Reliability Based Design Optimization of Systems with Dynamic Failure Probabilities of Components

by

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ABSTRACT

This research is to address the design optimization of systems for a specified reliability level, considering the dynamic nature of component failure rates. In case of designing a mechanical system (especially a load-sharing system), the failure of one component will lead to increase in probability of failure of remaining components. Many engineering systems like aircrafts, automobiles, and construction bridges will experience this phenomenon.

In order to design these systems, the Reliability-Based Design Optimization framework using Sequential Optimization and Reliability Assessment (SORA) method is developed. The dynamic nature of component failure probability is considered in the system reliability model. The Stress-Strength Interference (SSI) theory is used to build the limit state functions of components and the First Order Reliability Method (FORM) lies at the heart of reliability assessment. Also, in situations where the user needs to determine the optimum number of components and reduce component redundancy, this method can be used to optimally allocate the required number of components to carry the system load. The main advantage of this method is that the computational efficiency is high and also any optimization and reliability assessment technique can be incorporated. Different cases of numerical examples are provided to validate the methodology.

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INTRODUCTION

1.1 Background

System design optimization deals with developing efficient engineering systems, which should be competitive in terms of cost, performance, and its lifetime value. In the current competitive industrial world, almost every industry strive to improve the quality of their products with minimum cost and maximum safety. But even though there are lots of modern manufacturing tools available, the presence of uncertainties in terms of design parameters, material strength, and also some external factors like loads cannot be ignored. The assumption of deterministic constraints can be made for the simplified computing purpose, but this will certainly have a huge impact when the system is put into use. Many researchers are developing methods to incorporate these uncertainties that resulted in various probabilistic design methodologies. These methods are applied to design the system with given number of components, but have only limited usage. This is because in most of the system (parallel or mixed system), the components are linked with each other and the failure of one component might lead to redistribution of loads acting on the system, resulting in increased probability of failure of the remaining components. Most of the developed methods failed to account for this dependent nature of component failure probability, which laid the groundwork for this research.

1.2 Motivation and Problem Definition

Load sharing systems are those in which the entire system load is shared among components in different proportions to support the working of system [13]. The failure of one component will increase the probability of failure of remaining components as the load acting on the system gets redistributed and thereby increasing the probability of failure of entire system. Several epistemic and aleatory uncertainties exists during the design and manufacturing of these systems, which has to be quantified appropriately for designing a reliable system. Uncertainties to be considered for efficient design are categorized into objective and subjective types [5, 9, 10, 11]. Objective uncertainty (Aleatory) exists due to the natural variation in the performance of the system. For instance, humidity, temperature, or some material parameters like conductivity are examples of aleatory uncertainties. Subjective uncertainty (Epistemic) exists due to lack of knowledge and they can be reduced by understanding the design by obtaining more data [5]. Hence, developing a good reliability analysis procedure should play a major role in system design.

In case of mechanical systems, the reliability is calculated based on the Stress-Strength Interference (SSI) theory [8, 12]. According to this SSI model, the reliability can be defined as the probability that load or stress acting on the component is lower than the strength of the component, which is calculated based on the probability density function of stress and strength.

Several methods have been devised for evaluating the reliability of different types of systems. But these methods have been developed by considering the probability of failure of the components to be independent of each other. But in the case of load sharing systems with dependent failure rate, using these existing methods may lead to incorrect conclusions as the probability of component failure depends on the state of other components in the system. Figure 1.1 represents a simple load sharing parallel system with two components, C_1 and C_2, carrying a total load, P.

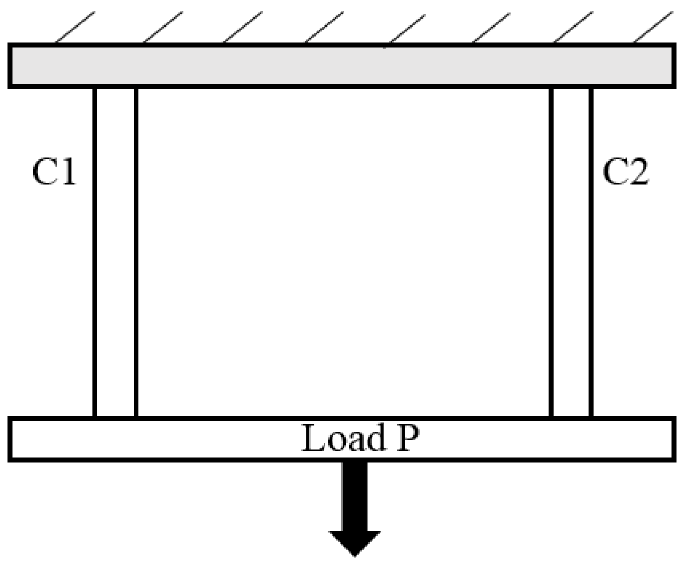

Figure 1.1: Load Sharing System with Two Components.

If we assume that the two components are non-identical and component 1 fails first, then Figure 1.2 shows the shift in limit state functions due to the redistribution of load ‘P’ [13].

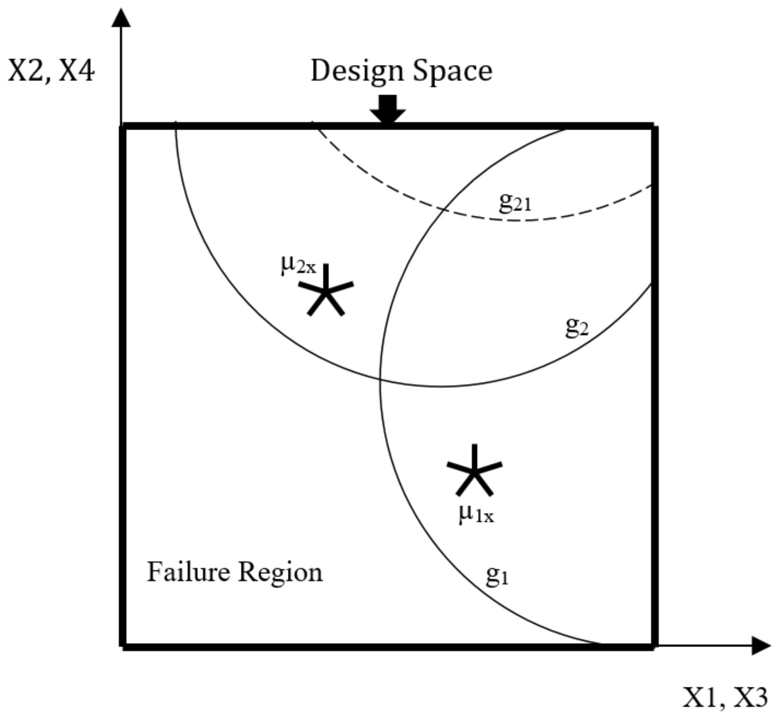

Figure 1.2: Limit State Functions of the Components 1 and 2.

The two points (μ_{1x} and μ_{2x}) from Figure 1.2 represent the optimal value (based on functions g_1 and g_2) for the two load sharing components 1 and 2 respectively [13]. But when component 1 fails, the entire load gets shifted to component 2 and the current optimal point for the surviving component may not satisfy the system reliability requirement. The limit state function of this surviving component would have shifted to a new position g_{21} represented using dotted line in figure 1.2 [13]. Now, the region below the function g_{21} represents the failure region of the entire system. One solution is to design each components separately to carry the full load for the given system reliability level. But, this solution deviates from the concept of load sharing and will result in increased cost of production and wastage of material. So far, many methods have been developed with the consideration of functions represented by g_1 and g_2 only. We might also end up in a situation where we need to increase the design space in order to achieve the required reliability level of the system for some cases. Some methods have been developed by considering the dependent failure probabilities, but resulted in high computational requirement.

1.3 Literature Review

The Reliability-Based Design Optimization (RBDO) method is used to overcome the problem in engineering design by considering the stochastic nature of the variables and find an optimum design point for each component to satisfy system reliability requirement. The main objective of RBDO is to achieve maximum reliability with minimum cost. RBDO methods are classified depending on how the reliability analysis is incorporated into the optimization process [1, 2]. There are many techniques that have been developed and they can be classified into nested double loop method and decoupled-loop method. The nested double loop method involves large number of computations to solve the problem. This is because, when nested RBDO is used, the reliability constraint of the given system can be evaluated after each optimization loop, but the computational cost is very high especially when the system is complex.

The decoupled loop method has less computational work when compared to that of nested method. Also, in order to compute the reliability of the system, there are two approaches. The first one is to replace the probabilistic constraint in the optimization loop with the Taylor series expansion along with updating the gradients of failure probabilities after each optimization iteration [2, 17]. The second approach is to use heuristic method to increase the component reliability levels until the system reliability target is achieved

[3, 4]. Initially, the design is optimized only for the given component reliability target and then evaluated to find whether the system level reliability requirement is met. If it is not met, then the component reliability targets are increased arbitrarily based on some knowledge about the components of the system and then the iteration is repeated until the goal is met.

RBDO using single loop approach (a decoupled loop method) is presented in [5, 14, 15, 16]. Usually, single loop algorithms have proven to be computationally efficient in case of RBDO and are mostly applied to design components for the required reliability level [3]. When single loop RBDO method is used, the optimization and reliability analysis method is carried out simultaneously to design the system. Single loop algorithms are proved to be computationally inexpensive and the accuracy of the solution will be reasonable when compared to nested loop methods [5].

