topology 1

if (top1==1)
    for i=1:1:n

       payback_mat(i) = (1/Benefit_per_year(i))*((Benefit_factor(i) -
Annual_cost_fid(i))*(life_FID) + SST_count*
Annual_cost_sst(i)*life_sst);
    end

     payback = real(payback_mat(abs(imag(payback_mat)) < 0.00001));
end

%Topology 2

if (top2==1)
    for i=1:1:n

       payback_mat(i) =
(1/Benefit_per_year(i))*(((0.5*Benefit_factor(i)+
(Benefit_factor(i)*Benefit_factor(i)/(16*Annual_cost_fid(i))))/2)*life_
FID + SST_count* Annual_cost_sst(i)*life_sst);

    end
     payback = real(payback_mat(abs(imag(payback_mat)) < 0.00001));
end

%Topology 3

if (top3==1)
    for i=1:1:n


      payback_mat(i) =
(1/Benefit_per_year(i))*(((0.375*Benefit_factor(i)+
(Benefit_factor(i)*Benefit_factor(i)/(128*Annual_cost_fid(i))))/2)*life
_FID + SST_count* Annual_cost_sst(i)*life_sst);

    end
           payback = real(payback_mat(abs(imag(payback_mat)) <
0.00001));
end


%*****************************************************************
%Plots histogram and bin and smooths the graph
subplot(3,1,pp)

A1 = (payback);
binranges= 0:1:200;
B1 = histc(A1,binranges);
S = smooth(B1);
bar(binranges,B1,'histc');
hold on;
```

```
plot(binranges,S,'color','r','linewidth',3.5);
xlabel('Payback time (years)');
ylabel('Frequency of payback period');
title('Payback period estimation');
set(gca,'fontname', 'Times New Roman', 'fontsize', 18)
xlim([0,100]);
% end
%*************************************************************
% Determining the probability of payback for a range

A1_sort= sort(A1);
lenA1 = length(A1);

% Years considered for probability determination

K = [25 35 45 55 65 75];
for kk = 1:1:6

count=0;
sum=0;

for hh=1:1:lenA1
        if(A1_sort(hh)>=0 && A1_sort(hh)<=K(kk))
            count=count+1;
            sum = sum + A1_sort(hh);
        end
 end

prob_result = count/lenA1;
display(prob_result);

% Conditional probability
cond_exp = sum/count;
display(cond_exp);

end
% mean and standard deviation

mean_payback = mean(A1_sort);
std_payback = std(A1_sort);

display( mean_payback);
display(std_payback);
```

APPENDIX F

DERIVATION OF THE PAYBACK PERIOD FOR CASE B, CHAPTER 4

Case B: Investment assumed constant, annual benefit increases as per ROI

The investment in the FREEDM system can be defined as the net cost of the two major components involved, namely the SSTs and FIDs. The expression is given by:

$$I = N_x C_{fid} T_{fid} + N_y C_{sst} T_{sst} ,$$

$$I_{n,years} = I(1 + ROI/100)^N .$$

Suppose the benefit acquired from the system at the end of every year is *B* and the rate of interest is *ROI%*. The annual maintenance is given by M_o. Table F.1 outlines the benefit accrued at the end of each year. At the end of *N* years, the benefit would be

Table F.1 Benefit acquired from the system

End of year	Benefit
1	$B - M_o$
2	$(B - M_o) + (B - M_o)(1 + ROI/100)$
3	$(B - M_o) + (B - M_o)(1 + ROI/100) + (B - M_o)(1 + ROI/100)^2$
Y	$(B - M_o) + (B - M_o)(1 + ROI/100) + ... + (B - M_o)(1 + ROI/100)^{Y-1}$

Thus at the end of Y years, the net benefit from the system can be defined as,

$$B_{n,years} = (B - M_o)(1 - (1 + ROI/100)^N)/(1 - (1 + ROI/100)) ,$$

At year Y, if the investment is equal to the net benefit from the system,

$$I = (B - M_o)(1 - (1 + ROI/100)^Y)/(1 - (1 + ROI/100)) ,$$

$$Y = (\ln((B - M_o) - I(1 - (1 + ROI/100))) - \ln(B - M_o))/\ln(1 + ROI/100) .$$

APPENDIX G

DETERMINATION OF THE PAYBACK PERIOD FOR CASE B AND C IN CHAPTER 4

Matlab code:

Case B and C:

