Determining Appropriate Sample Sizes and Their Effects on Key Parameters in Longitudinal Three-Level Models

by

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ABSTRACT

Through a two study simulation design with different design conditions (sample size at level 1 (L1) was set to 3, level 2 (L2) sample size ranged from 10 to 75, level 3 (L3) sample size ranged from 30 to 150, intraclass correlation (ICC) ranging from 0.10 to 0.50, model complexity ranging from one predictor to three predictors), this study intends to provide general guidelines about adequate sample sizes at three levels under varying ICC conditions for a viable three level HLM analysis (e.g., reasonably unbiased and accurate parameter estimates). In this study, the data generating parameters for the were obtained using a large scale longitudinal data set from North Carolina, provided by the National Center on Assessment and Accountability for Special Education (NCAASE). I discuss ranges of sample sizes that are inadequate or adequate for convergence, absolute bias, relative bias, root mean squared error (RMSE), and coverage of individual parameter estimates. The current study, with the help of a detailed two-part simulation design for various sample sizes, model complexity and ICCs, provides various options of adequate sample sizes under different conditions. This study emphasizes that adequate sample sizes at either L1, L2, and L3 can be adjusted according to different interests in parameter estimates, different ranges of acceptable absolute bias, relative bias, root mean squared error, and coverage. Under different model complexity and varying ICC conditions, this study aims to help researchers identify L1, L2, and L3 sample size or both as the source of variation in absolute bias, relative bias, RMSE, or coverage proportions for a certain parameter estimate. This assists researchers in making better decisions for selecting adequate sample sizes in a three-level HLM analysis. A limitation of the study was the use of only a single distribution for the dependent and explanatory

variables, different types of distributions and their effects might result in different sample size recommendations.

DEDICATION

This dissertation is dedicated to my mom (Dürdane Yel), dad (Halil İbrahim Yel), sister (Nurten Akan), brother (Nurettin Hasan Yel), Hüsnü Akan, Funda Yel, Beyza Akan, Ayfer Sena Akan, İbrahim Emre Yel, Deniz Sıla Akan, Deniz Berk Yel, Defne Akan, and my soul-mate Kerrie Wilkins.

Their unwavering love, support, and encouragement have made this journey possible…

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Chapter 1

INTRODUCTION

Statement of the Problem

A common question asked by researchers when designing a quantitative study is, "how many participants do I need?" The answer to this question depends on several criteria: in particular, the type of study, types of measurement scales, missing data, acceptable significance level, target power, and effect size. Numerous tables, formulas, and software have been developed to determine the optimal sample size for a study design given the acceptable significance level and target power. However, these are only generalizable to a relatively limited number of research designs (Faul, Erdfelder, Buchner, & Lang, 2009; Konstantopoulos, 2009; Raudenbush, Spybrook, Congdon, Liu, Martinez, Bloom, & Hill, 2011; Spybrook, Bloom, Congdon, Hill, Martinez, & Raudenbush, 2011). For example, G*Power 3.1 (Faul et al., 2009), one of the most commonly used software packages among social scientists, lets users calculate the sample size for fixed effects ANOVA, multiple regression, logistic regression, and other traditional designs. However, one of the key assumptions of these traditional research designs is the independence of the subjects. Violation of this assumption results in additional complications for the researcher and the research design. For instance, the achievement scores of students from the same classroom tend to be more similar than the achievement scores of students from another class. This type of structure, where students are nested within classrooms, teachers are nested within schools, and classrooms are nested within schools, is called a multilevel, nested structure, or hierarchically clustered data. This nesting structure typically violates the assumptions of independence of most classical statistics (Gelman & Hill, 2006; Hox & Stoel, 2005; Luke, 2004; Peugh, 2010; Snijders & Bosker, 2011). Additionally, this nesting structure of multilevel modeling complicates the sampling procedure because each level involves a different number of units. In longitudinal studies, for example, researchers are often faced with the question of sampling few subjects and measuring them often versus selecting more subjects and measuring them less often. There were some existing guidelines available for longitudinal studies with two-levels, but not for the three-level designs. Thus, further research is warranted to investigate sample size determination especially in cases where traditional assumptions such as the independence of subjects are violated.

Multilevel modeling is an analytic technique developed as a response to the shortcomings of traditional statistical approaches when they are applied to nested/hierarchical/multilevel data. For example, as Raudenbush and Bryk (1988) indicated, when the traditional statistical modeling techniques such as ANOVA are applied to nested data, inferential validity might be compromised due to the under or overestimated standard errors. Before the development of multilevel models, the popular approaches to analyzing nested data included aggregating the outcome scores or analyzing data at separate levels or clusters. These methods had several shortcomings, such as the loss of information or not having a clear interpretation when the results from different levels diverged. Though multilevel models can aid in addressing the shortcomings of traditional methods, the method is still developing. One such area is determining appropriate samples size for each level of the three-level model. To aid in this, Optimal Design Software for Multilevel and Longitudinal Research (Raudenbush et al., 2011) calculates power and sample size for certain multilevel models. However, the design choices in this software are limited to randomized controlled trials such as two-level cluster randomized trial. Unfortunately, the software does not provide options for non-experimental studies that these are prevalent in education research, in particular longitudinal research. Another software package that was developed for two-level models is called Power IN Two-level designs (PINT) developed by Bosker, Snijders, and Guldemond (2003). Stata's module LBPOWER calculates the approximate power and sample size for longitudinal studies, but it is very limited in scope and does not include three-level models. In the medical literature, Basagaña, Liao, and Spiegelman (2011) developed an R syntax called optitxs to calculate power and sample size for repeated measures and longitudinal models. However, the models addressed in optitxs do not include three-level models. MLPowSim Software Package was developed by Browne, Lahi, and Parker (2009) and provides sample size calculations for random effects models. The program can be used to calculate sample size for three-level random effects models, but it does not include three-level longitudinal models.

To date, only a small number of simulation studies have been conducted to focus on sample size issues for multilevel data for nonexperimental studies. The findings from these studies were unsatisfying and vary based on the nature of the factors modified, the effect being examined, and the complexity of the model being investigated. Consequently, no strong sample size guidelines are available for researchers to utilize in making multilevel design decisions. Lastly, the majority of existing studies have focused on two-level multilevel models with no examination of sample size requirement for longitudinal three-level models. Thus, further examination of sample size requirements for longitudinal three-level models is needed.

The two areas where sample size choice plays an important role are hypothesis testing and estimation. The former issue pertains to sufficient statistical power needed to obtain statistically significant results. G*Power (Faul et al., 2009) and Optimal Design Software (Raudenbush et al., 2011) aim to help researchers with these issues. The latter issue concerns the relationship between the number of cases/subjects and the quality of the estimates of parameters of multilevel models. Simulation studies are usually performed to determine the accuracy and efficiency of the parameter estimates. This study concentrates on the quality of parameter estimates rather than the sufficient statistical power to estimate statistically significant results. Thus, this study focuses on determining the impact of model complexity, intraclass correlations (ICC), and sample sizes on statistical estimates for three-level models, in particular, those used for longitudinal data structures commonly found in education, where measurement occasions are nested within students who are nested within classrooms or schools. This study contributes to our understanding of the effect of varying model complexity, ICC, and sample sizes at higher levels to parameter recovery for longitudinal three-level multilevel models.

Chapter 2

LITERATURE REVIEW

To situate the discussion of sample size and its requirements, this section first focuses on MLMs and introduces the notation. Next, there is a review of the existing research on sample size in two-level multilevel models. Given the dearth of the literature on sample size recommendations for three-level HLM models, the basis for understanding sample size requirements comes from existing research on two-level models.

Multilevel Models

Multilevel models (MLM) are known under a variety of names, including "random coefficient model" (de Leeuw & Kreft, 1986), "variance component model" (Longford, 1995), and "hierarchical linear model" (Raudenbush & Bryk, 1986, 1988). MLMs can include data from multiple levels to examine the impact of individual-level and group-level factors on an individual-level outcome measure. MLMs allow the researchers to examine the relationships between predictors at different levels as well as the cross-level relationships among predictors measured at different levels.

MLMs define an analytical framework for both fixed and random effects and, because of this are also called linear mixed models (West, Welch, & Gałecki, 2014). Fixed effects describe the relationships of the independent variables to the dependent variable for an entire population. On the other hand, random effects are specific to groups or subjects within a population.

The levels of a three-level model can be thought of as follows. At the first level, a regression equation uses a set of predictors to predict an outcome variable. The regression coefficients obtained at the first level are then used as outcomes in the second level, with an associated set of predictors. Similarly, the regression coefficients obtained at the second level of the analysis are then used as outcomes at the third level, to be predicted from a set of predictors.

Notation. A variety of notational schemes are usually employed in MLMs, which can be somewhat confusing. The following notation is used in this study to have consistency and clarity.

Level 1 (L1)	Variable	Time_{tij}
	Coefficients	π
	Error Term	ε
	Error variance	σ_e^2
L2 (L2)	Variable	X_{ij}
	Coefficients	β
	Error Term	r
	Error variance	σ_r^2
Level 3 (L3)	Variable	Z_j
	Coefficients	γ
	Error Term	u
	Error variance	σ_u^2

The subscripts used in MLMs identify the different levels. For example, the level 1 (L1) independent variable is $\text{Time}_{tij,}$ where *t* indicates L1 (time), *i* indicates L2 (individuals or students), and *j* indicates L3 (groups or schools). The L2 and L3 independent variables are written similar to L1 independent variables.

The coefficients of independent variables at L1 are written as $\pi_{tij.}$ At L1, *t* indicates the intercept or the unique predictor such as π_{0ij} (for the intercept) or π_{1ij} (for the first predictor). At L2, $\beta_{tij,}$ where t indicates which L1 coefficient variable is predicted. For example, β_{0ij} is predicting the L1 intercept π_{0ij}, β_{1ij} is predicting slope of the L1 variable. At L3, $\gamma_{tij,}$ where *t* indicates which L1 coefficient variable is predicted, *i* indicates which L2 coefficient variable is predicted. For example, γ_{000} is predicting the L1 intercept π_{0ij}, and the L2 intercept $\beta_{00j.}$

Three-level Model. As an example, consider the following basic three-level model structure for linear growth over time, which is the focus of this study. We assume that the L1 (within person) represents the repeated measures made on the same unit of analysis. L2 represents the units of analysis (between persons). L3 represents the grouping or cluster (between schools). The three-level model may be written as follows where index t symbolizes time; index i symbolizes individuals, and index j symbolizes the schools.

Level 1 $$Y_{tij} = \pi_{0ij} + \pi_{1ij}\text{Time}_{tij} + \varepsilon_{tij} \tag{1}$$

where,

Y_{tij} = the student score at time *t* for student *i* in school *j*.

$Time_{tij}$ = the student level time predictor at time *t* for student *i* in school *j*.

π_{0ij} = the initial status of student *i* in school *j* when time variable is centered at 0.

π_{1ij} = the linear growth rate across time for student *i* in school *j*.

ε_{tij} = time specific deviation from student's predicted growth line at time *t* for student *i* in school *j*. ε_{ijk} is assumed to be $N\ (0, \sigma_\varepsilon^2)$.

Level 2

$$\pi_{0ij} = \beta_{00j} + \beta_{01j} X_{ij} + r_{0ij} \qquad (2a)$$

$$\pi_{1ij} = \beta_{10j} + \beta_{11j} X_{ij} + r_{1ij} \qquad (2b)$$

where

β_{00j} = the intercept of the L2 equation that predicts L1 intercepts, π_{0ij}.

β_{01j} = the slope of the variable X at L2 equation that predicts L1 intercepts, π_{0ij}.

β_{10j} = the intercept of the L2 equation that predicts L1 slopes.

X_{ij} = L2 predictor.

β_{11j} = the slope of the variable X at L2 equation that predicts L1 slopes.

r_{0ij} = student level error term associated with intercept. The random student effect within school *j*, which is normally distributed with a mean of 0 and variance σ^2_{r0}. It is a random effect.

r_{1ij} = student level error term associated with the slope. The random student effect within school j which is normally distributed with a mean of 0 and variance σ^2_{r1}. It is a random effect.

r_{0ij} and r_{1ij} are assumed to be N (0, σ^2_{r0}) and N (0, σ^2_{r1}) with cov (r_{0ij} , r_{1ij}) = $\sigma_{r_0 r_1}$.

Level 3

$$\beta_{00j} = \gamma_{000} + \gamma_{001} Z_j + u_{00j} \qquad (3a)$$

$$\beta_{01j} = \gamma_{010} + \gamma_{011} Z_j + u_{01j} \qquad (3b)$$

$$\beta_{10j} = \gamma_{100} + \gamma_{101} Z_j + u_{10j} \qquad (3c)$$

$$\beta_{11j} = \gamma_{110} + \gamma_{111} Z_j + u_{11j} \qquad (3d)$$

where Z_j is a L3 predictor.

γ_{000} = is the grand mean. Also, it is the intercept of L3 equation that predicts the L2 term β_{00j} (fixed effect).

γ_{001} = is the corresponding L3 coefficient that represents the direction and strength of the association between school characteristics (Z_j). Also, it is the slope of L3 equation that predict the L2 term β_{00j} (fixed effect)

γ_{010} = is the intercept of L3 equation that predicts the L2 term β_{01j} (fixed effect).

γ_{011} = is the slope of L3 equation that predicts the L2 term β_{01j} (fixed effect). Also, it can be thought of as the regression coefficient for the interaction of L2 predictor X and L3 predictor Z.

γ_{100} = is the intercept of L3 equation that predicts the L2 term β_{10j} (fixed effect).

γ_{101} = is the slope of L3 equation that predicts the L2 term β_{10j} (fixed effect). Also, it can be thought of as the regression coefficient for the interaction of L1 predictor Time and L3 predictor Z.

γ_{110} = is the intercept of L3 equation that predicts the L2 term β_{11j} (fixed effect). Also, it can be thought as the regression coefficient for the interaction of L1 predictor Time and L2 predictor X.

γ_{111} = is the slope of L3 equation that predicts the L2 term β_{11j} (fixed effect). Also, it can be thought as the regression coefficient for the three-way interaction by L1 predictor Time, L2 predictor X, and L3 predictor Z.

u_{00j} = L3 error term associated with β_{00j}. It is a random effect.

u_{01j} = L3 error term associated with β_{01j}. It is a random effect.

u_{10j} = L3 error term associated with β_{10j} . It is a random effect.

u_{11j} = L3 error term associated with β_{11j} . It is a random effect.

The L3 error terms assumed to be

$$\begin{pmatrix} u_{00j} \\ u_{01j} \\ u_{10j} \\ u_{11j} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{u_{00}} & \sigma_{u_{00}u_{01}} & \sigma_{u_{00}u_{10}} & \sigma_{u_{00}u_{11}} \\ \sigma_{u_{00}u_{01}} & \sigma^2_{u_{01}} & \sigma_{u_{01}u_{10}} & \sigma_{u_{01}u_{11}} \\ \sigma_{u_{00}u_{10}} & \sigma_{u_{01}u_{10}} & \sigma^2_{u_{10}} & \sigma_{u_{10}u_{11}} \\ \sigma_{u_{00}u_{11}} & \sigma_{u_{01}u_{11}} & \sigma_{u_{10}u_{11}} & \sigma^2_{u_{11}} \end{pmatrix} \right)$$

The model may be written in combined form; substituting equations 2 and 3 into 1 gives equation 4

$$Y_{tij} = \gamma_{000} + \gamma_{100}\,\mathrm{Time}_{tij} + \gamma_{010}\,X_{ij} + \gamma_{001}\,Z_j + \gamma_{110}\,\mathrm{Time}_{tij}\,X_{ij} + \gamma_{101}\mathrm{Time}_{tij}\,Z_j + \gamma_{011}\,Z_jX_{ij} + \gamma_{111}\,\mathrm{Time}_{tij}\,X_{ij}\,Z_j + u_{01j}\,X_{ij} + u_{10j}\,\mathrm{Time}_{tij} + u_{11j}\,\mathrm{Time}_{tij}\,X_{ij} + r_{1ij}\mathrm{Time}_{tij} + \varepsilon_{tij} + r_{0ij} + u_{00j}. \quad (4)$$

Therefore, in this three-level random model, there is a total of 15 parameters; eight fixed effects and seven random effects. Specifically, fixed effects are γ_{000}, γ_{100}, γ_{010}, γ_{001}, γ_{110}, γ_{101}, γ_{011}, γ_{111} and random effects are ε_{tij}, r_{0ij}, r_{1ij}, u_{00j}, u_{01j}, u_{10j}, u_{11j}.

Intraclass Correlations

An intraclass correlation (ICC) assesses the proportion of the outcome variation due to between-group differences in the intercept. The ICC values range between 0 and 1. Larger ICC values mean that there is a strong relationship between the data collected from individuals within the same group. In other words, each member of a group provides little unique information.

In a three-level model, an ICC for each level can be calculated. Using the same example model in the previous section, we can calculate ICC for each level using the equations in Table 2. It is assumed that for a null model with no predictors, the L1 residual has a constant variance of σ_e^2. Next, it is assumed that the L2 residual variance has a constant variance of σ_r^2. Finally, it is assumed that the L3 residual has a constant variance of σ_u^2.

Table 1.

ICC Calculations for Different Levels

ICC at Level 1	ICC at Level 2	ICC at Level 3
$ICC_{L1}=\frac{\sigma_e^2}{\sigma_e^2+\sigma_r^2+\sigma_u^2}$	$ICC_{L2}=\frac{\sigma_r^2}{\sigma_e^2+\sigma_r^2+\sigma_u^2}$	$ICC_{L3}=\frac{\sigma_u^2}{\sigma_e^2+\sigma_r^2+\sigma_u^2}$

The residual variance at each level for the baseline model is not useful for the prediction of the outcome variable, but it sets a baseline to determine the contribution of other variables when they are added to the model.

The variance partitioning across three levels for a Null model and with time as L1 predictor are illustrated in Figure 1. For example, the ICCs based on the null model work with the middle part of this figure, looking at the proportional contribution. Moreover, on the right-hand side of the figure, for a model with time predictor at L1, we now have residual variances of different kinds. These variances add up to the total variance minus the variance explained by the L1 predictor. The variances shown in the middle part are shared among the different parts on the right-hand side of Figure 1.

Researchers have looked at the effect of ICC on sample size requirements as part of their research designs. Donoghue and Jenkins (1992) found that ICC had no significant effect on sample size requirements. Other researchers found that the accuracy of parameter estimates are affected by ICCs (Muthen, Wisnicky, & Nelson, 1991; Snijders & Bosker, 2011). Kim's (1990) simulation study showed that high ICC values required a large number of observations within groups for accurate parameter estimates. Similarly, Bassiri (1988) concluded in her study that higher ICC values are associated with poorer precision in fixed and random effects estimates. Shih (2008) conducted a detailed simulation study using a two-level model and calculated the average bias for each of the parameter estimates. Shih found that average bias related to the fixed effects intercepts drops as the ICC increases. However, unstable bias estimates were obtained when the

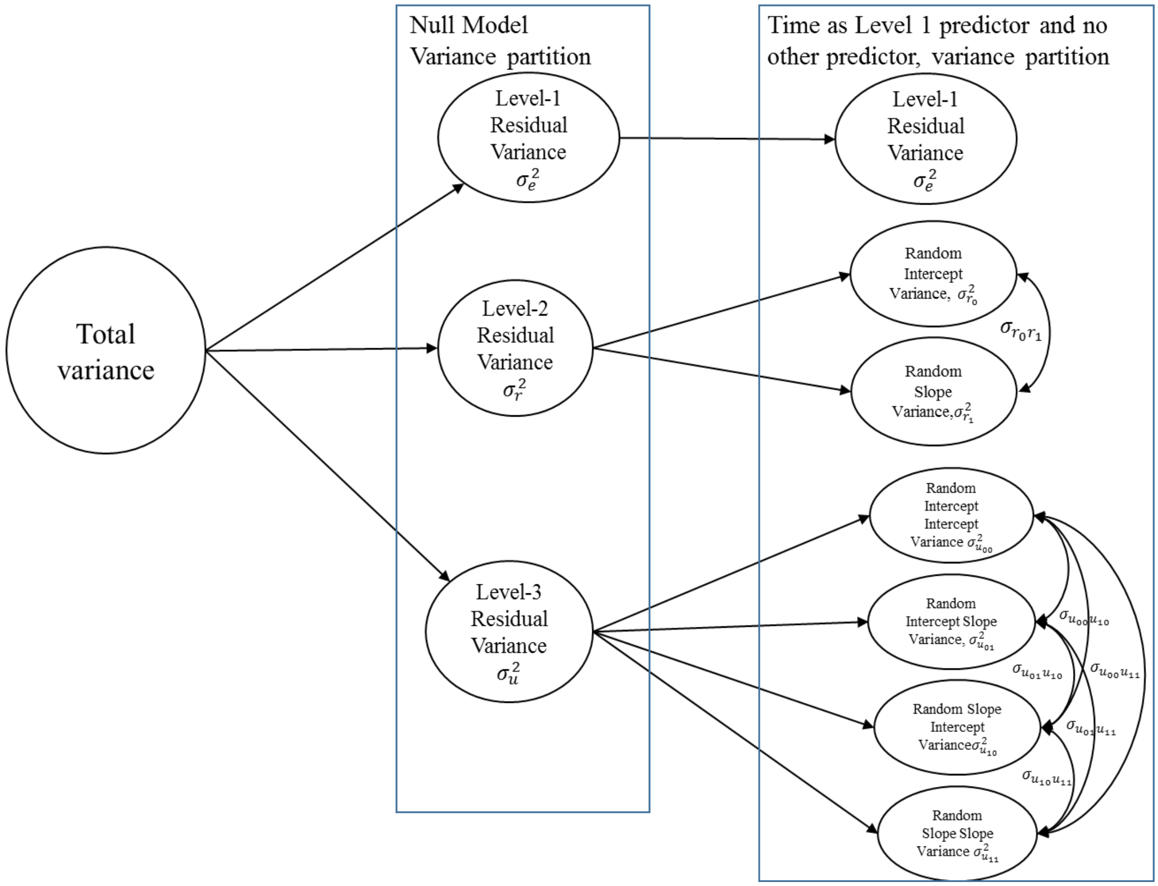

Figure 1. Partitioning Variance Across Levels for Null Model and Time as L1 Variable

focus was fixed effects slope estimates. For example, average bias dropped when ICC increased from 0.05 to 0.1 for γ_{11}. On the other hand, average bias increased for the same parameter (γ_{11}) when ICC increased from 0.10 to 0.15. It is not clear why different studies resulted in different conclusions. However, model complexity might be one of the reasons, but no systematic examination of the relationship between model complexity and ICC has been conducted.

In sum, findings of the previous research are inconsistent, and this inconsistency requires more examination of the effects of ICC on sample size requirements and parameter recovery. The relationship between ICC and model complexity are systematically examined in this study.

Sample Size in Two-level Models

Sample size choices affect parameter estimation. If the sample size is too small, the parameter estimates may be poor and usually lead to results that are inconclusive or worse, misleading. This would amount to a waste of valuable time and resources on the researcher's end. The effect of poor sample size choices is amplified and becomes more problematic in multilevel models, as more parameters are estimated in MLMs compared to models that involve only one level.

In a typical multilevel model, fixed effects (γ_{000}, γ_{100}, γ_{010}, γ_{001}, γ_{110}, γ_{101}, γ_{011}, γ_{111}), random effects (ε_{tij}, r_{0ij}, r_{1ij}, u_{00j}, u_{01j} , u_{10j}, u_{11j}), variance components (σ_e^2, $\sigma_{r_0}^2$, $\sigma_{r_1}^2$, $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, $\sigma_{u_{11}}^2$), and covariance components ($\sigma_{r_0}\sigma_{r_1}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}$ $\sigma_{u_{11}}$,$\sigma_{u_{10}}\sigma_{u_{11}}$) are generally the parameters of interest. Depending on the parameters of interest, sample size needs might change. For example, in two-level designs, the fixed effects can be estimated with more precision than the variance components and cross-level interactions (Hox, 2010; Scherbaum & Ferreter, 2009). The inclusion of covariates such as the student's gender (at L2) or proportion of students who receive free or reduced lunch in the school (at L3) can additionally impact the sample size calculations as covariates can reduce the between-group variance (Reise & Duan, 2003; Raudenbush, 1997). Although the available guidelines vary regarding the minimum sample size required for multilevel research, it is well known that multilevel research requires larger sample sizes compared to single-level studies. However, no commonly accepted guidelines for ideal sample sizes currently exist. Given this, several simulation studies have been designed to examine the effect of small sample sizes on various two-level models focusing on model convergence, variance estimates, fixed effects estimates, and standard errors. The results from these studies vary based on the focus of the study and the examined effect. The next section summarizes the studies by focus; fixed effects, random effects, and variance-covariance components.

Fixed Effects. Fixed effect (γ_{000}, γ_{100}, γ_{010}, γ_{001}, γ_{110}, γ_{101}, γ_{011}, γ_{111}) is a term borrowed from the analysis of variance designs (ANOVA). However, a clarification of the term might be needed in the MLM framework. In ANOVA designs, fixed effects mean that data collection involves all levels of a factor of interest. However, in the multilevel modeling context, the fixed effects refer to the intercept and slopes that remain constant across higher-order units. The regression weights that are constant for each subject in the sample in an MLM are called fixed effects (Heck & Thomas, 2015; Hoffman, 2014).

Two-level MLM simulation studies have consistently shown that fixed effects were estimated with near identical precision given the modified factors such as ICCs, L1 sample size, and L2 sample size (Bell, Ferron, & Kromrey, 2008; Clarke & Wheaton, 2007; Maas & Hox, 2005; Mok, 1995). Similarly, the same general pattern has also been observed for the standard errors of the fixed effects (Bell, Ferron, & Kromrey, 2008; Clarke & Wheaton, 2007; Maas & Hox, 2005; Mok, 1995).

One of the most cited two-level sample size related simulation studies is that of Mok (1995). In this study, 11 different observation sizes for L1 were used: 5, 10, 20, 30, 40, 50, 60, 70, 80, 100, and 150. Additionally, the same number of L2 units was used, resulting in 121 different sample size conditions when completely crossed. The minimum total size was 25, and the maximum was 22,500. One hundred datasets were generated and analyzed for each sample size condition. The estimation method used in this study was Restricted Iterative Generalized Least Squares (RIGLS), and the fixed ICC had a value of 0.15. This study found that all estimates of fixed effects were within one standard error of the true value, regardless of the distribution of the sample size among level one and level two units if the total sample size was more than 800. On the other hand, if the total sample size was less than 800, the results showed that when the number of L2 units was equal to the number of observations at L1, fixed effects were estimated close to the data generating values. Similarly, when the number of L2 units exceeded the number of observations at L1, fixed effects were estimated close to the data generating values. However, estimates showed strong bias when the number of observations at L1 per unit of L2 exceeded the number of units at L2. This is especially important in a typical school, where there are usually more students in a classroom than the number of classrooms in the school. For example, a school might have an average of 30 students per classroom, but only five third-grade classrooms. The study by Mok (1995) suggested that more bias is present in this kind of study design when estimating model parameters such as fixed effects. Therefore, it is imperative that researchers be cautious about the bias issues in their research design when designing educational studies where the number of classrooms at L2 are less than the number of students per classroom at L1. Although it might make more sense and be more cost effective to sample more students from a given classroom, instead of recruiting more classrooms for the study, bias in the parameter estimates might result in erroneous conclusions. In agreement with this, Mok (1995) concluded that it is better to maximize the L2 units to minimize estimation error.

In another study, Afshartous (1995) used a real dataset, which mainly focused on L2 estimates. In this case, a sub-sampling routine and randomly sampled L2 units were utilized with the following values: 40, 80, 160, and 320 schools, from a pool of 1,034 schools. The L1 sample size ranged from 1 to 70 units. Unfortunately, the ICC values used in the study were not reported. One hundred samplings were performed for each of the L2 units, and a full maximum likelihood estimation was utilized, resulting in unbiased estimates of the fixed effects for a minimum L2 sample size of 40. Unfortunately, guidelines regarding what the L1 sample size should be for unbiased estimates of fixed effects were not provided.

Afshartous conducted another study in 1997 and compared ordinary least squares estimates (ignoring the multilevel structure) and multilevel estimates regarding the prediction of a future observable. Afshartous (1997) modified five factors: the L2 sample size (10, 25, 50, 100, 300); the L1 sample size (5, 10, 25, 50, 100); ICC (0.2, 0.4, 0.6, 0.8); L2 variance components (0.125, 0.33, 0.75, 2.0.); and the covariance of the error terms (varied between 0.03 and 1.5). The L1 error terms were generated to be normal with a mean of zero and a variance of 0.5, and each condition was replicated 100 times. The results showed that the fixed effects were estimated with minimal bias when the L1 sample size was 10, and when the L2 sample size was 100. Similar to the previous findings, this study found that the number of groups was more important than the group size. On the other hand, the data revealed that it was better to have a larger L1 sample size when the focus of interest was a prediction of a future outcome.

Clarke and Wheaton (2007) conducted a Monte Carlo simulation using two-level MLM by varying three conditions: L2 sample size, L1 sample size, and ICC. This study used a relatively small number of L1 (2, 5, 10, 20) and L2 sample sizes (50, 100, 200) compared to Mok (1995). However, three different ICCs (0.1, 0.2, 0.3) were used instead of one fixed value. One thousand simulated datasets were generated for each of the 36 conditions and the SAS procedure MIXED was used with restricted maximum likelihood. They had some convergence issues with the smallest L1 sample size of 2 and the smallest L2 sample size of 50. Although the focus was data sparseness, no significant evidence of bias for fixed effects for L1 sample sizes greater than two, and L2 sample size greater than 50 were found. These results contradict the commonly cited L1 sample size suggestion of 30, and the L2 sample size suggestion of 30. Furthermore, it also contradicts Mok's (1995) suggestion of a minimum of 800 total sample size.

Bell, Ferron, and Kromrey (2008) conducted a Monte Carlo simulation study to explore the issues related to data sparseness by modifying six factors: (a) proportion of L2 sample size to L1 sample size (0, 0.10, 0.30, 0.50, 0.70); (b) L1 sample size varied, small (average 10) and large (average 50); (c) L2 sample size (50, 100, 200, 500); (d) levels of collinearity (0, 0.30); (e) L2 error variance (0.05, 0.10, 0.15, 0.30); and (f) model complexity. The 40 sample size conditions were crossed with 144 design conditions and generated 1000 data sets using SAS procedure IML. Then, these datasets were analyzed with the SAS procedure MIXED using maximum likelihood estimation.

Convergence was only an issue for approximately 2% of the conditions. The results showed very low levels of statistical bias for fixed effects estimates for both small and large sample sizes used at L1 and L2. It appears that they used sample sizes of less than 10 since the average small sample size was 10. This contradicts the results of Clarke and Wheaton's (2007) suggestion of a minimum L1 sample size of 10.

Maas and Hox (2005) conducted a simulation study to examine the parameter estimates and the corresponding standard errors for the number of groups and group size. Similar to Clarke and Wheaton (2007), ICCs were also modified in this study. They used a simple two-level model with one predictor at each level, and modified three factors: (a) L1 size (5, 30, and 50); (b) L2 sample size (30, 50, 100); and (c) ICC (0.10, 0.20, 0.30). One thousand simulated datasets were generated for each of the 27 conditions and used MLwiN (Rasbash, Charlton, Browne, Healy, & Cameron, 2005) using restricted maximum likelihood, without any convergence issues. They concluded that fixed effects were estimated with near precision. Considering the presented findings above, these are the lowest sample size suggestions so far.

One of the most recent simulation studies was conducted by Meinck and Vandenplas in 2012, which focused on the precision of parameter estimates for a two-level model. The total variance was fixed and distributed among levels depending on the ICC (0.1, 0.2, 0.3, 0.4) used in the study. Additionally, socioeconomic status (SES) was used as both L1 and L2 variables. The correlation between the outcome variable and SES was set to be 0.3. These values were obtained from two international large-scale assessments: the Trends in International Mathematics and Science Study (TIMSS), and the Program for International Student Assessment (PISA). L1 sample size was selected to be 5, 10, 15, 20, 25, and 30 while L2 sample size was selected to be 50,100,150, and 200, and each sample size condition was replicated 600 times using Mplus (Muthén & Muthén, 2015). The results of this study demonstrated that fixed effects were estimated with the highest precision.

Random Effects. Random effects (ε_{tij}, r_{0ij}, r_{1ij}, u_{00j}, u_{01j}, u_{10j}, u_{11j}) are also called random slope and random intercepts. As described above, fixed effect means that each individual gets the same effect. On the other hand, random effect means that each individual gets his or her effect (Hoffman, 2014). Unfortunately, the studies that clearly focus on random effects are limited. Meinck and Vandenplas (2012) introduced random intercepts in some of the models they examined, showing that the random intercept was the parameter that could be measured with the highest precision in all of the models examined. The slope of random intercepts was another parameter examined, demonstrating that the slope of random intercepts was harder to estimate with a high degree of precision compared to random intercepts and fixed effects. However, aligned with the literature (Mok, 1995), increasing L2 sample size resulted in higher precision gains in the estimates of the slope of random intercepts than increasing L1 sample size.

Variance-Covariance Components. Variance-covariance components are other parameters that have been investigated. These parameters are closely related to the random effects. Variance components include $\boldsymbol{\sigma_e^2}$, $\boldsymbol{\sigma_{r_0}^2}$, $\boldsymbol{\sigma_{r_1}^2}$, $\boldsymbol{\sigma_{u_{00}}^2}$, $\boldsymbol{\sigma_{u_{01}}^2}$, $\boldsymbol{\sigma_{u_{10}}^2}$, $\boldsymbol{\sigma_{u_{11}}^2}$. Covariance components include $\boldsymbol{\sigma_{r_0}\sigma_{r_1}}$, $\boldsymbol{\sigma_{u_{00}}\sigma_{u_{01}}}$, $\boldsymbol{\sigma_{u_{00}}\sigma_{u_{10}}}$, $\boldsymbol{\sigma_{u_{00}}\sigma_{u_{11}}}$, $\boldsymbol{\sigma_{u_{01}}\sigma_{u_{10}}}$, $\boldsymbol{\sigma_{u_{01}}\sigma_{u_{11}}}$, $\boldsymbol{\sigma_{u_{10}}\sigma_{u_{11}}}$. Some of the studies that focused on fixed effects also reported the results for variance components (Afshartous, 1995, 1997; Mok, 1995).

One of the early studies, as mentioned above, was that conducted by Afshartous (1995), which suggested that at least 320 L2 units would be needed to obtain unbiased estimates of L2 variance components. However, no suggestions for L1 sample size were provided, as the L1 sample size in his study varied between 1 and 70. Consequently, the L1 size has great variability and may not be as realistic for practical research designs.

Afshartous conducted another study in his 1997 project, which explored different L1 sample sizes in more detail. This study found that the variance components were estimated with minimal bias when the group size was 10, and the number of groups was 100. Given this suggestion, the total sample size should be at least 1,000. Afshartous's suggestion of 1,000 is lower than that of the previous studies in the literature. For example, Mok (1995) found that larger total sample sizes are required for variance components and reported different recommendations for L1 variance components and L2 variance components. Mok found that the variance estimates of L1 were more precise when the total sample size exceeded 4,000, and the variance estimates of L2 were less biased when the total sample size was greater than 2,500.

Clarke and Wheaton's (2007) findings agreed with those of Afshartous (1997). They recommended at least 10 L1 units, and at least 100 groups, for minimal bias in intercept variance. Their results showed a positive bias on the intercept and slope variance estimates when the total sample size was less than 1,000. However, these values doubled for the minimal bias in slope variance estimates: at least 20 L1 units and 200 L2 units were the recommended values, consistent with Mok (1995).

In their 2005 study, Maas and Hox found that L1 variance estimates were very accurate regardless of the sample sizes examined. However, L2 variance components were underestimated when the sample size was small, and the largest bias was observed when the L1 sample size was 5, L2 sample size was 30, and the ICC was the highest 0.3. They recommended using at least 100 L2 units for precise estimates of the L2 variances components.

A more recent study by Meinck and Vandenplas (2012) found that the L1 residual variance was estimated with high precision, even when the sample sizes were small, similar to the findings by Maas and Hox (2005). This finding is not surprising, because the parameter is measured at L1, and only the total sample size matters instead of L1 or L2 sample size. Meinck and Vandenplas' (2012) finding showed that the result for the L2 variance components was very different when the focus was not L1 residual variance. These studies suggest that researchers need to have significantly larger sample sizes when the main focus of interest is the estimation of variance components rather than of the fixed effects.

Summary

All multilevel sample size related research up to this point has focused on two-level models. The literature showed that the effect of sample sizes on different model estimates varied. So, not all model estimates in MLMs are equally affected by sample sizes at different levels and ICC choices. For example, fixed effects terms are generally estimated more accurately compared to variance-covariance terms even with the small sample sizes at different levels. However, no systematic examination of the effect of various ICCs and model complexity on different parameter estimates has been conducted so far. In this study, I specifically examined the effect of ICCs, model complexity, and sample size choices on different parameter estimates.

To date, there have not been any simulation studies published that focus on three-level models. Because the studies that focused on two-level models used different design choices, sample size recommendations varied among studies. In this study, the common design choices presented above guided the design choices.

Purpose of the Study

The purpose of this study is to determine the impact of sample size on statistical estimates for three-level models, in particular, those used for longitudinal data structures commonly found in education. The following variables are modified: the sample size at L2 and L3, ICC, and the model complexity. Sample size at L1 was set to 3 which is a typical number of repeated measures found in longitudianal studies in education. This study contributes to our understanding of the effect of varying model complexity, ICCs, and sample sizes at L2 and L3 on parameter recovery for three-level multilevel models.

Research Questions

This study addressed the following major research questions:

1. How does each of the modified factors (model complexity, ICC, L2, and L3 sizes) affect the intercept estimates ($\gamma_{000}, \gamma_{010}, \gamma_{100},$ and γ_{110})?
2. How does each of the modified factors influence the fixed effects slope estimates ($\gamma_{001}, \gamma_{011}, \gamma_{101},$ and γ_{111})?
3. How does each of the modified factors influence the variance components (σ_e^2, $\sigma_{r_0}^2$, $\sigma_{r_1}^2$, $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$)?
4. How does each of the modified factors influence the covariance components ($\sigma_{r_0 r_1}$, $\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$)?

The goal of this simulation study is to determine the impact of sample size on statistical estimates for three-level models, in particular, those used for longitudinal data structures commonly found in education. A range of conditions are examined in a two study design. In Study 1, a small set of conditions were examined, and the results of Study 1 guided the design choices for Study 2.

Chapter 3

STUDY 1

Researchers have investigated sample size issues in two level multilevel modeling using various sample sizes for level 1 (L1) and level 2 (L2), different intraclass correlation (ICC) values, and simple (at least one predictor) to more complicated models (two or more predictors) (Afshartous, 1995; Bell, Ferron, & Kromrey, 2008; Mok, 1995; Meinck & Vandenplas, 2012 ; Scherbaum & Ferreter, 2009). However, no study to date has examined the sample size requirements for three level models. Given the lack of empirical attention, the current study sought to examine the sample size requirements for three level models.

In Study 1, the focus was to examine the sample size choices commonly found in the literature on two level models and extend that to three level models. ICCs, model complexity, L2, and L3 sample sizes were modified.

Study 1: Method

The ICC, sample size, and model choices in Study 1 were guided by the published simulation and empirical studies found in the social sciences literature.

The first modified factor was ICC. Two different sets of ICC used in Study 1. Spybrook et al., 2011 reported that repeated measures studies usually have high ICC values, and they range between 0.5 and 0.7. In empirical studies, the results usually showed that the ICC at school level (L3 in our case) has an ICC values around 0.10 (Murray, Stevens, Hannan, Catellier, Schmitz, Dowda, Conway, Rice, And Yang, 2006; Siddiqui, Hedeker, Flay, & Hu, 1996). Guided by these values, the first set of ICC values were set to 0.50 for L1, 0.40 for L2, and 0.10 for L3. In medical literature, the hospital level (L3) ICC's found to be around 0.20 (Bell, Owens, Ferron, Kromrey, 2008; Hox, 2010). Considering this, in the second set of ICCs, the L3 ICC was set to 0.3, and L2 ICC was set to 0.2, and L1 ICC was kept same, 0.5. Table 2 showed the ICCs used in Study 1.

Table 2.

ICC Values Used in Study 1

Study	ICC	Level 1	Level 2	Level 3
1	1	0.5	0.40	0.10
1	2	0.5	0.20	0.30

The L1 sample size was set to be 3. Longitudinal growth models in educational studies typically require at least three repeated measures per individual. The typical classroom size in the United States based on 2011-2012 Schools and Staffing Survey (SASS) is about 20 students. However, most the studies mentioned above suggested either 30 or 50 as L2 sample size. Considering this varying information in the literature, L2 sample sizes were set to 10 and 50 in Study 1 to test the effect of relatively low and high L2 sample sizes on the parameter estimates. In the two-level simulation studies mentioned above, the sample size recommendations for the highest level varied between 30 and 200. Given this, a low and medium L3 sample size was selected. L3 sample sizes of 30 and 100 were examined.

Model complexity was the last factor modified. In this preliminary exploration, two different models were examined. The first model was relatively less complex (include only one predictor) compared to the second model (three predictors total). In the two level simulation literature, researchers usually used a single model with one predictor at each level such as Maas and Hox (2005). Following this, the first model only included the time predictor and L1. In the second model, the first model was modified by adding one predictor to both L2 and L3.

Table 3 includes the equations used in the first model. It has only one predictor at L1 and is called the L1 model throughout this study. All of available residual variance-covariance terms were estimated at all levels.

Table 3.

L1 Model

Level 1	$Y_{tij} = \pi_{0ij} + \pi_{1ij}\text{Time}_{tij} + \varepsilon_{tij}$	(5a)
Level 2	$\pi_{0ij} = \beta_{00j} + r_{0ij}$	(5b)
	$\pi_{1ij} = \beta_{10j} + r_{1ij}$	(5c)
Level 3	$\beta_{00j} = \gamma_{000} + u_{00j}$	(5d)
	$\beta_{10j} = \gamma_{100} + u_{10j}$	(5e)
Combined	$Y_{tij} = \gamma_{000} + u_{00j} + r_{0ij} + (\gamma_{100} + u_{10j} + r_{1ij})\, \text{Time}_{tij} + \varepsilon_{tij}$	(5f)

The second model in Study 1 was the most complex model used in this study. Two new predictors added to the L1 model; one predictor at L2 and one predictor at L3. This model is called L1L2L3 model since there is one predictor at each level. Table 4 shows the equations used in L1L2L3 model. All of available the residual variance-covariance terms were estimated at all levels.

Table 4.

L1L2L3 Model

Level 1	$Y_{tij} = \pi_{0ij} + \pi_{1ij}\text{Time}_{tij} + \varepsilon_{tij}$	(6a)
Level 2	$\pi_{0ij} = \beta_{00j} + \beta_{01j}\, X_{ij} + r_{0ij}$	(6b)
	$\pi_{1ij} = \beta_{10j} + \beta_{11j}\, X_{ij} + r_{1ij}$	(6c)
Level 3	$\beta_{00j} = \gamma_{000} + \gamma_{001}\, Z_j + u_{00j}$	(6d)
	$\beta_{01j} = \gamma_{010} + \gamma_{011}\, Z_j + u_{01j}$	(6e)
	$\beta_{10j} = \gamma_{100} + \gamma_{101} Z_j + u_{10j}$	(6f)
	$\beta_{11j} = \gamma_{110} + \gamma_{111}\, Z_j + u_{11j}$	(6g)
Combined	$Y_{tij} = \gamma_{000} + \gamma_{001}\, Z_j + u_{00j} + (\gamma_{010} + \gamma_{011}\, Z_j + u_{01j})\, X_{ij} + r_{0ij} + (\gamma_{100} + \gamma_{101} Z_j + u_{10j} + (\gamma_{110} + \gamma_{111}\, Z_j + u_{11j})\, X_{ij} + r_{1ij})\text{Time}_{tij} + \varepsilon_{tij}$	(6h)

Data model misfit and its effect on sample size requirements is not an interest in this simulation study. Because of that, the correct model was fit to the data generation model for both models.

Data simulation. This study modified Busing's (1993) two-level sampling to generate two-level data where first L2 values were generated, and then L1 values, and then the combined equation used to generate the outcome variable. Population values for data generation were taken from a real data set as described in the following section.

In this study, residual terms for each level were generated based on fitting each model to the real dataset and the ICC level used in this study. Then, L2 predictors were simulated where applicable. After that, L3 predictors were calculated based on the L2 predictors. Finally, the combined equation was used to obtain the outcome variable. The models that were used in Study 1 were described in Equations 5 and 6.

Centering predictors is a common practice in MLM, and there are two main centering methods; grand mean centering and centering within the cluster. The general purpose of centering is to obtain a meaningful zero point so that the interpretation of parameters is meaningful. In this study, the focus was the accuracy of parameter estimates rather than the interpretation of the parameter. However, the software used in this study, HLM7, requires centering and as a result, grand mean centering was used.

The datasets were generated using R 3.2.1 (R Development Core Team, 2015). 500 datasets were randomly generated from both L1 and L1L2L3 models for each condition.

Parameter specifications. To simulate realistic data in educational research, I fit a three-level model to a real dataset. The analytic sample used to obtain the data generating parameters came from North Carolina. This dataset included 3rd,4th,and 5th graders from 2010, 2011, and 2012, respectively. Students who met the following criteria were included the analytical sample: (a) in the 3rd grade in 2010; (b) followed the typical grade sequence from Grades 3-5; (c) stayed in the same school from Grades 3-5; (d) did not have any missing values on mathematics achievement scores from Grades 3-5. Additionally, schools that had less than 20 students were not included in the analytical sample. The resulting total number of students was 68,455. The total number of schools was 1176. It is assumed that the MLM assumptions were not violated. L1L2L3 Model was fitted to these data and the results for fixed effects are presented in Table 5 and the random effects presented in Table 6.

Table 5.

Fixed Effect Results for L1L2L3 Model

Fixed Effect		Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For Intercept1	π_0					
For Intercept2	β_{00}					
Intercept3	γ_{000}	347.3650	0.0508	6841.006	1174	<0.001
Z	γ_{001}	0.1740	0.0141	12.324	1174	<0.001
For X	β_{01}					
Intercept3	γ_{010}	0.7330	0.0031	236.749	1174	<0.001
Z	γ_{011}	0.0004	0.0008	2.631	1174	0.008
For Time slope	π_1					
For Intercept2	β_{10}					
Intercept3	γ_{100}	5.7660	0.0338	170.783	1174	<0.001
Z	γ_{101}	-0.0190	0.0091	2.081	1174	0.038
For X	β_{11}					
Intercept3	γ_{110}	-0.0138	0.0016	8.558	1174	<0.001
Z	γ_{111}	0.0004	0.0004	2.480	1174	0.013

Table 6.

Random Effect Results for L1L2L3 Model

Random Effect		*SD*	Variance Component	*d.f.*	χ^2	*p*-value
Level 1	e	3.640	13.249			
Intrcpt1	r_0	4.498	20.235	66103	187076.16	<0.001
Time slope	r_1	0.612	0.374	66103	69739.55	<0.001
Intrcpt1/Intrcpt2	u_{00}	1.487	2.210	1174	5079.82	<0.001
Intrcpt1/ X	u_{01}	0.052	0.003	1174	1608.19	<0.001
Time/Intrcpt2	u_{10}	1.056	1.115	1174	10023.96	<0.001
Time/ X	u_{11}	0.032	0.001	1174	1929.41	<0.001

Table 7 shows the L3 residual variance-covariance matrix obtained after fitting L1L2L3 model to the data

Table 7.

Variance-Covariance Matrix for L3 Residual Terms for L1L2L3 Model

Residual Term	1	2	3	4
1. u_{00j}	2.210	-0.033	-0.899	0.003
2. u_{01j}	-0.033	0.003	0.017	-0.001
3. u_{10j}	-0.899	0.017	1.115	-0.010
4. u_{11j}	0.003	-0.001	-0.010	0.001

The following steps were used to generate the raw data for the multilevel models. The values were adjusted based on the model and ICCs used.

1. The total variance was set to 40 based on the results of the real data analysis described above.
2. L1 error terms were generated from a normal distribution with mean 0, and variance 20, $\varepsilon_{tij} \sim N(0, 20)$ to have an ICC level of 0.5 at L1.
3. The correlation between L2 error terms were set to -.219 (obtained from the real data analysis). L2 error terms (r_{0ij} and r_{1ij}) were generated for each group using a multivariate normal distribution. The ratio between the variance of r_{0ij} and r_{1ij} (r_{0ij}/r_{1ij} =20.235/0.374) from the real data analysis was kept constant for the different values of ICCs as listed in Table 2. Table 8 shows the L2 residual variance values for both ICCs used in Study 1.

Table 8.

Level 2 Residual Variance Values for Different ICC Conditions

ICCs	$\sigma^2_{\mathrm{r}0ij}$	$\sigma^2_{\mathrm{r}1ij}$
ICC 1	15.71	0.29
ICC 2	3.93	0.07

4. The L3 error terms (u_{00j}, u_{01j}, u_{10j}, u_{11j}) for each school were generated by using a multivariate normal distribution with mean 0 and variance values listed in Table 9 for each ICCs listed in Table 2. The correlation values listed in Table 10 were constant across different ICC conditions.

Table 9.

L3 Residual Variance Values for Different ICC Conditions

ICCs	$\sigma^2_{u_{00j}}$	$\sigma^2_{u_{01j}}$	$\sigma^2_{u_{10j}}$	$\sigma^2_{u_{11j}}$
ICC 1	2.655	0.003	1.330	0.001
ICC 2	7.970	0.010	4.019	0.004

Table 10.

L3 Residual Correlation Used in the Current Study

	$\sigma_{u_{00j}}$	$\sigma_{u_{01j}}$	$\sigma_{u_{10j}}$	$\sigma_{u_{11j}}$
$\sigma_{u_{00j}}$	1.00	-0.43	-0.57	0.07
$\sigma_{u_{01j}}$	-0.43	1.00	0.32	-0.60
$\sigma_{u_{10j}}$	-0.57	0.32	1.00	-0.30
$\sigma_{u_{11j}}$	0.07	-0.60	-0.30	1.00

5. The fixed effects values for γ_{000}, γ_{001}, γ_{010}, γ_{011}, γ_{100}, γ_{101}, γ_{110}, and γ_{111} were obtained by fitting L1L2L3 Model to the North Carolina data described earlier

were used in data generation and were constant across different models and conditions. Table 5 includes these fixed effects values.

6. The L3 error terms generated using multivariate normal distribution combined with *γs* to compute βs for each L2 unit.
7. Continuous X_{ij} values for each L2 unit were generated with mean 350 and standard deviation 10.
8. The Zj values for each L3 unit was calculated by averaging X_{ij} values for the L2 unit.
9. The L2 error terms combined with X_{ij} and βs to compute π for each L2 unit.
10. The L1 error terms combined with πs to compute Y_{tij}.

To run the simulations, a large number of replications (500) were generated for each condition. Gammas (γs) were fixed across the different models and replications, while the second-level predictor values (Xs) were sampled, from pre-specified distributions for each sample size and ICCs. Zs were calculated based on the Xs values. All of the raw data were generated in R. Each model was applied to the raw data to get the multilevel parameter estimates.

Comparison of Estimates

All of the analyses were conducted using HLM 7 for Windows using maximum likelihood estimation. To compare the effect of sample size at different levels, ICCs, and model complexity, several qualities of the parameter estimates were used; (a) convergence, (b) absolute bias, (c) relative bias, (d) root mean squared error, and (e) parameter coverage proportions. Each of these values were calculated for each of the parameter estimates.

Convergence. Non-convergence rate was calculated for each design choices. Convergence of a model occurs when the estimation procedure stabilizes upon a unique solution. Problems can arise when data does not allow for estimation of a meaningful solution. Non-convergence were estimated by the number of replications in which no estimate is available for each of the fixed effect, random effect, and variance components. HLM 7 stops when the maximum number of iterations reached and provides the error message accordingly. However, it does not provide any information regarding which parameters converged and which did not converge.

Absolute bias. To obtain a measure of the magnitude of this bias, the following equation was used for each model:

$$Bias(\pi) = \frac{\sum_{i=1}^{\mathrm{R}} (\hat{\pi}_i - \pi)}{\mathrm{R}} \qquad (7)$$

where, $\hat{\pi}$ is the estimated parameter and π is the true value. R is the total number of replications, and *i* is the replication number. Absolute values for bias were used to prevent cancellation of negative and positive values.

Absolute bias values close to 0 indicates unbiased parameter estimates. However, no formal criteria is available for when a absolute is too big or not acceptable since the absolute bias values are sensitive to the magnitude of the data generating values.

Relative bias. The performance of each analytical solution was assessed with measures of relative bias (Stone & Sobel, 1990). Relative bias is a measure of the accuracy of the estimates and is calculated as the difference between the parameter estimates and the data generating value (true value or population value). An index of relative bias is computed by dividing the bias of a parameter by the data-generating value of that parameter to eliminate the effect of the magnitude of the parameter. Relative bias not only allows for the comparison of how bias in a parameter estimate may change depending on the size of the effect (Krull, 1997), but also takes into account the direction of bias. Negative values indicate underestimation, and positive values indicate overestimation. A smaller relative bias indicates a more accurate parameter estimate. Researchers used different criteria for acceptable bias such as Coleman's (2006) considered relative bias is between -0.01 and 0.01 as unbiased. Muthén, Kaplan, and Hollis (1987) suggested any bias with absolute value less than 0.10-0.15 is acceptable. Some recent studies considered a relative bias of .20 as acceptable (Vallejo, Fernández, Cuesta, & Livacic-Rojas, 2015). On the other hand, Hoogland and Boomsma (1998) used a relatively conservative approach of 0.05 or less as the acceptable bias. Following the suggestions by Muthén, Kaplan, and Hollis (1987), the acceptable bias in this study was set to be between -.15 and .15.

The relative bias of parameter estimates was calculated using equation 8

$$\textit{Relative Bias } (\pi) = \frac{\sum_{i=1}^{R} \frac{(\hat{\pi}_i - \pi)}{\pi}}{R} \tag{8}$$

where, $\hat{\pi}$ is the estimated parameter and π is the true value. R is the total number of replications, and *i* is the replication number. Relative Bias was calculated for each model, in each condition, across the 500 replications.

Root Mean Squared Error (RMSE). The average squared difference between the estimate and its true value is called Mean Squared Error (MSE). MSE provides a useful measure of the overall precision of the parameter estimate, but since we took the square of the difference, it is not in the original metric. The square root of the MSE transforms the MSE back into the same scale as the parameter. A smaller Root Mean Squared Error (RMSE) values indicates a more precise parameter estimates. However, there is a dearth in the literature regarding a formal criteria for when a RMSE is too big or not acceptable. In this section, the effect of different sample size and ICC conditions on RMSE values presented for both fixed effects and residual variance-covariance terms.

The precision of a parameter estimate can be expressed by the square root of the mean squared error, calculated as:

$$RMSE(\pi) = \sqrt{\frac{\sum_{i=1}^{R}(\hat{\pi}_i - \pi)^2}{R}} \qquad (9)$$

where, $\hat{\pi}$ is the estimated parameter and π is the true value. R is the total number of replications, and *i* is the replication number. RMSE are calculated for each model, in each condition, across the 500 replications.

Coverage. The coverage probability of a confidence interval is the proportion of times that true parameter value is contained in the interval to the nominal rate. The nominal rate was set to 0.95 in this study. Across modified factors, the 95% confidence interval was calculated for parameter estimates in each data set. The frequency that the true parameter is in this 95% interval counted and divided by the total number of replications for each parameter under each condition. If data-model fit is working well, the actual parameter coverage should be close to the .95. If the coverage is close to 0.95 and the parameter estimates are accurate, the standard errors of the estimates are unbiased and consequently the Type-I error rates are properly controlled. However, if the standard errors are biased upward, it leads to coverage rates larger than the nominal rate. These produce conservative rates of Type-I error. On the other hand, if the standard errors are biased downward, it leads to coverage rates less than the nominal rate. These produce more liberal rates of Type-I error.

Parameter coverage was only calculated for each of the fixed effects (γ_{000}, γ_{100}, γ_{010}, γ_{001}, γ_{110}, γ_{101}, γ_{011}, γ_{111}). The efforts to find methods to calculate the confidence intervals for the residual variance and covariance term at different levels in three level models proved fairly unproductive as such no clear guidelines were available. The results of simulation studies on constructing confidence intervals for variance components by using the Wald test (van der Leeden et al., 1997) and the chi-square test (Harwell, 1997; Sánchez-Meca & Marín-Martínez, 1997) suggest that with small numbers of groups, both tests suffer from a very low power. Given the lack of clear guidelines and the poor performance of existing methods, only the confidence interval for the fixed effects terms were calculated.

Study 1: Results and Discussion

This section presents how sample sizes, model complexity, and ICCs affect the estimates of fixed effects and error variance-covariance terms in a three-level HLM.

Table 11 shows which parameters were estimated in each model, where a '+' indicates that parameter was estimated in that model. The order of presentation for each of the parameter group is as follows: (a) convergence, (b) relative bias results, (c) RMSE results, and (d) parameter coverage. The tabular representation of relative bias and RMSE are presented for each of the parameter estimates. However, because it is often difficult to interpret tabular information in simulation studies, graphical summaries were also presented and examined when necessary.

There are four fixed effect intercept terms ($\gamma_{000}, \gamma_{010}, \gamma_{100}$, and γ_{110}) as seen in Table 11. γ_{000} and γ_{100} were estimated in both L1 and L2L2L3 models. γ_{010} and γ_{110} were only estimated in L1L2L3 model. γ_{000} is the intercept term for the equation predicting β_{00j}. γ_{010} is the intercept term for the equation predicting β_{01j}. γ_{100} is the intercept term for the equation predicting β_{10j}. γ_{110} is the intercept term for the equation predicting β_{11j}. γ_{110} is also called a cross-level interaction term between L1 and L2 predictor.

Table 11.

Estimated Parameters under Both Models

Parameter Group	Level	Parameter	Model L1	Model L1L2L3
Fixed Effects Intercept Terms	L3	γ_{000}	+	+
		γ_{010}		+
		γ_{100}	+	+
		γ_{110}		+
Fixed Effects Slope Terms	L3	γ_{001}		+
		γ_{011}		+
		γ_{101}		+
		γ_{111}		+
Residual Variance Terms	L1	σ_e^2	+	+
	L2	$\sigma_{r_0}^2$	+	+
		$\sigma_{r_1}^2$	+	+
	L3	$\sigma_{u_{00}}^2$	+	+
		$\sigma_{u_{10}}^2$	+	+
		$\sigma_{u_{01}}^2$		+
		$\sigma_{u_{11}}^2$		+
Residual Covariance Terms	L2	$\sigma_{r_0}\sigma_{r_1}$	+	+
	L3	$\sigma_{u_{00}}\sigma_{u_{01}}$		+
		$\sigma_{u_{00}}\sigma_{u_{10}}$		+
		$\sigma_{u_{00}}\sigma_{u_{11}}$		+
		$\sigma_{u_{01}}\sigma_{u_{10}}$		+
		$\sigma_{u_{01}}\sigma_{u_{11}}$		+
		$\sigma_{u_{10}}\sigma_{u_{11}}$		+

There are four fixed effect slope terms ($\gamma_{001}, \gamma_{011}, \gamma_{101}$, and γ_{111}) as seen in Table 11. Fixed effect slope terms were estimated only in L1L2L3 model. γ_{001} is the slope term for the equation predicting β_{00j}. γ_{001} is the only non-interaction slope term. γ_{011} is the slope term for the equation predicting β_{01j}. It is also called a cross-level interaction term between L2 and L3 predictor. γ_{101} is the slope term for the equation predicting β_{10j}. It is also called a cross-level interaction term between L1 and L3 predictor. γ_{111} is the slope term for the equation predicting β_{11j}. It is also called a three-way cross-level interaction term between L1, L2, and L3 predictor.

There are seven residual variance terms (σ_e^2, $\sigma_{r_0}^2$, $\sigma_{r_1}^2$, $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) as seen in Table 11. σ_e^2 is the L1 residual variance term. $\sigma_{r_0}^2$ and $\sigma_{r_1}^2$ are the L2 residual variance term. $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$ are the L3 residual variance terms. There are also seven residual covariance terms ($\sigma_{r_0 r_1}$, $\sigma_{u_{00} u_{01}}$, $\sigma_{u_{00} u_{10}}$, $\sigma_{u_{00} u_{11}}$, $\sigma_{u_{01} u_{10}}$, $\sigma_{u_{01} u_{11}}$, and $\sigma_{u_{10} u_{11}}$) as seen in Table 11. $\sigma_{r_0 r_1}$is the L2 residual covariance term and the remaining residual covariance terms are the L3 residual covariance terms.

Convergence, absolute bias, relative bias, RMSE, and parameter coverage proportion results are presented in the next section. The results are presented follow in the order of parameter groups presented in Table 11.

Convergence. Model convergence was not a substantial problem with any of the conditions examined in study. There were only maximum of two replications under the lowest sample size condition (N_{L1}=3, N_{L2}=10, N_{L3}=100) for the L1L2L3 model. For these non-convergence conditions, new data were generated to keep the number of replications at 500.

Absolute bias. The absolute bias values for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effects intercept terms. The absolute bias values for the fixed effect intercept terms are presented in Table 12. As clearly seen in Table 12, the absolute bias

Table 12.

Absolute Bias for Fixed Effect Intercept Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
γ_{000}	10	30	0.273	0.228	0.281	0.242
	50	30	0.118	0.107	0.116	0.105
	10	100	0.146	0.125	0.151	0.138
	50	100	0.062	0.055	0.061	0.058
γ_{010}	10	30			0.031	0.028
	50	30			0.013	0.010
	10	100			0.017	0.015
	50	100			0.007	0.006
γ_{100}	10	30	0.14	0.15	0.16	0.152
	50	30	0.06	0.06	0.07	0.068
	10	100	0.09	0.08	0.08	0.085
	50	100	0.04	0.03	0.04	0.036
γ_{110}	10	30			0.018	0.018
	50	30			0.007	0.007
	10	100			0.010	0.009
	50	100			0.004	0.004

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

values for the γ_{010}, γ_{100}, and γ_{110} are all relatively small and close to 0 compared to the γ_{000}. The highest absolute bias values are associated with γ_{000} where N_{L2}=10 and N_{L3}=30 regardless of varying model complexity and ICCs. It is also clear in the table that as L2 and L3 sample sizes increase absolute bias values decrease.

Fixed effects slope terms. The absolute bias values for the fixed effect slope terms are presented in Table 13. As clearly seen in the Table 13, the absolute bias values

Table 13.

Absolute Bias for Fixed Effect Slope Terms

Parameter	Sample Size*		L1L2L3 Model	
	L2	L3	ICC1	ICC2
γ_{001}	10	30	0.133	0.174
	50	30	0.214	0.337
	10	100	0.067	0.093
	50	100	0.117	0.186
γ_{011}	10	30	0.011	0.012
	50	30	0.013	0.015
	10	100	0.006	0.005
	50	100	0.007	0.008
γ_{101}	10	30	0.084	0.127
	50	30	0.153	0.247
	10	100	0.044	0.069
	50	100	0.076	0.129
γ_{111}	10	30	0.006	0.008
	50	30	0.007	0.010
	10	100	0.003	0.004
	50	100	0.004	0.005

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

for the $\gamma_{011}$and γ_{111} are all relatively small and close to 0 compared to the γ_{001} and γ_{101}. The highest absolute bias values are associated with γ_{001} where N_{L2}=50 and N_{L3}=30 regardless of varying model complexity and ICCs. Similar to the fixed effects intercept terms, as L3 sample sizes increase absolute bias values decrease. However, unlike fixed effects intercept terms, as L2 sample sizes increase absolute bias values increase.

Residual variance terms. The absolute bias values for the residual variance terms are presented in Table 14. Table 14 shows that absolute bias values across conditions ranged from 0.001 to 6.796. The absolute bias values for L3 residual variance terms $\sigma^2_{u_{10}}$and $\sigma^2_{u_{11}}$are very close to 0. However, the absolute bias values for σ^2_e (L1 residual variance), $\sigma^2_{r_0}$ (L2 residual variance associated with L1 equation intercept, π_{0ij}), $\sigma^2_{r_1}$ (L2 residual variance associated with L1 equation intercept, π_{1ij}), $\sigma^2_{u_{00}}$ (L3 residual variance associated with L2 intercept for equation, π_{0ij}), and $\sigma^2_{u_{10}}$ (L3 residual variance associated with L2 intercept for equation, π_{1ij}) are relatively higher compare to the absolute bias values found in the fixed effects intercept and slope terms.

The highest absolute bias values are associated with $\sigma^2_{u_{00}}$ (L3 residual variance associated with L2 intercept for equation, π_{0ij}) where N_{L2}=50 and N_{L3}=100 regardless of varying model complexity and ICCs. Similar to the fixed effects intercept terms, as L2 sample sizes increase absolute bias values decrease for σ^2_e, $\sigma^2_{r_0}$, $\sigma^2_{r_1}$, $\sigma^2_{u_{10}}$, and $\sigma^2_{u_{11}}$. For $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$, only for L1 model as L2 sample sizes increase absolute bias values decrease. However, for both $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{00}}$, absolute bias vales actually increase for ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) conditions as L2 sample sizes increase. Under L1 model for ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) condition, for the $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{00}}$ absolute bias values decrease as L2 sample size increase when L3 sample size 30.

However, absolute bias values increase as L2 sample size increase when L3 sample size was 100. These relationship warrants more exploration in Study 2.

Table 14.

Absolute Bias for the Residual Variance Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
σ_e^2	10	30	1.059	1.170	1.150	1.084
	50	30	0.522	0.491	0.521	0.540
	10	100	0.618	0.582	0.647	0.610
	50	100	0.303	0.279	0.317	0.291
$\sigma_{r_0}^2$	10	30	2.321	1.844	2.192	1.732
	50	30	1.035	0.838	0.985	0.845
	10	100	1.330	1.023	1.226	1.026
	50	100	0.574	0.477	0.558	0.468
$\sigma_{r_1}^2$	10	30	0.569	0.538	0.473	0.448
	50	30	0.300	0.267	0.298	0.274
	10	100	0.376	0.312	0.345	0.284
	50	100	0.191	0.163	0.203	0.170
$\sigma_{u_{00}}^2$	10	30	1.002	1.370	0.981	1.811
	50	30	0.421	0.589	0.778	5.536
	10	100	0.620	0.759	0.731	1.382
	50	100	0.229	0.313	1.681	6.796
$\sigma_{u_{10}}^2$	10	30			0.005	0.006
	50	30			0.002	0.006
	10	100			0.003	0.004
	50	100			0.002	0.007
$\sigma_{u_{01}}^2$	10	30	0.396	0.656	0.424	0.857
	50	30	0.159	0.275	0.371	2.807
	10	100	0.223	0.332	0.305	0.656
	50	100	0.087	0.149	0.853	3.435
$\sigma_{u_{11}}^2$	10	30			0.002	0.002
	50	30			0.001	0.002
	10	100			0.001	0.002
	50	100			0.001	0.003

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

Residual covariance terms. The absolute bias values for the residual covariance terms are presented in Table 15. Table 15 shows that absolute bias values across conditions ranged from 0.001 to 3.877. The absolute bias values for $\sigma_{u_{00}}\sigma_{u_{01}}$(L3 residual covariance between β_{00j} and β_{01j} which are the intercept and the slope of the L2 equation π_{0ij}, respectively), $\sigma_{u_{00}}\sigma_{u_{11}}$(L3 residual covariance between β_{00j} and β_{11j} which are the intercept and the slope of the L2 equations π_{0ij} and π_{1ij}, respectively), $\sigma_{u_{01}}\sigma_{u_{10}}$(L3 residual covariance between β_{01j} and β_{10j} which are the intercept and the slope of the L2 equations π_{0ij} and π_{1ij}, respectively), $\sigma_{u_{01}}\sigma_{u_{11}}$(L3 residual covariance between β_{01j} and β_{11j} which are the slopes of the L2 equations π_{0ij} and π_{1ij}, respectively), and $\sigma_{u_{10}}\sigma_{u_{11}}$ (L3 residual covariance between β_{10j} and β_{11j} which are the intercept and slope of the L2 equation π_{1ij}, respectively) are all very low and less than 0.15. These absolute values are relatively low compared to the residual covariance terms for $\sigma_{r_0}\sigma_{r_1}$and $\sigma_{u_{00}}\sigma_{u_{10}}$. The two highest absolute values were associated with $\sigma_{r_0}\sigma_{r_1}$(L2 residual covariance term) and $\sigma_{u_{00}u_{10}}$(L3 residual covariance between β_{00j} and β_{10j} which are the intercepts of the L2 equations π_{0ij} and π_{1ij}, respectively). For $\sigma_{r_0}\sigma_{r_1}$, the absolute bias values decrease as L2 and L3 sample sizes increase. However, for the $\sigma_{u_{00}u_{10}}$, absolute bias values tend to increase as L2 sample size increase. The absolute bias values tend to decrease as L3 sample size increase for $\sigma_{u_{00}u_{10}}$.

The absolute bias values for the remaining L3 residual covariance terms tend to decrease as L2 sample size increases as well as L3 sample increases.

Table 15.

Absolute Bias for the Residual Covariance Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
$\sigma_{r_0}\sigma_{r_1}$	10	30	0.961	0.892	0.896	0.801
	50	30	0.458	0.411	0.454	0.415
	10	100	0.607	0.51	0.523	0.484
	50	100	0.271	0.241	0.269	0.234
$\sigma_{u_{00}}\sigma_{u_{01}}$	10	30			0.060	0.102
	50	30			0.035	0.173
	10	100			0.041	0.061
	50	100			0.047	0.156
$\sigma_{u_{00}}\sigma_{u_{10}}$	10	30	0.639	1.121	0.545	1.015
	50	30	0.339	0.939	0.547	3.877
	10	100	0.415	0.903	0.403	0.881
	50	100	0.286	0.873	0.747	2.708
$\sigma_{u_{00}}\sigma_{u_{11}}$	10	30			0.031	0.053
	50	30			0.014	0.027
	10	100			0.022	0.031
	50	100			0.007	0.01
$\sigma_{u_{01}}\sigma_{u_{10}}$	10	30			0.04	0.059
	50	30			0.013	0.018
	10	100			0.023	0.031
	50	100			0.008	0.037
$\sigma_{u_{01}}\sigma_{u_{11}}$	10	30			0.002	0.004
	50	30			0.001	0.005
	10	100			0.002	0.003
	50	100			0.002	0.005
$\sigma_{u_{10}}\sigma_{u_{11}}$	10	30			0.023	0.04
	50	30			0.013	0.061
	10	100			0.015	0.025
	50	100			0.016	0.053

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

Relative Bias. The relative bias values for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effect intercept terms. The relative bias values for the fixed effect intercept terms are presented in Table 16. As clearly seen in the Table 16, the relative bias values for the γ_{011}, γ_{111}, are all within acceptable values unlike γ_{000}, γ_{101}. The highest relative

Table 16.

Relative Bias for Fixed Effect Intercept Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
γ_{000}	10	30	0.347	0.279	0.351	0.306
	50	30	0.152	0.133	0.145	0.129
	10	100	0.184	0.155	0.193	0.171
	50	100	0.078	0.068	0.077	0.073
γ_{011}	10	30			0.038	0.035
	50	30			0.017	0.013
	10	100			0.021	0.019
	50	100			0.009	0.008
γ_{101}	10	30	0.182	0.185	0.194	0.191
	50	30	0.080	0.081	0.082	0.085
	10	100	0.108	0.104	0.105	0.108
	50	100	0.046	0.043	0.044	0.045
γ_{111}	10	30			0.023	0.022
	50	30			0.009	0.008
	10	100			0.012	0.012
	50	100			0.005	0.005

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

bias values were associated with the condition where N_{L2}=10 and N_{L3}=30. For all of the fixed effects intercept parameters, the relative bias values tend to drop as L2 sample sizes increase as well as L3 sample sizes increase.

Fixed effect slope terms. The relative bias values for the fixed effect slope terms are presented in Table 17. As clearly seen in the Table 17 the relative bias values for

Table 17.

Relative Bias for Fixed Effect Slope Terms

Parameter	Sample Size*		L1L2L3 Model	
	L2	L3	ICC1	ICC2
γ_{001}	10	30	0.167	0.219
	50	30	0.270	0.420
	10	100	0.085	0.118
	50	100	0.146	0.229
γ_{011}	10	30	0.013	0.015
	50	30	0.017	0.019
	10	100	0.007	0.007
	50	100	0.008	0.010
γ_{101}	10	30	0.106	0.158
	50	30	0.191	0.305
	10	100	0.056	0.086
	50	100	0.096	0.163
γ_{111}	10	30	0.008	0.010
	50	30	0.009	0.012
	10	100	0.004	0.004
	50	100	0.005	0.006

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

γ_{011} and γ_{111}, are all within acceptable range. On the other hand, the relative bias values for γ_{000} and γ_{111}, are not all within acceptable range. Similar to the fixed effects intercept terms, as L3 sample sizes increase relative bias values decrease. However, unlike fixed effects intercept terms, as L2 sample sizes increase relative bias values increase.

Residual variance terms. The relative bias values for the residual variance terms are presented in Table 18. As clearly seen in the Table 18, the relative bias results for L1 model under shows that level 1 residual variance (σ_e^2) were estimated with minimal bias. Similarly, the level 2 residual variance associated with the intercept ($\sigma_{r_0}^2$) were also estimated with minimal bias. However, there are some clear differences in the relative bias for the level 2 residual variance associated with the slope. The largest relative bias (3.25) was for $\sigma_{r_1}^2$ and it was the condition terms associated with level 2 intercept and slope. The largest relative bias (3.25) was for $\sigma_{r_1}^2$ and it was the condition when ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) and sample sizes for N_{L2}=10 and N_{L3}=30 are the lowest examined in Study 1. Similarly, when ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) the relative bias for $\sigma_{r_1}^2$ was the highest among all the sample size combinations. The magnitude of the relative bias values for $\sigma_{r_1}^2$ is around 3 times higher for ICC_2 than the ICC_1 condition. This difference warrants further exploration of the effect of varying ICC values on relative bias.

Table 18.

Relative Bias for Residual Variance Terms

Parameter	Sample Size* L2	L3	L1 Model ICC1	L1 Model ICC2	L1L2L3 Model ICC1	L1L2L3 Model ICC2
σ_e^2	10	30	-0.015	-0.022	-0.021	-0.021
	50	30	-0.004	-0.005	-0.007	-0.009
	10	100	-0.007	-0.010	-0.008	-0.013
	50	100	-0.002	-0.002	-0.003	-0.003
$\sigma_{r_0}^2$	10	30	0.020	0.067	-0.013	0.043
	50	30	0.008	0.016	0.007	0.022
	10	100	0.014	0.023	0.007	0.028
	50	100	-0.001	0.002	0.005	0.002
$\sigma_{r_1}^2$	10	30	1.247	3.247	0.993	2.662
	50	30	0.319	1.167	0.415	1.193
	10	100	0.646	1.482	0.491	1.377
	50	100	0.094	0.395	0.100	0.418
$\sigma_{u_{00}}^2$	10	30	-0.123	-0.048	-0.103	-0.135
	50	30	-0.039	-0.036	-0.224	-0.692
	10	100	-0.036	-0.010	-0.108	-0.100
	50	100	-0.015	-0.009	-0.621	-0.853
$\sigma_{u_{10}}^2$	10	30			1.408	0.072
	50	30			-0.086	-0.571
	10	100			0.476	-0.123
	50	100			-0.454	-0.774
$\sigma_{u_{01}}^2$	10	30	-0.096	-0.048	-0.140	-0.146
	50	30	-0.028	-0.037	-0.232	-0.697
	10	100	-0.036	-0.019	-0.120	-0.103
	50	100	-0.012	-0.010	-0.629	-0.855
$\sigma_{u_{11}}^2$	10	30			0.974	0.007
	50	30			-0.212	-0.576
	10	100			0.305	-0.139
	50	100			-0.468	-0.767

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

Residual covariance terms. The relative bias values for the residual variance terms are presented in Table 19. As clearly seen in the Table 19, the majority of the relative bias values are not within the acceptable range. The only condition where the relative bias values for $\sigma_{r_0}\sigma_{r_1}$ are within acceptable range was for the conditions where NL2=50 and NL3=100 regardless of varying model complexity and ICCs. None of the conditions for $\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$,$\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$ have acceptable relative bias values. On the other hand as seen in Table 19, only one or two conditions for $\sigma_{u_{00}}\sigma_{u_{10}}$ and $\sigma_{u_{01}}\sigma_{u_{10}}$have relative bias values within acceptable range. Overall, the relative bias estimates for the residual covariance terms are not within acceptable range especially for L3 residual covariance terms. This warrants further explorations such as fixing the L3 covariance to 0 and check how that effects the relative bias for other parameters in the model.

To summarize the results for L1 model, parameter estimates were grouped as fixed effects, L1 residual variance, L2 residual variance-covariance, and L3 residual variance-covariance for easy comparison. Figure 2 includes the visual representation of the relative bias results group by the type of estimates. The first two columns in Figure 2 represents the fixed effects for ICC_1 and ICC_2 respectively. Similarly, the third and fourth column represents the L1 residual variance for ICC_1 and ICC_2 respectively. The fifth and sixth column was for L2 residual variance-covariance terms, and finally, the last two columns are for L3 residual variance-covariance terms.

Table 19.

Relative Bias for Residual Covariance Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
$\sigma_{r_0}\sigma_{r_1}$	10	30	0.55	1.76	0.51	1.71
	50	30	0.13	0.61	0.24	0.54
	10	100	0.29	0.72	0.34	0.63
	50	100	0.00	0.12	0.09	0.12
$\sigma_{u_{00}}\sigma_{u_{01}}$	10	30			-0.37	-0.29
	50	30			-0.56	-1.44
	10	100			-0.31	-0.22
	50	100			-1.11	-1.32
$\sigma_{u_{00}}\sigma_{u_{10}}$	10	30	-0.39	-0.30	-0.11	-0.14
	50	30	-0.28	-0.29	-0.38	-1.19
	10	100	-0.30	-0.27	-0.13	-0.17
	50	100	-0.26	-0.27	-0.65	-0.82
$\sigma_{u_{00}}\sigma_{u_{11}}$	10	30			0.17	0.18
	50	30			0.76	1.97
	10	100			0.32	0.21
	50	100			1.11	0.80
$\sigma_{u_{01}}\sigma_{u_{10}}$	10	30			-0.35	-0.09
	50	30			-0.04	-0.26
	10	100			-0.08	-0.08
	50	100			-0.26	-0.59
$\sigma_{u_{01}}\sigma_{u_{11}}$	10	30			0.89	-0.23
	50	30			-0.71	-1.46
	10	100			-0.15	-0.30
	50	100			-1.25	-1.32
$\sigma_{u_{10}}\sigma_{u_{11}}$	10	30			-0.23	-0.29
	50	30			-0.57	-1.66
	10	100			-0.36	-0.23
	50	100			-1.22	-1.45

Note. *L1 sample size is 3.

If we look at the rows in Figure 2, the first two rows keep the L1 (N=3) and L3 (N=30) sample size constant and varies the L2 sample sizes from 10 to 50. In rows three and four, L3 sample size was increased to 100, and L2 sample size was 10 in row 3 and 50 in row 5.

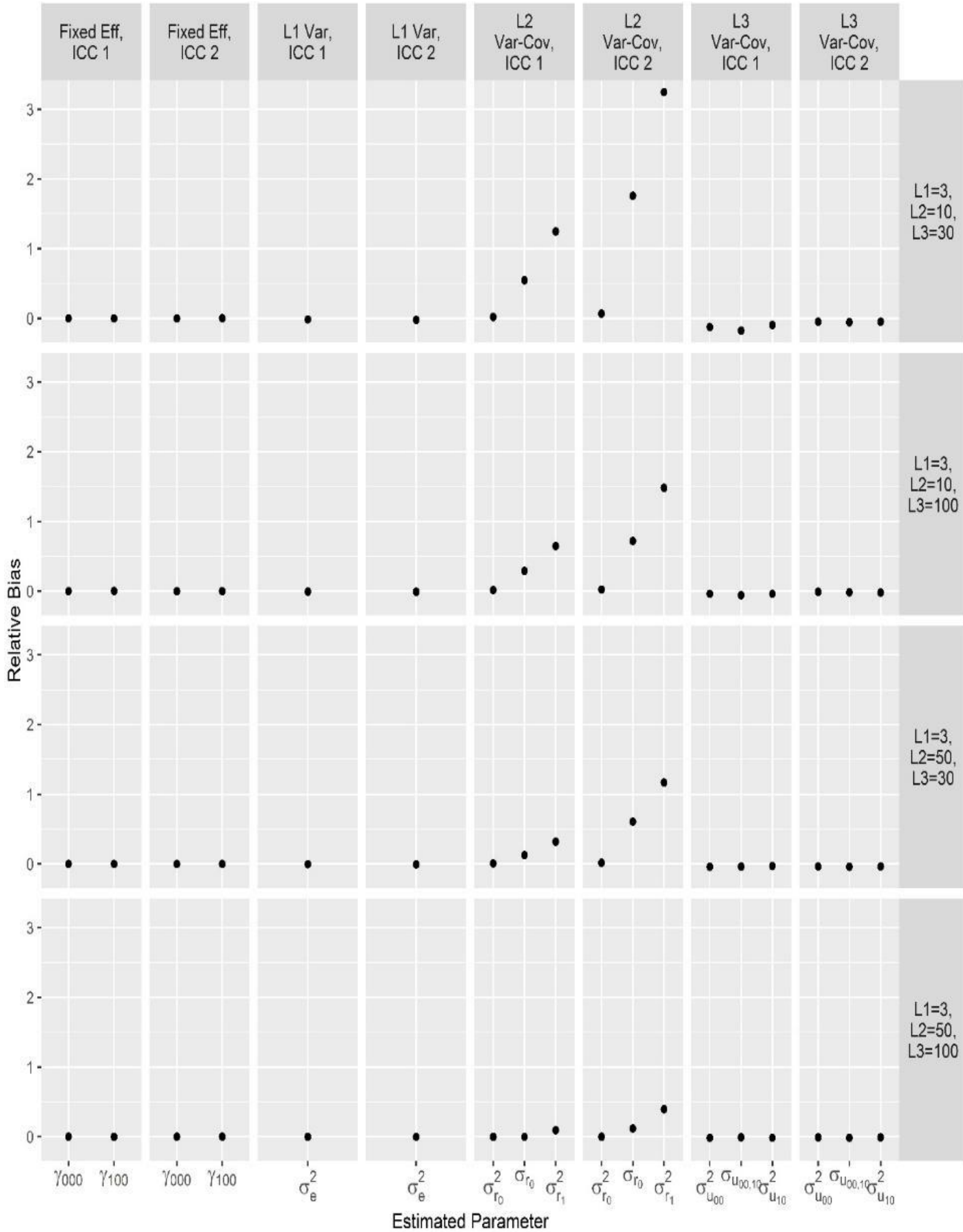

Figure 2. Relative Bias Grouped by Different Parameter Estimates for L1 Model

Using the rule, 0.15 (absolute values) as acceptable relative bias value, it is clear that regardless of sample size and ICC combinations, fixed effects, L1 residual variance, and L3 residual variance-covariance estimates are unbiased. However, when the focus is L2 residual variance-covariance terms, the same conclusion do not hold. The relative bias estimates for ICC_1 is less than the ICC_2 (Less variability at L2 compared to ICC_1). The only acceptable relative bias was for $\sigma_{r_0}^2$ under all sample size conditions. The other conclusion was as the sample size for L2 and L3 increase, the relative bias decreases.

For L1L2L3 model, Figure 3, Figure 4, and Figure 5 includes the visual representation of relative bias values for the fixed effects; L1 residual variance, L2 residual variance-covariance terms; and the L3 residual variance-covariance terms.

The close examination of Figure 3 revealed that the sample size conditions regardless of ICC that had the lowest relative bias was the condition where $N_{L1}=3$, $N_{L2}=10$, and $N_{L3}=100$. Against the expectation, increasing the L2 sample size did not provide lower relative bias values where $N_{L1}=3$, $N_{L2}=50$, and $N_{L3}=100$. Figure 3 also showed that of low relative bias values when L3 sample size was 100.

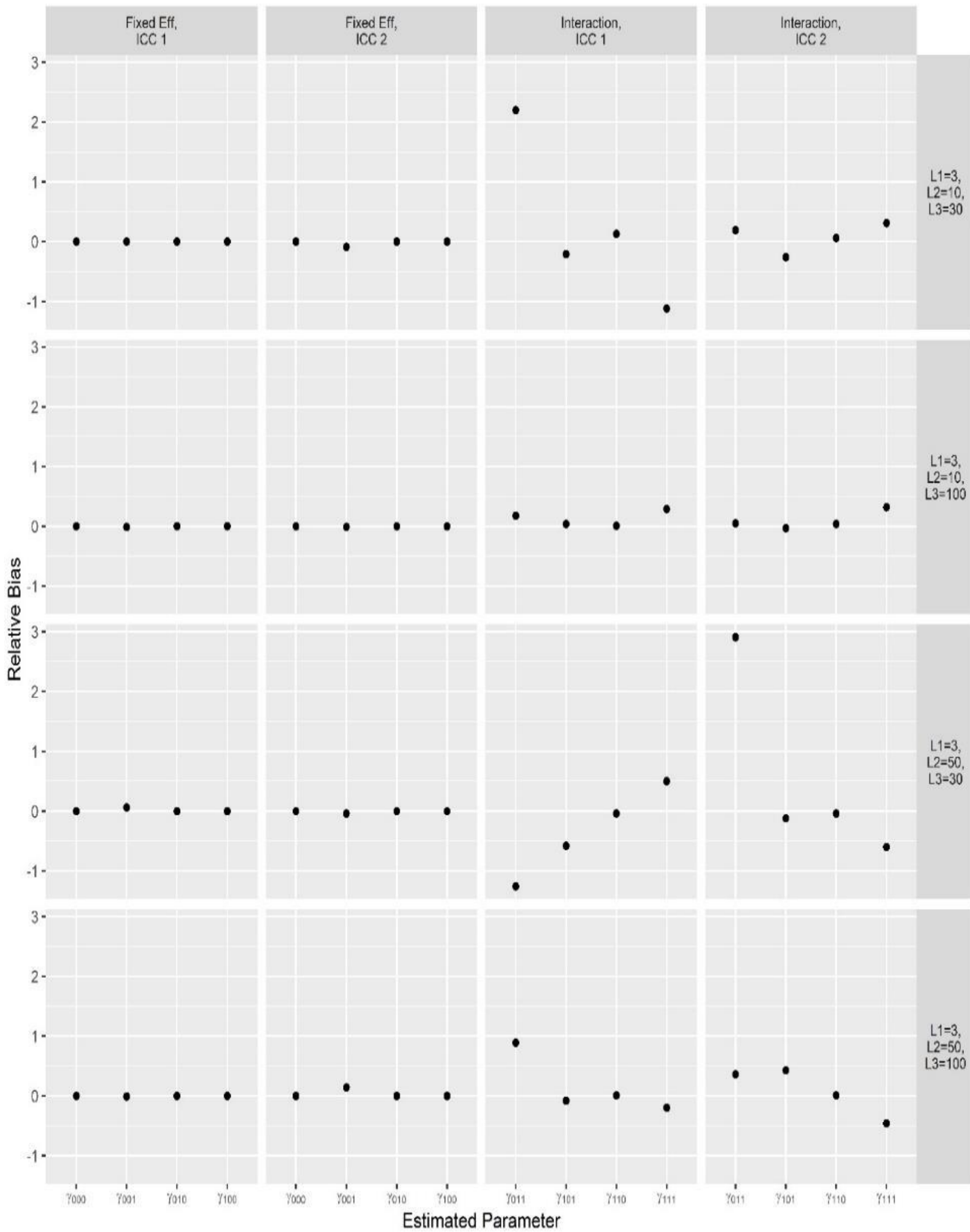

Figure 3. Fixed Effects Relative Bias Grouped for the L1L2L3 Model

Figure 4 shows that the L2 residual variance-covariance terms have higher relative bias values compared to the fixed effects terms presented in Figure 2.

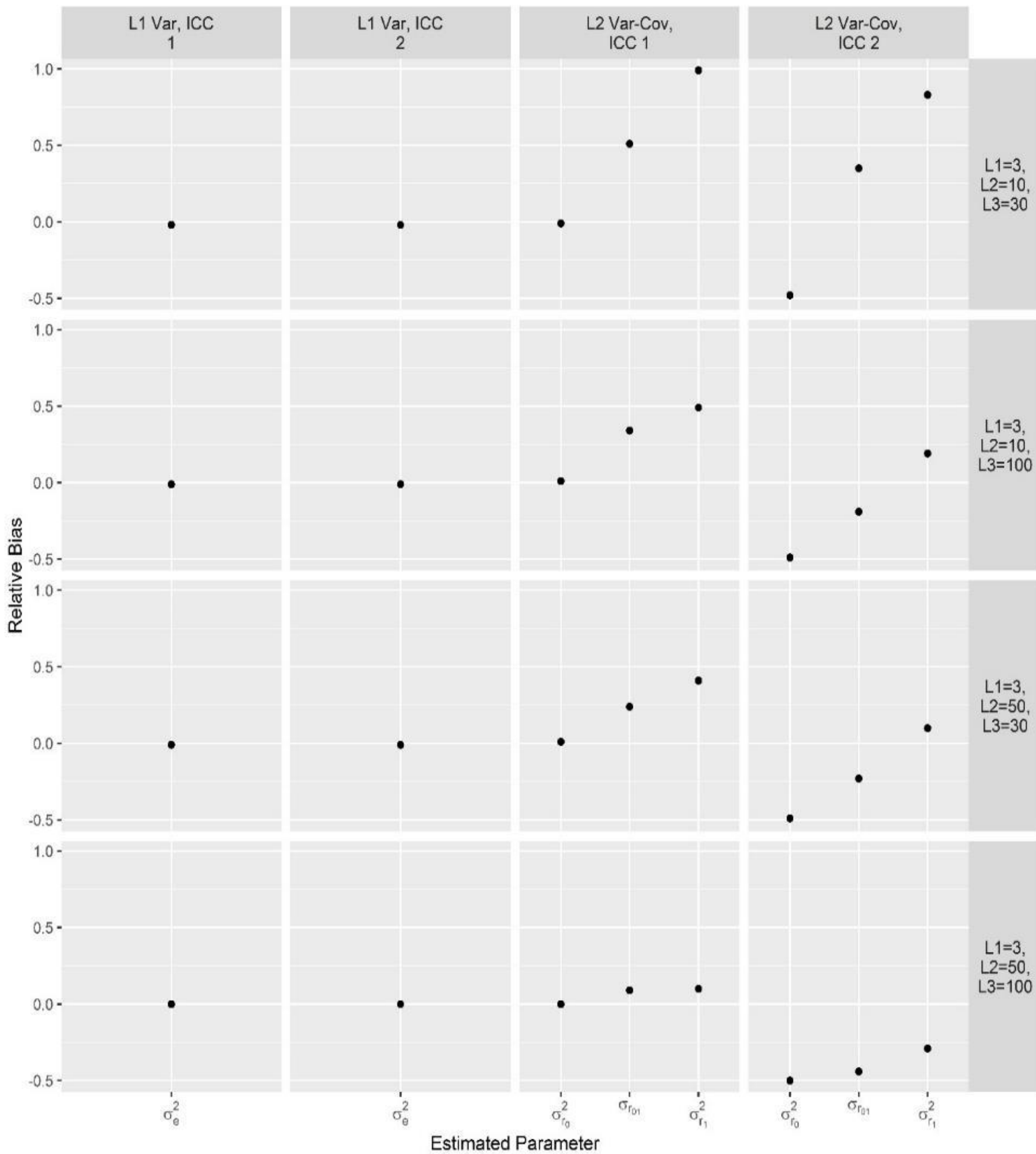

Figure 4. L1 Residual Variance and L2 Residual Variance-Covariance Terms for the L1L2L3 Model

Visual inspection of Figure 5 shows that the condition where $N_{L2} = 10$, and $N_{L3} = 100$ for ICC_1 had the lowest relative bias values compared the other conditions. It was also generally true that ICC_1 conditions have lower relative bias values compared to ICC_2 conditions.