Reliability-Based Design Optimization of load sharing parallel or mixed systems is computationally intensive due to the dependence between probabilities of failure of components. The problem especially gets intensified in evaluating the probabilistic constraints that are incorporated to quantify the uncertainties concerning the materials, load, geometry, etc. The Stress-Strength Interference (SSI) theory plays a major role to evaluate the system reliability, especially in case of mechanical systems as the stress and strength parameters are directly introduced in the model [8].

An efficient single loop RBDO formulation is developed in [2] which is capable of handling both component level reliability as well as system level reliability for different types of systems. The authors used a single loop RBDO formulation and an equivalent method that is effective in handling both system level and component level reliability constraints. This method helps the user to allocate optimum level of reliability for the individual components in order to satisfy both the component as well as the system level reliability targets. Various numerical examples are provided to validate the developed methodology.

The method developed in [2] also proved that single loop method is computationally efficient way to solve RBDO problem with system reliability constraint. But the authors have assumed the probability of failure of the components to be independent of each other. When the failure probabilities of components are not independent to each other with system consisting of large number of non-identical components, then there will be more complexity in arriving at the optimum design [13].

Another approach for evaluating the reliability of the system based on the failure dependence of the components and redistribution of the load is presented in [8]. The authors have considered the varying nature of failure rates with respect to stress and strength parameters. The authors took account of the Strength Degradation Path Dependence (SDPD) of the various components in a system due to repeated application of random load using state probabilities. The Markov chain theory is used to represent the various states of the components and Monte Carlo Simulation is used in order to verify the proposed models.

The reliability evaluation of load sharing power system is proposed in [18]. The authors developed models considering a number of subsystems and used supplementary variable technique in order to estimate the state probabilities of the system. A method for evaluating the reliability of load sharing k out of n: G system with imperfect switching is developed in [19]. The authors used Markov theory to develop a reliability model of system with exponential lifetime [8].

The reliability analysis of load sharing system subjected to different load behavior is provided in [20]. The authors considered a standby system with two components under varying load and used Weibull probability distribution of time to failure to derive models. The investigation of load sharing systems is also done in [21, 22] in which the authors studied about different methods of computing system reliabilities and the impact of different loads on system reliability evaluation methods [20].

Most of the research stated above has not considered the failure dependence of components. For those that have considered failure dependency, their computational requirements were very high. Therefore, an efficient method needs to be developed in order to overcome the problem in designing the load sharing system and to find an optimum design point for each component that satisfies system reliability requirement.

Sequential Optimization and Reliability Assessment (SORA) method developed by [1] is used in this research to optimally design the load sharing system. Traditional Monte Carlo Simulation is more accurate but its computational cost is very high especially when the reliability requirement is close to one [5, 23]. Taylor series method cannot deal with highly non-linear performance function and also it is too complex to handle high dimensional data [5, 26, 27]. The response surface method builds meta models using limited amount of samples and replace the true system response [28]. Numerical integration using dimension reduction method [29-33] is also applicable for some cases. The Most Probable Point (MPP) evaluation is based on First Order Reliability Method (FORM) and there are two approaches. The Reliability Index Approach (RIA) is a direct reliability analysis method in which the MPP is obtained by formulating an optimization problem, but the convergence of this method is low [6, 34, 36, 37]. Another method, Performance Measure Approach (PMA) which is an indirect method [35, 38] is more robust and efficient than RIA method.

1.4 Research Organization

The report is organized as follows. The traditional approaches for reliability based design of engineering system and their drawbacks are discussed. Then Reliability-Based Design Optimization (RBDO) framework to design the load sharing systems using the Sequential Optimization and Reliability Assessment (SORA) method is proposed. The procedure is explained using the First Order Reliability Method (FORM) for reliability analysis as it can produce good results with minimum computational requirements which will be validated using numerical examples. Also the following assumptions are included in our approach for designing the load sharing system.

1. Failure of components are mutually exclusive, ie., if there are two components in the system, these two components cannot fail at the same time.
2. The time dependent degradation of the component is not considered. Whenever the stress exceeds the strength, the component fails immediately.
3. Only the system level reliability is provided by the customer.
4. The order in which the components fail is known.
5. Normality assumption is maintained throughout this report.

CHAPTER 2

RELIABILITY-BASED DESIGN OF MECHANICAL SYSTEMS

Today, a variety of probabilistic design methods have been developed in order to aid the efficient design of mechanical systems. The most common methods like robust design [39-43] and reliability-based design [44-47] have been in practice for a long time. The objective of the reliability-based design is to ensure the satisfaction of the probabilistic constraints at the required level whereas the robust design focuses on ensuring the system to be working under abrupt input conditions. Both of them can be achieved by simultaneously optimizing the mean performance and performance variance.

The first and foremost task in probabilistic design is uncertainty analysis, which gives the knowledge about the impacts of various uncertainties that the system inputs have on the output. These characteristics are formulated mathematically and optimization is performed in order to obtain the optimum design values for the system to withstand the given amount of uncertainties caused by the input variation.

Having explained about the uncertainties in previous section, one of the most vital challenge with probabilistic design optimization is the computational efficiency. The evaluation of probabilistic constraints poses a major requirement of high computational power, which is very challenging for the implementation of probabilistic design. In order to have knowledge about the probabilistic characteristics of the system at a particular design point, a large number of iterations of deterministic optimization have to be carried out with respect to the nominal point. This can be done by using simulation approaches, such as Monte Carlo Simulation (MCS), or by using some deterministic approximation methods for probabilistic constraint analysis. A plenty of research has been conducted particularly concentrating on improving the computational efficiency of the probabilistic constraints for complex engineering systems. A brief summary of different types of system is given below.

2.1 Series Systems

In case of series system, the failure of one component will lead to the total system failure. The reliability of the system is defined as the probability that component 1 is working and component 2 is working and so on to all the components present in the system are working. If the reliability of the individual components is denoted by R_i, then the reliability of series system with 'n' number of components is given by,

$$R_{series} = \prod_{i=1}^{n} R_i \tag{2.1}$$

So, for a series system, all the components must be in working condition for the system to function. The series configuration of components is shown in Figure 2.1.

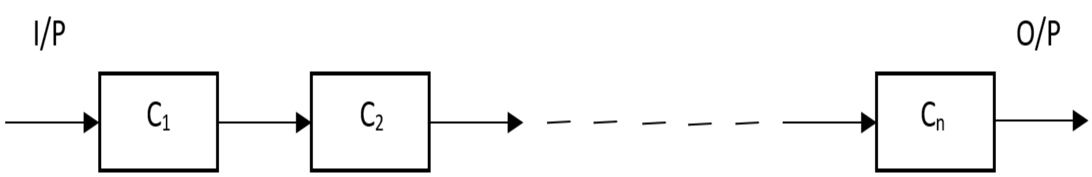

Figure 2.1: Series System

For the series system, if there are 3 components with the reliability of individual components being 0.9, then the reliability of the system is 0.73.

2.2 Parallel Systems

In case of parallel system, the system works until all the components fail. The reliability of the system is defined as the probability that the component 1 is working and/or component 2 is working and/or any component(s) present in the system is working. If the reliability of the individual components is denoted by R_i, then the reliability of parallel system with 'n' number of components is given by,

$$R_{parallel} = 1 - \prod_{i=1}^{n} (1 - R_i) \tag{2.2}$$

So, for a parallel system, atleast one of the components must be in working condition for the system to function. The parallel configuration of components is shown in Figure 2.2. For the system, if there are 3 components with the reliability of individual components being 0.9, then the reliability of the system is 0.99.

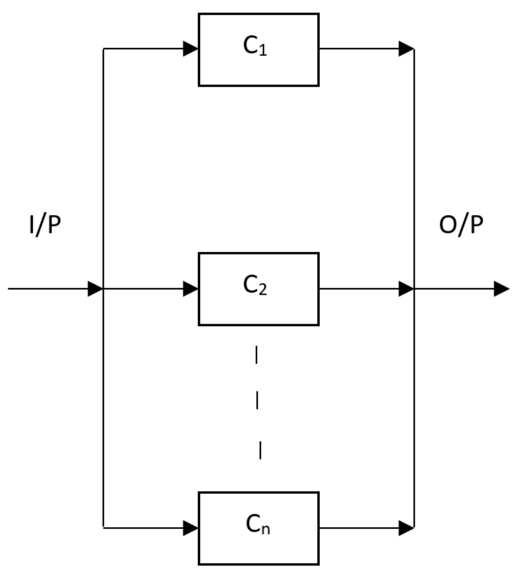

Figure 2.2: Parallel System

It is to be noted that the reliability of the series system is lower than that of its individual components but the reliability of parallel system is higher than that of the individual components. Although the parallel system offers higher reliability, it is difficult to build the system because of its redundancy in number of components [2].

2.3 Mixed Systems

There are some systems in which some components are configured in series while others are in parallel configuration. Such systems are called mixed systems. Most of the consumer products are mixed system.

2.4 K out of n: G Systems

Some systems are designed in such a way that certain components can fail without damaging the system but more than ‘n’ components (n>1) need to function well for the system to work [48]. Such systems are called k out of n: G systems. Examples of this type of system is aircraft engine which requires 2 out of 4 engines to work for the aircraft to be stable.

In the above-mentioned parallel system in section 2.2, the failure of components is assumed to be independent of each other. But, for real world applications, especially in case of parallel and mixed systems, the failure of components are not independent to each other, thereby causing difficulty in obtaining the real estimate of the reliability of the system. This causes trouble in optimally designing the components for the given load. The probability of failure of the system for dependent component failure is given by,

$$Pf_{system} = P(C_1) * P(C_2/C_1) * P(C_3/C_1, C_2) \ldots\ldots P(C_n/C_1, C_2, \ldots. C_{n-1}) \qquad (2.3)$$

where, $P(C_i)$ denotes the probability of failure of i^{th} component. Also if we assume all the components have equal chances of being failed initially and then the probability of failure of other components gets varied depending on the component that has failed, then the evaluation of system reliability increases many fold with increase in number of components.