```
% Calculation of payback period for all the three topologies
% Method 2 - Approximations: Maintenance = 1000 and ROI = 1.5%.
% Ranges assumed for the parameters
% Benefit_factor = 2000-5000
% Annual benefit = 8000-14000
% Cost of an FID = 10000-25000
% Cost of SST = 15000-30000
% Fixed for all the distribution

% Other parameters assumed to constant such as
% Life of FID = 15 years
% Life of CB = 15 years

clc;
clear;
close all;
%Number of samples
n=1000000;

% rate of interest
ROI = 1.5;

k = 1 + ROI/100;
%Maintenance
Mo = 1000;
%Service life of FID and SST
life_FID=15;
life_sst=15;

% Feeder rating = 1 MVA
% Assumed peak demand per residential load = 15kVA
% Three phase SST rating = 25 kVA
% Assumed customers on the line = 40

SST_rating = 25e03;
Resid_peak_demand = 15e03;

%Customers on line and SST count
Cust_line = 40;
SST_count = Cust_line*Resid_peak_demand/SST_rating;

% distribution of input variables - Triangular

Benefit_factor = (3500+500*(randn(n,1)));
Annual_cost_fid =((15000+10000*(rand(n,1))))/life_FID;
Annual_cost_sst =((15000+15000*(rand(n,1))))/life_sst;
Benefit_per_year = 8000 + 6000*0.5*(rand(n,1)+rand(n,1));

% Annual cost according to topologies  - cost of circuit breaker is not
% considered
```

```
%   for pp = 1:1:3

%Topology selection
top1=1;
top2=0;
top3=-1;

% Topology 1

if (top1==1)

    for i=1:1:n
           Opt_Nx(i) = (Benefit_factor(i)/(Annual_cost_fid(i)))-1;

           FID_number(i) = ceil(Opt_Nx(i));

           Investment_FID(i) =
(Annual_cost_fid(i)*life_FID)*FID_number(i);

           Investment_SST(i) = (Annual_cost_sst(i)*life_sst)*SST_count;

           Investment (i) = Investment_FID(i) + Investment_SST(i);


  % Investment considered constant

             Xb = (Benefit_per_year(i) - Mo);
             Xa = (Xb) - (Investment(i)*(1-k));


           payback_mat(i) = (log(Xa) - log(Xb))/(log(k));

          % Investment considered increasing

          Xa = Benefit_per_year(i) - Mo;
          Xb = Investment(i)*(1-k) + Xa;

          payback_mat(i) = (log(Xa) - log(Xb))/(log(k));

    end
     payback = real(payback_mat(abs(imag(payback_mat)) < 0.00001));
end

%Topology 2

if (top2==1)
    for i=1:1:n

           Opt_Nx(i) = ((0.5*Benefit_factor(i)-
2*Annual_cost_fid(i))+sqrt(4*(Annual_cost_fid(i))^2+0.25*(Benefit_facto
r(i))^2))/(2*Annual_cost_fid(i));

            FID_number(i) = ceil(Opt_Nx(i));
```

```
           Investment_FID(i) =
(Annual_cost_fid(i)*life_FID)*FID_number(i);

           Investment_SST(i) = (Annual_cost_sst(i)*life_sst)*SST_count;

           Investment (i) = Investment_FID(i) + Investment_SST(i);

  % Investment considered constant

             Xb = (Benefit_per_year(i) - Mo);
             Xa = (Xb) - (Investment(i)*(1-k));


           payback_mat(i) = (log(Xa) - log(Xb))/(log(k));


               % Investment considered increasing

          Xa = Benefit_per_year(i) - Mo;
          Xb = Investment(i)*(1-k) + Xa;

          payback_mat(i) = (log(Xa) - log(Xb))/(log(k));



    end
     payback = real(payback_mat(abs(imag(payback_mat)) < 0.00001));
end

%Topology 3

if (top3==1)
    for i=1:1:n

            Opt_Nx(i) = ((0.25*Benefit_factor(i)-
4*Annual_cost_fid(i))+sqrt(16*(Annual_cost_fid(i))^2+0.0625*(Benefit_fa
ctor(i))^2 +
Benefit_factor(i)*Annual_cost_fid(i)))/(2*Annual_cost_fid(i));