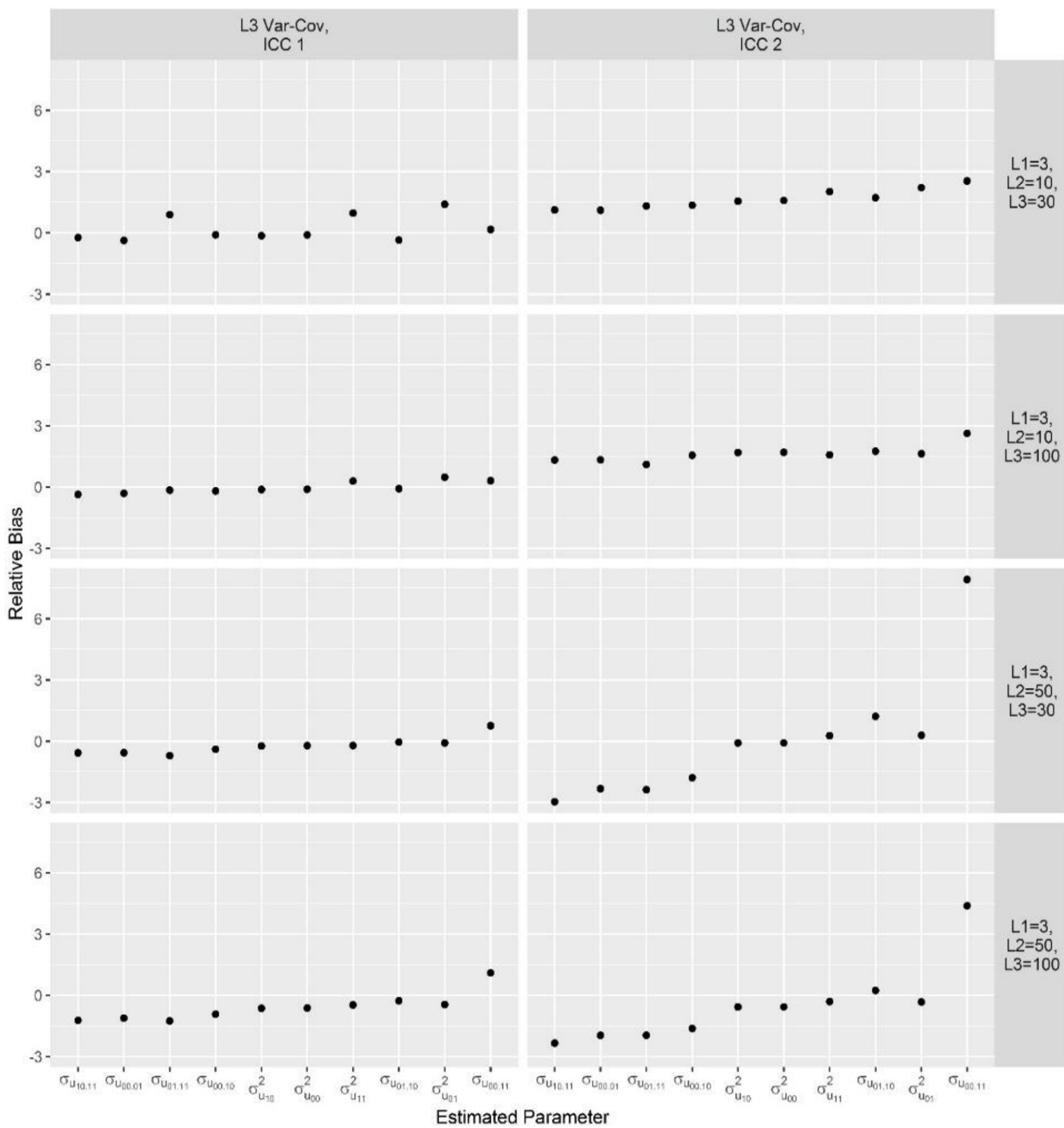

Figure 5. L3 Residual Variance-Covariance for the L1L2L3 Model

RMSE. The RMSE values for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effect intercept terms. The RMSE values for the fixed effect intercept terms are presented in Table 20. As clearly seen in the Table 20, the RMSE values for the

Table 20.

RMSE for Fixed Effect Intercept Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
γ_{000}	10	30	0.347	0.279	0.351	0.306
	50	30	0.152	0.133	0.145	0.129
	10	100	0.184	0.155	0.193	0.171
	50	100	0.078	0.068	0.077	0.073
γ_{011}	10	30			0.038	0.035
	50	30			0.017	0.013
	10	100			0.021	0.019
	50	100			0.009	0.008
γ_{101}	10	30	0.182	0.185	0.194	0.191
	50	30	0.080	0.081	0.082	0.085
	10	100	0.108	0.104	0.105	0.108
	50	100	0.046	0.043	0.044	0.045
γ_{111}	10	30			0.023	0.022
	50	30			0.009	0.008
	10	100			0.012	0.012
	50	100			0.005	0.005

Note. *L1 sample size is 3.

γ_{011}, γ_{111}, were all very low compared to γ_{000}, γ_{101}. The highest RMSE values were associated with the condition where N_{L2}=10 and N_{L3}=30. For all of the fixed effects intercept parameters, the RMSE values tend to drop as L2 sample size increase as well as L3 sample size increases.

Fixed effect slope terms. The RMSE values for the fixed effect slope terms are presented in Table 21. As clearly seen in the Table 21, the RMSE values for γ_{011} and γ_{111}, are all relatively low compared to the RMSE values for γ_{000} and γ_{111}. Similar to the fixed effects intercept terms, as L3 sample sizes increase RMSE values decrease.

Table 21.

RMSE for Fixed Effect Slope Terms

Parameter	Sample Size*		L1L2L3 Model	
	L2	L3	ICC1	ICC2
γ_{001}	10	30	0.167	0.219
	50	30	0.270	0.420
	10	100	0.085	0.118
	50	100	0.146	0.229
γ_{011}	10	30	0.013	0.015
	50	30	0.017	0.019
	10	100	0.007	0.007
	50	100	0.008	0.010
γ_{101}	10	30	0.106	0.158
	50	30	0.191	0.305
	10	100	0.056	0.086
	50	100	0.096	0.163
γ_{111}	10	30	0.008	0.010
	50	30	0.009	0.012
	10	100	0.004	0.004
	50	100	0.005	0.006

However, unlike fixed effects intercept terms, as L2 sample sizes increase RMSE values increase.

Residual variance terms. The RMSE values for the residual variance terms are presented in Table 22. As clearly seen in the Table 22, almost all of the RMSE values are relatively high compared to the RMSE values for the fixed effects intercepts and slopes terms. The minimum RMSE values were associated with $\sigma^2_{u_{10}}$and $\sigma^2_{u_{11}}$. The two largest RMSE values (5.706 and 6.797) were linked to $\sigma^2_{u_{00}}$ under L1L2L3 (one predictor at each level) where the ICC at L2 was lowest and ICC at L3 was highest ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) among examined conditions. These largest RMSE values observed for $\sigma^2_{u_{00}}$ when N_{L2}=50 and N_{L3}=30 or N_{L3}=100. The magnitude of the RMSE values for $\sigma^2_{u_{00}}$ around 4 times higher for ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) than the ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) condition. The difference between ICC_1 and ICC_2 was that more of the shared variability shifts from L2 to L3 in ICC_2. This difference in where the shared variability and its potential effect on the RMSE values warrants further exploration of the effect of varying ICC values on RMSE in Study 2.

RMSE values for the residual variance terms tend to decrease as L2 sample size increases as well as L3 sample size increases also. However, RMSE values of $\sigma^2_{u_{00}}$ violates this pattern.

Residual covariance terms. The RMSE values for the residual covariance terms are presented in Table 23. As clearly seen in the Table 23, the majority of the RMSE values were very small and close to 0. The only conditions where the RMSE values were

Table 22.

RMSE for Residual Variance Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
σ_e^2	10	30	1.338	1.437	1.448	1.382
	50	30	0.659	0.619	0.646	0.669
	10	100	0.761	0.737	0.825	0.760
	50	100	0.380	0.347	0.392	0.369
$\sigma_{r_0}^2$	10	30	2.980	2.338	2.771	2.162
	50	30	1.317	1.037	1.247	1.051
	10	100	1.684	1.287	1.546	1.274
	50	100	0.731	0.596	0.687	0.596
$\sigma_{r_1}^2$	10	30	0.832	0.841	0.753	0.744
	50	30	0.392	0.387	0.392	0.399
	10	100	0.503	0.466	0.469	0.430
	50	100	0.237	0.209	0.249	0.223
$\sigma_{u_{00}}^2$	10	30	1.235	1.698	1.215	2.35
	50	30	0.524	0.747	1.024	5.706
	10	100	0.759	0.958	0.918	2.321
	50	100	0.285	0.394	1.892	6.797
$\sigma_{u_{10}}^2$	10	30			0.009	0.008
	50	30			0.002	0.006
	10	100			0.004	0.005
	50	100			0.002	0.007
$\sigma_{u_{01}}^2$	10	30	0.493	0.821	0.514	1.145
	50	30	0.197	0.344	0.512	2.894
	10	100	0.277	0.413	0.41	1.163
	50	100	0.108	0.186	0.968	3.435
$\sigma_{u_{11}}^2$	10	30			0.003	0.003
	50	30			0.001	0.002
	10	100			0.001	0.002
	50	100			0.001	0.003

Note. *L1 sample size is 3.

Table 23.

RMSE for Residual Covariance Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
$\sigma_{r_0}\sigma_{r_1}$	10	30	1.228	1.151	1.132	1.049
	50	30	0.585	0.529	0.58	0.532
	10	100	0.766	0.66	0.679	0.613
	50	100	0.337	0.297	0.335	0.294
$\sigma_{u_{00}}\sigma_{u_{01}}$	10	30			0.077	0.128
	50	30			0.044	0.177
	10	100			0.052	0.082
	50	100			0.051	0.156
$\sigma_{u_{00}}\sigma_{u_{10}}$	10	30	0.763	1.347	0.687	1.494
	50	30	0.400	1.006	0.809	4.136
	10	100	0.495	1.014	0.580	1.518
	50	100	0.316	0.898	0.964	3.220
$\sigma_{u_{00}}\sigma_{u_{11}}$	10	30			0.041	0.066
	50	30			0.016	0.028
	10	100			0.026	0.038
	50	100			0.008	0.010
$\sigma_{u_{01}}\sigma_{u_{10}}$	10	30			0.050	0.074
	50	30			0.017	0.019
	10	100			0.029	0.039
	50	100			0.008	0.037
$\sigma_{u_{01}}\sigma_{u_{11}}$	10	30			0.004	0.005
	50	30			0.002	0.005
	10	100			0.002	0.003
	50	100			0.002	0.005
$\sigma_{u_{10}}\sigma_{u_{11}}$	10	30			0.029	0.050
	50	30			0.016	0.063
	10	100			0.018	0.032
	50	100			0.017	0.053

Note. *L1 sample size is 3.

high were under $\sigma_{r_0}\sigma_{r_1}$, $\sigma_{u_{00}}\sigma_{u_{01}}$ and $\sigma_{u_{00}}\sigma_{u_{10}}$. All other residual covariance terms have a RMSE value less than 0.10. Generally, the RMSE values decrease as the L2 or L3 sample size increases.

All of the conditions under $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$,$\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$ had RMSE values very close to 0. Overall, four out of seven residual covariance terms had relatively high RMSE values. This warrants further explorations such as how fixing the L3 covariance to 0 effects the RMSE for other parameters in the model. Another point worth mentioning was that almost all of the RMSE values for level 3 residual variance-covariance terms were negative which means they were all underestimated.

Coverage. The parameter coverage proportions for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effect intercept terms. Table 24 shows that parameter coverage proportions for the fixed effect intercept terms. Almost all of the coverage proportions are greater than the nominal level 0.95 except the two conditions under L1L2L3 model for γ_{010} where N_{L2}=10 and N_{L3}=30.

The coverage rates larger than the nominal rate is a sign of upward biased standard errors given the parameter estimates are accurate. On the other hand, upward biased standard errors tend to produce conservative rates of Type-I error errors given the parameter estimates are accurate.

Table 24.

Parameter Coverage Proportions for Fixed Effect Intercept Terms

Parameter	Sample Size*		L1 Model		L1L2L3 Model	
	L2	L3	ICC1	ICC2	ICC1	ICC2
γ_{000}	10	30	0.98	1.00	0.98	1.00
	50	30	1.00	1.00	1.00	1.00
	10	100	0.99	1.00	0.98	1.00
	50	100	1.00	1.00	1.00	1.00
γ_{010}	10	30			0.95	0.95
	50	30			1.00	1.00
	10	100			0.96	0.96
	50	100			0.98	1.00
γ_{100}	10	30	0.99	1.00	0.99	1.00
	50	30	1.00	1.00	1.00	1.00
	10	100	0.99	1.00	1.00	1.00
	50	100	1.00	1.00	1.00	1.00
γ_{110}	10	30			0.96	0.97
	50	30			0.96	1.00
	10	100			0.96	0.97
	50	100			0.97	1.00

Parameter coverage proportions for γ_{000} and γ_{100} were relatively higher than the parameter coverage proportions for γ_{010} and γ_{110}. This uncertainty in estimation may be due to the much larger scale of γ_{000} and γ_{100} intercept terms relative to the γ_{010} and γ_{110}.

Fixed effects slope terms. Table 25 shows that fixed effects slope terms have more conditions that had the coverage proportions at the nominal level compared to the fixed effects intercept terms. Unlike the fixed effect intercept terms, coverage proportions are general lower than the nominal level of .95. The coverage rates less than the nominal rate are a sign of downward biased standard errors given the parameter estimates are accurate. These produce relatively conservative rates of Type-I error given the parameter estimates are accurate.

Table 25.

Parameter Coverage Proportions for Fixed Effect Slope Terms

Parameter	Sample Size*		L1L2L3 Model	
	L2	L3	ICC1	ICC2
γ_{001}	10	30	0.91	0.92
	50	30	0.91	0.90
	10	100	0.95	0.92
	50	100	0.93	0.95
γ_{011}	10	30	0.90	0.86
	50	30	0.88	0.91
	10	100	0.93	0.94
	50	100	0.95	0.95
γ_{101}	10	30	0.90	0.88
	50	30	0.90	0.92
	10	100	0.94	0.93
	50	100	0.94	0.93
γ_{111}	10	30	0.91	0.88
	50	30	0.90	0.90
	10	100	0.94	0.95
	50	100	0.93	0.93

Summary of Study 1 Results

Convergence rate was not an issue for the models examined. Because of that, it was not examined in this section.

Fixed Effects Intercept Terms. The absolute bias values, relative bias values for fixed effects intercept terms were generally acceptable, and the RMSE values were generally low under the studied conditions for both L1 and L1L2L3 model. However, $\boldsymbol{\gamma_{000}}$ and $\boldsymbol{\gamma_{100}}$ had higher absolute bias values and RMSE values for both models and ICCs when the L3 sample size was 30 regardless of L2 sample size.

The parameter coverage proportions for the fixed effect intercept terms were generally above the nominal level across the models, sample sizes, and ICCs examined under Study 1.

In all of the calculated summary statistics (absolute bias, relative bias and RMSE results) for fixed effects intercept terms, increasing the sample size both at L2 and L3 reduced the absolute bias, relative bias and RMSE values regardless of the model complexity and ICCs.

Fixed Effects Slope Terms. Fixed effects slope terms were only estimates under the L1L2L3 model (one predictor at each level). The absolute bias values for fixed effects slope terms were generally acceptable except for the γ_{001} and γ_{101} where L3 sample size was 30 regardless of ICCs and L2 sample sizes. On the other hand, the relative bias values were only acceptable for γ_{001} in which the data generating value was relatively higher compared to the other fixed effect slope terms. Relative bias values tend to be smaller if the data generating values are higher. On the hand, relative bias values tend to be higher if the data generating values were between -1 and +1 since the calculations of relative bias values involve dividing by the data generating values. Similar to the absolute bias values, RMSE values generally acceptable except for the γ_{001} and γ_{101} where L3 sample size was 30 regardless of ICCs and L2 sample sizes.

The parameter coverage proportions for the fixed effect slope terms were generally below the nominal level across the models, sample sizes, and ICCs examined under Study 1.

In all of the calculated summary statistics (absolute bias, relative bias and RMSE results) for fixed effects slope terms, increasing the sample size at L3 reduced the absolute bias, relative bias and RMSE values regardless of the model complexity and ICCs. However, the same pattern was not observed for the L2 sample size. This difference requires further examined of this pattern in Study 2.

To test the effect of how increasing both L2 and L3 sample sizes affected the absolute bias, relative bias, RMSE, and parameter coverage proportion results, the maximum L2 sample size was set to 75, and L3 sample size was set to 150. Additionally, the L2 sample size of 25 was examined in Study 2 which is a typical classroom size.

Residual Variance Terms. The relative bias estimates for L1 residual variance term ($\boldsymbol{\sigma_e^2}$) were within acceptable range for both the L1 model and L1L2L3 model under the examined conditions but not for the absolute bias and RMSE.

The absolute bias and RMSE values L2 residual variance term $\sigma_{r_1}^2$ were relatively higher compared to the L1 residual variance term (σ_e^2). On the other hand, the absolute bias and RMSE values for the second L2 residual variance term $\sigma^2_{r_0}$ were relatively lower compared to the L1 residual variance term (σ^2_e).

The relative bias values for $\sigma^2_{r_1}$ were relatively higher compared to both σ^2_e and $\sigma^2_{r_0}$. The magnitude of data generating parameter for $\sigma^2_{r_1}$was lower than the both σ^2_e and $\sigma^2_{r_0}$. This discrepancy in the RMSE values might be related to that.

The absolute bias and RMSE values of $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$were all very low and close to 0. However, the same was not true for the RMSE values of $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$. All three summary statistics (absolute bias, relative bias, and RMSE) for $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$have relatively higher values compared to the $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$.

Generally, absolute bias, relative bias, and RMSE values tend to decrease as both L2 and L3 sample size increases. However, this pattern was violated by $\sigma^2_{u_{00}}$and $\sigma^2_{u_{10}}$for absolute bias and RMSE specifically for the L2 sample sizes. Similarly, this pattern was violated for relative bias for all of the L3 residual variances regardless of model complexity and ICCs.

Residual Covariance Terms. Most of the L3 residual covariance terms ($\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$) had very low absolute bias and RMSE values but almost all of the residual covariance terms did not have relative bias values within acceptable range. The remaining residual covariance terms $\sigma_{r_0}\sigma_{r_1}$and $\sigma_{u_{00}}\sigma_{u_{10}}$had higher values in all of the thee summary statistics examined.

Increasing the sample size on both L2 and L3 generally decreased the relative bias values and RMSE values. However, changing ICC from ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.40,

ICC_{L3}=0.10) to ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) caused two or three times increase in the absolute bias and RMSE values for $\sigma_{u_{00}}\sigma_{u_{10}}$ in the L1L2L3 model. In the L1L2L3 model. In other words, as more of shared variability shifts from L2 to L3, the magnitude of the absolute bias and RMSE values increased drastically for $\sigma_{u_{00}}\sigma_{u_{10}}$under L1L2L3 model but not under L1 model.

This drastic rise in the absolute bias and RMSE values warrants more exploration of the effects of varying the magnitude of the ICC values. In both ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) and ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30), L1 was set to be the same and 0.5. However, most of the shared variability was shifted from L2 to L3 in the ICC_2.

Naturally, the question of what happens if the shared variability and both L2 and L3 was equal. Keeping this question in mind, the next ICC condition was set to equal ICC at L2 and L3 was tested. To do this, ICC for L2 was set to .25, and L3 was set to .25. In the second new ICC, to test the effect of lowering L2 ICC slightly (0.10) instead of drastically (0.20), ICC for L2 was set to .30, and L3 was set to .20.

Majority of the absolute bias and RMSE values were relatively small for the L3 residual covariance terms. This warranted further explorations of the L3 residual covariance terms and their effect on the accuracy of parameter estimates. To test this, three new models proposed to be included in Study 2.

In the first new model that was called L1L2 variable model, the L3 predictor was be dropped. L1L2 model equations were shown in Table 26. In the second new model, the L3 residual covariance terms was set to 0 for the L1L2 model, and it was called

L1L2 No L3 Covariance Model. In the third new model, the L3 residual covariance terms was set to 0 for the L1L2L3 model, and it was called L1L2L3 No L3 Covariance Model. The model equations for the L1L2 No L3 Covariance Model was the same as L1L2 model equations since we only constrained the L3 covariance terms to be 0. Similarly, the model equations for the L1L2L3 No L3 Covariance Model was the same as L1L2L3 model equations since we only constrained the L3 covariance terms to be 0.

Table 26.

L1L2 Model (Level 1 and Level 2 predictor)

Level 1	$Y_{tij} = \pi_{0ij} + \pi_{1ij}\text{Time}_{tij} + \varepsilon_{tij}$	(8a)
Level 2	$\pi_{0ij} = \beta_{00j} + \beta_{01j}\, X_{ij} + r_{0ij}$	(8b)
	$\pi_{1ij} = \beta_{10j} + \beta_{11j}\, X_{ij} + r_{1ij}$	(8c)
Level 3	$\beta_{00j} = \gamma_{000} + u_{00j}$	(8d)
	$\beta_{01j} = \gamma_{010} + u_{01j}$	(8e)
	$\beta_{10j} = \gamma_{100} + u_{10j}$	(8f)
	$\beta_{11j} = \gamma_{110} + u_{11j}$	(8g)
Combined	$Y_{tij} = \gamma_{000} + u_{00j} + (\gamma_{010} + u_{01j})\, X_{ij} + r_{0ij} + (\gamma_{100} + u_{10j} + (\gamma_{110} + u_{11j})\, X_{ij} + r_{1ij})\, \text{Time}_{tij} + \varepsilon_{tij}$	(8h)

Chapter 4

STUDY 2

Method

The same data generation procedure in Study 1 was also used in Study 2. Similarly, the data generating values used in Study 1 was also used in Study 2. As described in Study 1 under “comparison of estimates”, the same summary statistics was also employed in Study 2.

Results of Study 1 are used to make design choices for Study 2. Study 1 results can be summarized as follows. First, results showed that the model parameters have different sensitivity to modified conditions depending on whether the parameter was on L1, L2 or L3. Second, fixed effects intercept terms were accurately estimated compared to the fixed effects slope terms. Third, both fixed effects intercept and slope parameters were more accurately estimated compared to the residual variance-covariance terms. Fourth, varying ICC levels affected the residual variance-covariance parameters more than the fixed effects parameters. Finally, increasing model complexity resulted in higher relative bias and RMSE values for the model’s fixed effects and residual variance-covariance terms.

In Study 1, two ICC conditions were examined, a) L1=.5, L2=.40, L3=0.10, (b) L1=.50, L2=.20, L3=.30. The results of Study 1 showed that varying ICCs affected the summary statistics, but the change from ICC_1 (L1=.5, L2=.40, L3=0.10) to ICC_2 (L1=.50, L2=.20, L3=.30) was drastic especially for L2 ICC. The L3 ICC was tripled from ICC_1 to ICC_2.

The first new ICC examined the equality of L2 and L3 ICCs, and the new ICCs was set to be L1=.5, L2=.25, L3=0.25, which was referred as ICC_3 throughout the study. Next, instead of dropping ICC at L2 from 0.40 to 0.20, ICC at L2 was dropped 0.10 and the second new set of ICCs (b) L1=.50, L2=.30, L3=.20, which was referred as ICC_4 throughout the study. Table 27 showed the ICC values used in Study 2. Highlighted cells show the ICC values examined in Study 1.

Table 27.

ICC Values Used in Both the Study 1 and Study 2

Study	ICC	ICC_{L1}	ICC_{L2}	ICC_{L3}
1	1	0.50	0.40	0.10
1	2	0.50	0.20	0.30
2	3	0.50	0.25	0.25
2	4	0.50	0.30	0.20

Note. Highlighted cells indicate the conditions examined in Study 1.

Study 2 addressed the following questions, (a) How does increasing the highest sample sizes at L2 and L3 affects the individual parameter estimates under different model and ICC combinations? (b) How does fixing L3 residual variance-covariance terms to 0 affects the individual parameter estimates under different model and ICC combination? (c) How does varying ICCs with a small increment or decrement at L2 and L3 affects parameter estimates?

To answer these questions, the conditions in Study 2 included two new L2 sample sizes which were 25 and 75. The L2 sample sizes examined in Study 1 was 10 and 50. The 75 was selected because for some of the parameters did not provide acceptable absolute bias, relative bias and RMSE results with the highest L2 sample size of 50. The results generally showed that increasing L2 sample size generally resulted in lower summary statistics examined throughout the study. I decided to increase the maximum L2 size to be 75. L2 sample size of 25 included because the typical classroom size is 25 and HLM is a widely used method in education settings.

One new L3 sample size used in Study 2 was 150. In Study 1, increasing L3 sample size generally resulted in lower summary statistics, but not all of the parameter reached to the acceptable level of relative bias, or lower absolute bias and RMSE. Because of that, the maximum L3 sample size was set to 150 in Study 2.

Lastly, the three new models proposed in Study 1 examined in Study 2. Study 1 showed that model complexity played a role in the accuracy of parameter estimates, especially for L3 residual covariance. In the first new model, L3 predictor dropped from the L1L2L3 model. This model was referred as L1L2 model or model 2 throughout the study. Next, the L3 residual variance dropped from the L1L2 model to test the effect of it on the accuracy of other parameter estimates. This model was called L1L2 No L3 Covariance Model or Model 3 throughout the study. Lastly, the L3 residual variance dropped from the L1L2L3 model to test the effect of it on the accuracy of other parameter estimates. This new model was called L1L2L3 No L3 Covariance model or model 5 throughout the study. The two models used in Study 1 was also used in Study 2. L1 model which was also referred as model 1 and L1L2L3 model which was also referred as model 4.

The models and sample sizes used in Study 2 listed in Table 28 and conditions that were examined in Study 1 were highlighted.

Table 28.

Model, L1, L2, and L3 Sample Sizes Used in Study 2

Study	Model	N_{L1}	N_{L2}	N_{L3}
1	L1	3	10	30
1	L1L2L3		50	100
2	L1L2		25	150
2	L1L2 No L3 Residual Covariance		75	
2	L1L2L3 No L3 Residual Covariance			

Note. Highlighted cells indicate the conditions examined in Study 1. L1 model had one predictor at L1. L1L2 model had one predictor at both L1 and L2. L1L2L3 model had one predictor at each level. L1L2 No L3 Covariance model did not estimate L3 residual covariance. Similarly, L1L2L3 No L3 Covariance model did not estimate L3 residual covariance.

In sum, the sample size conditions that are examined in this study (both Study 1 and Study 2) were as follows; N_{L1}=3, N_{L2}=10, N_{L2}=25, N_{L2}=50, N_{L2}=75, N_{L3}=30, N_{L3}=100, N_{L3}=150. Five models were examined (a) L1 Model, (b) L1L2L3 Model, (c) L1L2 Model, (d) L1L2 No L3 Residual Covariance Model, (e) L1L2L3 No L3 Residual Covariance Model. Table 27 shows the four set of ICC values included in Study 2.

Study 2: Results and Discussion

Following the order of research questions presented in Chapter 1, this section presents how sample sizes, model complexity and ICCs affect the estimates of fixed effects intercept, fixed effects slope, residual variance, residual covariance terms in a three-level HLM.

Similar to the Study 1, model convergence was not a substantial problem with any of the conditions examined in this study. Convergence was only an issue for a maximum of three replications under the lowest sample size conditions (N_{L1}=3, N_{L2}=10, N_{L3}=30) for each model and ICC. In the replications where the model did not converge to a solution, new data were generated and analyzed to keep the number of replications at 500.

Table 29 showed which parameters are estimated in each model, where a '+' indicates that parameter was estimated in that model. The order of presentation for each of the parameter estimates as follows: (a) absolute bias, (c) relative bias, (c) RMSE results, and (d) parameter coverage.

Estimates of absolute bias, relative bias, RMSE, and parameter coverage (only for fixed effects intercept and slope terms) were calculated across replications for each of the parameter under each model, ICC, L2, and L3 sample sizes. The total number of estimated parameters was 22 and instead of presenting the absolute bias values for each of the parameter separately, the parameters were grouped into 4 categories based on the order of research questions: (a) the fixed effect intercept terms ($\gamma_{000}, \gamma_{010}, \gamma_{100}$, and γ_{110}), (b) fixed effects slope terms ($\gamma_{001}, \gamma_{011}, \gamma_{101}$, and γ_{111}), (c) variance terms (σ_e^2,

$\sigma^2_{r_0}$, $\sigma^2_{r_1}$, $\sigma^2_{u_{00}}$, $\sigma^2_{u_{01}}$, $\sigma^2_{u_{10}}$, and $\sigma^2_{u_{11}}$), (d) covariance terms ($\sigma_{r_0}\sigma_{r_1}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$).

Table 29.

Estimated Parameters Under Each Model

Parameter Group	Level	Parameter	Model				
			1	2	3	4	5
Fixed Effects Intercept Terms	L3	γ_{000}	+	+	+	+	+
		γ_{010}		+	+	+	+
		γ_{100}	+	+	+	+	+
		γ_{110}		+	+	+	+
Fixed Effects Slope Terms	L3	γ_{001}				+	+
		γ_{011}				+	+
		γ_{101}				+	+
		γ_{111}				+	+
Residual Variance Terms	L1	σ^2_e	+	+	+	+	+
	L2	$\sigma^2_{r_0}$	+	+	+	+	+
		$\sigma^2_{r_1}$	+	+	+	+	+
	L3	$\sigma^2_{u_{00}}$	+	+	+	+	+
		$\sigma^2_{u_{10}}$	+	+	+	+	+
		$\sigma^2_{u_{01}}$		+	+	+	+
		$\sigma^2_{u_{11}}$		+	+	+	+
Residual Covariance Terms	L2	$\sigma_{r_0}\sigma_{r_1}$	+	+	+	+	+
	L3	$\sigma_{u_{00}}\sigma_{u_{01}}$		+		+	
		$\sigma_{u_{00}}\sigma_{u_{10}}$	+	+		+	
		$\sigma_{u_{00}}\sigma_{u_{11}}$		+		+	
		$\sigma_{u_{01}}\sigma_{u_{10}}$		+		+	
		$\sigma_{u_{01}}\sigma_{u_{11}}$		+		+	
		$\sigma_{u_{10}}\sigma_{u_{11}}$		+		+	

Note. Model 1 is L1 model. Model 2 is L1L2 model. Model 3 is L1L2 No L3 Covariance model. Model 4 is L1L2L3 model. Model 5 is L1L2L3 No L3 Covariance model.

There are four fixed effect intercept terms ($\gamma_{000}, \gamma_{010}, \gamma_{100}$, and γ_{110}) as seen in Table 29. γ_{000} and γ_{100} are estimated in all examined models. γ_{010} and γ_{110} were also estimated in all models except the L2 model. γ_{000} is the intercept term for the equation predicting β_{00j}. γ_{010} is the intercept term for the equation predicting β_{01j}. γ_{100} is the intercept term for the equation predicting β_{10j}. γ_{110} is the intercept term for the equation predicting β_{11j}. γ_{110} is also called a cross-level interaction term between L1 and L2 predictor.

There are four fixed effect slope terms ($\gamma_{001}, \gamma_{011}, \gamma_{101}$, and γ_{111}) as seen in Table 29. Fixed effect slope terms are estimated only in L1L2L3 and L1L2L3 no L3 residual Covariance model. γ_{001} is the slope term for the equation predicting β_{00j}. γ_{001} is the only non-interaction slope term. γ_{011} is the slope term for the equation predicting β_{01j}. It is also called a cross-level interaction term between L2 and L3 predictor. γ_{101} is the slope term for the equation predicting β_{10j}. It is also called a cross-level interaction term between L1 and L3 predictor. γ_{111} is the slope term for the equation predicting β_{11j}. It is also called a three way cross-level interaction term between L1, L2, and L3 predictor.

There are seven residual variance terms (σ_e^2, $\sigma_{r_0}^2$, $\sigma_{r_1}^2$, $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$). σ_e^2 is the L1 residual variance term. $\sigma_{r_0}^2$ and $\sigma_{r_1}^2$ are the L2 residual variance terms. $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$ are the L3 residual variance terms.

There are also seven residual covariance terms ($\sigma_{r_0}\sigma_{r_1}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$). $\sigma_{r_0}\sigma_{r_1}$ is the L2 residual covariance term and

$\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$ are the L3 residual covariance terms.

The tabular representation of the absolute bias, relative bias, RMSE, and parameter coverage (only for fixed effects intercept and slope terms) values are created for each of the parameter estimates under each group. However, because it is often difficult to interpret tabular information in simulation studies. Graphical summaries are also created for each parameter.However, due to space limitations, similarity of results in some parameter estimates, and the large number of parameter estimates. The tabular representation of the absolute values, relative bias, RMSE, and parameter coverage for each of the parameter estimates were only included in Appendix A, Appendix B, Appendix C, and Appendix G. Similarly, not all of the visual representation of the absolute bias, relative bias, RMSE, and parameter coverage are presented in this section. However, all of the visual representation of results of Study 2, were included in Appendix D, Appendix E, Appendix F and Appendix H.

Absolute bias. The absolute bias values for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effect intercept terms. The overall descriptive statistics for the absolute bias of fixed effects intercept terms are presented in Table 30. As shown in Table 30, the true value for the γ_{110} is very small. To show the actual value γ_{110} the numerical values are presented with three decimal points.

Table 30

Absolute Bias Descriptive Statistics for Fixed Effect Intercept Terms

Parameter	True Value	Minimum	Maximum	Mean	*SD*
γ_{000}	347.365	0.037	0.281	0.102	0.058
γ_{010}	0.733	0.004	0.085	0.032	0.023
γ_{100}	5.766	0.023	0.161	0.061	0.034
γ_{110}	-0.014	0.003	0.019	0.008	0.004

The absolute bias values for the fixed effect intercept terms are generally very low and across conditions ranged from 0.003 to 0.281. The highest absolute bias values are found with γ_{000} and γ_{100}. Figure 6 shows the absolute bias values of γ_{000}. Tables of the absolute bias statistics for γ_{000} and all other parameters can be found in Appendix A. As seen in the Figure 6, the highest absolute bias values for γ_{000} are associated with the condition where N_{L2}=10 and N_{L3}=30, bottom left panel. On the other hand, the lowest absolute bias values for γ_{000} are associated with the condition where N_{L2}=75 and N_{L3}=150, top right panel. The effect of ICC and model complexity on absolute bias is very low. However, the absolute bias values that are associated with ICC_1 (solid red line, ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) is generally the highest followed by ICC_4 (dash-dotted black line, ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (dashed blue line, ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (dotted magenta line, ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). In other words, as more of shared variability shifts from L2 to L3, the absolute bias values increase slightly.

Each row of Figure 6 represents an L2 sample size, and each row has three panels that represent L3 sample sizes. Looking within each row, the highest absolute bias values are associated with the panel where N_{L3}=30, the first column, and the lowest absolute bias values are associated with the panel where N_{L3}=150, the last column. In other words, keeping all else constant, when the L3 sample size increases from 30 to 150, the absolute bias values decreases.

Similarly, each column of Figure 6 represents an L3 sample size, and each column has four panels that represent L2 sample sizes. Looking within each column, the highest absolute bias values are associated with the panel where N_{L2}=10, the bottom row. On the other hand, the lowest absolute bias values are associated with the panel where N_{L2}=75, the top row. In other words, keeping all else constant, when the L2 sample size increases from 10 to 75, the absolute values decreases.

Similar to the γ_{000}, absolute bias values for γ_{100} are higher compared to the remaining fixed effect intercept terms. γ_{100} is also examined to check whether similar pattern observed in γ_{000} also observed in γ_{100}, Although Figure 6 is very similar to Figure 7, and the pattern seen in γ_{000} holds for γ_{100}, the absolute bias values for γ_{100} is nearly half of the absolute bias values for γ_{000}. The same general pattern holds for the remaining fixed effect terms. Thus, the results are not presented here, but the tables and figures for those can be found in Appendix A and Appendix D, respectively.

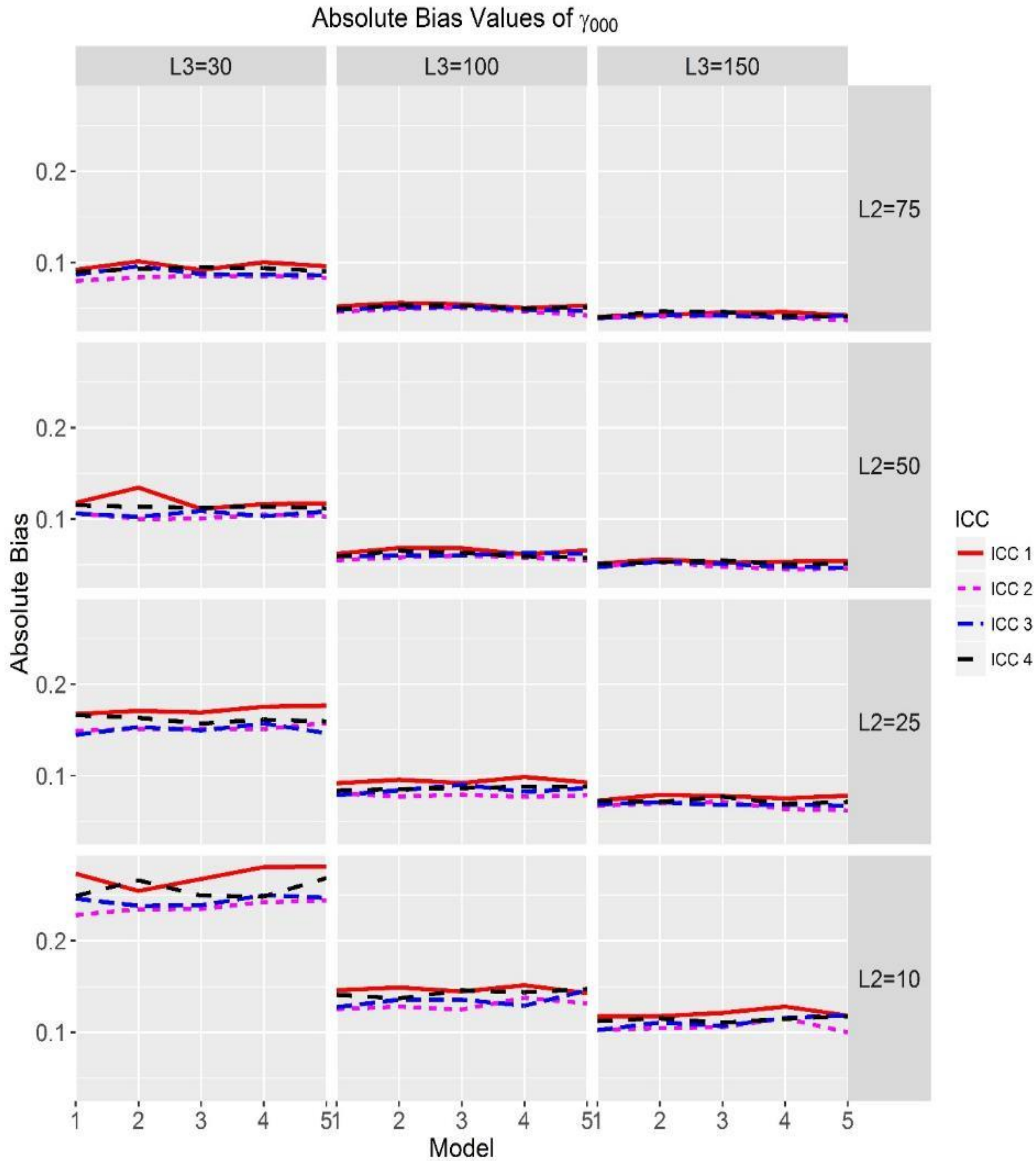

Figure 6. Plot of Absolute Bias Across Manipulated Factors for γ_{000}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

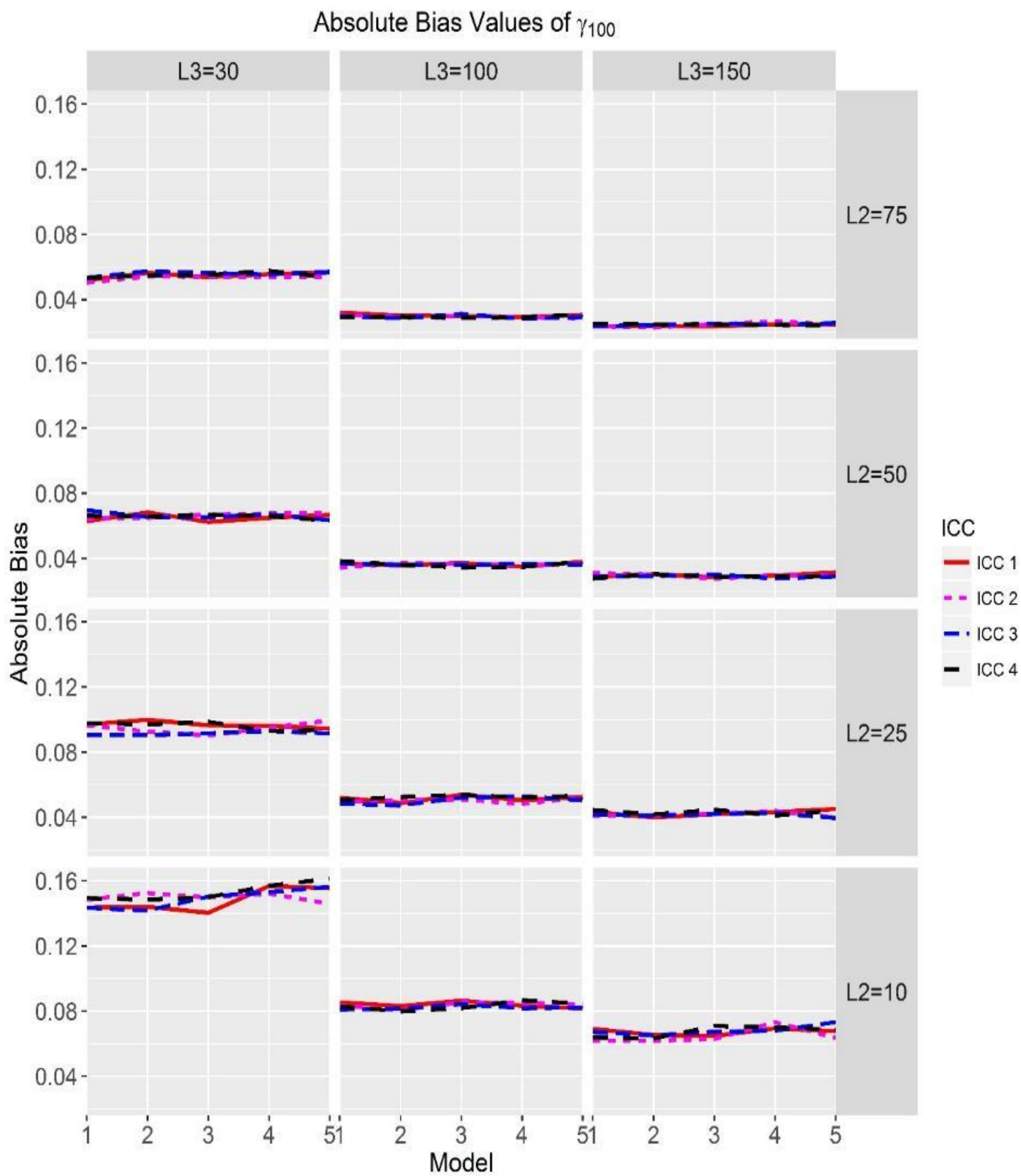

Figure 7. Plot of Absolute Bias Across Manipulated Factors for γ_{100}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

Fixed effect slope terms. The overall descriptive statistics for the absolute bias of fixed effects slope terms are presented in Table 31. As shown in Table 31, a number of true values are very small. To show the actual numerical value, each true value is presented with four decimal points.

Table 31.

Absolute Bias Descriptive Statistics for Fixed Effect Slope Terms

Parameter	True Value	Minimum	Maximum	Mean	SD
γ_{001}	0.1738	0.0544	0.3946	0.1662	0.0876
γ_{011}	0.0004	0.0045	0.0173	0.0085	0.0035
γ_{101}	-0.0190	0.0337	0.3020	0.1156	0.0618
γ_{111}	0.0004	0.0025	0.0113	0.0052	0.0022

The absolute bias values for the fixed effect slope terms are generally very low and across conditions ranged from 0.0025 to 0.3946. The highest absolute bias values are found with γ_{001} and γ_{101}, 0.3946 and 0.3020. Figure 8 shows the absolute bias values of γ_{001} . As seen in the Figure 8, the highest absolute bias values for γ_{001} are associated with the condition where N_{L2}=75 and N_{L3}=30, top left panel. On the other hand, the lowest absolute bias values for γ_{001} are associated with the condition where N_{L2}=10 and N_{L3}=150, bottom right panel. The effect of model complexity is very low but keep in mind that fixed effect slope terms are estimated only in L1L2L3 and L1L2L3 with no L3 covariance models. The separation between the lines is an indication of the effects of ICCs on the absolute bias values of γ_{001}. Unlike the general pattern observed in the fixed effect intercept terms, the absolute values that are associated with ICC_1 (solid red line, ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) is generally the highest followed by ICC_4 (dash-dotted black line, ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (dashed blue line,

ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (dotted magenta line, ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

Unlike the fixed effect intercept terms, increasing L2 sample size resulted in higher absolute values. Looking within each column of Figure 8, the highest absolute bias values are associated with the panel where N_{L2}=75 (the top row), and the lowest absolute bias values are associated with the panel where N_{L2}=10, the bottom row. In other words, keeping all else constant, when the L2 sample size increases from 10 to 75, the absolute values also increase.

Similar to the fixed effect intercept terms, increasing L3 sample size resulted in lower absolute values. Looking within each column of Figure 8, the highest absolute bias values are associated with the panel where N_{L3}=30 (the first column), and the lowest absolute bias values are associated with the panel where N_{L3}=150, the last column. In other words, keeping all else constant, when the L3 sample size increases from 30 to 150, the absolute values decrease. Similar to the γ_{001}, absolute bias values for γ_{101} are higher compared to the remaining fixed effect intercept terms. γ_{101} is also examined to check whether similar patterns observed as seen in γ_{001}. Figure 9 shows the absolute bias values of γ_{101}. Although, Figure 9 is very similar to the Figure 8 and the pattern seen in γ_{001} holds for γ_{101}, the absolute bias values for γ_{100} is generally lower compared to the absolute bias values for γ_{001}. The same general pattern holds for the remaining fixed effect slope terms. Thus, the results are not presented here but the tables and figures for those can be found in Appendix A and Appendix D, respectively.

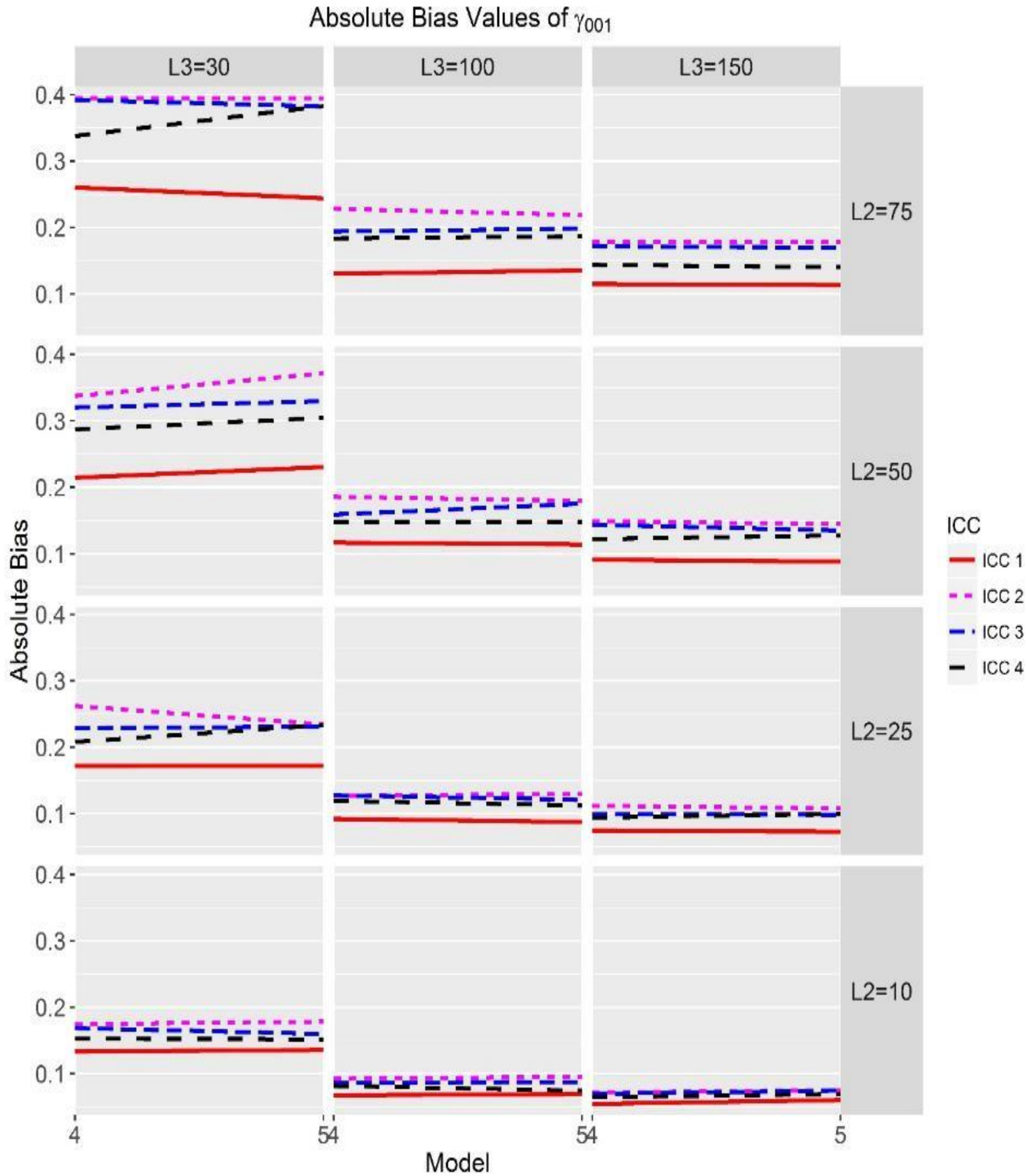

Figure 8. Plot of Absolute Bias Across Manipulated Factors for γ_{001}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

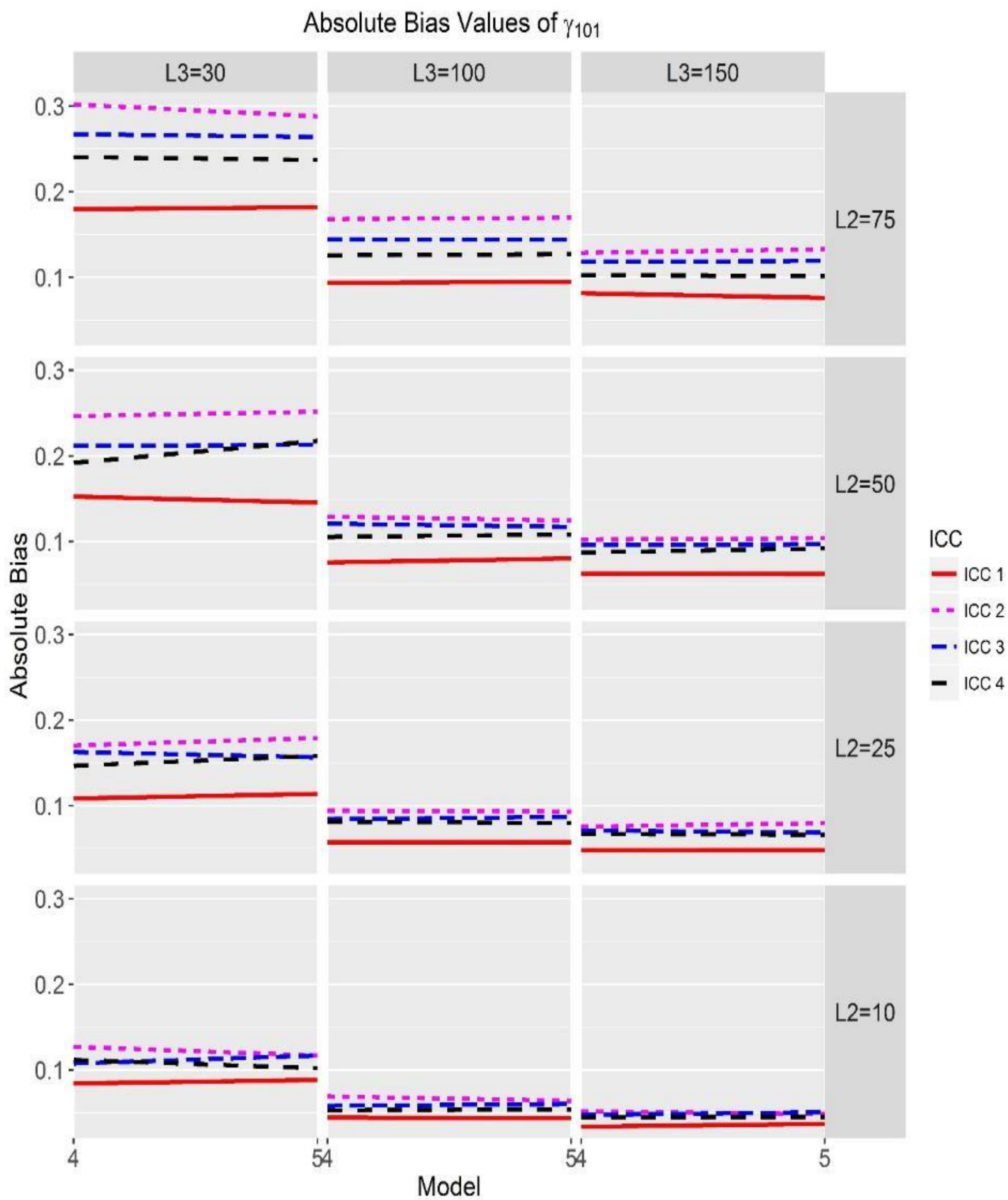

Figure 9. Plot of Absolute Bias Across Manipulated Factors for γ_{101}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

Residual variance terms. The overall descriptive statistics for the absolute bias of residual variance terms are presented in Table 32. The absolute bias values for the residual variance terms are generally very low except for σ_e^2 (L1 residual variance), $\sigma_{r_0}^2$ (L2 residual variance associated with L1 equation intercept, π_{0ij}), $\sigma_{u_{00}}^2$ (L3 residual variance associated with L2 intercept for equation, π_{0ij}), and $\sigma_{u_{10}}^2$ (L3 residual variance associated with L2 intercept for equation, π_{1ij}). Table 32 shows that absolute bias values across conditions ranged from 0.0004 to 19.2777. The highest absolute bias values are found with $\sigma_{r_0}^2$ and $\sigma_{u_{10}}^2$.

Table 32.

Absolute Bias Descriptive Statistics for Residual Variance Terms

Parameter	Minimum	Maximum	Mean	SD
σ_e^2	0.1872	1.1859	0.4662	0.2508
$\sigma_{r_0}^2$	0.3060	19.2777	3.5804	4.6188
$\sigma_{r_1}^2$	0.1149	0.5685	0.2586	0.1004
$\sigma_{u_{00}}^2$	0.1436	7.0267	2.5218	1.9027
$\sigma_{u_{01}}^2$	0.0011	0.0082	0.0043	0.0018
$\sigma_{u_{10}}^2$	0.0572	3.5466	1.2463	0.9773
$\sigma_{u_{11}}^2$	0.0004	0.0031	0.0016	0.0007

Note. The true values for each of the residual variance terms are presented in Table 8 and Table 9.

Compared to the fixed effects intercept and slope terms, absolute bias values for the residual variance terms are generally higher. Similar patterns observed for the absolute bias values in the fixed effect slope terms are also observed in σ_e^2, $\sigma_{r_1}^2$, $\sigma_{u_{01}}^2$, and $\sigma_{u_{01}}^2$. The absolute bias values for these terms can be found as both tables and figures in Appendix A and Appendix D, respectively.

The pattern observed in $\sigma^2_{r_0}$, $\sigma^2_{u_{00}}$, and $\sigma^2_{u_{10}}$ varied based on the model examined. For example, the absolute bias values for these residual variances showed a similar pattern as the fixed effect intercept terms under L1 model. Figure 10, Figure 11, Figure 12 shows the absolute bias values for in $\sigma^2_{r_0}$, $\sigma^2_{u_{00}}$, and $\sigma^2_{u_{10}}$, respectively.

The straight line in the figures is an indication that there is not a difference in the absolute bias values from one model to the next. However, lines with an angle indicates that there is a difference in absolute bias value from one model to the next. Figure 10 demonstrates this very well. For example, the top right panel where N_{L2}=75 and N_{L2}=150, there is an angle from model 1 to model 2 which indicates model 2 absolute values are relatively higher compared model 1. The difference between model 1 and model 2 is the addition of a L2 predictor in model 2. On the other hand, the line is relatively straight from model 2 to model 3 which indicates that the absolute bias values are relatively similar in both models. The difference between model 2 and model 3 is that in model 2 all of the L3 residual covariance are estimated but not in model 3. From model 3 to model 4, the line had a sudden drop which means the relative bias values decrease. An important difference between model 3 and model 4 is that model 3 includes an additional predictor at L3. Finally, similar to the lines from model 2 to model 3, the line is relatively straight from model 4 to model 5. The difference between model 4 and model 5 is that in model 4 all of the L3 residual covariance are estimated but not in model 5.

To summarize, for $\sigma^2_{r_0}$, introducing new predictors affect the absolute bias values but removing the L3 covariance terms do not affect the absolute bias values. Although, it is not as obvious for the N_{L3}=30 conditions because of the y axis scale used in the plots, the same general pattern is observed in other panels. Figure 11 shows the absolute bias values for $\sigma^2_{u_{00}}$.

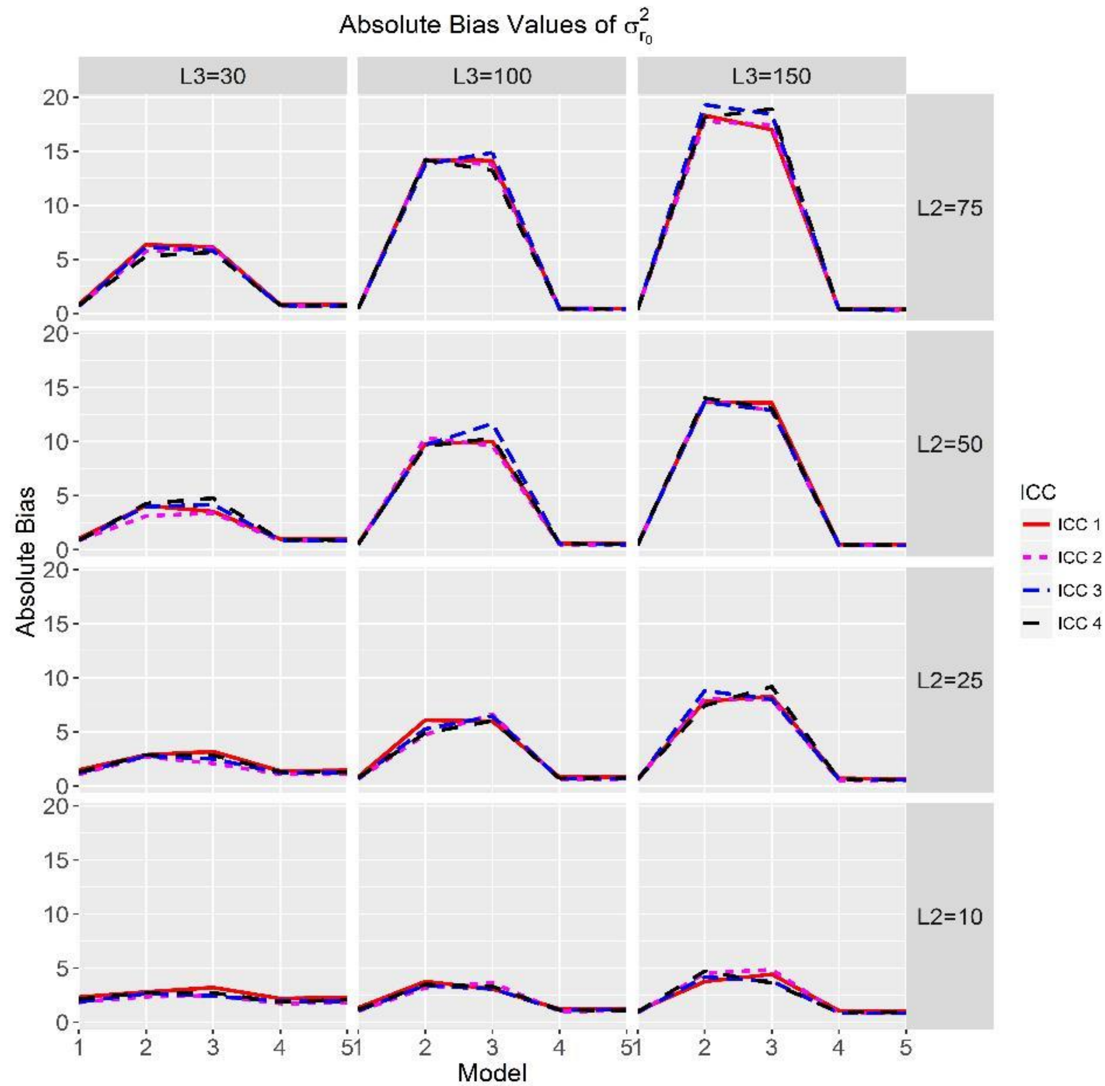

Figure 10. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{r_0}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

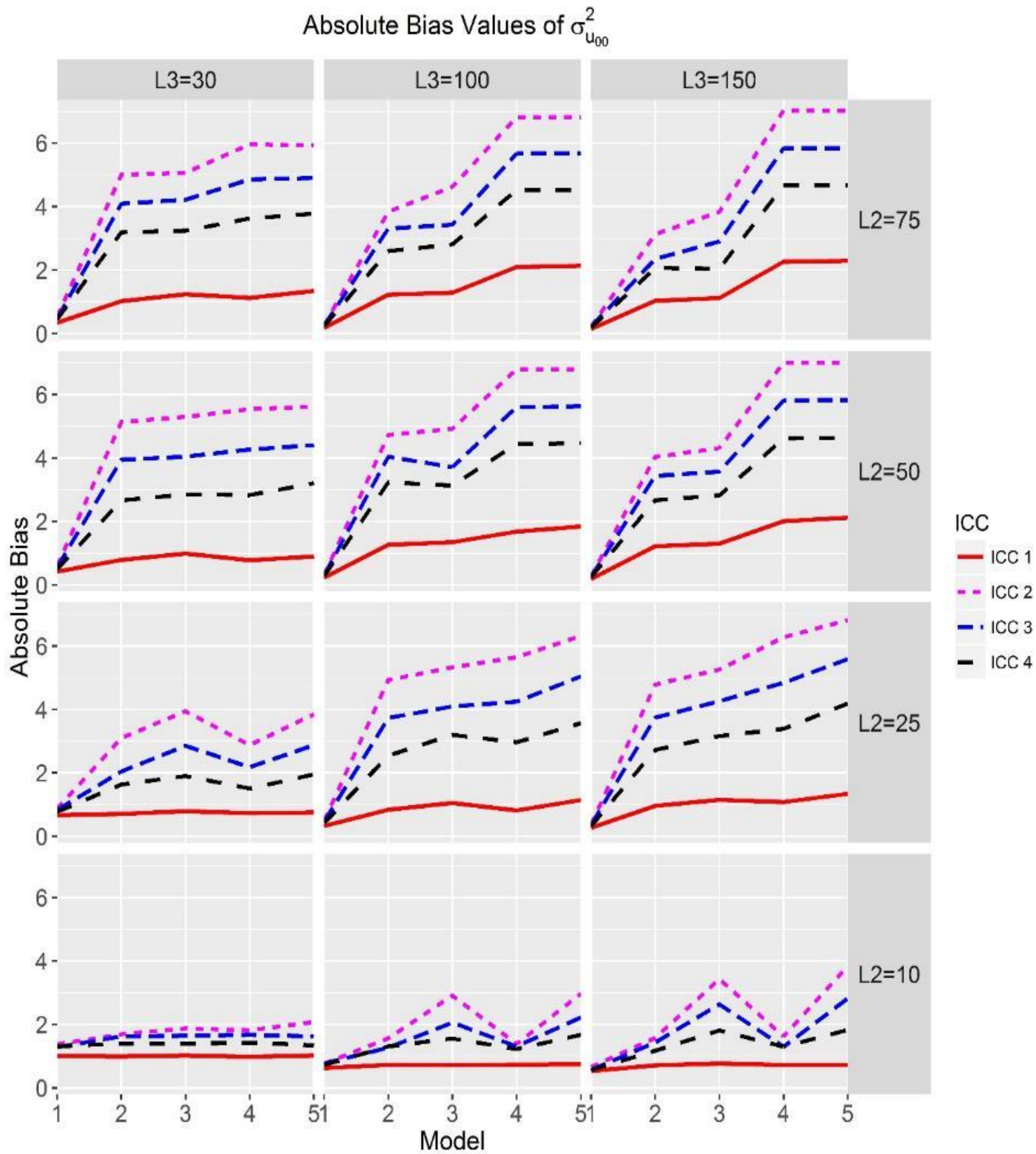

Figure 11. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{u_{00}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

Similar to the absolute bias values of $\sigma^2_{r_0}$, Figure 11 shows that model complexity affects the absolute bias values. However, unlike the absolute bias values in $\sigma^2_{r_0}$, absolute bias values increase from model 3 (L1L2 model no L3 covariance) to model 4 (L1L2L3 model). Figure 11 also shows that ICC also had an effect on the absolute bias values. The absolute values that are associated with ICC_1 (solid red line, ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) is generally the lowest followed by ICC_4 (dash-dotted black line, ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (dashed blue line, ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (dotted magenta line, ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). In other words, keeping ICC_{L1} constant as the ICC_{L2} decrease or ICC_{L3} increase absolute bias for $\sigma^2_{u_{00}}$ increases.

The bottom three panels in Figure 11 shows the lowest absolute bias values for $\sigma^2_{u_{00}}$. These panels are associated with N_{L2}=10 conditions. Figure 12 shows the absolute bias values for $\sigma^2_{u_{10}}$. Although the absolute values for $\sigma^2_{u_{10}}$ is less than the $\sigma^2_{u_{00}}$, similar patterns observed in both L3 residual variance terms.

Residual covariance terms. The overall descriptive statistics for the absolute bias of residual covariance terms are presented in Table 33. The average absolute bias values for $\sigma_{u_{00}}\sigma_{u_{01}}$(L3 residual covariance between β_{00j} and β_{01j} which are the intercept and the slope of the L2 equation π_{0ij}, respectively), $\sigma_{u_{00}}\sigma_{u_{11}}$(L3 residual covariance between β_{00j} and β_{11j} which are the intercept and the slope of the L2 equations π_{0ij} and π_{1ij}, respectively), $\sigma_{u_{01}}\sigma_{u_{10}}$(L3 residual covariance between β_{01j} and β_{10j} which are the

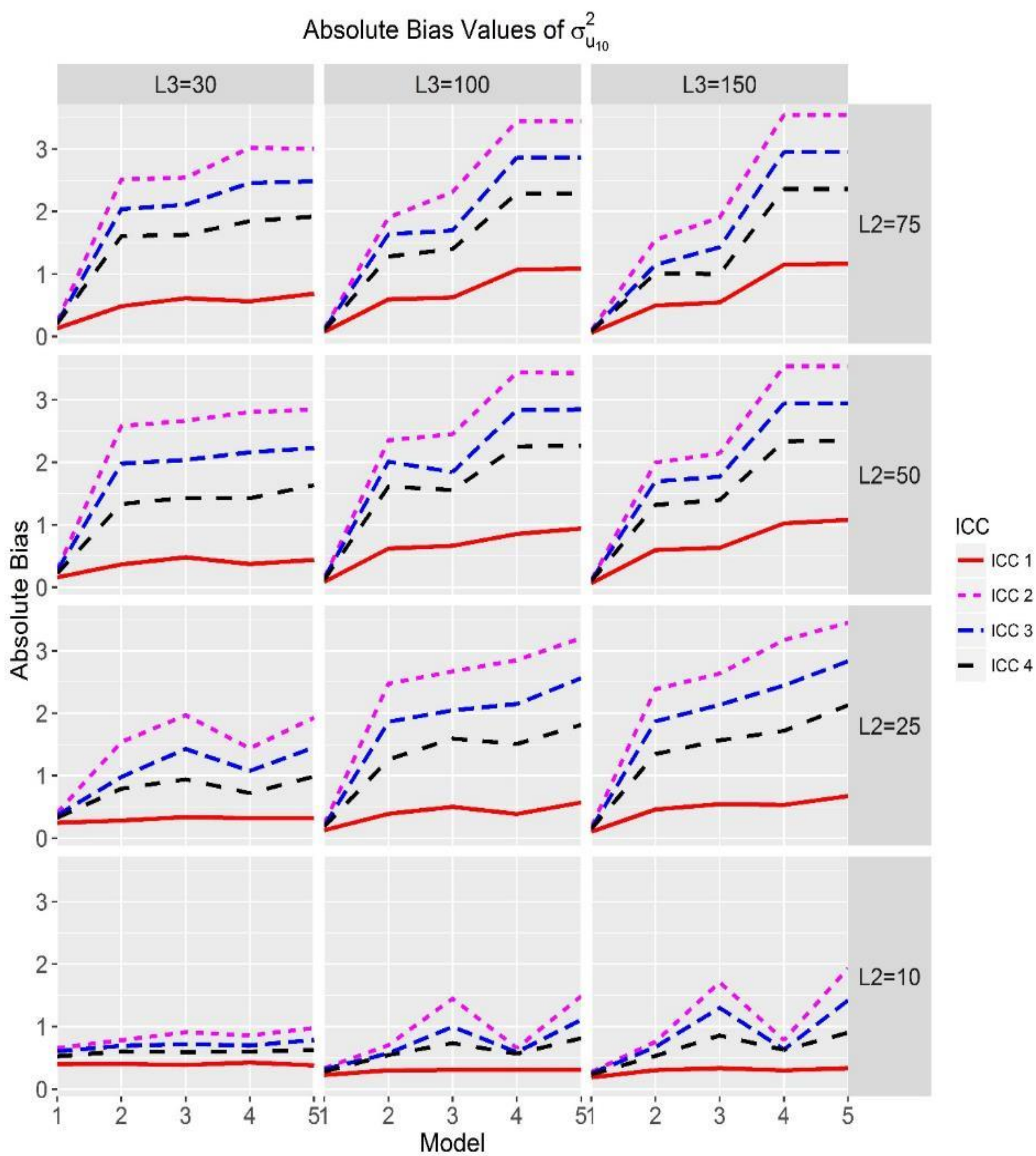

Figure 12. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

intercept and the slope of the L2 equations π_{0ij} and π_{1ij}, respectively), $\sigma_{u_{01}}\sigma_{u_{11}}$(L3 residual covariance between β_{01j} and β_{11j} which are the slopes of the L2 equations π_{0ij} and π_{1ij}, respectively), and $\sigma_{u_{10}}\sigma_{u_{11}}$ (L3 residual covariance between β_{10j} and β_{11j} which are the intercept and slope of the L2 equation π_{1ij}, respectively) are all very low and less than 0.10. These absolute values are relatively low compared to the residual variance terms examined in the previous section.

Table 33.

Absolute Bias Descriptive Statistics for Residual Covariance Terms

Parameter	Minimum	Maximum	Mean	SD
$\sigma_{r_0}\sigma_{r_1}$	0.1528	0.9610	0.4275	0.1818
$\sigma_{u_{00}}\sigma_{u_{01}}$	0.0315	0.1791	0.0872	0.0383
$\sigma_{u_{00}}\sigma_{u_{10}}$	0.2806	4.3984	1.3834	1.0547
$\sigma_{u_{00}}\sigma_{u_{11}}$	0.0035	0.0577	0.0200	0.0123
$\sigma_{u_{01}}\sigma_{u_{10}}$	0.0077	0.0644	0.0253	0.0119
$\sigma_{u_{01}}\sigma_{u_{11}}$	0.0012	0.0053	0.0030	0.0011
$\sigma_{u_{10}}\sigma_{u_{11}}$	0.0107	0.0625	0.0314	0.0132

Note. The correlation among the L3 covariance terms are presented in Table 10.

The two highest absolute values are associated with $\sigma_{r_0}\sigma_{r_1}$(L2 covariance term) and $\sigma_{u_{00}u_{10}}$(L3 residual covariance between β_{00j} and β_{10j} which are the intercepts of the L2 equations π_{0ij} and π_{1ij}, respectively). Visual representation of the absolute bias values for these two parameters are shown in Figure 13 and Figure 14.

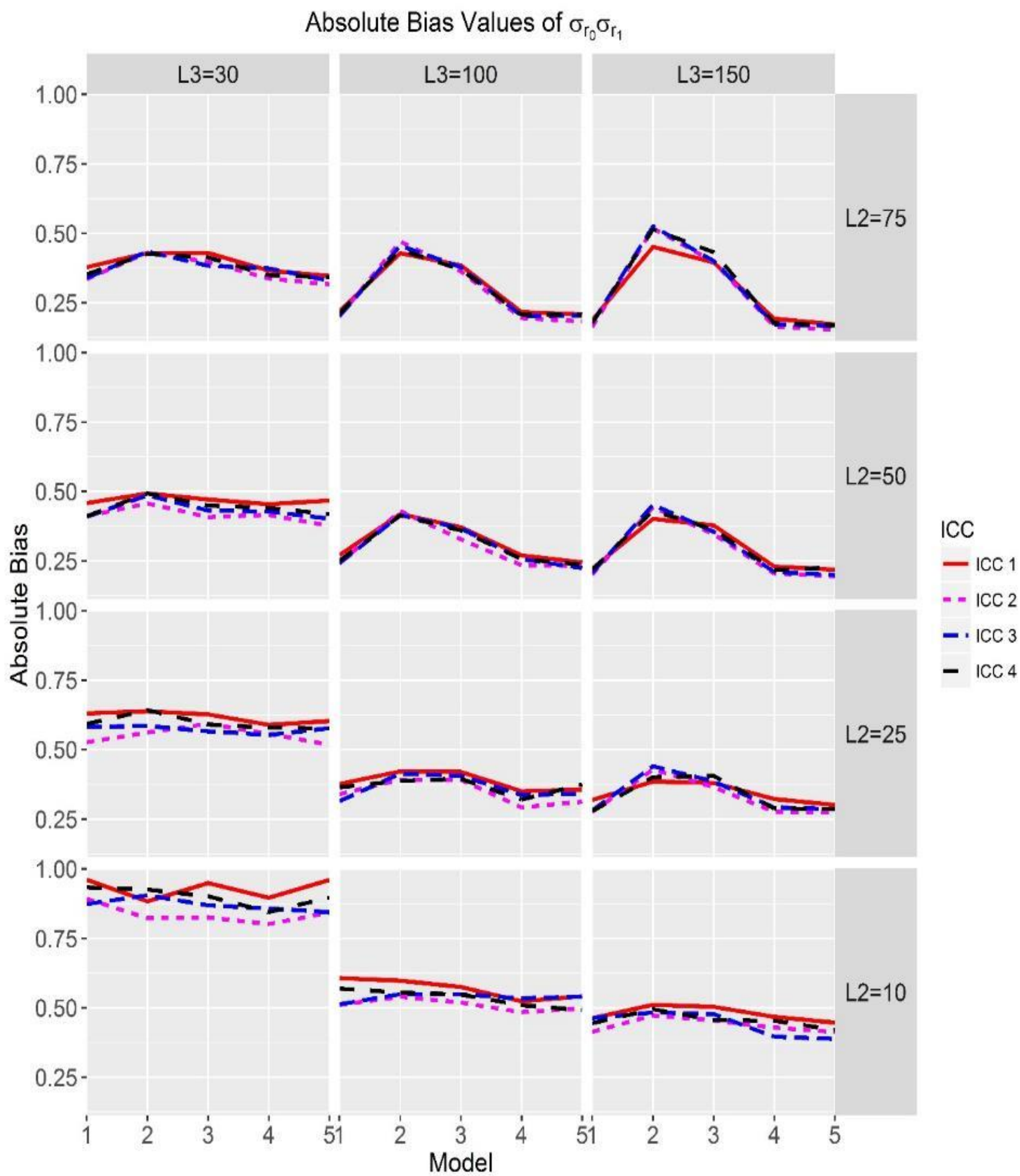

Figure 13. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{r_0}\sigma_{r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

Figure 13 shows that for N_{L2}=10 and N_{L2}=25 (the bottom two rows), the absolute bias values for $\sigma_{r_0}\sigma_{r_1}$ decrease as the L3 sample size increases from 30 to 150. Similarly, for N_{L3}=30 (panels on the first/left column) the absolute bias values decrease as the L2 sample size increases from 10 to 75. For the conditions where N_{L2}=10 and N_{L2}=25 or N_{L3}=30, the effect of ICCs and model complexity on absolute bias is not as apparent as it is for the residual variance $\sigma^2_{u_{10}}$ and $\sigma^2_{u_{00}}$. On the other hand, for the four conditions where L2 sample size is either 50 or 75 and L3 sample sizes either 100 and 150, the relative bias values are almost identical. However, in these four conditions, the inclined lines or lines with an angle clearly demonstrates the effect of model complexity compared to the other conditions under $\sigma_{r_0}\sigma_{r_1}$.

Figure 14 represents the absolute bias values for $\sigma_{u_{00}}\sigma_{u_{10}}$. The bottom row in Figure 14 shows that he absolute bias values are relatively stable and the effect of ICC, L3 sample size, and model complexity is minimal when N_{L2}=10. For the remaining conditions under $\sigma_{u_{00}}\sigma_{u_{10}}$, the absolute values increase as the model complexity increase. The effect of ICC on these conditions are also clearer as the separation between the lines more apparent. The absolute values that are associated with ICC_1 (solid red line, ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) is generally the lowest followed by ICC_4 (dash-dotted black line, ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (dashed blue line, ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (dotted magenta line, ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30).

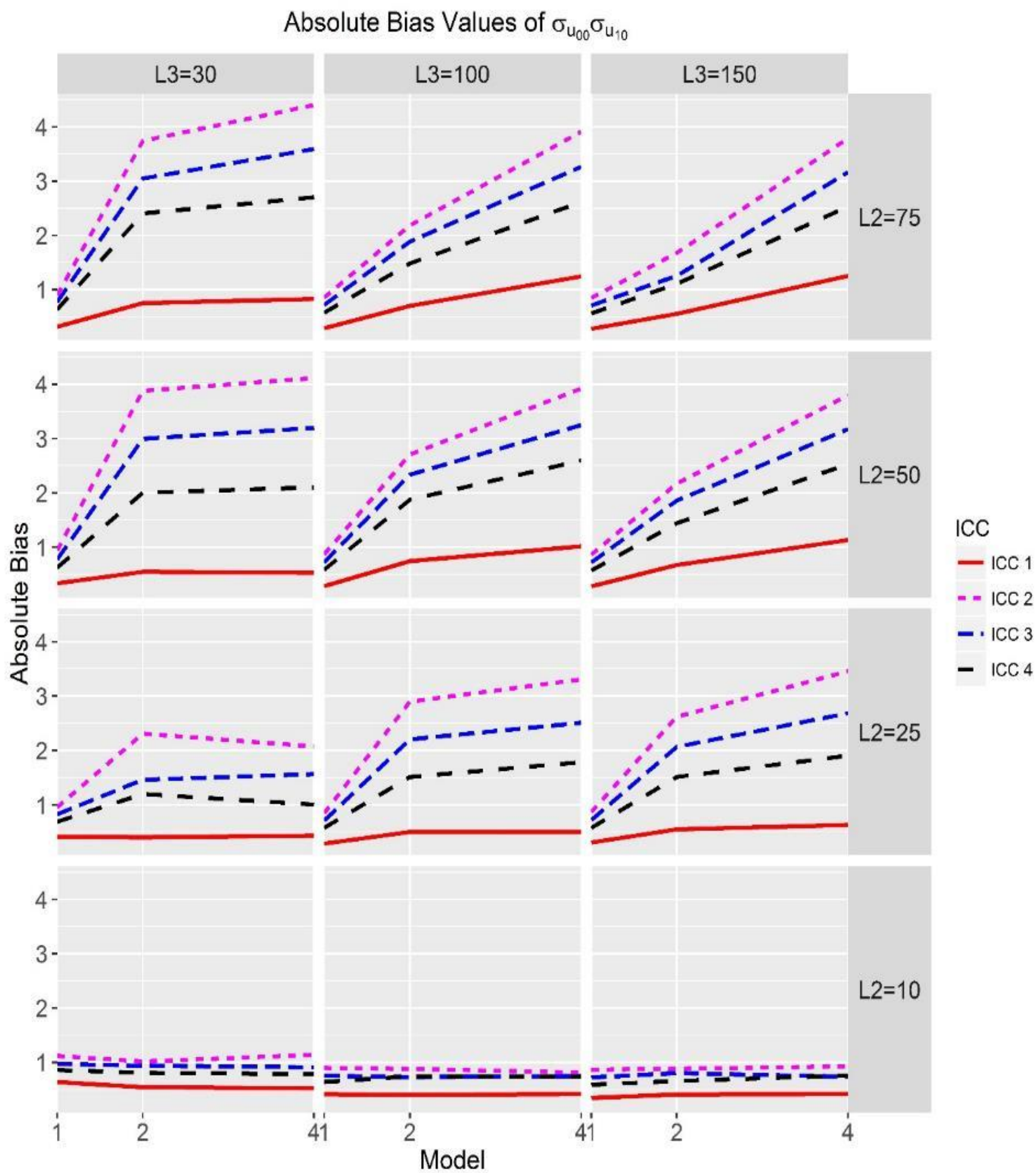

Figure 14. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{00}}\sigma_{u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

The remaining covariance terms showed similar patterns and the results are not examined here. However, both the tabular and visual representation of the absolute bias values for the remaining residual covariance terms included in Appendix A and Appendix D.

Summary of absolute bias. The absolute bias statistic is generally low for the majority (12 out of 22) of the parameter estimates (γ_{010}, γ_{110}, γ_{011}, γ_{111}, $\sigma^2_{r_1}$, $\sigma^2_{u_{01}}$, $\sigma^2_{u_{11}}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$) and relatively high for the remaining γ_{000}, γ_{100}, γ_{001}, γ_{101}, σ^2_e, $\sigma^2_{r_0}$, $\sigma^2_{u_{00}}$, $\sigma^2_{u_{10}}$, $\sigma_{r_0}\sigma_{r_1}$, $\sigma_{u_{00}}\sigma_{u_{10}}$. Other than the modified factors and their effect on absolute bias, one potential reasons for this difference is the magnitude of the data generating or true value. It is observed that the high absolute values are usually associated with higher true value.

Researchers usually control for effect of the true value magnitude by calculating the relative bias. The drawback of relative bias is that it can be magnified when the true value is between -1 and 1. Next, the results are relative bias is discussed.

Relative bias. The relative bias values for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effect intercept terms. The overall descriptive statistics for the relative bias of fixed effects intercept terms are presented in Table 34. The relative bias values for the fixed effect intercept terms are all very small and within the acceptable range for γ_{000}, γ_{010} and γ_{100}. Due to space limitations, they are not examined here. The tables and figures related to these parameters are included in Appendix B and Appendix E, respectively.

Table 34.

Relative Bias Descriptive Statistics for Fixed Effect Intercept Terms

Parameter	True Value	Minimum	Maximum	Mean	*SD*
γ_{000}	347.365	-0.0001	0.0001	0.0000	0.0000
γ_{010}	0.733	-0.0041	0.1155	0.0282	0.0369
γ_{100}	5.766	-0.0022	0.0025	0.0000	0.0006
γ_{110}	-0.014	-0.1062	0.1700	0.0262	0.0456

The relative bias values across conditions ranged from -0.1062 to 0.1700. The highest relative bias values are associated with γ_{110} which is estimated all models except L1 model. It is the intercept term for the equation predicting β_{11j}. It is an interaction term and represents the interaction between Time and L2 predictor.

Figure 15 presented the relative bias values of γ_{110}, tables of the relative bias statistics for γ_{110} and all other parameters included in Appendix B. As seen in the Figure 15, all of the relative bias except one is within the acceptable range. The bottom left panel shows the condition where the relative bias is not within the acceptable range. It is the condition where N_{L2}=10, N_{L3}=30, ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30), under L1L2 model (one predictor at L1 and L2). The bottom left and middle panels where N_{L2}=10 and N_{L3}=30 or N_{L3}=100 had the largest variability in relative bias values.

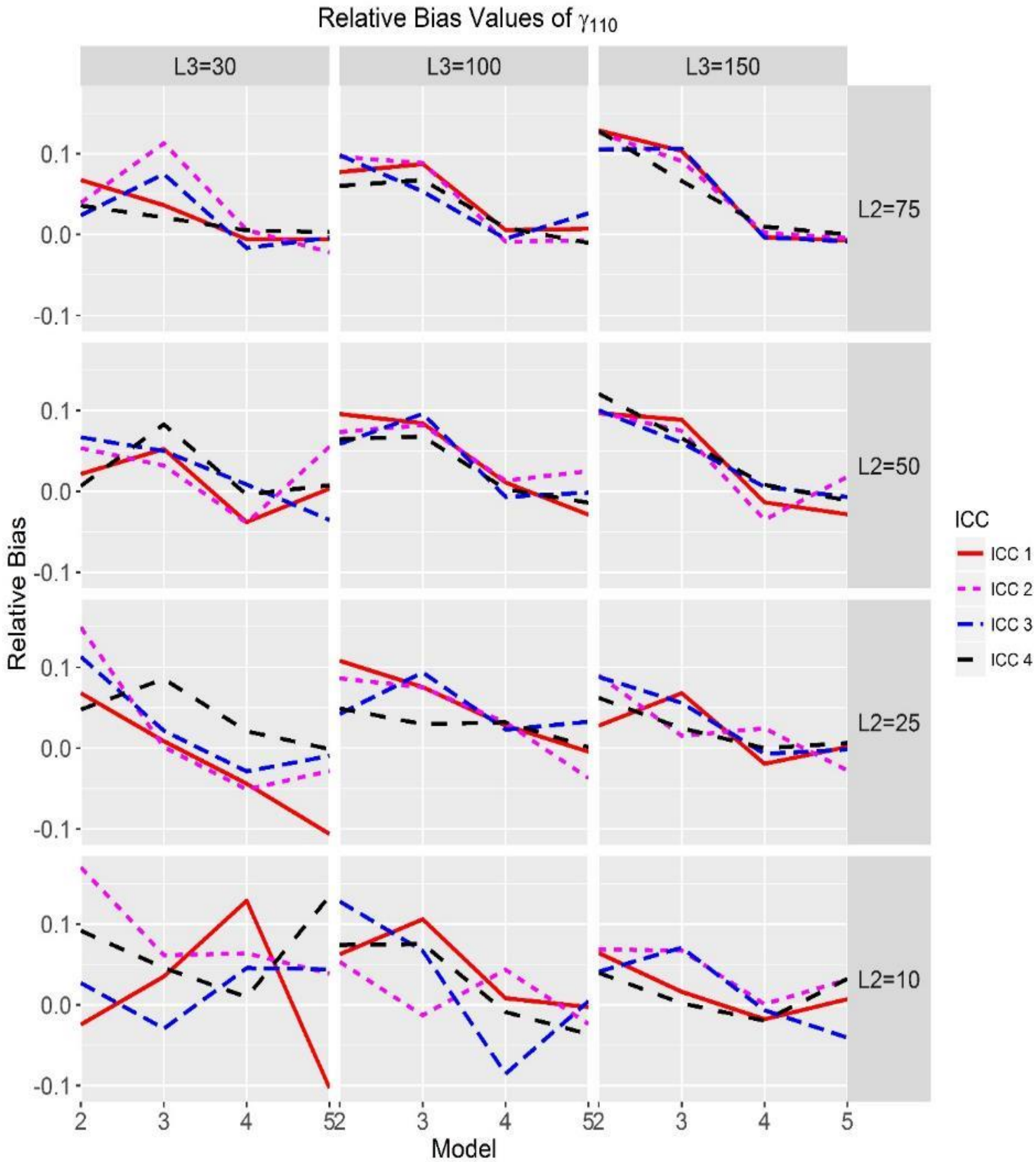

Figure 15. Plot of Relative Bias Across Manipulated Factors for γ_{110}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

The effect of ICC on relative bias is minimal for the conditions in the panels where N_{L2}=50 or N_{L2}=75and N_{L3}=100 or N_{L3}=150 as the separation between the lines are very small. However, in the remaining conditions, the separation between the lines are clearer, but the effect of ICC is unstable. For example, following ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) in the top left panel where N_{L2}=75 and N_{L3}=30 shows that it had the highest relative bias values for model 2 (one predictor at L1 and L2). However, for the same model in the panel where N_{L2}=50 and N_{L3}=30 shows that it had the highest relative bias values.

The non-straight line in Figure 15 shows that model complexity effects the relative bias values. However, the effect is unstable. For example, following ICC1 (solid red line, ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) in the bottom left panel where N_{L2}=10 and N_{L3}=30 shows that relative bias values increased from model 2 (one predictor at L1 and L2) to model 4 (one predictor at each level). However, looking at the panel above the bottom left panel where N_{L2}=25 and N_{L3}=30 shows that the relative bias values for ICC1 (red solid line, ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) constantly decreased from model 2 (one predictor at L1 and L2) to model 5 (one predictor at each level with no L3 covariance).

Fixed effect slope terms. The overall descriptive statistics for the relative bias of fixed effects intercept terms are presented in Table 35. All of the fixed effect slope terms are only estimated in L1L2L3 and L1L2L3 No L3 covariance models. The relative bias values for the fixed effect slope terms are generally higher than fixed effect intercept terms. Relative bias values across conditions ranged from -2.6038 to 3.4736.

Table 35.

Relative Bias Descriptive Statistics for Fixed Effect Slope Terms

Parameter	True Value	Minimum	Maximum	Mean	*SD*
γ_{001}	0.1738	-0.2156	0.2858	-0.0002	0.0639
γ_{011}	0.0004	-2.6038	3.4736	0.0548	0.6758
γ_{101}	-0.0190	-1.1478	0.7978	-0.0004	0.2110
γ_{111}	0.0004	-2.1360	1.3688	-0.0388	0.4085

Almost all of the relative bias values for γ_{001} are with the acceptable range from -.15 to .15 (93 out of 96) regardless of the sample size, ICC, and model complexity. The tabular and visual representation of relative bias values included in Appendix B and Appendix E.

Unlike γ_{001}, the majority the relative bias of γ_{011} are either less than -.15 or greater than 0.15. Also, the largest relative bias difference between the minimum and maximum is for the γ_{001} (3.4736- (-2.6038)=6.0774). Figure 16 shows the the relative bias values for γ_{011}. It is difficult to see any pattern regarding how the relative bias affected by varying ICC, model complexity, L2, and L3 sample sizes. For example, for the condition where N_{L3}=30, ICC_1, and L1L2L3 model, absolute relative bias decreased as N_{L2} increased.

The same pattern is not observed in any other condition under the L1L2L3 and L1L2L3 No L3 Covariance models. The number of conditions where relative bias values are negative and positive are almost equal. There is no obvious distinction between under and over estimation for the γ_{011}. The relative bias of γ_{011} for the conditions where N_{L2}=10 and N_{L3}=100, N_{L2}=75 and N_{L3}=150 are relatively low compared to the other conditions.