While most of the research has assumed the failure independence between the components, only some research is dedicated to developing methodology to design the system with dependent failure rate of components. This is because the computational requirement for evaluating probabilistic constraint is high, which becomes much higher when we consider system with dependent failure rate between components. One such proven methodology that has been used to efficiently deal with probabilistic constraint optimization problem is Sequential Optimization and Reliability Assessment (SORA). Before explaining about the probabilistic optimization and SORA technique, some useful concepts are discussed.

2.5 Drawbacks with Deterministic Assumption

Using a deterministic approach in system design and analysis (i.e., if the physical parameters like diameter of the rod is assumed to be deterministic say 25cm), this will lead to erroneous conclusion because not all the components can be manufactured to the exact diameter due to the manufacturing variations. Hence probabilistic approach is necessary to accommodate the variations. A large number of important points should be considered while designing the system. Though the probability of failure decreases by increasing the safety factor, the utilization of safety factor approach does not guarantee zero failure rate [49]. In addition to narrowing down the region of random variables, adjusting the mean value of the random variable may also provide assistance in reducing the probability of failure [49]. The manufacturing tolerance can also be tightened in order to reduce the geometric dimension variation [49]. In the field of engineering design, the final aim is to have a better trade-off between system cost and the probability of failure, as failure happens at some point of time even for the worst-case design [49]. Hence, it is vital to bring the probability theory into system design so as to accommodate for the uncertainties in physical parameters that have effects on the performance of the system.

2.6 Stress-Strength Interference (SSI) Theory

It is always important to identify and handle the uncertain parameters induced during the design or manufacturing process as it is vital for reliability analysis. In case of mechanical systems, especially for the reliability analysis, the Stress-Strength Interference (SSI) theory aids the purpose [12]. According to this SSI model, the reliability can be defined as the probability that load or stress acting on the component is lower than the strength of the component, which is calculated based on the probability density function of stress and strength. The wide spread application of this SSI model is due to the fact that both stress and strength parameters are directly introduced into the model which aids the designer during the design and analysis of the mechanical components [8].

The Stress Strength Interference theory is discussed in detail in [12, 50, 51], which mathematically represents these parameters by probability distributions.

$$\text{Probability of failure} = \text{Probability}\ (\text{Stress} \geq \text{Strength}) \tag{2.4}$$

In mechanical sense, the term stress represents the mechanical force or load that is applied on the system, and the term strength denotes the yield strength of the physical unit that is subjected to the loads in order to perform its intended function [12]. The Figure 2.3 from [12] represents the concept of this stress strength interference theory.

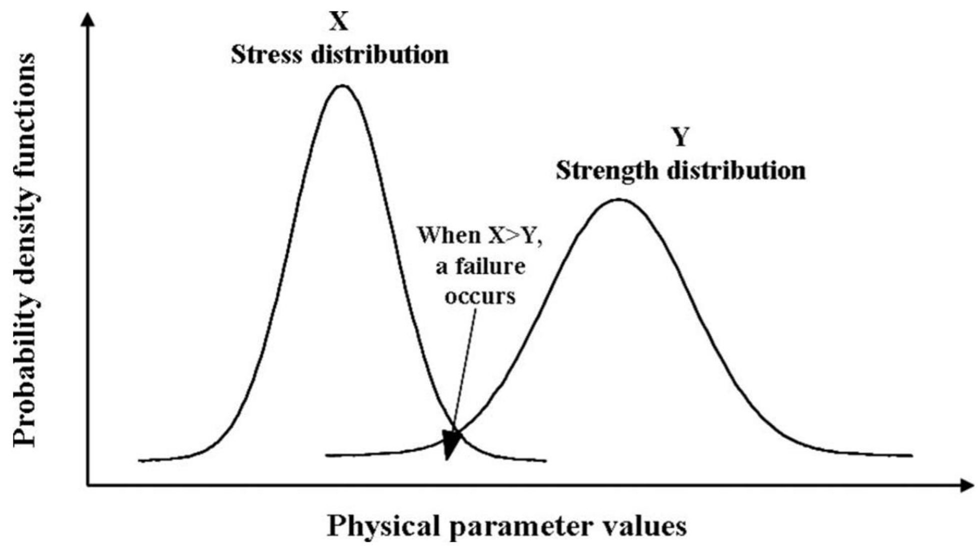

Figure 2.3: Stress-Strength Interference Theory from Huang et al [12].

In cases where there is a single parameter of stress (denoted by random variable X) and strength (denoted by random variable Y) variables, the reliability can be found by,

$$\mathrm{R} = \mathrm{P}(\text{Stress} < \text{Strength}) \tag{2.5}$$

$$\mathrm{R} = \mathrm{P}(\mathrm{X} < \mathrm{Y}) \tag{2.6}$$

$$\mathrm{R} = \int_{-\infty}^{\infty} f_y(y) \left[\int_{-\infty}^{y} f_x(x)dx\right] dy \tag{2.7}$$

In cases when there are two or more strength and stress parameters, the threshold becomes multidimensional, which is termed as the limit state function [12]. The Figure 2.4 illustrates the concept of the limit state function.

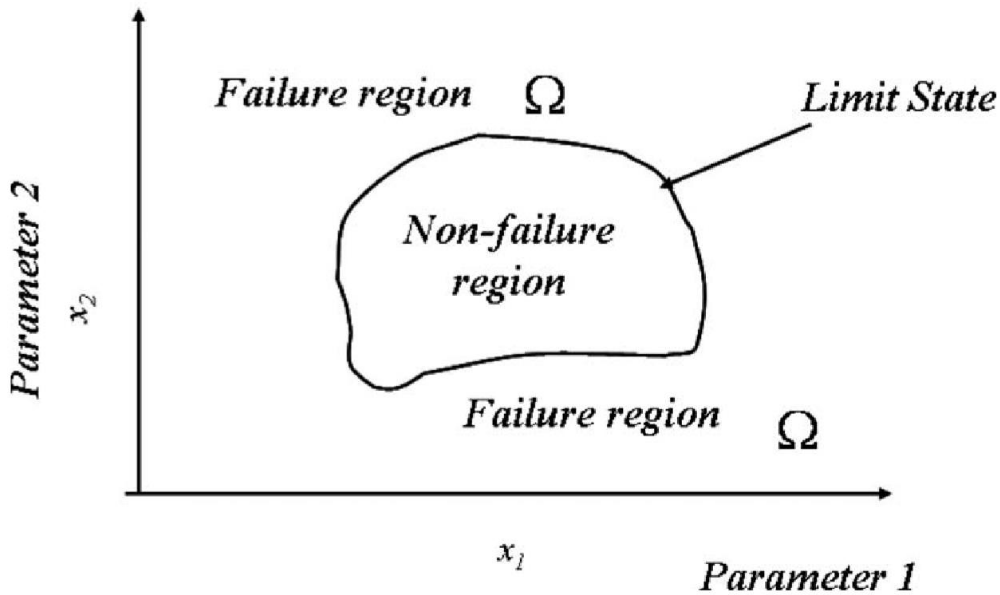

Figure 2.4: Limit State Function from Huang et al [12].

It is noted that when the parameters (x_1, x_2) falls outside the limit state region, the component fails. In this case, the probability of failure can be mathematically represented by,

$$P_f = \iint_{(x_1,\, x_2) \in F} f(x_1, x_2) dx_1 dx_2 \tag{2.8}$$

The failure region is denoted by F and the function f(x_1, x_2) denotes the joint probability density function of the random variables x_1 and x_2.

2.7 Reliability-Based Design Optimization

Having studied about the problems in deterministic approach, uses of probability theory, stress-strength interference theory and evaluating the probability of failure from the given characteristics, the RBDO problem formulation is discussed in this section.

In mechanical or construction engineer's point of view, the Reliability Based Design is an important aspect of design optimization, as it plays a critical role in maintaining the design feasibility under various uncertainties. A typical RBDO formulation considers the uncertainties in the design variables and guarantees the system reliability by utilizing the probabilistic constraint functions for the system safety requirement [5, 6, 7]. The generic formulation of RBDO is given below.

$$\text{Objective: Minimize } f(d, \mu_X, \mu_P) \tag{2.9}$$

$$\text{Subject To: Probability } [G_i(d, \mu_X, \mu_P) \geq 0] \geq R_i \tag{2.10}$$

$$\mu_x^L \leq \mu_x \leq \mu_x^U \tag{2.11}$$

$$\mu_p^L \leq \mu_p \leq \mu_p^U \tag{2.12}$$

$$d^L \leq d \leq d^U \tag{2.13}$$

$$i = 1,2, \dots, m \tag{2.14}$$

The objective function $f(d, \mu_X, \mu_P)$ given in the above formulation can be interpreted as the cost function of the system, evaluated at the means of X and P. The cost function can be linear as well as non-linear. In this formulation, 'd' denotes the vector of deterministic parameters, ‘X’ represents the vector of random variables and ‘P’ denotes the vector of random parameters. The most important part that is a major difference from other regular optimization problem is the presence of probabilistic constraint function, which ensures the system safety (reliability). The function $G_i(d, \mu_X, \mu_P)$ is the performance function of the system that emphasizes the reliability requirement of the system. The condition $G_i(d, \mu_X, \mu_P) > 0$ denotes the safety region of the system and $G_i(d, \mu_X, \mu_P) < 0$ denotes the failure region of the system. Also, $G_i(d, \mu_X, \mu_P) = 0$ represents the limit state surface that represents the boundary between the safe and failure region of the system. The variable ‘R_i’ denotes the target reliability of the system.