            FID_number(i) = ceil(Opt_Nx(i));

           Investment_FID(i) =
(Annual_cost_fid(i)*life_FID)*FID_number(i);

           Investment_SST(i) = (Annual_cost_sst(i)*life_sst)*SST_count;

           Investment (i) = Investment_FID(i) + Investment_SST(i);

  % Investment considered constant

             Xb = (Benefit_per_year(i) - Mo);
             Xa = (Xb) - (Investment(i)*(1-k));


           payback_mat(i) = (log(Xa) - log(Xb))/(log(k));
```

```
            % Investment considered increasing


          Xa = Benefit_per_year(i) - Mo;
          Xb = Investment(i)*(1-k) + Xa;

          payback_mat(i) = (log(Xa) - log(Xb))/(log(k));


           end
           payback = real(payback_mat(abs(imag(payback_mat)) <
0.00001));
    end


%*********************************************************************
% %Plots histogram and bin and smooths the graph
%    esubplot(3,1,pp)
% % subplot(3,1,PL);
 A1 = (payback);
binranges= -50:1:300;
B1 = histc(A1,binranges);
S = smooth(B1);
bar(binranges,B1,'histc');
hold on;
plot(binranges,S,'color','r','linewidth',3.5);

set(gca,'fontname', 'Times New Roman', 'fontsize', 18)

xlabel('Payback time (years)');
ylabel('Frequency of payback period');
title('Payback period estimation - Topology 1');
xlim([0,300]);
%    end
% %*******************************************************************
% Determining the probability of payback for a range
for kk = 1:1:6

L = [35 70 105 140 175 210];
count=0;
sum=0;
A1_sort= sort(A1);
lenA1 = length(A1);

for hh=1:1:lenA1
        if(A1_sort(hh)>=0 && A1_sort(hh)<=L(kk))
            count=count+1;
            sum = sum + A1_sort(hh);
        end
 end

prob_result = count/lenA1;
display(prob_result);
```

```
% mean and standard deviation

% mean_payback = mean(A1_sort);
% std_payback = std(A1_sort);
%
% display( mean_payback);
% display(std_payback);

% Conditional probability
cond_exp = sum/count;
display(cond_exp);

end
% mean and standard deviation

mean_payback = mean(A1_sort);
std_payback = std(A1_sort);

display( mean_payback);
display(std_payback);
```

APPENDIX H

DETERMINATION OF THE PAYBACK PERIOD FOR CASE D IN CHAPTER 4

Matlab code:

```
% Calculation of payback period for all the three topologies
% Method D - Approximations: Maintenance = 1000 and ROI = 1.5%.
% Ranges assumed for the parameters
% Benefit_factor = 2000-5000
% Annual benefit = 8000-14000
% Cost of an FID = 10000-25000
% Cost of SST = 15000-30000
% Fixed for all the distribution

% Other parameters assumed to constant such as
% Life of FID = 15 years
% Life of CB = 15 years

clc;
clear;
close all;
%Number of samples
n=1000000;

% rate of interest
ROI = 1.5;

k = 1 + ROI/100;
%Maintenance
Mo = 1000;
%Service life of FID and SST
life_FID=15;
life_sst=15;

% Feeder rating = 1 MVA
% Assumed peak demand per residential load = 15kVA
% Three phase SST rating = 25 kVA
% Assumed customers on the line = 40

SST_rating = 25e03;
Resid_peak_demand = 15e03;

%Customers on line and SST count
Cust_line = 40;
SST_count = Cust_line*Resid_peak_demand/SST_rating;

% distribution of input variables - Triangular

Benefit_factor =  2000 + 1000*0.5*(rand(n,1)+ rand(n,1)) +
1000*rand(n,1)+ 500 + 150*randn(n,1);

Annual_cost_fid = (15000 + 3000*0.5*(rand(n,1)+ rand(n,1)) +
3000*rand(n,1)+ 2000 + 750*randn(n,1))/life_FID;