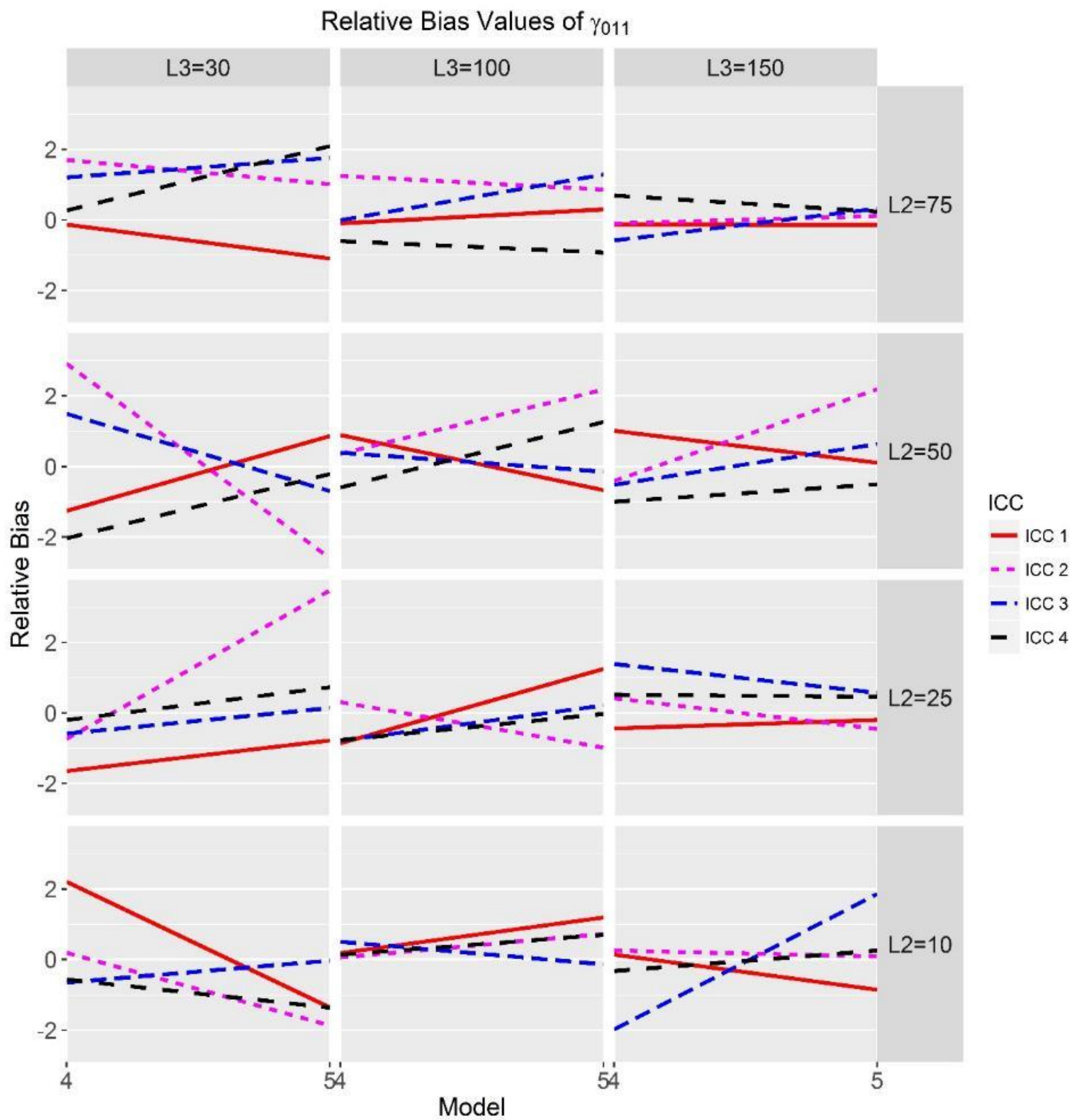

Figure 16. Plot of Relative Bias Across Manipulated Factors for γ_{110}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

The effect of ICC on relative bias values for γ_{011} is not clear. For example, examining the solid red line for ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) demonstrated that for some L2 (N_{L2}=25) and L3(N_{L3}=30) size combinations, the relative bias increased from L1L2L3 model to L1L2L3 No L3 Covariance model. On the other hand, it decreased from L1L2L3 Model to L1L2L3 No L3 Covariance model when (N_{L2}=10) and L3(N_{L3}=30).

The relative bias values for γ_{101} and γ_{111} are very similar to the relative bias values of γ_{011}. The tabular and visual representaion of relative bias values for γ_{101} and γ_{111} included in Appendix B and Appendix E.

Residual variance terms. The overall descriptive statistics for the relative bias of residual variance terms are presented in Table 36.

Table 36.

Relative Bias Descriptive Statistics for Residual Variance Terms

Parameter	Minimum	Maximum	Mean	SD
σ_e^2	-0.0260	0.0009	-0.0070	0.0058
$\sigma_{r_0}^2$	-0.0131	2.2512	0.2790	0.4722
$\sigma_{r_1}^2$	-0.0057	3.3784	0.8184	0.6327
$\sigma_{u_{00}}^2$	-0.8819	0.0019	-0.3818	0.2913
$\sigma_{u_{01}}^2$	-0.8465	1.8340	-0.2830	0.3954
$\sigma_{u_{10}}^2$	-0.8825	-0.0008	-0.3842	0.2925
$\sigma_{u_{11}}^2$	-0.8354	1.5193	-0.2440	0.3291

Note. The true values for each of the residual variance terms are presented in Table 8 and Table 9.

As seen in Table 36, the relative bias of σ_e^2 are all between -0.026 and 0.0009 regardless of varying ICCs, model complexity, L2, and L3 sample sizes. Since all of the relative bias values for σ_e^2 are all within the acceptable range, the tabular and visual presentation of the relative bias values included only in the Appendix B and Appendix E.

The relative bias of σ^2_{r0} are all within acceptable range for the following models; L1 model where there is only one predictor at L1, L1L2L3 model where there is one predictor at each level, and L1L2L3 No L3 Covariance model where there is one predictor at each level but no L3 residual covariance . However, the majority of the relative bias values are not within acceptable range for the models L1L2 model where there is one predictor at both L1 and L2, and L1L2 No L3 Covariance model where there is one predictor at both L1 and L2 but no L3 residual covariance.

As seen in Figure 17, the relative bias values for L1L2 and L1L2 No L3 Covariance models are relatively higher compared to the other models. In both L1L2 and L1L2 with no L3 Covariance models included one predictor at both L1 and L2. It is also clear that for these two models, the relative bias values increased as L2 sample size or L3 sample size increased. There is also a clear separation between the lines for different ICC values. In other words, ICC had an effect on the relative bias values for σ^2_{r0}. The lowest relative bias values are associated with ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.4, ICC_{L3}=0.1), followed by ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). Non-straight lines in Figure 17 indicated that model complexity affected the relative bias values for σ^2_{r0}.

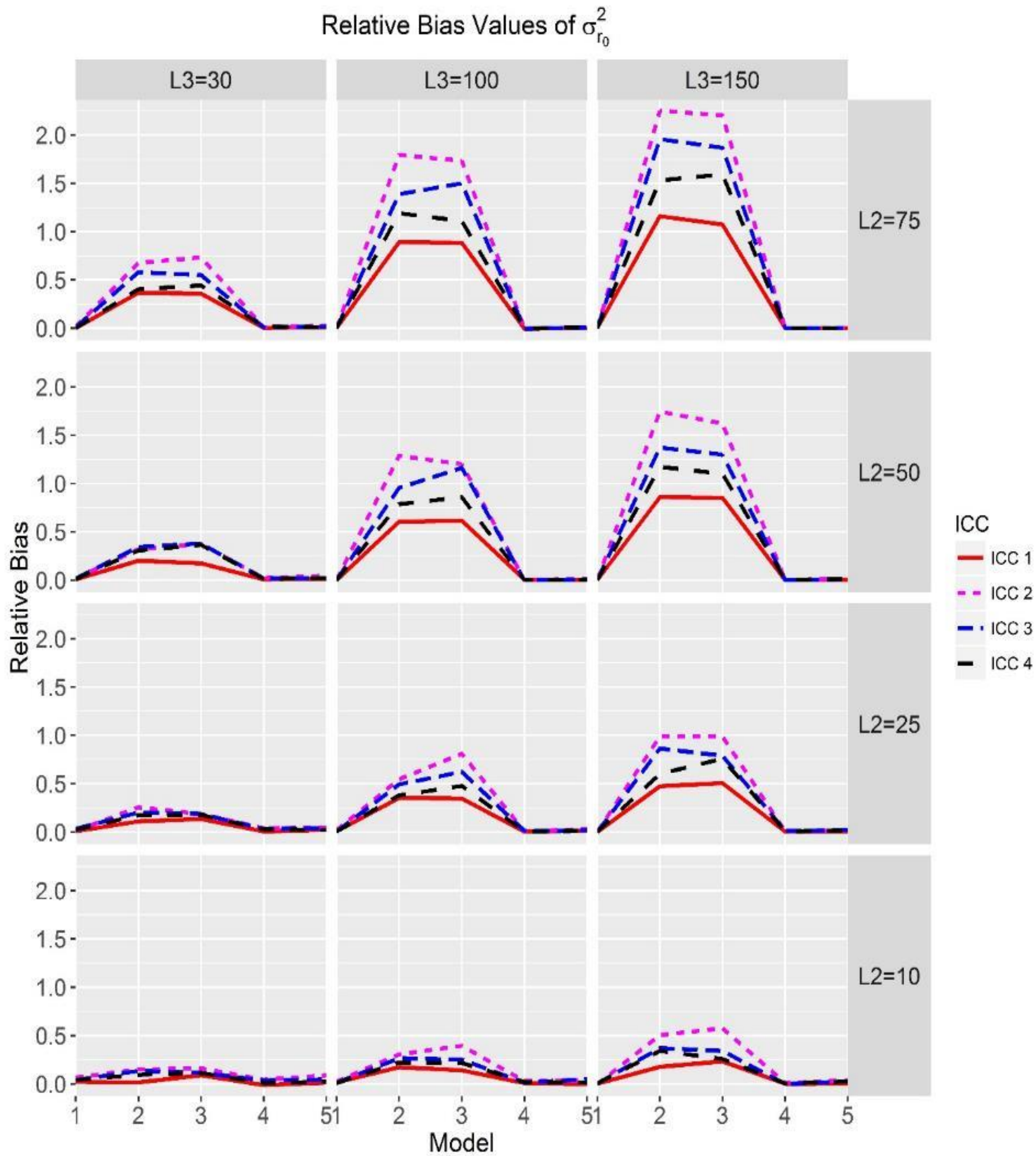

Figure 17. Plot of Relative Bias Across Manipulated Factors for σ^2_{r0}.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

The majority of the relative bias values of σ_{r1}^2 are not within acceptable range regardless of varying ICCs, model complexity, L2, and L3 sample sizes. However, the relative bias values of σ_{r1}^2 decreased as the L2 sample sizes increased. Similarly, the relative bias values of σ_{r1}^2 decreased as the L3 sample sizes increased.

Figure 18 shows that there is a clear separation between the ICC values. In other words, varying ICC had an effect on the relative bias values of σ_{r1}^2. ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) had generally the highest relative bias values followed by ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), and ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.4, ICC_{L3}=0.1). In other words, the higher the ICC at ICC_{L2}, the lower the relative bias of σ_{r1}^2. The non-straight lines show that model complexity also affected the relative bias values of σ_{r1}^2. Overall, one can say that model complexity and varying ICCs affected the relative bias of σ_{r1}^2 similar to the σ_{r0}^2.

All of the relative bias values of $\sigma_{u_{00}}^2$ under the L1 model are within the acceptable range. Unlike the L2 residual variance terms, the relative bias generally increased as the L2 or L3 sample size increased regardless of varying ICCs, model complexity for all the models except the L1 model. Figure 19 shows the relative bias values for $\sigma_{u_{00}}^2$. It could be easily seen in Figure 19 that all of the relative bias values are negative which means they are all underestimated. The non-straight lines in Figure 19 shows that model complexity had an effect on the relative bias of $\sigma_{u_{00}}^2$. As the model complexity increased, the absolute relative bias values generally also increase.

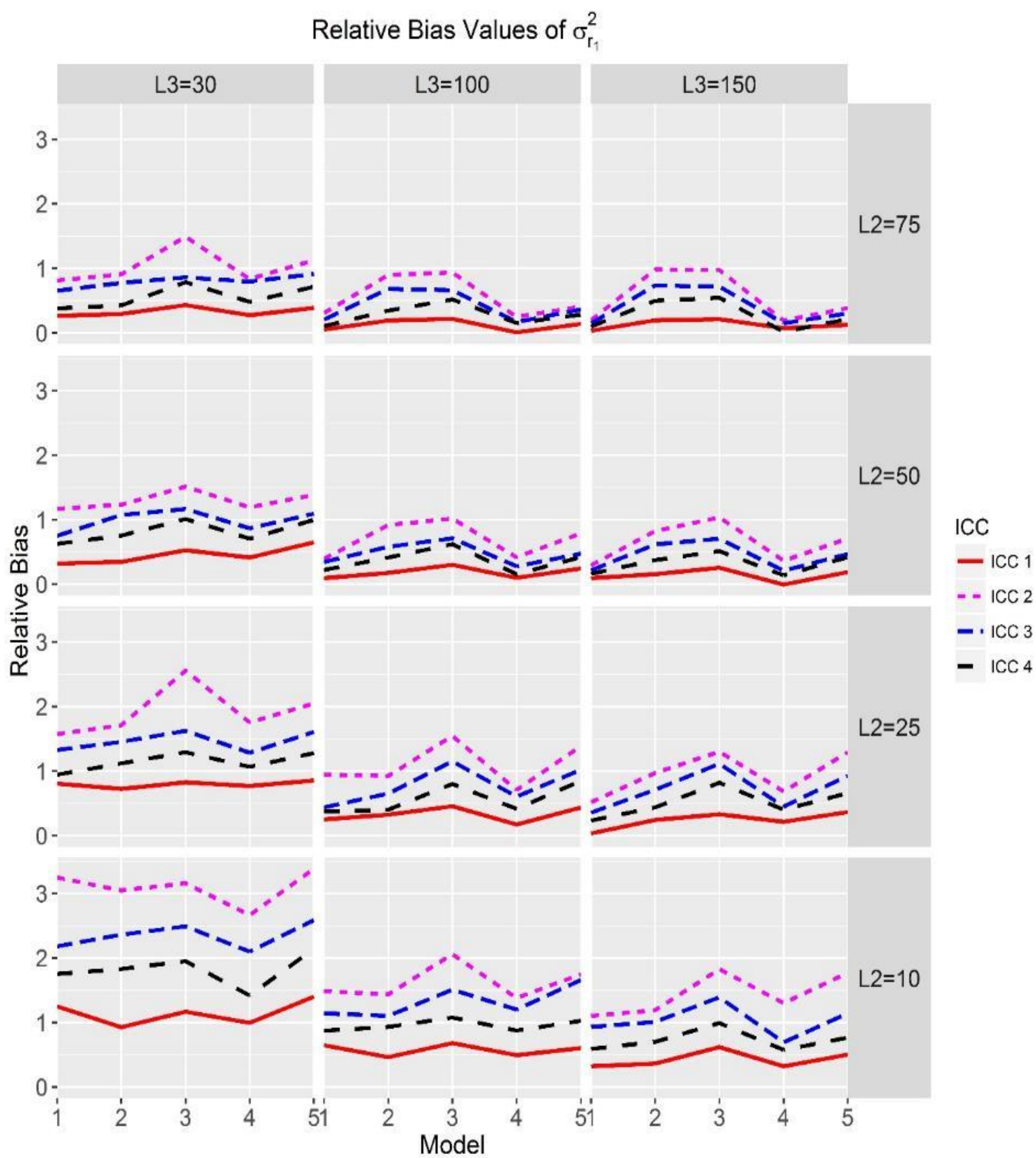

Figure 18. Plot of Relative Bias Across Manipulated Factors for σ^2_{r1}.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

The Figure 19 also shows that the relative bias differences between the ICCs disappeared as the L2 and L3 sample sizes increased.

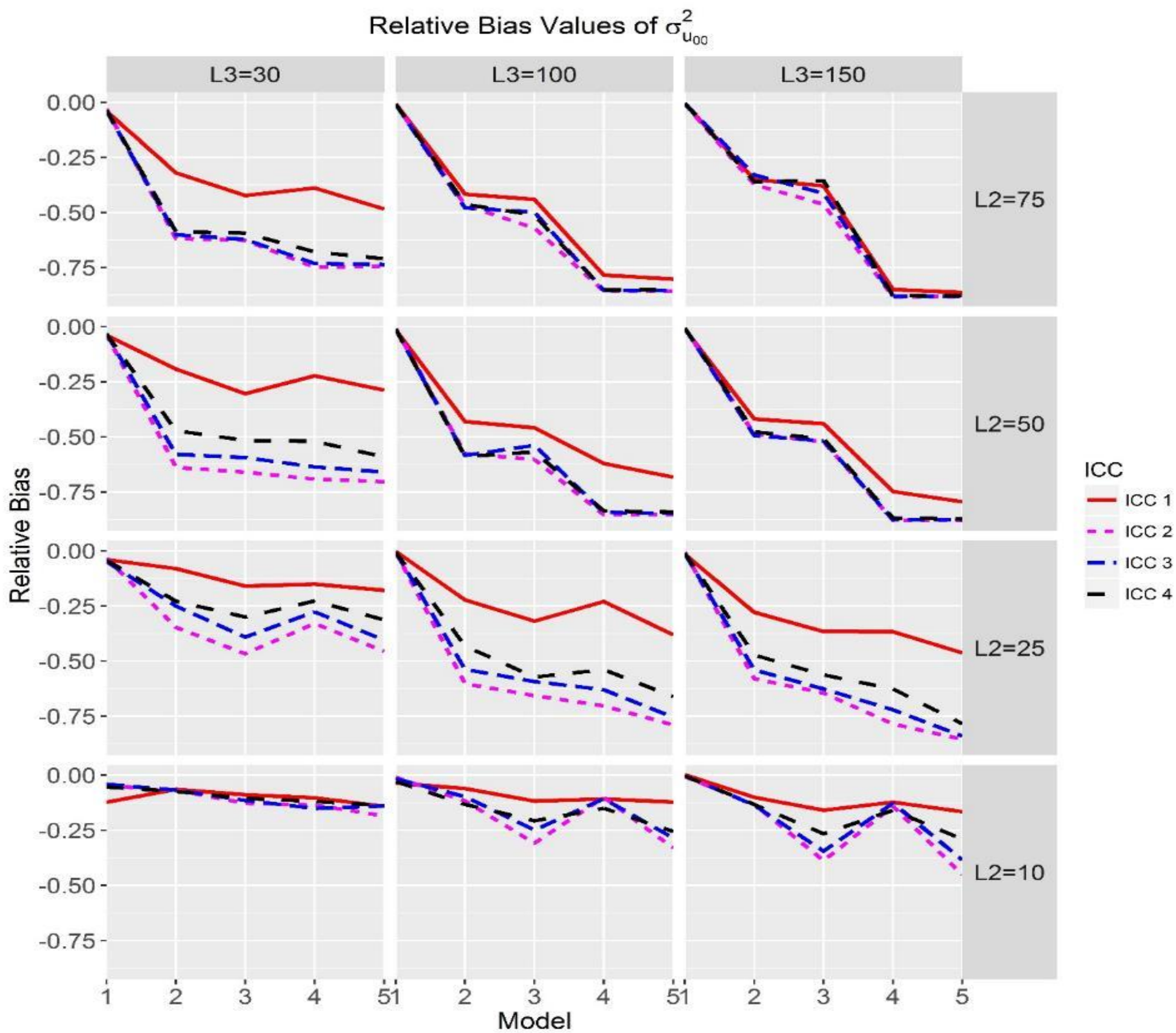

Figure 19. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{u_{00}}$

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

The patterns observed for the relative bias $\sigma^2_{u_{01}}$, $\sigma^2_{u_{10}}$, and $\sigma^2_{u_{11}}$ are very similar to the relative bias for $\sigma^2_{u_{00}}$. Because of that the relative bias values for these terms are not presented here. The tabular and graphical representation of the relative bias results $\sigma^2_{u_{01}}$, $\sigma^2_{u_{10}}$, and $\sigma^2_{u_{11}}$ included in Appendix B and Appendix E, respectively.

Residual covariance terms. The overall descriptive statistics for the relative bias of residual covariance terms are presented in Table 37. The relative bias values for the residual covariance terms are generally higher than fixed effect intercept terms. Relative bias values across conditions ranged from -1.7160 to 2.1587.

Table 37.

Relative Bias Descriptive Statistics for Residual Covariance Terms

Parameter	Minimum	Maximum	Mean	*SD*
$\sigma_{r_0}\sigma_{r_1}$	-0.0493	2.1587	0.5772	0.4498
$\sigma_{u_{00}}\sigma_{u_{01}}$	-1.5186	0.0000	-0.3096	0.4641
$\sigma_{u_{00}}\sigma_{u_{10}}$	-1.3565	0.0000	-0.3212	0.3895
$\sigma_{u_{00}}\sigma_{u_{11}}$	-0.1347	2.1443	0.3193	0.5209
$\sigma_{u_{01}}\sigma_{u_{10}}$	-0.6838	0.0383	-0.0884	0.1660
$\sigma_{u_{01}}\sigma_{u_{11}}$	-1.4784	1.2808	-0.3025	0.5035
$\sigma_{u_{10}}\sigma_{u_{11}}$	-1.7160	0.0000	-0.3375	0.5122

Note. The correlation among the L3 covariance terms is presented in Table 1.

Figure 20 shows the relative bias values of $\sigma_{\mathrm{r0}}\sigma_{\mathrm{r1}}$.The majority of the relative bias values of $\sigma_{\mathrm{r0}}\sigma_{\mathrm{r1}}$ are not within acceptable range, given the varying ICCs, model complexity, L2, and L3 sample sizes. However, like the σ^2_{r1}, the relative bias values of $\sigma_{\mathrm{r0}}\sigma_{\mathrm{r1}}$ decreased as the L2 sample size increased regardless of varying ICCs, model complexity, and L2 sample sizes. Similarly, the relative bias values of $\sigma_{\mathrm{r0}}\sigma_{\mathrm{r1}}$ decreased

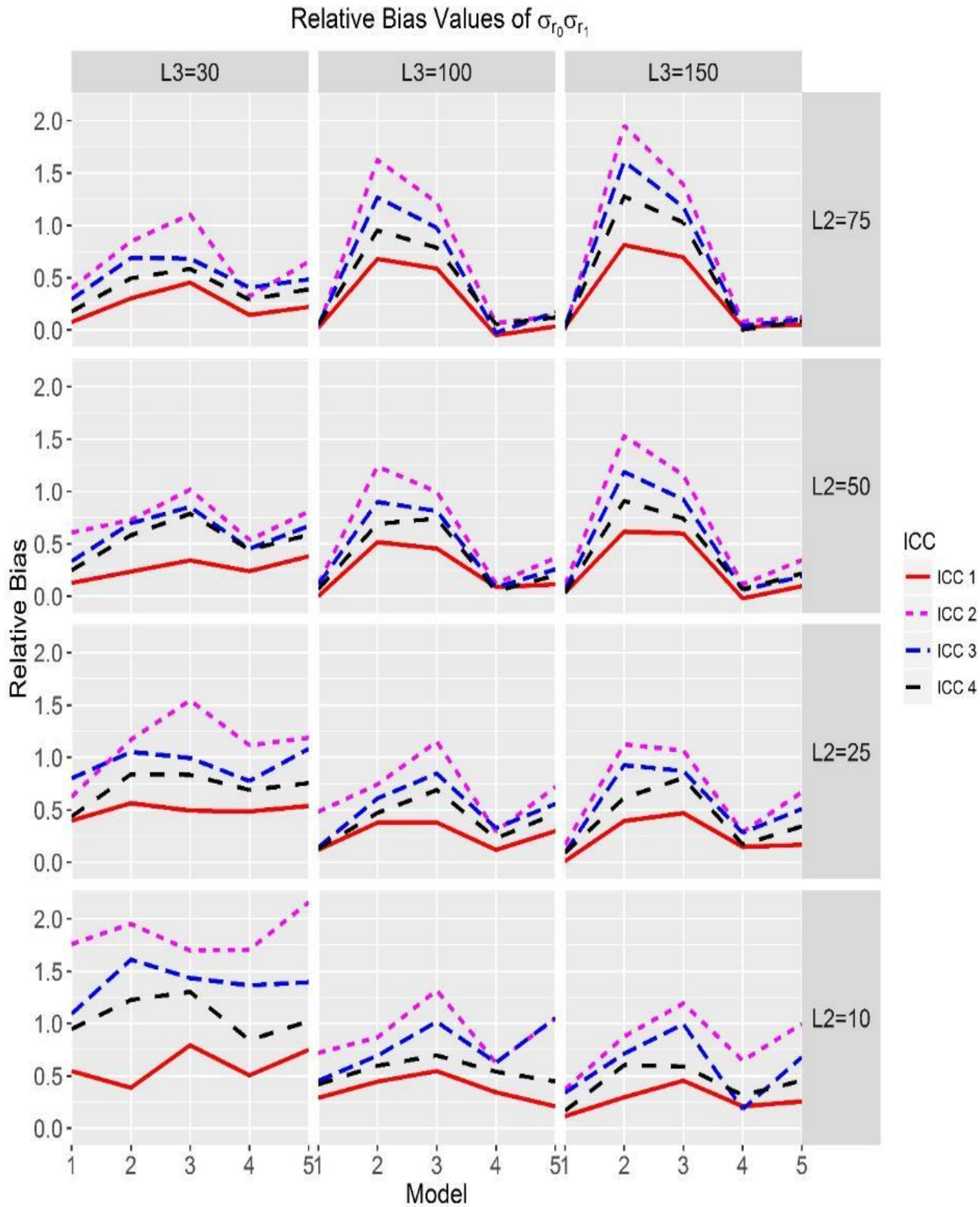

Figure 20. Plot of Relative Bias Across Manipulated Factors for $\sigma_{r_0}\sigma_{r_1}$.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

as the L3 sample size increased. Model 2 and model 3 had no conditions where the relative bias values are within acceptable range. Both model 2 and model 3 included one predictor at L1 and L2 but model 2 included the L3 residual covariance and model 3 did not.

Unlike model 2 and model 3, model 1, model 4, and model 5 had a few conditions for $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ where the relative bias values are within acceptable range. The diffference between model 1 and both model 2 and 3 is that model 1did not have the L2 predictor and only included one predictor at L1. Although, it seemed including a new predictor resulted in increase in relative bias values from model 1 to model 2 and model 3, the same relationship did not hold from model 2 and model 3 to model 4 and model 5. The relative bias values decreased from model 2 and model 3 to model 4 and model 5. Model 4 and model 5 had one more predictor which is at L3 compared to model 2 and model 3 which did not have the L3 predictor. The table of relative bias for $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ are included in Appendix B highlights the relative bias values that are within acceptable range.

The nonstraight lines in Figure 20 clearly shows that model complexity had an effect on relative bias values for $\sigma_{r_0}\sigma_{r_1}$. However, the effect is unstable, relative bias values for L1L2 model is higher than both the less complicated L1 model and the more complicated L1L2L3 model. Similarly, varying ICC had an effect on the relative bias values of $\sigma_{r_0}\sigma_{r_1}$. ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30) had generally the highest relative bias values followed by ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), and ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.4, ICC_{L3}=0.1). In other words, the higher the ICC at ICC_{L2}, the lower the relative bias is for $\sigma_{r_0}\sigma_{r_1}$.

The relative bias values of $\sigma_{u_{00}}\sigma_{u_{10}}$ are presented in Figure 21. All of the relative bias values are negative which means the $\sigma_{u_{00}}\sigma_{u_{10}}$ are underestimated. The bottom three panel represents the relative bias values when the L2 sample size is 10 and it shows that the relative bias values for N_{L2}=10 are the lowest compared the other L2 sample size conditions.

The effect of ICC on relative bias values for $\sigma_{u_{00}}\sigma_{u_{10}}$ are minimal when the L2 sample size is 10 (bottom three panels). When the L2 sample size is 25, there is a clear separation between the ICC_1 and the other ICCs regardless of L3 sample size and model complexity. Similarly, when the L2 sample size is 50, although it is not as large as when N_{L2}=25, there is still a separation between the ICC_1 and the other ICCs regardless of L3 sample size and models. It is also clear that the difference in relative bias disappears when L2 is 75 and L3 is 100 or 150, but there is a difference when L3 is 30.

Increasing model complexity had varying effects on relative bias values for $\sigma_{u_{00}}\sigma_{u_{10}}$. For example, for the conditions when L2 is 50 or 75 and L3 is 100 or 150, the relative bias values decreased as the model complexity increased. However, when L2 is 25 and L3 is 30, the relative bias values increased for ICC_1 from model L1 to model L1L2 and decreased from model L1L2 to model L1L2L3.

Similar to the $\sigma_{u_{00}}\sigma_{u_{10}}$, the relative bias values of $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$ are almost all negative which means they are all generally underestimated regardless of model complexity, ICC, L2 and L3 sample sizes. The effect of increasing

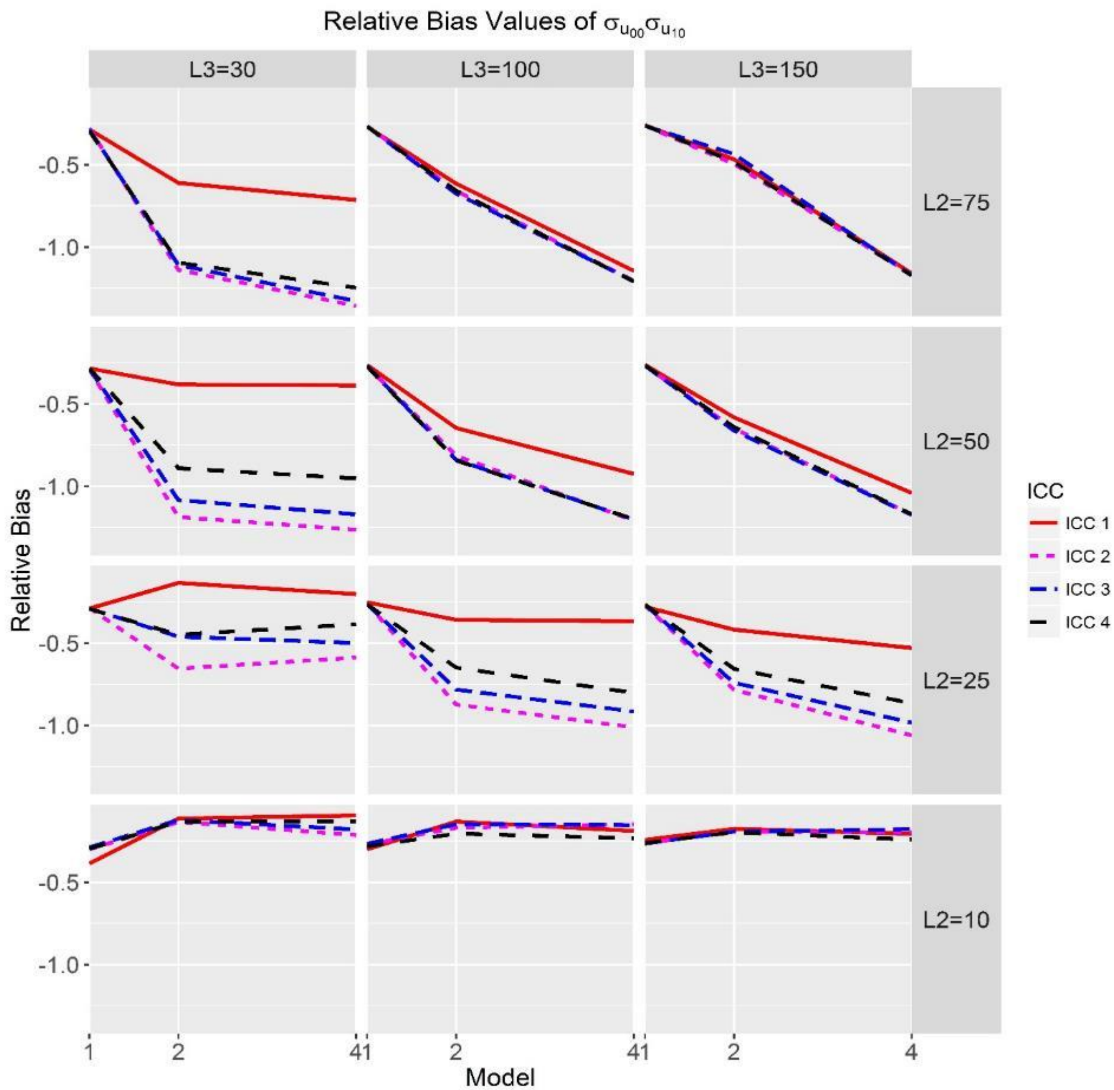

Figure 21. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{00}}\sigma_{u_{10}}$.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

L2 or L3 sample sizes on relative bias values are unstable. In other words, the relative bias sometimes decreased when L2 sample size increased and sometimes increased when

L2 sample size increased. The tabular and visual representation of relative bias results can be found in Appendix B and Appendix D, respectively.

Figure 22 shows the relative bias values of $\sigma_{u_{00}}\sigma_{u_{11}}$ and demonstrates that it is

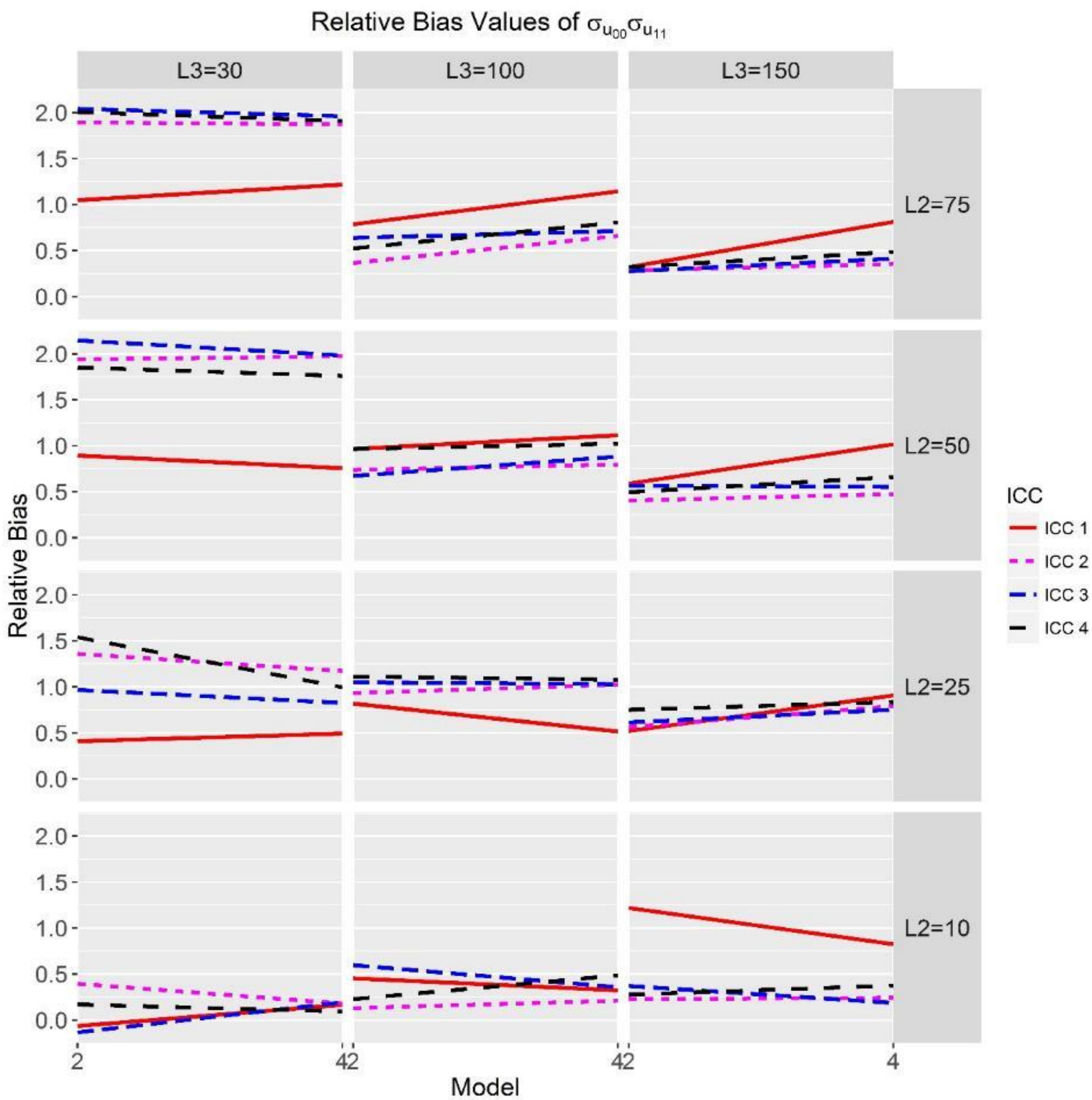

Figure 22. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{00}}\sigma_{u_{11}}$.

Note. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20) difficult to see a consistent pattern in terms of the effect of model complexity. For example, following the solid red lines (ICC_1 where ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) shows that relative values increased for some conditions and decreased for some other condition when the model complexity increased. Similarly, the effect of ICC is not clear, again following the solid red line (ICC_1), it is easy to see that relative bias values are lower for the solid red line (ICC_1) compared to the other ICCs for some conditions but not all conditions.

All of the relative bias values for $\sigma_{u_{00}}\sigma_{u_{11}}$ are positive except the condition where N_{L2}=10, N_{L3}=30, ICC_1 under L1L2 model. Almost all of the relative bias values are not within the acceptable range. The relative bias values are unstable in terms of L2 and L3 sample sizes. The effect of increasing L2 or L3 sample size is unstable. The increase in L2 o3 L3 sample sizes resulted in an increase in some conditions and decrease in some other conditions.

Summary of relative bias. The relative bias statistic is generally not within the acceptable range for the majority (17 out of 22) of the parameter estimates (γ_{001}, γ_{101}, γ_{011}, γ_{111}, $\sigma^2_{r_1}$, $\sigma^2_{r_0}$, $\sigma^2_{u_{00}}$, $\sigma^2_{u_{10}}$, $\sigma^2_{u_{01}}$, $\sigma^2_{u_{11}}$, $\sigma_{r_0}\sigma_{r_1}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$) and relatively low for the remaining γ_{000}, γ_{010}, γ_{100}, γ_{110}, and σ^2_e. It is important to note that relative bias is highly susceptible to true values between -1 and 1. Indeed, it is a clear indication that the relative bias values of the almost all of the parameters that had true values between -1 and 1 are not within the acceptable range.

RMSE. The RMSE values for fixed effects intercept terms, fixed effects slope terms, residual variance, and residual covariance terms are presented in this section.

Fixed effect intercepts terms. The overall descriptive statistics for the RMSE of fixed effects intercept terms are presented in Table 38. The RMSE values for the fixed effect intercept terms are generally very low and across conditions ranged from 0.00 to 0.3542. The highest RMSE values are associated with γ_{000} and γ_{100} and the lowest RMSE values are associated with γ_{010} and γ_{110}.

Table 38.

RMSE Descriptive Statistics for Fixed Effect Intercept Terms

Parameter	True Value	Minimum	Maximum	Mean	*SD*
γ_{000}	347.365	0.0462	0.3542	0.1278	0.0724
γ_{010}	0.733	0.0000	0.0943	0.0314	0.0292
γ_{100}	5.766	0.0289	0.2068	0.0762	0.0432
γ_{110}	-0.014	0.0000	0.0238	0.0077	0.0059

Figure 23 shows the RMSE values of γ_{000}, tables of the absolute bias statistics for γ_{000} and all other parameters included in Appendix C. Figure 24 illustarated that RMSE values of γ_{000} decreased when L2 sample size increased regardless of L3 sample sizes, ICCs, and model complexity. Similarly, RMSE values for γ_{000} decreased when L3 sample size increased regardless of L2 sample sizes, ICCs, and model complexity. The highest RMSE values are associated with the condition where L_{2N}=10, and L_{3N}=30 under each model and ICC condition. L2 sample size of 10 produced the highest RMSE values for each of the L3 sample sizes. Figure 23 also shows that L_{3N}=30 conditions had the highest RMSE values compared to L_{3N}=100 and L_{3N}=150.

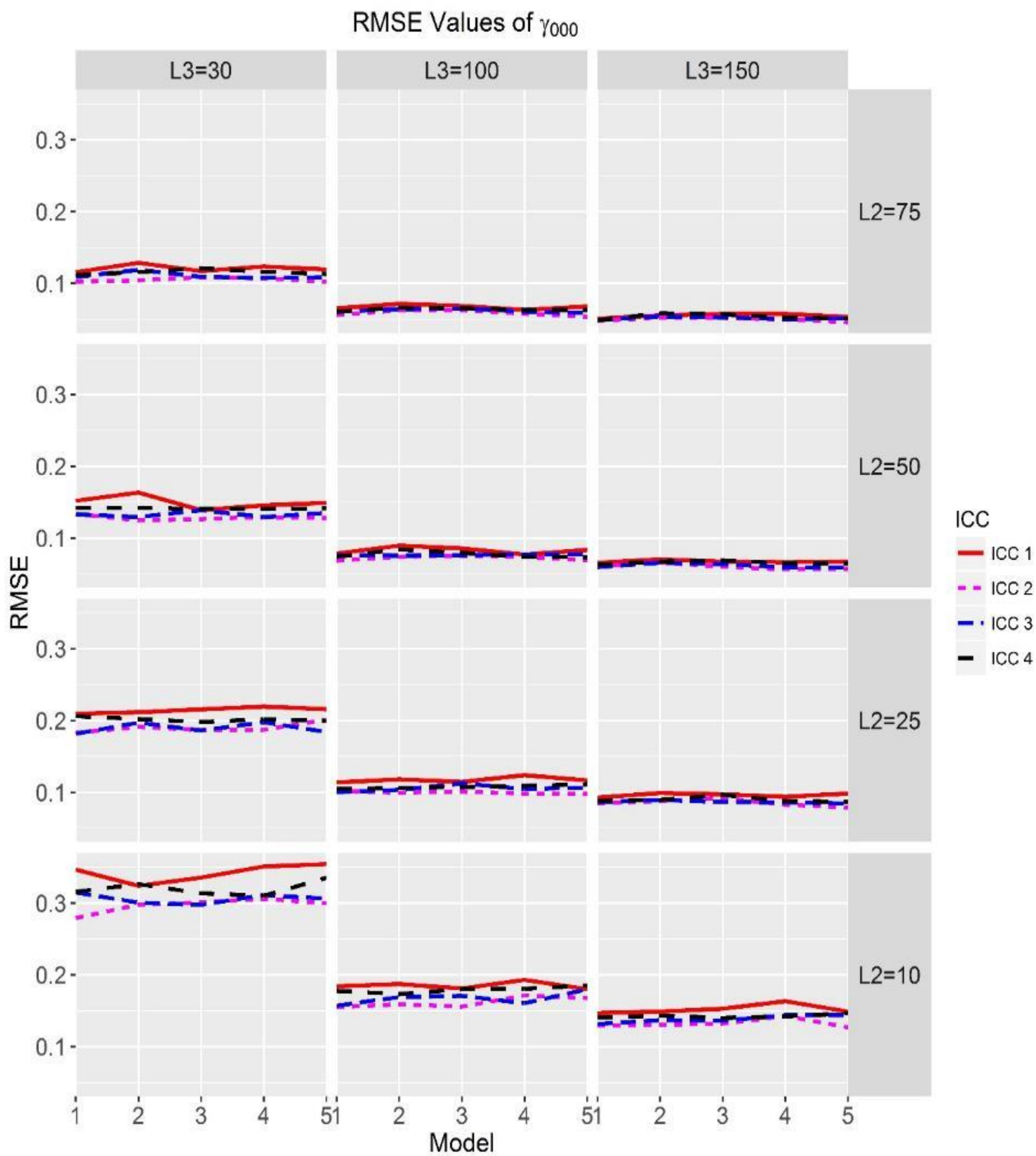

Figure 23. Plot of RMSE Across Manipulated Factors for γ_{000}.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

The straight lines indicated that RMSE values of γ_{000} are not affected by model complexity. The largest RMSE difference between models is 0.03. The largest RMSE difference between varying ICCs is 0.03 except where L_{2N}=10, and L_{3N}=30. The differences are relatively larger under this condition. For example, the RMSE difference under L1 model is 0.07 where L_{2N}=10, and L_{3N}=30 when the ICC changes from ICC_1 to ICC_2.

The RMSE values of γ_{010} and γ_{110} are all less than 0.10. The RMSE results for γ_{100} and the observed patterns are very similar to the γ_{000}. In order to save space in this section, the tabular and graphical representation of RMSE values for γ_{010}, γ_{110}, and γ_{100} are all presented in Appendix C and Appendix F.

Fixed effect slope terms. The overall descriptive statistics for the RMSE of fixed effects slope terms are presented in Table 39. The RMSE values for the fixed effect slope terms are generally very low and across conditions ranged from 0.00 to 0.5040. The highest RMSE values are association with γ_{001} and γ_{101} and the lowest RMSE values are associated with γ_{010} and γ_{110}.

Table 39.

RMSE Descriptive Statistics for Fixed Effect Slope Terms

Parameter	True Value	Minimum	Maximum	Mean	*SD*
γ_{001}	0.1738	0.0671	0.5040	0.2085	0.1098
γ_{011}	0.0004	0.0000	0.0215	0.0043	0.0060
γ_{101}	-0.0190	0.0000	0.3821	0.0580	0.0864
γ_{111}	0.0004	0.0000	0.0140	0.0026	0.0037

The RMSE values of γ_{001} are not as low as γ_{000}. However, similar to the RMSE values of γ_{000}, RMSE values of γ_{001} are reduced when L3 sample sizes increased regardless of L2 sample sizes, ICCs, and model complexity. On the other hand, RMSE values of γ_{001} increased when L2 sample sizes increased regardless of L3 sample sizes, ICCs, and model complexity.

Figure 24 shows the RMSE values of γ_{001}. As seen in Figure 24, the lowest RMSE values observed when N_{L2} is 10, and N_{L3} is 100 or 150 and the highest RMSE values are associated with the condition where N_{L2}=75 and N_{L3}=30 (top left panel). Although the difference is subtle, Figure 24 also revealed that RMSE values generally decreased from L1L2L3 Model to L1L2L3 with No L3 covariance terms. Moreover, ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.4, ICC_{L3}=0.1), the solid red line, had the lowest RMSE values compared to the other ICCs. Though it is not a large difference, there is also a clear separation between the ICCs. Following ICC_1, ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20) had the second lowest RMSE values followed by ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25) and ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). In other words, as the ICC at L2 drops and L3 increased keeping the ICC at L1 constant, the RMSE values increased.

The RMSE values for γ_{011} and γ_{111} are all less than 0.10. The RMSE results for γ_{011} and the observed patterns are very similar to the γ_{001}. In order to save space in this

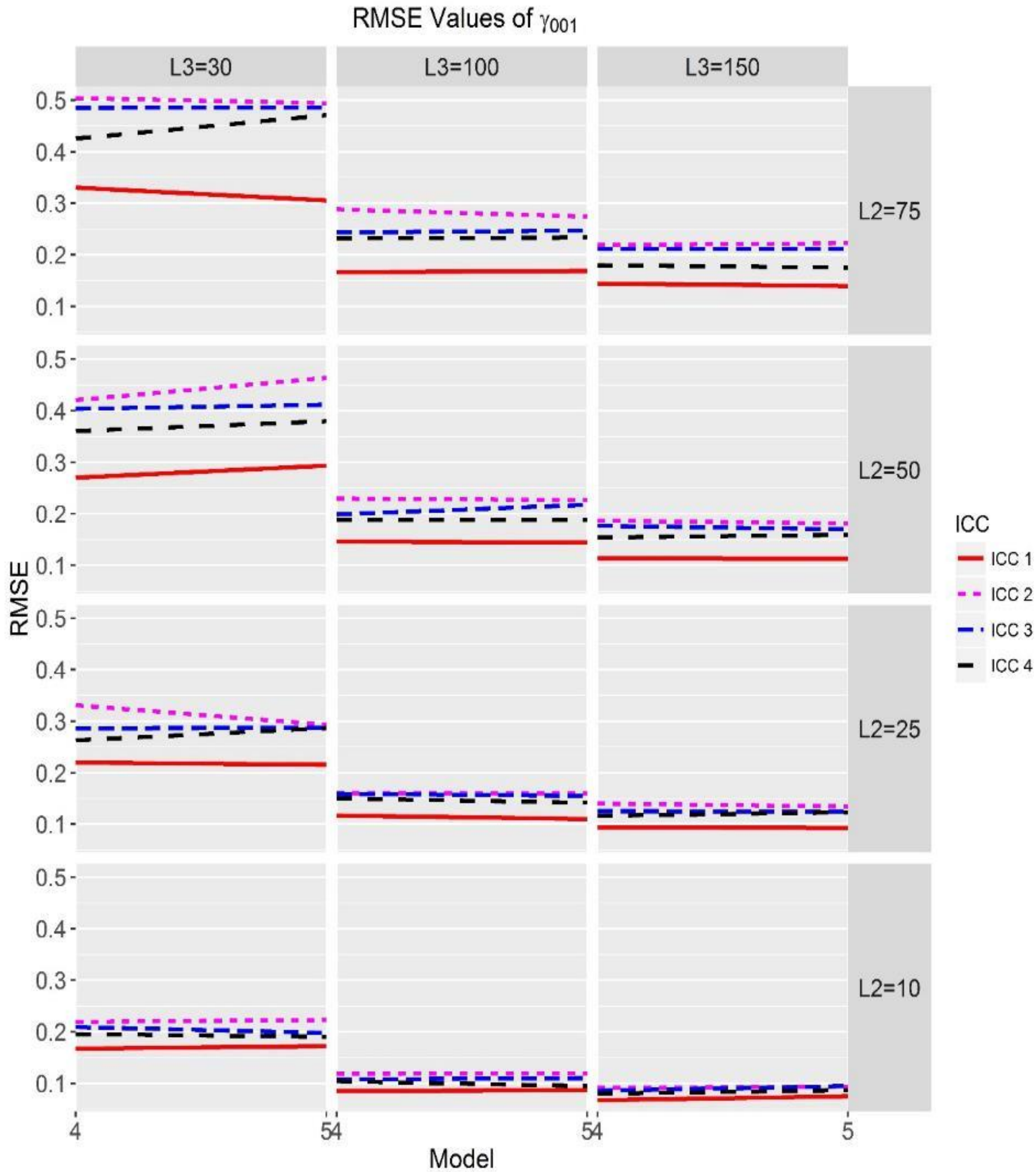

Figure 24. Plot of RMSE Across Manipulated Factors for γ_{001}.

Note. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20) section, the tabular and graphical representation of RMSE values for γ_{011}, γ_{101}, and γ_{111} are all presented in Appendix C and Appendix F.

Residual variance terms. The overall descriptive statistics for the RMSE of residual variance terms are presented in Table 40. The RMSE values for the residual variance terms are generally high and across conditions ranged from 0.00 to 24.105. The highest RMSE values are associated with $\sigma^2_{r_0}$, $\sigma^2_{u_{00}}$and $\sigma^2_{u_{10}}$ and the lowest RMSE values are associated with $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$.

Table 40.

Relative Bias Descriptive Statistics for Residual Variance Terms

Parameter	Minimum	Maximum	Mean	SD
σ^2_e	0.233	1.502	0.586	0.315
$\sigma^2_{r_0}$	0.379	24.105	6.037	7.071
$\sigma^2_{r_1}$	0.144	0.876	0.365	0.172
$\sigma^2_{u_{00}}$	0.181	7.027	2.902	1.944
$\sigma^2_{u_{01}}$	0.000	0.011	0.004	0.003
$\sigma^2_{u_{10}}$	0.071	3.547	1.450	1.001
$\sigma^2_{u_{11}}$	0.000	0.004	0.001	0.001

The RMSE values of σ^2_e are presented in Figure 25. As seen Figure 25, all of the RMSE values decreased as the L2 sample sizes increased regardless of varying ICCs, model complexity, and L3 sample sizes. Similarly, all of the RMSE values of σ^2_e decreased as the L3 sample sizes increased regardless of varying ICCs, model complexity, and L2 sample sizes. As seen in Figure 25, the highest RMSE values are associated with the condition (bottom left panel) where N_{L2}=10 and N_{L3}=30. The lowest

RMSE values are associated with the condition (bottom left panel) where N_{L2}=75 and N_{L3}=150.

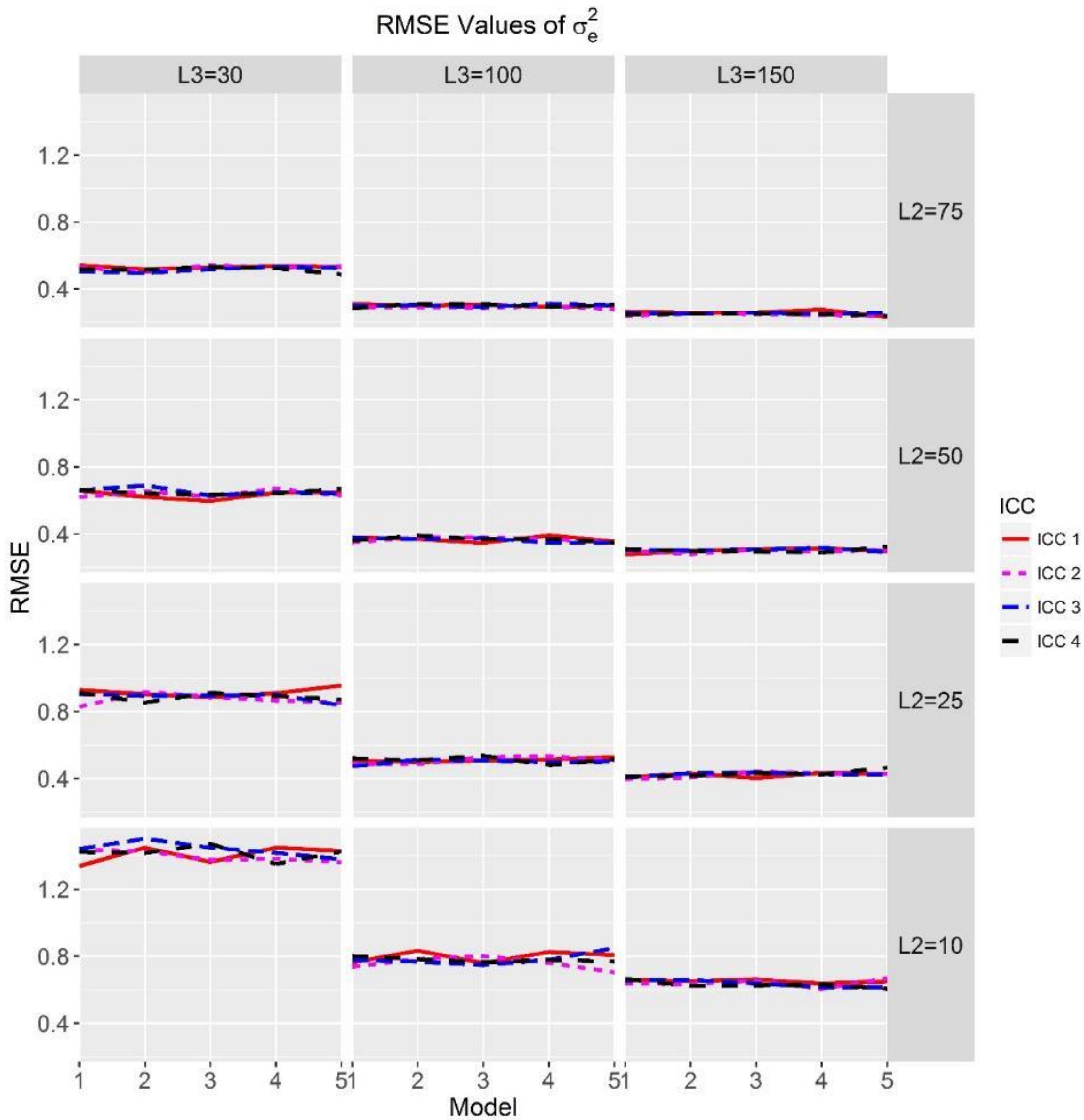

Figure 25. Plot of RMSE Across Manipulated Factors for σ_e^2.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

The RMSE of σ_{r0}^2 are presented in Figure 26. Similar to the relative bias values

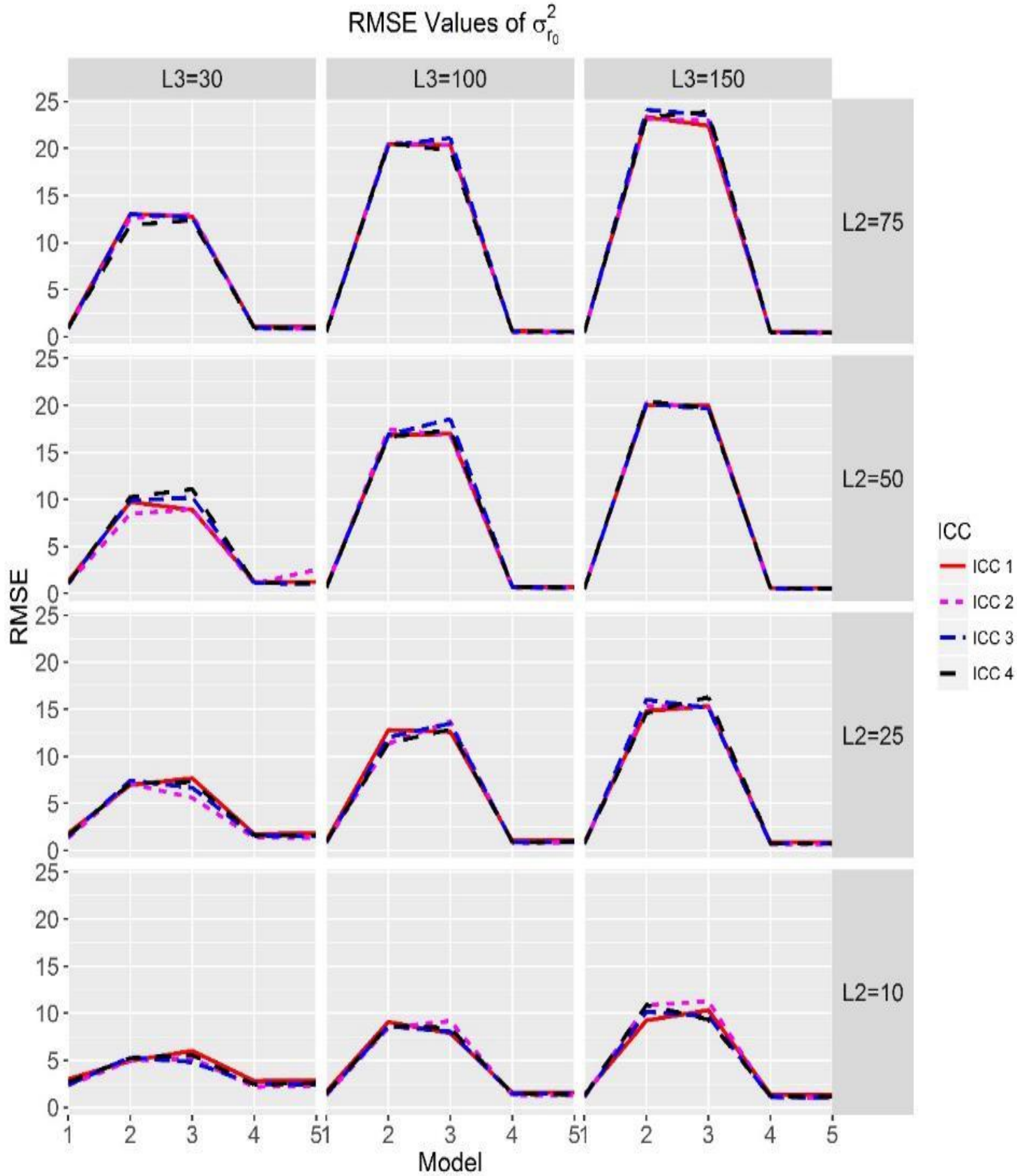

Figure 26. Plot of RMSE across manipulated factors for σ^2_{r0}.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

for $\sigma^2_{\mathrm{r}0}$, RMSE values under L1L2 and L1L2 No L3 Covariance models are relatively high. For the L1, L2L2L3, and L1L2L3 No L3 Covariance models, all of the RMSE values decreased as the L2 sample sizes increased regardless of varying ICCs, model complexity, and L3 sample sizes. Similarly, all of the RMSE values of $\sigma^2_{\mathrm{r}0}$ decrease as the L3 sample sizes increase regardless of varying ICCs, model complexity, and L2 sample sizes. The opposite of this relationship exists for the L1L2 and L1L2 No L3 Covariance models.

The patterns observed in the RMSE values of σ^2_{e} are also observed in the RMSE values of $\sigma^2_{\mathrm{r}1}$, $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$. Because of this, the tabular and graphical representation RMSE values for $\sigma^2_{\mathrm{r}1}$and $\sigma^2_{u_{10}}$ are only presented in Appendix C and Appendix F.

The RMSE values of $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$ and are all less than or equal to 0.02 regardless of varying ICCs, model complexity, L2, and L3 sample sizes. Because of this reason, the tabular and graphical representation RMSE values for $\sigma^2_{\mathrm{r}1}$and $\sigma^2_{u_{10}}$ are only presented in Appendix C and Appendix F.

Residual covariance terms. The overall descriptive statistics for the RMSE of residual covariance terms are presented in Table 41. The RMSE values for the residual variance terms are generally high and across conditions ranged from 0.00 to 4.399. The highest RMSE values are associated with $\sigma_{r_0}\sigma_{r_1}$ and $\sigma_{u_{00}}\sigma_{u_{10}}$. The rest of the residual covariance terms had RMSE values less than 0.10.

The RMSE values of $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ are presented in Figure 27. All of the RMSE values of $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ decreased as the L2 sample sizes increased regardless of varying ICCs, model complexity, and L2 sample sizes. Similarly, all of the the relative bias values of $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ decreased as the L3 sample sizes increased regardless of varying ICCs, model complexity, and L2 sample sizes.

Table 41.

RMSE Descriptive Statistics for Residual Covariance Terms

Parameter	Minimum	Maximum	Mean	*SD*
$\sigma_{r_0}\sigma_{r_1}$	0.191	1.228	0.551	0.238
$\sigma_{u_{00}}\sigma_{u_{01}}$	0.000	0.179	0.039	0.054
$\sigma_{u_{00}}\sigma_{u_{10}}$	0.000	4.399	0.970	1.157
$\sigma_{u_{00}}\sigma_{u_{11}}$	0.000	0.075	0.010	0.015
$\sigma_{u_{01}}\sigma_{u_{10}}$	0.000	0.082	0.012	0.018
$\sigma_{u_{01}}\sigma_{u_{11}}$	0.000	0.006	0.001	0.002
$\sigma_{u_{10}}\sigma_{u_{11}}$	0.000	0.063	0.014	0.019

Note. The correlation among the L3 covariance terms is presented in Table 13.

The non-straight lines shows that model complexity affects the RMSE values for $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$. Introducing new predictors to a model affected the RMSE bias values for $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ but removing the L3 covariance terms seemed to not affect the RMSE values. The RMSE values generally increased from L1 model to L1L2 model but decreased from L1L2L3 model. This difference is clearly seen especially for the top right four panels in Figure 27 where L2 sample size is either 50 or 75 and L3 sample size is either 100 and 150.

The ICC's effect on the RMSE values of $\sigma_{\mathrm{r}0}\sigma_{\mathrm{r}1}$ is minimal since the separation between the lines are very small. The largest separation is observed when the L2 sample size is 10 and L3 sample size is 30.

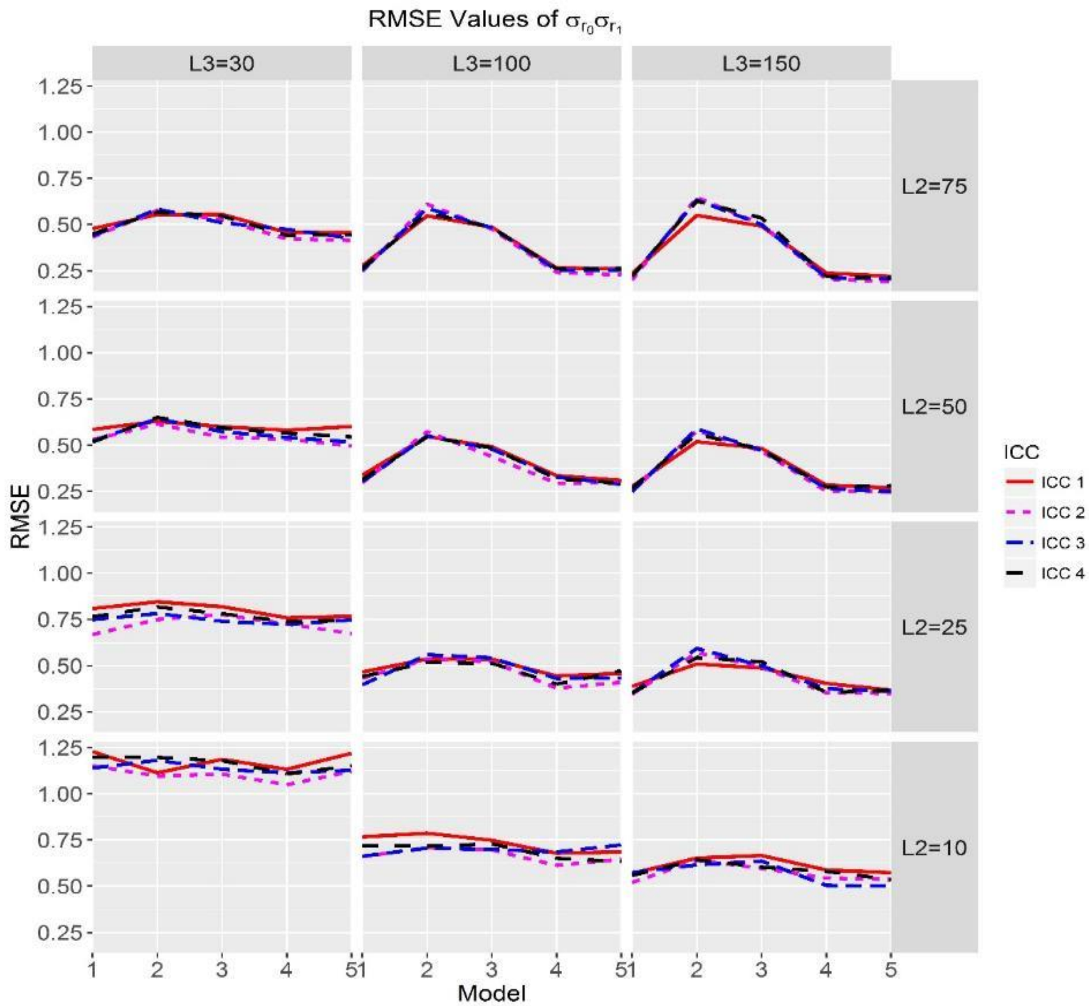

Figure 27. Plot of RMSE Across Manipulated Factors for $\sigma_{r0}\sigma_{r1}$.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2NoL3Cov Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3NoL3Cov Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20)

Figure 28 shows RMSE values of $\sigma_{u_{00}}\sigma_{u_{10}}$. The RMSE values decreased as the L2 or L3 sample sizes increased for L1 model. However, the same relationship did not hold for the L1L2 and L1L2L3 models. On the contrary, for some conditions, the RMSE values increased as the L2 or L3 sample sizes increased. For example, for ICC_2 under

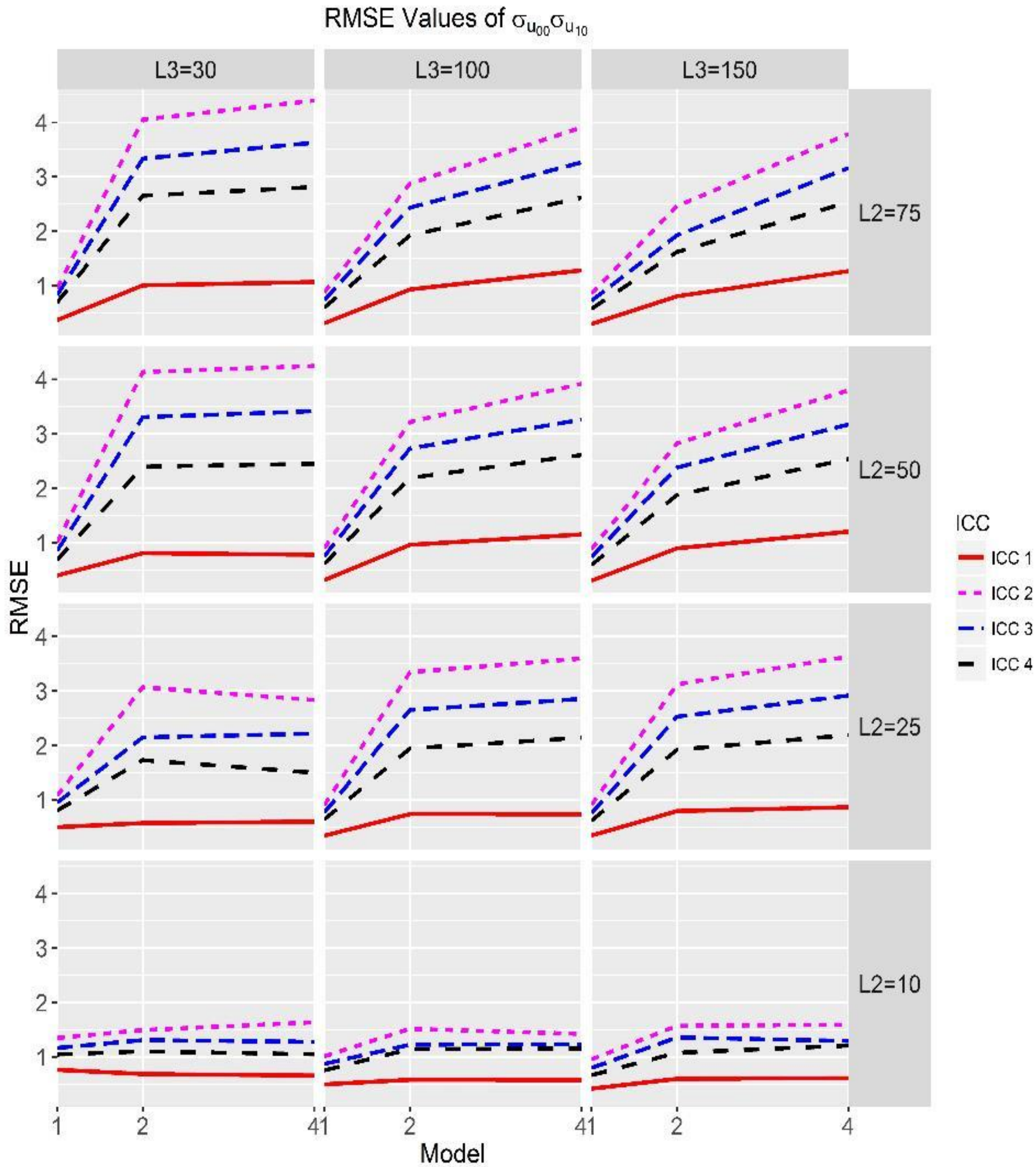

Figure 28. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{00}}\sigma_{u_{10}}$.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

L1L2 Model, the RMSE values increased when L3 sample size is 30 and L2 sample sizes increased from 10 to 25 or 50.

In Figure 28, it is easily seen that as the model complexity increased the RMSE values of $\sigma_{u_{00}}\sigma_{u_{10}}$ increased regardless of the ICCs, L2 and L3 sample sizes. The figure also shows that there is a clear separation in the ICC values. The lowest RMSE values are associated with the ICC_1 (ICC_{L1}=0.50, ICC_{L2}=0.4, ICC_{L3}=0.10) followed by ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). It is also very easy to see that the relative values are the lowest when L2 is 10 regardless of L3 sample sizes, model complexity, and ICCs.

Coverage. Parameter coverage proportions are only calculated for fixed effects intercept and slope terms.

Fixed effect intercepts terms. The parameter coverage proportions for γ_{000} and γ_{100} are generally very high and above the nominal level. Parameter coverage proportions across conditions ranged from 0.98 to 1. Similarly, the parameter coverage proportions for γ_{110} is generally above the nominal level of .95. The coverage rates larger than the nominal rate is a sign of upward biased standard errors, given the parameter estimates are accurate. These produce conservative rates of Type-I error, given the parameter estimates are accurate.

Since the parameter coverage proportions of γ_{000}, γ_{100}, and γ_{110} are mostly above the nominal level. Those are not examined here in this section. However, all of the tabular and visual representation of the parameter coverage proportions for the fixed effects intercept terms γ_{000}, γ_{100}, and γ_{110} are presented in Appendix G and Appendix H, respectively.

On the other hand, the parameter coverage proportions for γ_{010} shows greater variability. Across conditions, parameter coverage proportions for γ_{010} ranged from 0 to 1. Figure 29 shows the parameter coverage for γ_{010}. The non-straight line that goes from model 3 to model 4 shows that the parameter coverage drastically increased from model 3 to model 4. One of the major differences between model 3 and model 4 is the fact that model 4 includes a L3 predictor and model 3 did not. All of the 12 panels in Figure 29, revealed that the parameter coverage for model 2 (L1L2 model) and model 3 (L1L2 model with no L3 residual covariance) are always under the nominal level of 0.95. On the other hand, the parameter coverage for model 4 (L1L2L3 model) and model 5 (L1L2L3 model with no L3 residual covariance) is very close to 0, but it is not clear whether it is at .95, less or higher than .95. To clarify this, the parameter coverage of γ_{010} for only model 4 and model 5 illustrated in Figure 30. As seen in Figure 30, almost all of the parameter coverage proportions are above the nominal level. The bottom left panel where N_{L2}=10 and N_{L3}=30 is the only panel where the coverage proportion is close or at the nominal level except for ICC1 under model 5 (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10).

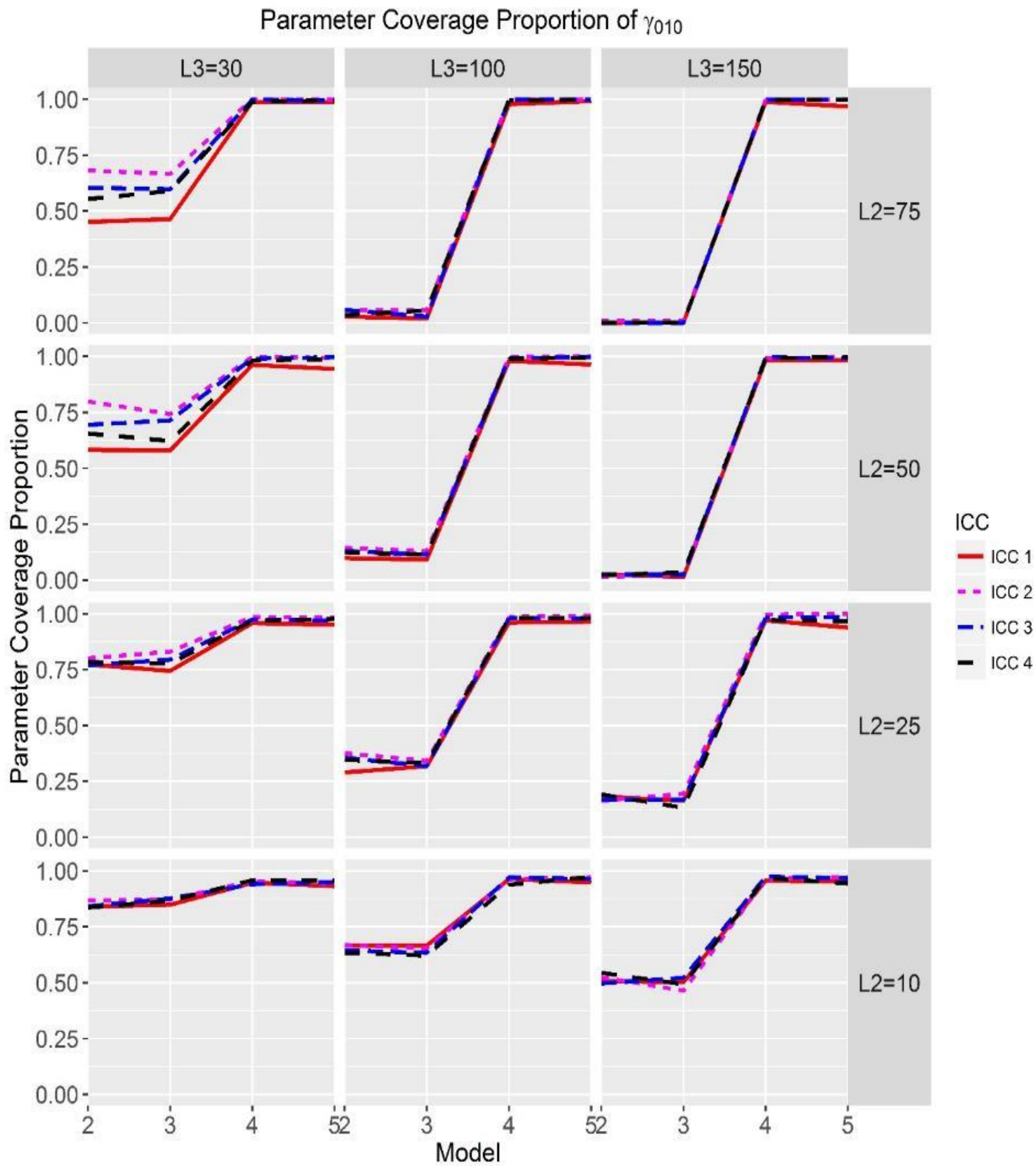

Figure 29. Plot of RMSE Across Manipulated Factors for γ_{010}.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

Fixed effects slope terms. Unlike the fixed effects intercept terms, the parameter coverage proportions for the fixed effects slope terms are generally lower than the nominal level of .95. The coverage rates less than the nominal rate are a sign of downward biased standard errors, given that the parameter estimates are accurate. These produce relatively liberal rates of Type-I error, given that the parameter estimates are accurate. The parameter coverage proportions of γ_{001} are presented in Figure 31. Generally, we expect the lines in the figure clustered around 0.95 but unfortunately that is not the case. γ_{001} is estimated in model 4 (L1L2L3 model) and model 5 (L1L2L3 model with no L3 covariance) which equates to 96 conditions. Out of these 96 conditions, parameter coverage is less than the nominal level (.95) 65 times, at nominal level 26 times, and higher than the nominal level 5 times. The four panels in the left column where N_{L3}=30 had no conditions where the parameter coverage proportion is at the nominal level. Similar results and patterns are observed for the remaining fixed effects slope terms γ_{011}, γ_{101}, and γ_{111}. Because of that, the parameter coverage proportions are not examined here but all of the tabular and visual representation of the parameter coverage proportions are presented in Appendix G and Appendix H, respectively.

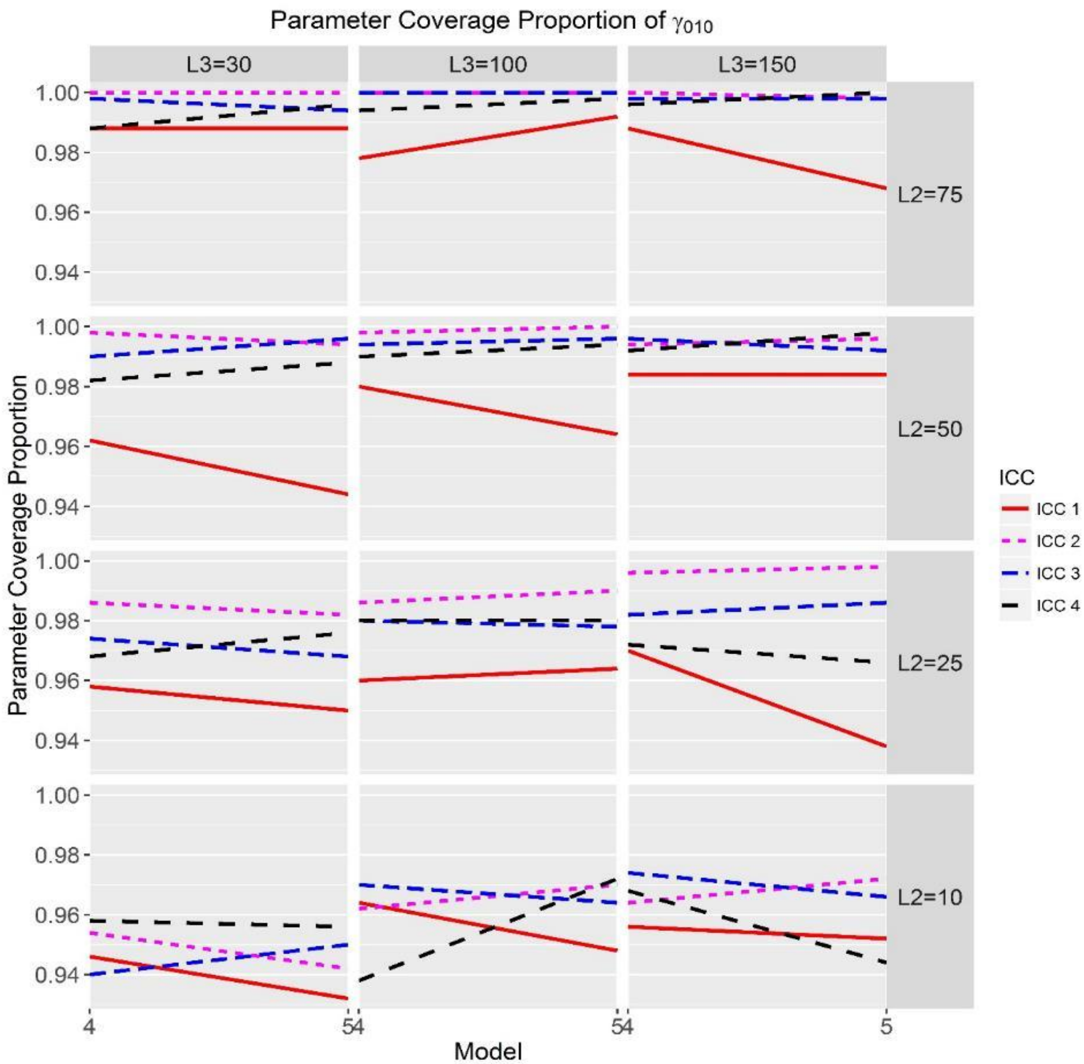

Figure 30. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{010} (M4-M5 only).

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

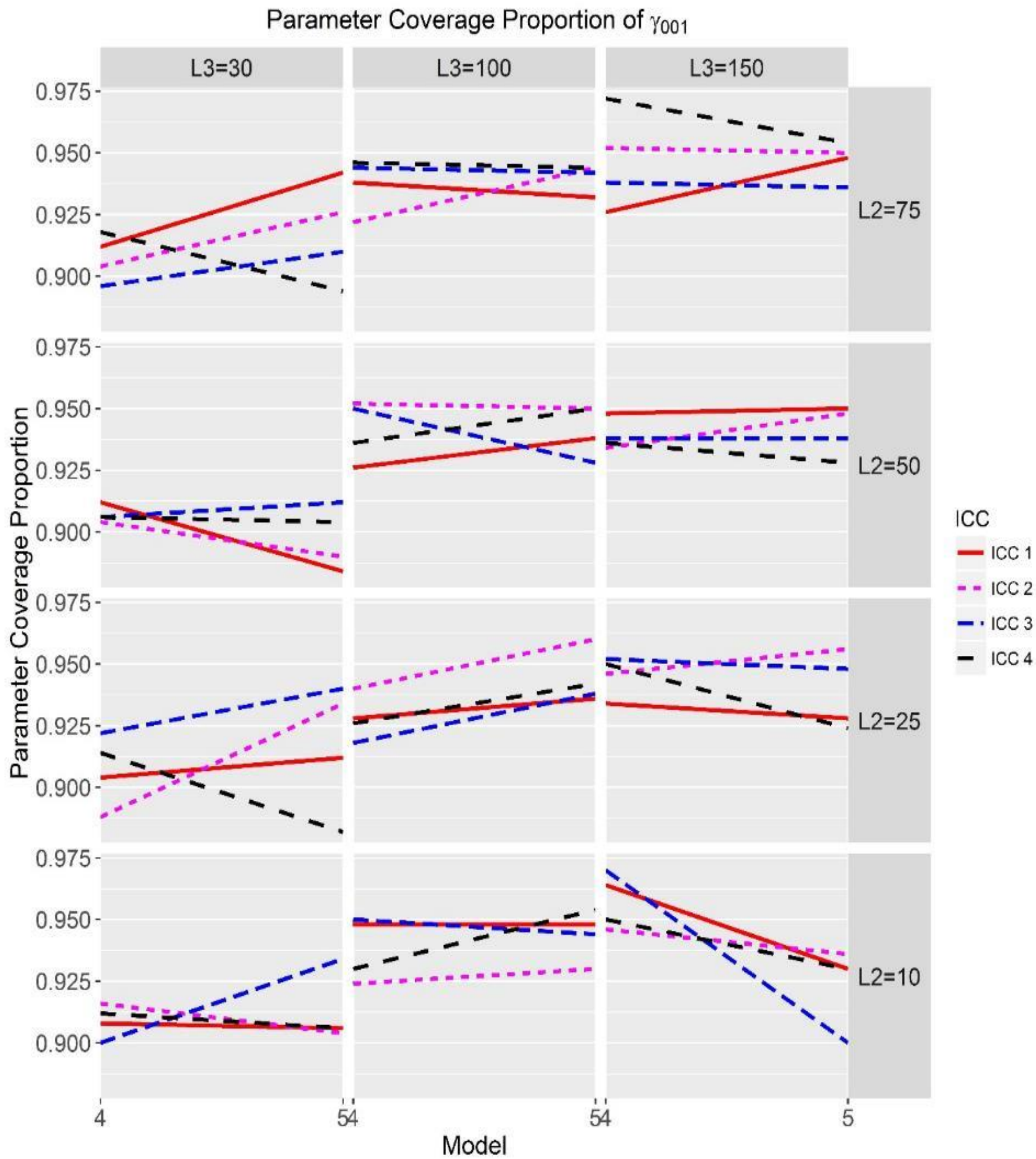

Figure 31. Plot of parameter coverage proportions Across Manipulated Factors for γ_{001}.

Note. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20).

Summary of Results

Fixed effect intercepts terms. Table 42 included a summary of the results for absolute bias, relative bias, RMSE, and parameter coverage proportion collapsed across model complexity and ICC. The fixed effects intercept and slope terms include four digits for each of the summary statistics summarized above in the following order (a) absolute bias, (b) relative bias, (c) RMSE, and (d) parameter coverage. For absolute bias, 1 indicated the absolute bias values are low for the L2 and L3 sample size combinations, 0 indicated the absolute bias values are high. Similarly, for relative bias, 1 indicated the relative bias values are within acceptable range and 0 meant they are not. For RMSE, 1 indicated the RMSE values are low for the L2 and L3 sample size combinations, 0 indicated the RMSE values are high. Lastly, for parameter coverage, "-" indicates that parameter coverage proportion is not estimated for that particular parameter. For parameter coverage, 1 meant parameter coverage proportion is at the nominal level for at least one model-ICC combinations and 0 indicated none of the model-ICC combinations produced a parameter coverage proportion at the nominal level. For example, in Table 42 for $\boldsymbol{\gamma_{010}}$ if we look at the column where NL3=150 and NL2=10, it shows that all of the examined statistics agrees that this L2 and L3 sample size combination produces an accurate parameter estimate for $\boldsymbol{\gamma_{010}}$.

Table 42 also provides a quick summary of the results but it might be overwhelming to track all of the numbers, and it is hard to see the patterns exist in the results. To solve this issue, for each cell in Table 42, I added 1s to produce an agreement summary. If all four summary statistics agrees the total is maximum 4 for fixed effects

Table 42.

Summary of Absolute Bias, Relative Bias, RMSE, Parameter Coverage

Parameter Group		L3=30				L3=100				L3=150			
		10*	25*	50*	75*	10*	25*	50*	75*	10*	25*	50*	75*
Fixed Effects Intercepts	γ_{000}	0100	0100	1110	1110	0110	1110	1110	1110	1110	1110	1110	1110
	γ_{010}	0111	0111	1110	1110	1111	1110	1110	1110	1111	1110	1110	1110
	γ_{100}	1110	1110	1110	1110	1110	1110	1110	1110	1110	1110	1110	1110
	γ_{110}	1110	1111	1110	1110	1111	1110	1110	1110	1111	1110	1110	1110
Fixed Effects Slopes	γ_{001}	0100	0100	0100	0100	1111	1110	0101	0101	1111	1111	0111	0111
	γ_{011}	1010	1010	1010	1010	1011	1011	1011	1011	1011	1011	1011	1011
	γ_{101}	1010	1000	1000	1000	1010	1011	1011	1011	1111	1011	1011	1011
	γ_{111}	1010	1010	1010	1010	1011	1011	1011	1010	1011	1011	1011	1011
Residual Variance	σ_e^2	010-	010-	010-	010-	010-	010-	010-	010-	010-	010-	010-	010-
	$\sigma_{r_0}^2$	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-
	$\sigma_{r_1}^2$	000-	000-	000-	000-	000-	000-	000-	001-	000-	000-	001-	001-
	$\sigma_{u_{00}}^2$	011-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-
	$\sigma_{u_{10}}^2$	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-
	$\sigma_{u_{01}}^2$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-
	$\sigma_{u_{11}}^2$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-
Residual Covariance	$\sigma_{r_0}\sigma_{r_1}$	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-
	$\sigma_{u_{00}}\sigma_{u_{01}}$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-
	$\sigma_{u_{00}}\sigma_{u_{10}}$	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-	000-
	$\sigma_{u_{00}}\sigma_{u_{11}}$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-
	$\sigma_{u_{01}}\sigma_{u_{10}}$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-
	$\sigma_{u_{01}}\sigma_{u_{11}}$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-
	$\sigma_{u_{10}}\sigma_{u_{11}}$	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-	101-

Note. *L2 sample sizes. Results collapsed across model complexity and ICC. The first digit indicates whether the absolute bias is acceptable or not. The second digit indicates whether the relative bias is acceptable or not. The third digit indicates whether the RMSE is acceptable or not. The last digit indicates whether the parameter coverage proportion is acceptable or not. 1 means Yes, 0 means No. “-” means not calculated.

intercept and slope terms. The maximum is 3 for the residual variance and covariance terms since parameter coverage proportions are not calculated for the residual variance and covariance terms. I used four different icons for fixed effects intercept and slope terms to indicate the level of agreement among the summary statistics (a) white check mark with a circle around indicates all four summary statistics agreed, (b) green check mark no circle around indicated that at least three out of four summary statistics agreed, (c) yellow exclamation point indicated two of the four summary statistics agreed, (d) red cross indicates maximum of one summary statistics had acceptable values. Similarly, for the variance and covariance terms, I used three different icons (a) green check mark no circle around indicates that at least two out of three summary statistics agrees, (b) yellow exclamation point indicated one summary statistics had acceptable values. (c) red cross meant none of the summary statistics had acceptable values.

A quick look at the first four rows in Table 43 shows that the summary statistics agrees on almost all of the fixed effect intercepts are within the acceptable range or close to 0 except the two L2 sample sizes of 10 and 25 under N_{L3}=30, and L2 sample size of 10 under N_{L3}=100.