The above formulation is for the reliability-based design optimization of a system consisting of only one component. If there are more than one component in the system that have to be optimally designed, different types of formulation of the probabilistic constraint is required based on whether the system is in series configuration, parallel configuration or mixed configuration.

In case of series system, the objective function will be the sum of cost of individual components and the probability of failure is given below [1].

$$Pf_{series} = Prob\{\bigcup_i G_i(d, \mu_X, \mu_P) < 0\} \tag{2.15}$$

As all the components in the system must be functional for the system to be operating, the union of all the performance functions of components is required to be in the safe region. But for parallel system, any one component needs to be operating for the system to be functional, so the probability of failure is as follows.

$$Pf_{parallel} = \text{Prob}\{\bigcap_{i} G_i(d, \mu_X, \mu_P) < 0\} \tag{2.16}$$

The mixed system can be represented as a combination of both series and parallel system (union and intersection function).

The above formulation has the assumption that the failure of each component is independent of each other, which means that the probability of failure is fixed for each component. In the above mentioned parallel system, if there are 2 components, then the formulation becomes,

$$Pf_{parallel} = P(G_1(x) < 0) \cap P(G_2(x) < 0) \tag{2.17}$$

In case of load sharing systems in which the failure of one component is dependent on the condition of other components, in addition to the above formulation, the conditional probability of the components' condition (working/failed) needs to be incorporated in the constraint. Also, when the number of components increase, the evaluation of this probabilistic constraint requires great effort in order to obtain the optimal design values.

As stated in [1, 3, 4], the double loop strategy can be employed to solve the probabilistic constraints to get accurate results. So far many methods like Fast Probability Integration [52] and two point adaptive non-linear approximation [46], have been developed in order to improve the efficiency of the double loop strategy as it is computationally infeasible for complex systems [44, 45]. Du et al. [53] provided a brief review of all the methods and modeling approaches for design under uncertainty. In recent years, the single loop strategy [54, 55, 56] is adopted as it avoids the nested loops of reliability assessment and optimization. The reliability constraints are formulated as deterministic constraints [54, 57] and approximating the condition of Most Probable Point (MPP) increased the computational efficiency. Du and Chen [1] doubted that the optimality is not satisfactory in some cases, as the active reliability constraint may not converge to the actual MPP, and developed a new probabilistic design method called Sequential Optimization and Reliability Assessment (SORA). This method has been proved as an efficient method for designing individual components [5, 7, 59] with single as well as multiple failure functions for the required reliability level.

Hence, in order to solve our above-mentioned problem of designing system with dependent component failures, an efficient framework is developed using the Sequential Optimization and Reliability Assessment (SORA). The details about this methodology and its implementation is discussed in next chapter.

CHAPTER 3

SEQUENTIAL OPTIMIZATION AND RELIABILITY ASSESSMENT (SORA) APPROACH FOR OPTIMAL SYSTEM DESIGN

The Sequential Optimization and Reliability Assessment (SORA) method is developed by Du and Chen [1] for solving the problem of design under uncertainty. This method has been used extensively for the optimal and computationally efficient design of mechanical components. The SORA method has been successfully utilized to solve the design optimization of individual component with great efficiency by Zhuang [5] and Zhuang et al. [6, 7]. This method is based on serial single loop strategy [54, 55, 56] which decouples optimization loop from reliability analysis loop. Also, the method can handle both deterministic as well as random variables and parameters very efficiently. Hence, this method is extended to solve our design optimization problem of the load sharing system with dependent failure probabilities.

The SORA method uses a single loop strategy with cycles of deterministic optimization followed by reliability assessment. The deterministic optimization is carried out first so as to verify the feasibility of the probabilistic constraint and then followed by reliability analysis [1, 5, 13]. The advantage of this method is discussed in Du et al. [1]. In SORA methodology, the optimization and reliability assessment are decoupled from each other which gives the freedom of choosing any optimization technique as well as reliability analysis technique appropriately [1, 59].

The reliability is evaluated only at the desired level of reliability percentile. Usually, if the required reliability level is high (close to 1), the computational requirement will be high as well, because the search region is large and that requires more function evaluations. So, it is essential to move the design point to its optimum as soon as possible to reduce the necessity for re-locating the most probable point. Hence, in order to overcome this problem, percentile formulation is used in SORA method to establish equivalence between deterministic optimization and probabilistic optimization [1]. The Figure 3.1 from [1] represents the concept of probabilistic constraint boundary and deterministic constraint boundary.

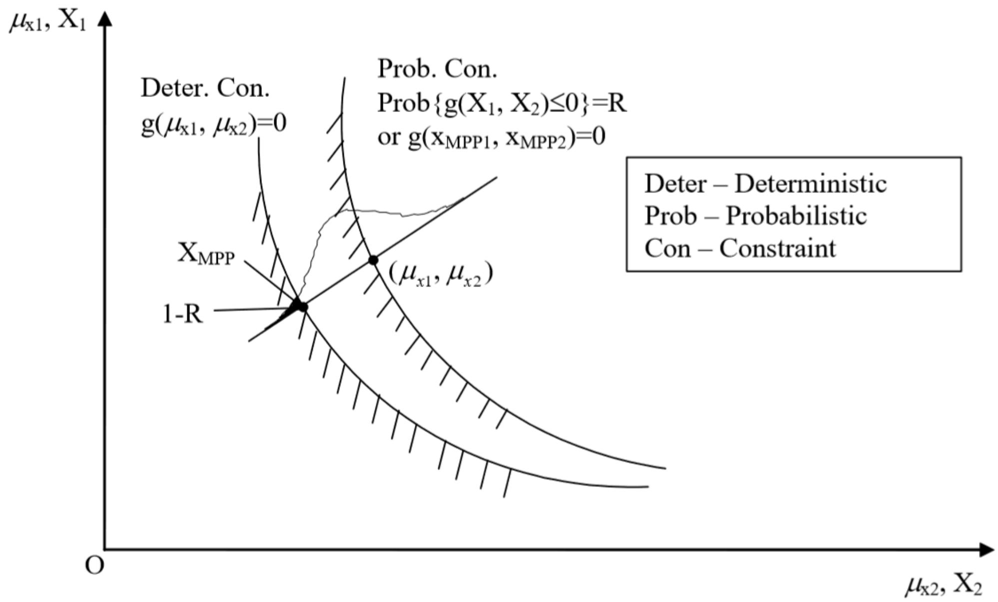

Figure 3.1: Deterministic and Probabilistic Constraint Boundary from Du et al [1].

In the Figure 3.1, two co-ordinate systems are plotted (design space μ_1, μ_2 and random space X_1, X_2) for two random design variables. If no uncertainty is considered, $g(\mu_{x1}, \mu_{x2}) = 0$ will be the constraint boundary for the deterministic design case. If uncertainty is considered, $\mathrm{Prob}\{g(\mu_{x1}, \mu_{x2}) \leq 0\} = R$ will be the constraint boundary [1]. The constraint of the probabilistic design is much stricter than the deterministic design as the reliability achieved by the deterministic design is lower than the probabilistic design. In other words, the failure region for probabilistic design is larger than that of the deterministic design [1, 59].

In the Figure 3.1, X_{MPP} is the inverse most probable point obtained by converting the x-space into standard normal u-space. The most probable point is the worst case point such that if this point satisfies the deterministic constraint, then all the other points will be feasible. So the $\mathrm{Prob}\{g(\mu1, \mu2) \leq 0\} = R$ is equivalent to $g(X_{MPP1}, X_{MPP2}) = 0$, which denotes that the evaluation of probabilistic constraint at design point is the same as evaluating the deterministic constraint at the inverse most probable point. [1]. It is important that if the probabilistic constraint is feasible, the inverse MPP for the design variables will be on the deterministic constraint boundary or inside the feasible boundary. A brief review of locating the most probable point using First Order Reliability method is discussed in section 3.1

3.1 First Order Reliability Method (FORM)

Some of the most commonly employed reliability analysis methods are Monte Carlo simulation, importance sampling, First Order Reliability Method (FORM), Second Order Reliability Method (SORM), and the Response Surface Method [49]. Ref [23] provided a summary of some reliability assessment approaches as the design solution based on deterministic approach would not be appropriate due to uncertainties.

Though the traditional Monte Carlo simulation gives accurate reliability estimate, the computational effort is high due to large sample data requirement. A number of methods have been developed to reduce the computational effort and aim to provide estimates of the integral form of failure probability [49]. In these methods, the joint probability density function $f_x(x)$ is simplified by transforming the probability density function into a standard normal distribution function of the random variables of the same dimension. Then, the limit state function g(x) = 0 is approximated by the Taylor series expansion and keeping the first few terms of the approximation. Ref [49] provides brief explanation about the process involved. If only the linear terms of this approximation are included, then it is First Order Reliability analysis Method (FORM) and if the second order terms are also included, then it is called as Second Order Reliability analysis Method (SORM). The Performance Measure Approach is one such FORM used in this research.