Annual_cost_sst = (15000 + 5000*0.5*(rand(n,1)+ rand(n,1)) +
5000*rand(n,1)+  2500 + 1000*randn(n,1))/life_sst;
```

```
Benefit_per_year =  8000 + 2000*0.5*(rand(n,1)+ rand(n,1)) +
2000*rand(n,1)+ 1000 + 350*randn(n,1);

% Annual cost according to topologies  - cost of circuit breaker is not
% considered

%  for pp = 1:1:3

%Topology selection
top1=1;
top2=0;
top3=0;

% Topology 1

if (top1==1)

    for i=1:1:n
           Opt_Nx(i) = (Benefit_factor(i)/(Annual_cost_fid(i)))-1;

           FID_number(i) = ceil(Opt_Nx(i));

           Investment_FID(i) =
(Annual_cost_fid(i)*life_FID)*FID_number(i);

           Investment_SST(i) = (Annual_cost_sst(i)*life_sst)*SST_count;

           Investment (i) = Investment_FID(i) + Investment_SST(i);

           % Investment considered increasing

           payback_mat(i) = (log((Benefit_per_year(i) - Mo)) -
log(Investment(i)*(1-k) + Benefit_per_year(i)))/(log(k));


           % Investment considered constant

%            payback_mat(i) = (log((Benefit_per_year(i) - Mo) -
(Investment(i)*(1-k))) -log( Benefit_per_year(i)))/(log(k));

    end
     payback = real(payback_mat(abs(imag(payback_mat)) < 0.00001));
end

%Topology 2

if (top2==2)
    for i=1:1:n

           Opt_Nx(i) = ((0.5*Benefit_factor(i)-
2*Annual_cost_fid(i))+sqrt(4*(Annual_cost_fid(i))^2+0.25*(Benefit_facto
r(i))^2))/(2*Annual_cost_fid(i));

            FID_number(i) = ceil(Opt_Nx(i));
```

```
           Investment_FID(i) =
(Annual_cost_fid(i)*life_FID)*FID_number(i);

           Investment_SST(i) = (Annual_cost_sst(i)*life_sst)*SST_count;

           Investment (i) = Investment_FID(i) + Investment_SST(i);

               % Investment considered increasing

           payback_mat(i) = (log((Benefit_per_year(i) - Mo)) -
log(Investment(i)*(1-k) + Benefit_per_year(i)))/(log(k));


           % Investment considered constant

%            payback_mat(i) = (log((Benefit_per_year(i) - Mo) -
(Investment(i)*(1-k))) -log( Benefit_per_year(i)))/(log(k));

    end
     payback = real(payback_mat(abs(imag(payback_mat)) < 0.00001));
end

%Topology 3

if (top3==3)
    for i=1:1:n

            Opt_Nx(i) = ((0.25*Benefit_factor(i)-
4*Annual_cost_fid(i))+sqrt(16*(Annual_cost_fid(i))^2+0.0625*(Benefit_fa
ctor(i))^2 +
Benefit_factor(i)*Annual_cost_fid(i)))/(2*Annual_cost_fid(i));

            FID_number(i) = ceil(Opt_Nx(i));

           Investment_FID(i) =
(Annual_cost_fid(i)*life_FID)*FID_number(i);

           Investment_SST(i) = (Annual_cost_sst(i)*life_sst)*SST_count;

           Investment (i) = Investment_FID(i) + Investment_SST(i);

           % Investment considered increasing

           payback_mat(i) = (log((Benefit_per_year(i) - Mo)) -
log(Investment(i)*(1-k) + Benefit_per_year(i)))/(log(k));


           % Investment considered constant

%            payback_mat(i) = (log((Benefit_per_year(i) - Mo) -
(Investment(i)*(1-k))) -log( Benefit_per_year(i)))/(log(k));

           end
           payback = real(payback_mat(abs(imag(payback_mat)) <
0.00001));
```

```
    end


%*********************************************************************
% %Plots histogram and bin and smooths the graph
%     subplot(3,1,pp)
% % subplot(3,1,PL);
 A1 = (payback);
binranges= -50:1:300;
B1 = histc(A1,binranges);
S = smooth(B1);
bar(binranges,B1,'histc');
hold on;
plot(binranges,S,'color','r','linewidth',3.5);

set(gca,'fontname', 'Times New Roman', 'fontsize', 18)

xlabel('Payback time (years)');
ylabel('Frequency of payback period');
title('Payback period estimation - Topology 1');
xlim([0,200]);
% %*******************************************************************
Determining the probability of payback for a range
for kk = 1:1:6