The second 4 row in Table 43 shows that all of the L2 sample sizes under N_{L3}=30 had issues. However, the summary statistics generally agrees for N_{L3}=100 and N_{L3}=150. γ_{001} had some issues under N_{L2}=50 or N_{L2}=75 and N_{L3}=100. Similarly, γ_{101} had some issues under N_{L2}=10 and N_{L3}=100 . Lastly, γ_{111} had some issues under N_{L2}=75 and N_{L3}=100 .

Table 43 shows that most of the residual variance terms are not within the acceptable range or limit across the examined summary statistics. All of the sample size combinations for $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{01}}$as well as one condition of $\sigma^2_{u_{01}}$ under N_{L2}=10 and N_{L3}=30 shows agreement on the two out of three summary statistics. Table 43 provided the details about which of the two summary statistics agree.

Two of the three summary statistics for residual covariance terms agrees except for the $\sigma_{r_0}\sigma_{r_1}$ and $\sigma_{u_{00}}\sigma_{u_{10}}$. Table 43 provided the details about which of the two summary statistics agreed.

Table 43.

Summary of Absolute Bias, Relative Bias, RMSE, Parameter Coverage

Parameter Group		L3=30				L3=100				L3=150			
		10*	25*	50*	75*	10*	25*	50*	75*	10*	25*	50*	75*
Fixed Effects Intercepts	γ_{000}	✘1	✘1	✔3	✔3	!2	✔3	✔3	✔3	✔3	✔3	✔3	✔3
	γ_{010}	✔3	✔3	✔3	✔3	✅4	✔3	✔3	✔3	✅4	✔3	✔3	✔3
	γ_{100}	✔3	✔3	✔3	✔3	✔3	✔3	✔3	✔3	✔3	✔3	✔3	✔3
	γ_{110}	✔3	✅4	✔3	✔3	✅4	✔3	✔3	✔3	✅4	✔3	✔3	✔3
Fixed Effects Slopes	γ_{001}	✘1	✘1	✘1	✘1	✅4	✔3	!2	!2	✅4	✅4	✔3	✔3
	γ_{011}	!2	!2	!2	!2	✔3	✔3	✔3	✔3	✔3	✔3	✔3	✔3
	γ_{101}	!2	✘1	✘1	✘1	!2	✔3	✔3	✔3	✅4	✔3	✔3	✔3
	γ_{111}	!2	!2	!2	!2	✔3	✔3	✔3	!2	✔3	✔3	✔3	✔3
Residual Variance	σ_e^2	!1	!1	!1	!1	!1	!1	!1	!1	!1	!1	!1	!1
	$\sigma_{r_0}^2$	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0
	$\sigma_{r_1}^2$	✘0	✘0	✘0	✘0	✘0	✘0	✘0	!1	✘0	✘0	!1	!1
	$\sigma_{u_{00}}^2$	✔2	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0
	$\sigma_{u_{10}}^2$	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0
	$\sigma_{u_{01}}^2$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2
	$\sigma_{u_{11}}^2$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2
Residual Covariance	$\sigma_{r_0}\sigma_{r_1}$	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0
	$\sigma_{u_{00}}\sigma_{u_{01}}$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2
	$\sigma_{u_{00}}\sigma_{u_{10}}$	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0	✘0
	$\sigma_{u_{00}}\sigma_{u_{11}}$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2
	$\sigma_{u_{01}}\sigma_{u_{10}}$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2
	$\sigma_{u_{01}}\sigma_{u_{11}}$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2
	$\sigma_{u_{10}}\sigma_{u_{11}}$	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2	✔2

Note. For fixed effects intercept and slope terms; (a) white check mark with a circle around indicates all four summary statistics agreed, (b) green check mark no circle around indicated that at least three out of four summary statistics agreed, (c) yellow exclamation point indicated two of the four summary statistics agreed, (d) red cross indicates maximum of one summary statistics had acceptable values. For the variance and covariance terms; (a) green check mark no circle around indicates that at least two out of three summary statistics agrees, (b) yellow exclamation point indicated one summary statistics had acceptable values. (c) red cross meant none of the summary statistics had acceptable values.

Chapter 5

DISCUSSION

Three-level models are applicable for analyzing multilevel data, and their use is growing in educational, psychological, and social science research. HLM allows modeling of clustered data such as students' achievement scores nested in students, students in classes, and classes in schools. Although three-level HLM has become increasingly popular, there is a lack of empirically-based guidelines about sample size choices at different levels, model complexity, and how varying ICCs affect the different parameter estimates. The primary purpose of this study was to determine the impact of sample size on statistical estimates for three-level models, in particular, those used for longitudinal data structures commonly found in education. This was conducted via a two study simulation design, using data generating parameters obtained from a large scale longitudinal data set from North Carolina, provided by the National Center on Assessment and Accountability for Special Education (NCAASE). The following variables were modified: the model complexity, the ICC, and the sample size at L2, and L3 to answer the following four questions;

1. How does each of the modified factors (model complexity, ICC, L2, and L3 sizes) affect the intercept estimates ($\gamma_{000}, \gamma_{010}, \gamma_{100},$ and γ_{110})?
2. How does each of the modified factors influence the fixed effects slope estimates ($\gamma_{001}, \gamma_{011}, \gamma_{101},$ and γ_{111})?
3. How does each of the modified factors influence the variance components (σ_e^2, $\sigma_{r_0}^2$, $\sigma_{r_1}^2$, $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$)?

4. How does each of the modified factors influence the covariance components ($\sigma_{r_0r_1}$, $\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$)?

This chapter aims to provide general guidelines for sample sizes to obtain accurate parameter estimates based on the results of the simulation study. First, I discuss the effect of model complexity on the relative bias, RMSE, absolute bias, and parameter coverage. Second, I examine the effect of varying ICCs on the relative bias, RMSE, absolute bias, and parameter coverage. Third, I discuss the effect of changing L2 sample sizes on the relative bias, RMSE, absolute bias, and parameter coverage. Fourth, I discuss the effect of varying L3 sample sizes on the relative bias, RMSE, absolute bias, and parameter coverage. Fifth, suggestions are made about how to choose adequate sample sizes for specific parameter estimates in three-level longitudinal models. Finally, the limitations and the directions for future research are discussed.

Model Complexity

The majority of simulation studies that examine two-level sample size requirements tend to focus on a single model. Moreover, model complexity is rarely of interest. In this study, I examined five three-level models with increasing complexity to determine the effect of model complexity on parameter estimates by calculating the relative bias, RMSE, absolute bias, and parameter coverage.

The results of RMSE and absolute bias were very close to each other. Because of that, the relative bias difference and RMSE values difference were calculated for each model pair, such as between L1 model and L1L2 model, for each estimated parameter while keeping ICC, L2, and L3 sample sizes the same to determine whether model complexity has an effect on the bias and accuracy of the parameter estimates. For example, the RMSE differences for σ_e^2 were calculated between L1 and L1L2 model by taking the difference of RMSE values when ICC_1, N_{L2}=10, and N_{L3} =30. This process was repeated for each ICC and sample size combinations focusing on one model pair at a time such as the L1 and L1L2 models or L1 and L1L2L3 models. The goal of this process was to determine the largest relative bias and RMSE within each model pair so that it shows us all the remaining differences between model pairs were less. As seen in Table 44, for the fixed effect intercept terms ($\gamma_{000,}$ $\gamma_{010,}$ $\gamma_{100,}$ and γ_{110}) and the L1 residual variance (σ_e^2), the difference is minimal and as low as 0.01. In other words, model complexity had minimal effect on the parameter estimates of the fixed effect intercept terms ($\gamma_{000,}$ $\gamma_{010,}$ $\gamma_{100,}$ and γ_{110}), fixed effect slope term (γ_{001}), and the L1 residual variance (σ_e^2). However, the relative bias difference or RMSE difference was considerably high for the remaining fixed effects slope terms ($\gamma_{011,}$ $\gamma_{101,}$ and γ_{111}), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance terms ($\sigma_{r_0 r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) and covariance terms ($\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}$ $\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$). The difference in relative bias and RMSE for L2 residual variance ($\sigma_{r_0}^2$) is as high as 23.70. In other words, model complexity had a relatively larger effect on the parameter estimates under the examined conditions for the remaining fixed effects slope terms ($\gamma_{011,}$ $\gamma_{101,}$ and γ_{111}), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance terms ($\sigma_{r_0 r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) and covariance terms ($\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$).

These results are similar to previous two-level studies that examined model complexity. For example, Meinck and Vandenplas (2012) found that model complexity did not affect the L1 residual variance (σ_e^2) but affected the other parameters in the two-level models they examined. Table 44 shows that model complexity has minimal effects on L1 residual variance σ_e^2 as well as the fixed effects intercept terms (γ_{000}, γ_{010}, γ_{000}, and γ_{110}) and one of the fixed effects slope terms (γ_{001}).

Table 44.

Biggest Relative Bias and RMSE Difference Based on Model Complexity

Parameter Group	Parameter	Biggest Relative Bias Difference	Biggest RMSE Difference
Fixed Effects Intercepts	γ_{000}	0.01	0.03
	γ_{001}	0.16	0.05
	γ_{010}	0.12	0.09
	γ_{011}	5.52	0.01
Fixed Effects Slopes	γ_{100}	0.01	0.02
	γ_{101}	0.79	0.03
	γ_{110}	0.23	0.01
	γ_{111}	2.31	0.01
Residual Variance	σ_e^2	0.01	0.12
	$\sigma_{r_0}^2$	2.25	23.70
	$\sigma_{r_1}^2$	0.85	0.17
	$\sigma_{u_{00}}^2$	0.55	6.76
	$\sigma_{u_{01}}^2$	1.57	0.01
	$\sigma_{u_{10}}^2$	0.56	3.42
	$\sigma_{u_{11}}^2$	1.55	0.01
Residual Covariance	$\boldsymbol{\sigma_{r_0}\sigma_{r_1}}$	1.87	0.46
	$\boldsymbol{\sigma_{u_{00}}\sigma_{u_{01}}}$	0.84	0.05
	$\boldsymbol{\sigma_{u_{00}}\sigma_{u_{10}}}$	1.08	3.45
	$\boldsymbol{\sigma_{u_{00}}\sigma_{u_{11}}}$	0.54	0.01
	$\boldsymbol{\sigma_{u_{01}}\sigma_{u_{10}}}$	0.51	0.01
	$\boldsymbol{\sigma_{u_{01}}\ \sigma_{u_{11}}}$	0.86	0.01
	$\boldsymbol{\sigma_{u_{10}}\sigma_{u_{11}}}$	0.95	0.01

The effect of model complexity on absolute bias values for the fixed effects intercept and slopes terms were minimal. However, it is important to note that the fixed effect slope terms were estimated only in model 4 and model 5 where each level had one predictor. The only difference between model 4 and model 5 was the fact that L3 residual covariance terms were not estimated. Thus, if a researcher is interested in the fixed effects intercept and slope estimates, under the examined conditions this work suggests that not estimating L3 residual covariance terms has minimal effect on the absolute bias values regardless of ICC, L2, and L3 sample sizes.

Although the effect of model complexity on fixed effects intercept and slope terms is minimal, the same pattern is not observed for the residual variance terms. Researchers should exercise caution if they are interested in the residual variance estimates of L1 (σ_e^2), L2 residual variance term ($\sigma_{r_0}^2$), and all of the L3 residual variance terms except $\sigma_{u_{11}}^2$. The effect of model complexity on L3 residual terms $\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, and $\sigma_{u_{10}}^2$ is minimal only in the ICC1 condition where L2 sample size is either 10 or 25 regardless of L3 sample sizes.

The effect of model complexity on the residual covariance terms is relatively large for two of the seven residual covariance terms, $\sigma_{r_0 r_1}$(L2 residual covariance term) and $\sigma_{u_{00}u_{10}}$ (L3 residual covariance term associated with intercept terms of π_{0ij} and π_{1ij}). The effect of model complexity for $\sigma_{u_{00}u_{10}}$ is only minimal for the L2 sample size is 10 regardless of L3 sample sizes and ICCs.

As mentioned before, the parameter coverage is only calculated for the fixed effects intercept and slope terms. The effect of model complexity on parameter coverage is minimal for the fixed effects intercept terms except for γ_{010}. There are substantial differences in the parameter coverage results from model 2 and model 3 to model 4 and model 5. The main difference between these four models is the additional predictor at L3. Thus, if a researcher is interested in the γ_{010}, under the examined conditions this work suggests that adding a predictor to L3 affects the parameter coverage results. However, further examination of this effect is needed to discover the causes of this difference and whether the same effect would have been observed with different data generating values. Although not formally tested, the magnitude of true values seemed to affect the summary statistics used in the study.

The effect of model complexity on parameter coverage is minimal for the fixed effects slope terms. However, the parameter coverage proportions were generally less than 0.95 which means the standard errors were biased downward given the parameters estimated accurately. These produce more liberal rates of Type-I error and researchers should exercise caution interpreting the standard errors for the fixed effect slope terms.

Intraclass Correlations

The difference in RMSE and the relative bias was calculated for each ICC pair such as ICC_1 (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10) and ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30) or ICC_2 and ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25) while keeping model complexity, L2 sample sizes, and L3 sample sizes constant to determine the effect of ICC on the bias and accuracy of the parameter estimates. As seen in Table 45, for the fixed effect intercept terms (γ_{000}, γ_{010}, γ_{100}, and γ_{110}) and the L1 residual variance (σ_e^2), the difference in relative bias and RMSE is minimal and as low as 0.00. However, the relative bias difference or RMSE difference is considerably higher for the fixed effects slope terms (γ_{001}, γ_{011}, γ_{101}, and γ_{111}), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance

Table 45.

Biggest Relative Bias and RMSE Difference Based on ICC

Parameter Group	Parameter	Biggest Relative Bias Difference	Biggest RMSE Difference
Fixed Effects Intercepts	γ_{000}	0.00	0.07
	γ_{001}	0.50	0.19
	γ_{010}	0.01	0.02
	γ_{011}	4.95	0.01
Fixed Effects Slopes	γ_{100}	0.00	0.02
	γ_{101}	1.95	0.16
	γ_{110}	0.24	0.00
	γ_{111}	2.11	0.00
Residual Variance	σ_e^2	0.01	0.15
	$\sigma_{r_0}^2$	1.13	2.20
	$\sigma_{r_1}^2$	2.12	0.14
	$\sigma_{u_{00}}^2$	0.47	5.22
	$\sigma_{u_{01}}^2$	1.65	0.01
	$\sigma_{u_{10}}^2$	0.47	2.63
	$\sigma_{u_{11}}^2$	1.32	0.00
Residual Covariance	$\sigma_{r_0}\sigma_{r_1}$	1.56	0.14
	$\sigma_{u_{00}}\sigma_{u_{01}}$	0.88	0.13
	$\sigma_{u_{00}}\sigma_{u_{10}}$	0.88	3.47
	$\sigma_{u_{00}}\sigma_{u_{11}}$	1.22	0.03
	$\sigma_{u_{01}}\sigma_{u_{10}}$	0.42	0.03
	$\sigma_{u_{01}}\sigma_{u_{11}}$	1.32	0.00
	$\sigma_{u_{10}}\sigma_{u_{11}}$	1.09	0.05

terms ($\sigma_{r_0 r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) and covariance terms ($\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$).

In other words, varying ICC levels had relatively larger effects on the parameter estimates under the examined conditions for the fixed effects slope terms (γ_{001}, γ_{011}, γ_{101}, and γ_{111}), L2 residual variance ($\sigma^2_{r_0}$ and $\sigma^2_{r_1}$) and covariance terms ($\sigma_{r_0 r_1}$), and L3 residual variance ($\sigma^2_{u_{00}}$, $\sigma^2_{u_{01}}$, $\sigma^2_{u_{10}}$, and $\sigma^2_{u_{11}}$) and covariance terms ($\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$) compared to the fixed effects intercept terms.

The effort to compare these findings with earlier research proved fairly unproductive since the previous studies focused on solely on two-level models. Given that, these results somewhat agree with the study conducted by Meinck and Vandenplas (2012). They concluded that more similar the units were within clusters (high ICC), the less precise the estimates were. So, increasing ICC levels introduces higher sampling errors, thus it affects the residual variance-covariance terms at L2 and L3.

The effect of varying ICC on the absolute bias is minimal for the fixed effect intercept terms and for the two of the fixed effect slope terms, γ_{011}, and γ_{111}. However, the same pattern is not observed for γ_{001} and γ_{101}. The results show that for γ_{001} and γ_{101} if the ICC at L3 is greater than the ICC at L2 keeping the L1 ICC 0.5, the absolute bias values tends to be higher.

The absolute bias values for L1 and L2 residual variance were minimally affected by the varying ICC levels. However, this is not true for the L3 residual variance terms. If researchers were interested in absolute bias and had a way to control the ICCs at different levels, under the examined conditions this work suggests it is better to have greater ICCs at L2 than at L3.

The effect of varying ICCs at parameter coverage is minimal for both fixed effect slope and intercept terms.

L2 Sample Size

The difference in RMSE and the relative bias was calculated for each L2 sample size pairs, for example, the RMSE difference between N_{L2}=10 and N_{L2}=75, while keeping model complexity, L1 sample sizes, and L3 sample sizes the same to determine the effect of L2 sample size on the bias and accuracy of the parameter estimates. For example, the relative bias values of γ_{111} under Model 2, ICC_1, L3 sample size of 30 is 1.37 when L2 sample size is 25 and 0.50 when L2 sample size is 50. The absolute value of the difference for γ_{110} is 0.87 (1.37-0.50=0.87). The same calculation was conducted for each parameter, model, ICC, L3 sample sizes and L2 pair. The highest difference in both RMSE and relative bias are presented in Table 46. As seen in this table, the difference in relative bias and RMSE for the fixed effect intercept terms (γ_{000}, γ_{010}, γ_{100}, and γ_{110}) is minimal and as low as 0.00. However, the relative bias difference or RMSE difference is considerably high for the fixed effects slope terms (γ_{001},γ_{011}, γ_{101}, and γ_{111}), L1 residual variance (σ_e^2), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance terms ($\sigma_{r_0r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) and covariance terms ($\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$). In other words, varying L2 sample size had a relatively larger effect on the parameter estimates under the examined conditions for the fixed effects slope terms (γ_{001}, γ_{011}, γ_{101}, and γ_{111}), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance terms ($\sigma_{r_0r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and

$\sigma^2_{u_{11}}$) and covariance terms ($\sigma_{u_{00}u_{01}}$, $\sigma_{u_{00}u_{10}}$, $\sigma_{u_{00}u_{11}}$, $\sigma_{u_{01}u_{10}}$, $\sigma_{u_{01}u_{11}}$, and $\sigma_{u_{10}u_{11}}$) compared to the fixed effects intercept terms (γ_{000}, γ_{010}, γ_{100}, and γ_{110}).

Table 46.

Biggest Relative Bias and RMSE Difference Based on L2 Sample Size

Parameter Group	Parameter	Biggest Relative Bias Difference	Biggest RMSE Difference
Fixed Effects Intercepts	γ_{000}	0.00	0.23
	γ_{001}	0.31	0.29
	γ_{010}	0.06	0.04
	γ_{011}	6.08	0.01
Fixed Effects Slopes	γ_{100}	0.00	0.14
	γ_{101}	1.58	0.22
	γ_{110}	0.17	0.02
	γ_{111}	2.49	0.01
Residual Variance	σ^2_e	0.02	1.01
	$\sigma^2_{r_0}$	1.75	14.58
	$\sigma^2_{r_1}$	2.44	0.54
	$\sigma^2_{u_{00}}$	0.76	4.50
	$\sigma^2_{u_{01}}$	1.97	0.01
	$\sigma^2_{u_{10}}$	0.75	2.28
	$\sigma^2_{u_{11}}$	1.61	0.00
Residual Covariance	$\sigma_{r_0}\sigma_{r_1}$	1.51	0.76
	$\sigma_{u_{00}}\sigma_{u_{01}}$	1.25	0.07
	$\sigma_{u_{00}}\sigma_{u_{10}}$	1.15	2.76
	$\sigma_{u_{00}}\sigma_{u_{11}}$	2.28	0.04
	$\sigma_{u_{01}}\sigma_{u_{10}}$	0.62	0.06
	$\sigma_{u_{01}}\ \sigma_{u_{11}}$	2.13	0.00
	$\sigma_{u_{10}}\sigma_{u_{11}}$	1.44	0.06

These findings regarding the fixed effect intercept term align with the results of previous simulation studies that focused on two-level models. Shih (2008) found that the accuracy of fixed effect intercept estimates did not change considerably across sample size conditions. Similarly, Donoghue and Jenkins' (1992) observed that adding subjects to each group did not affect fixed effects. Shih (2008) also found that the estimates for the fixed effects slope term were also unstable. However, there was only one slope term in the study of Shih (2008).

The present findings regarding the L1 residual variance term align with the two-level simulation results of Shih (2008) and Darandari (2004). They found that the accuracy of the L1 residual variance and L2 residual variance-covariance estimates did not change considerably across the varying sample size conditions they examined after the relative bias values fell below 0.10. In the current study, the relative bias values for an L1 residual variance for all of the conditions are between -.03 and 0. The variance of L1 residual variance in Shih (2008) ranged from 4 to 50, and it was set to 20 across conditions.

The results for L2 residual variance terms ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) are also very similar to the findings of Shih (2008) except for the L1L2 and L1L2 no L3 covariance models. The L2 residual variance terms for these two models consistently show an anomalous result in which as the L2 sample size increases, the relative bias, RMSE, and absolute bias values also increase. The same pattern is not observed in the L1L2L3 and L1L2L3 no L3 covariance models. The major difference between L1L2 and L1L2L3 models is the addition of an L3 predictor in the L1L2L3 model. Similarly, the major difference between L1L2 no L3 covariance and L1L2L3 no L3 covariance models is the addition of an L3 predictor in the L1L2L3 model. It is an anomaly that increasing sample size at L2 results in higher absolute bias, relative bias, and RMSE values. One potential reason for this anomaly is the addition of the L3 predictor. Although, it is not directly related to this anomaly, Mok (1995) also indicated that estimates shows strong bias when the number of observations at L1 per unit of L2 exceeded the number of units at L2. These results are an indication that more research needed to explain these different anomalies.

Keeping all else constant, increasing L2 sample sizes results in lower absolute bias values. This pattern is mostly observed in almost all of the parameters examined in this study. However, the same pattern is not observed for the two fixed effects slope terms (γ_{001} and γ_{101}), the level 2 residual variance ($\sigma^2_{r_0}$) associated with level 2 equation for π_{0ij} , the level 3 residual variance terms for $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$ and the level 3 covariance term ($\sigma_{u_{00}}\sigma_{u_{10}}$) between the L3 residual variance for $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$. The efforts to find why this is the case for these two parameters proved fairly unproductive. However, one commonality between these two parameters is that they were they were both the slope terms predicting the Level 2 intercept terms (β_{00j} and β_{10j}). But, it is not clear why the absolute values for two parameters increase as the L2 sample size increase.

In summary, increasing L2 sample sizes generally decreases the absolute bias, relative bias, and RMSE values for both fixed effects intercept and slope terms as well as residual variance and covariance terms. However, a few parameter estimates such as the L2 residual variance term ($\sigma^2_{r_0}$) did not follow this general rule. Further examination is required for better understanding of this anomaly.

L3 Sample Size

The difference in RMSE and the relative bias was calculated for each L3 sample size pair, such as the difference between N_{L3}=30 and N_{L3}=150, while keeping model complexity, ICCs, and L2 sample sizes same to determine the effect of L3 sample size on the bias and accuracy of the parameter estimates. For example, the relative bias values of γ_{011} under Model 5, ICC_1, L2 sample size of 10 is 2.20 when L3 sample size is 30 and 0.18 when L3 sample size is 100. The absolute value of the difference for γ_{110} is 2.02 (2.20-0.18=2.02). The same calculation was conducted for each parameter, model, ICC, L2 sample sizes and L3 pair. The highest difference in both RMSE and relative bias are presented in Table 47.

As seen in Table 47, the difference in relative bias and RMSE for the fixed effect intercept terms (γ_{000}, γ_{010}, γ_{100}, and γ_{110}) is minimal and as low as 0.00. However, the relative bias difference or RMSE difference is considerably higher for the fixed effects slope terms (γ_{001}, γ_{011}, γ_{101}, and γ_{111}), L1 residual variance (σ_e^2), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance terms ($\sigma_{r_0}\sigma_{r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) and covariance terms ($\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$).

In other words, varying L3 sample sizes had relatively larger effects on the parameter estimates under the examined conditions for the fixed effects slope terms (γ_{001}, γ_{011}, γ_{101}, and γ_{111}), L2 residual variance ($\sigma_{r_0}^2$ and $\sigma_{r_1}^2$) and covariance terms ($\sigma_{r_0 r_1}$), and L3 residual variance ($\sigma_{u_{00}}^2$, $\sigma_{u_{01}}^2$, $\sigma_{u_{10}}^2$, and $\sigma_{u_{11}}^2$) and covariance terms ($\sigma_{u_{00}}\sigma_{u_{01}}$, $\sigma_{u_{00}}\sigma_{u_{10}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{10}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$) compared to the fixed effects intercept terms (γ_{000}, γ_{010}, γ_{100}, and γ_{110}).

Table 47.

Biggest Relative Bias and RMSE Difference Based on L3 Sample Size

Parameter Group	Parameter	Biggest Relative Bias Difference	Biggest RMSE Difference
Fixed Effects Intercepts	γ_{000}	0.00	0.21
	γ_{001}	0.30	0.30
	γ_{010}	0.05	0.04
	γ_{011}	4.79	0.01
Fixed Effects Slopes	γ_{100}	0.00	0.12
	γ_{101}	1.20	0.22
	γ_{110}	0.17	0.01
	γ_{111}	2.45	0.01
Residual Variance	σ_e^2	0.02	0.85
	$\sigma_{r_0}^2$	1.58	11.85
	$\sigma_{r_1}^2$	2.15	0.50
	$\sigma_{u_{00}}^2$	0.52	2.83
	$\sigma_{u_{01}}^2$	1.23	0.01
	$\sigma_{u_{10}}^2$	0.53	1.44
	$\sigma_{u_{11}}^2$	1.01	0.00
Residual Covariance	$\sigma_{r_0}\sigma_{r_1}$	1.39	0.66
	$\sigma_{u_{00}}\sigma_{u_{01}}$	0.85	0.07
	$\sigma_{u_{00}}\sigma_{u_{10}}$	0.67	1.59
	$\sigma_{u_{00}}\sigma_{u_{11}}$	1.77	0.04
	$\sigma_{u_{01}}\sigma_{u_{10}}$	0.49	0.04
	$\sigma_{u_{01}}\sigma_{u_{11}}$	1.26	0.00
	$\sigma_{u_{10}}\sigma_{u_{11}}$	1.02	0.03

The difference in the accuracy of estimation between the fixed effects intercept and slope term might be related to the fact that estimating slope parameters is harder than intercept parameters since the estimator of the intercept parameter is required in the estimation of the slope parameter. These differences might also be related to the fact that many of the fixed effect slope terms also represent cross-level interactions and varying sample sizes at these levels might cause the unstable parameter estimates.

Similar to the results discussed under L2 sample size, the absolute bias values generally drop as the L3 sample size increases for almost all of the parameter estimates except for the L2 residual variance term, $\sigma^2_{r_0}$. But, it is not clear why the absolute values for $\sigma^2_{r_0}$ parameters increase as the L3 sample size increase.

To summarize, as L3 sample sizes increase, generally the absolute bias, relative bias, and RMSE values for both fixed effects intercept and slope terms as well as residual variance and covariance terms decrease. Although majority of the parameters follow this pattern, a few exceptions exist such as the L2 residual variance term ($\sigma^2_{r_0}$) that do not follow this general pattern. Follow up analysis is required to better understand this issue.

Adequate Sample Sizes for Fixed Effect Intercept Terms

Considering all of the summary statistics examined (absolute bias, relative bias, RMSE, and coverage), if researchers are interested in fixed effect intercept terms regardless of varying ICCs and model complexity, this work suggests sample size combinations as small as 3/10/30 (N_{L1}=3, N_{L2}=10, N_{L3}=30) can result in relatively accurate and precise parameter estimates. Although the absolute bias and RMSE values are slightly higher for γ_{000} compared to the remaining fixed effects intercept terms (γ_{010}, γ_{100}, and γ_{110}), it is important to keep in mind that the magnitude of the data generating parameters for γ_{000} are relatively higher compared to the remaining fixed effects intercept parameters. It is known that absolute bias and RMSE values are sensitive to the magnitude of the data generating parameter. In almost all instances examined in the current study, estimating or not estimating L3 residual covariance had minimal effects on parameter accuracy and precision, so it is up to the researcher to make the decision of estimating the L3 residual covariance terms in the model given the minimal effect of not doing so.

Adequate Sample Sizes for Fixed Effect Slope Terms

The fixed effect slope terms require slightly larger sample sizes since estimating slope parameters are harder than that of the intercept parameter because the estimator of the intercept parameter is required in the estimation of the slope parameter. Given that and considering all the summary statistics used, if researchers are interested in fixed effects slope terms, regardless of varying ICCs and model complexity, this work suggests sample size combinations as small as 3/10/100 (N_{L1}=3, N_{L2}=10, N_{L3}=100) can still result in relatively accurate parameter estimates. It is also important to note that as expected increasing L3 sample sizes resulted in better accuracy and precision values but increasing L2 sample sizes did not. This finding aligns with Mok (1995)'s suggestions that increasing the higher level sample size is better than increasing the lower level sample sizes. Although Mok (1995) studied two level models, the same principles apply here. This is especially important for educational researchers that collect data from schools and students. In a typical school, where there are usually more students in a classroom than the number of classrooms in the school. It is easier and cost effective to collect more data from the same school compared to finding more schools and collecting data. However, it is important to note that increasing L2 sample size rather than L3 sample size adds additional bias and lowers the precision for fixed effects slope terms. Therefore, it is imperative that researchers be cautious about the bias and precision issues in their research design when designing educational studies. Although it might make more sense and be more cost effective to sample more students from a given classroom, instead of recruiting more classrooms for the study, bias in the parameter estimates might result in erroneous conclusions.

Overall, the absolute bias, relative bias, and RMSE values for fixed effects terms are better than residual variance-covariance terms. However, it is important to point out that determining the type of fixed effects that researchers are interested in is an integral part of making sample size decisions. In general, fixed effects intercept terms have lower absolute bias, relative bias, and RMSE values compared to the fixed effects slope terms. Given that, fixed effects intercept terms do not require sample sizes as high as fixed effects slope terms.

Adequate Sample Sizes for Residual Variance Terms

The relative bias values of L1 residual variance term (σ_e^2) are all within acceptable range and the absolute value of relative bias values are all less than 0.04 regardless of ICCs, model complexity, L2, and L3 sample sizes. In other words, if the researchers are interested in relative bias of σ_e^2, this work suggests sample size combinations as small as 3/10/30 (N_{L1}=3, N_{L2}=10, N_{L3}=30) can result in relatively low relative bias values. This finding aligns with the two-level literature. Maas and Hox (2005) pointed out that the L1 residual variance estimates were generally very accurate. However, if the researchers are interested in absolute bias and RMSE values close to 0, larger sample sizes are needed. The lowest observed absolute bias and RMSE values for σ_e^2 are ranged from 0.19 to 0.25 under the highest L2 and L3 sample sizes examined in the current study 3/75/150 (N_{L1}=3, N_{L2}=75, N_{L3}=150). Given the highest L2 and L3 sample size examined did not result in absolute bias and RMSE values close to 0 and the absolute bias and RMSE values decrease as both L2 and L3 sample sizes increase, this work suggests even larger sample sizes than the ones examined in this study are required to obtain absolute bias and RMSE values close to 0.

The relative bias results for the L2 intercept residual variance ($\sigma^2_{r_0}$) are similar to the L1 residual terms for the three out of five models; (a) L2, (b) L1L2L3, and (c) L1L2L3 no L3 residual covariance models regardless of ICCs, L2, and L3 sample sizes. Consequently, researchers can choose a sample size combination as small as 3/10/30 (N_{L1}=3, N_{L2}=10, N_{L3}=30) to obtain acceptable relative bias values for L2 intercept residual variance ($\sigma^2_{r_0}$) estimates. However, the same is not true for the remaining two models; (a) L1L2 and (b) L1L2 no L3 residual covariance models. The results for these two models are unstable, and none of the study conditions consistently produces relative bias values within acceptable range. This is an anomaly that L1L2 model produces higher bias values considering that L1L2 is a less complicated model compared to L1L2L3 model. It appears though adding an L3 predictor helped reduce the relative bias values for $\sigma^2_{r_0}$. The lowest absolute bias and RMSE values for $\sigma^2_{r_0}$ are ranged from 0.31 to 0.50 under the highest L2 and L3 sample sizes examined in the current study 3/75/150 (N_{L1}=3, N_{L2}=75, N_{L3}=150) under L1, L1L2L3, and L1L2L3 no L3 covariance models. In other words, the highest L2 and L3 sample size examined in the current study did not result in absolute bias and RMSE values close to 0. However, one conclusion of the current study is that the absolute bias and RMSE values decrease as both L2 and L3 sample sizes increase for $\sigma^2_{r_0}$. As a result of this observation, this work suggests even larger sample sizes than the ones examined in this study are required to obtain absolute bias and RMSE values close to 0 for $\sigma^2_{r_0}$. Model complexity and shifting variability between L2 and L3 does not seem to effect the sample size choices at both L2 and L3 for $\sigma^2_{r_0}$.

Unlike for $\sigma^2_{r_0}$, the relative bias values for $\sigma^2_{r_1}$ are generally not within acceptable ranges. On the other hand, the absolute bias and RMSE values for $\sigma^2_{r_1}$ are relatively small compared to the $\sigma^2_{r_0}$. As more of shared variability shifts from L2 to L3 (in terms of the ICCs), the relative bias values for $\sigma^2_{r_1}$ increase. Although, ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) produced the lowest relative bias values followed by ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30), the examined sample size conditions did not result in acceptable relative bias values for $\sigma^2_{r_1}$. However, as L2 and L3 sample sizes increase, the relative bias values for $\sigma^2_{r_1}$ decrease. In other words, if researchers are interested in relative bias values for $\sigma^2_{r_1}$, they need larger sample sizes than the 3/75/150 (N_{L1}=3, N_{L2}=75, N_{L3}=150). The anomaly observed in $\sigma^2_{r_0}$ regarding the increase in sample size results in increase in relative bias values under L1L2 model was not observed for $\sigma^2_{r_1}$. Similar to the relative bias values for $\sigma^2_{r_1}$, if researchers are interested in absolute bias or RMSE values for $\sigma^2_{r_1}$, they need larger sample sizes than the 3/75/150 (N_{L1}=3, N_{L2}=75, N_{L3}=150). These findings were somewhat consistent with two-level models literature. Mok (1995), Clarke and Wheaton (2007), Maas and Hox (2005) found that L2 variance components were sometimes underestimated.

L3 residual variance terms $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$ were the only two L3 variance terms estimated in all models. The relative bias values were the only statistics that fell within the acceptable range for all the ICCs, and L2 and L3 sample sizes for the L1 model. Consequently, for L1 model (time as L1 predictor) researchers can choose a sample size combination as small as 3/10/30 (N_{L1}=3, N_{L2}=10, N_{L3}=30) to obtain acceptable relative bias values for L3 intercept residual variance $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$ terms. For other models, the majority of the relative bias values were not within acceptable range. Similarly, the absolute bias and RMSE values were relatively higher and not close to 0. There was not a clear pattern observed regarding the minimum sample size for acceptable relative bias given the examined models, ICCs, L2 and L3 sample sizes.

$\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$ are the other two L3 residual variance terms. They are estimated in four out of five models. L1 model is the only model that they were not estimated. Similar to the $\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$, the majority of the relative bias values were not within acceptable range. However, the absolute bias and RMSE bias values were all very close to 0 for $\sigma^2_{u_{01}}$ and $\sigma^2_{u_{11}}$. Consequently, if researchers are interested in absolute bias and RMSE values, this study suggests that they can choose a sample size combination as small as 3/10/30 (N_{L1}=3, N_{L2}=10, N_{L3}=30) to obtain relatively small absolute bias and RMSE values for L3 residual variances ($\sigma^2_{u_{00}}$ and $\sigma^2_{u_{10}}$).

Adequate Sample Sizes for Residual Covariance Terms

The majority of the relative bias values for $\sigma_{r_0}\sigma_{r_1}$ are not within acceptable range and, there is not a clear pattern observed regarding the minimum sample size for acceptable relative bias given the examined models, varying ICCs, L2 and L3 sample sizes. Similarly, there is not a clear pattern observed for the absolute bias and RMSE values. Absolute bias and RMSE values are very high and not close to 0. On the other hand, keeping L2 sample size constant and increasing the L3 sample sizes generally lowers the relative bias estimates for $\sigma_{r_0}\sigma_{r_1}$. Similarly, keeping L3 sample size constant and increasing the L2 sample sizes generally lowers the relative bias estimates for $\sigma_{r_0}\sigma_{r_1}$. These findings were somewhat consistent with two-level models literature. Mok (1995), Clarke and Wheaton (2007), Maas and Hox (2005) found that L2 variance components were sometimes underestimated. As more of shared variability shifts from L2 to L3 (in terms of the ICCs), the relative bias values for $\sigma_{r_0}\sigma_{r_1}$ increase. ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10) produced the lowest relative bias values followed by ICC_4 (ICC_{L1}=0.50, ICC_{L2}=0.30, ICC_{L3}=0.20), ICC_3 (ICC_{L1}=0.50, ICC_{L2}=0.25, ICC_{L3}=0.25), and ICC_2 (ICC_{L1}=0.50, ICC_{L2}=0.20, ICC_{L3}=0.30). Given that, if researchers are interested in relative bias values and have ICC values similar to the ICC_1, (ICC_{L1}=0.50, ICC_{L2}=0.40, ICC_{L3}=0.10), they can choose a sample size combination as small as 3/50/30 (N_{L1}=3, N_{L2}=50, N_{L3}=30) to obtain relatively small relative bias values for L2 residual covariance ($\sigma_{r_0}\sigma_{r_1}$) under L1 model. Under ICC_1 and L1L2L3 model, they can choose a sample size combination as small as 3/75/30 (N_{L1}=3, N_{L2}=50, N_{L3}=30) to obtain relatively small relative bias values. Researchers need to pick sample sizes of at least 50 for L2 and 100 for L3 to obtain relatively small relative bias values under ICC_1 and the L1L2L3 no L3 residual covariance model. On the other hand, for ICC_3 and ICC_4 conditions under L1 model, researchers need lower sample sizes for L2, at least 25 for L2 and at least 100 for L3 to obtain relatively small relative bias values. ICC_2 has the lowest ICC at L2 and highest ICC at L3 compared to the other ICCs examined in this study. The conditions under L1 model for ICC_2 requires sample size of at least 50 for L2 and at least 100 for L3 to obtain relatively small relative bias values.

Under L1L2L3 model for ICC_1 conditions, researchers need a sample size of at least 50 for L2 and at least 100 for L3 to obtain relatively small relative bias values. On the other hand, ICC_2, ICC_3, and ICC4 under L1L2L3 model requires at least 75 for L2 and 150 for L3 to obtain relatively small relative bias values.

If the researchers are interested in absolute bias and RMSE values for L2 residual covariance ($\sigma_{r_0}\sigma_{r_1}$) term, none of the sample sizes combinations examined in this study provided acceptable absolute bias and RMSE values. However, as L2 and L3 sample sizes increase, the absolute bias and RMSE values decrease. So researchers need sample sizes larger than N_{L2}=75 and N_{L3}=150.

The absolute bias and RMSE values for L3 residual covariance terms ($\sigma_{u_{01}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$) are all very small and close to 0. Consequently, if researchers are interested in absolute bias and RMSE values, this study suggests that they can choose a sample size combination as small as 3/10/30 (N_{L1}=3, N_{L2}=10, N_{L3}=30) to obtain relatively small absolute bias and RMSE values for L3 residual covariance terms ($\sigma_{u_{01}}\sigma_{u_{11}}$, $\sigma_{u_{01}}\sigma_{u_{11}}$, $\sigma_{u_{00}}\sigma_{u_{11}}$, $\sigma_{u_{00}}\sigma_{u_{01}}$, and $\sigma_{u_{10}}\sigma_{u_{11}}$).

$\sigma_{u_{01}}\sigma_{u_{10}}$ was the only L3 residual covariance term that did not have absolute bias and RMSE values close to 0 under the examined conditions. If researchers are interested in absolute bias and RMSE values, this study suggests that they need to choose a sample size combination that is greater than 3/75/150 (N_{L1}=3, N_{L2}=75, N_{L3}=150).

Generally, the relative bias values for all of the L3 residual covariance terms are not within acceptable range, and they do not follow any particular pattern. This work suggests that researchers who are interested in the relative bias values for the L3 residual covariance terms should exercise caution as the results might misguide their interpretations.

Follow-up Analysis

It is an anomaly that for some of the estimated parameters such as γ_{010}, the calculated statistics (absolute bias, relative bias, and RMSE values) increase as L2 and L3 sample sizes increase. To better understand the issue, three different follow-up analyses were conducted using L1L2 model and 50 replications. In the first follow-up analysis, the L2 sample size was increased to 500, and L3 sample size increased to 750, providing 1,125,000 (3*500*750) data points to check whether increasing the sample size resulted in a decrease in the calculated statistics. Unfortunately, it did not reduce the calculated statistics. Next, given that some of the calculated statistics are sensitive to the magnitude of the data generating parameter, the data generating value for γ_{010} was increased from 0.7330 to 1.7330. Again, it did not reduce the calculated statistics. Lastly, the total variance in the models increased tenfold (from 40 to 400) to check whether the total variance was a reason for the anomaly. Unfortunately, increasing the total variance did not change the observed pattern in the calculated statistics.

At this point, it is unclear why increasing sample size results in increased relative bias, absolute bias, or RMSE values for some parameters and not others. Hence further explorations are warranted. Potential areas of explorations include the varying L1 sample sizes and the correlation between the outcome variable, L2, and L3 predictors. The typical L1 sample size (number of measurement occasions) of three, five, and ten has been found in the literature for typical longitudinal designs (Kwok, West, & Green, 2007). In this study, 3 was the only L1 sample size examined. This low sample size at L1 might offer a potential explanation for the anomaly. Another area of exploration involves varying the strength of the relationship between the outcome variable (math achievement score) and L2 predictor (average student reading score). The correlation between the outcome variable and the L2 predictor was about 0.75. This very high correlation maybe contributing to the anomaly. Consequently, exploration of alternative predictors may result in differing findings.

Limitations and Need for Further Research

Although this simulation study was complex, it was not exhaustive with respect to all relevant models, ICCs, and sample size choices. As a result, several limitations in this study are identified, some of which are related to the software used, while others are reflective of the design of the current study. For example, despite the vast usage of HLM 7 software in educational research, a major limitation of this modeling procedure is that for three-level models, the only available estimation algorithm available in HLM7 is the maximum likelihood algorithm (ML). ML estimates can be heavily biased for small samples. In other words, the optimality properties of ML might not apply for small sample sizes. As a result, one of the major limitations of the current study was the use of ML to obtain parameter estimates as the only method. Thus, future researchers may consider exploring the strengths and weaknesses of different estimation algorithms (e.g., restricted maximum likelihood, Bayesian, etc.) to produce unbiased or accurate parameter estimates, especially under small sample size conditions.

Additionally, the generalizability of the results to other settings that utilize three-level longitudinal models might be limited since the data generating values in the current study were obtained using a single large-scale educational dataset from North Carolina. For example, the correlation between the outcome and L2 predictor in the North Carolina data set was about 0.75 which might not be a typical correlation in an applied setting. Furthermore. the dataset only included one cohort of students (i.e., grade three to five). So, the results for the other grade levels might potentially differ. Additionally, the data from North Carolina included different numbers of students per school which made it unbalanced in nature. However, the current study was designed to be balanced whereby only the conditions where the number of students per school was equal across different conditions were examined. It is typical in applied studies to have unbalanced designs so examining only the balanced design conditions also limits the generalizability of the results.

As mentioned in the previous paragraph, using one set of data generating values limits the generalizability of results since varying the data generating values might lead to potentially different results. One potential solution to this issue is using standardized coefficients to generate data. There are two ways to standardize predictors; (a) group-level standardization (using each group's own mean and standard deviation) and (b) overall standardization (grand mean and standard deviation). However, these two standardization options and how they affect the interpretation has not yet been explored in the literature. Consequently, this study did not use any standardization.

Additionally, more time points at L1 would have been helpful in the parameter estimates. However, since the educational studies mainly use three-time points at L1, this study only focused on three-time points at L1. Future researchers might explore how additional time points (five or ten) at L1 affect the parameter estimates and its relation to sample size.

In the current study, five models were examined. However, none of the models examined had more than one predictor at each level. Having more than one predictor at a level might introduce within-level interactions and more cross-level interactions. Thus, it increases the model complexity. Future researchers might explore how introducing more than one predictor at each level affect the parameter estimates and its relation to sample size.

It is also very common in applied studies to fix one or more residual variance terms at L2 or L3 to zero, which eventually effects the model complexity. However, those models were not examined in this study. Fixing the residual variance terms might potentially impact the accuracy of parameter estimates. It is an area of exploration for future research.

Guided by the previous simulation studies in two-level literature, this study only used a maximum of one predictor at each level (total of three predictors). Examining fewer number of predictors is another limitation. In applied studies researchers typically have more predictors. For example, Subedi, Reese, and Powell (2015) used a total of 13 student level predictors and 2 teacher level predictors. Adding more predictors increases the number of estimated parameters and as a result increases model complexity. As mentioned above, the within-level interactions were not examined in this study.

This study only focused on sample size estimation for accurate parameter estimates and did not focus on obtaining specific power levels. Because of that, the suggestions in this study only based on obtaining accurate parameter estimates. Although, sample size estimation to obtain a specific power level has a different focus than accurate parameter estimation, considering both during sample size planning likely would result in better understanding of the effect that the researcher is interested in. In other words, sample size estimation for power and parameter accuracy complement each other and provides better explanations for the examined effect in focus.

The correct model was fit to the data generating model for each of the examined models. In other words, no misspecification was introduced in this study. Introducing misspecification potentially impacts the parameter estimates and standard errors. In turn, it might affect the sample size choices. Future researchers might explore how introducing misspecification affect the parameter estimates and its relation to sample size.

The discussion section was only limited to the available two-level sample size studies since there were no available three-level studies. Further investigations of three-level models may show whether the results hold true for distributions of the dependent and predictor variables other than the ones explored in this study.

Lastly, the current study did not explore the impact of different missing data mechanisms on the parameter estimates and the sample size suggestions. It is possible that different missing data mechanisms impact the parameter estimates and sample size suggestions.

Conclusion

To date, this work is the first to investigate the sample size requirements for three-level longitudinal models. The results indicate that sample size requirements for three-level longitudinal models are tied to the parameters of interest, ICC, and model complexity. However, the fixed effect intercepts parameters are estimated with highest accuracy followed by fixed effects slope terms. The variance-covariance terms generally required larger sample sizes than the ones examined in the current study. Given these results, it is recommended that researchers need to identify the possible ICC levels observed in literature and be clear about their research questions which in turn shapes their model and the potential parameters of interest.

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APPENDIX A

TABLE OF ABSOLUTE BIAS VALUES

Table A1.

Absolute Bias Values of γ_{000}

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.27	0.23	0.25	0.25	0.25	0.23	0.24	0.27	0.28	0.24	0.25	0.25
25	30	0.17	0.15	0.15	0.17	0.17	0.15	0.15	0.16	0.18	0.15	0.16	0.16
50	30	0.12	0.11	0.11	0.12	0.14	0.10	0.10	0.11	0.12	0.11	0.10	0.11
75	30	0.09	0.08	0.09	0.09	0.10	0.08	0.10	0.09	0.10	0.09	0.09	0.09
10	100	0.15	0.13	0.13	0.14	0.15	0.13	0.14	0.14	0.15	0.14	0.13	0.14
25	100	0.09	0.08	0.08	0.08	0.10	0.08	0.08	0.09	0.10	0.08	0.08	0.09
50	100	0.06	0.06	0.06	0.06	0.07	0.06	0.06	0.07	0.06	0.06	0.06	0.06
75	100	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05
10	150	0.12	0.10	0.10	0.11	0.12	0.11	0.11	0.12	0.13	0.12	0.12	0.12
25	150	0.07	0.07	0.07	0.07	0.08	0.07	0.07	0.07	0.08	0.06	0.07	0.07
50	150	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.05
75	150	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.04	0.04	0.04

Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.27	0.24	0.24	0.25	0.28	0.24	0.25	0.27
25	30	0.17	0.15	0.15	0.16	0.18	0.16	0.15	0.16
50	30	0.11	0.10	0.11	0.11	0.12	0.10	0.11	0.11
75	30	0.09	0.09	0.09	0.10	0.10	0.08	0.09	0.09
10	100	0.14	0.13	0.14	0.15	0.14	0.13	0.15	0.15
25	100	0.09	0.08	0.09	0.09	0.09	0.08	0.09	0.09
50	100	0.07	0.06	0.06	0.06	0.07	0.06	0.06	0.06
75	100	0.06	0.05	0.05	0.05	0.05	0.04	0.05	0.05
10	150	0.12	0.11	0.11	0.11	0.12	0.10	0.12	0.12
25	150	0.08	0.07	0.07	0.08	0.08	0.06	0.07	0.07
50	150	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05
75	150	0.05	0.04	0.04	0.05	0.04	0.04	0.04	0.04

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A2.

Absolute Bias Values of γ_{010}

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.043	0.041	0.044	0.044	0.031	0.028	0.030	0.028
25	30	0.038	0.040	0.039	0.038	0.018	0.017	0.018	0.017
50	30	0.042	0.038	0.042	0.041	0.013	0.010	0.013	0.013
75	30	0.048	0.046	0.048	0.044	0.010	0.009	0.010	0.010
10	100	0.039	0.039	0.040	0.042	0.017	0.015	0.015	0.017
25	100	0.048	0.044	0.045	0.044	0.010	0.009	0.010	0.010
50	100	0.059	0.060	0.058	0.058	0.007	0.006	0.007	0.006
75	100	0.071	0.071	0.069	0.071	0.006	0.005	0.005	0.006
10	150	0.041	0.043	0.042	0.042	0.013	0.012	0.012	0.013
25	150	0.051	0.053	0.056	0.051	0.008	0.007	0.007	0.008
50	150	0.069	0.070	0.069	0.071	0.006	0.005	0.005	0.006
75	150	0.084	0.081	0.085	0.082	0.005	0.004	0.004	0.004
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.044	0.041	0.040	0.042	0.031	0.027	0.029	0.030
25	30	0.040	0.036	0.038	0.039	0.019	0.016	0.018	0.017
50	30	0.041	0.041	0.042	0.044	0.015	0.012	0.011	0.012
75	30	0.047	0.049	0.046	0.046	0.011	0.010	0.010	0.010
10	100	0.040	0.040	0.039	0.041	0.016	0.015	0.015	0.016
25	100	0.047	0.050	0.049	0.048	0.011	0.010	0.009	0.010
50	100	0.059	0.058	0.064	0.059	0.007	0.006	0.006	0.007
75	100	0.071	0.069	0.072	0.067	0.006	0.005	0.005	0.006
10	150	0.042	0.046	0.041	0.040	0.014	0.012	0.013	0.014
25	150	0.054	0.052	0.053	0.056	0.009	0.007	0.007	0.008
50	150	0.069	0.067	0.067	0.067	0.006	0.005	0.006	0.006
75	150	0.080	0.080	0.082	0.083	0.005	0.004	0.004	0.004

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A3.

Absolute Bias Values of γ_{100}

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.14	0.15	0.14	0.15	0.14	0.15	0.14	0.15	0.16	0.15	0.15	0.16
25	30	0.10	0.10	0.09	0.10	0.10	0.09	0.09	0.10	0.10	0.10	0.09	0.09
50	30	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
75	30	0.05	0.05	0.05	0.05	0.06	0.05	0.06	0.06	0.06	0.05	0.06	0.06
10	100	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.08	0.09
25	100	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
50	100	0.04	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
75	100	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
10	150	0.07	0.06	0.07	0.06	0.07	0.06	0.07	0.06	0.07	0.07	0.07	0.07
25	150	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
50	150	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
75	150	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.03	0.03	0.03	0.02	0.03

Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.14	0.15	0.15	0.15	0.16	0.15	0.16	0.16
25	30	0.10	0.09	0.09	0.10	0.09	0.10	0.09	0.09
50	30	0.06	0.07	0.07	0.07	0.07	0.07	0.06	0.06
75	30	0.05	0.05	0.06	0.06	0.06	0.05	0.06	0.05
10	100	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.09
25	100	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
50	100	0.04	0.04	0.04	0.03	0.04	0.04	0.04	0.04
75	100	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
10	150	0.07	0.06	0.07	0.07	0.07	0.06	0.07	0.07
25	150	0.04	0.04	0.04	0.05	0.05	0.04	0.04	0.04
50	150	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
75	150	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.02

Note. *L1 sample size is 3. ICC$_1$, (ICC$_{L1}$=0.5, ICC$_{L2}$=0.40, ICC$_{L3}$=0.10). ICC$_2$ (ICC$_{L1}$=0.5, ICC$_{L2}$=0.20, ICC$_{L3}$=0.30). ICC$_3$ (ICC$_{L1}$=0.5, ICC$_{L2}$=0.25, ICC$_{L3}$=0.25). ICC$_4$ (ICC$_{L1}$=0.5, ICC$_{L2}$=0.30, ICC$_{L3}$=0.20)

Table A4.

Absolute Bias Values of γ_{110}

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.018	0.018	0.018	0.018	0.018	0.018	0.018	0.016
25	30	0.012	0.012	0.011	0.011	0.011	0.011	0.011	0.010
50	30	0.008	0.008	0.008	0.008	0.007	0.007	0.008	0.007
75	30	0.008	0.007	0.007	0.007	0.006	0.006	0.006	0.006
10	100	0.011	0.010	0.011	0.010	0.010	0.009	0.009	0.010
25	100	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.006
50	100	0.006	0.006	0.006	0.006	0.004	0.004	0.004	0.004
75	100	0.005	0.005	0.005	0.005	0.003	0.003	0.003	0.003
10	150	0.009	0.009	0.009	0.009	0.008	0.008	0.007	0.008
25	150	0.006	0.007	0.007	0.006	0.005	0.005	0.004	0.005
50	150	0.005	0.005	0.005	0.005	0.003	0.003	0.003	0.003
75	150	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.018	0.019	0.018	0.018	0.019	0.017	0.019	0.018
25	30	0.012	0.011	0.011	0.012	0.011	0.011	0.011	0.010
50	30	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.007
75	30	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.006
10	100	0.010	0.010	0.010	0.011	0.009	0.010	0.010	0.009
25	100	0.007	0.007	0.007	0.007	0.006	0.006	0.006	0.006
50	100	0.006	0.005	0.006	0.005	0.004	0.004	0.004	0.004
75	100	0.005	0.005	0.005	0.005	0.003	0.003	0.003	0.003
10	150	0.009	0.009	0.009	0.009	0.008	0.008	0.008	0.008
25	150	0.006	0.006	0.006	0.006	0.005	0.005	0.005	0.005
50	150	0.006	0.005	0.005	0.005	0.003	0.003	0.003	0.003
75	150	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A5.

Absolute Bias Values of γ_{001}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.13	0.17	0.17	0.15	0.14	0.18	0.16	0.15
25	30	0.17	0.26	0.23	0.21	0.17	0.23	0.23	0.23
50	30	0.21	0.34	0.32	0.29	0.23	0.37	0.33	0.31
75	30	0.26	0.40	0.39	0.34	0.24	0.39	0.38	0.38
10	100	0.07	0.09	0.09	0.08	0.07	0.10	0.09	0.07
25	100	0.09	0.13	0.13	0.12	0.09	0.13	0.12	0.11
50	100	0.12	0.19	0.16	0.15	0.11	0.18	0.18	0.15
75	100	0.13	0.23	0.19	0.18	0.14	0.22	0.20	0.19
10	150	0.05	0.07	0.07	0.06	0.06	0.08	0.07	0.07
25	150	0.08	0.11	0.10	0.09	0.07	0.11	0.10	0.10
50	150	0.09	0.15	0.14	0.12	0.09	0.15	0.14	0.13
75	150	0.12	0.18	0.17	0.14	0.11	0.18	0.17	0.14

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A6.

Absolute Bias Values of γ_{011}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.011	0.012	0.011	0.011	0.010	0.011	0.011	0.011
25	30	0.012	0.012	0.012	0.012	0.011	0.012	0.013	0.012
50	30	0.013	0.015	0.014	0.014	0.013	0.014	0.015	0.013
75	30	0.013	0.016	0.016	0.015	0.013	0.017	0.015	0.016
10	100	0.006	0.005	0.006	0.006	0.006	0.006	0.005	0.006
25	100	0.006	0.007	0.007	0.006	0.006	0.006	0.007	0.007
50	100	0.007	0.008	0.008	0.007	0.007	0.008	0.008	0.007
75	100	0.007	0.009	0.008	0.008	0.007	0.009	0.009	0.008
10	150	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.004
25	150	0.005	0.005	0.006	0.005	0.005	0.005	0.005	0.005
50	150	0.006	0.006	0.006	0.006	0.005	0.006	0.006	0.006
75	150	0.006	0.008	0.007	0.006	0.006	0.007	0.007	0.007

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A7.

Absolute Bias Values of γ_{101}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.08	0.13	0.11	0.11	0.09	0.12	0.12	0.10
25	30	0.11	0.17	0.16	0.15	0.11	0.18	0.16	0.16
50	30	0.15	0.25	0.21	0.19	0.15	0.25	0.21	0.22
75	30	0.18	0.30	0.27	0.24	0.18	0.29	0.26	0.24
10	100	0.04	0.07	0.06	0.05	0.04	0.06	0.06	0.05
25	100	0.06	0.09	0.08	0.08	0.06	0.09	0.09	0.08
50	100	0.08	0.13	0.12	0.11	0.08	0.13	0.12	0.11
75	100	0.09	0.17	0.15	0.13	0.10	0.17	0.14	0.13
10	150	0.03	0.05	0.05	0.05	0.04	0.05	0.05	0.05
25	150	0.05	0.08	0.07	0.07	0.05	0.08	0.07	0.07
50	150	0.06	0.10	0.10	0.09	0.06	0.10	0.10	0.09
75	150	0.08	0.13	0.12	0.10	0.08	0.13	0.12	0.10

Note. *L1 sample size is 3. L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A8.

Absolute Bias Values of γ_{111}

Sample Size*			L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3		ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30		0.006	0.008	0.007	0.007	0.006	0.007	0.007	0.007
25	30		0.007	0.008	0.008	0.007	0.007	0.008	0.007	0.007
50	30		0.007	0.010	0.009	0.009	0.007	0.009	0.009	0.008
75	30		0.008	0.011	0.010	0.009	0.008	0.011	0.010	0.009
10	100		0.003	0.004	0.004	0.003	0.003	0.004	0.004	0.003
25	100		0.004	0.004	0.004	0.004	0.003	0.004	0.004	0.004
50	100		0.004	0.005	0.005	0.005	0.004	0.005	0.005	0.005
75	100		0.004	0.005	0.005	0.005	0.004	0.006	0.005	0.005
10	150		0.003	0.003	0.003	0.003	0.002	0.003	0.003	0.003
25	150		0.003	0.004	0.003	0.003	0.003	0.003	0.003	0.003
50	150		0.003	0.004	0.004	0.004	0.003	0.004	0.004	0.004
75	150		0.003	0.005	0.004	0.004	0.003	0.005	0.004	0.004

Note. *L1 sample size is 3. L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A9.

Absolute Bias Values of σ_e^2

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.06	1.17	1.15	1.13	1.15	1.12	1.19	1.16	1.15	1.08	1.13	1.07
25	30	0.74	0.67	0.72	0.70	0.73	0.73	0.71	0.68	0.71	0.68	0.72	0.70
50	30	0.52	0.49	0.53	0.52	0.49	0.52	0.56	0.51	0.52	0.54	0.51	0.52
75	30	0.44	0.42	0.40	0.41	0.40	0.40	0.39	0.41	0.44	0.42	0.42	0.43
10	100	0.62	0.58	0.61	0.63	0.66	0.62	0.60	0.63	0.65	0.61	0.62	0.62
25	100	0.40	0.38	0.38	0.41	0.39	0.39	0.41	0.41	0.41	0.42	0.40	0.38
50	100	0.30	0.28	0.30	0.28	0.29	0.30	0.30	0.31	0.32	0.29	0.28	0.30
75	100	0.25	0.24	0.23	0.22	0.24	0.23	0.24	0.25	0.24	0.24	0.25	0.24
10	150	0.52	0.50	0.53	0.52	0.51	0.52	0.52	0.49	0.51	0.48	0.49	0.50
25	150	0.32	0.31	0.32	0.33	0.35	0.32	0.35	0.33	0.35	0.34	0.33	0.34
50	150	0.23	0.24	0.25	0.25	0.24	0.23	0.24	0.24	0.25	0.23	0.26	0.23
75	150	0.22	0.19	0.21	0.20	0.21	0.20	0.20	0.20	0.22	0.20	0.20	0.20
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.07	1.10	1.15	1.18	1.15	1.05	1.11	1.16
25	30					0.70	0.71	0.71	0.72	0.76	0.68	0.68	0.69
50	30					0.48	0.49	0.51	0.50	0.51	0.50	0.51	0.53
75	30					0.42	0.42	0.41	0.43	0.42	0.41	0.42	0.39
10	100					0.60	0.64	0.59	0.61	0.65	0.56	0.68	0.61
25	100					0.41	0.42	0.40	0.43	0.42	0.40	0.40	0.40
50	100					0.28	0.30	0.30	0.29	0.28	0.28	0.27	0.28
75	100					0.25	0.23	0.24	0.24	0.23	0.23	0.25	0.24
10	150					0.52	0.53	0.50	0.49	0.52	0.53	0.49	0.48
25	150					0.32	0.35	0.35	0.34	0.34	0.34	0.34	0.37
50	150					0.25	0.24	0.25	0.24	0.24	0.25	0.24	0.26
75	150					0.21	0.20	0.21	0.20	0.19	0.19	0.21	0.19

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A10.

Absolute Bias Values of $\sigma^2_{r_0}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	2.32	1.84	1.88	2.15	2.79	2.36	2.64	2.73	2.19	1.73	1.96	1.87
25	30	1.49	1.07	1.21	1.29	2.88	2.71	2.76	2.87	1.38	1.09	1.21	1.30
50	30	1.04	0.84	0.84	0.85	4.06	3.12	3.99	4.25	0.99	0.85	0.88	0.92
75	30	0.83	0.67	0.67	0.75	6.35	5.74	6.14	5.28	0.82	0.69	0.73	0.74
10	100	1.33	1.02	1.09	1.18	3.76	3.18	3.40	3.49	1.23	1.03	1.12	1.15
25	100	0.84	0.65	0.67	0.76	6.08	4.74	5.28	4.90	0.81	0.61	0.68	0.69
50	100	0.57	0.48	0.47	0.50	9.79	10.33	9.67	9.56	0.56	0.47	0.53	0.54
75	100	0.46	0.39	0.39	0.43	14.17	14.22	13.78	14.17	0.46	0.38	0.41	0.42
10	150	1.04	0.85	0.93	0.96	3.76	4.53	4.18	4.72	1.07	0.84	0.87	0.94
25	150	0.67	0.53	0.56	0.60	7.83	8.08	8.79	7.42	0.69	0.53	0.60	0.60
50	150	0.48	0.38	0.39	0.43	13.65	13.83	13.62	14.00	0.48	0.38	0.43	0.43
75	150	0.39	0.31	0.35	0.34	18.30	17.76	19.28	18.13	0.40	0.32	0.33	0.36
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					3.21	2.47	2.44	2.72	2.31	1.80	1.94	2.08
25	30					3.18	2.08	2.52	2.84	1.44	1.08	1.22	1.27
50	30					3.52	3.39	4.16	4.76	0.98	0.88	0.85	0.89
75	30					6.13	6.07	5.80	5.64	0.81	0.66	0.68	0.72
10	100					3.09	3.65	3.08	3.31	1.28	1.04	1.17	1.10
25	100					5.99	6.61	6.45	6.01	0.82	0.64	0.67	0.76
50	100					9.98	9.61	11.65	10.29	0.55	0.44	0.46	0.50
75	100					14.11	13.74	14.85	13.21	0.45	0.37	0.41	0.42
10	150					4.43	4.87	3.80	3.62	1.04	0.82	0.83	0.92
25	150					8.27	7.99	8.02	9.17	0.66	0.53	0.56	0.57
50	150					13.57	12.84	12.88	13.11	0.48	0.38	0.40	0.43
75	150					16.98	17.41	18.40	18.88	0.36	0.31	0.32	0.36

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A11.

Absolute Bias Values of $\sigma^2_{r_1}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.57	0.54	0.51	0.52	0.45	0.51	0.52	0.50	0.47	0.45	0.46	0.44
25	30	0.43	0.32	0.36	0.37	0.39	0.33	0.36	0.37	0.39	0.34	0.34	0.36
50	30	0.30	0.27	0.28	0.29	0.28	0.28	0.31	0.30	0.30	0.27	0.29	0.30
75	30	0.26	0.22	0.25	0.25	0.27	0.23	0.25	0.24	0.26	0.22	0.28	0.25
10	100	0.38	0.31	0.33	0.35	0.35	0.30	0.31	0.33	0.35	0.28	0.33	0.33
25	100	0.27	0.24	0.22	0.24	0.26	0.23	0.24	0.23	0.24	0.20	0.23	0.23
50	100	0.19	0.16	0.18	0.19	0.20	0.21	0.20	0.21	0.20	0.17	0.18	0.19
75	100	0.17	0.14	0.15	0.15	0.18	0.19	0.19	0.17	0.17	0.14	0.15	0.16
10	150	0.31	0.26	0.29	0.29	0.30	0.25	0.29	0.28	0.30	0.28	0.25	0.28
25	150	0.21	0.18	0.20	0.20	0.23	0.22	0.23	0.22	0.24	0.20	0.20	0.22
50	150	0.16	0.15	0.15	0.16	0.18	0.18	0.19	0.18	0.17	0.15	0.15	0.16
75	150	0.14	0.12	0.13	0.13	0.16	0.19	0.19	0.18	0.15	0.12	0.13	0.13
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.48	0.49	0.51	0.51	0.56	0.52	0.52	0.54
25	30					0.38	0.40	0.34	0.36	0.38	0.33	0.33	0.36
50	30					0.29	0.26	0.27	0.28	0.31	0.24	0.25	0.28
75	30					0.24	0.25	0.22	0.24	0.24	0.21	0.22	0.23
10	100					0.33	0.33	0.33	0.31	0.32	0.28	0.35	0.30
25	100					0.25	0.26	0.26	0.24	0.24	0.24	0.23	0.25
50	100					0.18	0.19	0.19	0.20	0.18	0.17	0.16	0.16
75	100					0.16	0.18	0.18	0.18	0.14	0.12	0.14	0.15
10	150					0.30	0.30	0.30	0.29	0.29	0.29	0.25	0.25
25	150					0.20	0.23	0.25	0.24	0.22	0.22	0.21	0.21
50	150					0.17	0.19	0.18	0.18	0.15	0.15	0.15	0.16
75	150					0.15	0.18	0.17	0.18	0.12	0.12	0.13	0.12

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A12.

Absolute Bias Values of $\sigma^2_{u_{00}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.00	1.37	1.28	1.31	0.99	1.69	1.62	1.40	0.98	1.81	1.67	1.42
25	30	0.67	0.87	0.84	0.77	0.71	3.10	2.04	1.64	0.73	2.88	2.18	1.51
50	30	0.42	0.59	0.56	0.50	0.79	5.13	3.94	2.67	0.78	5.54	4.26	2.83
75	30	0.34	0.53	0.47	0.45	1.02	5.00	4.08	3.19	1.12	5.96	4.86	3.63
10	100	0.62	0.76	0.77	0.72	0.73	1.56	1.26	1.31	0.73	1.38	1.31	1.22
25	100	0.33	0.48	0.46	0.42	0.84	4.93	3.73	2.54	0.82	5.65	4.24	2.96
50	100	0.23	0.31	0.31	0.28	1.27	4.72	4.04	3.23	1.68	6.80	5.60	4.44
75	100	0.18	0.26	0.25	0.23	1.23	3.84	3.31	2.60	2.09	6.82	5.67	4.52
10	150	0.53	0.65	0.56	0.56	0.71	1.57	1.43	1.17	0.72	1.62	1.30	1.30
25	150	0.27	0.37	0.36	0.32	0.96	4.79	3.74	2.72	1.08	6.28	4.84	3.39
50	150	0.19	0.27	0.26	0.24	1.22	4.03	3.43	2.67	2.01	7.00	5.82	4.61
75	150	0.14	0.21	0.21	0.18	1.03	3.12	2.35	2.06	2.26	7.03	5.84	4.66
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.02	1.87	1.64	1.39	1.02	2.07	1.62	1.34
25	30					0.80	3.94	2.86	1.90	0.75	3.84	2.87	1.95
50	30					0.99	5.30	4.04	2.85	0.90	5.62	4.40	3.20
75	30					1.24	5.07	4.21	3.24	1.34	5.93	4.90	3.78
10	100					0.71	2.91	2.06	1.55	0.74	2.96	2.22	1.67
25	100					1.05	5.32	4.10	3.20	1.14	6.31	5.04	3.57
50	100					1.35	4.92	3.71	3.13	1.84	6.80	5.62	4.47
75	100					1.28	4.62	3.43	2.81	2.13	6.82	5.67	4.52
10	150					0.77	3.43	2.63	1.81	0.72	3.84	2.81	1.83
25	150					1.15	5.26	4.26	3.16	1.34	6.82	5.59	4.19
50	150					1.30	4.31	3.57	2.82	2.12	7.01	5.82	4.63
75	150					1.12	3.83	2.90	2.05	2.29	7.02	5.84	4.66

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A13.

Absolute Bias Values of $\sigma^2_{u_{01}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.007	0.006	0.007	0.007	0.005	0.006	0.006	0.006
25	30	0.003	0.004	0.004	0.003	0.002	0.004	0.003	0.003
50	30	0.002	0.005	0.004	0.003	0.002	0.006	0.004	0.003
75	30	0.001	0.006	0.005	0.003	0.001	0.006	0.005	0.004
10	100	0.004	0.004	0.004	0.004	0.003	0.004	0.004	0.003
25	100	0.001	0.006	0.004	0.003	0.001	0.006	0.004	0.003
50	100	0.002	0.006	0.005	0.004	0.002	0.007	0.006	0.005
75	100	0.002	0.006	0.005	0.004	0.002	0.008	0.006	0.005
10	150	0.003	0.004	0.004	0.003	0.002	0.004	0.003	0.003
25	150	0.001	0.006	0.005	0.003	0.001	0.007	0.005	0.003
50	150	0.002	0.006	0.005	0.004	0.002	0.008	0.006	0.005
75	150	0.002	0.006	0.004	0.003	0.002	0.008	0.007	0.005
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.005	0.006	0.006	0.006	0.004	0.005	0.005	0.005
25	30	0.003	0.004	0.003	0.003	0.002	0.004	0.003	0.002
50	30	0.001	0.005	0.004	0.003	0.001	0.006	0.004	0.003
75	30	0.001	0.006	0.005	0.004	0.001	0.006	0.005	0.004
10	100	0.003	0.004	0.004	0.003	0.003	0.004	0.003	0.003
25	100	0.001	0.007	0.005	0.004	0.001	0.006	0.005	0.003
50	100	0.002	0.008	0.006	0.005	0.002	0.007	0.006	0.005
75	100	0.002	0.008	0.006	0.005	0.002	0.008	0.006	0.005
10	150	0.003	0.005	0.004	0.003	0.002	0.004	0.003	0.003
25	150	0.002	0.007	0.006	0.004	0.001	0.007	0.005	0.004
50	150	0.002	0.008	0.007	0.005	0.002	0.008	0.006	0.005
75	150	0.003	0.008	0.007	0.005	0.002	0.008	0.007	0.005

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A14.

Absolute Bias Values of $\sigma^2_{u_{10}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.40	0.66	0.61	0.53	0.40	0.78	0.69	0.60	0.42	0.86	0.70	0.60
25	30	0.25	0.41	0.37	0.33	0.28	1.54	0.98	0.79	0.32	1.44	1.08	0.72
50	30	0.16	0.28	0.27	0.22	0.37	2.58	1.98	1.33	0.37	2.81	2.16	1.43
75	30	0.13	0.24	0.21	0.20	0.48	2.52	2.04	1.60	0.56	3.02	2.46	1.85
10	100	0.22	0.33	0.32	0.28	0.30	0.70	0.57	0.54	0.31	0.66	0.59	0.57
25	100	0.13	0.22	0.19	0.19	0.39	2.47	1.86	1.26	0.39	2.85	2.15	1.51
50	100	0.09	0.15	0.14	0.12	0.62	2.36	2.01	1.61	0.85	3.44	2.83	2.26
75	100	0.07	0.12	0.11	0.10	0.59	1.90	1.64	1.28	1.07	3.45	2.87	2.29
10	150	0.19	0.27	0.25	0.23	0.30	0.75	0.67	0.53	0.30	0.78	0.63	0.63
25	150	0.10	0.18	0.15	0.15	0.46	2.38	1.87	1.35	0.53	3.17	2.45	1.72
50	150	0.07	0.12	0.11	0.10	0.60	2.00	1.70	1.32	1.02	3.54	2.94	2.34
75	150	0.06	0.10	0.09	0.08	0.49	1.55	1.15	1.01	1.15	3.55	2.95	2.36
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.38	0.91	0.72	0.59	0.38	0.98	0.79	0.63
25	30					0.34	1.97	1.43	0.94	0.32	1.93	1.45	0.99
50	30					0.48	2.67	2.04	1.43	0.44	2.84	2.23	1.64
75	30					0.61	2.54	2.11	1.63	0.68	3.00	2.48	1.92
10	100					0.30	1.45	1.00	0.73	0.31	1.49	1.11	0.81
25	100					0.50	2.67	2.05	1.60	0.57	3.20	2.56	1.82
50	100					0.66	2.45	1.84	1.56	0.94	3.43	2.85	2.27
75	100					0.62	2.31	1.70	1.40	1.09	3.44	2.87	2.29
10	150					0.33	1.71	1.30	0.86	0.33	1.93	1.42	0.90
25	150					0.55	2.63	2.13	1.57	0.67	3.45	2.83	2.13
50	150					0.63	2.14	1.77	1.40	1.08	3.54	2.94	2.35
75	150					0.54	1.90	1.43	1.00	1.16	3.55	2.95	2.36

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A15.

Absolute Bias Values of $\sigma^2_{u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002
25	30	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.001
50	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
75	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
10	100	0.001	0.002	0.002	0.001	0.001	0.002	0.001	0.001
25	100	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
50	100	0.001	0.002	0.002	0.002	0.001	0.003	0.002	0.002
75	100	0.001	0.002	0.002	0.002	0.001	0.003	0.002	0.002
10	150	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001
25	150	0.001	0.002	0.002	0.001	0.001	0.003	0.002	0.001
50	150	0.001	0.002	0.002	0.002	0.001	0.003	0.002	0.002
75	150	0.001	0.002	0.002	0.001	0.001	0.003	0.003	0.002
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.002	0.003	0.002	0.002	0.001	0.002	0.002	0.002
25	30	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001
50	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
75	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
10	100	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
25	100	0.001	0.002	0.002	0.001	0.000	0.002	0.002	0.001
50	100	0.001	0.002	0.002	0.002	0.001	0.003	0.002	0.002
75	100	0.001	0.002	0.002	0.001	0.001	0.003	0.002	0.002
10	150	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001
25	150	0.001	0.002	0.002	0.001	0.001	0.003	0.002	0.002
50	150	0.001	0.002	0.002	0.002	0.001	0.003	0.002	0.002
75	150	0.001	0.002	0.002	0.001	0.001	0.003	0.003	0.002

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table A16.

Absolute Bias Values of $\sigma_{r_0 r_1}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.96	0.89	0.87	0.93	0.88	0.82	0.91	0.93	0.90	0.80	0.86	0.85
25	30	0.63	0.53	0.58	0.59	0.64	0.56	0.59	0.64	0.59	0.56	0.55	0.58
50	30	0.46	0.41	0.41	0.41	0.49	0.46	0.49	0.49	0.45	0.42	0.43	0.44
75	30	0.38	0.34	0.34	0.35	0.43	0.43	0.44	0.43	0.37	0.34	0.37	0.35
10	100	0.61	0.51	0.51	0.57	0.60	0.54	0.55	0.55	0.52	0.48	0.53	0.51
25	100	0.38	0.34	0.31	0.36	0.42	0.39	0.41	0.39	0.35	0.29	0.34	0.32
50	100	0.27	0.24	0.24	0.25	0.42	0.43	0.42	0.41	0.27	0.23	0.26	0.26
75	100	0.22	0.20	0.20	0.21	0.43	0.47	0.46	0.44	0.22	0.19	0.20	0.21
10	150	0.46	0.41	0.46	0.45	0.51	0.47	0.48	0.49	0.47	0.43	0.40	0.45
25	150	0.32	0.28	0.28	0.28	0.39	0.43	0.44	0.40	0.32	0.28	0.29	0.29
50	150	0.22	0.20	0.21	0.22	0.40	0.44	0.45	0.43	0.23	0.20	0.21	0.22
75	150	0.19	0.16	0.18	0.18	0.45	0.52	0.53	0.51	0.19	0.16	0.17	0.18
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.95	0.83	0.87	0.90	0.96	0.84	0.84	0.90
25	30					0.63	0.59	0.57	0.59	0.60	0.52	0.58	0.58
50	30					0.47	0.41	0.43	0.45	0.47	0.38	0.40	0.42
75	30					0.43	0.40	0.38	0.41	0.35	0.32	0.33	0.34
10	100					0.58	0.52	0.55	0.55	0.54	0.50	0.54	0.49
25	100					0.42	0.39	0.41	0.39	0.36	0.31	0.34	0.37
50	100					0.37	0.33	0.37	0.36	0.24	0.23	0.22	0.24
75	100					0.38	0.36	0.38	0.37	0.21	0.18	0.20	0.21
10	150					0.50	0.45	0.48	0.46	0.45	0.41	0.39	0.42
25	150					0.38	0.37	0.39	0.41	0.30	0.27	0.29	0.29
50	150					0.38	0.35	0.36	0.36	0.22	0.19	0.20	0.23
75	150					0.40	0.40	0.40	0.43	0.17	0.15	0.17	0.17

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A17.

Absolute Bias Values of $\sigma_{u_{00}u_{01}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.06	0.10	0.10	0.09	0.06	0.10	0.09	0.08
25	30	0.04	0.12	0.09	0.08	0.04	0.11	0.09	0.07
50	30	0.04	0.17	0.14	0.10	0.04	0.17	0.14	0.10
75	30	0.05	0.16	0.14	0.11	0.04	0.18	0.15	0.12
10	100	0.04	0.07	0.07	0.07	0.04	0.06	0.06	0.06
25	100	0.04	0.13	0.11	0.08	0.03	0.14	0.11	0.08
50	100	0.04	0.12	0.11	0.09	0.05	0.16	0.13	0.11
75	100	0.04	0.10	0.09	0.07	0.05	0.15	0.13	0.10
10	150	0.04	0.07	0.06	0.05	0.04	0.06	0.05	0.05
25	150	0.04	0.12	0.10	0.08	0.04	0.14	0.11	0.08
50	150	0.04	0.10	0.09	0.07	0.05	0.15	0.13	0.10
75	150	0.03	0.08	0.06	0.06	0.05	0.15	0.12	0.10

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table A18.

Absolute Bias Values of $\sigma_{u_{00}u_{10}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC 1	ICC 2	ICC 3	ICC 4	ICC 1	ICC 2	ICC 3	ICC 4	ICC 1	ICC 2	ICC 3	ICC 4
10	30	0.64	1.12	0.97	0.86	0.55	1.02	0.94	0.81	0.52	1.14	0.91	0.79
25	30	0.41	0.96	0.83	0.69	0.40	2.31	1.46	1.20	0.43	2.08	1.56	1.01
50	30	0.34	0.94	0.78	0.62	0.55	3.88	3.00	2.01	0.53	4.11	3.20	2.10
75	30	0.32	0.90	0.78	0.63	0.76	3.74	3.05	2.40	0.83	4.40	3.59	2.70
10	100	0.42	0.90	0.75	0.64	0.40	0.88	0.73	0.74	0.42	0.81	0.75	0.74
25	100	0.29	0.85	0.71	0.58	0.50	2.90	2.20	1.51	0.50	3.31	2.51	1.79
50	100	0.29	0.87	0.73	0.59	0.75	2.71	2.34	1.88	1.02	3.92	3.25	2.60
75	100	0.29	0.86	0.72	0.58	0.71	2.19	1.89	1.49	1.24	3.91	3.26	2.61
10	150	0.35	0.87	0.73	0.59	0.41	0.88	0.80	0.66	0.42	0.93	0.74	0.76
25	150	0.31	0.87	0.72	0.58	0.55	2.62	2.07	1.51	0.63	3.46	2.68	1.91
50	150	0.28	0.87	0.72	0.57	0.68	2.17	1.86	1.45	1.14	3.80	3.17	2.53
75	150	0.28	0.85	0.71	0.56	0.56	1.68	1.26	1.11	1.26	3.79	3.16	2.53

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A19.

Absolute Bias Values of $\sigma_{u_{00}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.037	0.058	0.054	0.051	0.031	0.053	0.049	0.048
25	30	0.022	0.039	0.036	0.031	0.020	0.034	0.032	0.027
50	30	0.017	0.031	0.028	0.024	0.014	0.027	0.024	0.020
75	30	0.014	0.028	0.025	0.021	0.012	0.024	0.021	0.018
10	100	0.024	0.033	0.035	0.031	0.022	0.031	0.029	0.027
25	100	0.014	0.020	0.019	0.019	0.012	0.016	0.015	0.015
50	100	0.010	0.016	0.014	0.013	0.007	0.010	0.009	0.009
75	100	0.009	0.015	0.012	0.011	0.005	0.008	0.007	0.007
10	150	0.021	0.031	0.029	0.027	0.018	0.025	0.022	0.022
25	150	0.013	0.017	0.016	0.015	0.010	0.011	0.011	0.011
50	150	0.009	0.014	0.012	0.011	0.005	0.006	0.006	0.006
75	150	0.007	0.012	0.011	0.010	0.004	0.004	0.004	0.004

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A20.

Absolute Bias Values of $\sigma_{u_{01}u_{10}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.043	0.064	0.058	0.055	0.040	0.059	0.056	0.053
25	30	0.025	0.023	0.029	0.028	0.023	0.025	0.024	0.026
50	30	0.015	0.017	0.015	0.015	0.013	0.018	0.013	0.012
75	30	0.011	0.021	0.017	0.013	0.008	0.021	0.016	0.011
10	100	0.027	0.034	0.033	0.029	0.023	0.031	0.030	0.025
25	100	0.012	0.029	0.026	0.018	0.011	0.029	0.022	0.016
50	100	0.010	0.033	0.028	0.021	0.008	0.037	0.029	0.022
75	100	0.012	0.030	0.026	0.021	0.009	0.038	0.031	0.024
10	150	0.022	0.031	0.027	0.026	0.018	0.027	0.024	0.022
25	150	0.013	0.033	0.027	0.022	0.009	0.035	0.026	0.019
50	150	0.013	0.033	0.028	0.022	0.010	0.041	0.033	0.025
75	150	0.012	0.030	0.024	0.021	0.012	0.043	0.035	0.027

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A21.

Absolute Bias Values of $\sigma_{u_{01}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.003	0.004	0.004	0.003	0.002	0.004	0.003	0.003
25	30	0.002	0.005	0.004	0.003	0.001	0.004	0.003	0.003
50	30	0.001	0.005	0.004	0.003	0.001	0.005	0.004	0.003
75	30	0.002	0.005	0.004	0.003	0.002	0.005	0.004	0.004
10	100	0.002	0.003	0.003	0.003	0.002	0.003	0.002	0.002
25	100	0.002	0.004	0.004	0.003	0.001	0.005	0.004	0.003
50	100	0.002	0.004	0.003	0.003	0.002	0.005	0.004	0.003
75	100	0.001	0.003	0.003	0.002	0.002	0.005	0.004	0.003
10	150	0.002	0.003	0.003	0.002	0.002	0.003	0.002	0.002
25	150	0.002	0.004	0.004	0.003	0.001	0.005	0.004	0.003
50	150	0.001	0.003	0.003	0.002	0.002	0.005	0.004	0.003
75	150	0.001	0.003	0.002	0.002	0.002	0.004	0.004	0.003

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table A22.

Absolute Bias Values of $\sigma_{u_{10}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.025	0.044	0.041	0.035	0.023	0.040	0.038	0.034
25	30	0.016	0.047	0.036	0.031	0.015	0.041	0.035	0.026
50	30	0.016	0.061	0.049	0.037	0.013	0.061	0.050	0.035
75	30	0.016	0.058	0.048	0.039	0.016	0.062	0.052	0.040
10	100	0.017	0.030	0.028	0.024	0.015	0.025	0.025	0.023
25	100	0.013	0.046	0.038	0.028	0.012	0.049	0.039	0.030
50	100	0.015	0.042	0.036	0.030	0.016	0.053	0.044	0.036
75	100	0.013	0.035	0.030	0.024	0.018	0.052	0.043	0.035
10	150	0.015	0.028	0.023	0.021	0.013	0.024	0.021	0.020
25	150	0.013	0.042	0.034	0.027	0.013	0.048	0.039	0.029
50	150	0.013	0.034	0.030	0.024	0.017	0.050	0.042	0.034
75	150	0.011	0.029	0.022	0.020	0.017	0.049	0.041	0.033

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

APPENDIX B

TABLE OF RELATIVE BIAS VALUES

Table B1.

Relative Bias Values of γ_{000}

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	30					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	30					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	30					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	100					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	100					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	100					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	100					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	150					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	150					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	150					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	150					0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table B2.

Relative Bias Values of γ_{010}

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00
25	30	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00
50	30	0.06	0.05	0.06	0.06	0.00	0.00	0.00	0.00
75	30	0.07	0.06	0.06	0.06	0.00	0.00	0.00	0.00
10	100	0.05	0.05	0.05	0.06	0.00	0.00	0.00	0.00
25	100	0.07	0.06	0.06	0.06	0.00	0.00	0.00	0.00
50	100	0.08	0.08	0.08	0.08	0.00	0.00	0.00	0.00
75	100	0.10	0.10	0.09	0.10	0.00	0.00	0.00	0.00
10	150	0.06	0.06	0.06	0.06	0.00	0.00	0.00	0.00
25	150	0.07	0.07	0.08	0.07	0.00	0.00	0.00	0.00
50	150	0.09	0.10	0.09	0.10	0.00	0.00	0.00	0.00
75	150	0.11	0.11	0.12	0.11	0.00	0.00	0.00	0.00
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.05	0.05	0.04	0.05	0.00	0.00	0.00	0.00
25	30	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00
50	30	0.06	0.06	0.06	0.06	0.00	0.00	0.00	0.00
75	30	0.06	0.07	0.06	0.06	0.00	0.00	0.00	0.00
10	100	0.05	0.05	0.05	0.05	0.00	0.00	0.00	0.00
25	100	0.06	0.07	0.07	0.06	0.00	0.00	0.00	0.00
50	100	0.08	0.08	0.09	0.08	0.00	0.00	0.00	0.00
75	100	0.10	0.09	0.10	0.09	0.00	0.00	0.00	0.00
10	150	0.06	0.06	0.06	0.05	0.00	0.00	0.00	0.00
25	150	0.07	0.07	0.07	0.08	0.00	0.00	0.00	0.00
50	150	0.09	0.09	0.09	0.09	0.00	0.00	0.00	0.00
75	150	0.11	0.11	0.11	0.11	0.00	0.00	0.00	0.00

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B3.

Relative Bias Values of γ_{100}

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note. * L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

Table B4.

Relative Bias Values of γ_{110}

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.02	0.17	0.03	0.09	-0.10	0.04	0.04	0.13
25	30	0.07	0.15	0.11	0.05	-0.11	-0.03	-0.01	0.00
50	30	0.02	0.05	0.07	0.01	0.00	0.06	-0.04	0.01
75	30	0.07	0.04	0.02	0.04	-0.01	-0.02	0.00	0.00
10	100	0.06	0.05	0.13	0.07	0.00	-0.02	0.00	-0.04
25	100	0.11	0.09	0.04	0.05	0.00	-0.04	0.03	0.00
50	100	0.10	0.07	0.06	0.06	-0.03	0.03	0.00	-0.01
75	100	0.08	0.10	0.10	0.06	0.01	-0.01	0.03	-0.01
10	150	0.06	0.07	0.04	0.04	0.01	0.03	-0.04	0.03
25	150	0.03	0.09	0.09	0.06	0.00	-0.03	0.00	0.01
50	150	0.10	0.10	0.10	0.12	-0.03	0.02	-0.01	-0.01
75	150	0.13	0.13	0.11	0.13	-0.01	0.00	-0.01	0.00
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.04	0.06	-0.03	0.05	0.13	0.06	0.05	0.01
25	30	0.01	0.00	0.02	0.09	-0.04	-0.05	-0.03	0.02
50	30	0.05	0.03	0.05	0.08	-0.04	-0.04	0.01	0.00
75	30	0.04	0.11	0.07	0.02	-0.01	0.01	-0.02	0.01
10	100	0.11	-0.01	0.07	0.08	0.01	0.04	-0.09	-0.01
25	100	0.08	0.08	0.09	0.03	0.03	0.03	0.02	0.03
50	100	0.08	0.08	0.10	0.07	0.01	0.01	-0.01	0.00
75	100	0.09	0.09	0.05	0.07	0.01	-0.01	-0.01	0.01
10	150	0.02	0.07	0.07	0.00	-0.02	0.00	-0.01	-0.02
25	150	0.07	0.02	0.06	0.02	-0.02	0.02	-0.01	0.00
50	150	0.09	0.07	0.06	0.07	-0.01	-0.04	0.01	0.01
75	150	0.10	0.09	0.11	0.07	0.00	0.00	0.00	0.01

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B5.

Relative Bias Values of γ_{001}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.00	-0.09	0.06	-0.02	-0.03	0.04	-0.05	-0.07
25	30	-0.07	-0.02	0.03	0.05	0.09	-0.02	-0.13	-0.01
50	30	0.06	-0.04	-0.11	0.10	-0.15	0.29	-0.22	0.00
75	30	0.00	-0.10	0.02	0.01	0.05	0.16	0.05	0.03
10	100	-0.01	-0.01	-0.02	0.01	0.03	-0.01	0.05	0.01
25	100	0.04	-0.04	-0.03	0.07	0.01	0.01	0.01	0.02
50	100	-0.01	0.14	0.05	0.06	-0.04	-0.02	-0.05	0.03
75	100	-0.06	0.01	0.01	-0.05	0.05	-0.02	0.04	0.07
10	150	-0.01	0.00	0.03	0.02	0.00	-0.02	0.04	-0.05
25	150	-0.01	-0.07	-0.03	0.00	0.06	-0.04	0.04	0.01
50	150	-0.01	-0.01	-0.04	0.03	-0.02	0.02	0.01	0.03
75	150	-0.03	-0.12	0.04	-0.06	-0.01	-0.01	-0.05	-0.03

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B6.

Relative Bias Values of γ_{011}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
25	30	-1.65	-0.73	-0.59	-0.20	-0.78	3.47	0.14	0.73
50	30	-1.26	2.91	1.49	-2.04	0.86	-2.60	-0.70	-0.22
75	30	-0.13	1.70	1.20	0.27	-1.09	1.01	1.76	2.09
10	100	0.18	0.05	0.50	0.13	1.19	0.72	-0.14	0.70
25	100	-0.86	0.31	-0.79	-0.77	1.25	-0.98	0.21	-0.02
50	100	0.89	0.36	0.39	-0.60	-0.67	2.18	-0.15	1.26
75	100	-0.10	1.24	-0.01	-0.60	0.30	0.86	1.28	-0.92
10	150	0.14	0.25	-1.99	-0.33	-0.86	0.09	1.85	0.25
25	150	-0.44	0.41	1.39	0.52	-0.20	-0.45	0.57	0.46
50	150	1.01	-0.41	-0.52	-1.00	0.10	2.19	0.64	-0.51
75	150	-0.14	-0.10	-0.58	0.69	-0.14	0.11	0.32	0.24

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B7.