In Performance Measure Approach (PMA), the R-Percentile is assessed by employing an optimization problem in u-space to find the MPP of inverse reliability. After the random variable x is transformed to independent and standard normal random variable u, the mean becomes the origin and the most probable point should be a point on the limit state boundary that has distance 'β' from the origin. As the output of the performance function is assumed to follow normal distribution. Ref [5] gives the relation between the probabilistic constraint function and the reliability index as,

$$\mathrm{Prob}[\mathrm{G_i}(\mathrm{d},\mathrm{x},\mathrm{p}) \geq 0] = \int_0^\infty \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(\mathrm{t}^2)\right] \mathrm{dt} \qquad (3.1)$$

$$= 1 - \emptyset(-\beta_i) \qquad (3.2)$$

$$= \emptyset(-\beta_i) \qquad (3.3)$$

$$\text{where } t = \frac{g_i - \mu_{g_i}}{\sigma_{g_i}} \text{ and } \beta_i = \frac{\mu_{g_i}}{\sigma_{g_i}} \tag{3.4}$$

The value of beta is the reliability index and it can be shown that $\mu_{g_i} = \beta_i . \sigma_{g_i}$, when standard deviation is assumed to be constant, then the distance between the mean margin and the limit state boundary is given by the reliability index [5].

It is to be noted that if most probable point can satisfy the reliability level, then all the other points can satisfy the required reliability target. Also, the MPP should be at a minimum distance from the origin. Hence, the evaluation of the probabilistic constraint becomes an optimization problem in order to find the most probable point [59]. The problem formulation is given below.

$$\text{Minimize } G(u) \tag{3.5}$$

$$\text{S.T: } ||u|| = \beta \tag{3.6}$$

This MPP in u-space is again transformed to x-space using the mean and standard deviation of the random variable. Some of the traditional optimization techniques is given below.

3.2 Optimization Techniques

The Engineering design problems can be mathematically formulated as single-objective optimization problem or multi-objective optimization problem depending on the number of criteria involved. The ultimate aim is to either minimize or maximize the objective function subjected to some constraints, though there may be some unconstrained problems as well. The variables can be continuous, discrete (including binary) and also, based on the nature of variables, the problem can be formulated as either deterministic or stochastic optimization problems [49].

The solution methodology for these optimization problems (either constrained or unconstrained) can be classified as graphical method, optimality criteria method and search methods using algorithms [49]. The Graphical methods does not involve numerical algorithms and provides graphical visualization of the problem and the optimal solution. The optimality conditions reveals the necessary and sufficient conditions for the optimum value, including Lagrange Multipliers, Karush Kuhn Tucker (KKT) conditions, but the method is not simple and straightforward. The search methods includes gradient based search, line search method etc. The Gradient based approach utilizes search method depending upon the gradients of the objective and constraint function and arrives at the optimum solution. Steepest descent method, conjugate gradient method, Quasi-Newton method and other line search methods like secant method comes under search methods. Also, there are some non-gradient approach like genetic algorithm, simulated annealing etc.

3.3 SORA Procedure

A single SORA cycle consists of an optimization part and reliability assessment part. In each cycle, the deterministic optimization problem is solved. The design solution is updated and the reliability analysis is carried out to check whether the reliability level is satisfied by locating the inverse most probable points. If the reliability requirement is not satisfied, then the new inverse MPP’s are used to formulate the constraint function for the next cycle of deterministic optimization. In this new loop, the constraint boundary would have been shifted to new location and the MPP will be in the feasible region [59]. If not, the cycle is repeated and this method arrives at the optimum design by progressively improving the design solution. The detailed flowchart of this methodology is represented in Figure 3.2.

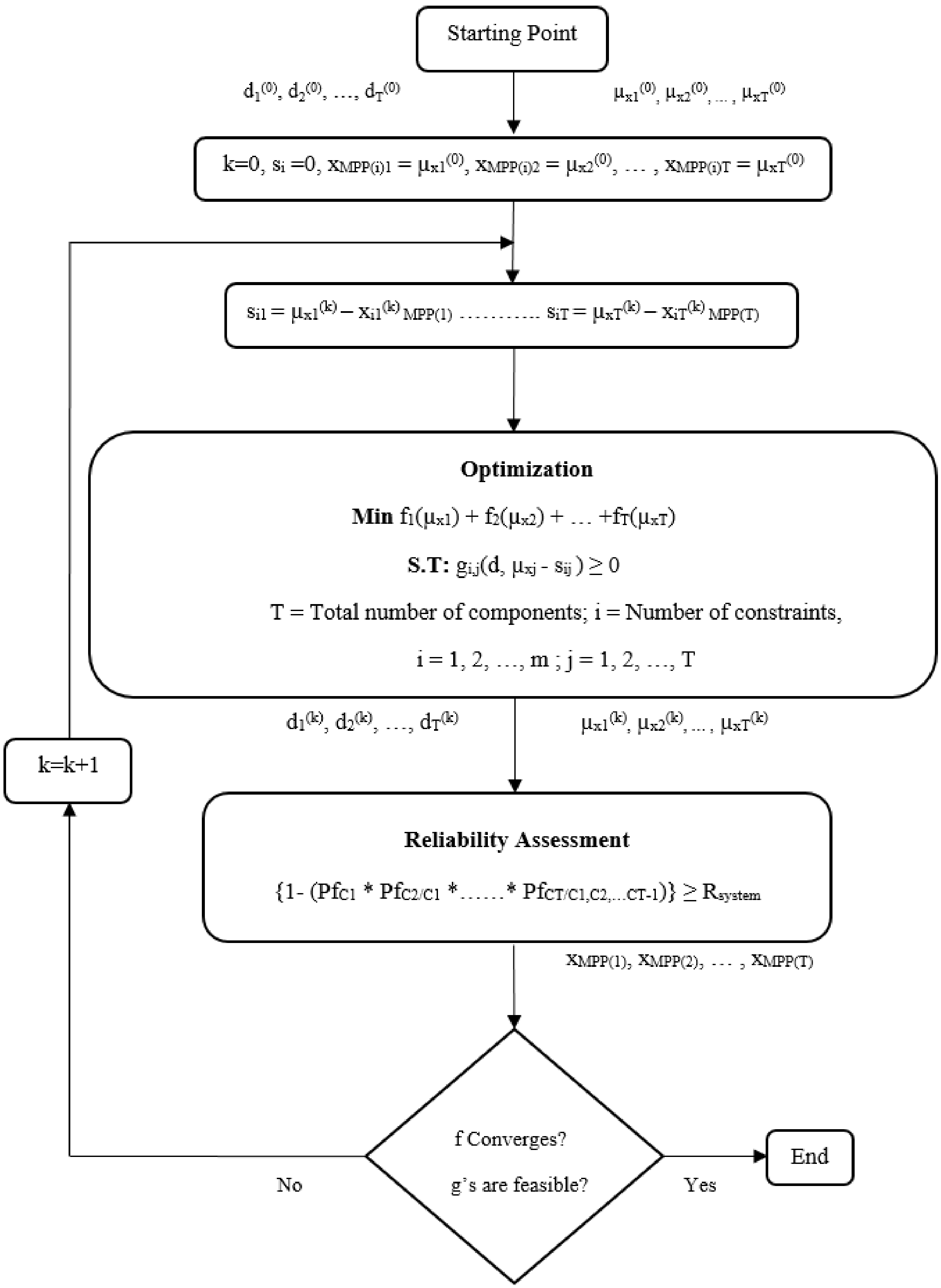

Figure 3.2: SORA Flowchart for Load Sharing Systems

In the flowchart of this methodology, there are two loops, optimization loop and reliability assessment loop. Initially, the value of most probable points are not available, so the mean of the random design variables and design parameters are selected as X_{MPP} and P_{MPP}. So, the value of shifting vector, which will be discussed later, is zero for the first cycle. In each cycle of this method, the optimization problem is solved first in order to find the value of μ_x and μ_p for each component. If we consider only the presence of random variables μ_x, then the problem formulation for first cycle is given as follows.

$$\text{Objective: Min} \sum_{j=1}^{T} f_j(d_j, \mu_{xj}) \tag{3.7}$$

$$\text{Subject To: } g_{i,j}(d_j, \mu_{xj}) \geq 0 \tag{3.8}$$

$$Lb \leq \mu_{xj} \leq Ub \tag{3.9}$$

$$Lb \leq d_j \leq Ub \tag{3.10}$$

$$i = 1, 2, \dots, m; \quad j = 1, 2, \dots, T; \tag{3.11}$$

In the above formulation, the total number of components is denoted as 'T', and $f(d_j, \mu_{xj})$ is the objective function that represents the total cost of the system as the sum of the cost of individual components. The deterministic design variable is denoted by 'd' and the mean of the random design variable X is denoted by 'μ_x'. The number of constraints is denoted by 'm'. In equation 3.8, $g_{i,j}$ denotes the performance function 'i' for component 'j'.

Now, for the first cycle, once the deterministic optimization given in the above formulation is solved, some of the constraints will be active and the optimum point μ_x for the system will be on the deterministic constraint function boundary. From Figure 3.3 [1], it is shown that the actual probability that this design variable μ_x will be feasible under uncertainty is approximately 0.5.