L = [25 50 75 100 125 150];
count=0;
sum=0;
A1_sort= sort(A1);
lenA1 = length(A1);

for hh=1:1:lenA1
        if(A1_sort(hh)>=0 && A1_sort(hh)<=L(kk))
            count=count+1;
            sum = sum + A1_sort(hh);
        end
 end

prob_result = count/lenA1;
display(prob_result);

% mean and standard deviation

% mean_payback = mean(A1_sort);
% std_payback = std(A1_sort);
%
% display( mean_payback);
% display(std_payback);

% Conditional probability
cond_exp = sum/count;
display(cond_exp);

end
% mean and standard deviation
```

```
mean_payback = mean(A1_sort);
std_payback = std(A1_sort);

display( mean_payback);
display(std_payback);
```

APPENDIX I

MATLAB CODE FOR DETERMINATION OF *PDF* USING SYSTEM THEORETIC APPROACH

Matlab code

```
%Test -- ratio of c / b
clear all;
close all;
'______________________'
%Number of samples;
n = 10e+07;
%mm = number of bins in the histogram
mm=1000;
%The mean and s.d.
mc=650000;
mb=10000;
sc=10000;
sb=500;
%generate monte carlo samples
c=mc+sc*randn(n,1);
b=mb+sb*randn(n,1);
y=c./b;
%make plot
[nnn,histogram]=hist(y, mm);
area=trapz(histogram, nnn/n);
% subplot(2,1,1);
plot(histogram, nnn/(area*n),'r');
% legend('Monte Carlo method');

hold on;

%Calculate the mean and sd of y

'Mean and S.D. of Y calculated by Monte Carlo'

mean(y)
std(y)
%
%
%Using formula
k=0;
%Get the scale factors the same as for the monte carlo simulation
yfixmin=min(histogram);
yfixmax=max(histogram);
for yy=yfixmin:0.025:yfixmax;
    k=k+1;
    num=mb*sc^2+mc*sb^2*yy;
    den=sqrt(sc^2+sb^2*yy^2);
    ff=(1/sqrt(2*pi))*(num/den);
    ee=exp(-0.5*(mc-mb*yy)^2/(sc^2+sb^2*yy^2));
    fy(k)=ff*ee;
    yyy(k)=yy;
end;
area2=trapz(yyy,fy);
fy=fy/area2;
% subplot(2,1,2);
plot(yyy,fy, 'k')
legend('Monte Carlo simulation','System Theoretic result');
%Calculate the mean and SD of Y using the formula
```

```
'Mean and S.D. of Y calculated by system theoretic formula'

meanofy =trapz(yyy,yyy.*fy)
q=size(yyy);
qq=q(2);
sdinteg=(yyy-ones(1,qq)*meanofy).*(yyy-ones(1,qq)*meanofy);
sdofy=sqrt(trapz(yyy,sdinteg.*fy))

set(gca,'fontname', 'Times New Roman', 'fontsize', 18)
xlabel('Payback time (years)');
ylabel('PDF of payback period');

%% Probability determination

h=0.001;

sum=0;
for jj=0:h:75

    num1=mb*sc^2+mc*sb^2*jj;
    den1=sqrt(sc^2+sb^2*jj^2);
    ff1=(1/sqrt(2*pi))*(num1/den1);
    ee1=exp(-0.5*(mc-mb*jj)^2/(sc^2+sb^2*jj^2));
    fy1=(ff1*ee1)/area2;

    jj = jj + h;

    num2=mb*sc^2+mc*sb^2*jj;
    den2=sqrt(sc^2+sb^2*jj^2);
    ff2=(1/sqrt(2*pi))*(num2/den2);
    ee2=exp(-0.5*(mc-mb*jj)^2/(sc^2+sb^2*jj^2));
    fy2=(ff2*ee2)/area2;

    sum = sum + (fy1+fy2)*h/2;


    jj = jj - h;
end;

display(sum);
```