Relative Bias Values of γ_{101}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.21	-0.26	-0.09	-0.23	-0.11	0.36	0.59	0.07
25	30	0.13	-0.28	0.24	-0.21	0.63	-0.76	-0.52	0.20
50	30	-0.58	-0.12	-0.34	0.43	-0.18	-0.38	-0.07	-0.21
75	30	0.65	0.80	0.35	-1.15	-0.03	0.01	0.30	-0.57
10	100	0.04	-0.03	0.00	-0.04	0.21	-0.20	-0.18	0.15
25	100	0.02	0.06	0.03	0.51	0.24	0.26	0.61	-0.25
50	100	-0.08	0.43	0.05	0.08	0.23	0.46	-0.22	0.33
75	100	0.11	0.35	-0.83	-0.28	0.51	0.50	-0.64	-0.05
10	150	0.00	0.07	0.04	0.22	-0.02	0.00	-0.12	0.07
25	150	-0.10	-0.25	-0.30	0.04	0.32	0.14	0.36	0.03
50	150	-0.23	0.34	-0.27	-0.11	0.15	0.00	-0.02	-0.16
75	150	-0.15	-0.40	0.00	-0.24	-0.26	0.15	-0.38	0.13

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B8.

Relative Bias Values of γ_{111}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-1.12	0.31	0.05	0.12	1.10	0.28	0.15	0.73
25	30	1.37	0.25	0.63	0.16	0.56	-1.28	-0.01	-0.54
50	30	0.50	-0.60	-0.03	-1.60	-0.21	-0.80	1.21	-0.12
75	30	-0.26	-1.75	-1.78	-2.14	0.88	-0.72	0.53	-0.46
10	100	0.29	0.32	-0.87	-0.47	-0.48	-0.44	-0.47	0.03
25	100	0.24	-0.69	0.84	0.04	-0.38	0.20	0.07	-0.56
50	100	-0.20	-0.46	-0.13	0.08	0.47	-0.91	-0.05	0.09
75	100	0.58	0.17	-0.61	0.05	-0.12	-1.29	-0.54	-0.28
10	150	0.32	-0.04	0.89	0.61	0.33	0.52	-0.28	-0.93
25	150	-0.23	-0.03	-0.13	-0.32	0.13	0.63	0.00	-0.55
50	150	-0.71	0.14	0.50	0.17	-0.66	-0.46	0.50	-0.10
75	150	-0.61	-0.28	0.66	-0.09	-0.15	0.77	-0.01	0.22

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B9.

Relative Bias Values of σ_e^2

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02
25	30	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.02
50	30	0.00	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00
75	30	0.00	-0.01	-0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00
10	100	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
25	100	0.00	-0.01	0.00	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	-0.01
50	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	150	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01
25	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
75	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03	-0.02	-0.03
25	30	-0.01	-0.02	-0.01	-0.01	-0.02	-0.02	-0.01	-0.01
50	30	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
75	30	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
10	100	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
25	100	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
50	100	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01
75	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	150	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
25	150	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
50	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
75	150	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Note. * L1 sample size is 3. ICC$_1$, (ICC$_{L1}$=0.5, ICC$_{L2}$=0.40, ICC$_{L3}$=0.10). ICC$_2$ (ICC$_{L1}$=0.5, ICC$_{L2}$=0.20, ICC$_{L3}$=0.30). ICC$_3$ (ICC$_{L1}$=0.5, ICC$_{L2}$=0.25, ICC$_{L3}$=0.25). ICC$_4$ (ICC$_{L1}$=0.5, ICC$_{L2}$=0.30, ICC$_{L3}$=0.20)

Table B10.

Relative Bias Values of $\sigma^2_{r_0}$

Sample Size		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.02	0.07	0.04	0.04	0.02	0.15	0.13	0.10	-0.01	0.04	0.04	0.01
25	30	0.01	0.01	0.03	0.02	0.11	0.26	0.20	0.17	0.01	0.04	0.03	0.03
50	30	0.01	0.02	0.02	0.01	0.20	0.32	0.34	0.31	0.01	0.02	0.02	0.01
75	30	0.00	0.01	0.01	0.00	0.37	0.67	0.58	0.40	0.00	0.01	0.01	0.02
10	100	0.01	0.02	0.01	0.01	0.17	0.31	0.26	0.22	0.01	0.03	0.01	0.02
25	100	0.01	0.02	0.00	0.00	0.35	0.54	0.49	0.38	0.00	0.01	0.01	0.01
50	100	0.00	0.00	0.00	0.00	0.60	1.29	0.95	0.79	0.00	0.00	0.00	0.00
75	100	0.00	0.00	0.00	0.00	0.89	1.79	1.39	1.19	0.00	0.00	0.00	0.00
10	150	0.00	0.00	0.00	0.00	0.18	0.50	0.37	0.34	0.00	0.01	0.00	0.01
25	150	0.00	0.00	0.00	0.00	0.47	0.99	0.86	0.60	0.00	0.01	0.01	0.01
50	150	0.00	0.00	0.00	0.00	0.86	1.74	1.37	1.17	0.00	0.00	0.00	0.00
75	150	0.00	0.00	0.00	0.00	1.16	2.25	1.96	1.53	0.00	0.00	0.00	0.00
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.09	0.16	0.12	0.11	0.01	0.09	0.04	0.02
25	30					0.13	0.19	0.19	0.18	0.02	0.05	0.04	0.02
50	30					0.17	0.38	0.38	0.37	0.01	0.04	0.02	0.02
75	30					0.36	0.73	0.55	0.44	0.01	0.03	0.02	0.01
10	100					0.14	0.39	0.25	0.22	0.00	0.04	0.05	0.02
25	100					0.35	0.81	0.62	0.48	0.01	0.03	0.02	0.01
50	100					0.62	1.20	1.16	0.86	0.00	0.01	0.01	0.00
75	100					0.88	1.73	1.50	1.11	0.00	0.00	0.01	0.01
10	150					0.23	0.57	0.34	0.26	0.00	0.04	0.03	0.02
25	150					0.51	0.99	0.79	0.76	0.01	0.02	0.02	0.01
50	150					0.85	1.62	1.30	1.10	0.01	0.01	0.01	0.01
75	150					1.07	2.20	1.87	1.59	0.00	0.00	0.00	0.00

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B11.

Relative Bias Values of $\sigma^2_{r_1}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.25	3.25	2.18	1.75	0.92	3.04	2.36	1.83	0.99	2.66	2.10	1.42
25	30	0.81	1.57	1.33	0.94	0.72	1.71	1.45	1.12	0.77	1.75	1.28	1.07
50	30	0.32	1.17	0.75	0.62	0.35	1.23	1.07	0.75	0.41	1.19	0.87	0.71
75	30	0.26	0.81	0.65	0.38	0.29	0.91	0.77	0.43	0.27	0.84	0.79	0.48
10	100	0.65	1.48	1.14	0.87	0.46	1.43	1.10	0.93	0.49	1.38	1.20	0.87
25	100	0.25	0.94	0.44	0.37	0.32	0.92	0.65	0.40	0.17	0.71	0.60	0.42
50	100	0.09	0.40	0.35	0.22	0.18	0.92	0.58	0.41	0.10	0.42	0.28	0.16
75	100	0.05	0.31	0.21	0.10	0.19	0.90	0.68	0.35	0.01	0.25	0.17	0.15
10	150	0.32	1.10	0.93	0.59	0.36	1.19	1.01	0.70	0.32	1.30	0.69	0.57
25	150	0.03	0.52	0.36	0.23	0.24	0.97	0.71	0.44	0.21	0.68	0.45	0.40
50	150	0.09	0.29	0.22	0.16	0.16	0.82	0.62	0.37	-0.01	0.36	0.21	0.14
75	150	0.03	0.20	0.15	0.10	0.19	0.98	0.73	0.50	0.07	0.19	0.15	0.02
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.17	3.15	2.49	1.95	1.40	3.38	2.58	2.14
25	30					0.82	2.56	1.62	1.29	0.85	2.05	1.61	1.28
50	30					0.53	1.51	1.17	1.01	0.65	1.38	1.09	1.00
75	30					0.43	1.49	0.86	0.79	0.39	1.13	0.92	0.72
10	100					0.68	2.06	1.51	1.08	0.60	1.75	1.66	1.02
25	100					0.45	1.54	1.15	0.80	0.43	1.39	1.02	0.85
50	100					0.30	1.02	0.71	0.62	0.25	0.79	0.47	0.42
75	100					0.22	0.94	0.66	0.52	0.14	0.41	0.36	0.28
10	150					0.62	1.83	1.39	0.99	0.50	1.77	1.14	0.76
25	150					0.33	1.30	1.12	0.82	0.36	1.29	0.93	0.66
50	150					0.26	1.04	0.70	0.51	0.19	0.71	0.47	0.42
75	150					0.21	0.97	0.72	0.54	0.12	0.39	0.30	0.21

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B12.

Relative Bias Values of $\sigma^2_{u_{00}}$

Sample Size		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.12	-0.05	-0.04	-0.05	-0.06	-0.07	-0.07	-0.07	-0.10	-0.14	-0.15	-0.12
25	30	-0.04	-0.03	-0.05	-0.04	-0.08	-0.35	-0.25	-0.23	-0.15	-0.33	-0.28	-0.23
50	30	-0.04	-0.04	-0.03	-0.03	-0.19	-0.64	-0.58	-0.47	-0.22	-0.69	-0.64	-0.52
75	30	-0.04	-0.03	-0.04	-0.04	-0.32	-0.62	-0.60	-0.59	-0.39	-0.75	-0.73	-0.68
10	100	-0.04	-0.01	-0.02	-0.03	-0.06	-0.12	-0.10	-0.13	-0.11	-0.10	-0.11	-0.15
25	100	0.00	-0.01	-0.01	-0.01	-0.22	-0.60	-0.54	-0.43	-0.23	-0.70	-0.63	-0.54
50	100	-0.01	-0.01	-0.01	-0.01	-0.43	-0.58	-0.58	-0.59	-0.62	-0.85	-0.84	-0.84
75	100	-0.01	-0.01	-0.01	-0.01	-0.42	-0.47	-0.48	-0.46	-0.78	-0.86	-0.85	-0.85
10	150	0.00	-0.01	0.00	0.00	-0.10	-0.13	-0.14	-0.13	-0.12	-0.14	-0.13	-0.16
25	150	-0.02	-0.01	-0.01	-0.01	-0.28	-0.58	-0.54	-0.47	-0.37	-0.79	-0.72	-0.63
50	150	-0.01	-0.01	-0.01	-0.01	-0.42	-0.49	-0.49	-0.48	-0.75	-0.88	-0.88	-0.87
75	150	-0.01	-0.01	0.00	-0.01	-0.35	-0.37	-0.33	-0.36	-0.85	-0.88	-0.88	-0.88
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					-0.09	-0.13	-0.11	-0.10	-0.14	-0.18	-0.14	-0.14
25	30					-0.16	-0.47	-0.39	-0.30	-0.18	-0.46	-0.40	-0.31
50	30					-0.30	-0.66	-0.59	-0.52	-0.29	-0.70	-0.66	-0.59
75	30					-0.42	-0.63	-0.62	-0.59	-0.48	-0.74	-0.74	-0.71
10	100					-0.12	-0.31	-0.25	-0.21	-0.12	-0.33	-0.29	-0.26
25	100					-0.32	-0.66	-0.59	-0.57	-0.38	-0.79	-0.75	-0.66
50	100					-0.46	-0.60	-0.54	-0.57	-0.68	-0.85	-0.85	-0.84
75	100					-0.44	-0.57	-0.50	-0.51	-0.80	-0.86	-0.85	-0.85
10	150					-0.16	-0.39	-0.34	-0.27	-0.17	-0.45	-0.38	-0.29
25	150					-0.37	-0.64	-0.63	-0.56	-0.46	-0.85	-0.84	-0.78
50	150					-0.44	-0.52	-0.52	-0.51	-0.79	-0.88	-0.88	-0.87
75	150					-0.38	-0.46	-0.41	-0.36	-0.86	-0.88	-0.88	-0.88

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B13.

Relative Bias Values of $\sigma^2_{u_{01}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.83	0.19	0.41	0.65	1.41	0.07	0.18	0.37
25	30	0.49	-0.21	-0.08	0.05	0.30	-0.28	-0.25	-0.14
50	30	0.00	-0.50	-0.41	-0.30	-0.09	-0.57	-0.51	-0.42
75	30	-0.13	-0.55	-0.52	-0.47	-0.31	-0.66	-0.62	-0.56
10	100	0.82	-0.03	0.06	0.10	0.48	-0.12	-0.02	0.02
25	100	-0.01	-0.52	-0.42	-0.30	-0.15	-0.58	-0.50	-0.39
50	100	-0.29	-0.62	-0.60	-0.56	-0.45	-0.77	-0.74	-0.70
75	100	-0.42	-0.61	-0.57	-0.54	-0.61	-0.80	-0.79	-0.76
10	150	0.61	-0.03	-0.01	0.07	0.42	-0.13	-0.09	-0.05
25	150	-0.08	-0.57	-0.51	-0.40	-0.20	-0.66	-0.59	-0.49
50	150	-0.35	-0.59	-0.57	-0.54	-0.55	-0.81	-0.79	-0.76
75	150	-0.44	-0.57	-0.53	-0.50	-0.69	-0.84	-0.82	-0.80
Sample Size		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.61	0.03	0.06	0.07	0.27	-0.15	-0.07	-0.04
25	30	0.05	-0.26	-0.19	-0.11	-0.04	-0.32	-0.26	-0.19
50	30	-0.13	-0.55	-0.50	-0.42	-0.19	-0.57	-0.51	-0.42
75	30	-0.33	-0.63	-0.59	-0.55	-0.31	-0.65	-0.62	-0.56
10	100	0.21	-0.18	-0.04	-0.02	0.09	-0.19	-0.13	-0.06
25	100	-0.22	-0.69	-0.60	-0.52	-0.19	-0.64	-0.57	-0.45
50	100	-0.54	-0.80	-0.79	-0.75	-0.44	-0.77	-0.74	-0.71
75	100	-0.71	-0.78	-0.79	-0.78	-0.62	-0.80	-0.79	-0.76
10	150	0.14	-0.31	-0.18	-0.13	0.16	-0.27	-0.21	-0.15
25	150	-0.32	-0.76	-0.71	-0.65	-0.21	-0.73	-0.67	-0.58
50	150	-0.68	-0.85	-0.83	-0.81	-0.57	-0.81	-0.79	-0.76
75	150	-0.78	-0.83	-0.83	-0.84	-0.70	-0.84	-0.83	-0.80

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B14.

Relative Bias Values of $\sigma^2_{u_{10}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.10	-0.05	-0.04	-0.05	-0.07	-0.09	-0.08	-0.09	-0.14	-0.15	-0.12	-0.10
25	30	-0.03	-0.04	-0.03	-0.04	-0.07	-0.35	-0.25	-0.24	-0.13	-0.33	-0.29	-0.23
50	30	-0.03	-0.04	-0.04	-0.03	-0.21	-0.64	-0.58	-0.48	-0.23	-0.70	-0.64	-0.52
75	30	-0.04	-0.03	-0.03	-0.04	-0.32	-0.62	-0.60	-0.59	-0.40	-0.75	-0.73	-0.69
10	100	-0.04	-0.02	-0.01	-0.02	-0.07	-0.11	-0.10	-0.12	-0.12	-0.10	-0.11	-0.14
25	100	-0.01	-0.01	-0.01	-0.01	-0.23	-0.60	-0.54	-0.44	-0.24	-0.70	-0.63	-0.55
50	100	-0.01	-0.01	-0.01	-0.01	-0.44	-0.58	-0.59	-0.59	-0.63	-0.85	-0.85	-0.84
75	100	-0.01	-0.01	-0.01	-0.01	-0.43	-0.46	-0.48	-0.47	-0.79	-0.86	-0.86	-0.85
10	150	0.00	-0.01	-0.02	-0.01	-0.12	-0.14	-0.14	-0.14	-0.13	-0.15	-0.13	-0.17
25	150	-0.02	-0.01	-0.01	-0.01	-0.30	-0.58	-0.54	-0.48	-0.36	-0.79	-0.73	-0.63
50	150	0.00	-0.01	-0.01	-0.01	-0.42	-0.49	-0.50	-0.48	-0.76	-0.88	-0.88	-0.87
75	150	-0.01	-0.01	0.00	-0.01	-0.35	-0.37	-0.33	-0.36	-0.86	-0.88	-0.88	-0.88
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					-0.10	-0.13	-0.11	-0.09	-0.12	-0.18	-0.14	-0.15
25	30					-0.15	-0.47	-0.39	-0.31	-0.16	-0.46	-0.41	-0.34
50	30					-0.31	-0.66	-0.60	-0.52	-0.29	-0.71	-0.66	-0.60
75	30					-0.43	-0.63	-0.63	-0.60	-0.50	-0.75	-0.74	-0.72
10	100					-0.10	-0.32	-0.24	-0.21	-0.14	-0.34	-0.30	-0.24
25	100					-0.33	-0.65	-0.60	-0.58	-0.39	-0.79	-0.76	-0.67
50	100					-0.47	-0.60	-0.54	-0.57	-0.69	-0.85	-0.85	-0.85
75	100					-0.44	-0.57	-0.50	-0.51	-0.81	-0.86	-0.86	-0.85
10	150					-0.16	-0.40	-0.36	-0.28	-0.16	-0.46	-0.39	-0.30
25	150					-0.37	-0.65	-0.63	-0.57	-0.48	-0.86	-0.84	-0.79
50	150					-0.45	-0.52	-0.52	-0.51	-0.80	-0.88	-0.88	-0.88
75	150					-0.38	-0.46	-0.42	-0.36	-0.87	-0.88	-0.88	-0.88

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B15.

Relative Bias Values of $\sigma^2_{u_{11}}$

Sample Size			L1L2 Model				L1L2L3 Model			
L2	L3		ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30		1.52	0.20	0.23	0.49	0.97	0.01	0.10	0.33
25	30		0.36	-0.19	-0.11	0.08	0.04	-0.33	-0.25	-0.15
50	30		0.02	-0.46	-0.40	-0.28	-0.21	-0.58	-0.52	-0.43
75	30		-0.09	-0.46	-0.45	-0.41	-0.35	-0.65	-0.62	-0.56
10	100		0.72	-0.02	0.05	0.13	0.30	-0.14	-0.13	0.00
25	100		0.08	-0.44	-0.37	-0.26	-0.19	-0.59	-0.51	-0.44
50	100		-0.17	-0.43	-0.45	-0.39	-0.47	-0.77	-0.74	-0.71
75	100		-0.22	-0.30	-0.32	-0.26	-0.64	-0.80	-0.79	-0.77
10	150		0.51	0.03	0.04	0.10	0.17	-0.16	-0.12	-0.10
25	150		0.03	-0.43	-0.35	-0.29	-0.26	-0.67	-0.61	-0.53
50	150		-0.16	-0.32	-0.33	-0.31	-0.59	-0.81	-0.79	-0.76
75	150		-0.12	-0.17	-0.14	-0.16	-0.72	-0.84	-0.82	-0.81
Sample Size			L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3		ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30		0.31	-0.03	0.00	0.07	-0.03	-0.24	-0.21	-0.07
25	30		0.10	-0.25	-0.17	-0.10	-0.13	-0.35	-0.29	-0.27
50	30		-0.13	-0.47	-0.41	-0.34	-0.24	-0.57	-0.52	-0.45
75	30		-0.20	-0.48	-0.44	-0.41	-0.34	-0.64	-0.62	-0.57
10	100		0.14	-0.16	-0.14	-0.04	0.06	-0.24	-0.19	-0.14
25	100		-0.07	-0.51	-0.44	-0.40	-0.22	-0.64	-0.59	-0.50
50	100		-0.33	-0.47	-0.38	-0.42	-0.51	-0.77	-0.74	-0.71
75	100		-0.26	-0.39	-0.32	-0.35	-0.65	-0.80	-0.79	-0.77
10	150		0.09	-0.26	-0.15	-0.09	-0.07	-0.31	-0.26	-0.23
25	150		-0.18	-0.51	-0.49	-0.44	-0.31	-0.72	-0.68	-0.60
50	150		-0.30	-0.38	-0.39	-0.35	-0.61	-0.81	-0.79	-0.77
75	150		-0.22	-0.28	-0.23	-0.16	-0.72	-0.83	-0.82	-0.81

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B16.

Relative Bias Values of $\sigma_{r_0 r_1}$

Sample Size		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.55	1.76	1.09	0.95	0.39	1.95	1.61	1.23	0.51	1.71	1.36	0.84
25	30	0.40	0.62	0.80	0.44	0.56	1.17	1.05	0.84	0.48	1.12	0.78	0.69
50	30	0.13	0.61	0.33	0.25	0.23	0.73	0.70	0.58	0.24	0.54	0.45	0.44
75	30	0.08	0.40	0.29	0.18	0.30	0.85	0.69	0.50	0.15	0.32	0.41	0.29
10	100	0.29	0.72	0.45	0.42	0.45	0.87	0.69	0.60	0.34	0.63	0.63	0.54
25	100	0.12	0.48	0.14	0.13	0.38	0.74	0.61	0.47	0.12	0.29	0.33	0.23
50	100	0.00	0.12	0.11	0.07	0.52	1.24	0.90	0.69	0.09	0.12	0.08	0.05
75	100	0.02	0.02	0.05	0.05	0.68	1.63	1.27	0.95	-0.05	0.07	-0.03	0.06
10	150	0.12	0.37	0.34	0.17	0.30	0.88	0.71	0.60	0.21	0.65	0.18	0.32
25	150	0.01	0.17	0.09	0.09	0.39	1.13	0.93	0.61	0.15	0.29	0.28	0.17
50	150	0.03	0.11	0.06	0.03	0.62	1.53	1.19	0.91	-0.02	0.11	0.06	0.07
75	150	0.03	0.00	0.04	0.02	0.81	1.95	1.61	1.28	0.03	0.08	0.04	0.00

Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.79	1.70	1.44	1.30	0.75	2.16	1.40	1.02
25	30	0.49	1.55	1.00	0.83	0.54	1.19	1.08	0.76
50	30	0.34	1.02	0.85	0.79	0.38	0.81	0.67	0.59
75	30	0.45	1.10	0.68	0.59	0.22	0.65	0.49	0.39
10	100	0.55	1.32	1.02	0.70	0.21	1.05	1.05	0.45
25	100	0.38	1.15	0.85	0.69	0.30	0.72	0.56	0.46
50	100	0.46	1.00	0.81	0.74	0.11	0.36	0.26	0.20
75	100	0.59	1.22	0.98	0.79	0.04	0.14	0.17	0.12
10	150	0.45	1.19	1.00	0.59	0.26	1.00	0.68	0.46
25	150	0.47	1.07	0.87	0.81	0.17	0.67	0.51	0.34
50	150	0.60	1.16	0.93	0.74	0.10	0.34	0.19	0.22
75	150	0.70	1.39	1.18	1.03	0.05	0.12	0.10	0.09

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B17.

Relative Bias Values of $\sigma_{u_{00}u_{01}}$

Sample Size*			L1L2 Model					L1L2L3 Model			
L2	L3		ICC1	ICC2	ICC3	ICC4		ICC1	ICC2	ICC3	ICC4
10	30		-0.34	-0.17	-0.16	-0.14		-0.37	-0.29	-0.25	-0.27
25	30		-0.17	-0.79	-0.59	-0.60		-0.33	-0.67	-0.68	-0.47
50	30		-0.57	-1.38	-1.26	-1.09		-0.56	-1.44	-1.36	-1.13
75	30		-0.81	-1.28	-1.26	-1.28		-0.95	-1.52	-1.50	-1.45
10	100		-0.31	-0.22	-0.16	-0.30		-0.31	-0.22	-0.19	-0.29
25	100		-0.49	-1.00	-0.90	-0.76		-0.52	-1.14	-1.04	-0.95
50	100		-0.79	-0.88	-0.90	-0.93		-1.11	-1.32	-1.33	-1.35
75	100		-0.69	-0.69	-0.68	-0.70		-1.33	-1.30	-1.31	-1.33
10	150		-0.28	-0.19	-0.24	-0.21		-0.34	-0.23	-0.25	-0.31
25	150		-0.59	-0.89	-0.85	-0.74		-0.68	-1.18	-1.12	-1.00
50	150		-0.70	-0.66	-0.66	-0.67		-1.22	-1.26	-1.28	-1.29
75	150		-0.50	-0.46	-0.41	-0.45		-1.31	-1.24	-1.25	-1.27

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B18.

Relative Bias Values of $\sigma_{u_{00}u_{10}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.39	-0.30	-0.29	-0.30	-0.11	-0.13	-0.12	-0.13	-0.09	-0.21	-0.18	-0.13
25	30	-0.29	-0.29	-0.29	-0.29	-0.13	-0.66	-0.46	-0.45	-0.20	-0.59	-0.50	-0.39
50	30	-0.28	-0.29	-0.29	-0.28	-0.38	-1.19	-1.08	-0.89	-0.39	-1.26	-1.17	-0.95
75	30	-0.29	-0.28	-0.29	-0.29	-0.61	-1.14	-1.11	-1.09	-0.71	-1.36	-1.33	-1.25
10	100	-0.30	-0.27	-0.27	-0.28	-0.13	-0.17	-0.15	-0.20	-0.19	-0.15	-0.15	-0.23
25	100	-0.25	-0.26	-0.26	-0.27	-0.36	-0.87	-0.78	-0.65	-0.37	-1.01	-0.92	-0.80
50	100	-0.26	-0.27	-0.27	-0.27	-0.65	-0.82	-0.84	-0.84	-0.92	-1.21	-1.20	-1.20
75	100	-0.27	-0.26	-0.27	-0.27	-0.61	-0.66	-0.67	-0.66	-1.14	-1.20	-1.21	-1.21
10	150	-0.24	-0.26	-0.27	-0.26	-0.18	-0.19	-0.19	-0.20	-0.21	-0.20	-0.18	-0.24
25	150	-0.28	-0.27	-0.27	-0.27	-0.42	-0.78	-0.74	-0.66	-0.53	-1.06	-0.98	-0.87
50	150	-0.26	-0.27	-0.27	-0.26	-0.58	-0.65	-0.66	-0.64	-1.04	-1.17	-1.17	-1.17
75	150	-0.26	-0.26	-0.26	-0.26	-0.47	-0.49	-0.43	-0.48	-1.16	-1.17	-1.17	-1.17

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B19.

Relative Bias Values of $\sigma_{u_{00}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.06	0.39	-0.13	0.17	0.17	0.18	0.19	0.09
25	30	0.41	1.36	0.97	1.54	0.49	1.17	0.83	1.00
50	30	0.89	1.94	2.14	1.85	0.76	1.97	1.98	1.76
75	30	1.05	1.89	2.04	2.01	1.22	1.87	1.96	1.91
10	100	0.45	0.13	0.60	0.23	0.32	0.21	0.36	0.49
25	100	0.82	0.93	1.05	1.11	0.51	1.02	1.03	1.08
50	100	0.97	0.74	0.67	0.97	1.11	0.80	0.88	1.02
75	100	0.79	0.37	0.64	0.52	1.14	0.66	0.71	0.81
10	150	1.22	0.23	0.37	0.28	0.82	0.24	0.19	0.37
25	150	0.52	0.56	0.61	0.75	0.91	0.79	0.75	0.84
50	150	0.59	0.40	0.57	0.50	1.02	0.48	0.55	0.66
75	150	0.32	0.28	0.28	0.32	0.81	0.35	0.41	0.49

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B20.

Relative Bias Values of $\sigma_{u_{01}u_{10}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.18	-0.06	0.01	0.02	-0.35	-0.09	-0.11	-0.13
25	30	-0.01	-0.04	0.02	-0.01	-0.11	-0.09	-0.14	-0.03
50	30	-0.01	-0.18	-0.11	-0.04	-0.04	-0.26	-0.21	-0.11
75	30	0.04	-0.21	-0.20	-0.13	-0.06	-0.33	-0.30	-0.25
10	100	0.01	-0.05	0.00	-0.03	-0.08	-0.08	-0.09	-0.09
25	100	0.01	-0.31	-0.23	-0.14	-0.06	-0.41	-0.33	-0.25
50	100	-0.16	-0.35	-0.31	-0.31	-0.26	-0.59	-0.56	-0.52
75	100	-0.16	-0.28	-0.26	-0.28	-0.43	-0.61	-0.60	-0.57
10	150	0.01	-0.01	0.00	-0.02	0.03	-0.06	-0.06	-0.09
25	150	-0.07	-0.37	-0.28	-0.22	-0.10	-0.53	-0.46	-0.37
50	150	-0.14	-0.30	-0.28	-0.25	-0.42	-0.66	-0.64	-0.60
75	150	-0.15	-0.20	-0.16	-0.18	-0.55	-0.68	-0.67	-0.65

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B21.

Relative Bias Values of $\sigma_{u_{01}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.28	-0.04	0.02	0.24	0.89	-0.23	-0.08	0.12
25	30	0.04	-0.94	-0.67	-0.63	-0.24	-0.87	-0.81	-0.63
50	30	-0.59	-1.40	-1.35	-1.15	-0.71	-1.46	-1.43	-1.25
75	30	-0.85	-1.25	-1.28	-1.28	-1.15	-1.47	-1.48	-1.45
10	100	0.26	-0.26	-0.17	-0.29	-0.15	-0.30	-0.33	-0.37
25	100	-0.57	-1.07	-0.98	-0.85	-0.75	-1.23	-1.16	-1.09
50	100	-0.79	-0.87	-0.93	-0.94	-1.25	-1.32	-1.34	-1.38
75	100	-0.71	-0.66	-0.67	-0.63	-1.39	-1.28	-1.29	-1.32
10	150	0.02	-0.18	-0.25	-0.24	-0.30	-0.35	-0.31	-0.44
25	150	-0.60	-0.93	-0.88	-0.84	-0.90	-1.24	-1.20	-1.12
50	150	-0.64	-0.63	-0.69	-0.69	-1.32	-1.26	-1.28	-1.32
75	150	-0.50	-0.43	-0.38	-0.42	-1.36	-1.22	-1.24	-1.27

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table B22.

Relative Bias Values of $\sigma_{u_{10}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	-0.26	-0.15	-0.27	-0.20	-0.23	-0.29	-0.28	-0.23
25	30	-0.18	-0.96	-0.68	-0.66	-0.40	-0.80	-0.75	-0.51
50	30	-0.59	-1.59	-1.46	-1.23	-0.57	-1.66	-1.57	-1.28
75	30	-0.91	-1.46	-1.42	-1.43	-1.04	-1.72	-1.70	-1.64
10	100	-0.20	-0.21	-0.19	-0.33	-0.36	-0.23	-0.27	-0.29
25	100	-0.52	-1.09	-0.98	-0.82	-0.53	-1.28	-1.17	-1.05
50	100	-0.82	-0.95	-1.00	-0.99	-1.22	-1.45	-1.46	-1.48
75	100	-0.68	-0.72	-0.72	-0.74	-1.45	-1.42	-1.43	-1.45
10	150	-0.32	-0.23	-0.22	-0.31	-0.29	-0.29	-0.26	-0.39
25	150	-0.60	-0.95	-0.89	-0.80	-0.70	-1.30	-1.23	-1.10
50	150	-0.62	-0.69	-0.70	-0.69	-1.32	-1.37	-1.38	-1.40
75	150	-0.52	-0.48	-0.40	-0.49	-1.41	-1.34	-1.35	-1.37

Note. *L1 sample size is 3. Highlighted cell indicates the condition where the relative bias is not within the acceptable range. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

APPENDIX C

TABLE OF RMSE VALUES

Table C1.

RMSE Values of γ_{000}

Sample Size		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.35	0.28	0.31	0.32	0.32	0.30	0.30	0.33	0.35	0.31	0.31	0.31
25	30	0.21	0.18	0.18	0.21	0.21	0.19	0.20	0.20	0.22	0.19	0.20	0.20
50	30	0.15	0.13	0.13	0.14	0.16	0.12	0.13	0.14	0.15	0.13	0.13	0.14
75	30	0.12	0.10	0.11	0.11	0.13	0.10	0.12	0.12	0.12	0.11	0.11	0.12
10	100	0.18	0.16	0.16	0.18	0.19	0.16	0.17	0.17	0.19	0.17	0.16	0.18
25	100	0.11	0.10	0.10	0.10	0.12	0.10	0.10	0.11	0.12	0.10	0.10	0.11
50	100	0.08	0.07	0.08	0.07	0.09	0.07	0.08	0.08	0.08	0.07	0.08	0.07
75	100	0.07	0.06	0.06	0.06	0.07	0.06	0.06	0.07	0.06	0.06	0.06	0.06
10	150	0.15	0.13	0.13	0.14	0.15	0.13	0.14	0.14	0.16	0.14	0.14	0.14
25	150	0.09	0.08	0.09	0.09	0.10	0.09	0.09	0.09	0.09	0.08	0.09	0.09
50	150	0.07	0.06	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06
75	150	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.06	0.06	0.05	0.05	0.05
Sample Size						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.35	0.30	0.31	0.34	0.34	0.30	0.30	0.31
25	30					0.22	0.20	0.18	0.20	0.22	0.19	0.19	0.20
50	30					0.15	0.13	0.13	0.14	0.14	0.13	0.14	0.14
75	30					0.12	0.10	0.11	0.11	0.12	0.11	0.11	0.12
10	100					0.18	0.17	0.18	0.19	0.18	0.16	0.17	0.18
25	100					0.12	0.10	0.11	0.11	0.11	0.10	0.11	0.11
50	100					0.08	0.07	0.08	0.07	0.09	0.08	0.08	0.08
75	100					0.07	0.05	0.06	0.06	0.07	0.06	0.07	0.07
10	150					0.15	0.13	0.14	0.15	0.15	0.13	0.14	0.14
25	150					0.10	0.08	0.08	0.09	0.10	0.09	0.09	0.10
50	150					0.07	0.06	0.06	0.06	0.07	0.06	0.06	0.07
75	150					0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.06

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C2.

RMSE Values of γ_{010}

Sample Size*		L1L2				L1L2L3			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.06	0.05	0.06	0.06	0.04	0.04	0.04	0.03
25	30	0.05	0.05	0.05	0.05	0.02	0.02	0.02	0.02
50	30	0.05	0.05	0.05	0.05	0.02	0.01	0.02	0.02
75	30	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01
10	100	0.05	0.05	0.05	0.05	0.02	0.02	0.02	0.02
25	100	0.06	0.05	0.06	0.05	0.01	0.01	0.01	0.01
50	100	0.07	0.07	0.07	0.07	0.01	0.01	0.01	0.01
75	100	0.08	0.08	0.08	0.08	0.01	0.01	0.01	0.01
10	150	0.05	0.06	0.05	0.05	0.02	0.02	0.02	0.02
25	150	0.06	0.07	0.07	0.06	0.01	0.01	0.01	0.01
50	150	0.08	0.08	0.08	0.08	0.01	0.01	0.01	0.01
75	150	0.09	0.09	0.09	0.09	0.01	0.00	0.01	0.01
Sample Size*		L1L2 No L3 Covariance				L1L2L3 No L3 Covariance			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.06	0.05	0.05	0.05	0.04	0.03	0.04	0.04
25	30	0.05	0.04	0.05	0.05	0.02	0.02	0.02	0.02
50	30	0.05	0.05	0.05	0.06	0.02	0.04	0.01	0.02
75	30	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01
10	100	0.05	0.05	0.05	0.05	0.02	0.02	0.02	0.02
25	100	0.06	0.06	0.06	0.06	0.01	0.01	0.01	0.01
50	100	0.07	0.07	0.08	0.07	0.01	0.01	0.01	0.01
75	100	0.08	0.08	0.08	0.08	0.01	0.01	0.01	0.01
10	150	0.05	0.06	0.05	0.05	0.02	0.01	0.02	0.02
25	150	0.07	0.06	0.07	0.07	0.01	0.01	0.01	0.01
50	150	0.08	0.08	0.08	0.08	0.01	0.01	0.01	0.01
75	150	0.09	0.09	0.09	0.09	0.01	0.01	0.01	0.01

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C3.

RMSE Values of γ_{100}

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.18	0.19	0.18	0.19	0.18	0.19	0.18	0.19	0.19	0.19	0.19	0.19
25	30	0.12	0.12	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.11	0.11
50	30	0.08	0.08	0.08	0.08	0.09	0.08	0.08	0.08	0.08	0.09	0.08	0.08
75	30	0.06	0.06	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
10	100	0.11	0.10	0.10	0.11	0.11	0.10	0.10	0.10	0.11	0.11	0.10	0.11
25	100	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.07	0.06	0.06	0.07	0.07
50	100	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.05	0.05	0.04
75	100	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
10	150	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.08	0.09
25	150	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06	0.05	0.05
50	150	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
75	150	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.18	0.19	0.19	0.19	0.19	0.18	0.20	0.21
25	30					0.12	0.11	0.11	0.12	0.12	0.13	0.12	0.12
50	30					0.08	0.08	0.08	0.08	0.08	0.09	0.08	0.08
75	30					0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
10	100					0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.11
25	100					0.07	0.06	0.07	0.07	0.07	0.07	0.06	0.07
50	100					0.05	0.05	0.05	0.04	0.05	0.05	0.05	0.05
75	100					0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
10	150					0.08	0.08	0.09	0.09	0.09	0.08	0.09	0.09
25	150					0.05	0.05	0.05	0.06	0.06	0.05	0.05	0.05
50	150					0.04	0.03	0.04	0.04	0.04	0.04	0.04	0.04
75	150					0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C4.

RMSE Values of γ_{110}

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.023	0.024	0.023	0.023	0.023	0.022	0.022	0.021
25	30	0.015	0.015	0.014	0.014	0.013	0.013	0.013	0.013
50	30	0.010	0.011	0.011	0.011	0.009	0.008	0.010	0.009
75	30	0.010	0.009	0.010	0.010	0.008	0.008	0.008	0.008
10	100	0.014	0.013	0.014	0.013	0.012	0.012	0.012	0.012
25	100	0.009	0.009	0.009	0.009	0.007	0.008	0.007	0.007
50	100	0.008	0.008	0.007	0.007	0.005	0.005	0.005	0.005
75	100	0.006	0.006	0.006	0.006	0.004	0.004	0.004	0.004
10	150	0.012	0.011	0.011	0.013	0.010	0.009	0.009	0.010
25	150	0.008	0.009	0.009	0.008	0.006	0.006	0.006	0.006
50	150	0.007	0.007	0.007	0.007	0.004	0.004	0.004	0.004
75	150	0.006	0.006	0.006	0.006	0.003	0.003	0.003	0.003
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.024	0.023	0.023	0.023	0.023	0.021	0.024	0.022
25	30	0.016	0.014	0.015	0.015	0.013	0.014	0.013	0.013
50	30	0.010	0.010	0.011	0.011	0.010	0.009	0.009	0.009
75	30	0.009	0.010	0.010	0.009	0.007	0.008	0.008	0.007
10	100	0.013	0.013	0.013	0.014	0.012	0.012	0.012	0.011
25	100	0.010	0.009	0.009	0.009	0.007	0.007	0.007	0.007
50	100	0.008	0.007	0.007	0.007	0.005	0.005	0.005	0.005
75	100	0.006	0.006	0.006	0.006	0.004	0.004	0.004	0.004
10	150	0.011	0.012	0.012	0.011	0.010	0.010	0.010	0.010
25	150	0.008	0.008	0.008	0.008	0.006	0.006	0.006	0.006
50	150	0.007	0.006	0.006	0.006	0.004	0.004	0.004	0.004
75	150	0.006	0.006	0.006	0.006	0.003	0.003	0.003	0.003

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C5.

RMSE Values of γ_{001}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.17	0.22	0.21	0.20	0.17	0.22	0.20	0.19
25	30	0.22	0.33	0.29	0.26	0.22	0.29	0.29	0.29
50	30	0.27	0.42	0.40	0.36	0.29	0.46	0.41	0.38
75	30	0.33	0.50	0.48	0.43	0.31	0.49	0.49	0.47
10	100	0.08	0.12	0.11	0.10	0.09	0.12	0.11	0.09
25	100	0.12	0.16	0.16	0.15	0.11	0.16	0.16	0.14
50	100	0.15	0.23	0.20	0.19	0.14	0.23	0.22	0.19
75	100	0.17	0.29	0.24	0.23	0.17	0.27	0.25	0.23
10	150	0.07	0.09	0.09	0.08	0.07	0.09	0.09	0.09
25	150	0.09	0.14	0.13	0.12	0.09	0.13	0.12	0.12
50	150	0.11	0.19	0.18	0.15	0.11	0.18	0.17	0.16
75	150	0.14	0.22	0.21	0.18	0.14	0.22	0.21	0.18

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C6.

RMSE Values of γ_{011}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.013	0.015	0.014	0.014	0.013	0.014	0.013	0.014
25	30	0.015	0.015	0.016	0.015	0.014	0.016	0.016	0.015
50	30	0.017	0.019	0.018	0.017	0.016	0.018	0.019	0.017
75	30	0.017	0.020	0.021	0.018	0.017	0.022	0.019	0.020
10	100	0.007	0.007	0.007	0.007	0.008	0.007	0.007	0.007
25	100	0.007	0.009	0.008	0.008	0.007	0.008	0.008	0.008
50	100	0.008	0.010	0.010	0.009	0.008	0.010	0.010	0.009
75	100	0.009	0.011	0.010	0.010	0.009	0.011	0.011	0.010
10	150	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
25	150	0.006	0.007	0.007	0.006	0.006	0.007	0.006	0.007
50	150	0.007	0.008	0.008	0.008	0.007	0.008	0.008	0.007
75	150	0.007	0.009	0.008	0.008	0.007	0.009	0.008	0.008

*Note. *L1 sample size is 3.* ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C7.

RMSE Values of γ_{101}

Sample Size*		L1L2L3				L1L2L3 No L3 Covariance			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.11	0.16	0.14	0.14	0.11	0.15	0.15	0.13
25	30	0.14	0.21	0.21	0.18	0.14	0.23	0.20	0.20
50	30	0.19	0.31	0.27	0.24	0.18	0.32	0.27	0.27
75	30	0.22	0.38	0.33	0.30	0.22	0.36	0.33	0.30
10	100	0.06	0.09	0.07	0.07	0.06	0.08	0.07	0.07
25	100	0.07	0.12	0.11	0.10	0.07	0.12	0.11	0.10
50	100	0.10	0.16	0.15	0.13	0.10	0.15	0.14	0.14
75	100	0.12	0.21	0.18	0.16	0.12	0.21	0.18	0.16
10	150	0.04	0.07	0.06	0.06	0.05	0.06	0.06	0.06
25	150	0.06	0.10	0.09	0.09	0.06	0.10	0.08	0.08
50	150	0.08	0.13	0.12	0.11	0.08	0.13	0.12	0.11
75	150	0.10	0.16	0.15	0.13	0.10	0.17	0.15	0.13

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C8.

RMSE Values of γ_{111}

Sample Size*		L1L2L3				L1L2L3 No L3 Covariance			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.008	0.010	0.009	0.008	0.008	0.009	0.009	0.009
25	30	0.009	0.010	0.010	0.010	0.008	0.010	0.009	0.009
50	30	0.009	0.012	0.011	0.011	0.009	0.012	0.011	0.010
75	30	0.010	0.014	0.012	0.011	0.010	0.014	0.012	0.012
10	100	0.004	0.004	0.004	0.004	0.004	0.005	0.004	0.004
25	100	0.004	0.005	0.005	0.005	0.004	0.005	0.005	0.005
50	100	0.005	0.006	0.006	0.006	0.005	0.006	0.006	0.006
75	100	0.005	0.007	0.007	0.006	0.005	0.007	0.007	0.006
10	150	0.004	0.004	0.004	0.003	0.003	0.004	0.004	0.003
25	150	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004
50	150	0.004	0.005	0.005	0.005	0.004	0.005	0.005	0.005
75	150	0.004	0.006	0.005	0.005	0.004	0.006	0.006	0.005

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C9.

RMSE Values of σ_e^2

Sample Size		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.34	1.44	1.44	1.42	1.45	1.43	1.50	1.42	1.45	1.38	1.42	1.35
25	30	0.93	0.83	0.90	0.91	0.90	0.92	0.89	0.85	0.91	0.87	0.90	0.89
50	30	0.66	0.62	0.66	0.66	0.62	0.65	0.69	0.64	0.65	0.67	0.65	0.64
75	30	0.54	0.53	0.51	0.52	0.52	0.50	0.50	0.52	0.54	0.53	0.53	0.53
10	100	0.76	0.74	0.78	0.80	0.83	0.78	0.77	0.78	0.83	0.76	0.78	0.78
25	100	0.51	0.49	0.47	0.52	0.50	0.49	0.51	0.51	0.51	0.53	0.50	0.48
50	100	0.38	0.35	0.38	0.36	0.37	0.38	0.37	0.39	0.39	0.37	0.35	0.37
75	100	0.31	0.29	0.30	0.29	0.30	0.29	0.30	0.31	0.29	0.30	0.31	0.29
10	150	0.66	0.64	0.65	0.66	0.65	0.63	0.66	0.62	0.64	0.60	0.62	0.63
25	150	0.41	0.40	0.41	0.41	0.43	0.41	0.43	0.42	0.43	0.43	0.43	0.42
50	150	0.28	0.30	0.31	0.31	0.30	0.28	0.30	0.30	0.31	0.29	0.32	0.29
75	150	0.26	0.24	0.26	0.25	0.26	0.25	0.25	0.25	0.28	0.24	0.25	0.25
Sample Size						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.36	1.37	1.45	1.47	1.43	1.36	1.38	1.42
25	30					0.88	0.89	0.89	0.91	0.95	0.85	0.84	0.87
50	30					0.59	0.62	0.63	0.63	0.65	0.63	0.64	0.67
75	30					0.53	0.54	0.52	0.53	0.53	0.54	0.52	0.49
10	100					0.76	0.80	0.75	0.76	0.81	0.70	0.85	0.77
25	100					0.51	0.53	0.51	0.54	0.53	0.51	0.50	0.52
50	100					0.34	0.38	0.38	0.37	0.35	0.35	0.35	0.35
75	100					0.31	0.29	0.29	0.31	0.30	0.28	0.30	0.30
10	150					0.66	0.65	0.64	0.63	0.65	0.67	0.62	0.61
25	150					0.40	0.44	0.43	0.43	0.43	0.43	0.42	0.46
50	150					0.31	0.31	0.31	0.30	0.30	0.31	0.29	0.32
75	150					0.26	0.25	0.26	0.25	0.23	0.24	0.26	0.24

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C10.

RMSE Values of $\sigma^2_{r_0}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	2.98	2.34	2.41	2.69	4.94	4.94	5.30	5.23	2.77	2.16	2.48	2.41
25	30	1.83	1.34	1.50	1.64	6.95	7.08	7.43	7.11	1.72	1.37	1.54	1.63
50	30	1.32	1.04	1.07	1.10	9.72	8.48	9.89	10.26	1.25	1.05	1.09	1.14
75	30	1.06	0.86	0.84	0.95	13.04	12.62	13.02	11.86	1.04	0.87	0.91	0.92
10	100	1.68	1.29	1.38	1.50	9.06	8.45	8.63	8.67	1.55	1.27	1.41	1.43
25	100	1.04	0.82	0.85	0.94	12.80	11.33	12.03	11.38	1.02	0.76	0.86	0.86
50	100	0.73	0.60	0.59	0.62	16.83	17.45	16.84	16.65	0.69	0.60	0.67	0.67
75	100	0.58	0.49	0.50	0.53	20.43	20.64	20.28	20.53	0.58	0.48	0.51	0.52
10	150	1.32	1.04	1.14	1.22	9.24	10.86	10.17	10.87	1.36	1.05	1.09	1.21
25	150	0.83	0.66	0.70	0.74	14.83	15.34	16.02	14.59	0.86	0.68	0.74	0.75
50	150	0.60	0.48	0.48	0.52	19.98	20.33	20.15	20.41	0.60	0.48	0.53	0.54
75	150	0.49	0.39	0.43	0.44	23.33	23.16	24.11	23.33	0.50	0.40	0.41	0.45
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					5.99	5.22	4.79	5.53	2.86	2.25	2.46	2.58
25	30					7.65	5.60	6.67	7.27	1.81	1.35	1.54	1.58
50	30					8.89	8.97	10.22	11.08	1.24	2.54	1.06	1.14
75	30					12.78	13.01	12.65	12.37	1.02	0.83	0.86	0.92
10	100					7.86	9.17	8.03	8.42	1.57	1.30	1.49	1.38
25	100					12.60	13.70	13.47	12.85	1.04	0.83	0.85	0.94
50	100					16.98	16.85	18.54	17.34	0.69	0.56	0.59	0.63
75	100					20.39	20.31	21.11	19.81	0.56	0.45	0.51	0.53
10	150					10.33	11.27	9.66	9.30	1.30	1.04	1.06	1.16
25	150					15.32	15.22	15.14	16.28	0.82	0.66	0.72	0.73
50	150					19.98	19.64	19.63	19.73	0.60	0.48	0.50	0.54
75	150					22.42	22.96	23.55	23.89	0.45	0.38	0.40	0.44

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C11.

RMSE Values of $\sigma^2_{r_1}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.83	0.84	0.82	0.80	0.69	0.82	0.84	0.77	0.75	0.74	0.75	0.72
25	30	0.59	0.49	0.54	0.55	0.54	0.52	0.53	0.55	0.54	0.54	0.52	0.54
50	30	0.39	0.39	0.39	0.38	0.37	0.40	0.42	0.40	0.39	0.40	0.40	0.40
75	30	0.34	0.31	0.34	0.32	0.34	0.32	0.34	0.31	0.33	0.31	0.38	0.32
10	100	0.50	0.47	0.47	0.49	0.48	0.48	0.46	0.48	0.47	0.43	0.49	0.47
25	100	0.33	0.32	0.29	0.31	0.34	0.32	0.33	0.30	0.31	0.29	0.31	0.29
50	100	0.24	0.21	0.23	0.24	0.25	0.28	0.26	0.27	0.25	0.22	0.23	0.23
75	100	0.20	0.18	0.19	0.19	0.22	0.25	0.24	0.21	0.20	0.18	0.19	0.20
10	150	0.39	0.39	0.42	0.39	0.40	0.37	0.41	0.40	0.39	0.40	0.35	0.39
25	150	0.26	0.25	0.25	0.25	0.28	0.30	0.29	0.29	0.29	0.26	0.27	0.28
50	150	0.19	0.18	0.18	0.20	0.22	0.24	0.24	0.22	0.21	0.19	0.19	0.20
75	150	0.17	0.15	0.16	0.16	0.19	0.24	0.23	0.23	0.19	0.15	0.16	0.16
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.74	0.84	0.86	0.82	0.86	0.85	0.83	0.88
25	30					0.54	0.61	0.53	0.56	0.55	0.52	0.52	0.55
50	30					0.41	0.38	0.39	0.41	0.44	0.35	0.37	0.41
75	30					0.33	0.37	0.31	0.35	0.33	0.31	0.32	0.33
10	100					0.47	0.50	0.49	0.47	0.46	0.45	0.53	0.46
25	100					0.32	0.36	0.36	0.33	0.33	0.34	0.32	0.35
50	100					0.24	0.25	0.26	0.26	0.23	0.23	0.20	0.21
75	100					0.20	0.23	0.23	0.23	0.19	0.16	0.18	0.19
10	150					0.41	0.43	0.43	0.42	0.41	0.42	0.38	0.37
25	150					0.27	0.31	0.33	0.31	0.28	0.29	0.28	0.28
50	150					0.22	0.25	0.23	0.23	0.19	0.19	0.18	0.20
75	150					0.19	0.23	0.22	0.22	0.16	0.14	0.16	0.15

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C12.

RMSE Values of $\sigma^2_{u_{00}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.24	1.70	1.58	1.61	1.22	2.19	2.03	1.75	1.21	2.35	2.07	1.74
25	30	0.82	1.09	1.04	0.96	0.89	3.96	2.77	2.17	0.90	3.75	2.87	1.99
50	30	0.52	0.75	0.69	0.63	1.03	5.44	4.31	3.10	1.02	5.71	4.53	3.24
75	30	0.42	0.65	0.59	0.55	1.29	5.38	4.41	3.47	1.38	5.96	4.90	3.77
10	100	0.76	0.96	0.98	0.91	0.92	2.49	1.98	1.84	0.92	2.32	2.01	1.78
25	100	0.41	0.59	0.57	0.53	1.16	5.67	4.45	3.21	1.15	6.12	4.81	3.54
50	100	0.29	0.39	0.38	0.35	1.59	5.57	4.70	3.75	1.89	6.80	5.62	4.47
75	100	0.22	0.32	0.32	0.29	1.58	5.00	4.23	3.32	2.15	6.82	5.67	4.52
10	150	0.65	0.81	0.70	0.71	0.94	2.72	2.31	1.81	0.95	2.75	2.20	2.00
25	150	0.34	0.45	0.45	0.40	1.33	5.66	4.54	3.41	1.45	6.59	5.25	3.89
50	150	0.23	0.34	0.32	0.30	1.60	5.20	4.37	3.43	2.13	7.00	5.82	4.63
75	150	0.18	0.26	0.26	0.23	1.46	4.56	3.55	2.98	2.28	7.03	5.84	4.66
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.29	2.51	2.11	1.74	1.24	2.71	2.10	1.68
25	30					0.99	4.60	3.48	2.44	0.93	4.54	3.52	2.47
50	30					1.22	5.52	4.37	3.23	1.16	5.76	4.63	3.48
75	30					1.46	5.42	4.50	3.51	1.55	5.95	4.92	3.85
10	100					0.92	4.05	2.97	2.21	0.94	4.12	3.15	2.34
25	100					1.36	5.91	4.69	3.68	1.46	6.50	5.28	3.93
50	100					1.65	5.70	4.48	3.67	2.00	6.80	5.63	4.48
75	100					1.62	5.53	4.30	3.49	2.18	6.82	5.67	4.52
10	150					1.02	4.60	3.60	2.54	0.98	4.92	3.76	2.58
25	150					1.49	5.96	4.88	3.71	1.66	6.88	5.67	4.36
50	150					1.65	5.39	4.46	3.53	2.19	7.01	5.82	4.64
75	150					1.53	5.08	4.00	2.97	2.30	7.02	5.84	4.66

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C13.

RMSE Values of $\sigma^2_{u_{01}}$

Sample Size*			L1L2 Model				L1L2L3 Model			
L2	L3		ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30		0.011	0.009	0.009	0.010	0.009	0.008	0.008	0.008
25	30		0.004	0.005	0.005	0.005	0.003	0.005	0.004	0.004
50	30		0.002	0.005	0.004	0.003	0.002	0.006	0.004	0.003
75	30		0.002	0.006	0.005	0.004	0.001	0.006	0.005	0.004
10	100		0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.004
25	100		0.002	0.006	0.005	0.003	0.002	0.006	0.005	0.003
50	100		0.002	0.007	0.005	0.004	0.002	0.007	0.006	0.005
75	100		0.002	0.006	0.005	0.004	0.002	0.008	0.006	0.005
10	150		0.004	0.005	0.005	0.004	0.003	0.005	0.004	0.004
25	150		0.002	0.006	0.005	0.004	0.001	0.007	0.005	0.004
50	150		0.002	0.006	0.005	0.004	0.002	0.008	0.006	0.005
75	150		0.002	0.006	0.005	0.004	0.002	0.008	0.007	0.005
Sample Size*			L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3		ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30		0.007	0.007	0.008	0.007	0.006	0.007	0.007	0.007
25	30		0.003	0.004	0.004	0.004	0.003	0.004	0.004	0.003
50	30		0.002	0.006	0.004	0.003	0.002	0.006	0.004	0.003
75	30		0.002	0.006	0.005	0.004	0.001	0.006	0.005	0.004
10	100		0.004	0.005	0.005	0.004	0.003	0.004	0.004	0.004
25	100		0.002	0.007	0.005	0.004	0.001	0.006	0.005	0.003
50	100		0.002	0.008	0.006	0.005	0.002	0.007	0.006	0.005
75	100		0.002	0.008	0.007	0.005	0.002	0.008	0.006	0.005
10	150		0.003	0.005	0.004	0.004	0.003	0.004	0.004	0.003
25	150		0.002	0.007	0.006	0.004	0.001	0.007	0.006	0.004
50	150		0.002	0.008	0.007	0.005	0.002	0.008	0.006	0.005
75	150		0.003	0.008	0.007	0.005	0.002	0.008	0.007	0.005

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C14.

RMSE Values of $\sigma^2_{u_{10}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.49	0.82	0.75	0.65	0.49	1.04	0.89	0.76	0.51	1.15	0.91	0.76
25	30	0.30	0.50	0.45	0.41	0.37	2.00	1.38	1.09	0.41	1.89	1.46	0.99
50	30	0.20	0.34	0.33	0.28	0.51	2.75	2.18	1.57	0.51	2.89	2.30	1.65
75	30	0.16	0.29	0.26	0.25	0.64	2.72	2.23	1.76	0.71	3.02	2.48	1.91
10	100	0.28	0.41	0.40	0.35	0.41	1.22	0.97	0.88	0.41	1.16	0.98	0.89
25	100	0.16	0.28	0.24	0.23	0.59	2.87	2.25	1.63	0.58	3.10	2.44	1.80
50	100	0.11	0.19	0.17	0.15	0.81	2.81	2.37	1.90	0.97	3.44	2.84	2.27
75	100	0.09	0.15	0.14	0.12	0.80	2.52	2.13	1.68	1.10	3.45	2.87	2.29
10	150	0.23	0.34	0.31	0.29	0.44	1.36	1.16	0.89	0.45	1.39	1.11	1.01
25	150	0.13	0.22	0.19	0.19	0.68	2.86	2.30	1.73	0.74	3.33	2.66	1.98
50	150	0.08	0.15	0.14	0.12	0.81	2.62	2.20	1.73	1.09	3.54	2.94	2.34
75	150	0.07	0.12	0.11	0.10	0.73	2.30	1.79	1.50	1.16	3.55	2.95	2.36
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.47	1.25	0.98	0.76	0.46	1.32	1.02	0.80
25	30					0.45	2.32	1.77	1.24	0.42	2.30	1.80	1.27
50	30					0.63	2.80	2.22	1.64	0.59	2.92	2.34	1.78
75	30					0.74	2.74	2.27	1.78	0.79	3.01	2.49	1.95
10	100					0.43	2.04	1.51	1.11	0.44	2.09	1.60	1.19
25	100					0.69	2.99	2.37	1.87	0.75	3.29	2.68	2.00
50	100					0.84	2.88	2.27	1.86	1.02	3.43	2.85	2.27
75	100					0.82	2.79	2.17	1.76	1.11	3.44	2.87	2.29
10	150					0.50	2.34	1.83	1.29	0.49	2.50	1.92	1.32
25	150					0.76	3.01	2.47	1.88	0.85	3.48	2.87	2.22
50	150					0.84	2.72	2.25	1.78	1.12	3.54	2.94	2.35
75	150					0.77	2.57	2.02	1.50	1.17	3.55	2.95	2.36

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C15.

RMSE Values of $\sigma^2_{u_{11}}$

Sample Size		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.003
25	30	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.001
50	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
75	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
10	100	0.002	0.002	0.002	0.002	0.001	0.002	0.002	0.002
25	100	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
50	100	0.001	0.003	0.002	0.002	0.001	0.003	0.002	0.002
75	100	0.001	0.002	0.002	0.002	0.001	0.003	0.002	0.002
10	150	0.002	0.003	0.002	0.002	0.001	0.002	0.002	0.001
25	150	0.001	0.003	0.002	0.002	0.001	0.003	0.002	0.001
50	150	0.001	0.003	0.002	0.002	0.001	0.003	0.002	0.002
75	150	0.001	0.002	0.002	0.002	0.001	0.003	0.003	0.002
Sample Size		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.002	0.003	0.003	0.002	0.002	0.003	0.003	0.003
25	30	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.001
50	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
75	30	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
10	100	0.001	0.002	0.001	0.001	0.001	0.002	0.002	0.002
25	100	0.001	0.002	0.002	0.001	0.001	0.002	0.002	0.001
50	100	0.001	0.003	0.002	0.002	0.001	0.002	0.002	0.002
75	100	0.001	0.003	0.002	0.002	0.001	0.002	0.002	0.002
10	150	0.001	0.002	0.001	0.001	0.001	0.002	0.002	0.001
25	150	0.001	0.003	0.002	0.002	0.001	0.002	0.002	0.002
50	150	0.001	0.003	0.002	0.002	0.001	0.002	0.002	0.002
75	150	0.001	0.003	0.003	0.002	0.001	0.002	0.002	0.001

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C16.

RMSE Values of $\sigma_{r_0 r_1}$

Sample Size		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	1.23	1.15	1.14	1.20	1.11	1.10	1.18	1.20	1.13	1.05	1.11	1.11
25	30	0.81	0.67	0.75	0.76	0.85	0.75	0.78	0.82	0.76	0.73	0.72	0.74
50	30	0.58	0.53	0.52	0.52	0.63	0.61	0.64	0.65	0.58	0.53	0.54	0.57
75	30	0.48	0.43	0.44	0.45	0.55	0.58	0.58	0.57	0.46	0.42	0.47	0.44
10	100	0.77	0.66	0.66	0.72	0.79	0.71	0.71	0.72	0.68	0.61	0.69	0.65
25	100	0.47	0.43	0.40	0.44	0.54	0.53	0.56	0.52	0.44	0.38	0.43	0.40
50	100	0.34	0.30	0.30	0.31	0.55	0.57	0.55	0.54	0.33	0.29	0.33	0.32
75	100	0.28	0.25	0.25	0.26	0.55	0.61	0.59	0.56	0.27	0.24	0.25	0.26
10	150	0.57	0.52	0.57	0.56	0.65	0.64	0.62	0.64	0.59	0.54	0.50	0.58
25	150	0.39	0.35	0.35	0.35	0.51	0.57	0.59	0.54	0.40	0.35	0.38	0.36
50	150	0.28	0.25	0.25	0.27	0.52	0.58	0.59	0.56	0.28	0.25	0.27	0.27
75	150	0.23	0.20	0.22	0.22	0.55	0.65	0.63	0.63	0.24	0.20	0.21	0.22
Sample Size						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.19	1.11	1.13	1.18	1.22	1.12	1.13	1.15
25	30					0.82	0.78	0.74	0.78	0.77	0.67	0.75	0.75
50	30					0.60	0.54	0.57	0.59	0.60	0.50	0.52	0.54
75	30					0.56	0.53	0.51	0.55	0.46	0.41	0.43	0.45
10	100					0.75	0.70	0.70	0.73	0.68	0.65	0.72	0.63
25	100					0.53	0.52	0.54	0.51	0.46	0.41	0.43	0.47
50	100					0.49	0.44	0.48	0.48	0.31	0.30	0.29	0.29
75	100					0.49	0.48	0.48	0.49	0.26	0.23	0.25	0.26
10	150					0.67	0.60	0.64	0.60	0.57	0.54	0.50	0.54
25	150					0.49	0.49	0.50	0.52	0.37	0.35	0.36	0.37
50	150					0.48	0.47	0.47	0.48	0.27	0.25	0.25	0.28
75	150					0.49	0.50	0.50	0.54	0.22	0.19	0.21	0.21

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C17.

RMSE Values of $\sigma_{u_{00}u_{01}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.08	0.13	0.12	0.12	0.08	0.13	0.11	0.10
25	30	0.05	0.15	0.12	0.10	0.05	0.13	0.11	0.09
50	30	0.05	0.18	0.15	0.11	0.04	0.18	0.15	0.11
75	30	0.05	0.17	0.14	0.12	0.05	0.18	0.15	0.12
10	100	0.06	0.09	0.09	0.08	0.05	0.08	0.08	0.07
25	100	0.05	0.14	0.12	0.09	0.04	0.15	0.12	0.10
50	100	0.05	0.13	0.12	0.09	0.05	0.16	0.13	0.11
75	100	0.04	0.12	0.10	0.08	0.05	0.15	0.13	0.10
10	150	0.05	0.09	0.08	0.07	0.05	0.08	0.07	0.07
25	150	0.05	0.13	0.11	0.09	0.04	0.15	0.12	0.09
50	150	0.04	0.12	0.10	0.08	0.05	0.15	0.13	0.10
75	150	0.04	0.10	0.08	0.07	0.05	0.15	0.12	0.10

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C18.

RMSE Values of $\sigma_{u_{00}u_{10}}$

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.76	1.35	1.17	1.05	0.69	1.49	1.31	1.10	0.66	1.64	1.28	1.05
25	30	0.50	1.08	0.95	0.80	0.57	3.07	2.15	1.73	0.60	2.83	2.22	1.50
50	30	0.40	1.01	0.85	0.69	0.81	4.14	3.31	2.40	0.78	4.25	3.41	2.45
75	30	0.37	0.95	0.83	0.69	1.01	4.05	3.33	2.65	1.07	4.40	3.62	2.81
10	100	0.50	1.01	0.87	0.75	0.58	1.52	1.23	1.15	0.58	1.42	1.23	1.16
25	100	0.34	0.90	0.77	0.64	0.74	3.35	2.65	1.94	0.74	3.60	2.85	2.14
50	100	0.32	0.90	0.75	0.61	0.96	3.22	2.73	2.19	1.15	3.92	3.26	2.61
75	100	0.31	0.87	0.74	0.60	0.93	2.88	2.43	1.93	1.28	3.91	3.26	2.61
10	150	0.42	0.96	0.80	0.67	0.60	1.57	1.36	1.08	0.61	1.59	1.29	1.21
25	150	0.35	0.91	0.76	0.62	0.79	3.12	2.53	1.92	0.87	3.63	2.91	2.19
50	150	0.30	0.88	0.74	0.59	0.90	2.82	2.38	1.88	1.20	3.80	3.17	2.54
75	150	0.30	0.86	0.73	0.58	0.81	2.46	1.93	1.62	1.27	3.79	3.16	2.53

Note. *L1 sample size is 3. ICC1, (ICCL1=0.5, ICCL2=0.40, ICCL3=0.10). ICC2 (ICCL1=0.5, ICCL2=0.20, ICCL3=0.30). ICC3 (ICCL1=0.5, ICCL2=0.25, ICCL3=0.25). ICC4 (ICCL1=0.5, ICCL2=0.30, ICCL3=0.20).

Table C19.

RMSE Values of $\sigma_{u_{00}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.046	0.075	0.067	0.064	0.041	0.066	0.063	0.061
25	30	0.028	0.045	0.042	0.037	0.024	0.039	0.037	0.033
50	30	0.022	0.034	0.031	0.026	0.016	0.028	0.025	0.022
75	30	0.017	0.030	0.028	0.023	0.014	0.024	0.021	0.019
10	100	0.031	0.042	0.044	0.039	0.026	0.038	0.036	0.033
25	100	0.017	0.023	0.024	0.024	0.014	0.017	0.017	0.016
50	100	0.013	0.020	0.017	0.016	0.008	0.010	0.010	0.009
75	100	0.011	0.019	0.016	0.015	0.006	0.008	0.008	0.007
10	150	0.026	0.041	0.038	0.036	0.021	0.031	0.027	0.027
25	150	0.018	0.022	0.021	0.020	0.011	0.012	0.012	0.012
50	150	0.012	0.019	0.016	0.015	0.005	0.006	0.006	0.006
75	150	0.010	0.016	0.015	0.013	0.004	0.004	0.004	0.004

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C20.

RMSE Values of $\sigma_{u_{01}u_{10}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.053	0.082	0.072	0.071	0.050	0.074	0.070	0.067
25	30	0.032	0.036	0.041	0.039	0.029	0.036	0.035	0.035
50	30	0.021	0.021	0.024	0.026	0.017	0.019	0.016	0.017
75	30	0.017	0.026	0.023	0.020	0.012	0.021	0.016	0.012
10	100	0.034	0.044	0.045	0.040	0.029	0.039	0.038	0.033
25	100	0.018	0.032	0.031	0.022	0.015	0.030	0.023	0.018
50	100	0.014	0.035	0.030	0.024	0.008	0.037	0.029	0.022
75	100	0.014	0.033	0.028	0.024	0.010	0.038	0.031	0.024
10	150	0.029	0.041	0.037	0.036	0.024	0.033	0.030	0.029
25	150	0.018	0.036	0.030	0.026	0.011	0.036	0.027	0.020
50	150	0.015	0.036	0.031	0.025	0.010	0.041	0.033	0.025
75	150	0.014	0.034	0.027	0.024	0.012	0.043	0.035	0.027

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C21.

RMSE Values of $\sigma_{u_{01}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.004
25	30	0.002	0.005	0.004	0.004	0.002	0.005	0.004	0.003
50	30	0.002	0.006	0.005	0.004	0.002	0.005	0.005	0.004
75	30	0.002	0.005	0.004	0.004	0.002	0.005	0.005	0.004
10	100	0.003	0.004	0.004	0.003	0.002	0.003	0.003	0.003
25	100	0.002	0.005	0.004	0.003	0.002	0.005	0.004	0.003
50	100	0.002	0.004	0.004	0.003	0.002	0.005	0.004	0.003
75	100	0.002	0.004	0.003	0.003	0.002	0.005	0.004	0.003
10	150	0.003	0.004	0.003	0.003	0.002	0.003	0.003	0.003
25	150	0.002	0.004	0.004	0.003	0.002	0.005	0.004	0.003
50	150	0.002	0.004	0.003	0.003	0.002	0.005	0.004	0.003
75	150	0.001	0.003	0.003	0.002	0.002	0.004	0.004	0.003

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table C22.

RMSE Values of $\sigma_{u_{10}u_{11}}$

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.031	0.055	0.051	0.045	0.029	0.050	0.047	0.043
25	30	0.020	0.056	0.044	0.038	0.019	0.050	0.042	0.033
50	30	0.019	0.063	0.052	0.040	0.016	0.063	0.052	0.039
75	30	0.019	0.060	0.051	0.041	0.018	0.063	0.052	0.041
10	100	0.021	0.038	0.035	0.030	0.018	0.032	0.031	0.028
25	100	0.016	0.050	0.041	0.032	0.015	0.052	0.042	0.033
50	100	0.016	0.046	0.039	0.032	0.017	0.053	0.044	0.036
75	100	0.015	0.040	0.035	0.028	0.018	0.052	0.043	0.035
10	150	0.019	0.035	0.030	0.027	0.016	0.031	0.027	0.025
25	150	0.016	0.046	0.038	0.030	0.015	0.050	0.041	0.032
50	150	0.015	0.039	0.034	0.027	0.017	0.050	0.042	0.034
75	150	0.013	0.034	0.027	0.024	0.017	0.049	0.041	0.033

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

APPENDIX D

VISUAL REPRESENTATION OF ABSOLUTE BIAS VALUES

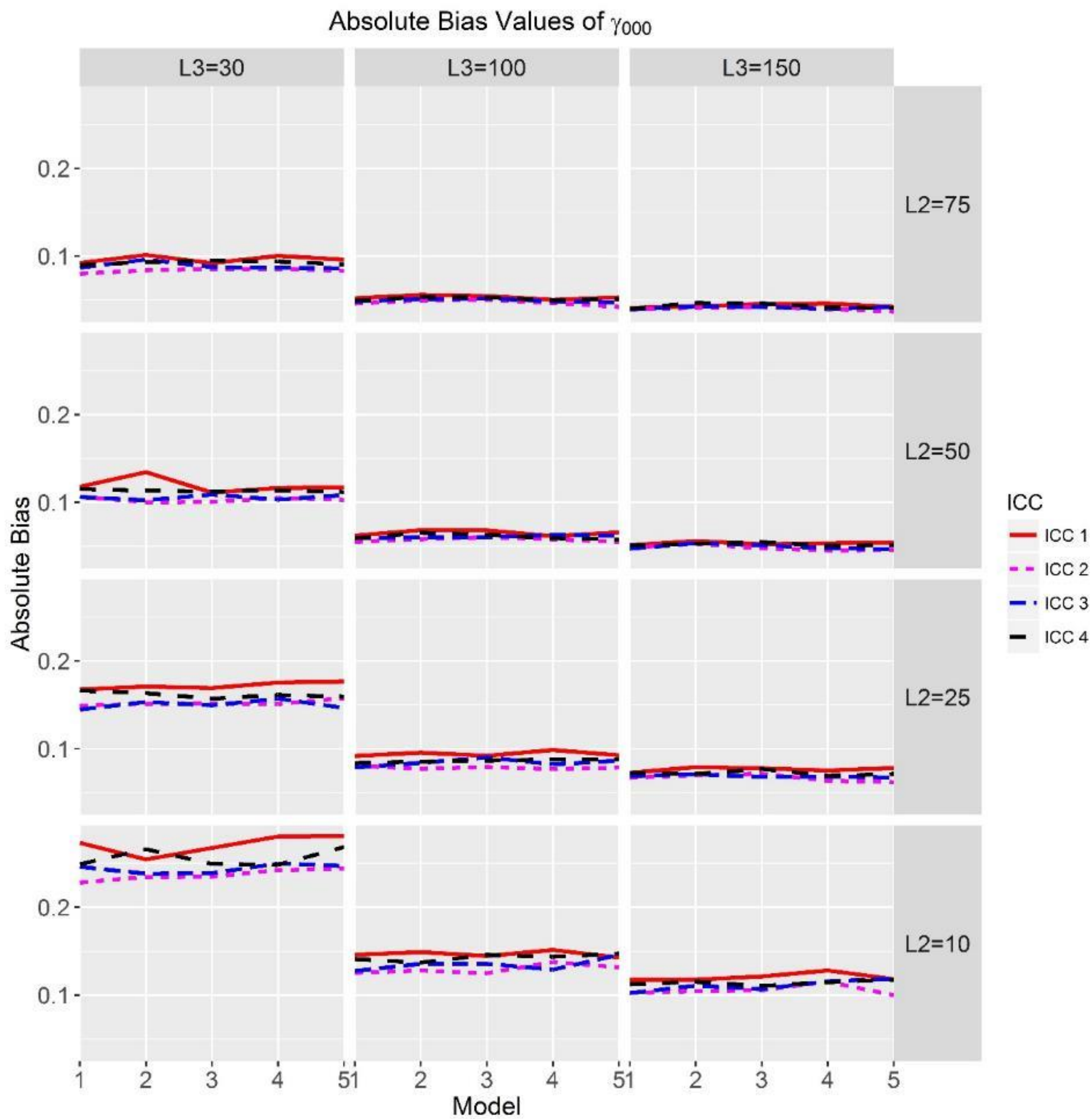

Figure D1. Plot of Absolute Bias Across Manipulated Factors for γ_{000}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

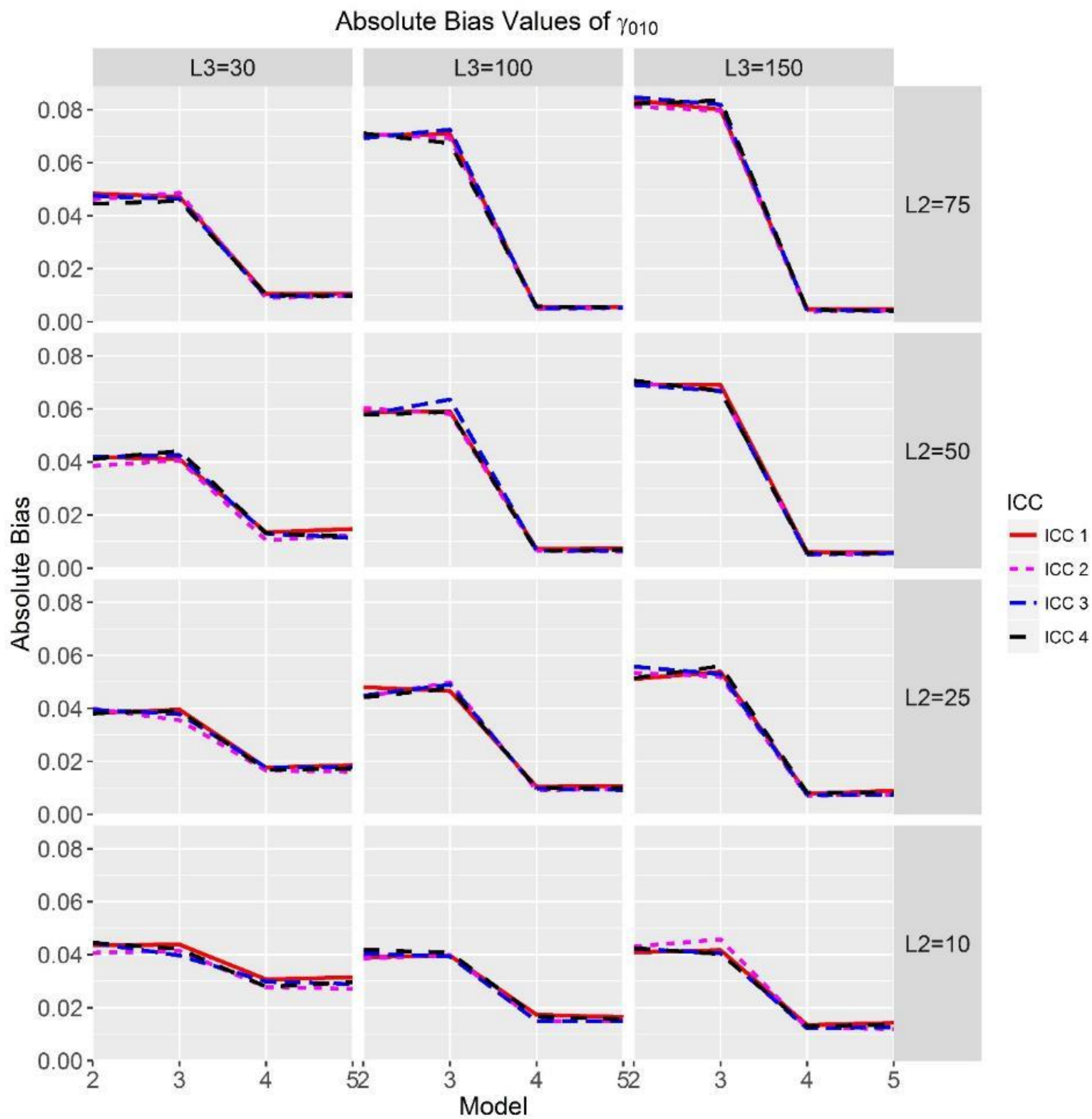

Figure D2. Plot of Absolute Bias Across Manipulated Factors for γ_{010}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

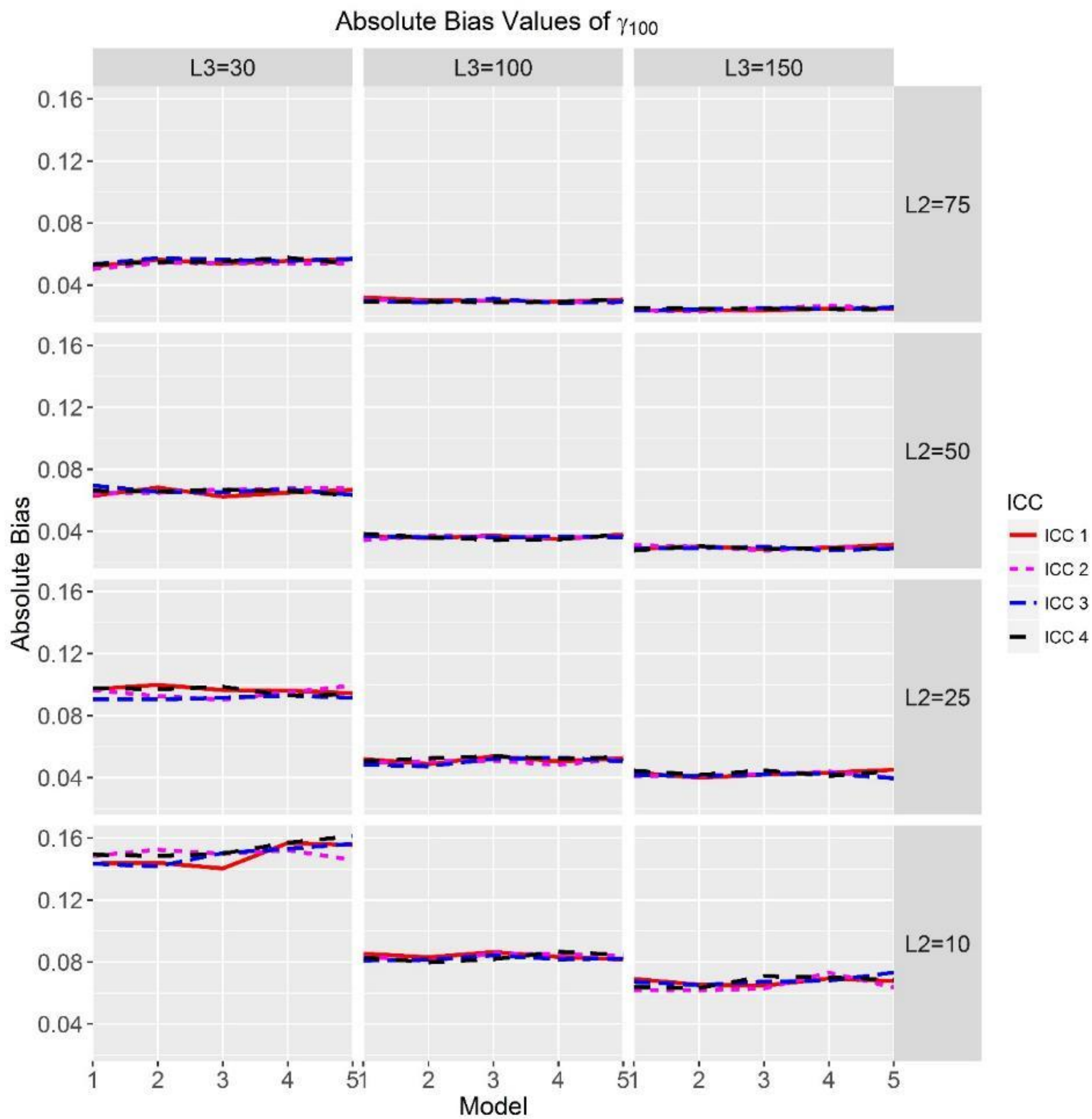

Figure D3. Plot of Absolute Bias Across Manipulated Factors for γ_{100}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

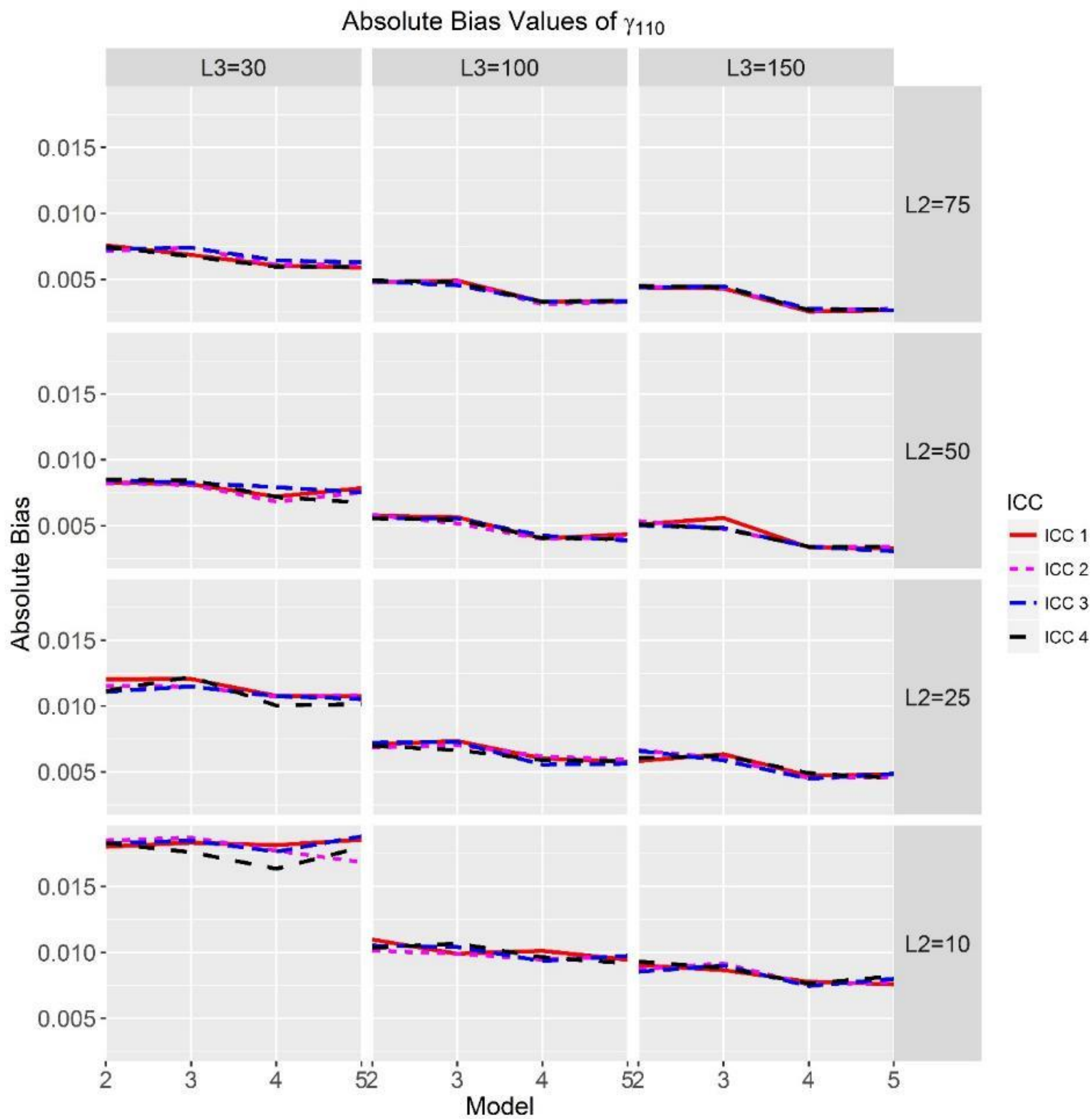

Figure D4. Plot of Absolute Bias Across Manipulated Factors for γ_{110}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

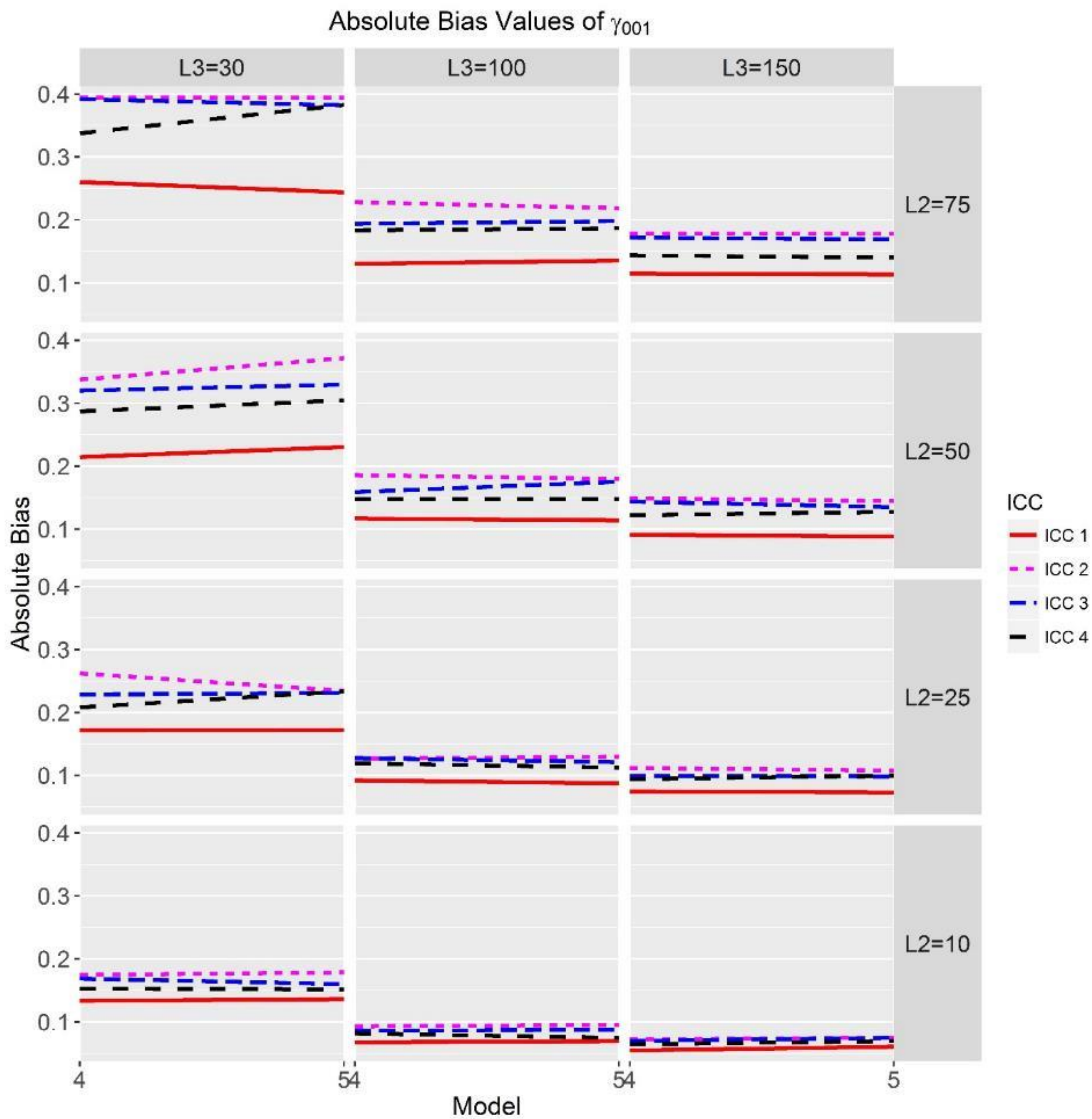

Figure D5. Plot of Absolute Bias Across Manipulated Factors for γ_{001}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