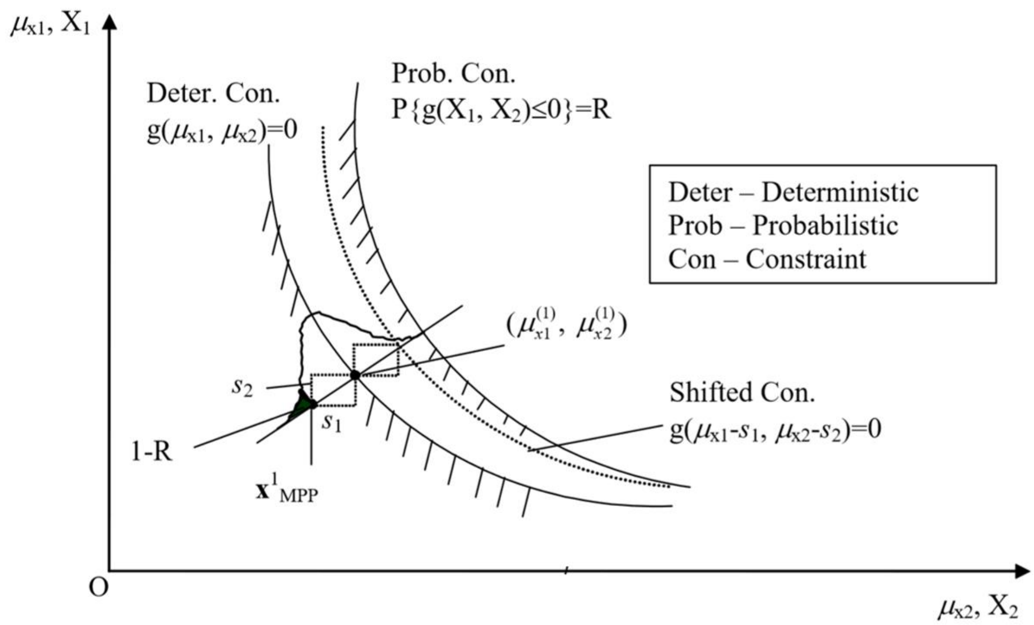

Figure 3.3: Shifting the Constraint Boundary from Du et al [1].

Now after the deterministic optimization loop is completed and the optimum is found, the reliability assessment is carried out for the solution obtained from the optimization phase. Assume that the system has two components with load ‘P’ acting on the system is divided equally into ‘P/2’ on the two components and the load gets redistributed if component 1 has failed. This forms a simple load sharing parallel system. So, the system reliability is as follows.

$$\text{System Reliability} = 1 - \text{Prob}\{\text{both components fail}\} \quad (3.12)$$

$$R_{system} = 1 - \{Pf_{C1} * Pf_{C2/C1}\} \quad (3.13)$$

Where Pf_{C1} denotes the probability of failure of component 1 and $Pf_{C2/C1}$ denotes the conditional probability of failure of component 2 given that component 1 has failed. In order to find all these probabilities, the PMA optimization, which is a first order reliability analysis method is used.

The x-space is transformed to standard normal u-space based on the mean and standard deviation of the random variable X. Then the inverse most probable points for the arbitrary component reliability level are located for the constraint $g_{i,j}$ from the problem formulation given below.

$$\text{Minimize } G(u) \quad (3.14)$$

$$\text{S.T: } ||u|| = \beta_{comp} \quad (3.15)$$

As it is already shown in figure 3.3 that the most probable point will lie in failure region and the reliability of this design point is around 0.5. Also the reliability of the design point with respect to the constraint $g_{i,j/k}$, where $g_{i,j/k}$ denotes the performance function 'i' with component 'j' working given that component(s) 'k' failed is found using,

$$\text{Minimize } ||u|| \quad (3.16)$$

$$\text{S.T: } G_{i,j/k}(u) = 0 \quad (3.17)$$

The value of conditional probability can be found from solving the above optimization problem. This process is repeated to calculate the conditional probabilities for all the components and then the system reliability is measured from equation 3.13.

If the reliability target is not met, the second cycle of the deterministic optimization needs to be implemented. Each active constraint should be modified to shift the most probable point at least onto the deterministic boundary. If 's' is denoted as the shifting vector, then each limit state function for the next optimization cycle will be as follows [1, 13, 59].

$$g(\mu_x - s) \geq 0 \tag{3.18}$$

The shifting vector should ensure the most probable point lies on the deterministic boundary and its value can be found from equation 3.13 below from [1, 13, 59].

$$s = \mu_x - x_{MPP} \tag{3.19}$$

The dotted line in Figure 3.3 shows the shifted deterministic boundary for system with identical components. The feasible region for the second cycle will be narrower when compared to the first cycle of the optimization. The optimum solution is obtained for second optimization cycle and reliability is assessed. The results should improve drastically from the first cycle. If the required target is not met, the process is continued until the objective converges and the reliability target is achieved [1].

CHAPTER 4

NUMERICAL EXAMPLES

In order to demonstrate and validate the developed methodology, four numerical example cases are discussed in this section. A simple case of a system with two identical components is modelled, followed by designing a system with non-identical components and finally the formulation for finding the optimum number of components for a system is discussed. In all the cases, the component 1 is assumed to fail first.

4.1 Case 1 - System with Identical Components

A simple load sharing system consisting of two identical components is considered. Figure 4.1 shows two identical I-Beams loaded with a bar at the top. Assuming the system to be a machine bed and a load of 600KN is applied at the top which splits equally to two beams, so that each beam experiences a load of 300KN.

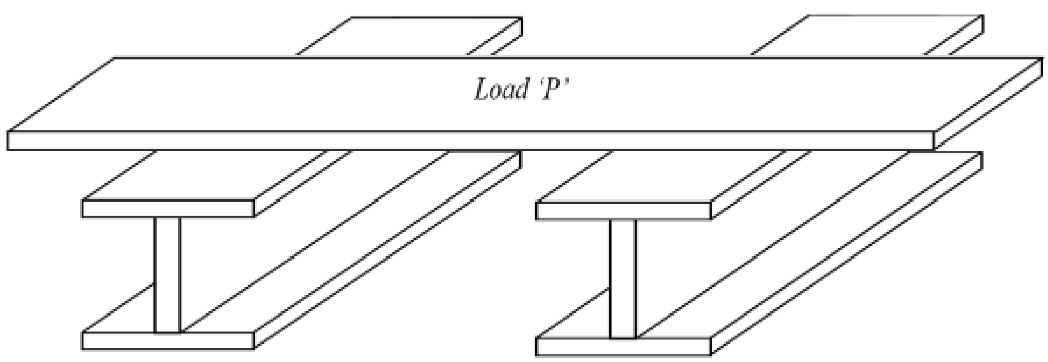

Figure 4.1: A Simple Load Sharing System

The beam fails when load exceeds its yield strength and the system fails if both components fail. The objective is to design the beams with minimum cost so that the reliability level of system is atleast 99.87%. As the components are identical, Figure 4.2 represents a single beam and its design parameters.

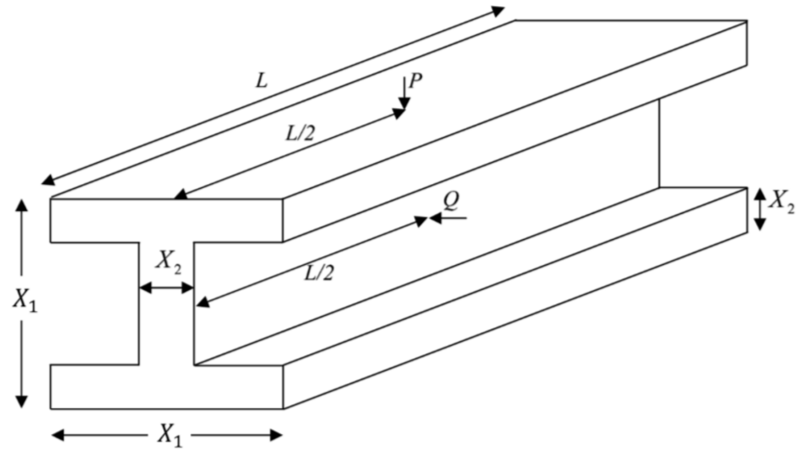

Figure 4.2: Dimensions of I-Beam from Zhuang [5].

To design the beam, two random variables X_1 and X_2 needs to be determined [5, 60, 13]. Due to manufacturing variability, these two variables X_1 and X_2 are random and are normally distributed with $\sigma_1 = 2.025$ cm and $\sigma_2 = 0.225$ cm respectively. The length of the beam is 200 cm. The maximum bending stress 'σ' for each beam is taken as 16 KN/ cm^2. Also, an unshared external axial load 'Q' of 50KN acts on each beam. Both the vertical and lateral loads are assumed to be normally distributed, i.e., P ~ N(600, 10) KN and Q ~ N(50, 1) KN. The target reliability index 'β' for the system is 3.0115 (Probability of failure= 0.0013). Both the components are identical and have same performance function ($g_1(x) = g_2(x)$).