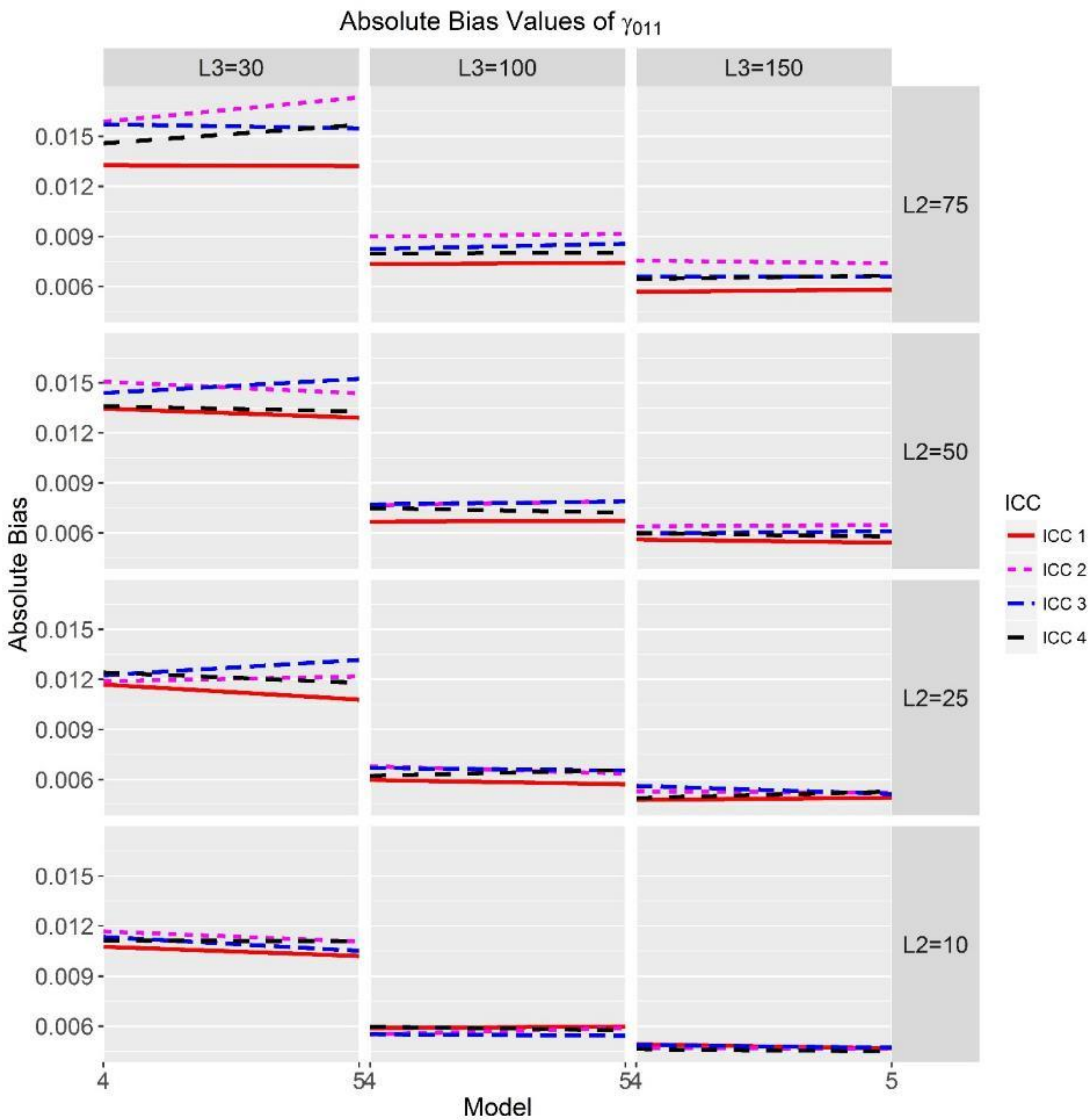

Figure D6. Plot of Absolute Bias Across Manipulated Factors for γ_{011}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

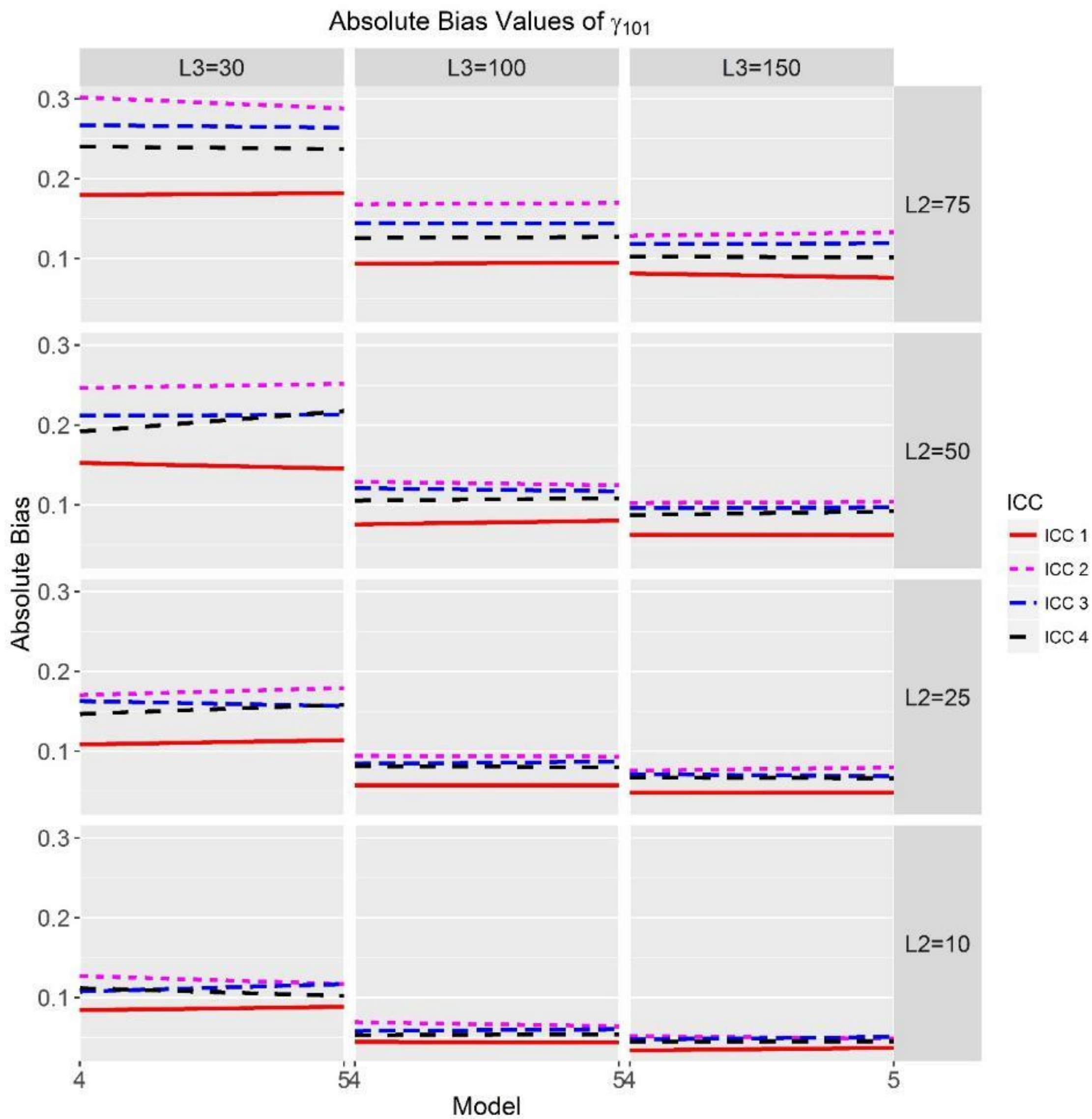

Figure D7. Plot of Absolute Bias Across Manipulated Factors for γ_{101}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

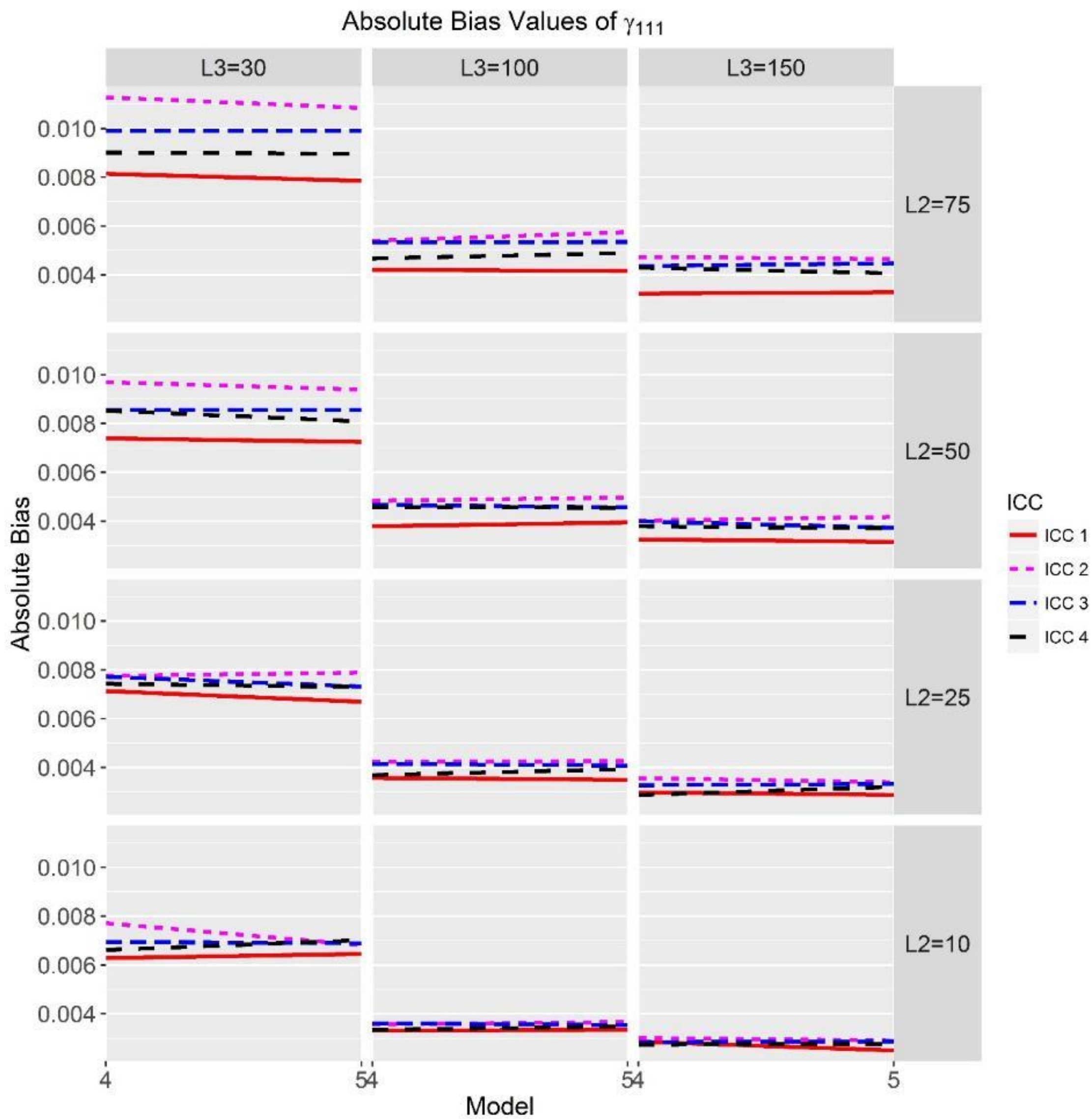

Figure D8. Plot of Absolute Bias Across Manipulated Factors for γ_{111}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

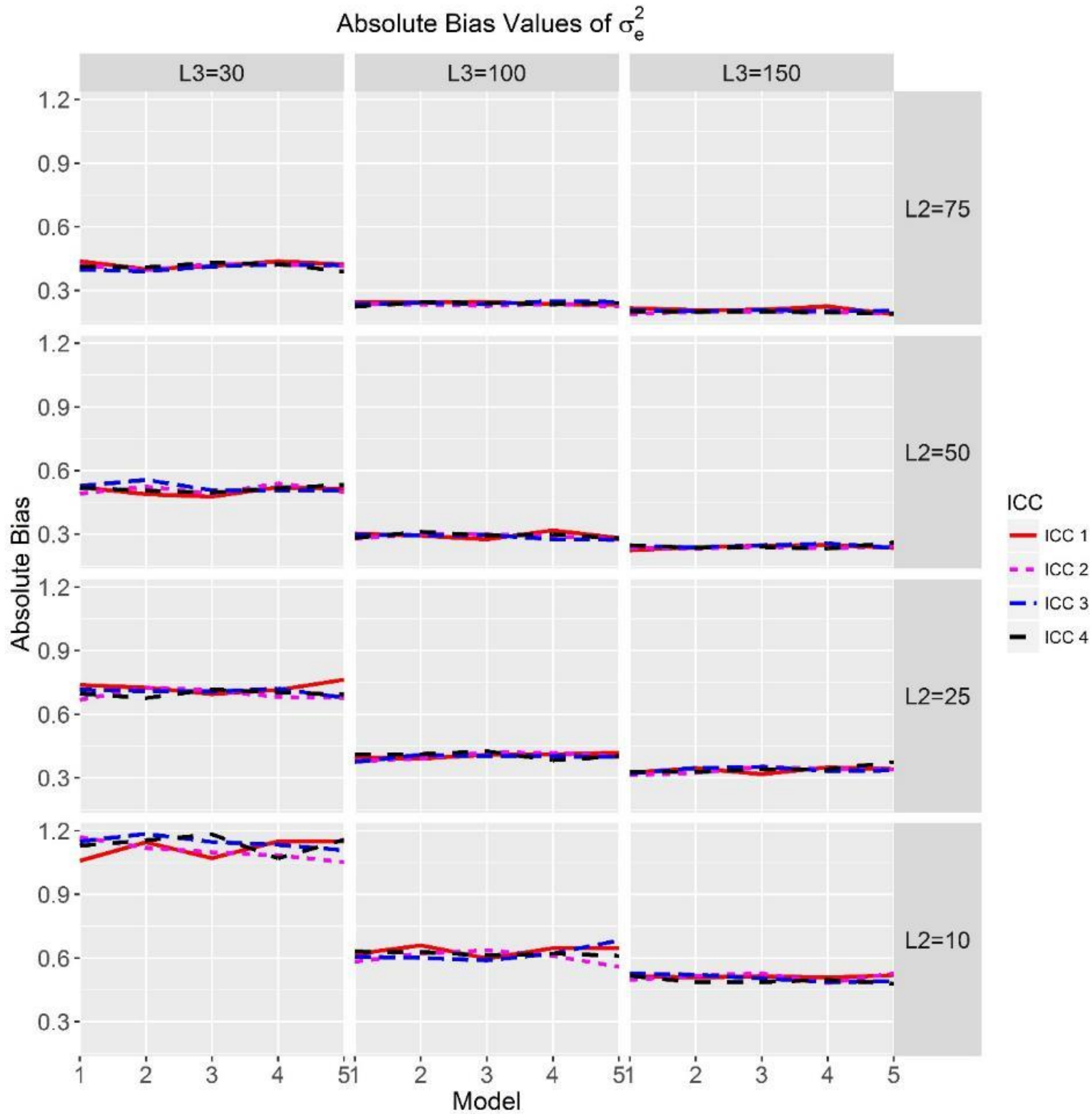

Figure D9. Plot of Absolute Bias Across Manipulated Factors for σ_e^2.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

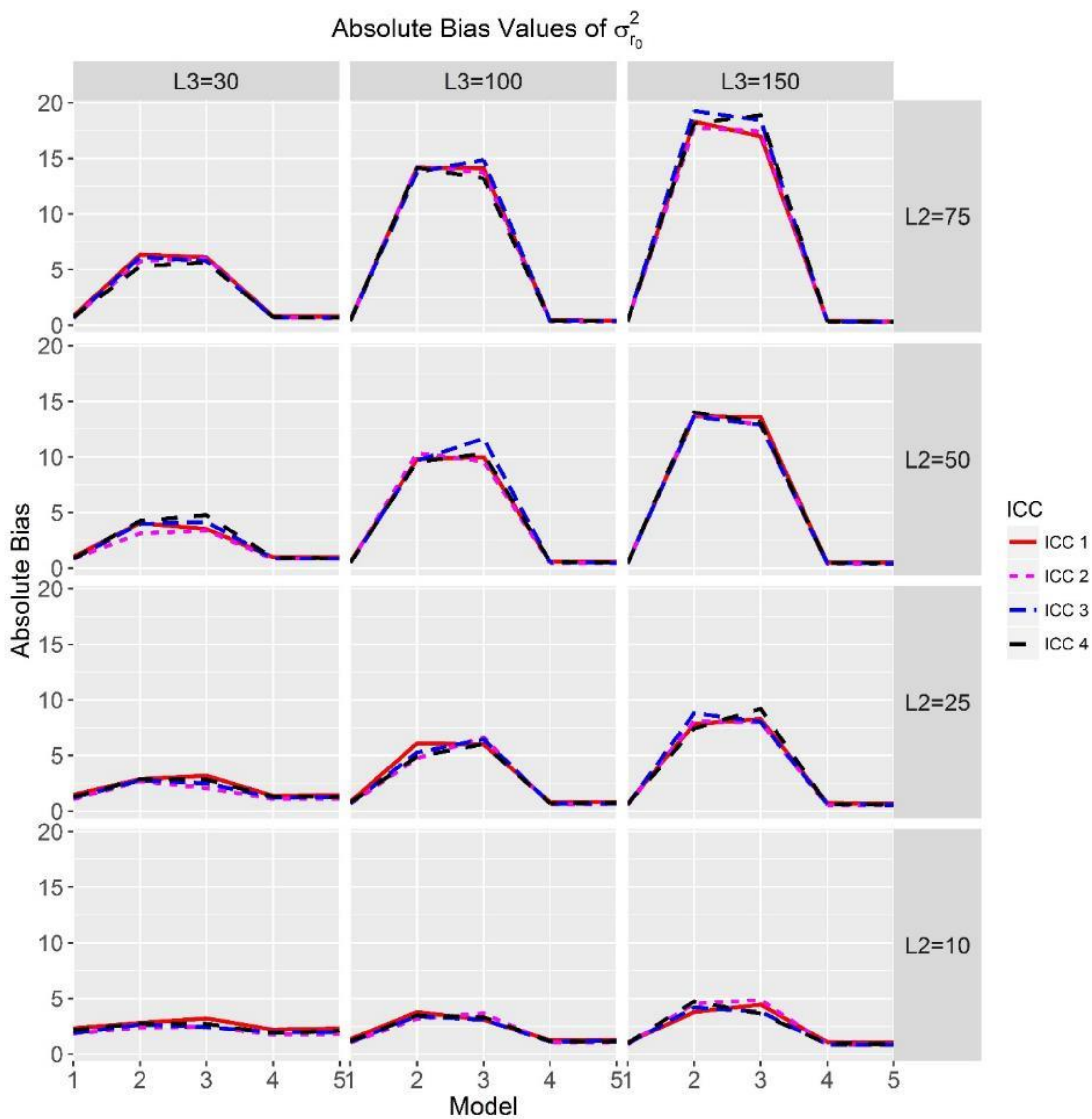

Figure D10. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{r_0}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

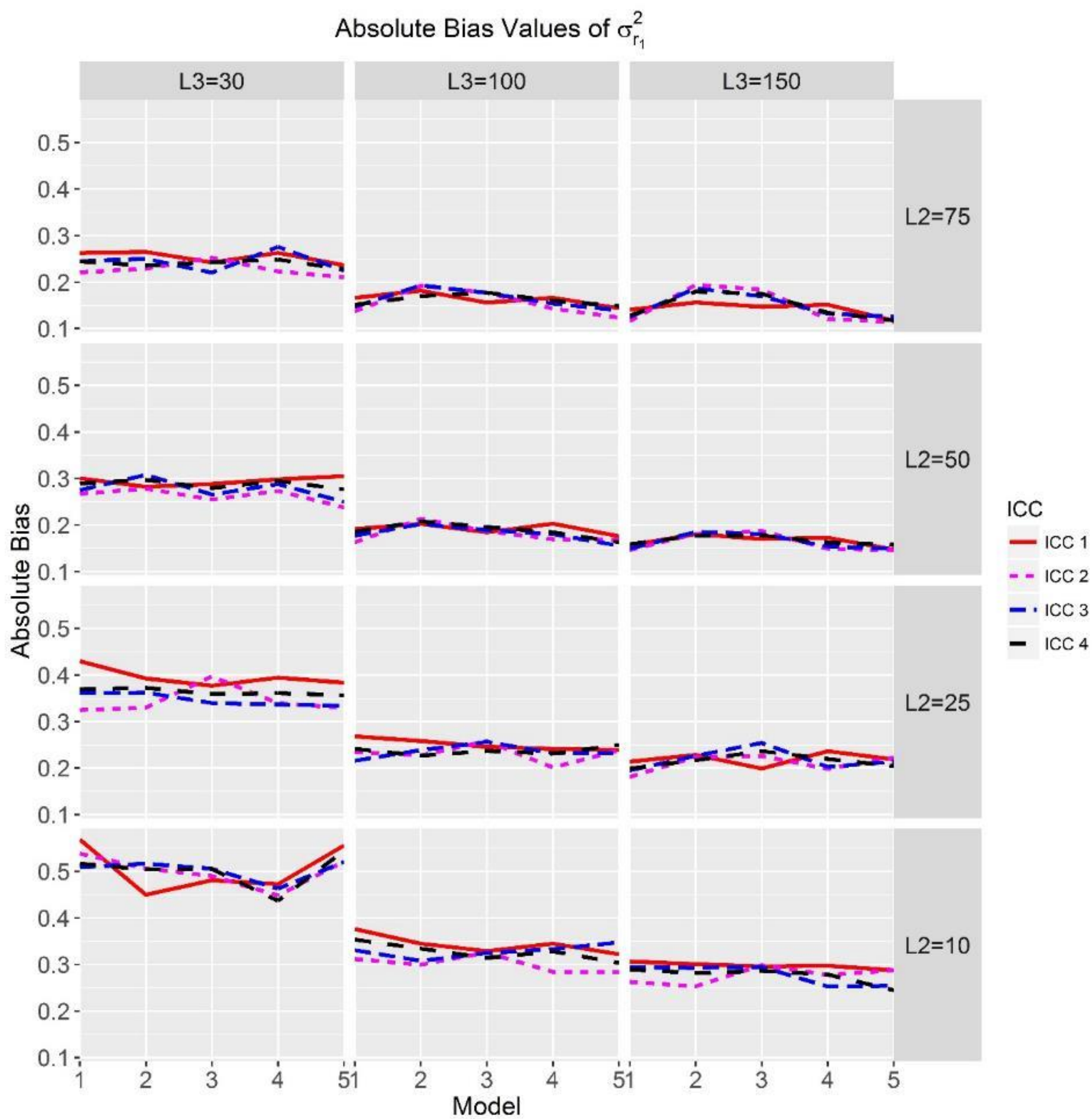

Figure D11. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

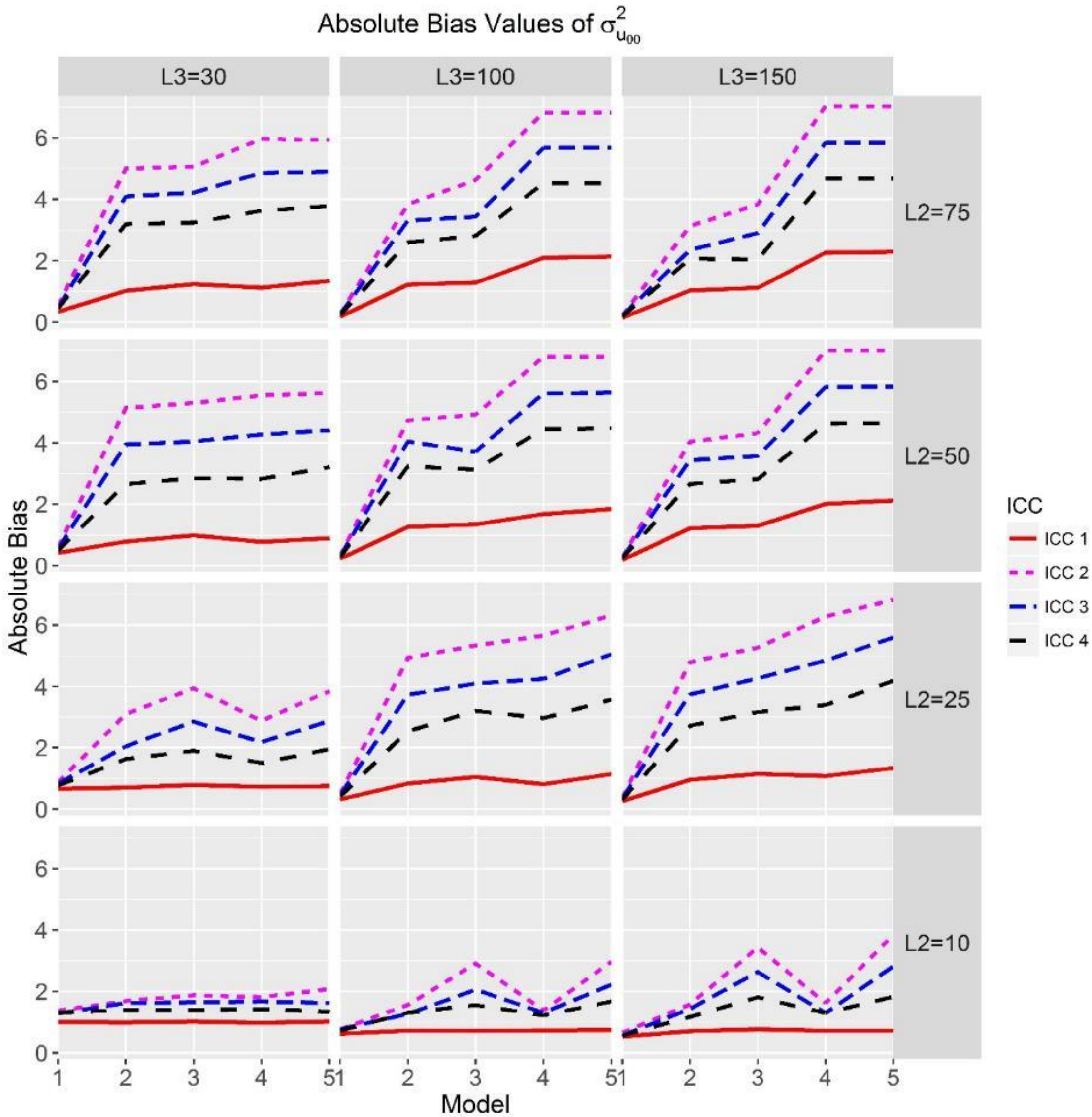

Figure D12. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{u_{00}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

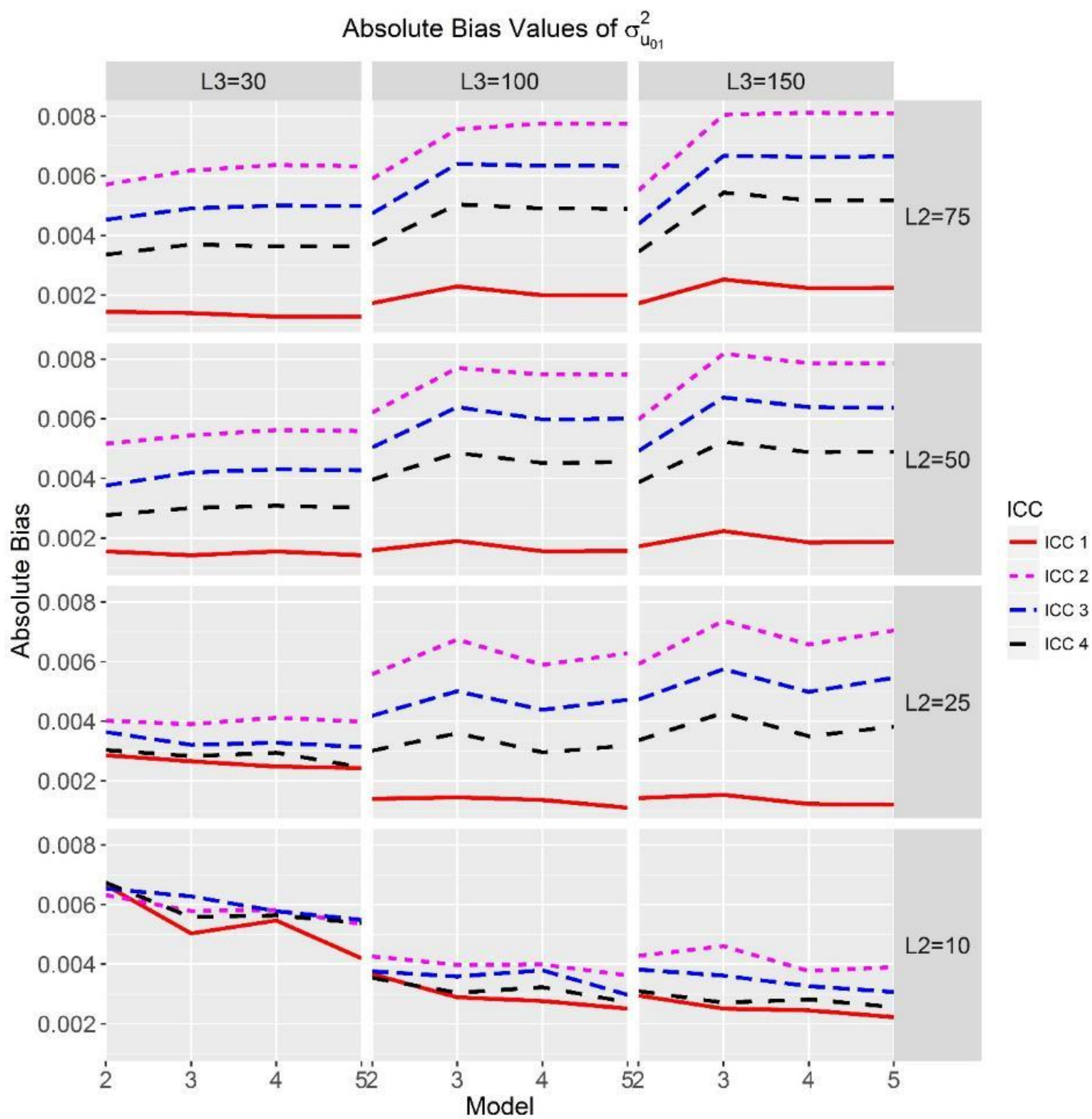

Figure D13. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{u_{01}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

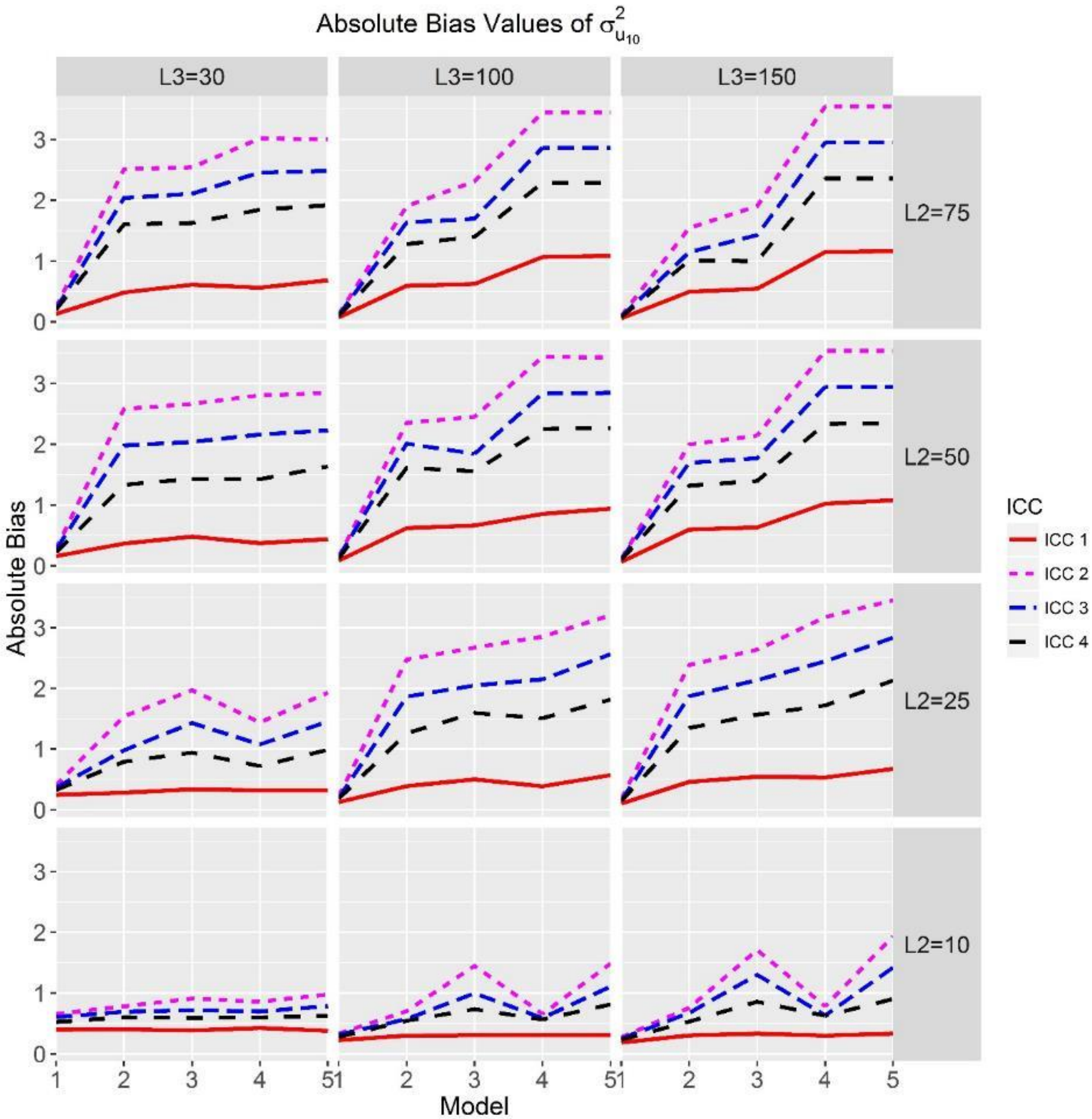

Figure D14. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

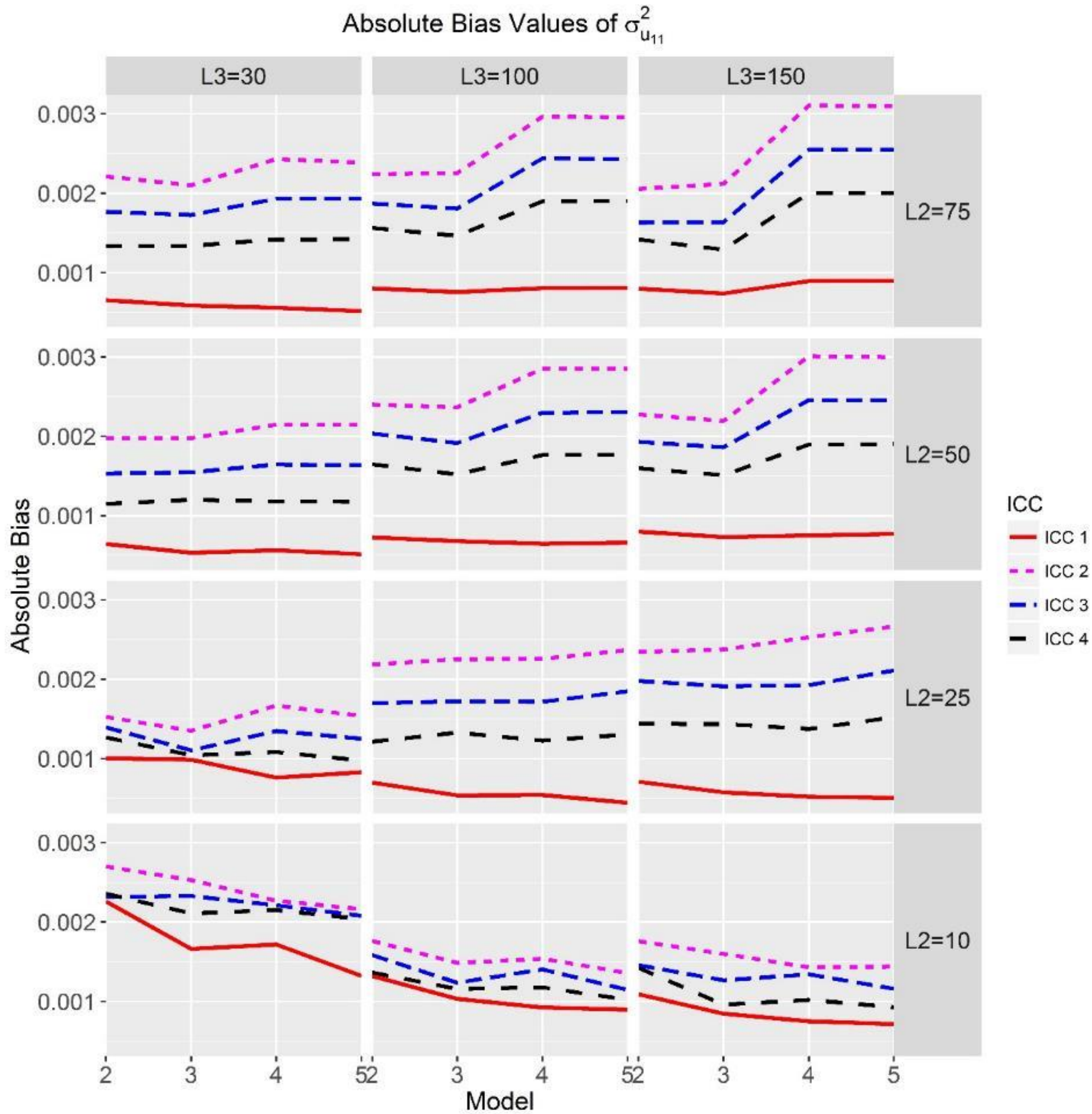

Figure D15. Plot of Absolute Bias Across Manipulated Factors for $\sigma^2_{u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

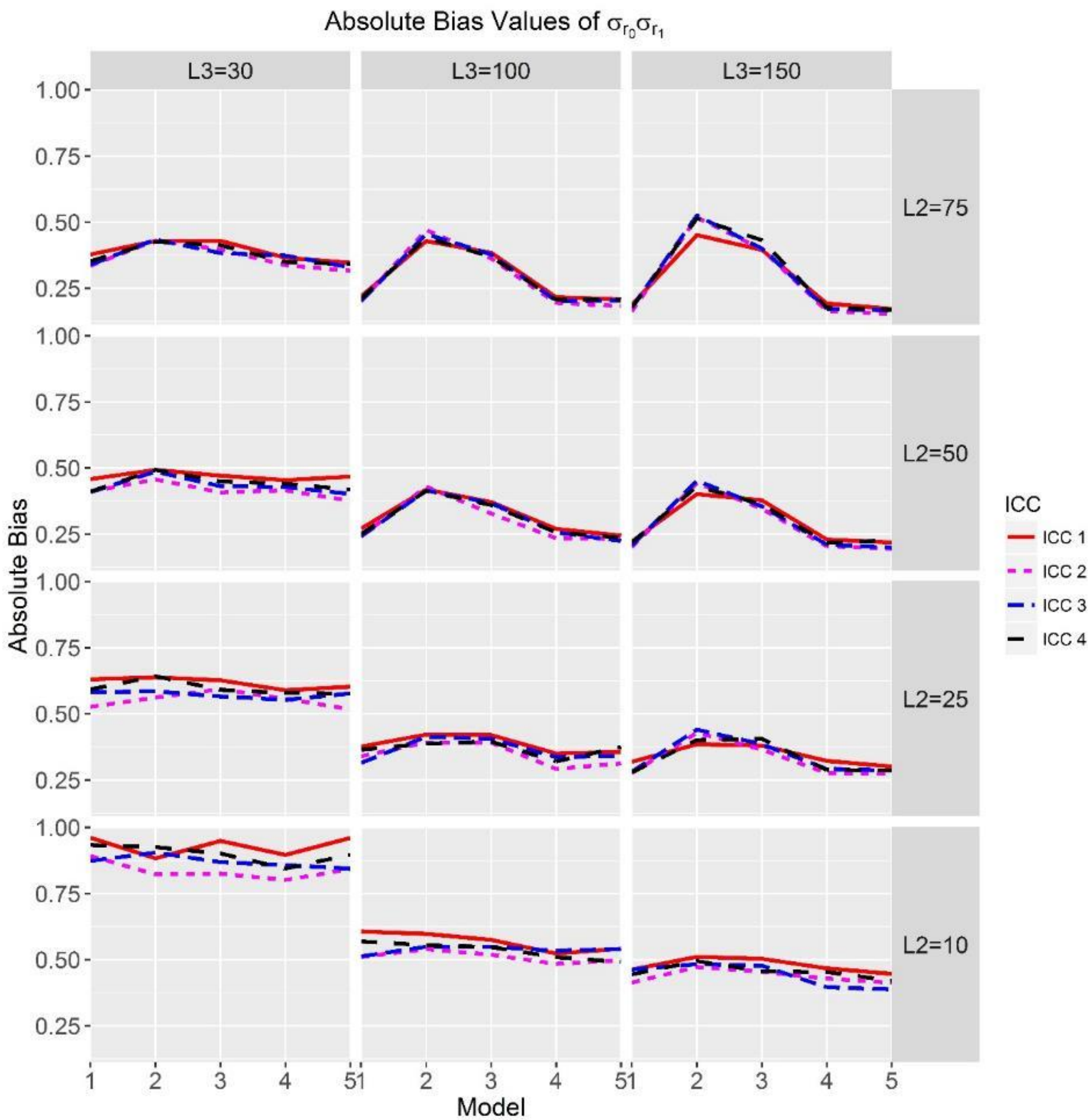

Figure D16. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{r_0 r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

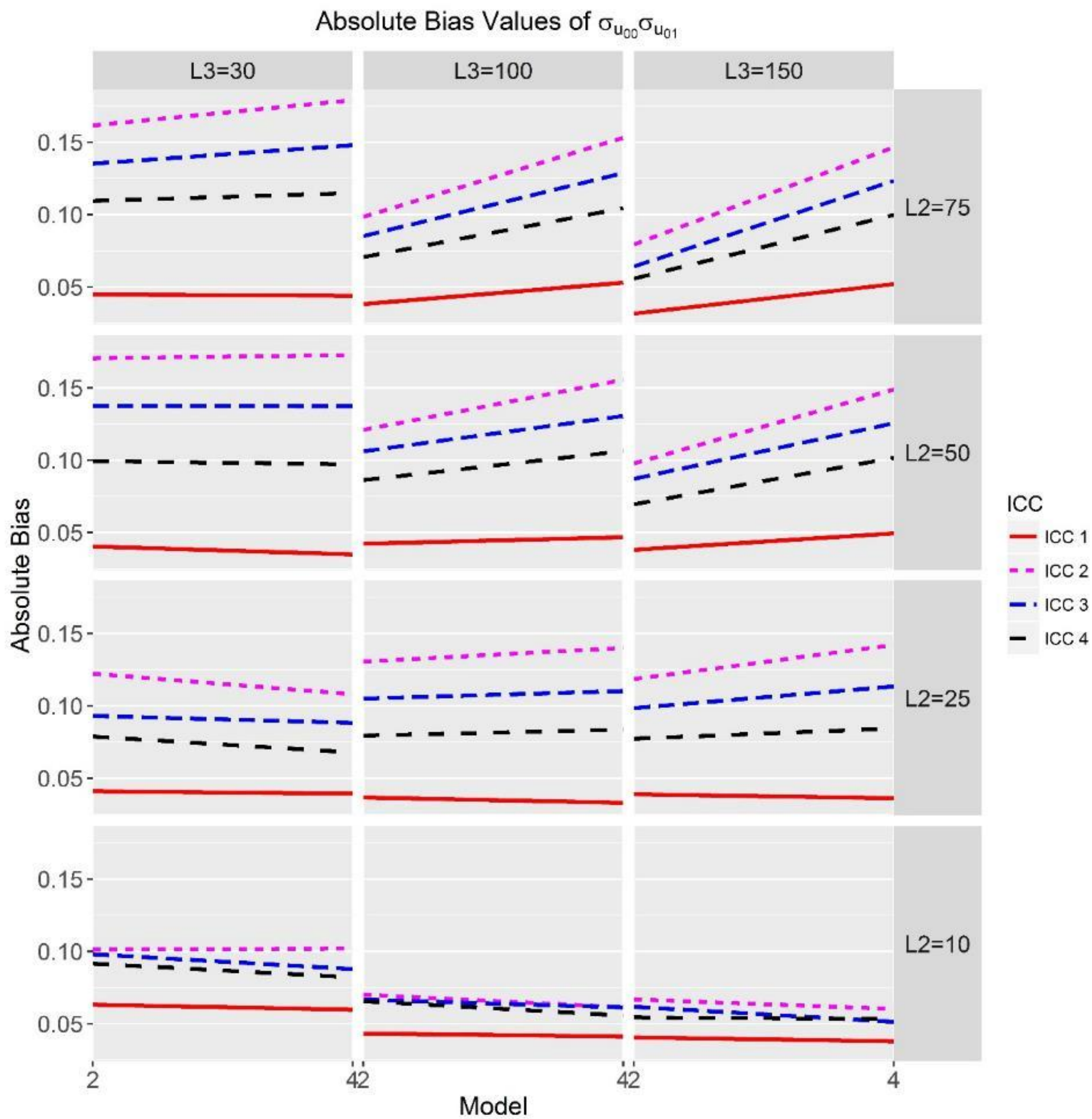

Figure D17. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{00}u_{01}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

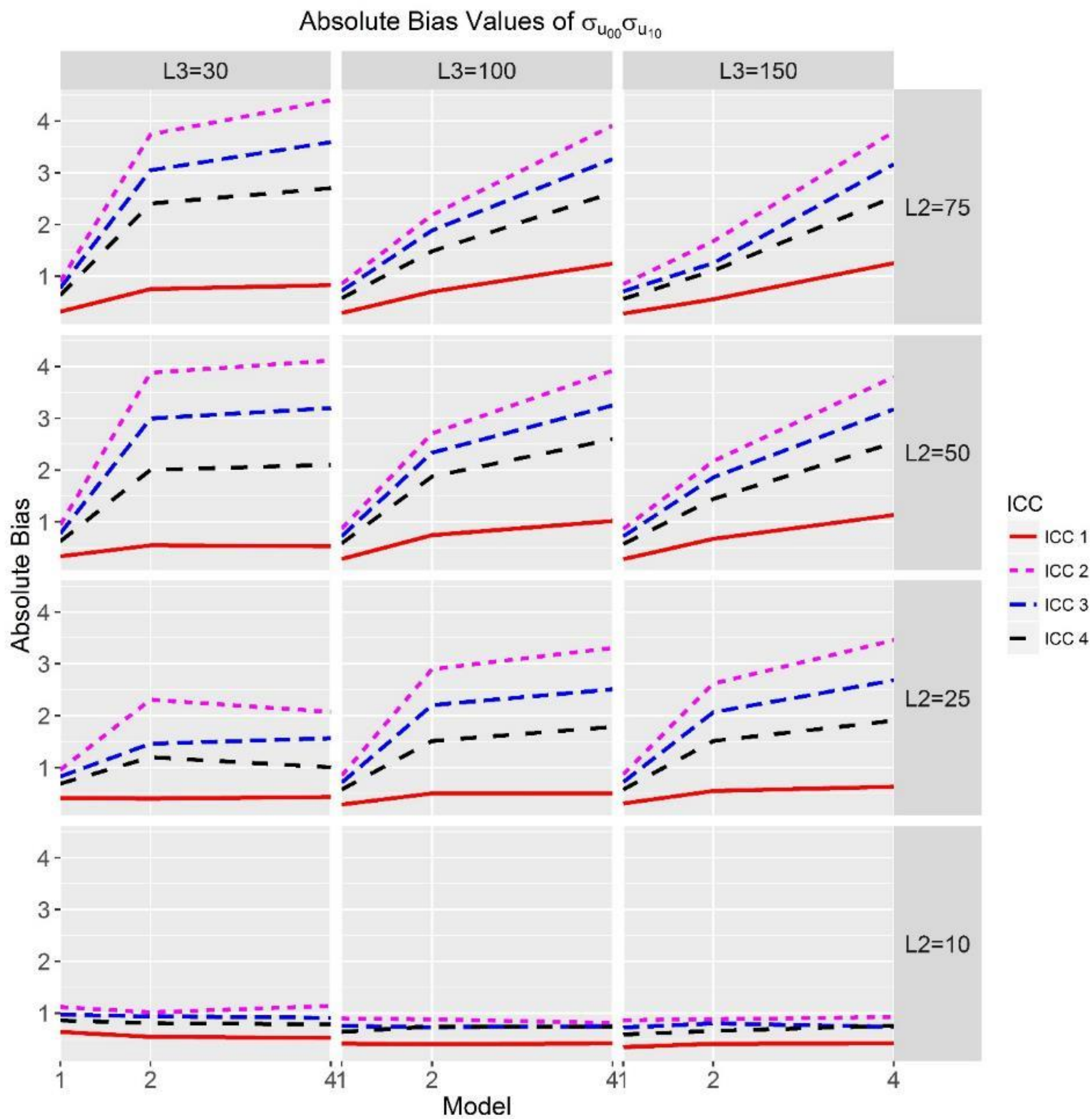

Figure D18. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{00}u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

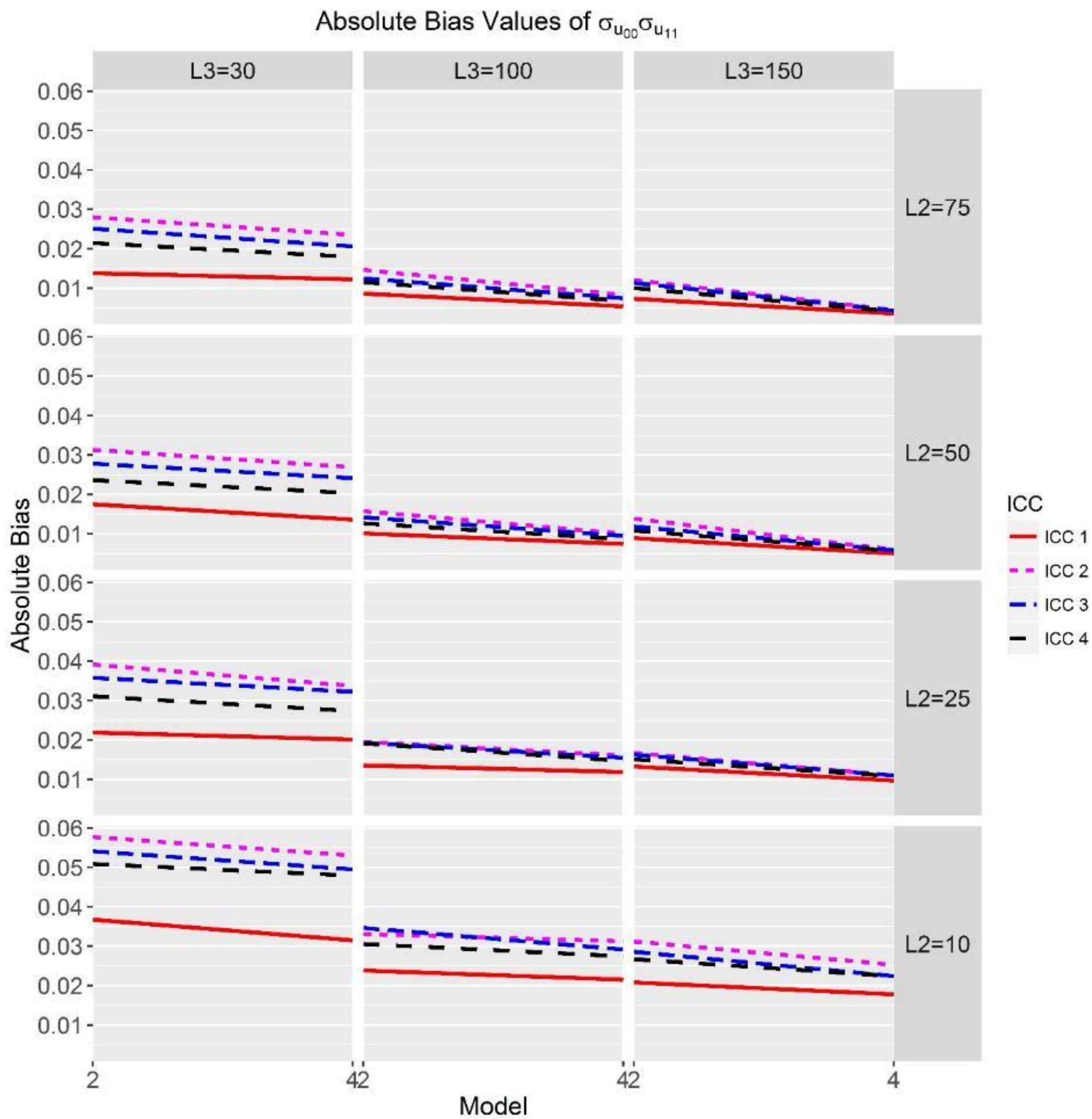

Figure D19. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{00}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

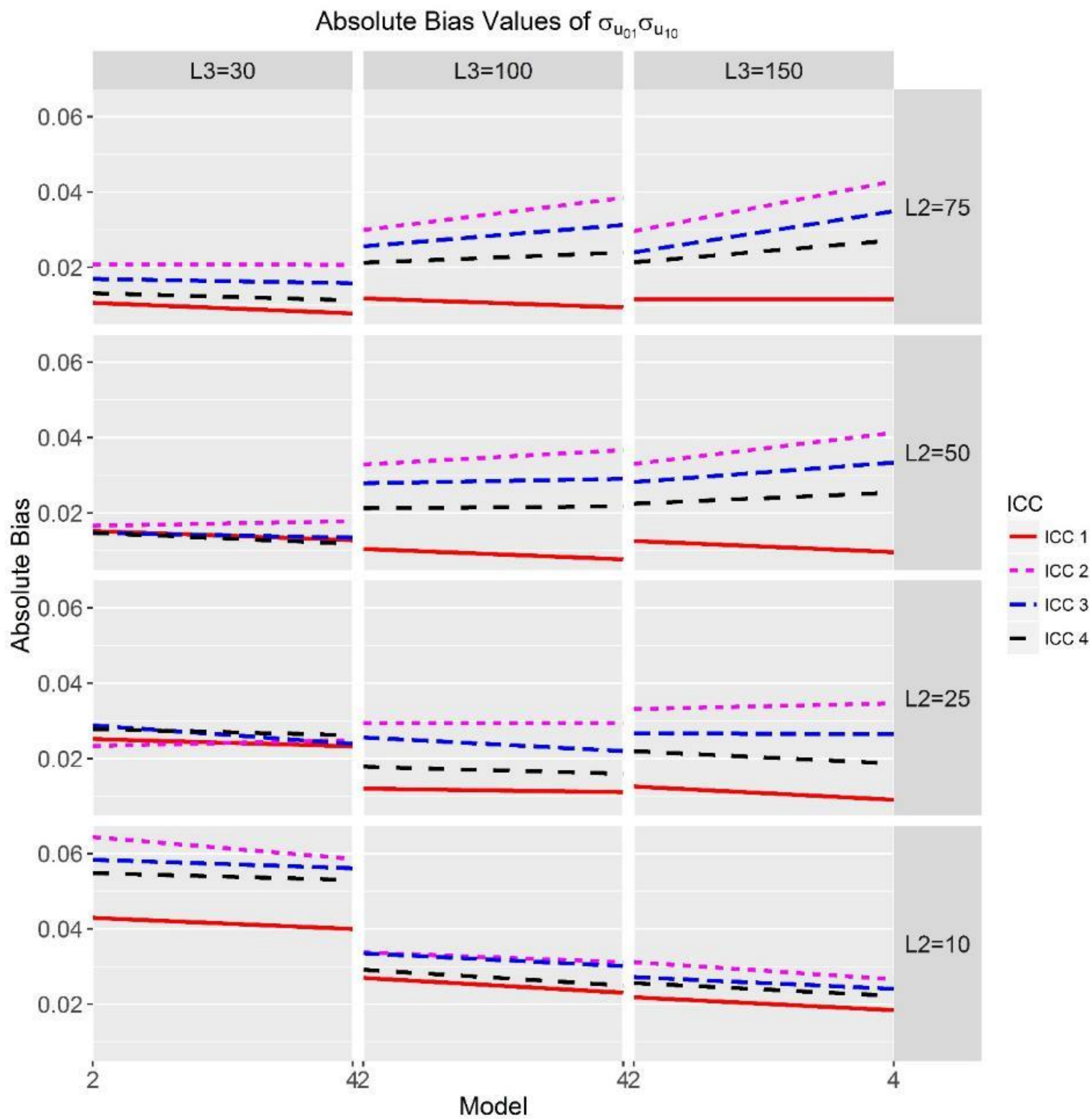

Figure D20. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{01}u_{10}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

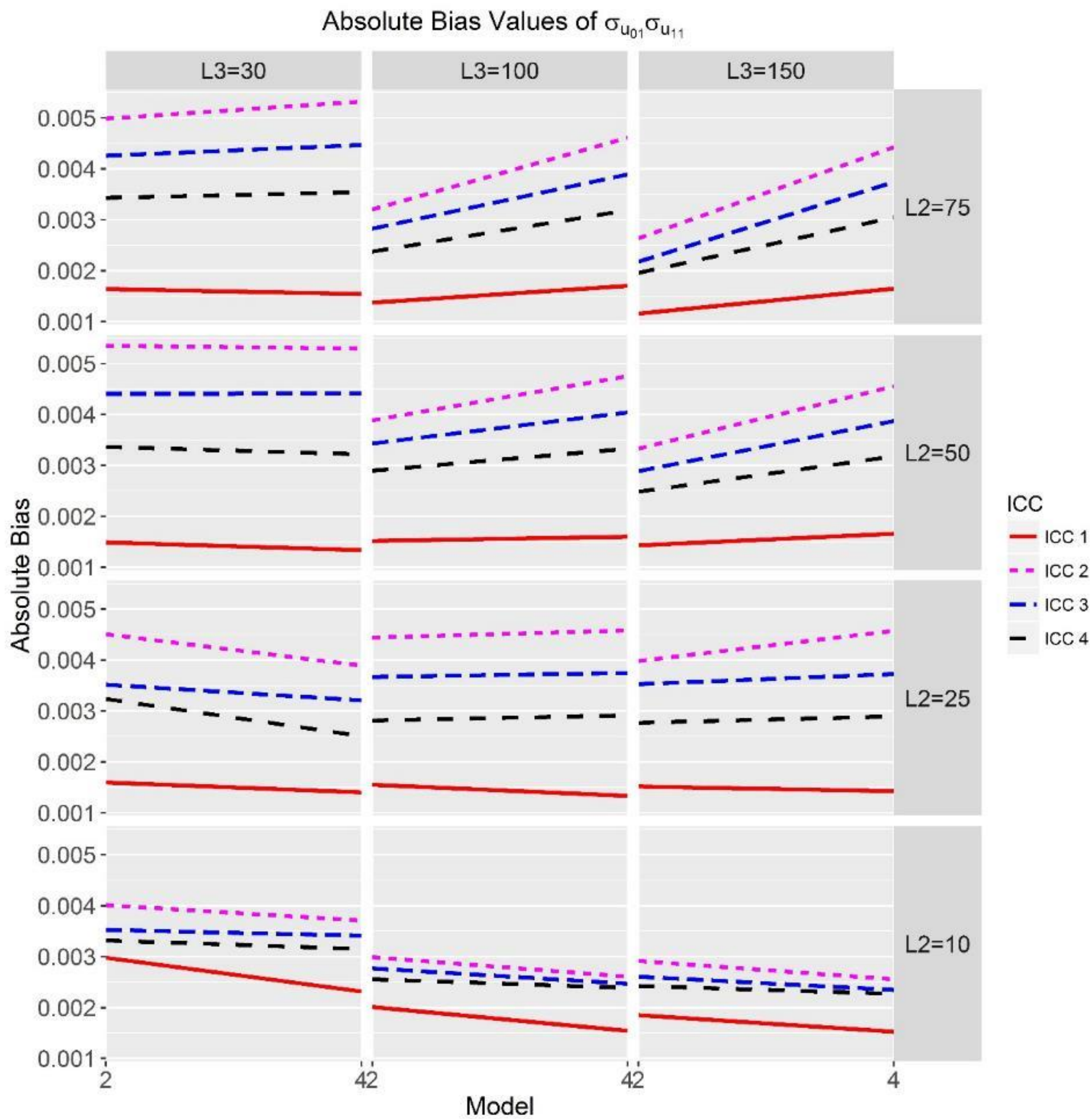

Figure D21. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{01}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

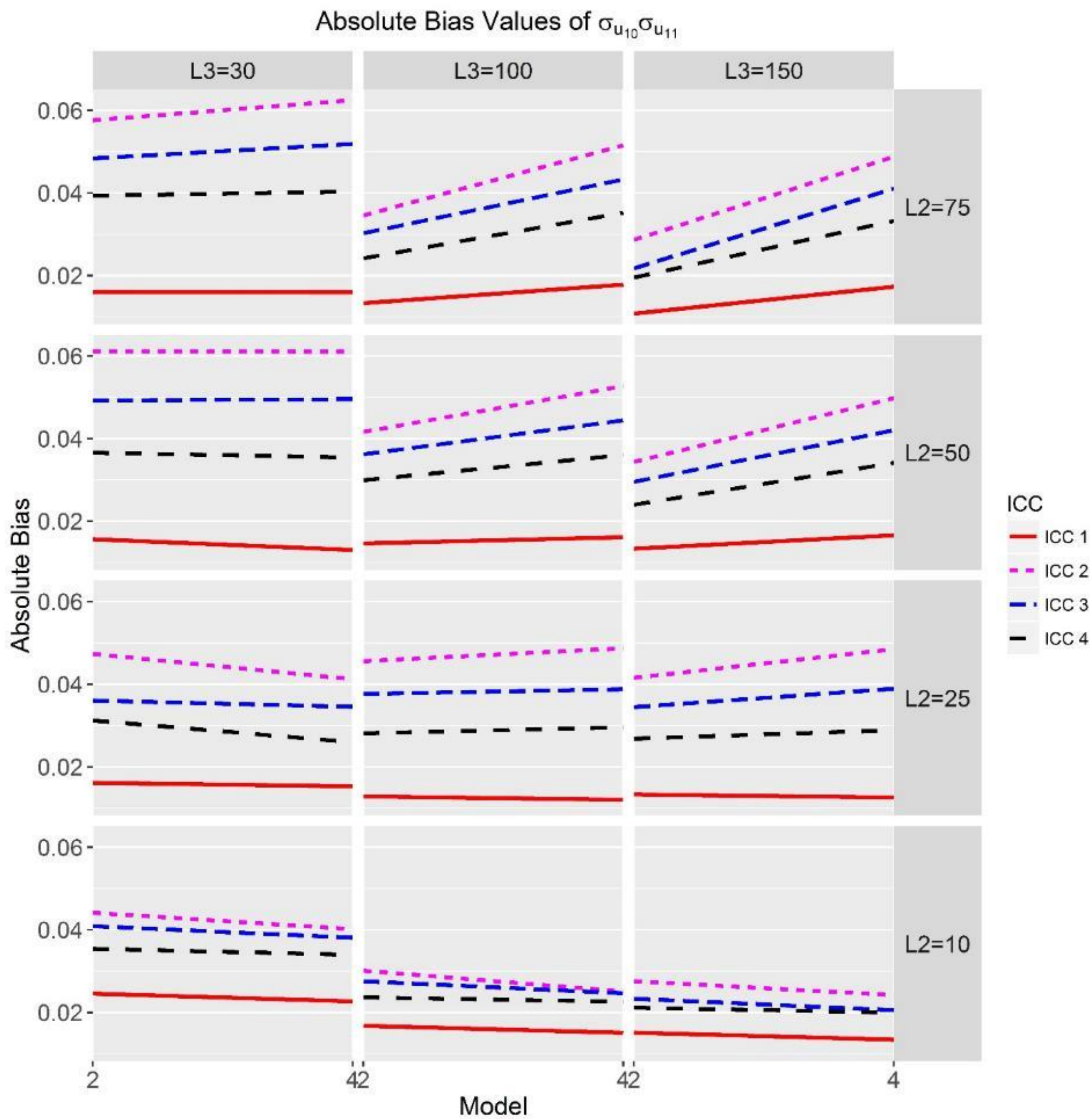

Figure D22. Plot of Absolute Bias Across Manipulated Factors for $\sigma_{u_{10}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

APPENDIX E

VISUAL REPRESENTATION OF RELATIVE BIAS VALUES

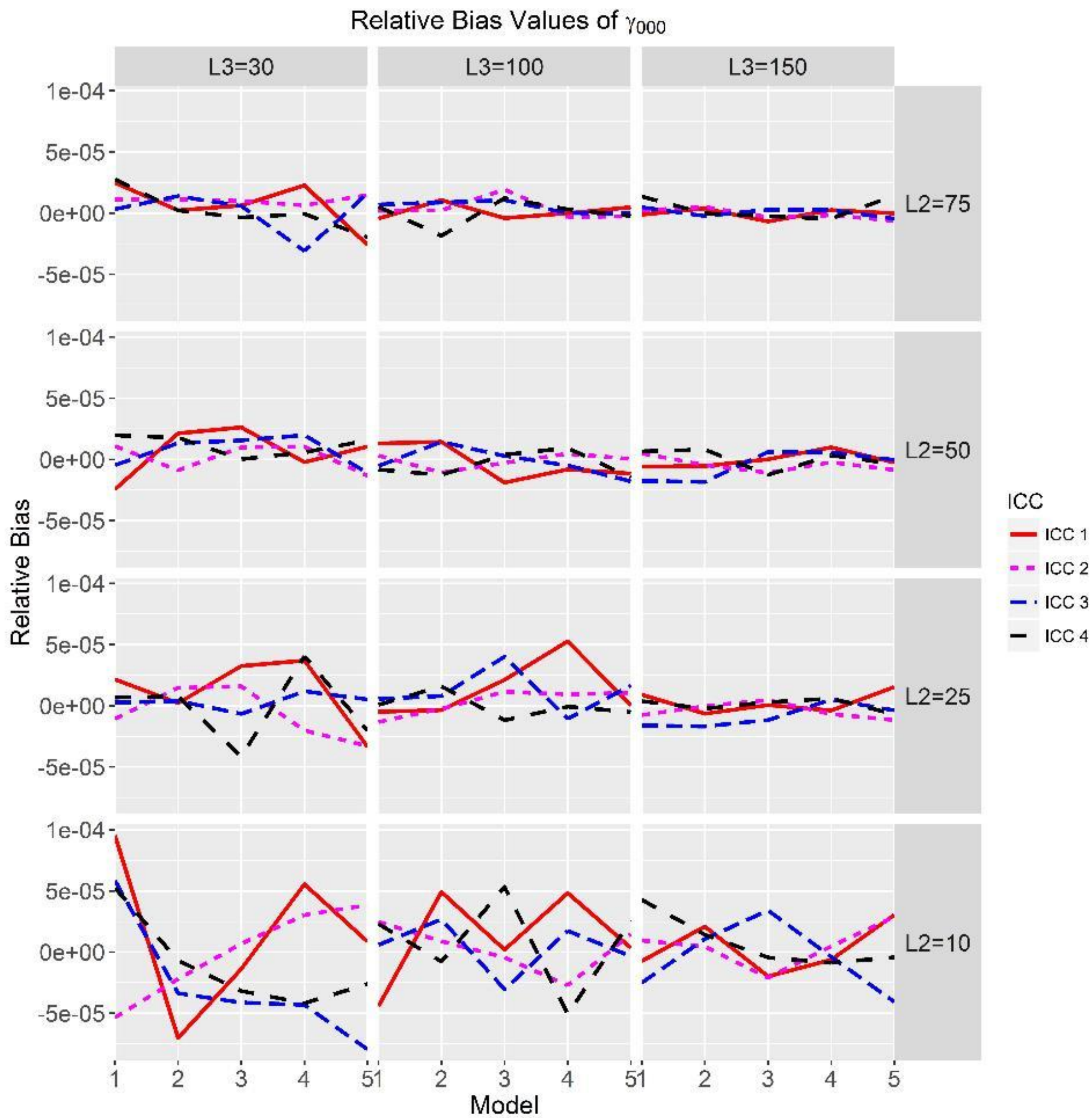

Figure E1. Plot of Relative Bias Across Manipulated Factors for γ_{000}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

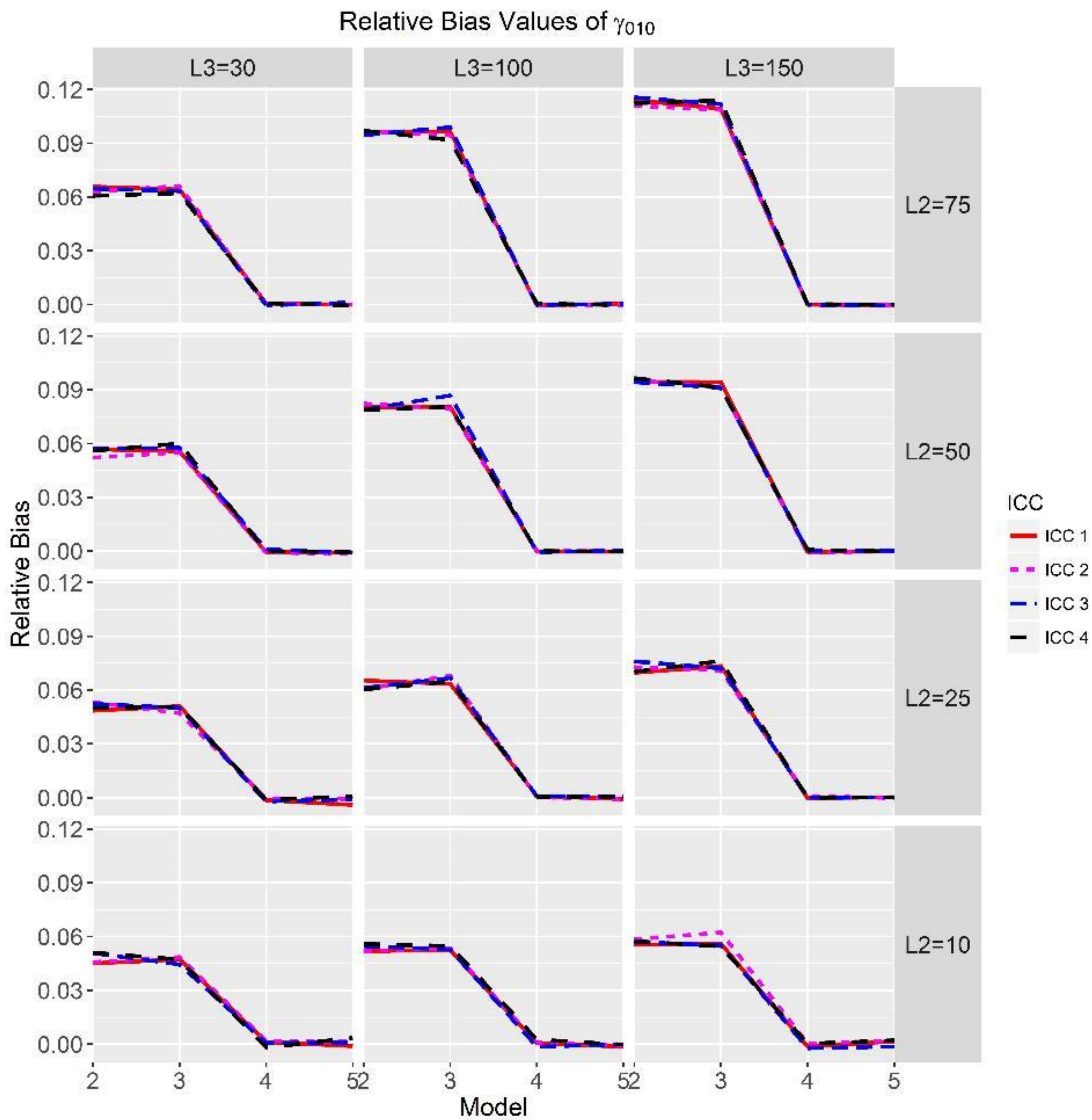

Figure E2. Plot of Relative Bias Across Manipulated Factors for γ_{010}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

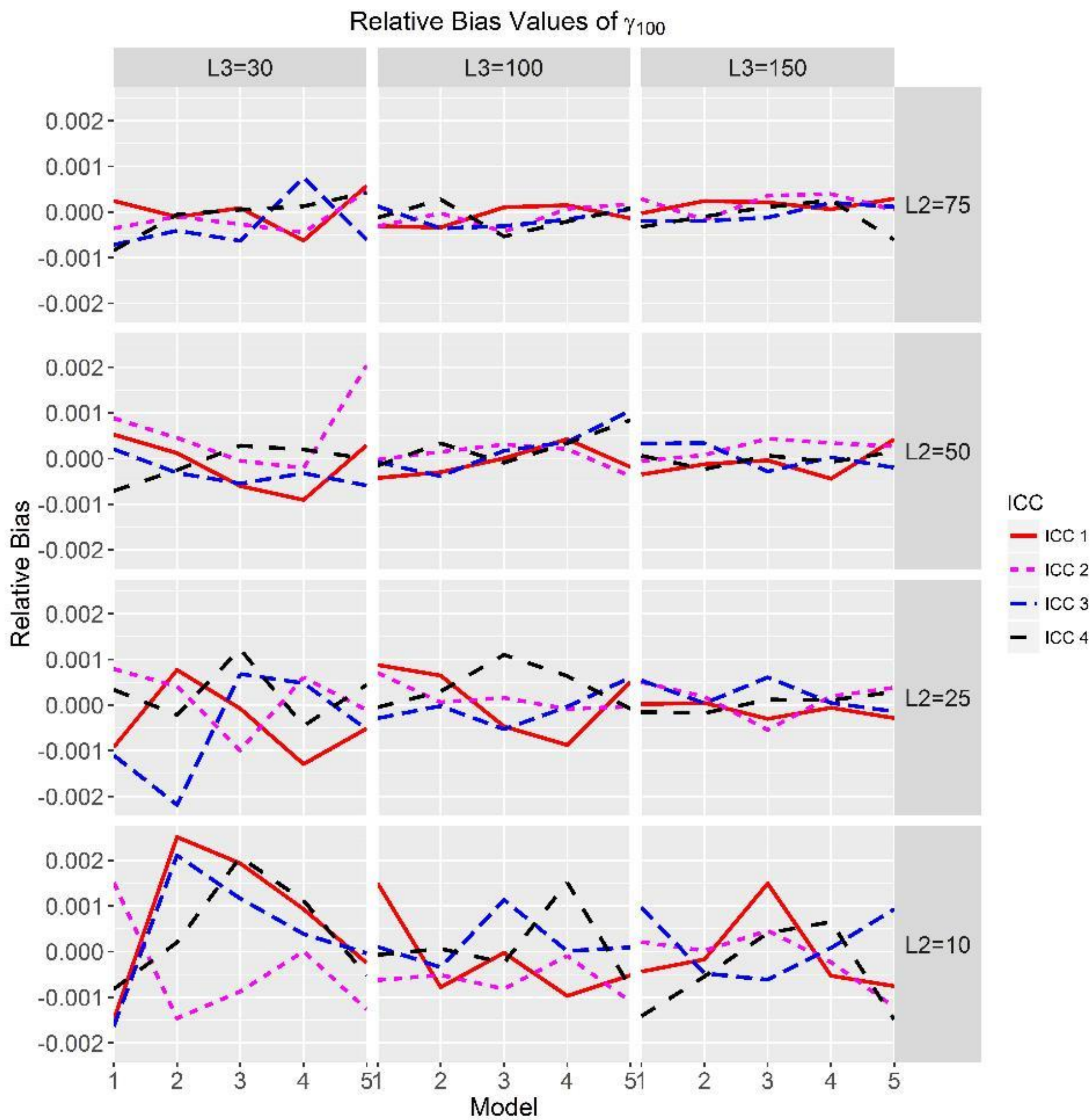

Figure E3. Plot of Relative Bias Across Manipulated Factors for γ_{100}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

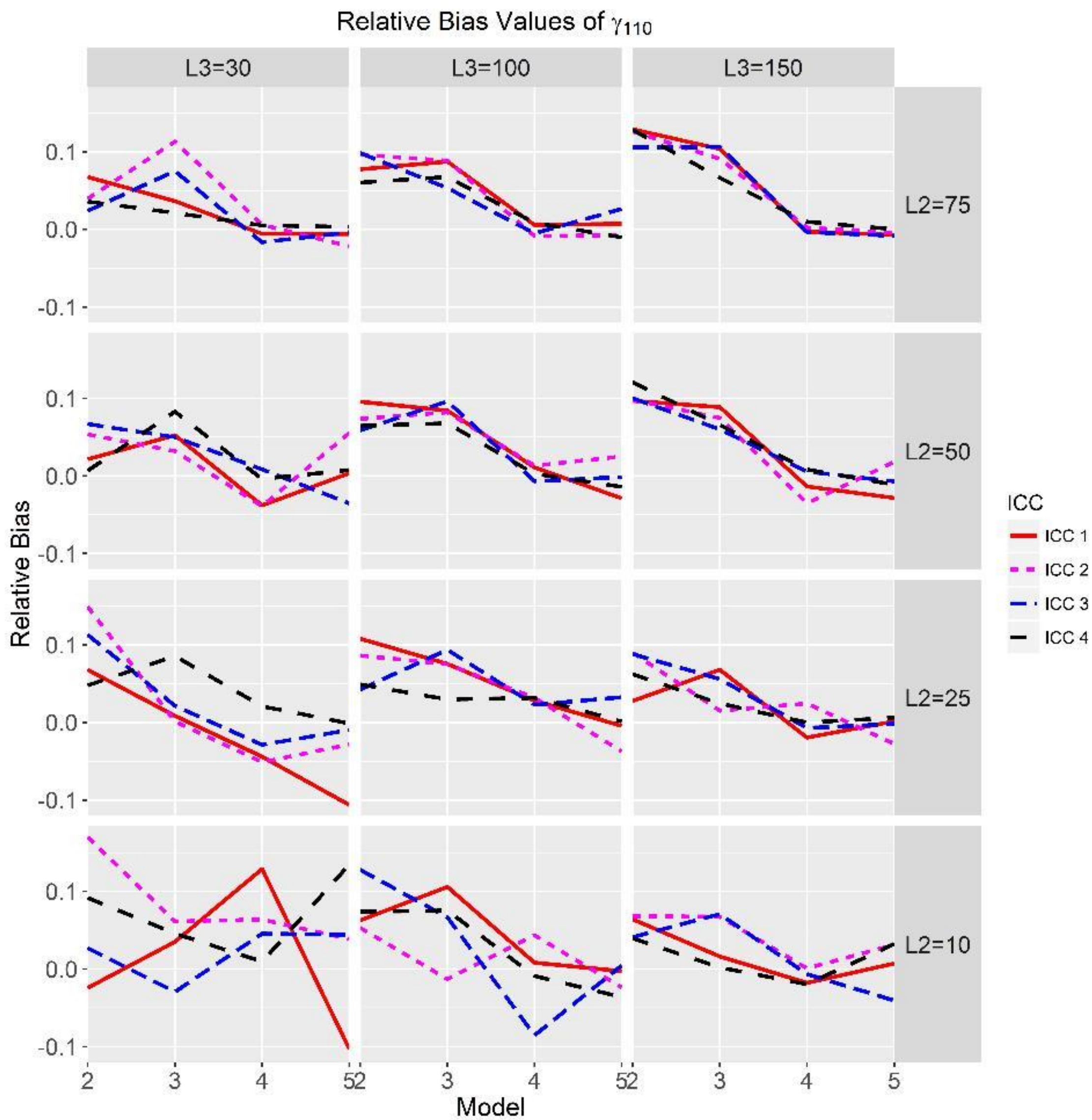

Figure E4. Plot of Relative Bias Across Manipulated Factors for γ_{110}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

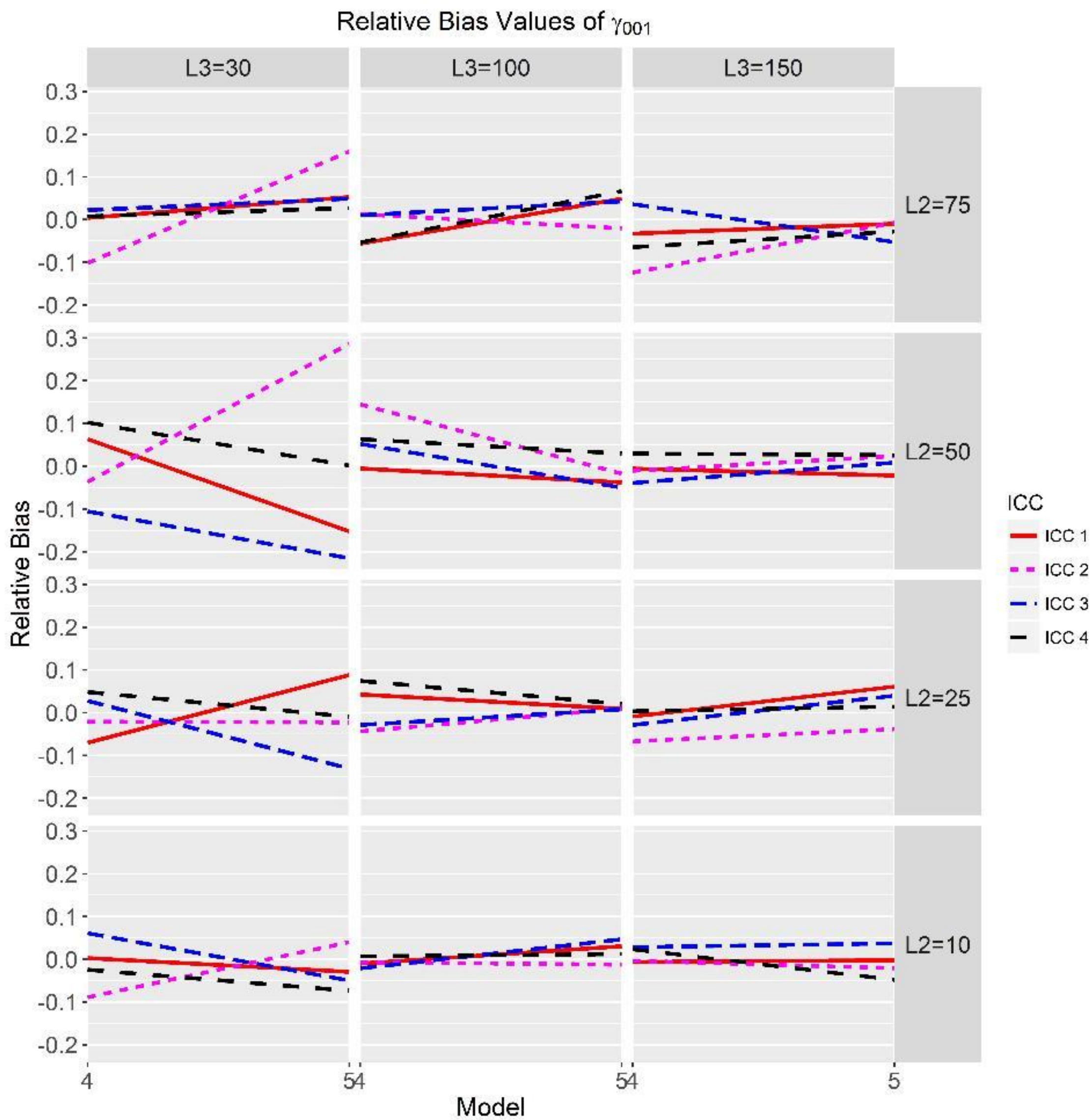

Figure E5. Plot of Relative Bias Across Manipulated Factors for γ_{001}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

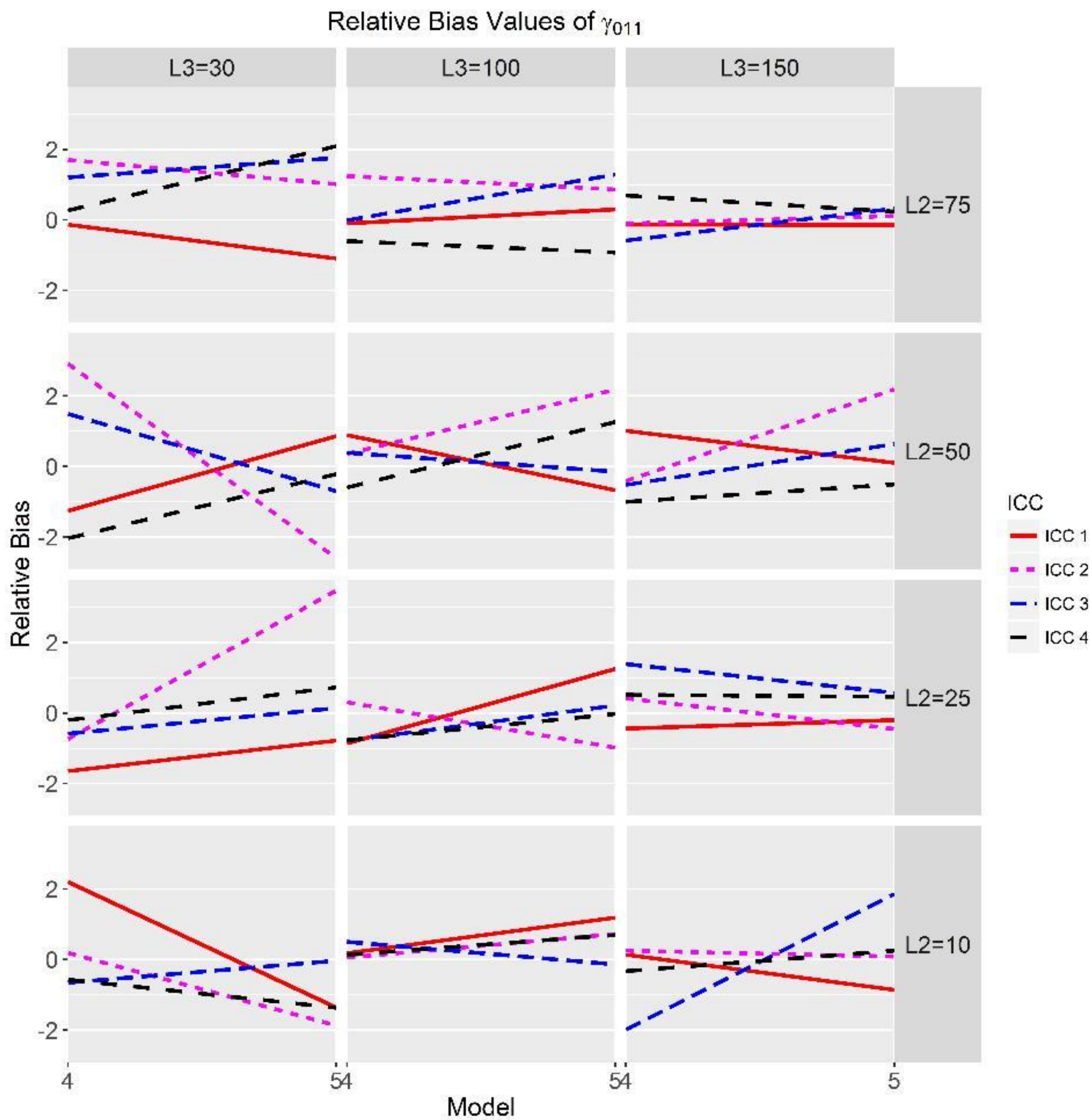

Figure E6. Plot of Relative Bias Across Manipulated Factors for γ_{011}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

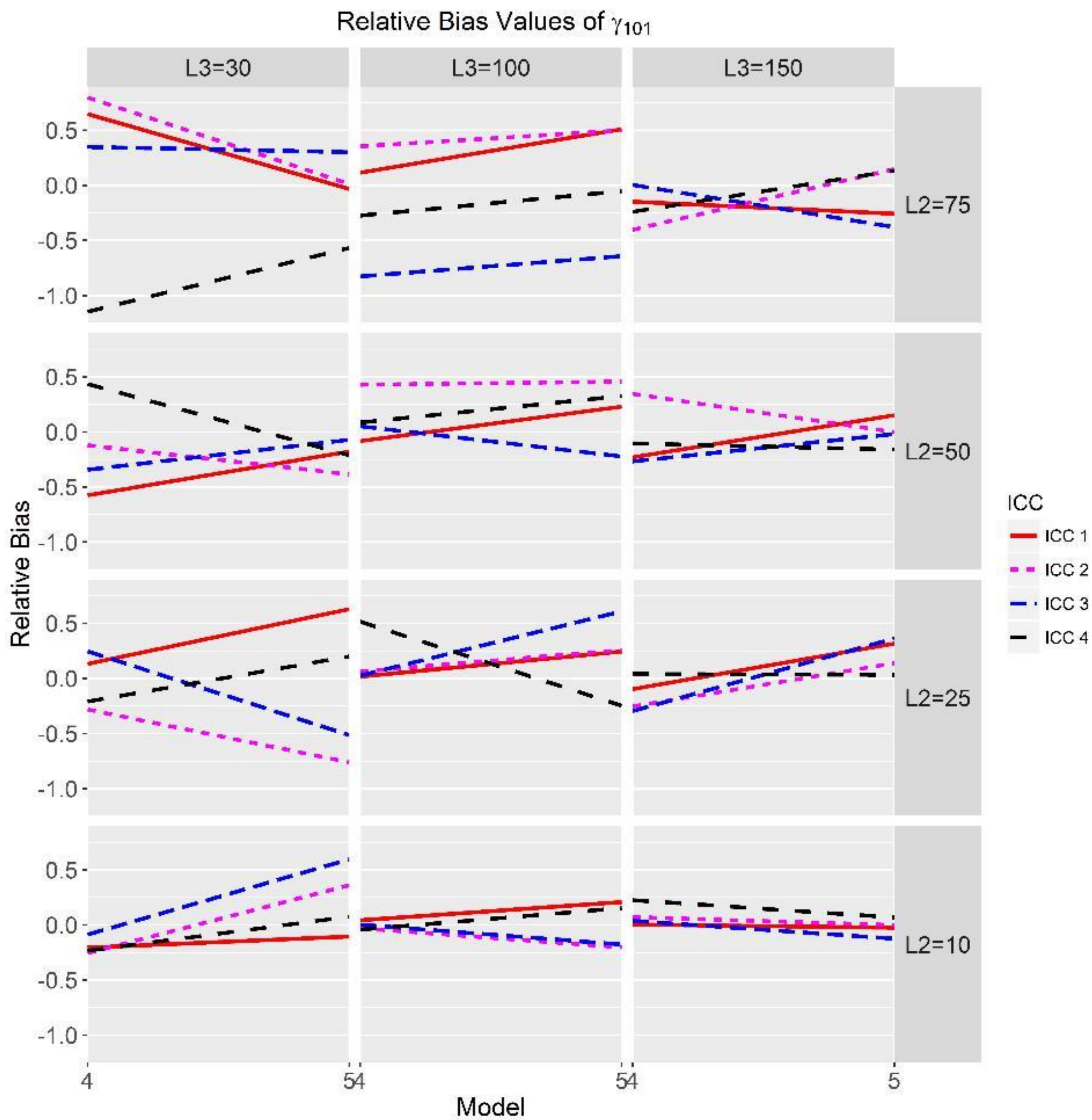

Figure E7. Plot of Relative Bias Across Manipulated Factors for γ_{101}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