The overall objective of this problem is to reduce the system cost, i.e., the weight of the individual components which contributes to overall system cost and on the other hand should satisfy the required system reliability level. For simplicity, the beam length and material density are assumed to be constant, so minimizing this function will be equivalent to minimizing the cross sectional area of the beam [5]. Now, the objective will be to minimize the function, $f(x_1, x_2) = 2x_1x_2 + x_2(x_1 - 2x_2)$. The limit state function $g(x_1, x_2) \geq 0$ for each component is the difference between bending threshold and the actual bending stress. $G(x_1, x_2)$ is defined as,

$$G(x_1, x_2) = \sigma - \left(\frac{M_y}{Z_y} + \frac{M_z}{Z_z}\right) \tag{4.1}$$

For the purpose of simplicity, the loads P and Q are assumed to be equal to their mean value and each component has only one performance function (i =1). As the beams are identical and have similar performance functions, the conditional probability functions $g_{1/2}$ and $g_{2/1}$ will also be the same. These two identical components are taken as two different components with same values for the variables for better understanding. The problem formulation is as follows.

$$\text{Objective: Minimize } 2\mu_1\mu_2 + \mu_2(\mu_1 - 2\mu_2) + 2\mu_3\mu_4 + \mu_4(\mu_3 - 2\mu_4) \tag{4.2}$$

$$\text{S.T:} \{1 - P[g_1(\mu_1, \mu_2) < 0] * P[g_{2/1}(\mu_3, \mu_4) < 0]\} \geq 0.9987 \tag{4.3}$$

$$10 \leq \mu_1 \leq 80, 0.9 \leq \mu_2 \leq 5 \,;\, 10 \leq \mu_3 \leq 80, 0.9 \leq \mu_4 \leq 5 \tag{4.4}$$

$$\mu_1, \mu_2, \mu_3, \mu_4 \geq 0 \tag{4.5}$$

$$\mu_1 = \mu_3 \,;\; \mu_2 = \mu_4 \tag{4.6}$$

$$g_1(\mu_1,\mu_2) = \sigma - \frac{0.3(\frac{p}{2})\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4{\mu_2}^2 + 3{\mu_1}^2 - 6\mu_1\mu_2)} + \frac{0.3q\mu_2}{(\mu_1 - 2\mu_2){\mu_2}^3 + 2\mu_2{\mu_1}^3} \tag{4.7}$$

$$g_2(\mu_3,\mu_4) = \sigma - \frac{0.3(\frac{p}{2})\mu_3}{\mu_4(\mu_3 - 2\mu_4)^3 + 2\mu_3\mu_4(4{\mu_4}^2 + 3{\mu_3}^2 - 6\mu_3\mu_4)} + \frac{0.3q\mu_4}{(\mu_3 - 2\mu_4){\mu_4}^3 + 2\mu_4{\mu_3}^3} \tag{4.8}$$

$$g_{1/2}(\mu_1,\mu_2) = \sigma - \frac{0.3p\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4{\mu_2}^2 + 3{\mu_1}^2 - 6\mu_1\mu_2)} + \frac{0.3q\mu_2}{(\mu_1 - 2\mu_2){\mu_2}^3 + 2\mu_2{\mu_1}^3} \tag{4.9}$$

$$g_{2/1}(\mu_3,\mu_4) = \sigma - \frac{0.3p\mu_3}{\mu_4(\mu_3 - 2\mu_4)^3 + 2\mu_3\mu_4(4{\mu_4}^2 + 3{\mu_3}^2 - 6\mu_3\mu_4)} + \frac{0.3q\mu_4}{(\mu_3 - 2\mu_4){\mu_4}^3 + 2\mu_4{\mu_3}^3} \tag{4.10}$$

The probability of failure of the system is nothing but the probability that both components fail. For this case, the failure probability should be less than 0.0013 or in other words, the reliability should be greater than or equal to 99.87%. In this problem, even though the components are identical and has same performance function, they are treated as two different components with same values for variables. But the final answer for the variables x_1, x_3 and x_2, x_4 will be identical.

The Genetic Algorithm is used for the Optimization process and Performance Measure Approach (PMA) is used for reliability analysis. Using the above discussed methodology, the optimum mean values of the design variables is found within few iterations. The solution obtained from MATLAB software for each cycle is tabulated in Table 4.1.

Table 4.1: Results for Case 1

Cycle	μ_1	μ_2	μ_3	μ_4	Cost	R_{system}
1	31.4745	0.9470	31.4745	0.9470	175.25	0.6583
2	34.2925	0.9259	34.2925	0.9259	187.07	0.8342
3	38.4987	1.0298	38.4987	1.0298	233.63	0.9988

The optimum design value for the beam is found to be μ_1 = 38.4987cm, μ_2 = 1.0298 cm with total system cost of 233.63 sq.cm. In order to validate the results obtained for the given system, the Monte Carlo Simulation is carried out for the given performance function by generating random samples from the given distribution. The average reliability of the system is found to be 99.80% and the solution is acceptable. If the conditional probabilities are not considered, then the actual reliability is only around 95.2 % which is erroneous due to load sharing property.

4.2 Case 2 - System with Identical Components but with Different Performance Functions

There are situations where the manufacturer can afford to produce only one type of component which is interchanged for different purposes. In these cases, the design of the components in the system is identical but the performance function for each component varies depending on the loading condition or location where it is installed for usage. The developed methodology can be extended to this type of system with identical components but with different performance functions for each component. To validate the claim, the same system shown in Figure 4.1 and 4.2 is used but the performance function is altered based on the applied load 'P'. In this case, the load 'P' is not equally distributed and the load 'Q' is acting on only one component and hence the failure function will not be the same even though the components are identical. The objective function remains the same but the performance function constraint is changed in the problem formulation as follows.

$$\text{Objective: Minimize } 2\mu_1\mu_2 + \mu_2(\mu_1 - 2\mu_2) + 2\mu_3\mu_4 + \mu_4(\mu_3 - 2\mu_4) \tag{4.11}$$

$$\text{S.T:} \{1 - P[g_1(\mu_1, \mu_2) < 0] * P[g_{2/1}(\mu_3, \mu_4) < 0]\} \geq 0.9987 \tag{4.12}$$

$$10 \leq \mu_1 \leq 80, 0.9 \leq \mu_2 \leq 5\,; 10 \leq \mu_3 \leq 80, 0.9 \leq \mu_4 \leq 5 \tag{4.13}$$

$$\mu_1, \mu_2, \mu_3, \mu_4 \geq 0 \tag{4.14}$$

$$\mu_1 = \mu_3\,;\ \mu_2 = \mu_4 \tag{4.15}$$

$$g_1(\mu_1, \mu_2) = \sigma - \frac{0.3(\frac{2p}{3})\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4{\mu_2}^2 + 3{\mu_1}^2 - 6\mu_1\mu_2)} + \frac{0.3q\mu_2}{(\mu_1 - 2\mu_2){\mu_2}^3 + 2\mu_2{\mu_1}^3} \quad (4.16)$$

$$g_2(\mu_3, \mu_4) = \sigma - \frac{0.3(\frac{p}{3})\mu_3}{\mu_4(\mu_3 - 2\mu_4)^3 + 2\mu_3\mu_4(4{\mu_4}^2 + 3{\mu_3}^2 - 6\mu_3\mu_4)} \quad (4.17)$$

$$g_{1/2}(\mu_1, \mu_2) = \sigma - \frac{0.3p\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4{\mu_2}^2 + 3{\mu_1}^2 - 6\mu_1\mu_2)} + \frac{0.3q\mu_2}{(\mu_1 - 2\mu_2){\mu_2}^3 + 2\mu_2{\mu_1}^3} \quad (4.18)$$

$$g_{2/1}(\mu_3, \mu_4) = \sigma - \frac{0.3p\mu_3}{\mu_4(\mu_3 - 2\mu_4)^3 + 2\mu_3\mu_4(4{\mu_4}^2 + 3{\mu_3}^2 - 6\mu_3\mu_4)} \quad (4.19)$$

The reliability of the system should be greater than or equal to 99.87%. This problem is also formulated as previous case such that, even though the components are identical, they are treated as two different components with same values for variables. The final answer for the variables x_1, x_3 and x_2, x_4 will be identical. The Genetic Algorithm is used for the Optimization process and Performance Measure Approach (PMA) is used for reliability analysis. The optimum mean values of the design variables is found within a couple of iterations. The solution obtained from MATLAB software for each cycle is tabulated in Table 4.2.

The optimum design value for the beam is found to be μ_1 = 40.5532 cm, μ_2 = 1.0902 cm with total system cost of 260.53 sq.cm. The average reliability of the system obtained with Monte Carlo Simulation is 98.92%. If the conditional probabilities are not considered, then the actual reliability is only around 94.6% which is erroneous due to load sharing property.

Table 4.2: Results for Case 2.

Cycle	μ_1	μ_2	μ_3	μ_4	Cost	R_{system}
1	36.0270	1.0136	36.0270	1.0136	208.92	0.6975
2	37.0654	1.0141	37.0654	1.0141	221.43	0.8790
3	40.5532	1.0902	40.5532	1.0902	260.53	0.9989

4.3 Case 3 - System with Non-Identical Components

Most of the engineering systems consists of components that are non-identical but their combined performance is necessary for the system to operate. In these cases, the above developed methodology can be used to design the components optimally, ensuring the system safety. In order to demonstrate this case, one of the I-Beam in the system shown in Figure 4.1 is replaced by a beam of rectangular cross section shown in Figure 4.3. Also, the external load ‘Q’ is ignored.