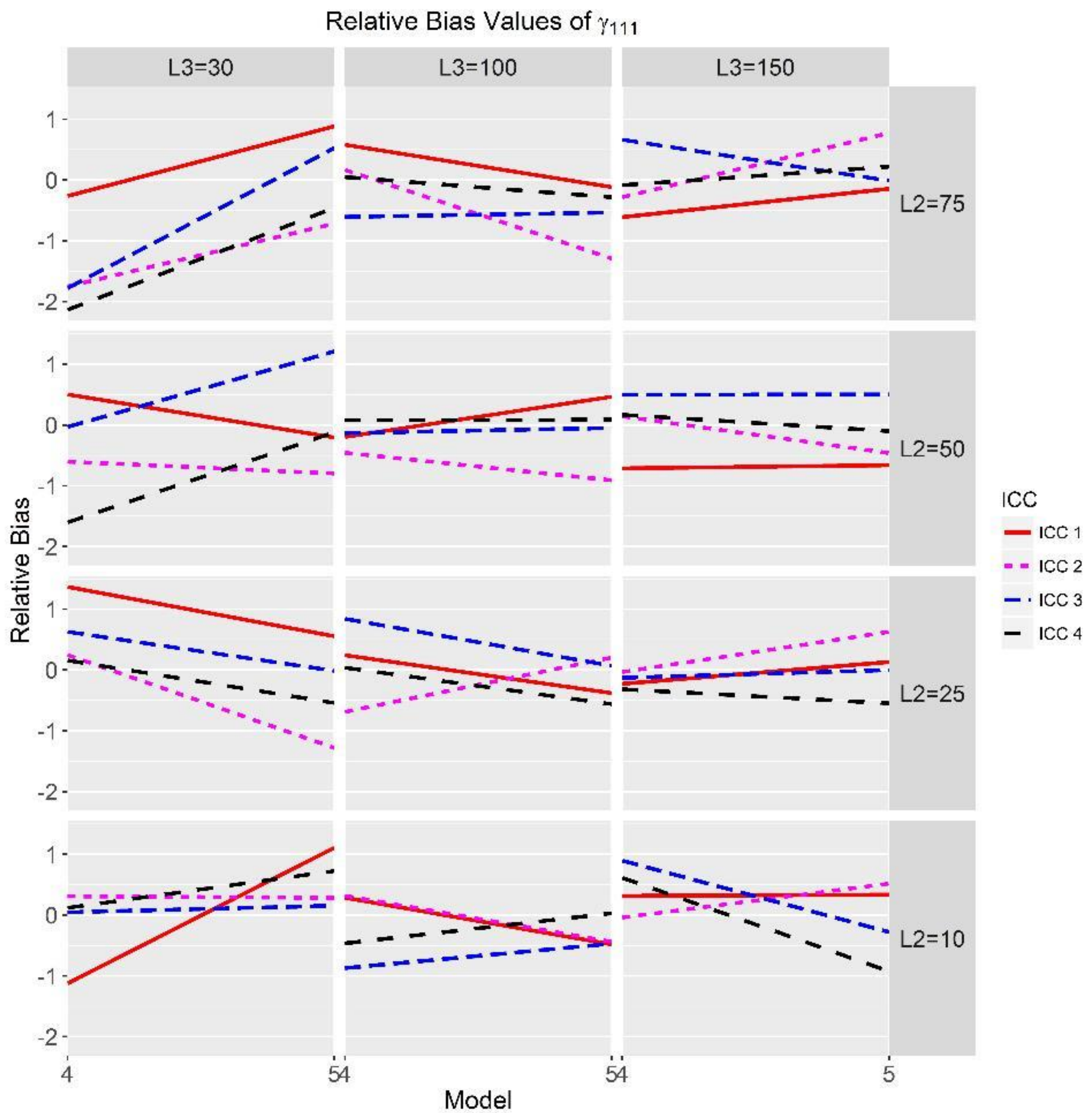

Figure E8. Plot of Relative Bias Across Manipulated Factors for γ_{111}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

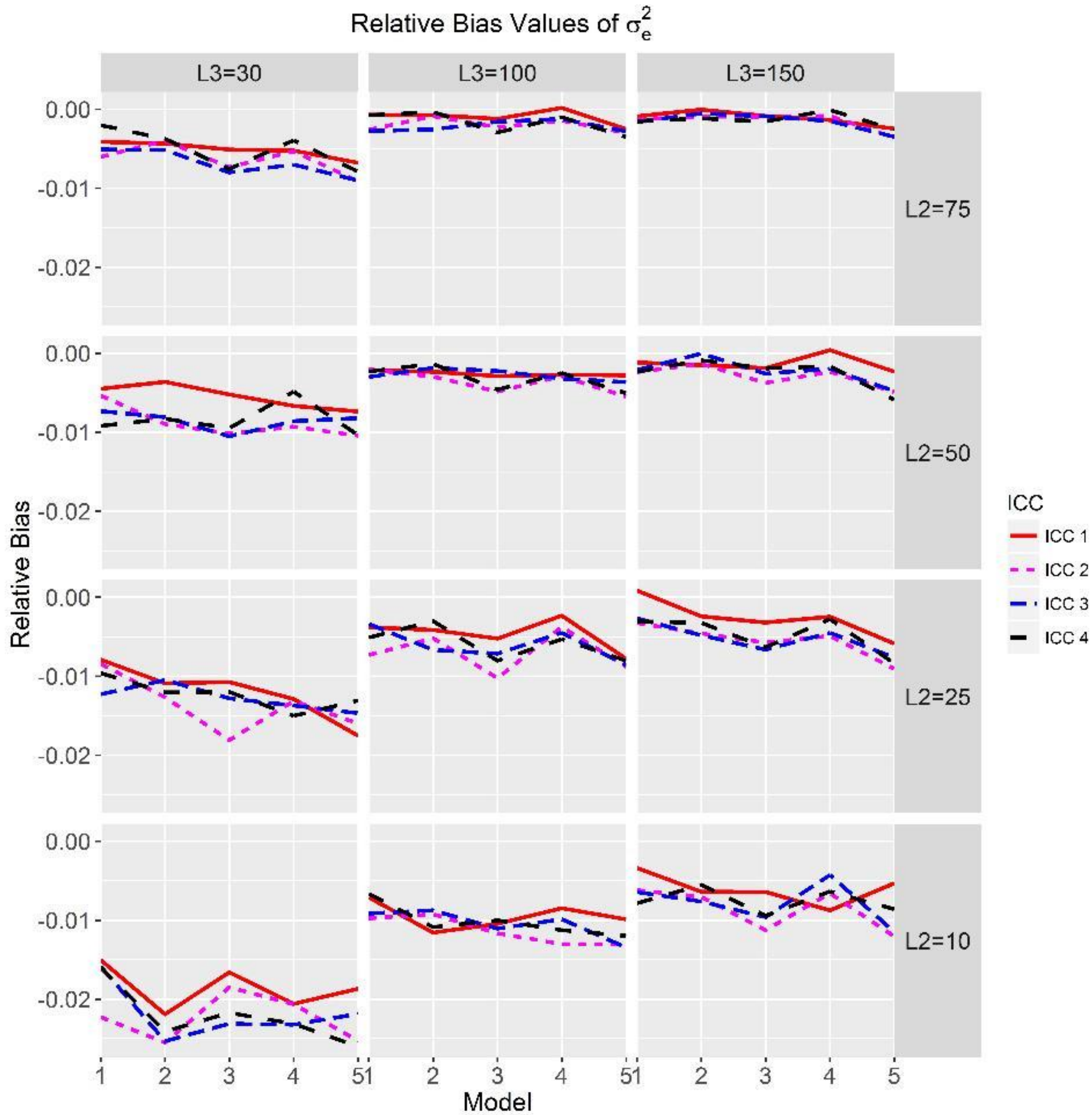

Figure E9. Plot of Relative Bias Across Manipulated Factors for σ_e^2.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

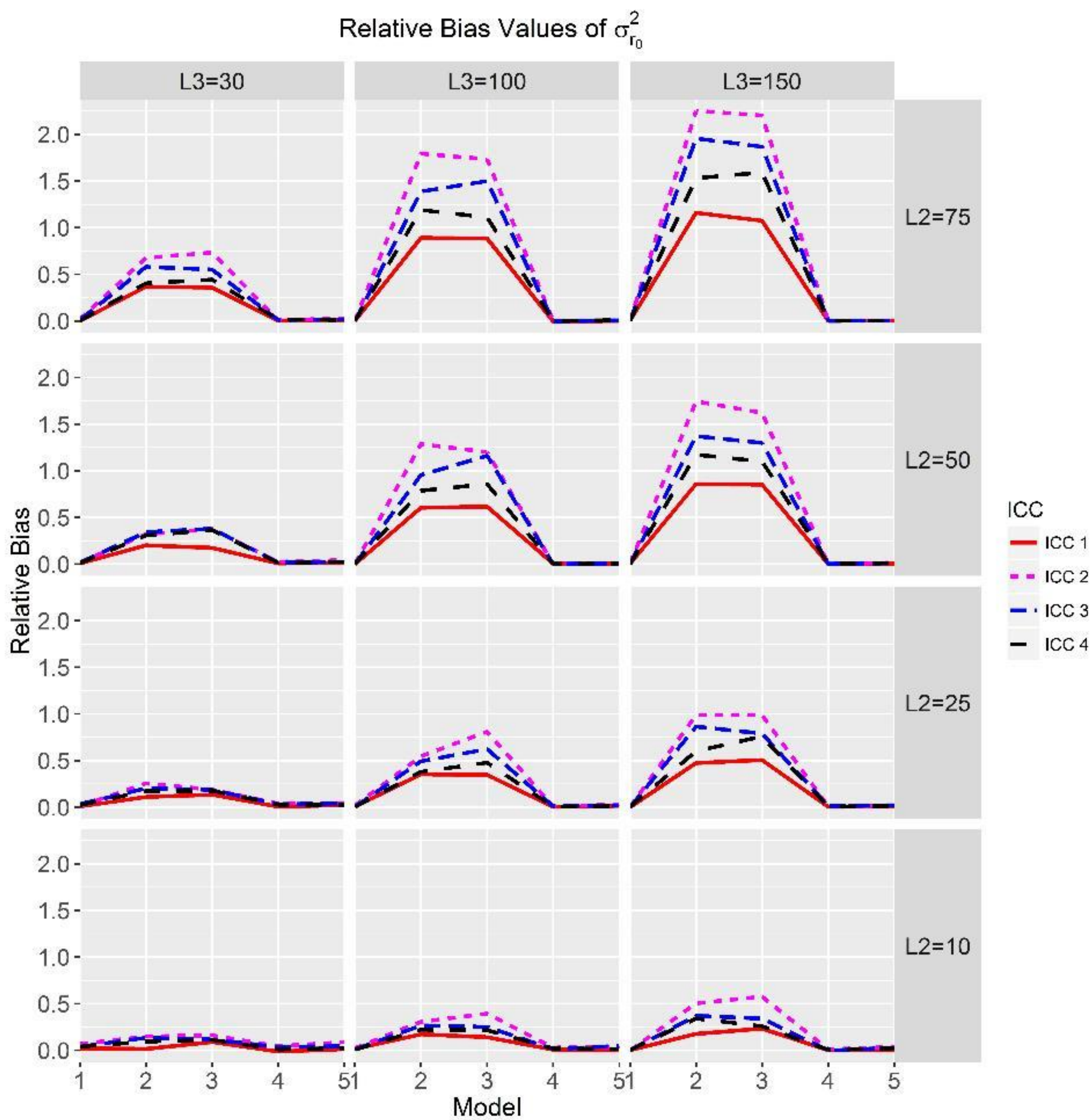

Figure E10. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{r_0}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

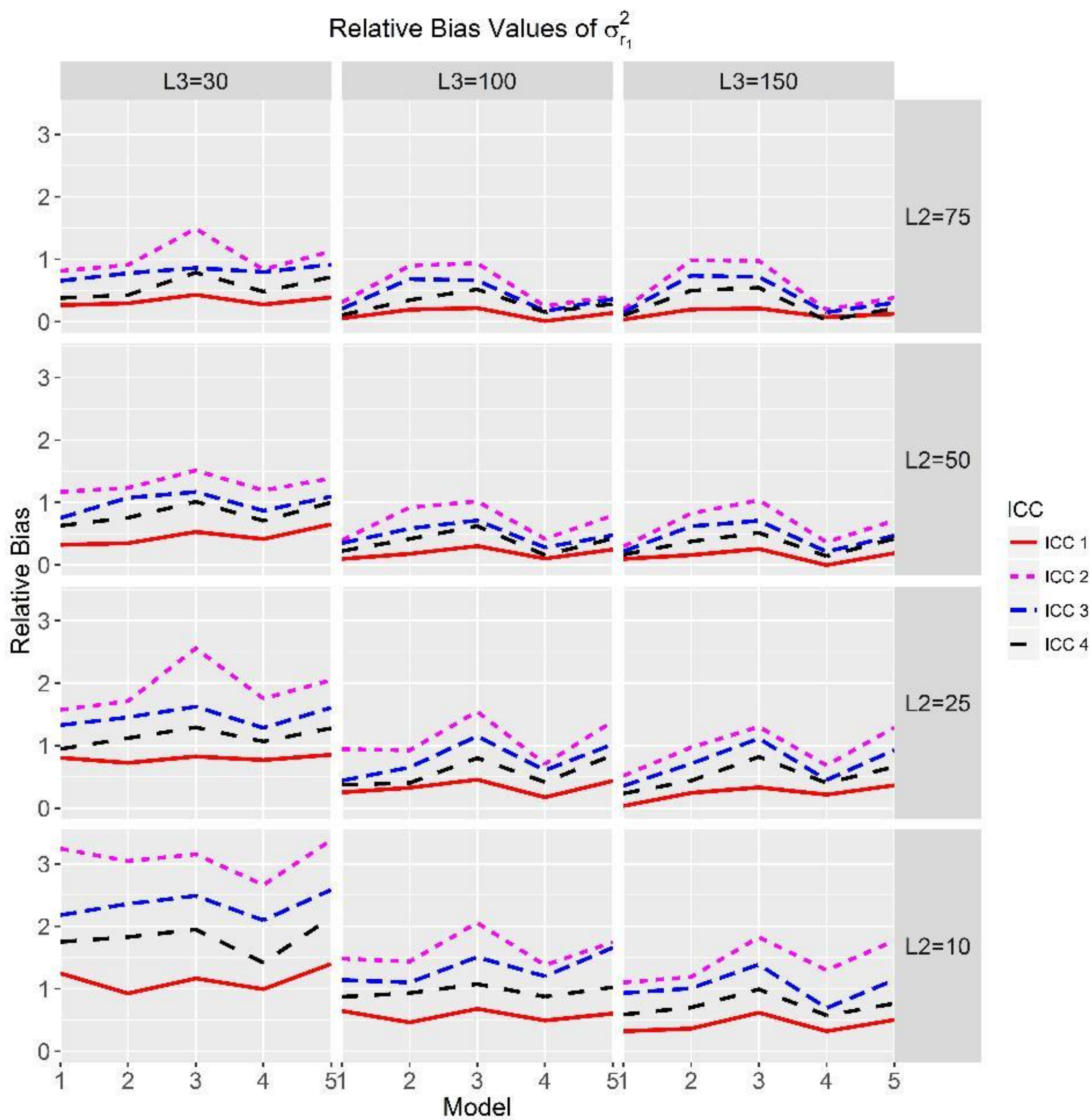

Figure E11. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

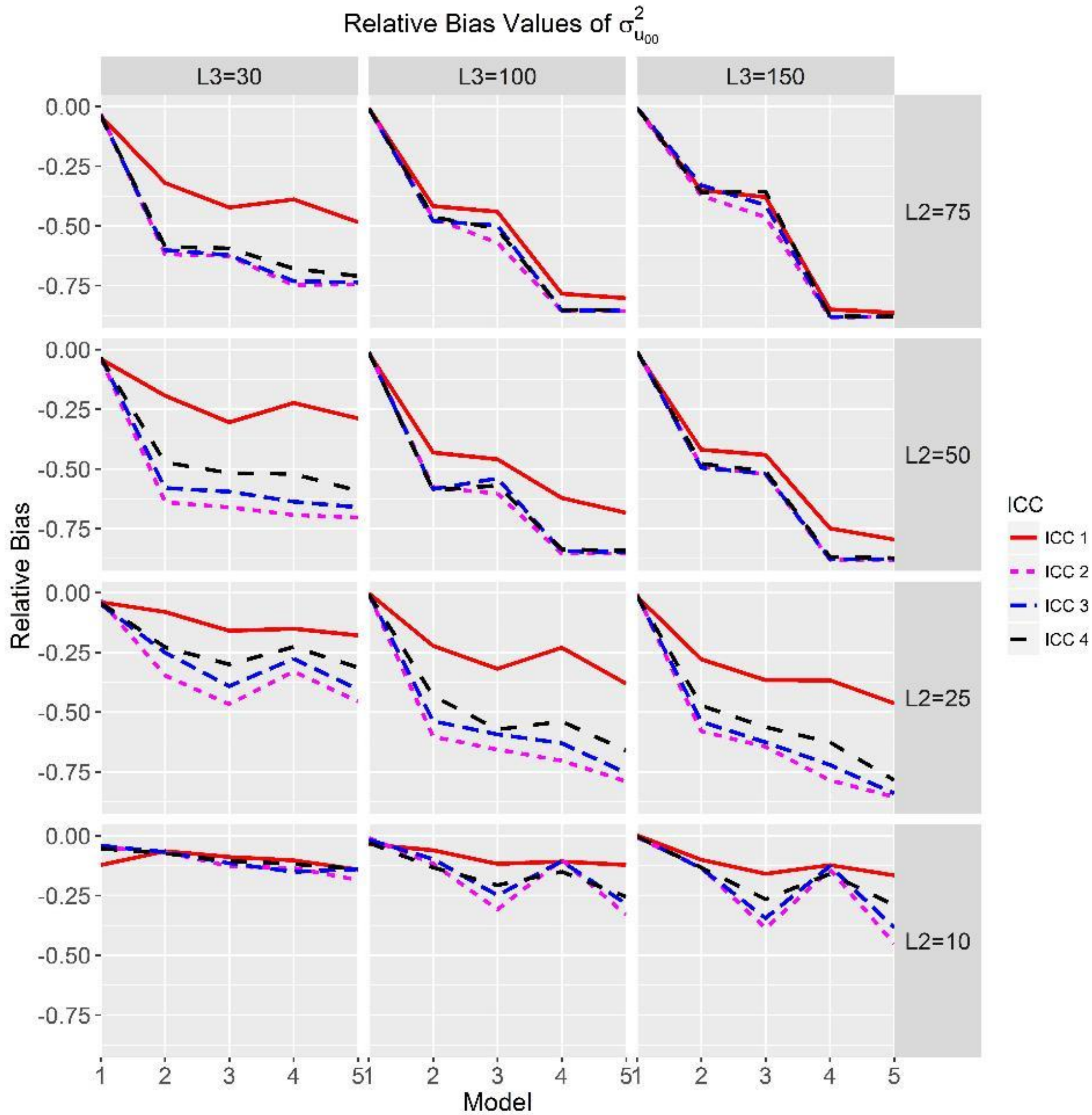

Figure E12. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{u_{00}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

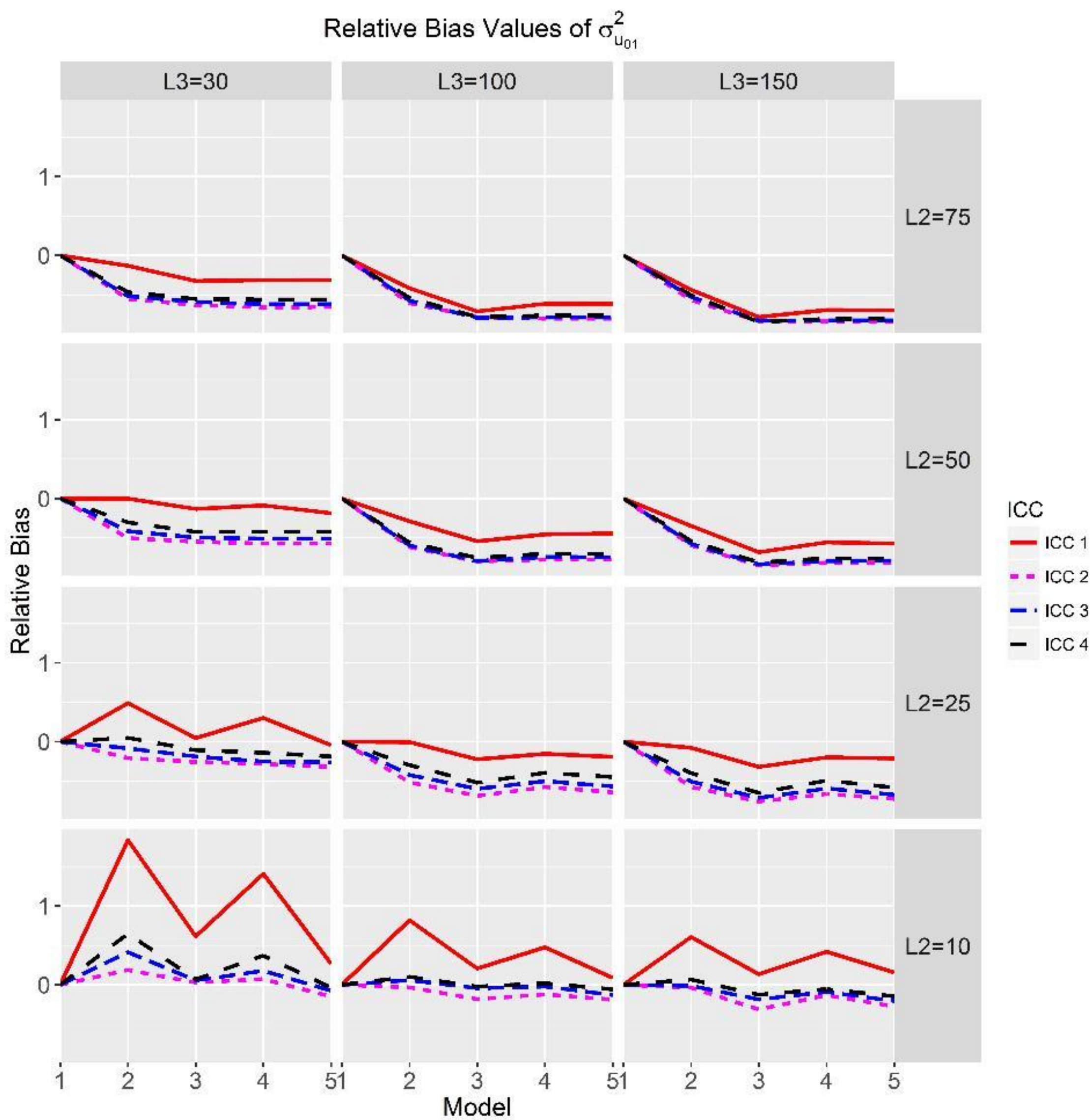

Figure E13. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{u_{01}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

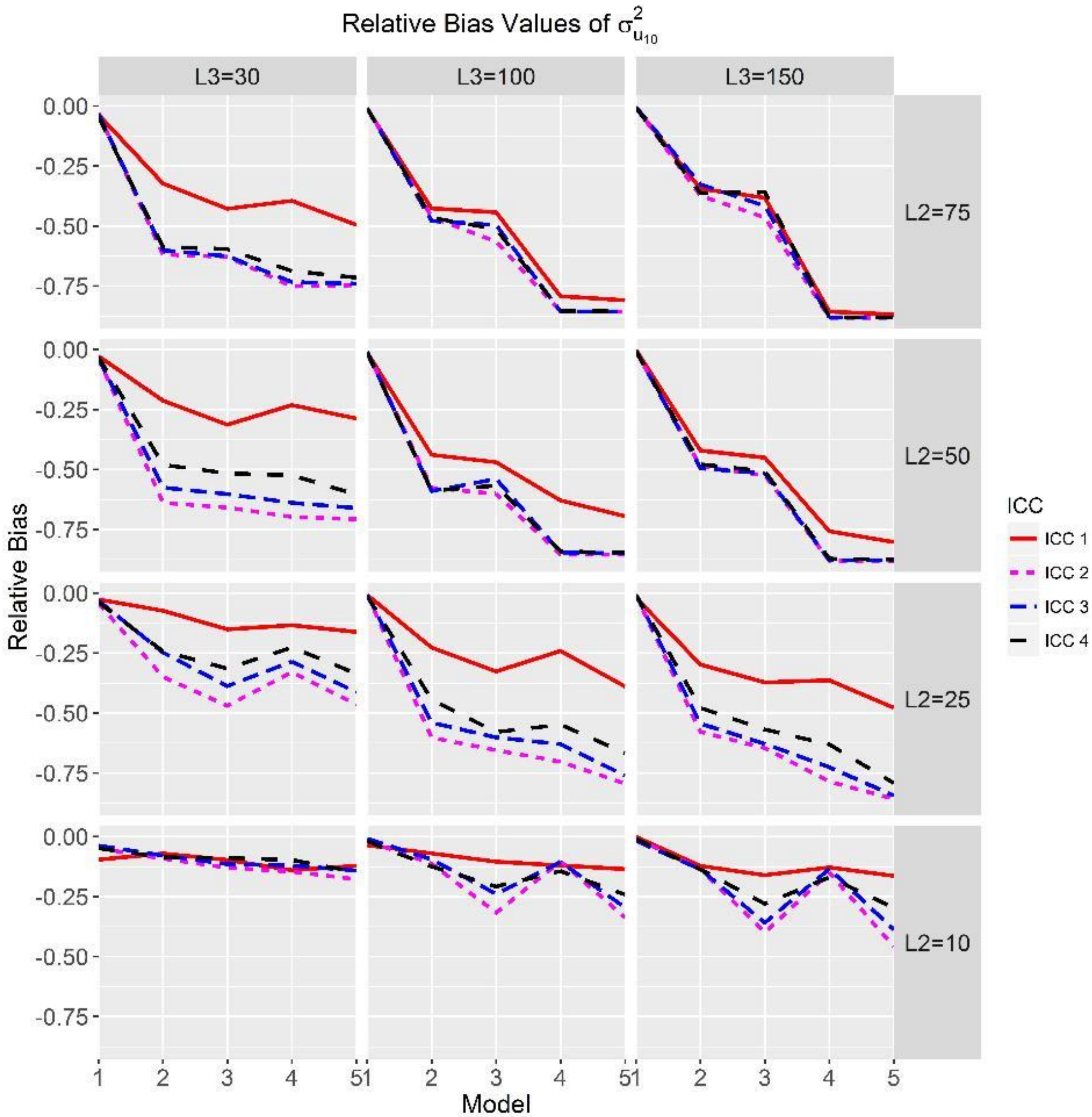

Figure E14. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

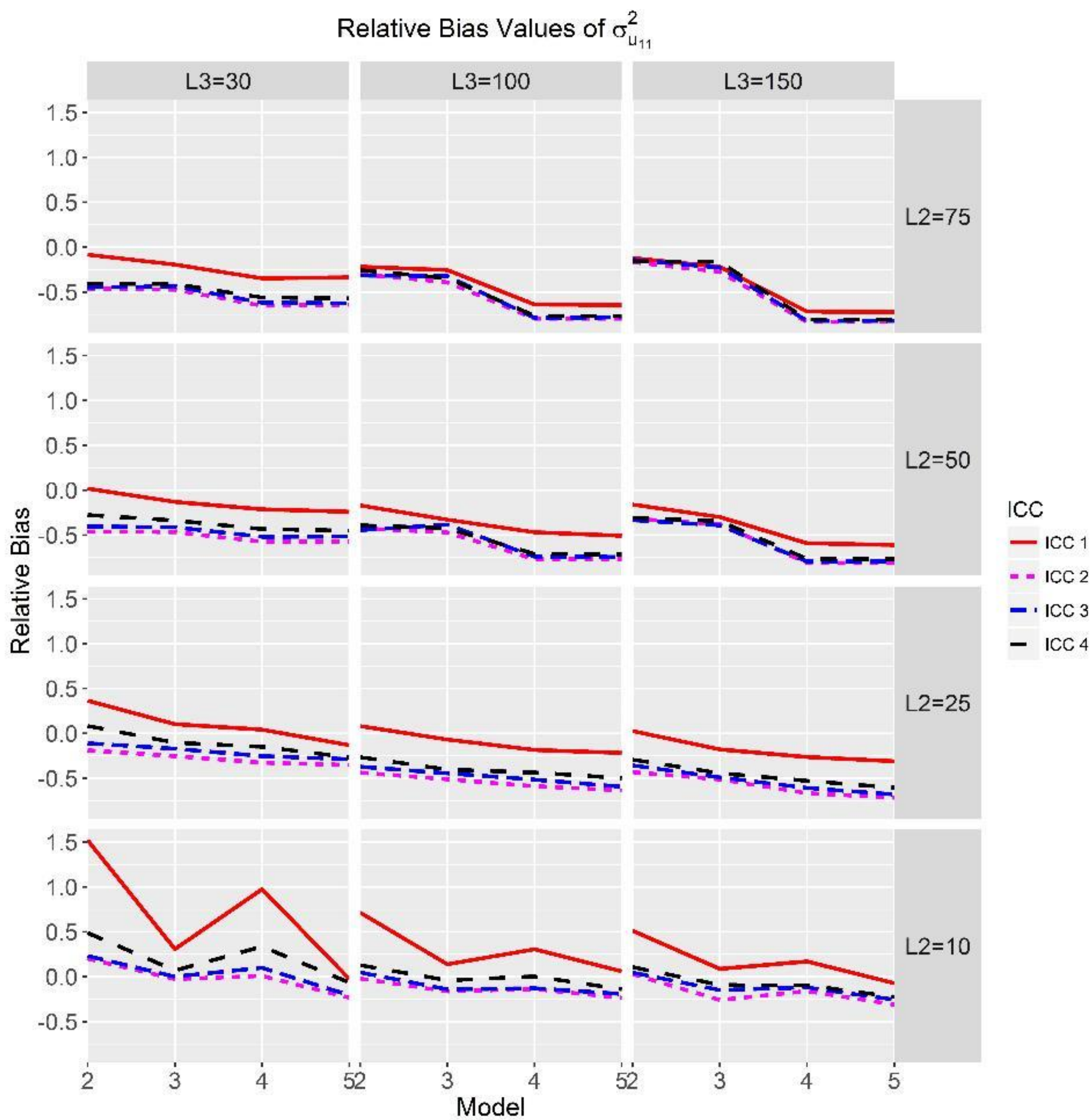

Figure E15. Plot of Relative Bias Across Manipulated Factors for $\sigma^2_{u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

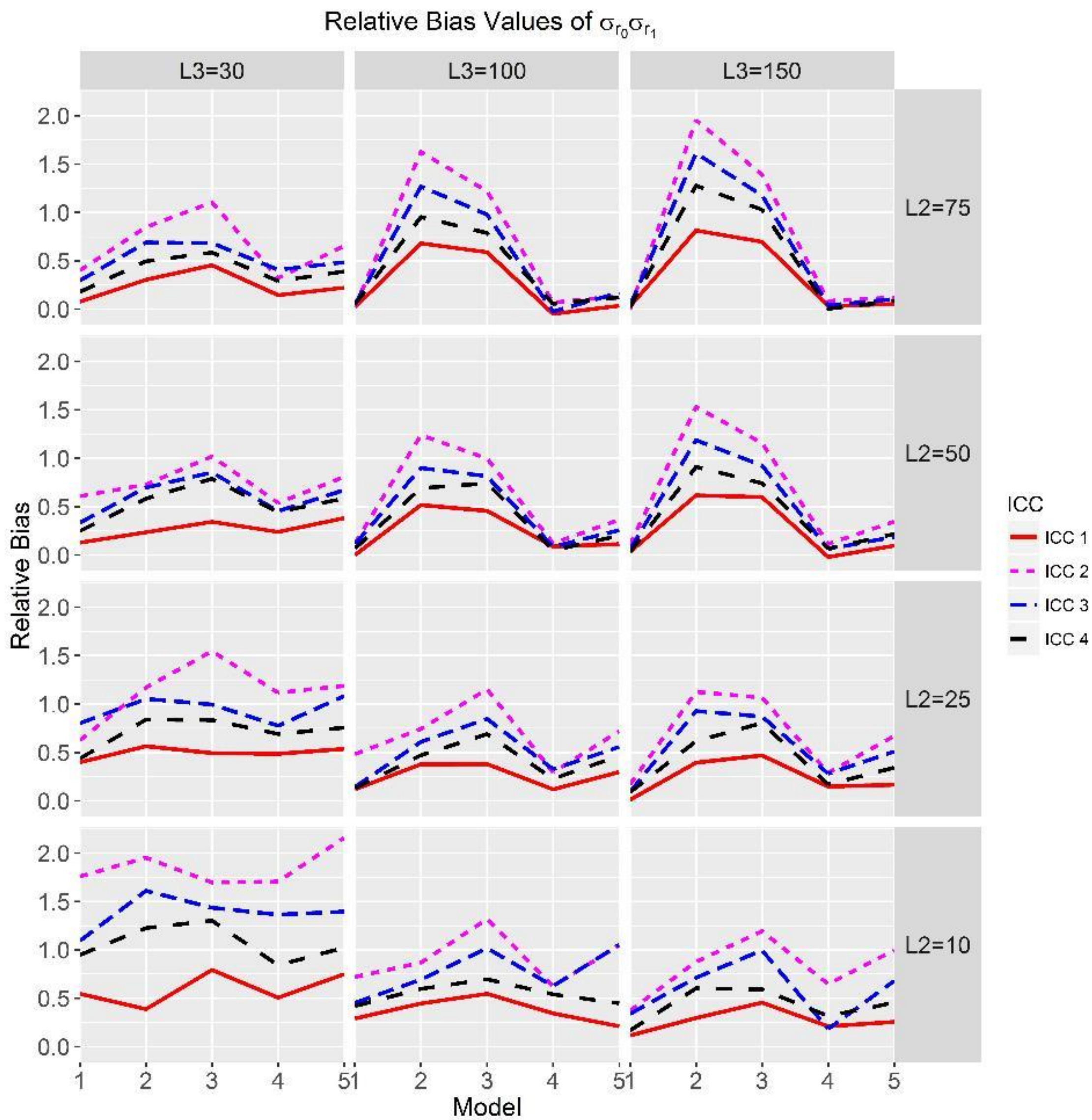

Figure E16. Plot of Relative Bias Across Manipulated Factors for $\sigma_{r_0 r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

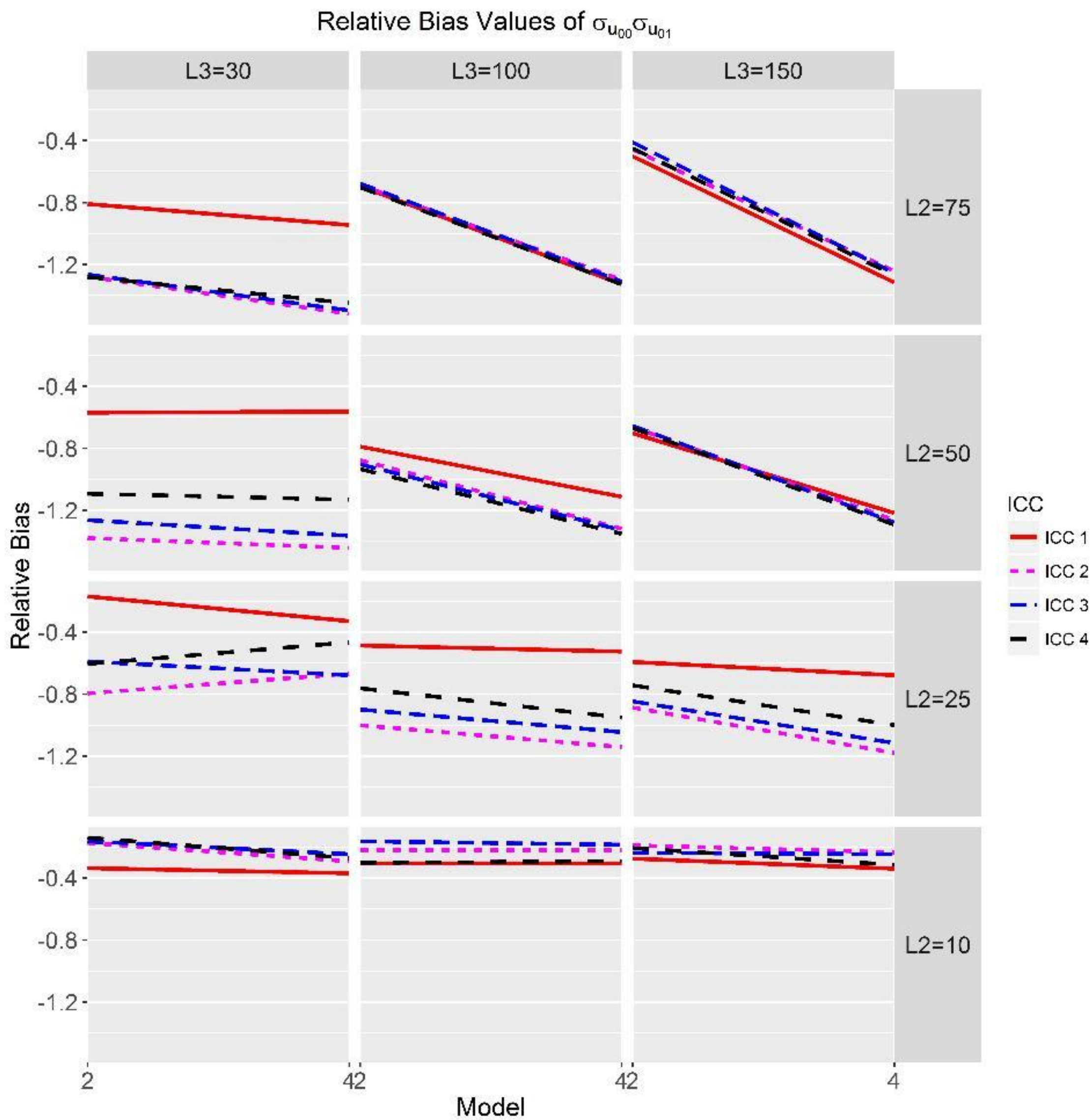

Figure E17. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{00}u_{01}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

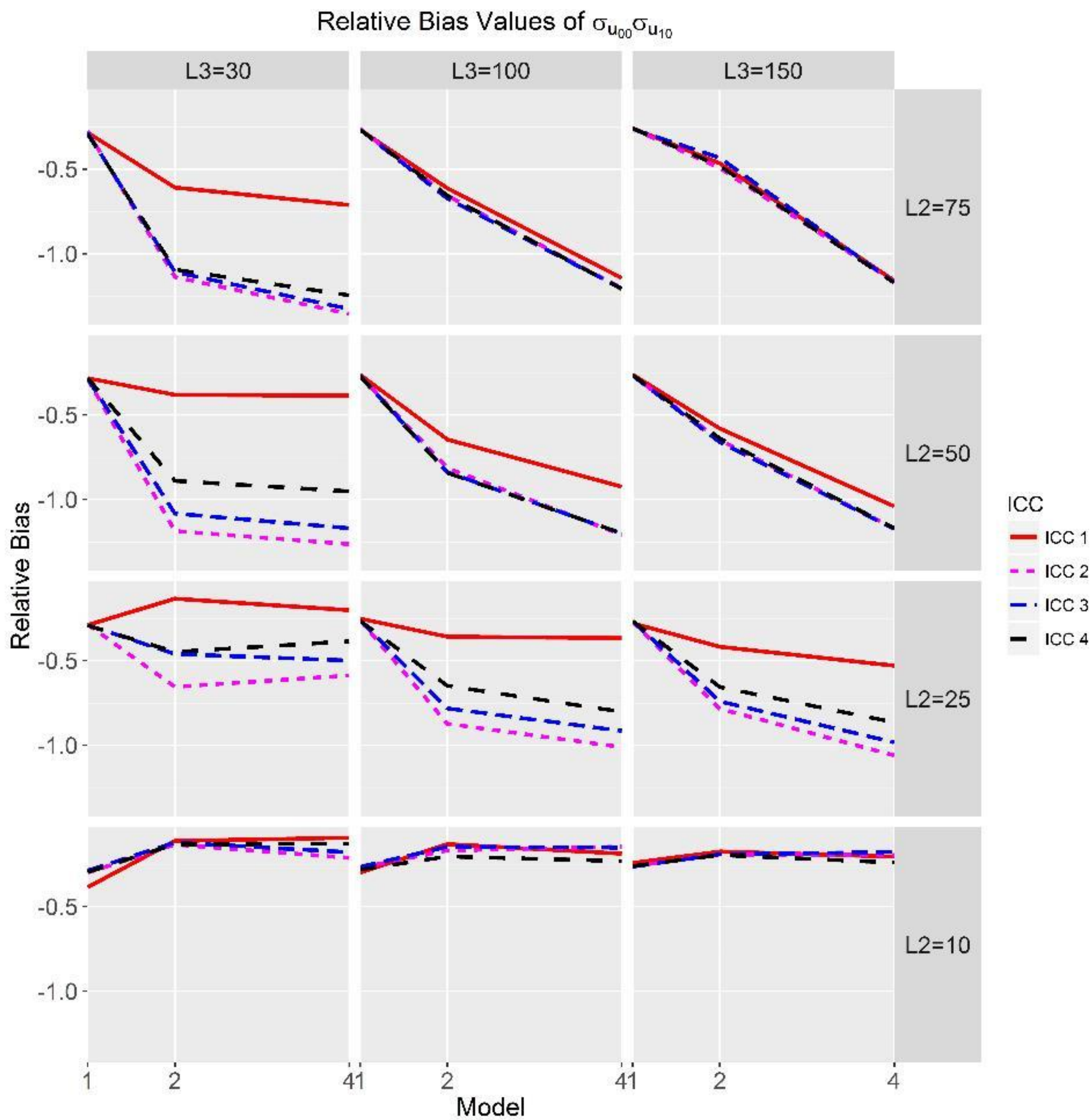

Figure E18. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{00}u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

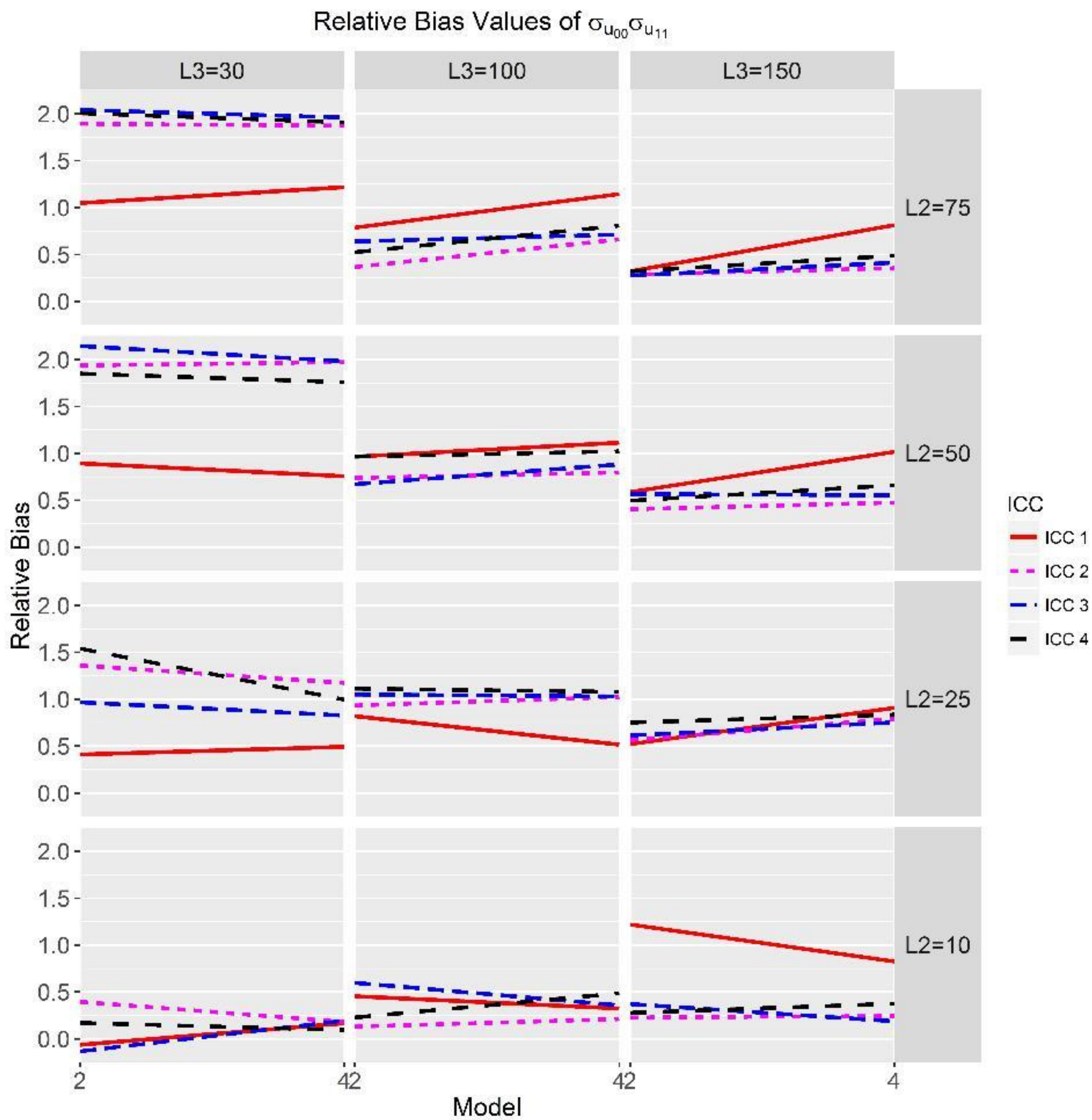

Figure E19. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{00}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

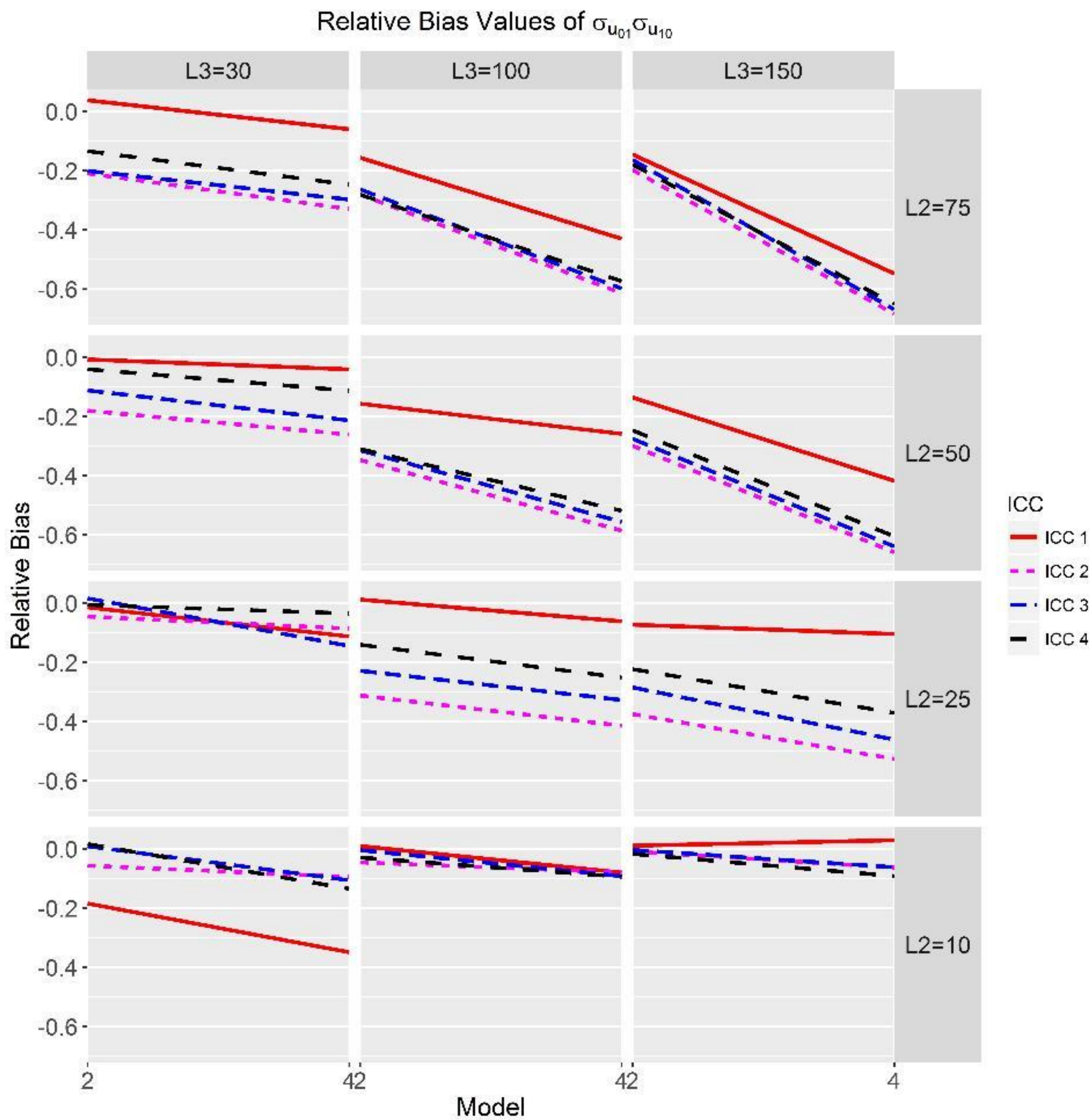

Figure E20. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{01}u_{10}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

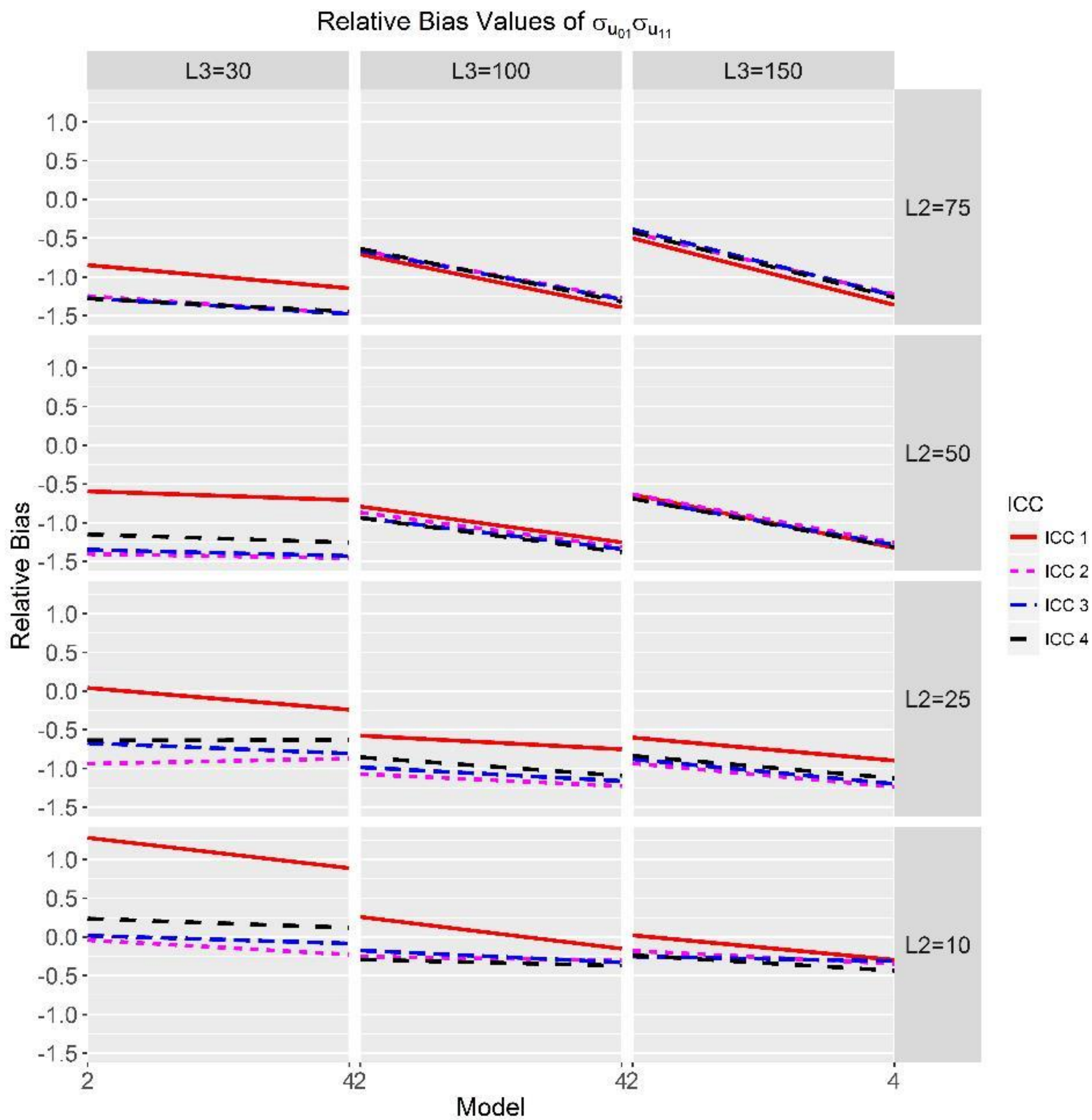

Figure E21. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{01}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

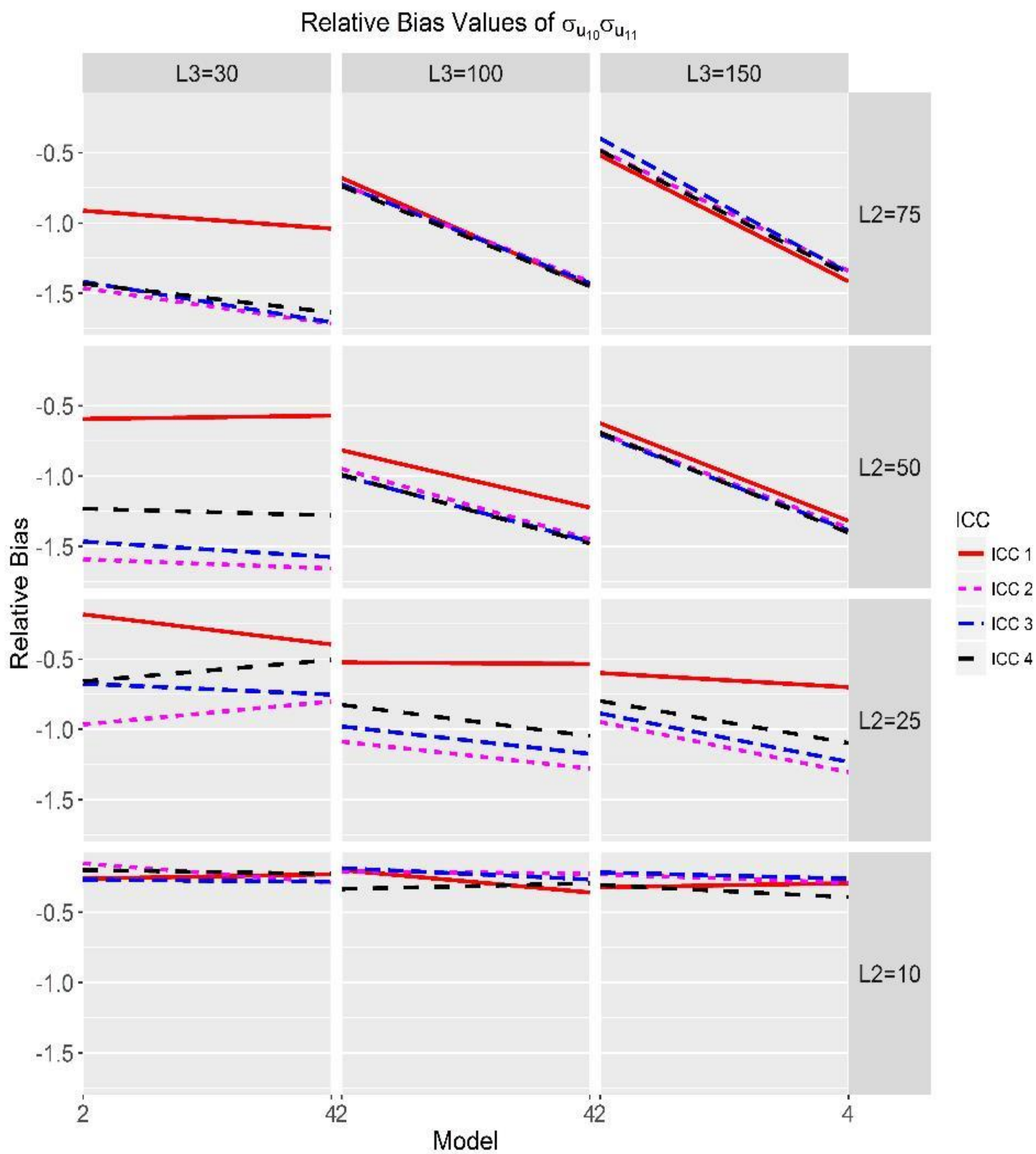

Figure E22. Plot of Relative Bias Across Manipulated Factors for $\sigma_{u_{10}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

APPENDIX F

VISUAL REPRESENTATION OF RMSE VALUES

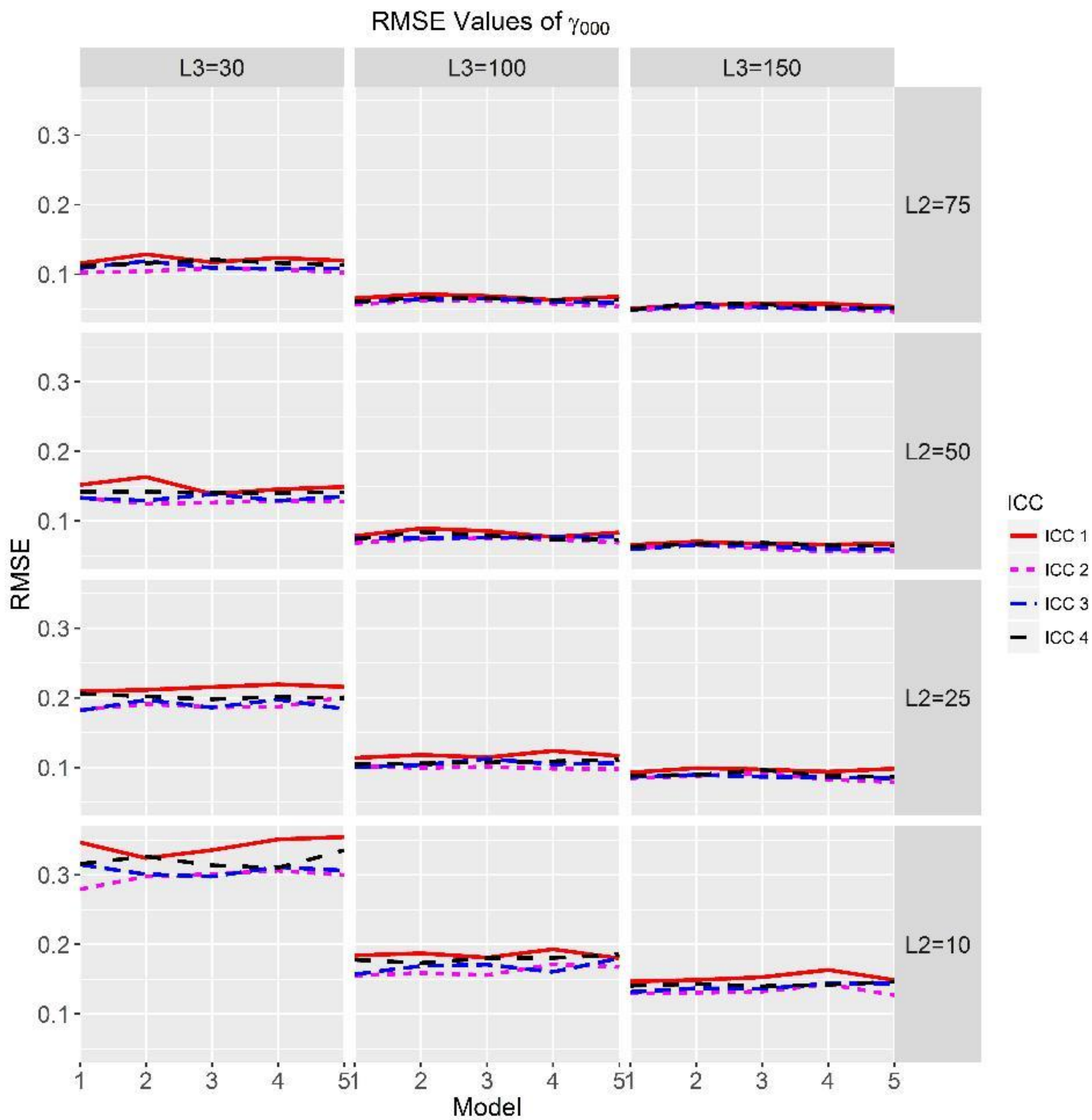

Figure F1. Plot of RMSE Across Manipulated Factors for γ_{000}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

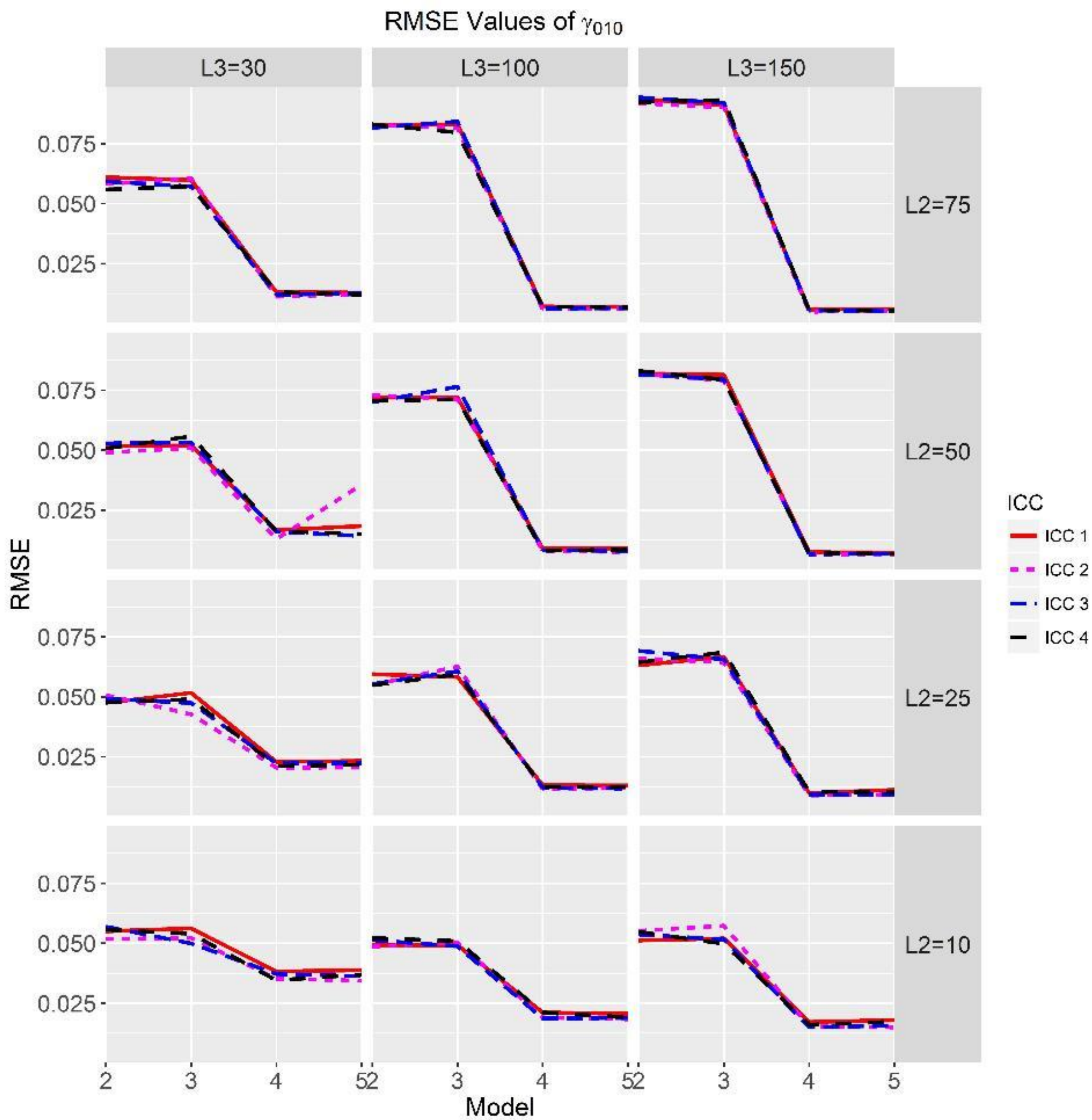

Figure F2. Plot of RMSE Across Manipulated Factors for γ_{010}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

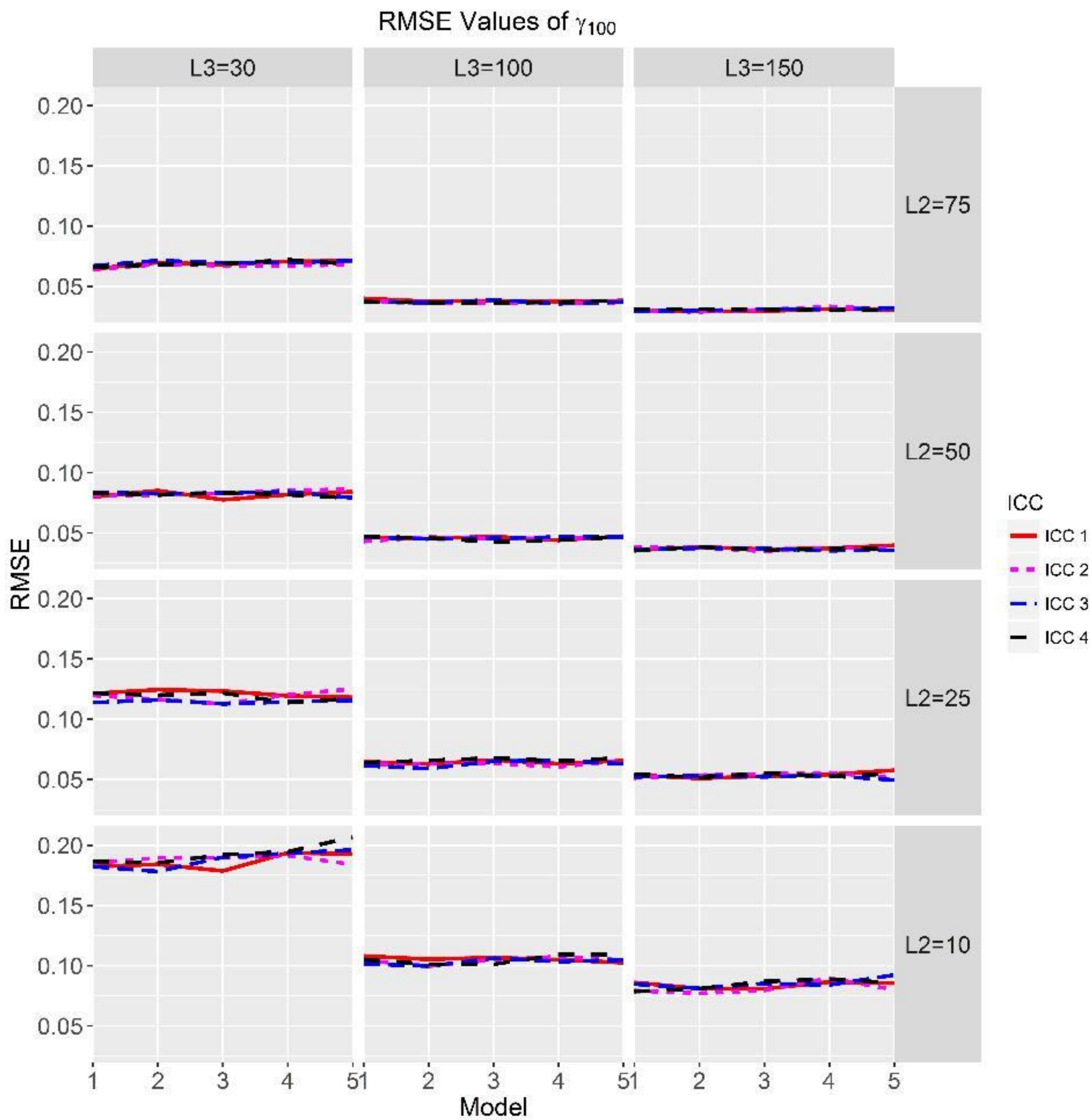

Figure F3. Plot of RMSE Across Manipulated Factors for γ_{100}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

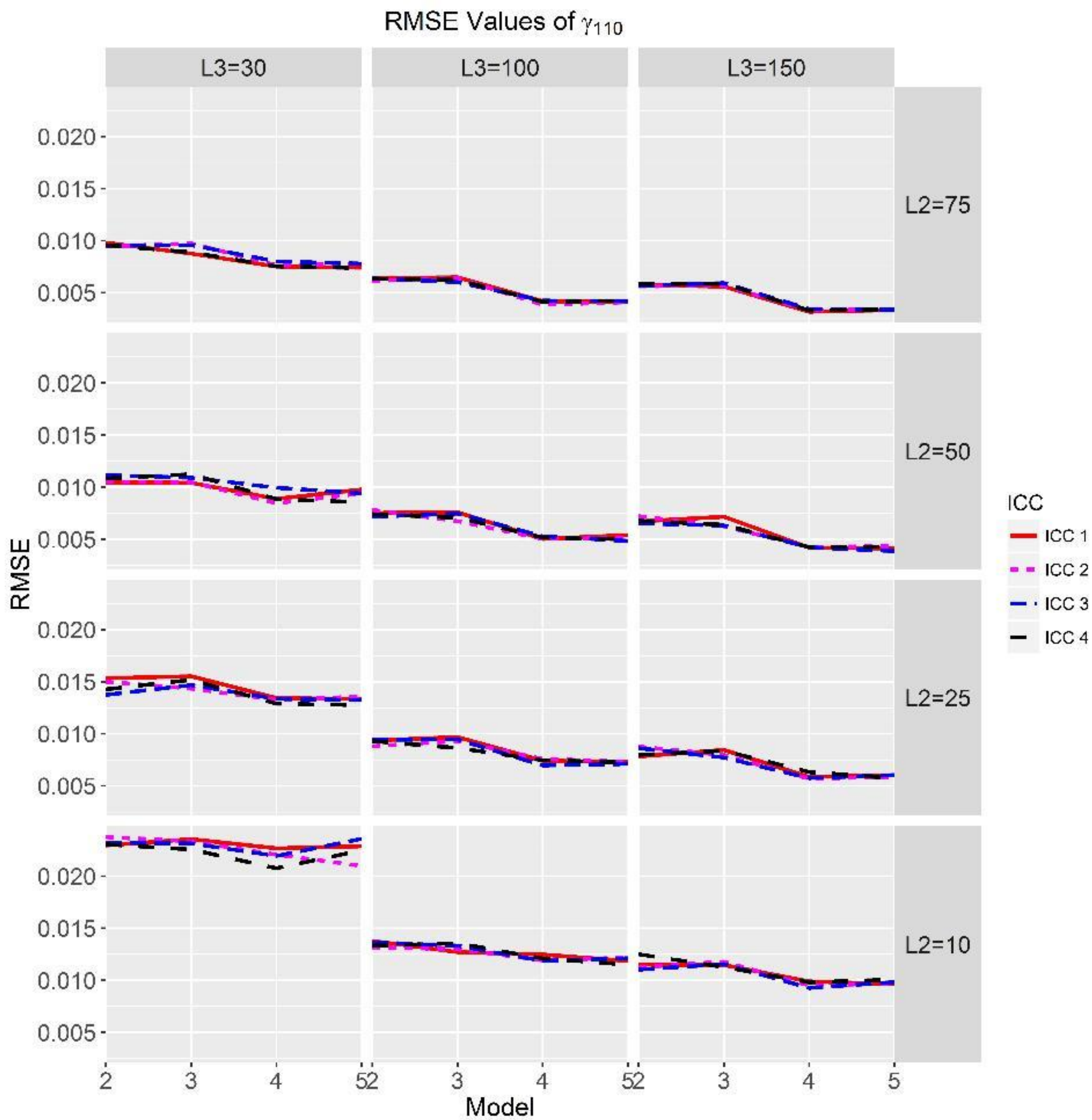

Figure F4. Plot of RMSE Across Manipulated Factors for γ_{110}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

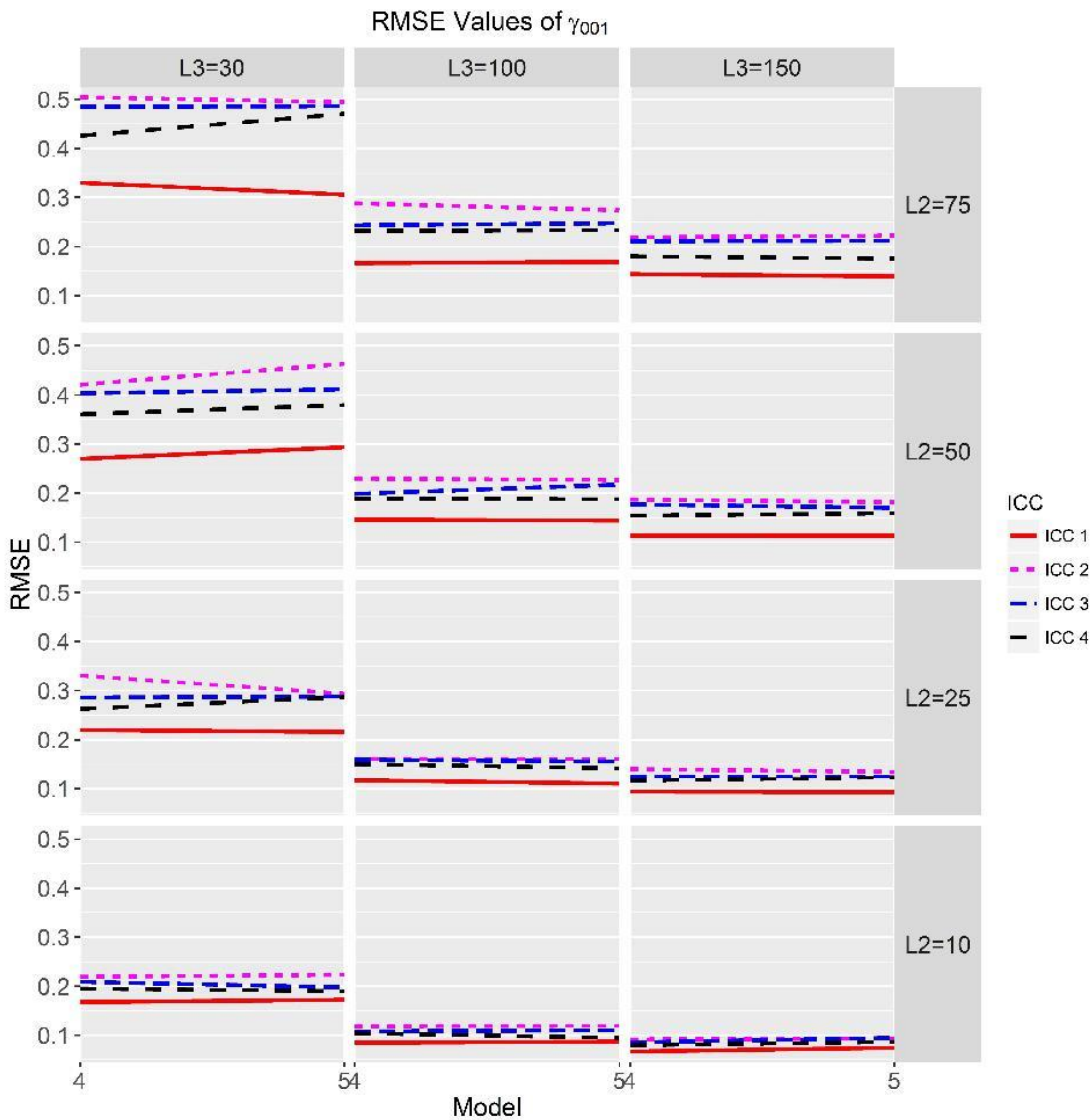

Figure F5. Plot of RMSE Across Manipulated Factors for γ_{001}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

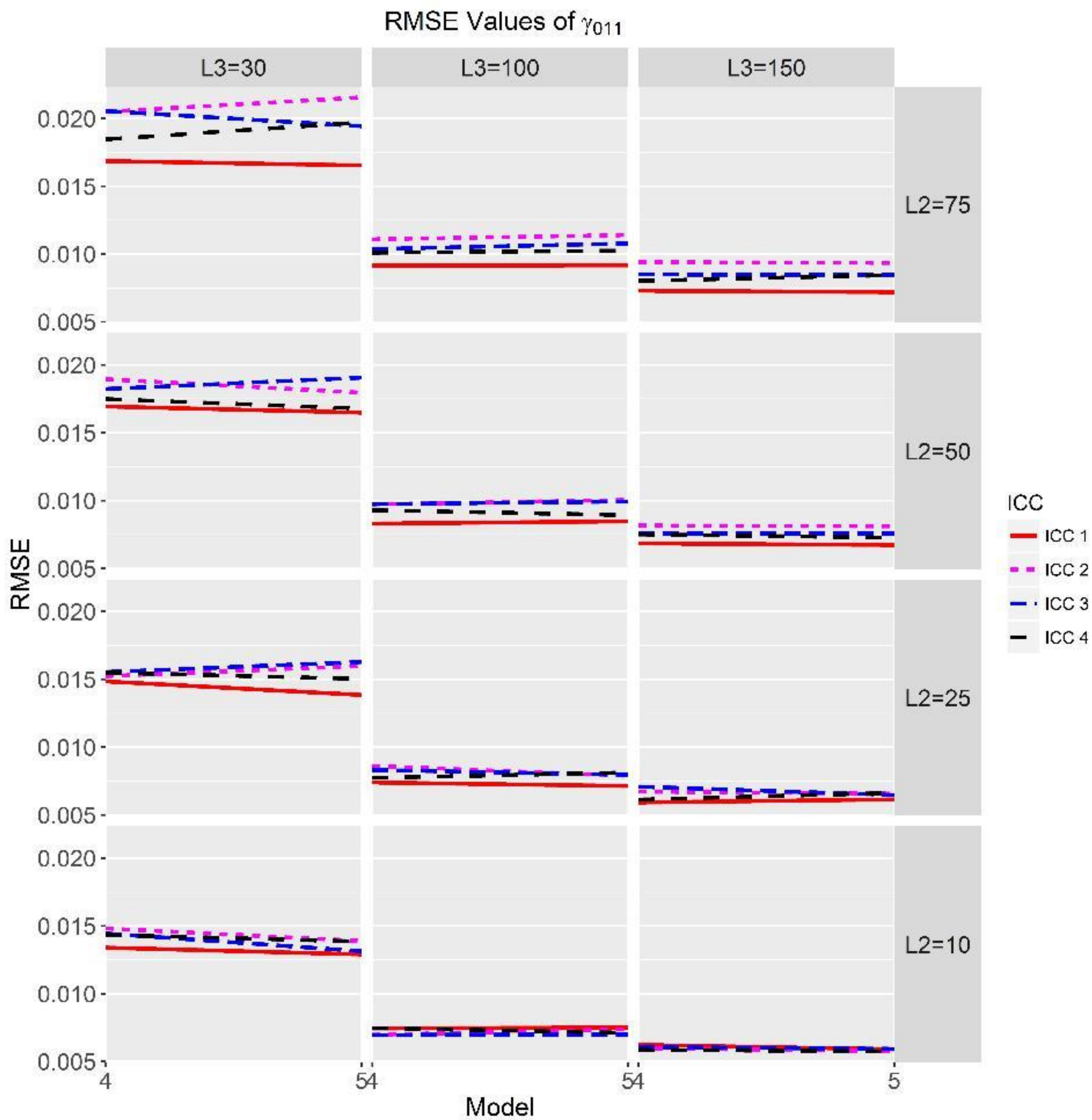

Figure F6. Plot of RMSE Across Manipulated Factors for γ_{011}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

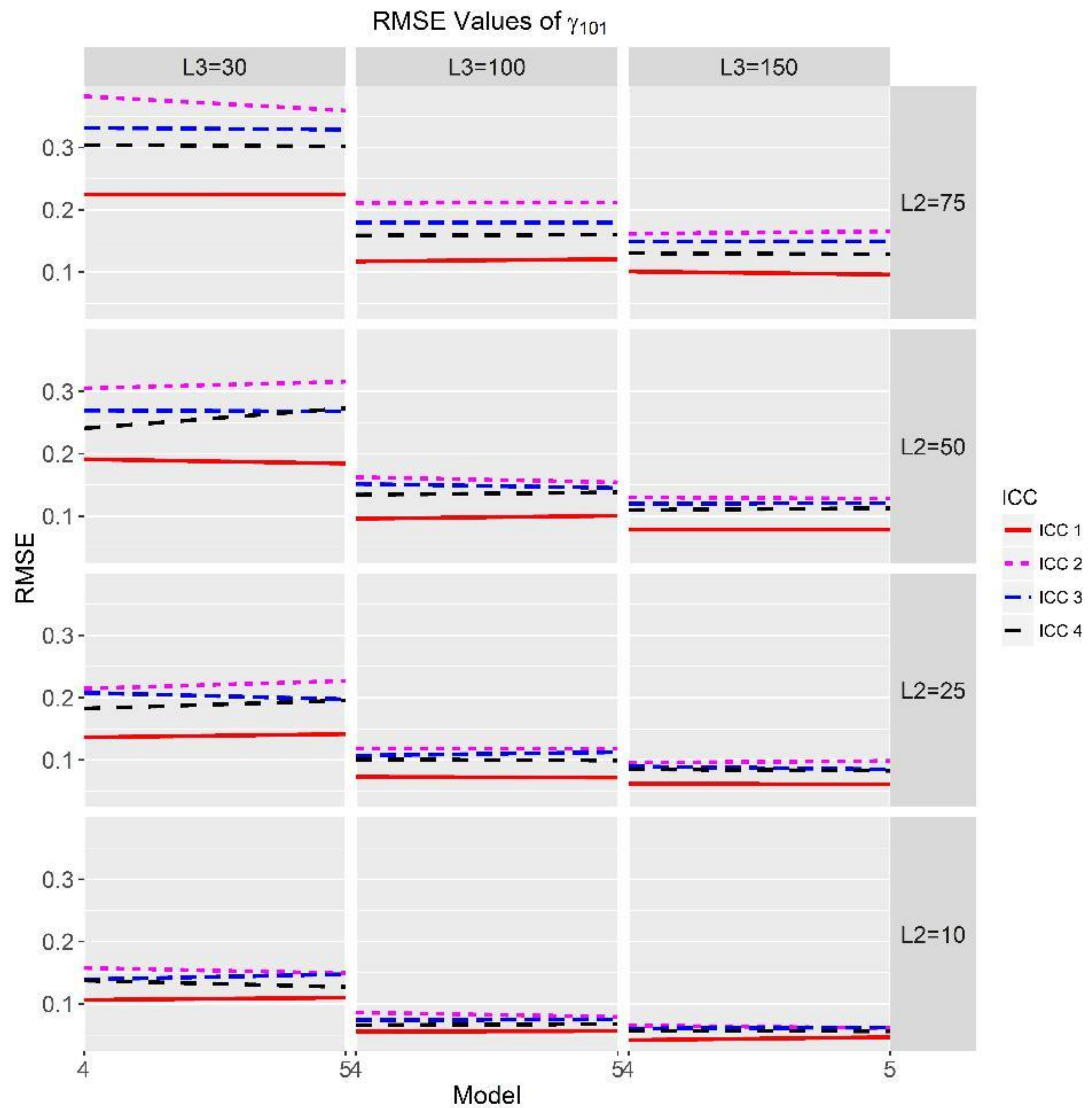

Figure F7. Plot of RMSE Across Manipulated Factors for γ_{101}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

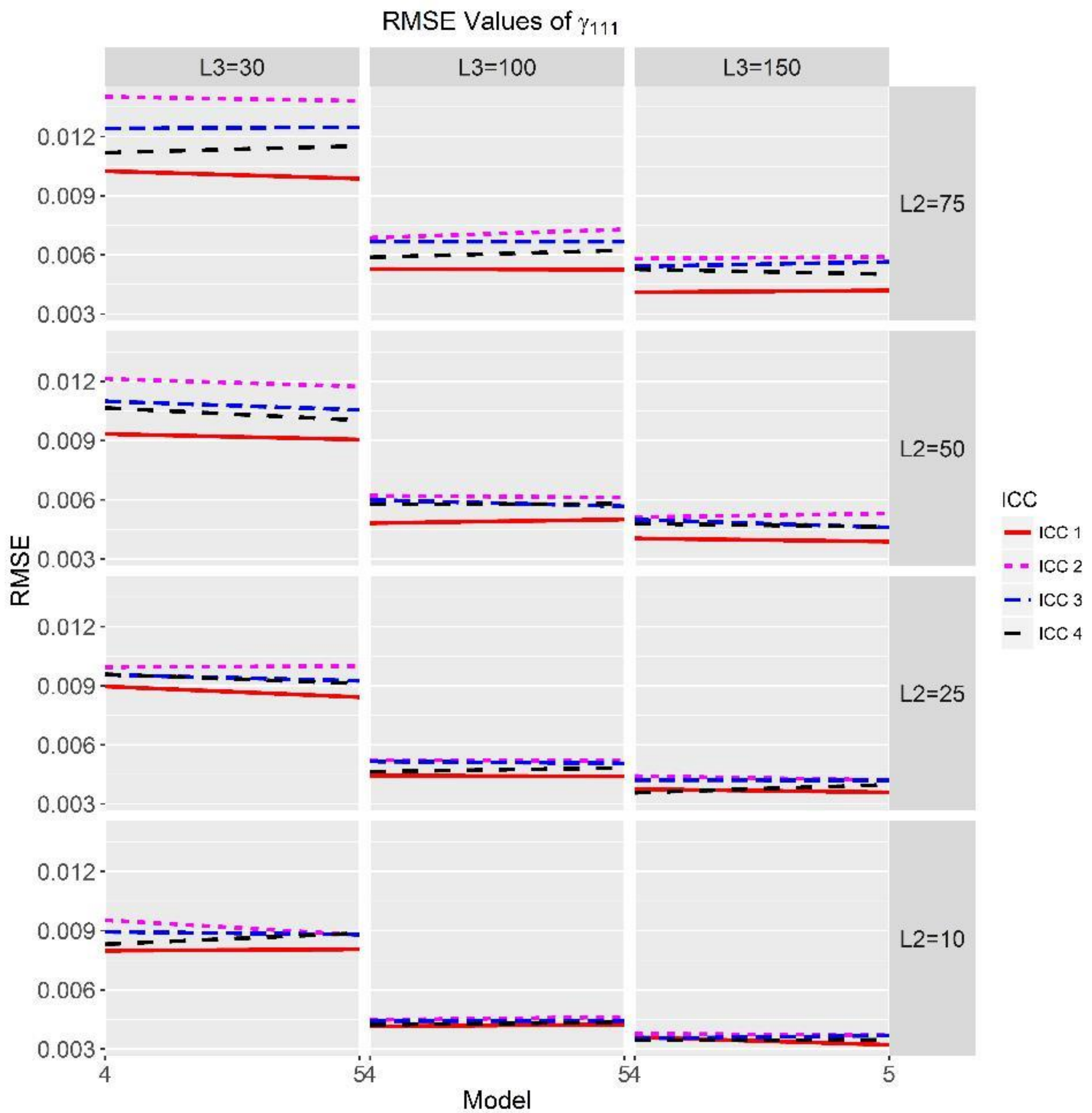

Figure F8. Plot of RMSE Across Manipulated Factors for γ_{111}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

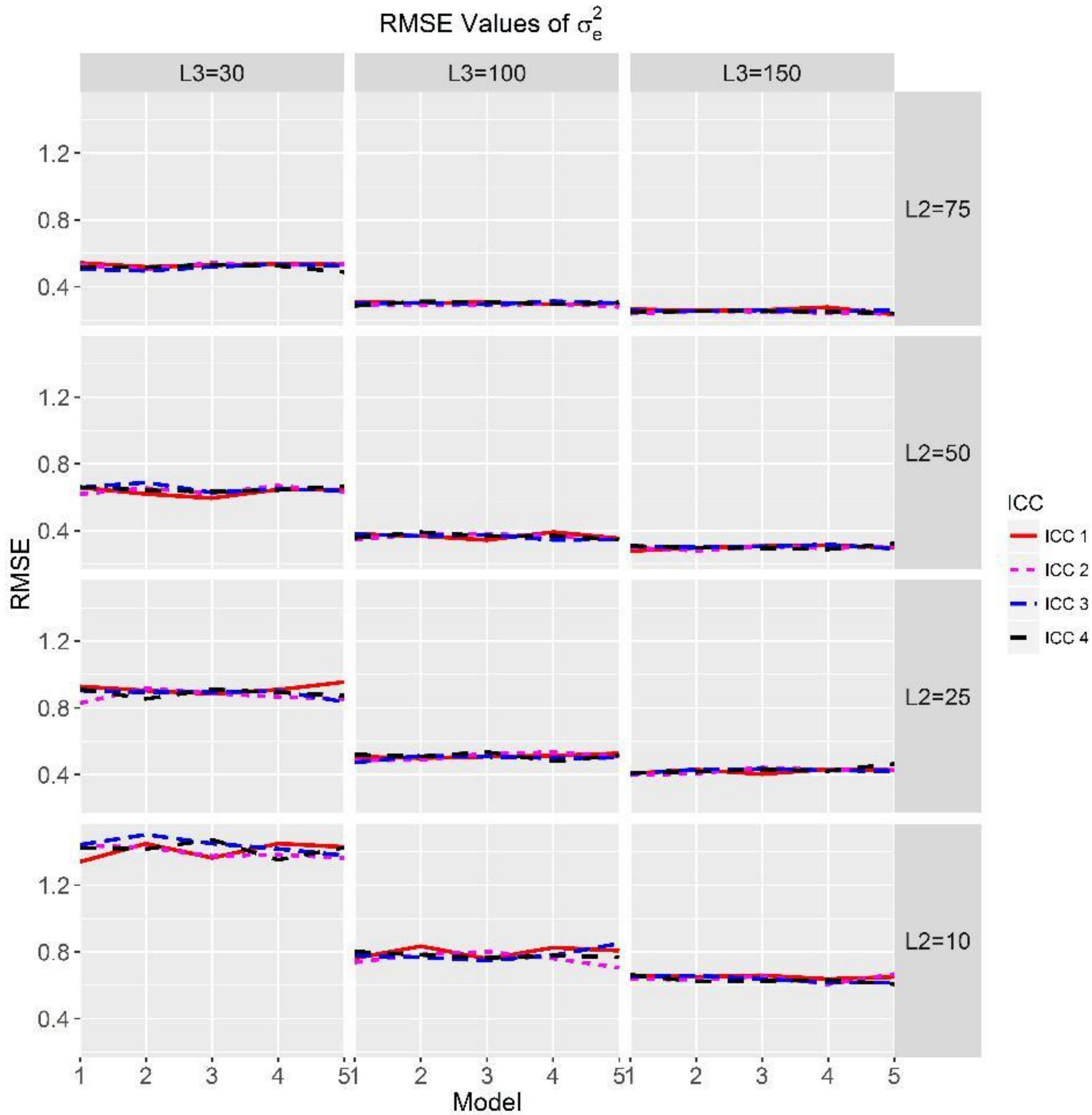

Figure F9. Plot of RMSE Across Manipulated Factors for σ_e^2.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

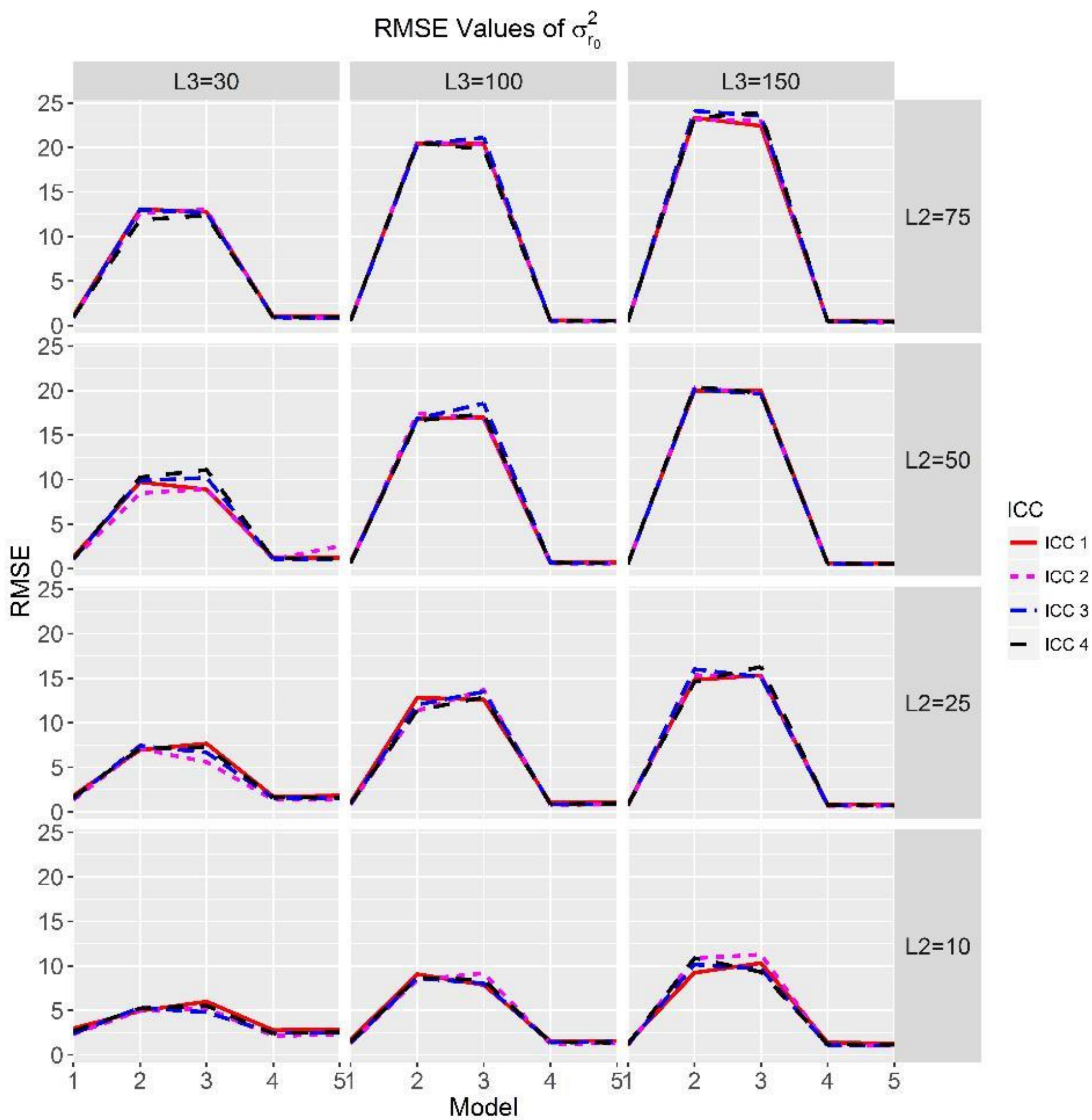

Figure F10. Plot of RMSE Across Manipulated Factors for $\sigma^2_{r_0}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