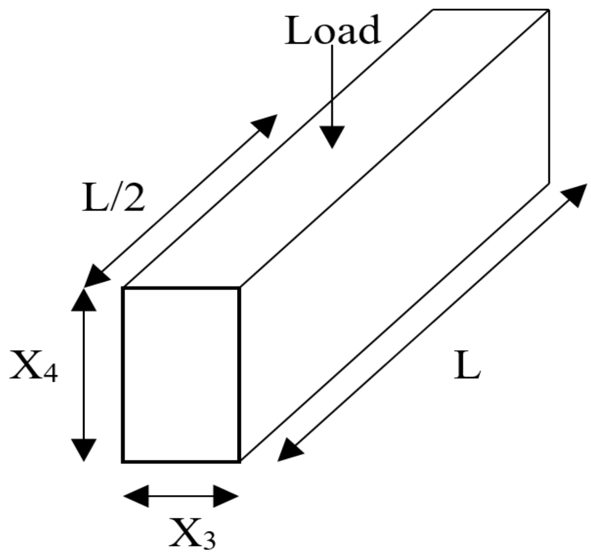

Figure 4.3: Beam of Rectangular Cross Section

The geometrical dimensions of this component is given in Figure 4.3. The length of the beam is 200cm. The random variables X_3 and X_4 are normally distributed with σ_3 = 2.025 cm and σ_4 = 0.225. The other parameters are also taken to be the same as the I-Beam, with load 'Q' removed. The cost function for rectangular beam is $f(x_3, x_4) = x_3.x_4$. The performance function $G(x_3, x_4)$ is given in equation 4.20.

$$G(x_3, x_4) = \sigma - \frac{6PL}{4 * x_3 {x_4}^2} \quad (4.20)$$

The complete problem formulation is as follows.

$$\text{Objective: Minimize } 2\mu_1\mu_2 + \mu_2(\mu_1 - 2\mu_2) + (\mu_3 * \mu_4) \quad (4.21)$$

$$S.T: \{1 - P[g_1(\mu_1, \mu_2) < 0] * P\left[g_{2/1}(\mu_3, \mu_4) < 0\right]\} \geq 0.9987 \quad (4.22)$$

$$10 \leq \mu_1 \leq 80, 0.9 \leq \mu_2 \leq 5 ; 1 \leq \mu_3 \leq 25, 10 \leq \mu_4 \leq 50 \quad (4.23)$$

$$\mu_1, \mu_2, \mu_3, \mu_4 \geq 0 \tag{4.24}$$

$$g_1(\mu_1, \mu_2) = \sigma - \frac{0.3(\frac{p}{2})\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4{\mu_2}^2 + 3{\mu_1}^2 - 6\mu_1\mu_2)} \tag{4.25}$$

$$g_2(\mu_3, \mu_4) = \sigma - \frac{300(\frac{p}{2})}{\mu_3{\mu_4}^2} \tag{4.26}$$

$$g_{1/2}(\mu_1, \mu_2) = \sigma - \frac{0.3p\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4{\mu_2}^2 + 3{\mu_1}^2 - 6\mu_1\mu_2)} \tag{4.27}$$

$$g_{2/1}(\mu_3, \mu_4) = \sigma - \frac{300p}{\mu_3{\mu_4}^2} \tag{4.28}$$

The Genetic Algorithm is used for the Optimization process and Performance Measure Approach (PMA) is used for reliability analysis. The optimum mean values of the design variables found from MATLAB software for each cycle is tabulated in Table 4.3.

Table 4.3: Results for Case 3.

Cycle	μ_1	μ_2	μ_3	μ_4	Cost	R_{system}
1	31.4275	0.9307	2.9527	43.6260	214.83	0.6709
2	32.5215	1.0124	3.7869	44.5877	265.58	0.8542
3	35.9531	1.1120	4.5016	45.9437	324.28	0.9990

The optimum value for the component 1 (I-Beam) is μ_1 = 35.9531 cm, μ_2 = 1.1120 cm and for rectangular bar is μ_3 = 4.5016 cm, μ_4 = 45.9437 cm with total system cost of 324.38 sq.cm. The average reliability of the system obtained with Monte Carlo Simulation is around 98.86%. If the conditional probabilities are not considered, then the actual reliability is only around 93%.

4.4 Case 4 - Selection of Components Required for the System

In addition to optimal design of components in a system, the user might be interested to know whether to include all the components or else eliminate some components so that the system cost could be reduced while the reliability target is still attainable. When the system consists of non-identical components, then the cost of each components will play a vital role in designing and allocating the number of components to the system. For instance, it is efficient to design components with lower cost function that perform at the same level than to design the one with higher cost function, provided that there is no restriction for the number of components to be manufactured.

In order to solve this case, a mixed integer programming problem is formulated with I-Beam and rectangular box beam used for the previous case. The final solution will provide a knowledge whether the system has an optimum cost by including both components or by designing any one component to carry full load for the given reliability level. The problem formulation is given below.

$$\text{Objective: Minimize } b_1[2\mu_1\mu_2 + \mu_2(\mu_1 - 2\mu_2)] + b_2[(\mu_3 * \mu_4)] \quad (4.29)$$

$$[b_1 g_1(\mu_1, \mu_2) \geq 0] \quad (4.30)$$

$$[b_2 g_2(\mu_3, \mu_4) \geq 0] \tag{4.31}$$

$$10 \leq \mu_1 \leq 80, 0.9 \leq \mu_2 \leq 5 ; 1 \leq \mu_3 \leq 25, 10 \leq \mu_4 \leq 50 \tag{4.32}$$

$$b_1, b_2 \in [0,1]; \ \mu_1, \mu_2, \mu_3, \mu_4 \geq 0 \tag{4.33}$$

$$g_1(\mu_1, \mu_2) = \sigma - \frac{0.3(\frac{p}{2})\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4\mu_2{}^2 + 3\mu_1{}^2 - 6\mu_1\mu_2)} \tag{4.34}$$

$$g_2(\mu_3, \mu_4) = \sigma - \frac{300(\frac{p}{2})}{\mu_3\mu_4{}^2} \tag{4.35}$$

$$g_{1/2}(\mu_1, \mu_2) = \sigma - \frac{0.3p\mu_1}{\mu_2(\mu_1 - 2\mu_2)^3 + 2\mu_1\mu_2(4\mu_2{}^2 + 3\mu_1{}^2 - 6\mu_1\mu_2)} \tag{4.36}$$

$$g_{2/1}(\mu_3, \mu_4) = \sigma - \frac{300p}{\mu_3\mu_4{}^2} \tag{4.37}$$

The reliability assessment is done according to the results of the above formulation. In order to keep the system in working condition, either the component 1 should be designed to carry the entire load or the component 2 should be designed to carry the entire load or components 1 and 2 should be combined to carry the entire load provided that even if one component fails, the other component holds out. The sum of the binary variables gives us the optimum number of components required for the system. Also, with the knowledge of these binary variables, the components that are to be included in the system can be easily identified. The result is that b_1 =1 and b_2 = 0 with design parameters are μ_1 = 53.8166 cm, μ_2 = 0.9524 cm, with total system cost of 151.95

sq.cm. In this case, the component 1 is optimum to carry the entire load depending on the overall cost. But, in terms of K out of n: G system, if it is essential to place at least two components, then one more constraint given in equation 4.38 is added to the above formulation.

$$b_1 + b_2 \geq k \tag{4.38}$$

If k=2, then b_1 =1 and b_2 = 1 and both the components are included in our design leading to optimum value of μ_1 = 35.6959 cm, μ_2 = 1.0105 cm for I-Beam and for rectangular bar μ_3 = 4.6404 cm, μ_4 = 45.7061 cm with total system cost of 320.26 sq.cm. In systems where the number of components is large, then this method will be helpful in selecting the components needed to be included in the system.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

This research proposes a novel approach for the design optimization of load sharing systems using Sequential Optimization and Reliability Assessment (SORA) framework. The optimal design of the components to satisfy the required system level reliability target can be arrived using this method with least computational requirement. This is because the objective converges rapidly by employing sequential cycles of optimization and reliability assessment. Several numerical examples are provided in order to validate this method by designing components for different types of systems. The number of function evaluations were less than 1000 for all the four cases of numerical examples. Also, the formulation for finding the optimum number of components for a given system is discussed at the end.

In terms of accuracy of the optimum solution, Monte Carlo Simulation is conducted from the given distribution and the reliability level achieved for each case is as follows.

1. Reliability of system with identical components is around 99.80%,
2. Reliability of system with identical components but with different performance function is around 99.52%,
3. Reliability of system with non-identical components is around 98.86%

The solutions obtained for these cases are acceptable as the error percentage is low. This error is due to the fact that the cost function and performance function for the components are non-linear. So, the first order approximation during reliability analysis leads to loss in accuracy of the result. Also, for a system with non-identical components, the convergence of the objective might take a long time with increased cycles of optimization and reliability analysis. This drawback may be due to the difference in component design variables as well as the constraint and objective functions are non-linear. So, the activities of the deterministic constraint changes drastically for each cycle leading to increased computation. But, sometimes, the results were conservative that led to increased system cost rather than allowing the system to fail. This is due to the nature of shifting vector strategy, which might have moved the most probable point far inside the feasible region than required. Also, this shifting strategy will work reliably only for the random variables with normal distribution. If there is a mix of random variables with normal distribution and other distributions, or if the random variables are non-normally distributed, this method of using FORM with SORA is not applicable.

In order to overcome the problems with FORM method, Du [58] proposed saddle point approximation method for SORA, which could be tried for our system. Also, there is a high need to reduce the curse of dimensionality while formulating the system reliability constraint for components with dependent probability of failure. As the number of components increase, the conditional probability for each and every components' working or failed state needs to be incorporated, which will increase the computational requirement of the single constraint many fold. So, if the system has many constraints, the computation will be much complex and also reaching an optimum solution will be difficult. Also, as the computational demand of most probable point based approach and the number of random variables are approximately proportional to each other, the random variables which are of least importance or inessential for the component design can be sorted out by developing methodology using design of experiment (DOE) techniques, which might be the objective for future research. This will result in reduced problem size and might contribute towards alleviating the curse of dimensionality problem, leading to increased computational efficiency.

But in case of designing simple system with less number of components or designing individual components for the given reliability, this method is reliable as well as efficient. Other usefulness with this method is that the design objective is deterministic and there is no need to perform probability analysis during the optimization process. Also, the reliability is measured only at the desired level (R-Percentile) and the use of robust inverse MPP search algorithm will makes it more computationally efficient [1]. Finally, the optimum number of components required for the system is found by formulating a mixed integer programming problem.

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