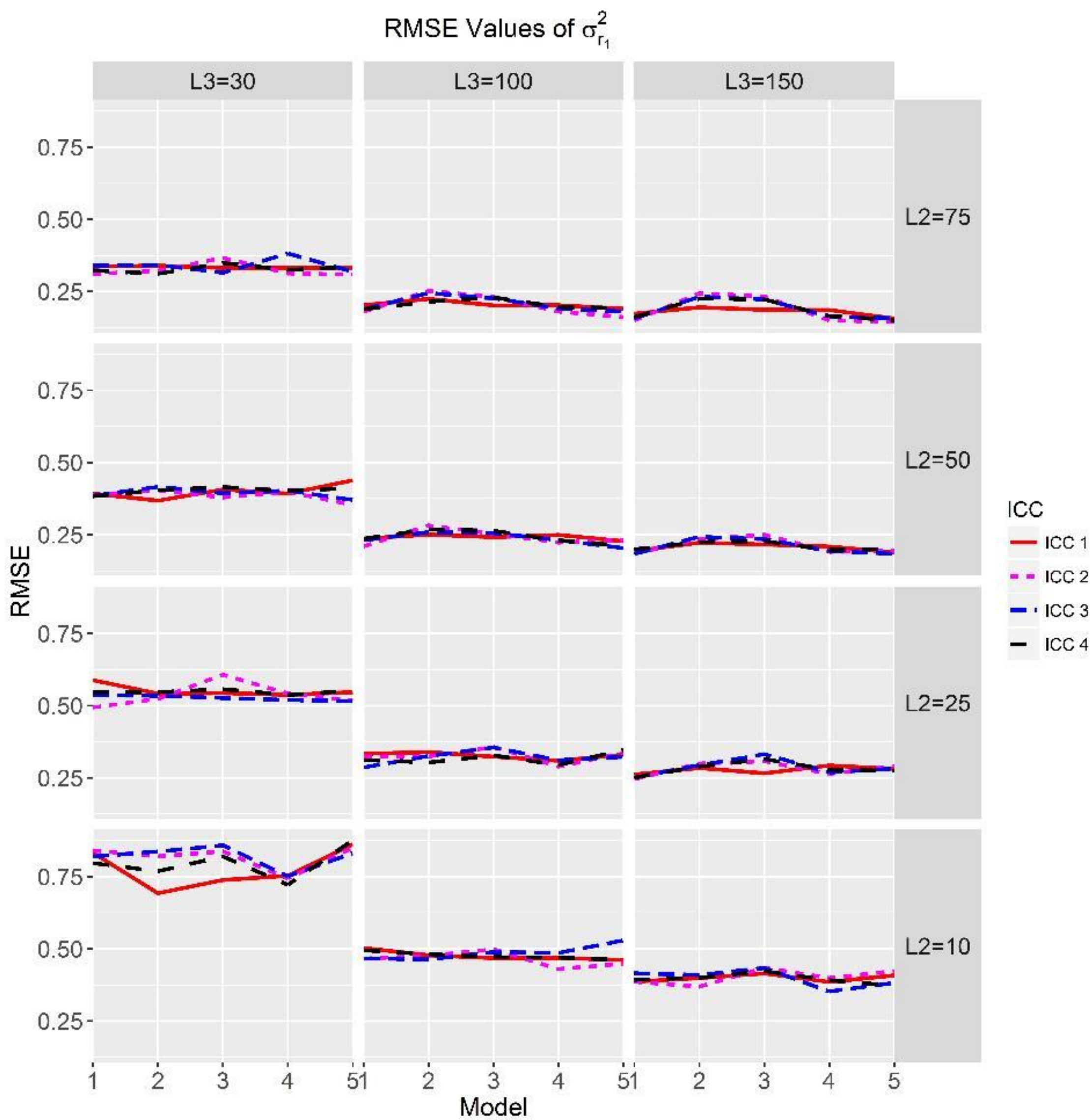

Figure F11. Plot of RMSE Across Manipulated Factors for $\sigma^2_{r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

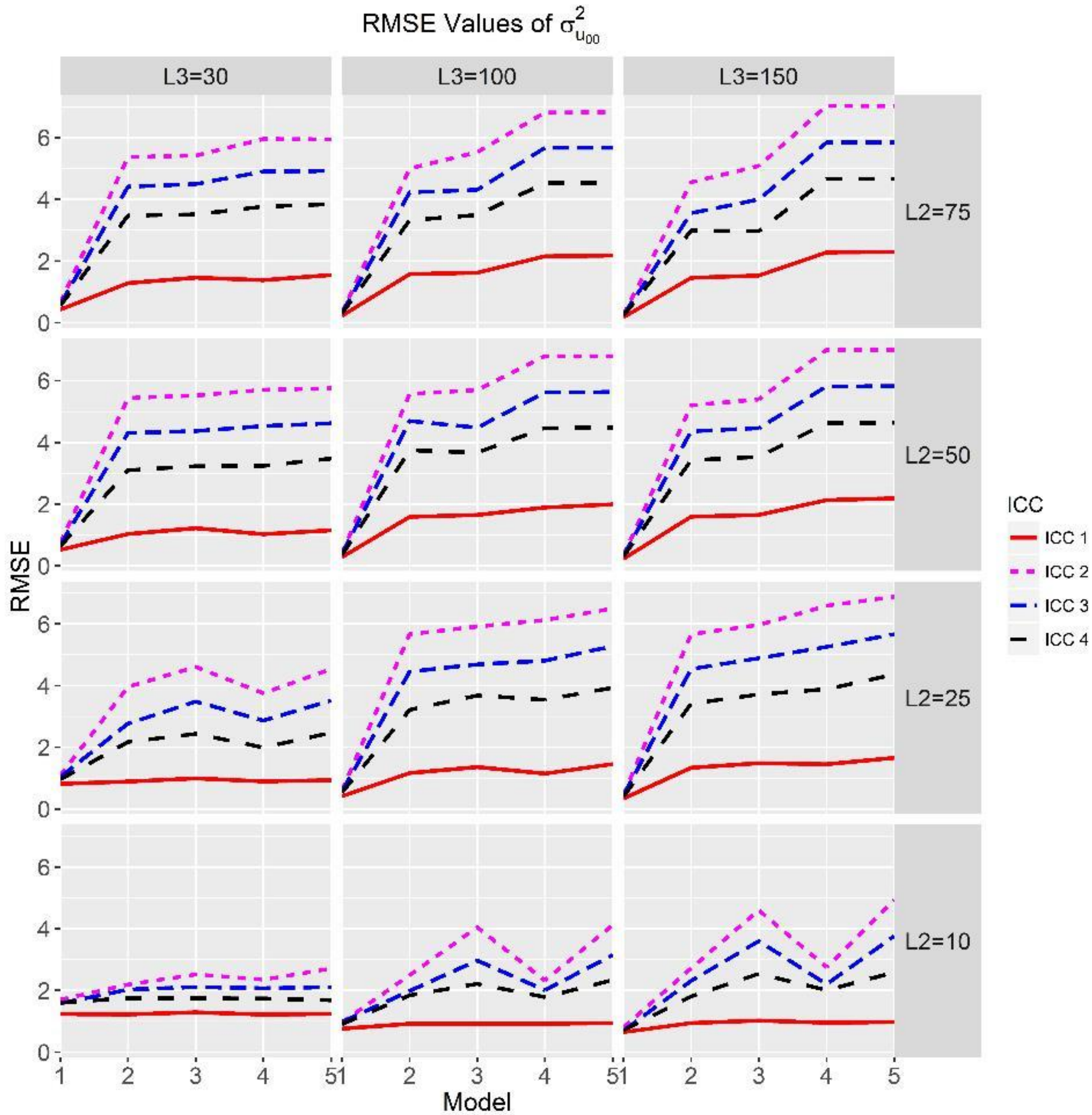

Figure F12. Plot of RMSE Across Manipulated Factors for $\sigma^2_{u_{00}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

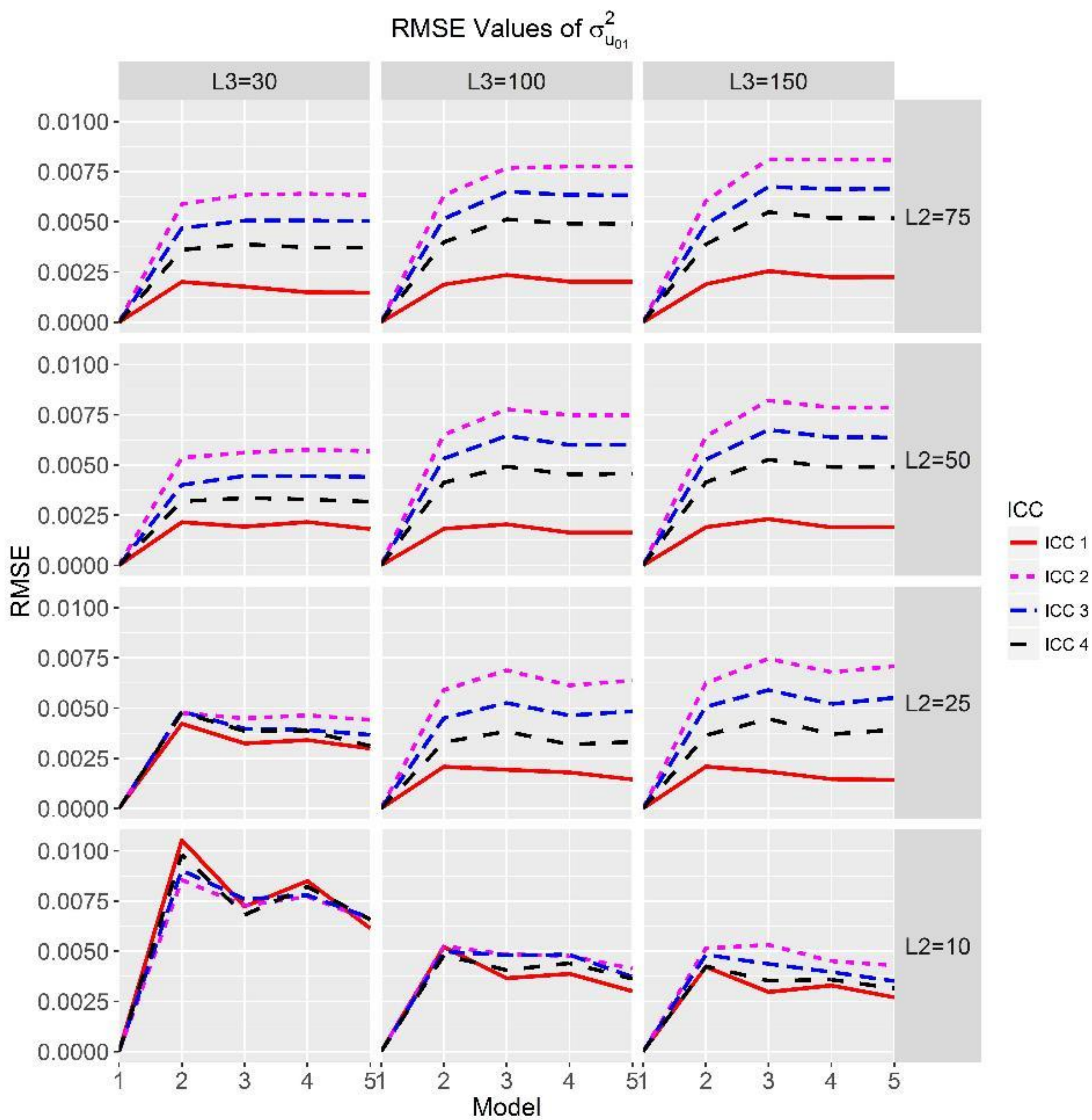

Figure F13. Plot of RMSE Across Manipulated Factors for $\sigma^2_{u_{01}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

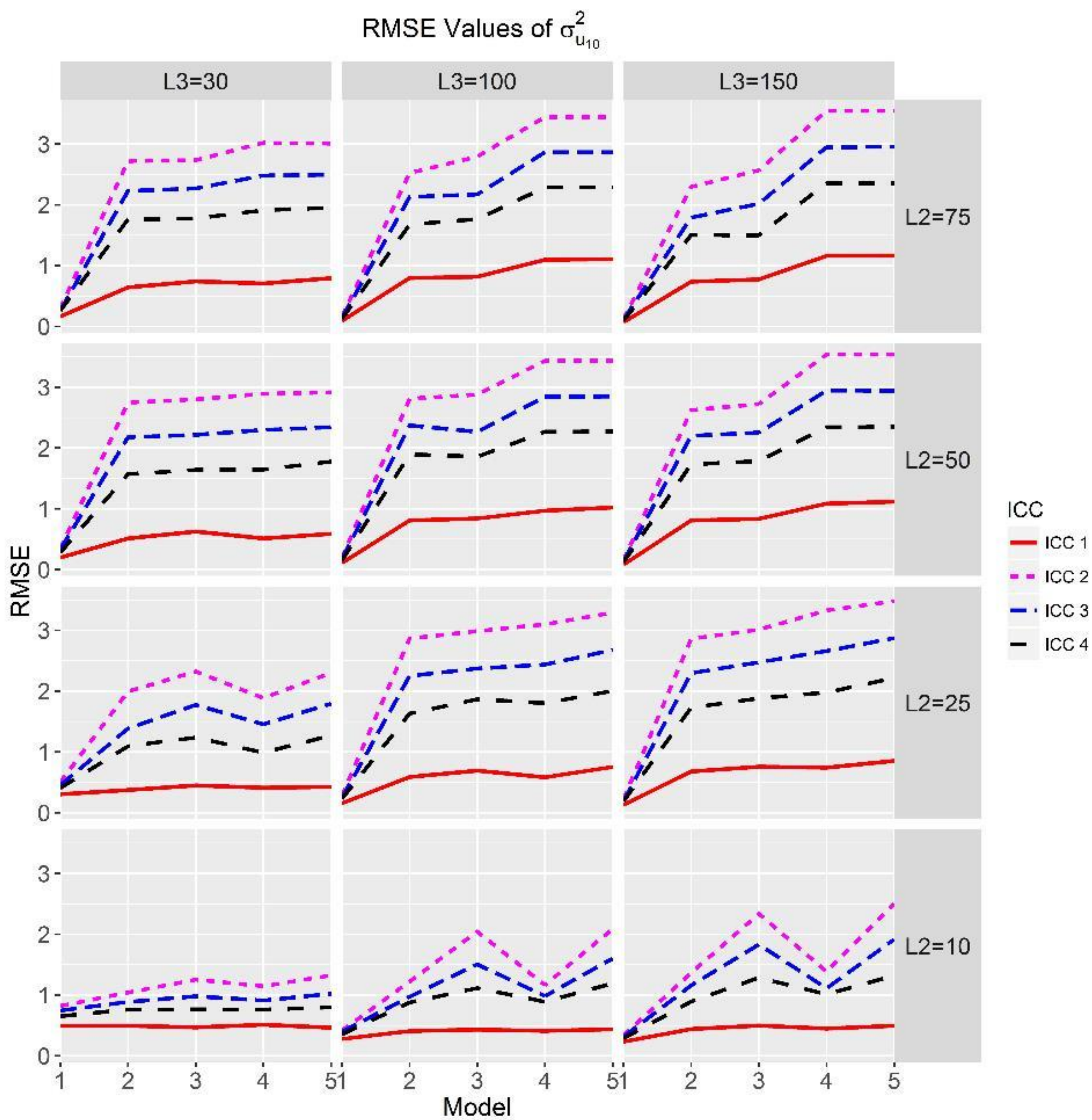

Figure F14. Plot of RMSE Across Manipulated Factors for $\sigma^2_{u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

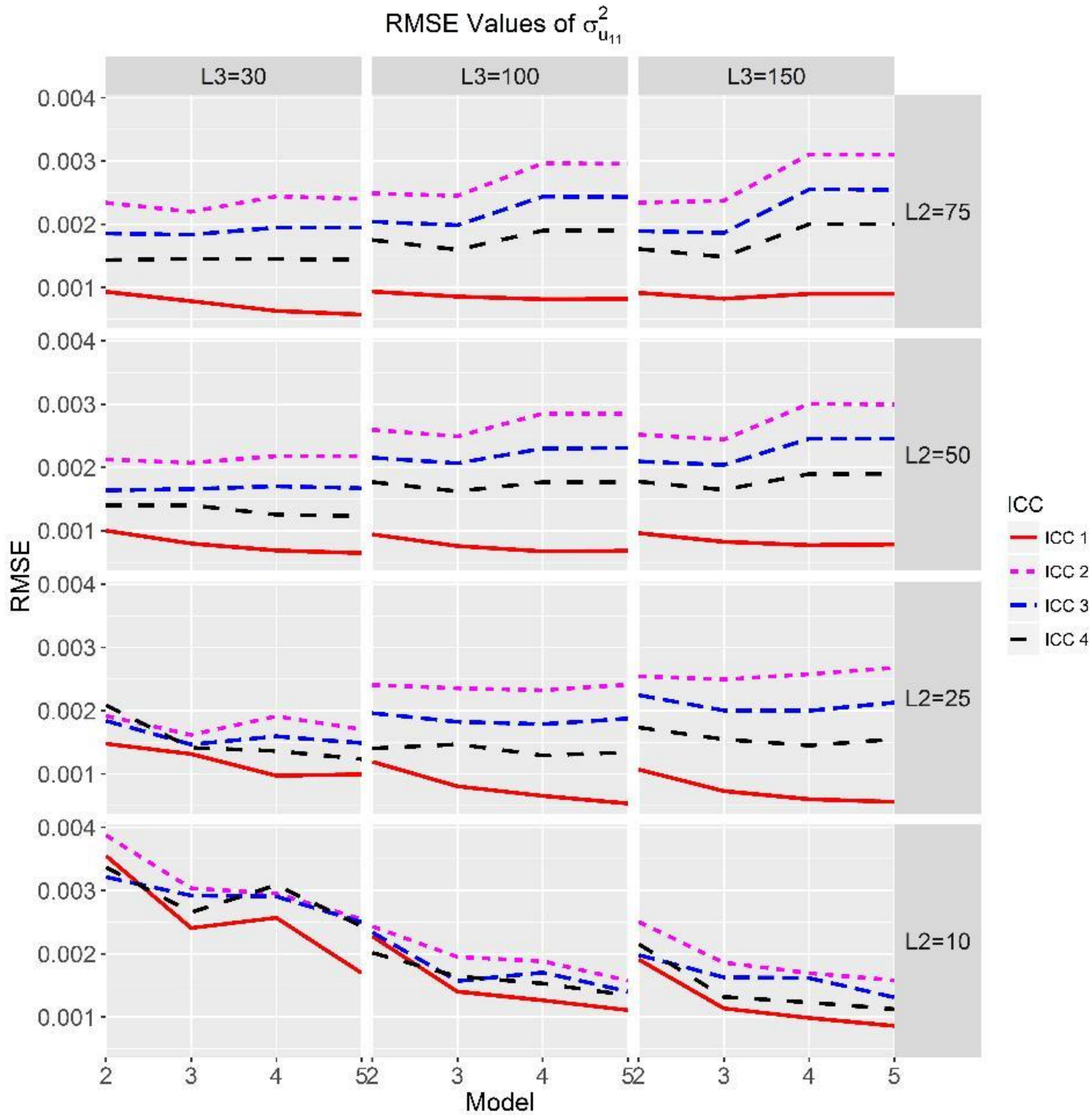

Figure F15. Plot of RMSE Across Manipulated Factors for $\sigma^2_{u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

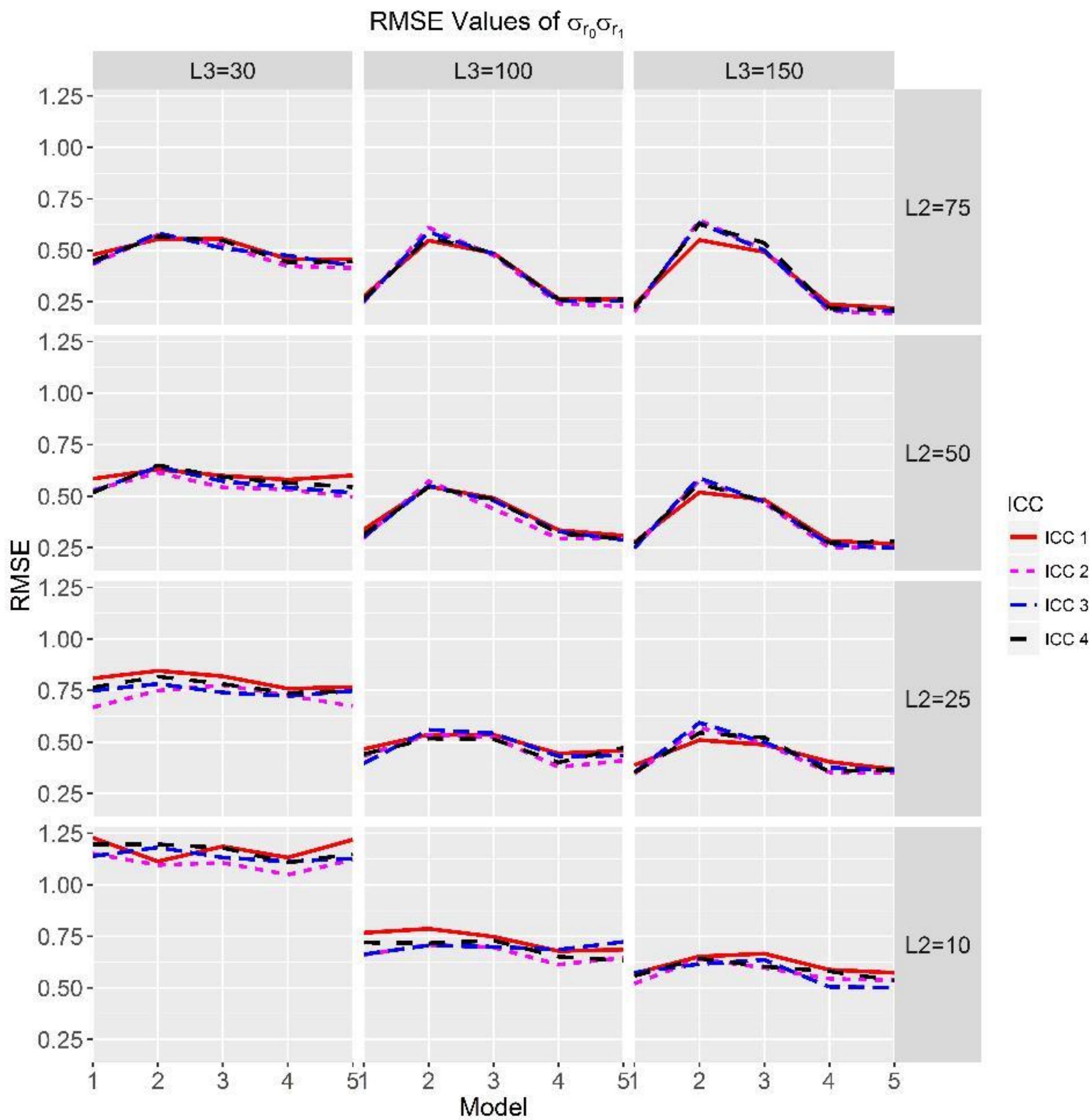

Figure F16. Plot of RMSE Across Manipulated Factors for $\sigma_{r_0 r_1}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

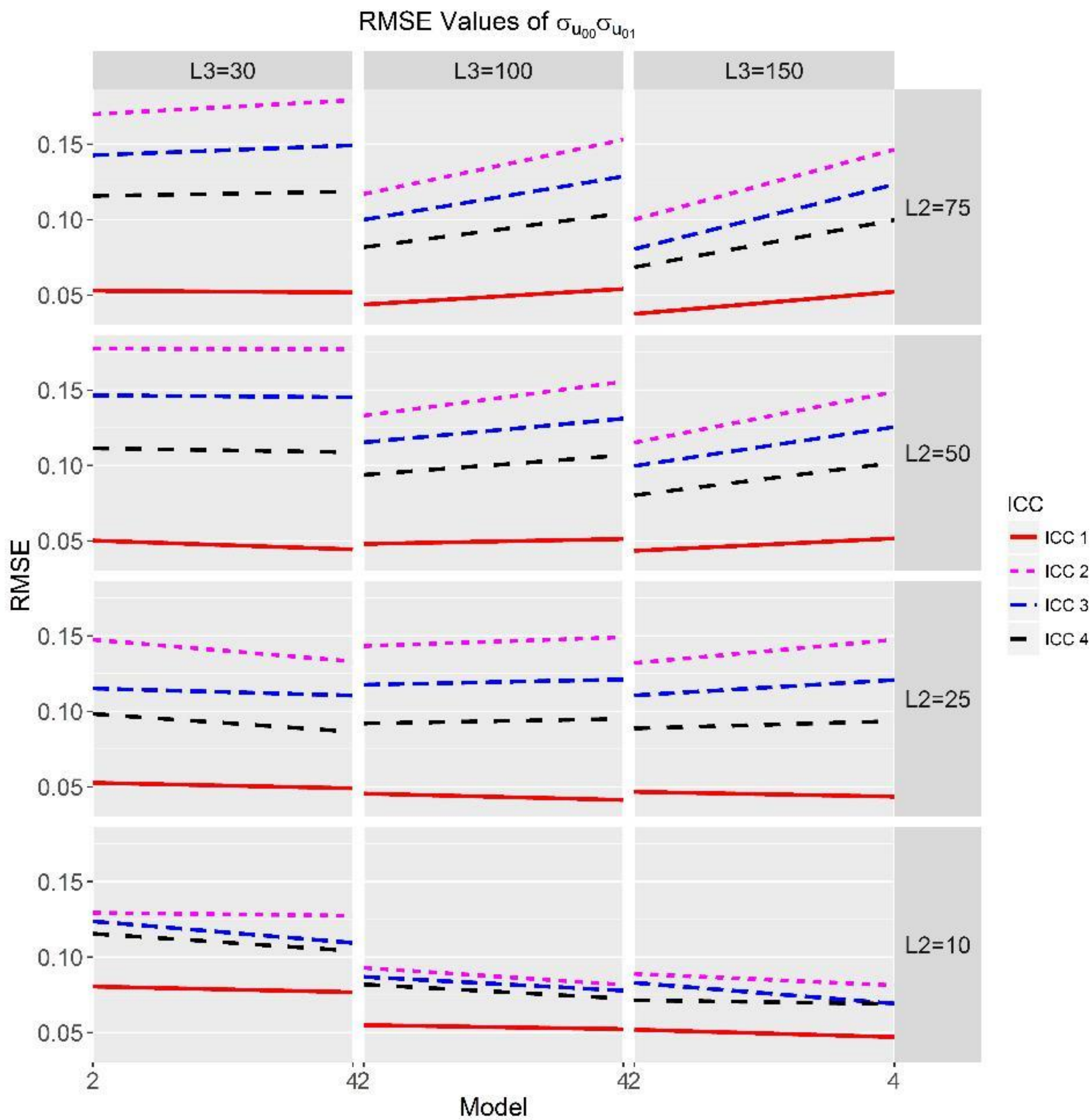

Figure F17. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{00}u_{01}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

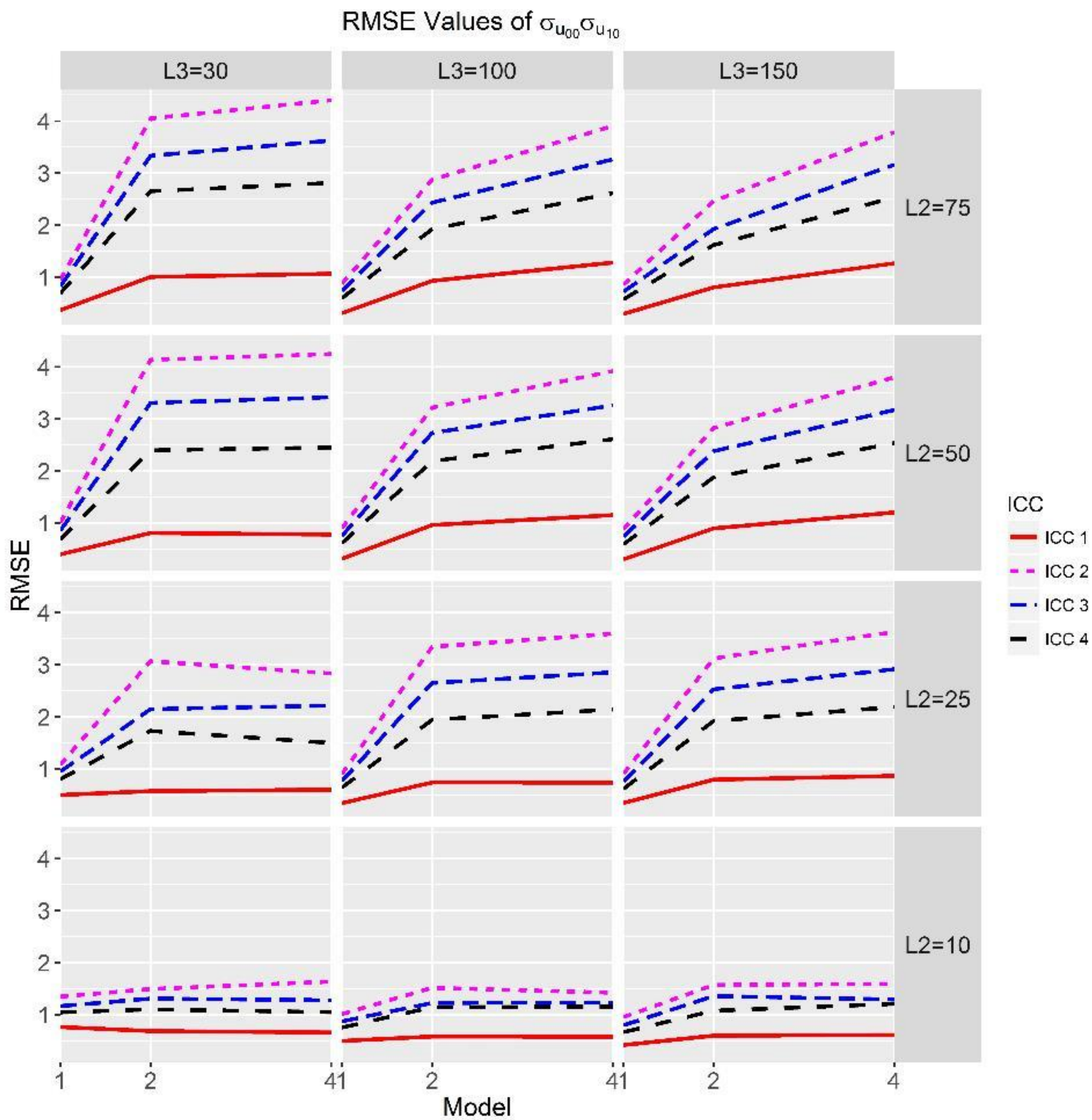

Figure F18. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{00}u_{10}}$.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

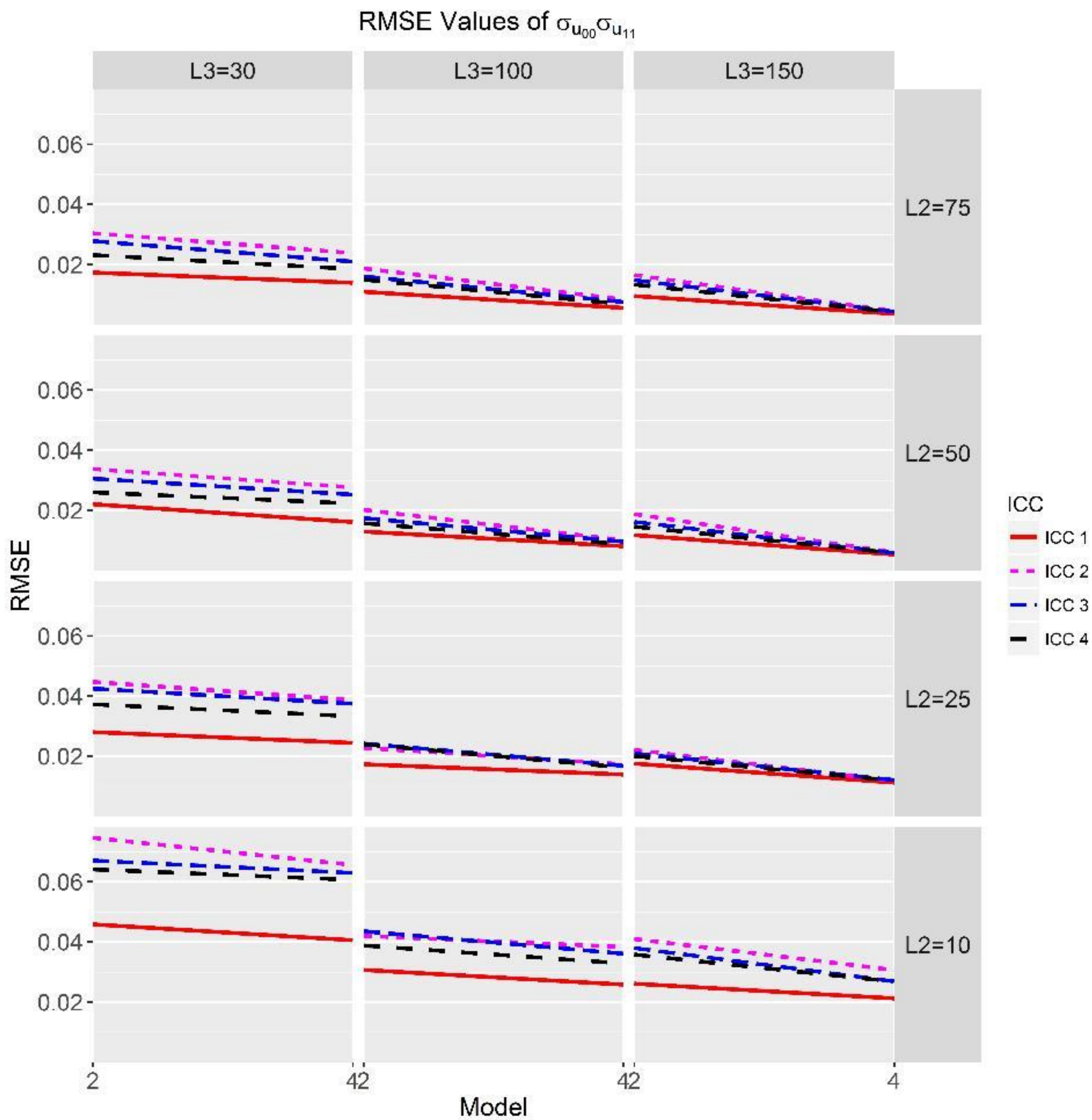

Figure F19. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{00}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

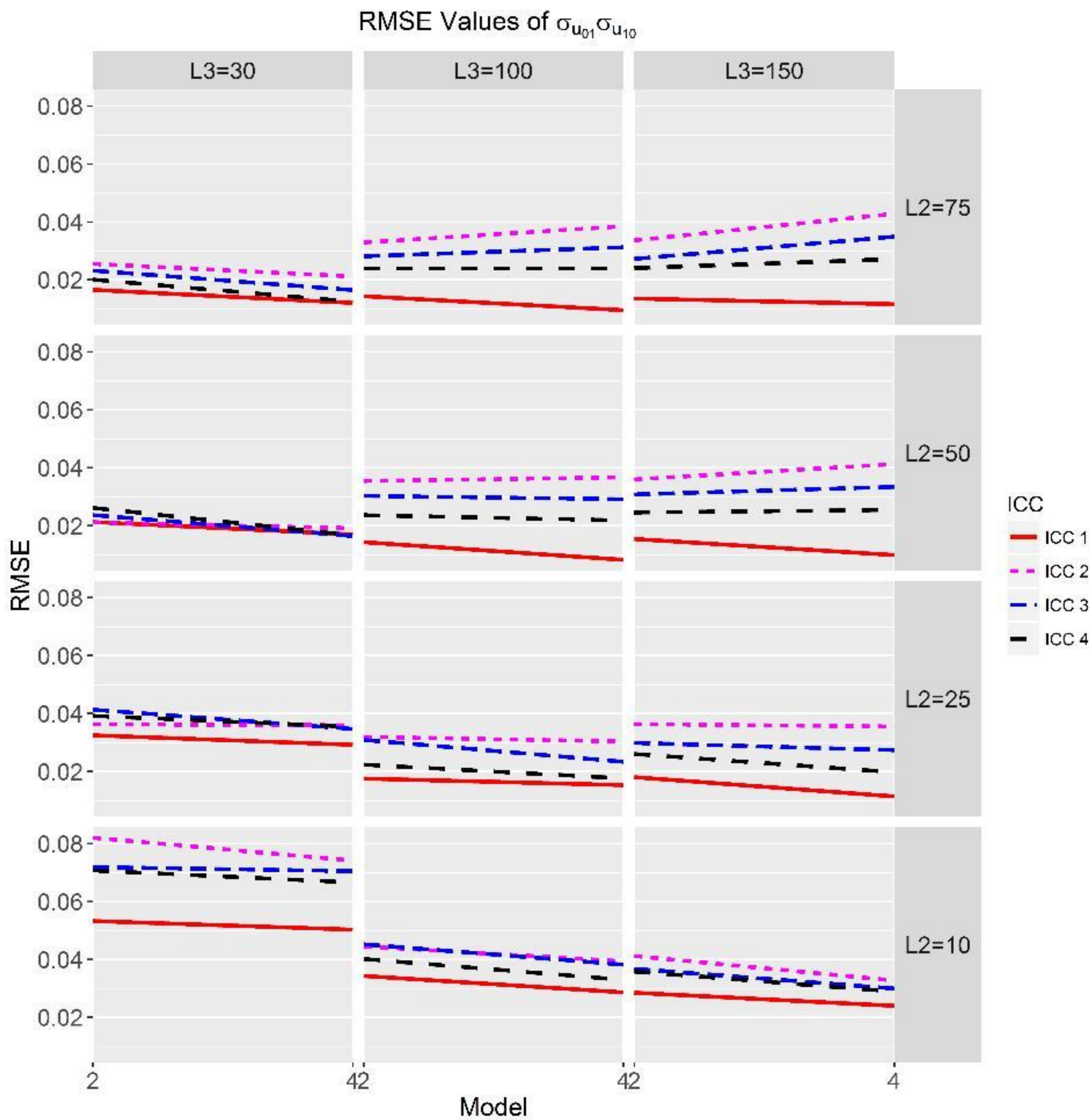

Figure F20. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{01}u_{10}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

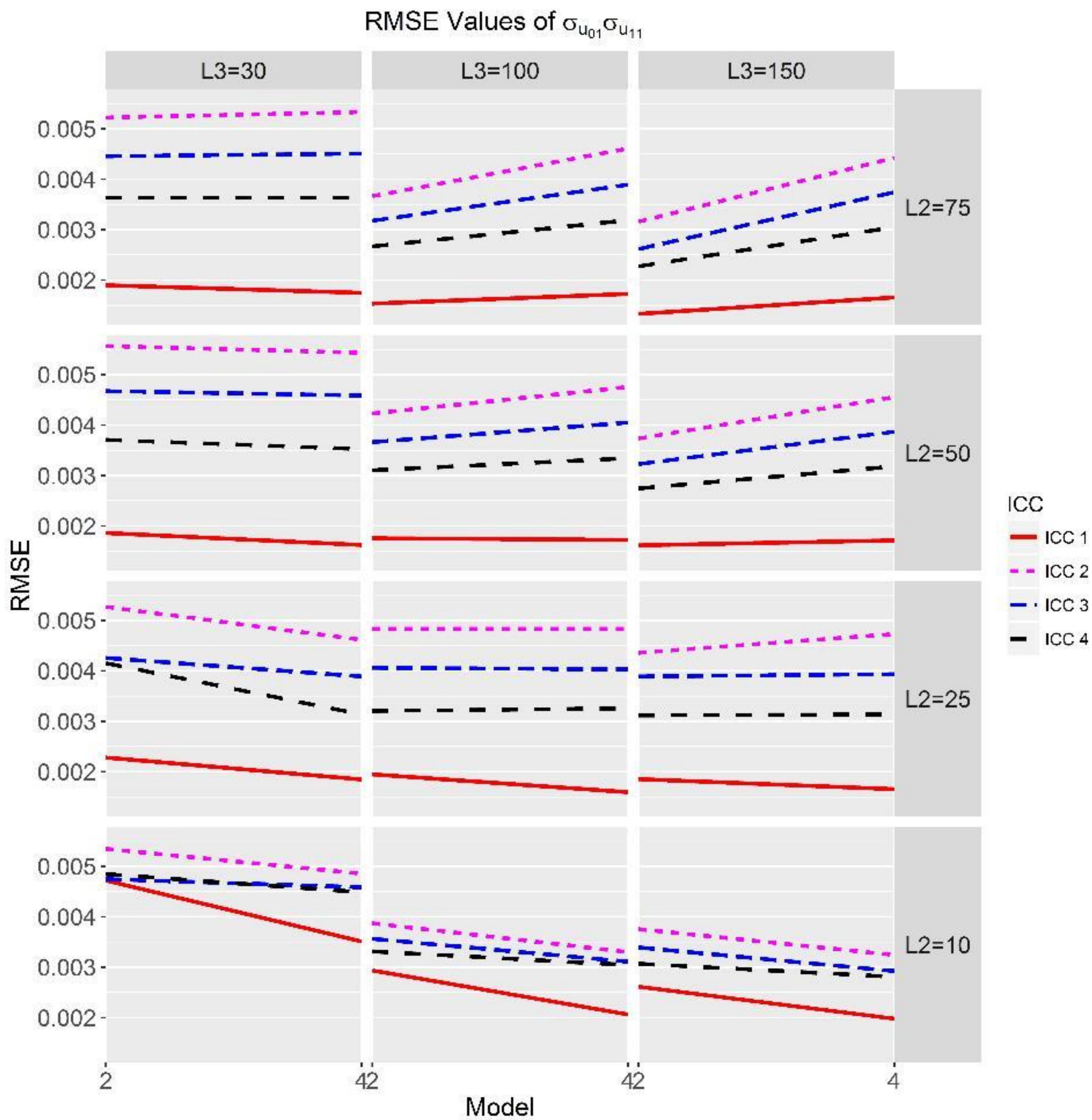

Figure F21. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{01}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

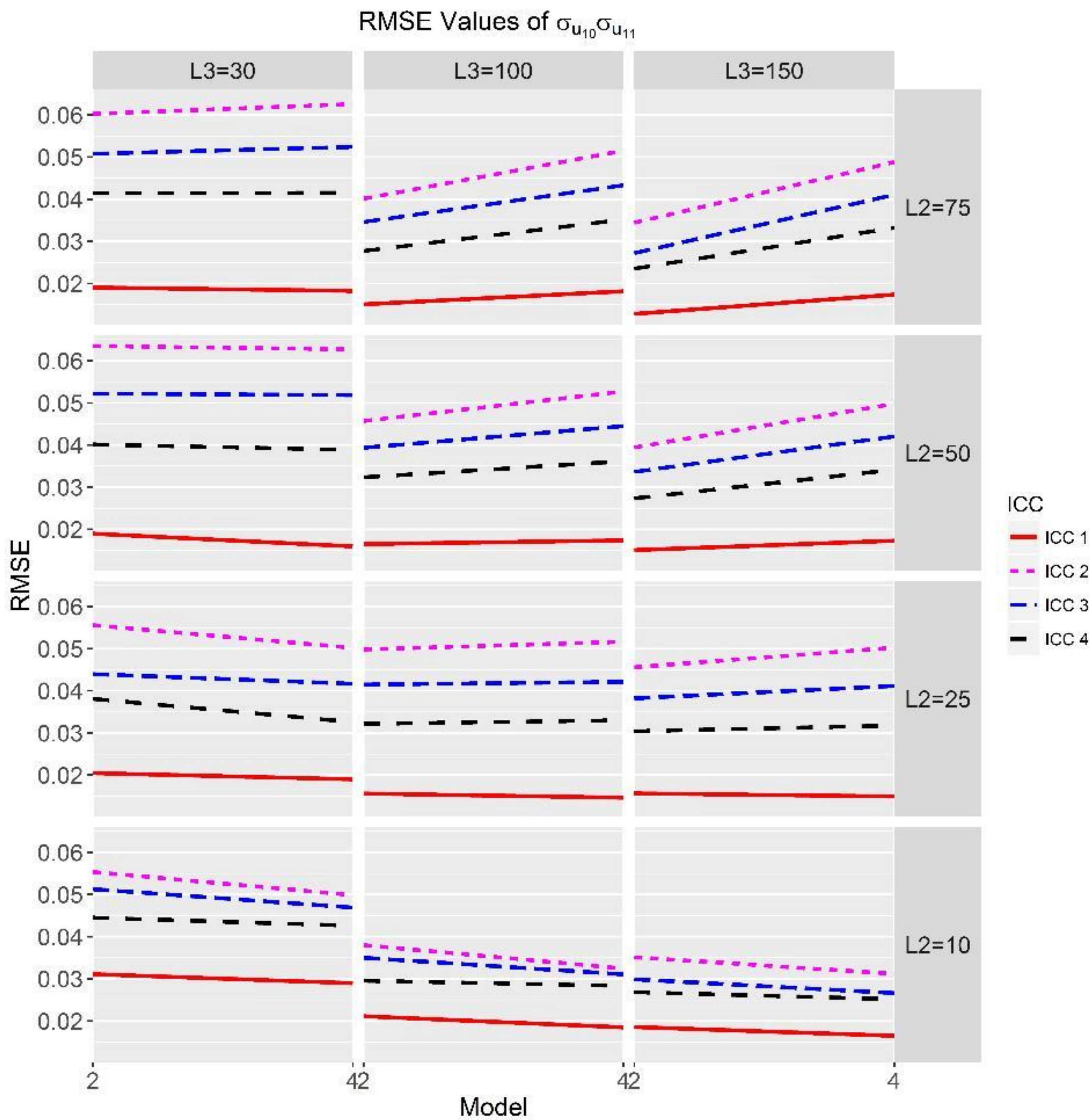

Figure F22. Plot of RMSE Across Manipulated Factors for $\sigma_{u_{10}u_{11}}$.

Notes. Model 2 is L1L2 Model. Model 4 is L1L2L3 Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

APPENDIX G

TABLE OF PARAMETER COVERAGE PROPORTIONS

Table G1.

Parameter Coverage Proportion of γ_{000}

Sample Size*		L1 Model				L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.98	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.98	1.00	1.00	1.00
25	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	100	0.99	1.00	1.00	1.00	0.98	1.00	1.00	1.00	0.98	1.00	1.00	1.00
25	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	150	1.00	1.00	1.00	0.99	0.99	1.00	1.00	1.00	0.98	1.00	1.00	1.00
25	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					0.98	1.00	1.00	1.00	0.98	1.00	1.00	1.00
25	30					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	30					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	30					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	100					0.99	1.00	1.00	1.00	0.99	1.00	1.00	0.99
25	100					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	100					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	100					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	150					0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
25	150					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	150					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	150					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G2.

Parameter Coverage Proportion of γ_{010}

Sample Size*		L1L2 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.84	0.87	0.84	0.84	0.95	0.95	0.94	0.96
25	30	0.77	0.80	0.77	0.78	0.96	0.99	0.97	0.97
50	30	0.58	0.80	0.69	0.66	0.96	1.00	0.99	0.98
75	30	0.45	0.68	0.60	0.55	0.99	1.00	1.00	0.99
10	100	0.67	0.67	0.64	0.63	0.96	0.96	0.97	0.94
25	100	0.29	0.38	0.36	0.35	0.96	0.99	0.98	0.98
50	100	0.10	0.14	0.13	0.12	0.98	1.00	0.99	0.99
75	100	0.03	0.06	0.06	0.03	0.98	1.00	1.00	0.99
10	150	0.51	0.52	0.50	0.54	0.96	0.96	0.97	0.97
25	150	0.18	0.16	0.17	0.19	0.97	1.00	0.98	0.97
50	150	0.03	0.01	0.02	0.02	0.98	0.99	1.00	0.99
75	150	0.00	0.01	0.00	0.00	0.99	1.00	1.00	1.00
Sample Size*		L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.85	0.87	0.88	0.87	0.93	0.94	0.95	0.96
25	30	0.74	0.83	0.79	0.78	0.95	0.98	0.97	0.98
50	30	0.58	0.74	0.71	0.62	0.94	0.99	1.00	0.99
75	30	0.46	0.67	0.60	0.59	0.99	1.00	0.99	1.00
10	100	0.67	0.65	0.63	0.62	0.95	0.97	0.96	0.97
25	100	0.32	0.34	0.32	0.33	0.96	0.99	0.98	0.98
50	100	0.09	0.13	0.11	0.11	0.96	1.00	1.00	0.99
75	100	0.02	0.06	0.03	0.06	0.99	1.00	1.00	1.00
10	150	0.50	0.46	0.52	0.49	0.95	0.97	0.97	0.94
25	150	0.16	0.19	0.17	0.13	0.94	1.00	0.99	0.97
50	150	0.01	0.02	0.02	0.03	0.98	1.00	0.99	1.00
75	150	0.00	0.01	0.00	0.00	0.97	1.00	1.00	1.00

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G3.

Parameter Coverage Proportion of γ_{100}

Sample Size*		L1L2L3 Model				L1L2L3 Model				L1L2L3 Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00
25	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	100	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	100	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	150	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Sample Size*						L1L2 No L3 Covariance Model				L1L2L3 No L3 Covariance Model			
L2	L3					ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30					1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00
25	30					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	30					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	30					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	100					0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25	100					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	100					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	100					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
10	150					1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00
25	150					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
50	150					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
75	150					1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G4.

Parameter Coverage Proportion of γ_{110}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.93	0.96	0.96	0.94	0.93	0.97	0.92	0.94
25	30	0.95	0.98	0.98	0.97	0.97	0.97	0.97	0.97
50	30	0.99	1.00	0.99	0.99	0.96	1.00	0.99	0.99
75	30	0.99	0.99	0.99	0.99	0.98	1.00	1.00	0.99
10	100	0.95	0.97	0.96	0.95	0.96	0.97	0.97	0.96
25	100	0.96	0.99	0.99	0.96	0.97	0.99	0.98	0.97
50	100	0.98	0.99	0.99	0.99	0.97	1.00	1.00	0.99
75	100	0.98	1.00	1.00	1.00	0.98	1.00	1.00	1.00
10	150	0.95	0.97	0.98	0.96	0.94	0.97	0.97	0.96
25	150	0.97	0.99	0.99	0.97	0.96	0.98	0.98	0.98
50	150	0.97	1.00	0.99	0.99	0.99	0.99	0.99	0.99
75	150	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G5.

Parameter Coverage Proportion of γ_{001}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.91	0.92	0.90	0.91	0.91	0.90	0.93	0.91
25	30	0.90	0.89	0.92	0.91	0.91	0.93	0.94	0.88
50	30	0.91	0.90	0.91	0.91	0.88	0.89	0.91	0.90
75	30	0.91	0.90	0.90	0.92	0.94	0.93	0.91	0.89
10	100	0.95	0.92	0.95	0.93	0.95	0.93	0.94	0.95
25	100	0.93	0.94	0.92	0.93	0.94	0.96	0.94	0.94
50	100	0.93	0.95	0.95	0.94	0.94	0.95	0.93	0.95
75	100	0.94	0.92	0.94	0.95	0.93	0.94	0.94	0.94
10	150	0.96	0.95	0.97	0.95	0.93	0.94	0.90	0.93
25	150	0.93	0.95	0.95	0.95	0.93	0.96	0.95	0.92
50	150	0.95	0.93	0.94	0.94	0.95	0.95	0.94	0.93
75	150	0.93	0.95	0.94	0.97	0.95	0.95	0.94	0.95

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G6.

Parameter Coverage Proportion of γ_{011}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.90	0.86	0.89	0.90	0.92	0.90	0.93	0.91
25	30	0.91	0.92	0.90	0.92	0.93	0.90	0.89	0.92
50	30	0.88	0.91	0.91	0.92	0.90	0.93	0.90	0.93
75	30	0.92	0.91	0.90	0.93	0.92	0.90	0.92	0.91
10	100	0.93	0.94	0.95	0.94	0.91	0.93	0.94	0.95
25	100	0.94	0.93	0.93	0.95	0.96	0.94	0.95	0.94
50	100	0.95	0.95	0.93	0.93	0.94	0.93	0.94	0.95
75	100	0.93	0.94	0.95	0.94	0.94	0.93	0.94	0.94
10	150	0.93	0.94	0.93	0.94	0.95	0.94	0.93	0.95
25	150	0.96	0.94	0.94	0.95	0.94	0.94	0.95	0.93
50	150	0.96	0.94	0.94	0.95	0.95	0.95	0.95	0.94
75	150	0.93	0.94	0.95	0.95	0.94	0.94	0.94	0.94

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G7.

Parameter Coverage Proportion of γ_{101}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.90	0.88	0.92	0.90	0.90	0.90	0.90	0.92
25	30	0.93	0.91	0.90	0.93	0.91	0.90	0.92	0.88
50	30	0.90	0.92	0.91	0.93	0.91	0.91	0.92	0.88
75	30	0.89	0.90	0.92	0.91	0.91	0.92	0.92	0.92
10	100	0.94	0.93	0.93	0.96	0.93	0.93	0.93	0.94
25	100	0.94	0.93	0.93	0.93	0.95	0.93	0.92	0.93
50	100	0.94	0.93	0.92	0.94	0.93	0.95	0.94	0.92
75	100	0.95	0.92	0.95	0.94	0.93	0.92	0.94	0.94
10	150	0.97	0.93	0.94	0.93	0.94	0.95	0.94	0.95
25	150	0.94	0.95	0.94	0.93	0.93	0.93	0.96	0.93
50	150	0.95	0.94	0.94	0.94	0.96	0.94	0.94	0.93
75	150	0.93	0.94	0.93	0.94	0.94	0.93	0.95	0.95

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

Table G8.

Parameter Coverage Proportion of γ_{111}

Sample Size*		L1L2L3 Model				L1L2L3 No L3 Covariance Model			
L2	L3	ICC1	ICC2	ICC3	ICC4	ICC1	ICC2	ICC3	ICC4
10	30	0.91	0.88	0.87	0.90	0.89	0.89	0.90	0.89
25	30	0.88	0.90	0.93	0.91	0.91	0.91	0.92	0.90
50	30	0.90	0.90	0.92	0.91	0.90	0.91	0.93	0.92
75	30	0.89	0.91	0.91	0.93	0.91	0.88	0.91	0.92
10	100	0.94	0.95	0.94	0.94	0.92	0.93	0.94	0.93
25	100	0.94	0.95	0.94	0.94	0.94	0.95	0.93	0.93
50	100	0.93	0.93	0.94	0.93	0.90	0.93	0.95	0.92
75	100	0.92	0.94	0.94	0.94	0.94	0.94	0.94	0.93
10	150	0.92	0.94	0.95	0.96	0.95	0.93	0.92	0.95
25	150	0.94	0.94	0.94	0.96	0.94	0.94	0.92	0.94
50	150	0.93	0.93	0.94	0.94	0.95	0.93	0.96	0.94
75	150	0.95	0.95	0.94	0.95	0.93	0.94	0.95	0.94

Note. *L1 sample size is 3. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

APPENDIX H

VISUAL REPRESENTATION OF PARAMETER COVERAGE PROPORTIONS

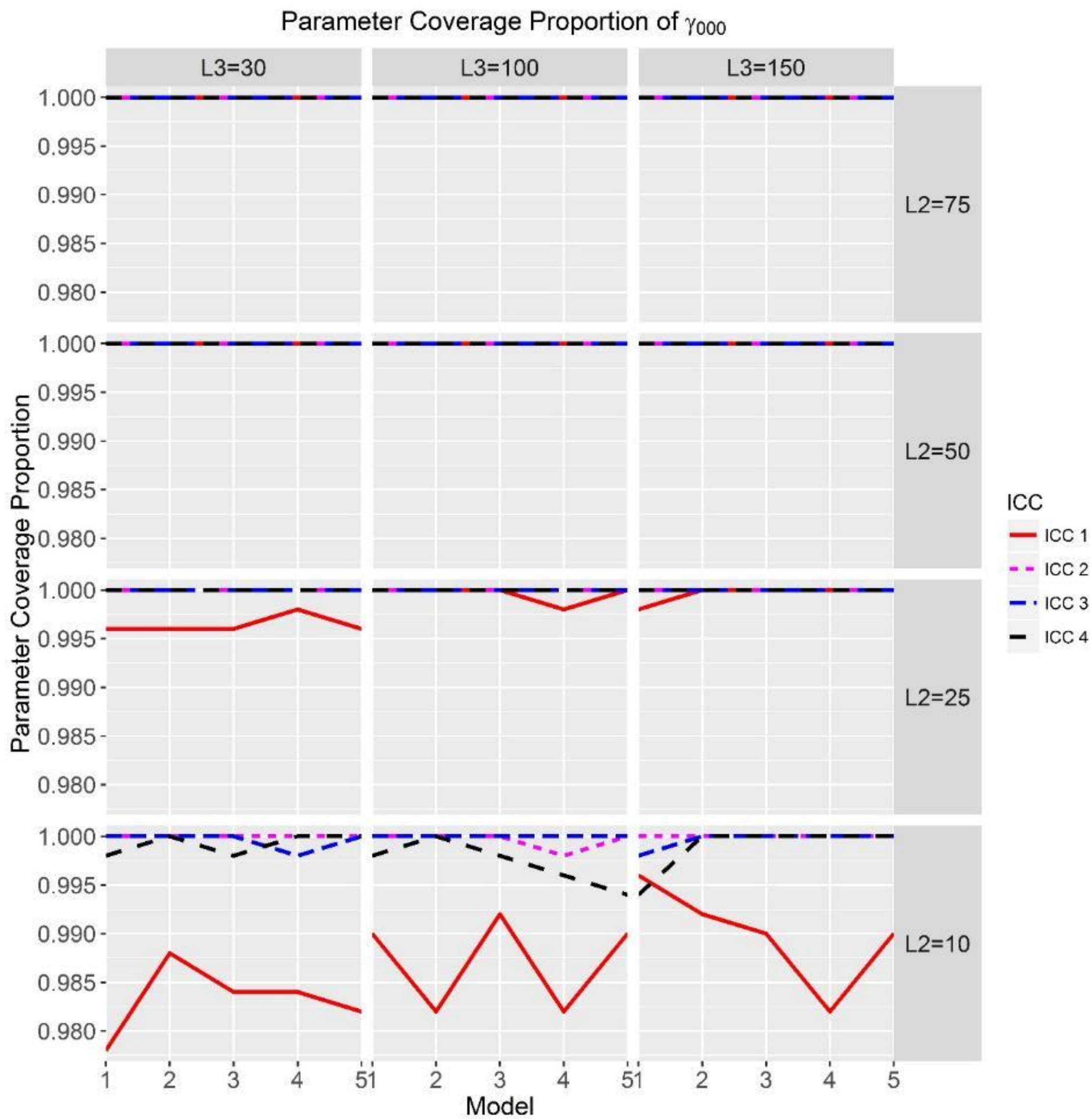

Figure H1. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{000}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

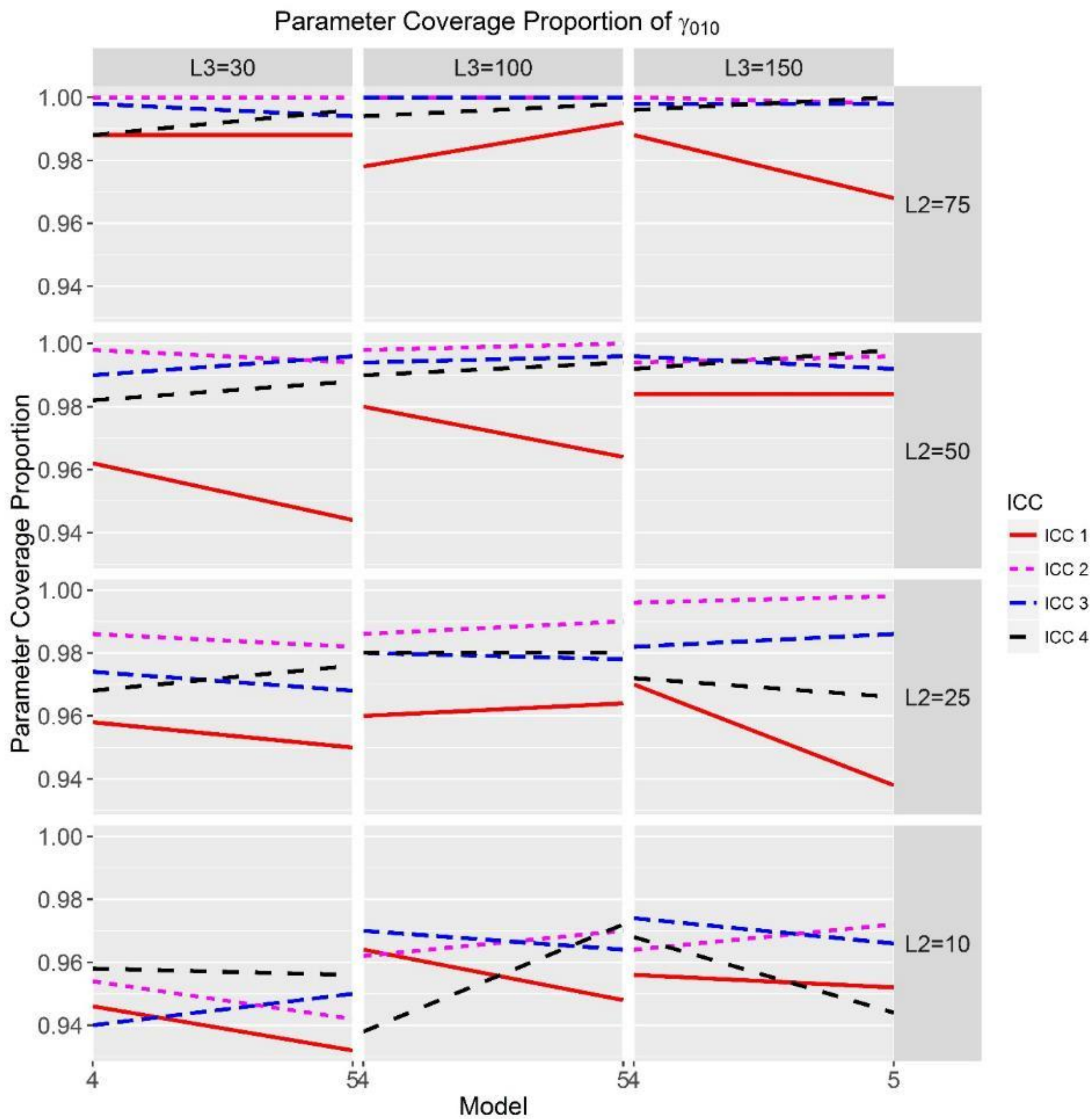

Figure H2. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{010}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

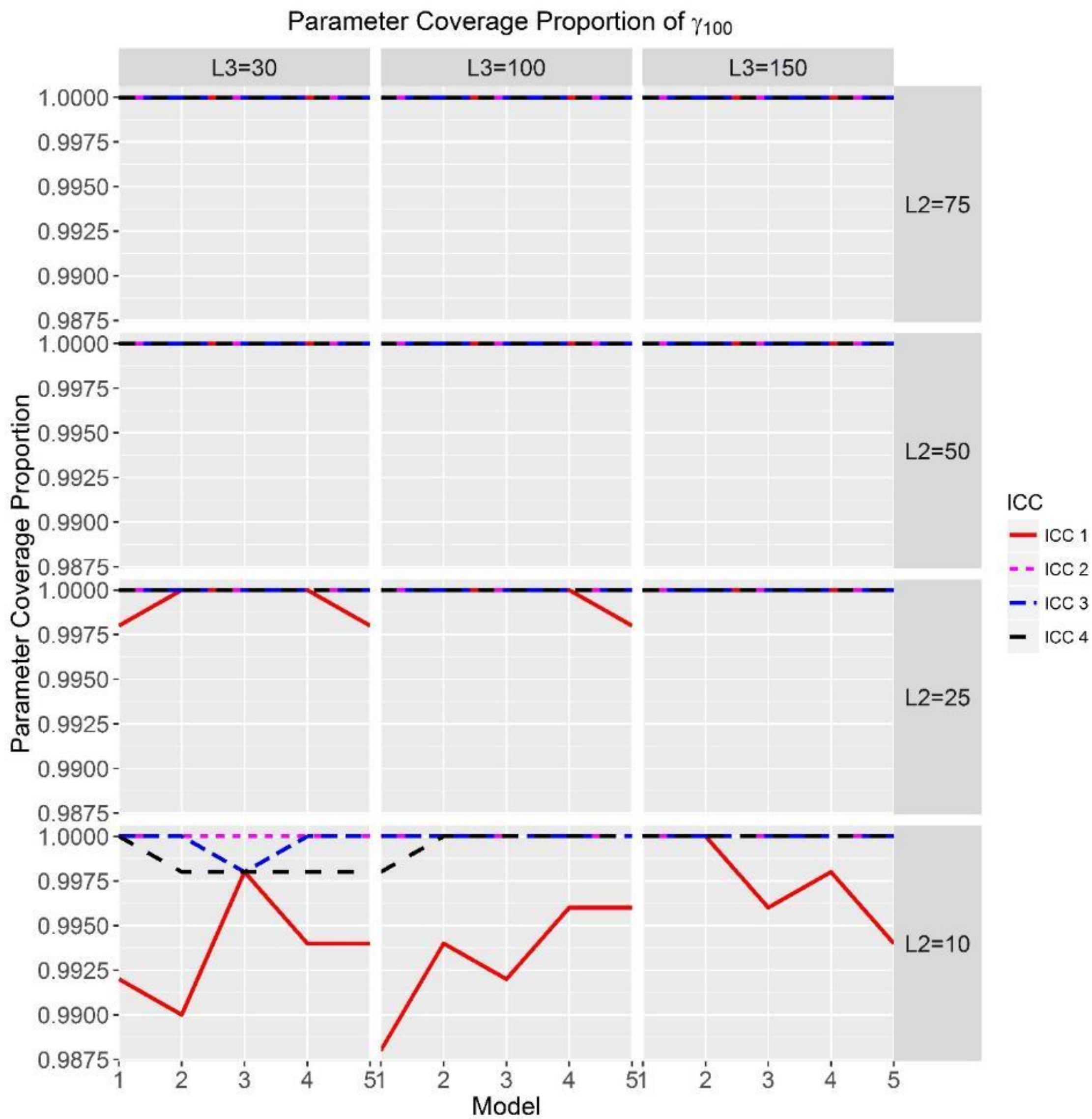

Figure H3. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{100}.

Notes. Model 1 is L1 Model. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20)

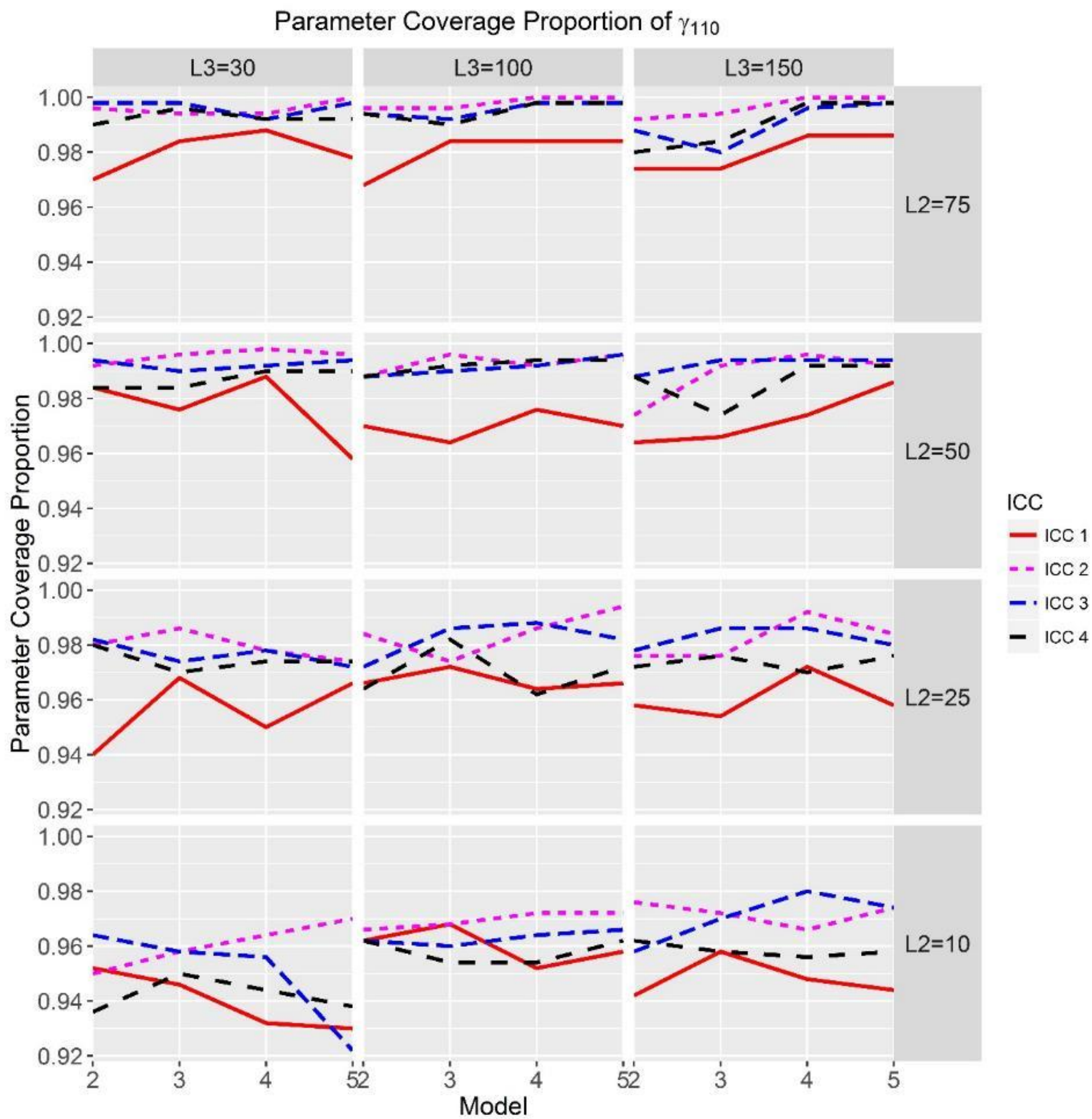

Figure H4. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{110}.

Notes. Model 2 is L1L2 Model. Model 3 is L1L2 No L3 Covariance Model. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

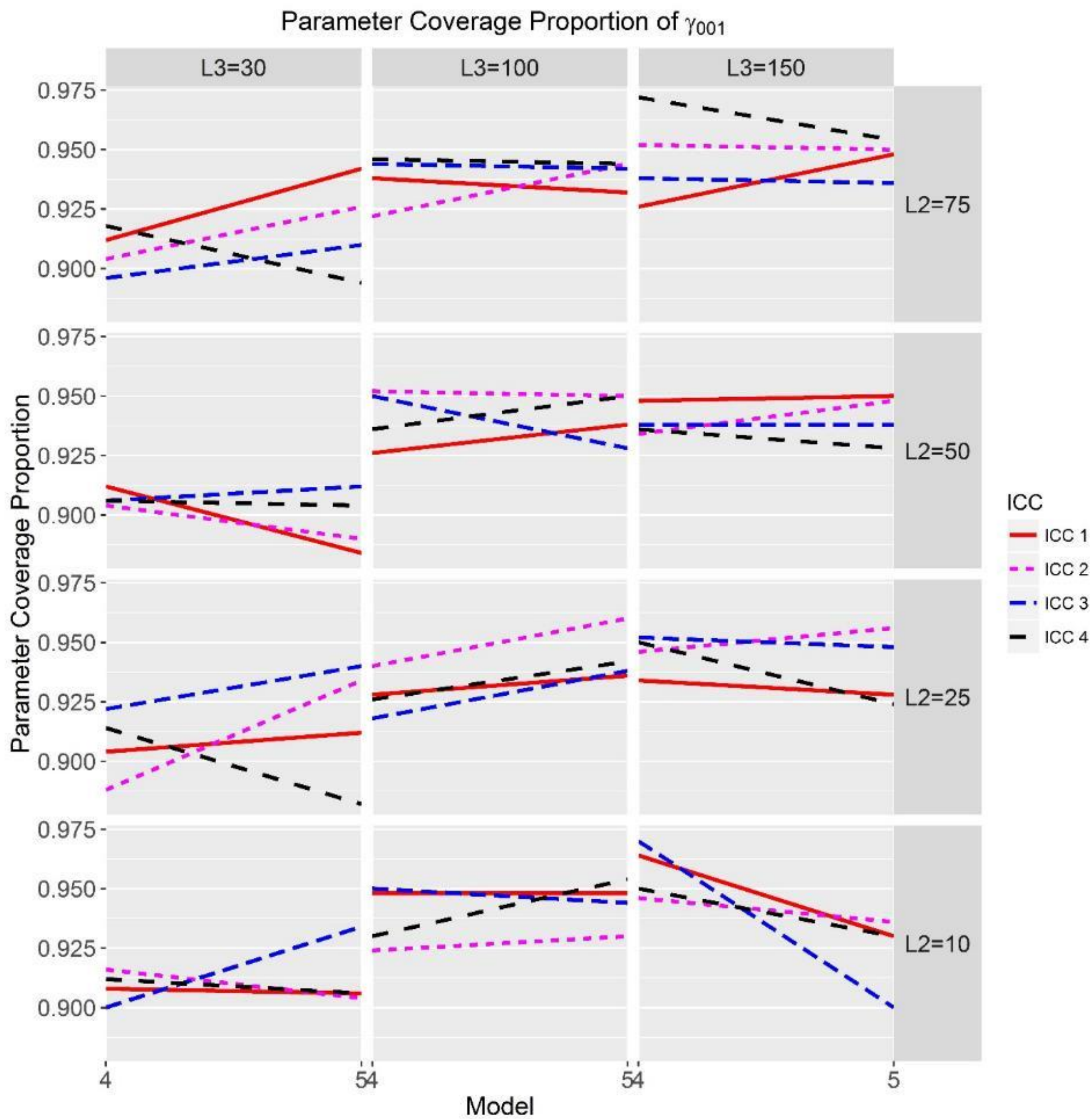

Figure H5. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{001}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

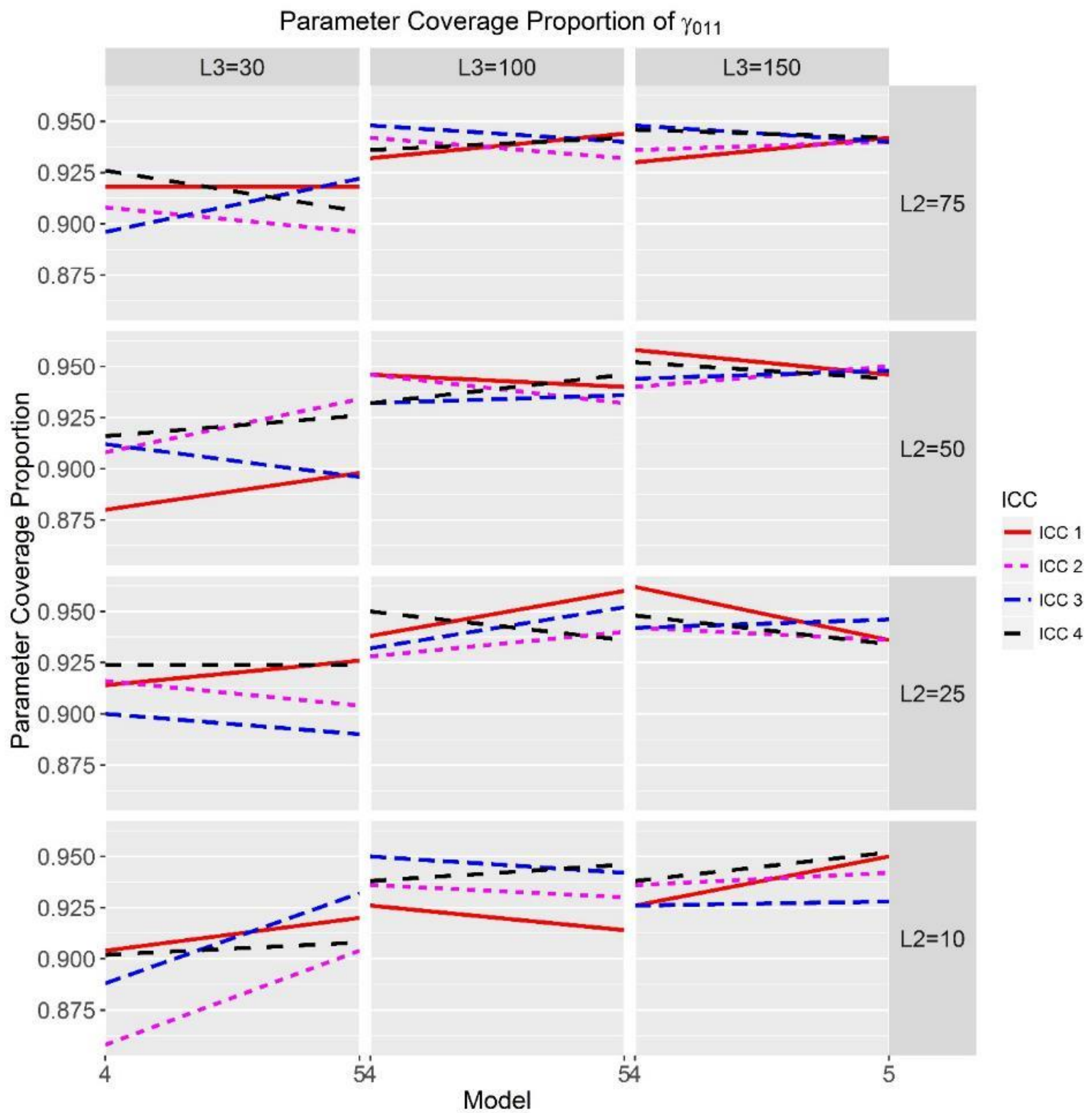

Figure H6. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{011}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

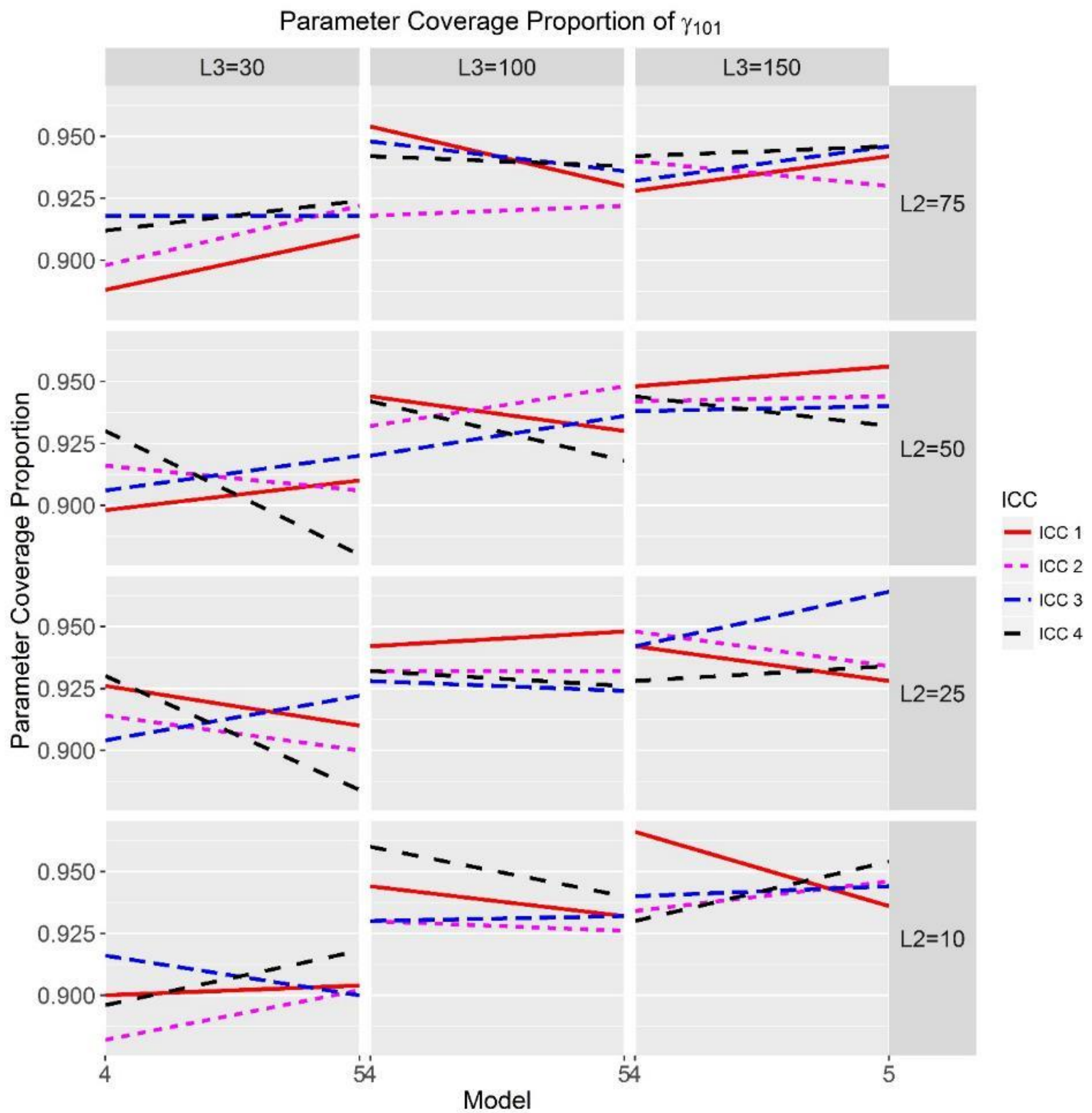

Figure H7. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{101}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

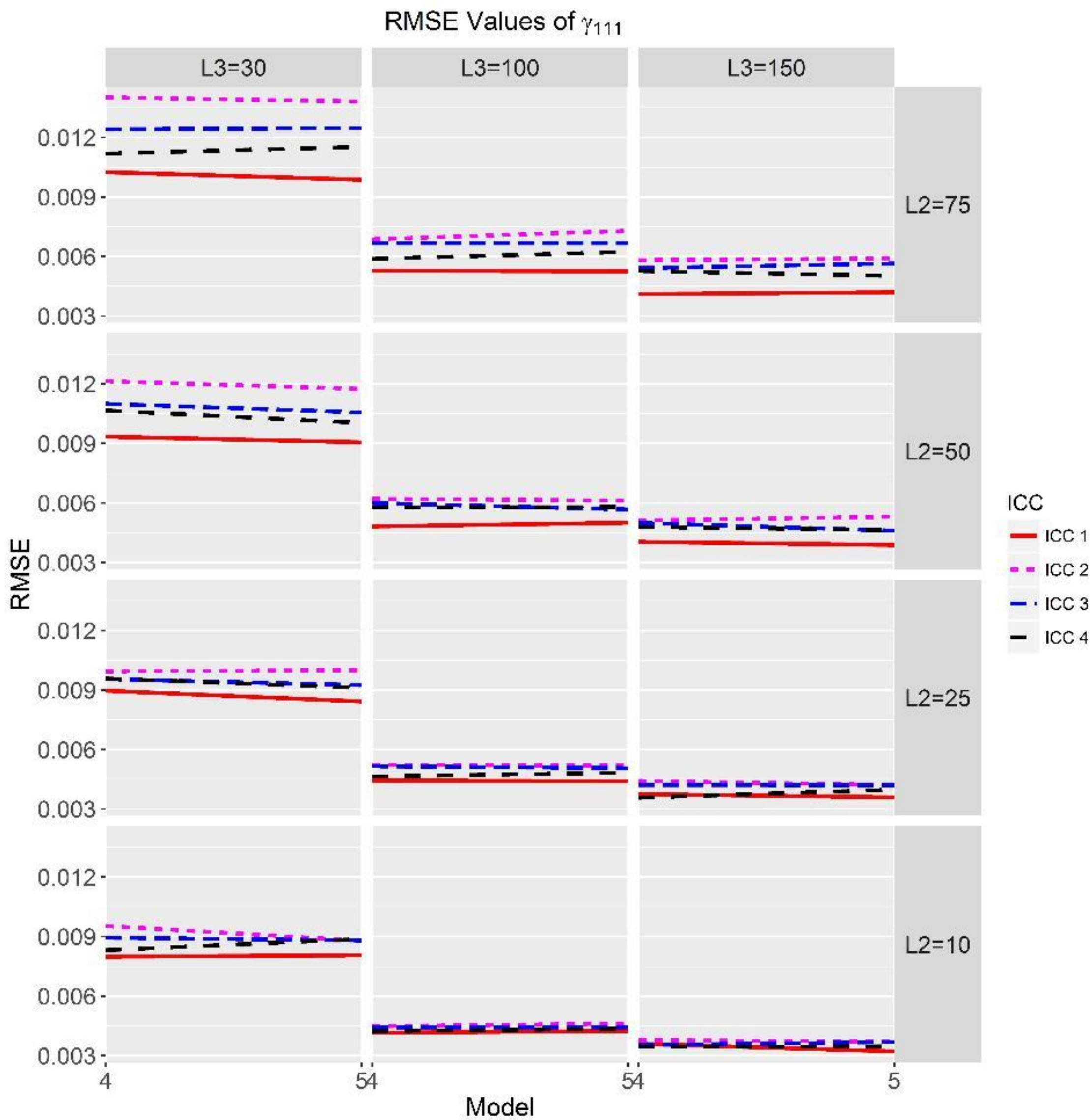

Figure H8. Plot of Parameter Coverage Proportions Across Manipulated Factors for γ_{111}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).

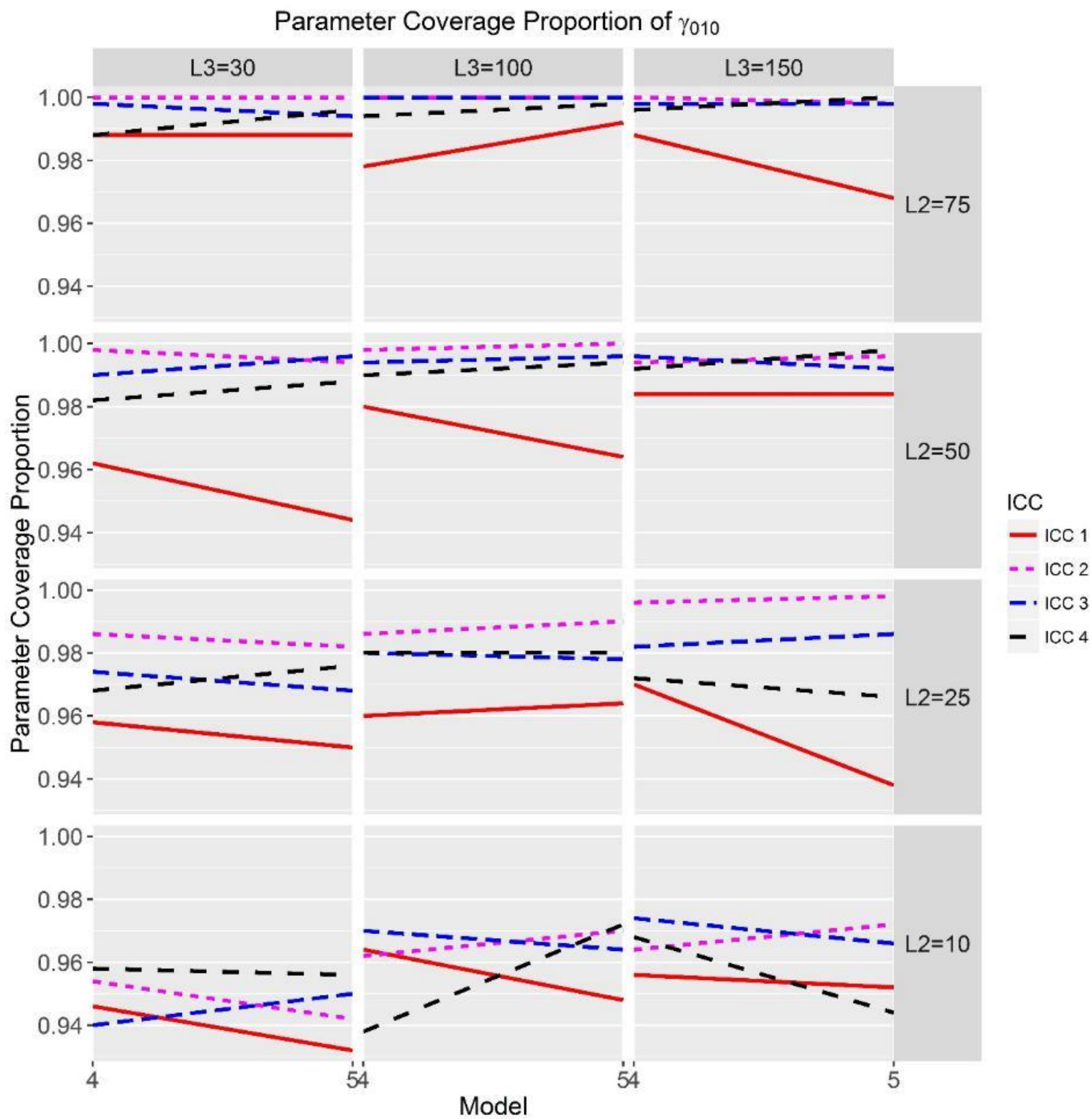

Figure H9. Plot of Parameter Coverage Proportions for Only M4 and M5 Across Manipulated Factors for γ_{111}.

Notes. Model 4 is L1L2L3 Model. Model 5 is L1L2L3 No L3 Covariance Model. ICC_1, (ICC_{L1}=0.5, ICC_{L2}=0.40, ICC_{L3}=0.10). ICC_2 (ICC_{L1}=0.5, ICC_{L2}=0.20, ICC_{L3}=0.30). ICC_3 (ICC_{L1}=0.5, ICC_{L2}=0.25, ICC_{L3}=0.25). ICC_4 (ICC_{L1}=0.5, ICC_{L2}=0.30, ICC_{L3}=0.20).