Obtaining Accurate Estimates of the Mediated Effect with and without Prior Information

by

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A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Arts

Approved July 2014 by the
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August 2014

ABSTRACT

Research methods based on the frequentist philosophy use prior information in a priori power calculations and when determining the necessary sample size for the detection of an effect, but not in statistical analyses. Bayesian methods incorporate prior knowledge into the statistical analysis in the form of a prior distribution. When prior information about a relationship is available, the estimates obtained could differ drastically depending on the choice of Bayesian or frequentist method. Study 1 in this project compared the performance of five methods for obtaining interval estimates of the mediated effect in terms of coverage, Type I error rate, empirical power, interval imbalance, and interval width at N = 20, 40, 60, 100 and 500. In Study 1, Bayesian methods with informative prior distributions performed almost identically to Bayesian methods with diffuse prior distributions, and had more power than normal theory confidence limits, lower Type I error rates than the percentile bootstrap, and coverage, interval width, and imbalance comparable to normal theory, percentile bootstrap, and the bias-corrected bootstrap confidence limits. Study 2 evaluated if a Bayesian method with true parameter values as prior information outperforms the other methods. The findings indicate that with true values of parameters as the prior information, Bayesian credibility intervals with informative prior distributions have more power, less imbalance, and narrower intervals than Bayesian credibility intervals with diffuse prior distributions, normal theory, percentile bootstrap, and bias-corrected bootstrap confidence limits. Study 3 examined how much power increases when increasing the precision of the prior distribution by a factor of ten for either the action or the conceptual path in mediation analysis. Power generally increases with increases in precision but there are many sample

size and parameter value combinations where precision increases by a factor of 10 do not lead to substantial increases in power.

DEDICATION

Za baba Jelku, i ostatak porodice

ACKNOWLEDGMENTS

I thank my committee chair, David MacKinnon, and the rest of my committee for their help in the formulation and execution of the project idea. I thank my family and friends for their support. Last, but not least, I thank Biggie for keeping me company while I was writing the code for the simulations in this project.

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CHAPTER 1

INTRODUCTION

The goal of many research projects is to identify and describe a relation between two variables, X and Y. Sometimes a third variable can improve the understanding of the relation between two variables. When a third variable is intermediate between X and Y in a causal chain, it is called a mediator (James & Brett, 1984; MacKinnon, 2008). More specifically, mediators are operationally defined as variables that transmit the influence that one variable (X) exerts on another (Y). According to Judd and Kenny (1981), there are three main reasons to do mediation analysis: 1) in order to investigate the process through which X affects Y, 2) in order to better predict the relationship between X and Y in different settings and populations, and 3) in order to learn which variables were key in a process and to then use this knowledge to design more effective interventions. MacKinnon (1994) and MacKinnon and Dwyer (1993) listed additional reasons for analyzing mediating variables: 4) as a manipulation check to make sure that the program changes the intervening variables it was supposed to change, 5) in cases where there is no program effect on the mediator, to find out whether the measures of the mediator require improvement, 6) in cases where program effects on mediators but not on dependent variables may suggest that effects on the dependent variable will occur later. Overall, there are two main uses of mediation models (MacKinnon, 2008): mediation for explanation and mediation for design. Mediation for explanation is used once a relation has been established between an independent variable and a dependent variable, and the researchers want to explain this relation in more detail. Mediation for design is often used in prevention studies, and its goal is to select mediating variables that are causally related to the dependent variable and then design a manipulation that will target the mediator and indirectly affect the dependent variable. Mediation analysis is used in many research areas, from psychology, sociology, communications, agriculture, and political science to epidemiology. Upcoming sections will offer an introduction to the Single Mediator Model, followed by methods for obtaining interval estimates for the mediated effect, and an explanation of Bayesian methods and their application to mediation. Properties of interval estimates are discussed, followed by a review of the literature on interval estimation of the mediated effect. Finally, the expected contributions of the current study are outlined, and the hypotheses of each of the three studies are listed.

Single Mediator Model

The simplest mediation model is the single mediator model that consists of three variables: an independent variable (X) related to the mediator (M), which is related to the dependent variable (Y) (MacKinnon, 2008). This model is described using the following three regression equations:

$$Y = i_1 + cX + e_1 \tag{1}$$
$$M = i_2 + aX + e_2 \tag{2}$$
$$Y = i_3 + bM + c'X + e_3 \tag{3}$$

where *c* represents the total effect of X on Y, *c'* represents the effect of X on Y adjusted for the effect of the mediator M, *b* measures the relation between the mediator M and the dependent variable Y adjusted for the independent variable X, and *a* measures the relation between X and M. i_1, i_2, and i_3 represent intercepts, and it is assumed that the three error terms, e_1, e_2, and e_3 follow a normal distribution with a mean of zero and variance σ_1^2, σ_2^2 and σ_3^2, respectively. There are three general ways to test for mediation as outlined by MacKinnon, Lockwood, Hoffman, West, and Sheets (2002): causal steps, product of coefficients, and difference in coefficients.

Causal steps. Baron and Kenny (1986) outlined four causal steps one can use to test for complete mediation. First, one should establish that there is a significant relation between X and Y (*c* coefficient). Secondly, one needs to determine whether the relation between X and M (the *a* path) is significant. If yes, one should proceed to the third step and test whether the relation between M and Y is significant when controlling for X (the *b* path). Finally, one should establish full mediation by making sure that X affects Y only through M, and thus that the *c'* path is zero (Judd, & Kenny, 1981). The fourth requirement was subsequently relaxed by Baron and Kenny (1986) allowing for partial mediation. Also, the requirement of a significant relation between X and Y is problematic in the case of inconsistent mediation models where the *ab* product is of the opposite sign from the *c'* path, thus making the total effect zero or close to zero even though the mediated and direct effects are different from zero.

Product of coefficients. One can also test for mediation by computing the product of coefficients *ab,* dividing it by the standard error of the mediated effect, *ab*, and comparing this value with the suitable critical value from the normal distribution. There are a few ways of computing the standard error of the mediated effect that will be outlined below, as well as problems with treating the distribution of *ab* as normal.

Difference in coefficients. It is also possible to obtain the value of the mediated effect by subtracting the direct effect from the total effect *c-c'* and test the significance using the standard error of *c-c'*. The product of coefficients and difference in coefficients methods of estimating the mediated effect produce the same results in linear single mediator models with continuous variables.

Interval estimates for the mediated effect

Wilkinson and the APA Task Force on Statistical Inference recommended that researchers report an interval estimate whenever possible (1999). Krantz (1999) outlined a few characteristics of confidence intervals that make them a better choice than null hypothesis significance tests: 1) the confidence interval includes a point estimate, whereas some articles simply report a *p*-value, 2) confidence intervals have valid procedural probability interpretations, 3) wide confidence intervals communicate uncertainty, which a *p*-value does not do, 4) if two confidence intervals overlap heavily, then it cannot be said that one experiment does not replicate the other, and 5) confidence intervals encourage the researchers to think about the magnitude of the parameter, whereas a *p*-value does not.

In the case of the mediated effect, there are multiple ways of constructing an interval estimate and several standard error formulas for the mediated effect. The general form of the confidence interval for the product of coefficients estimate of the mediated effect is as follows:

$$\hat{a}\hat{b} - z_{\alpha/2}(s_{\hat{a}\hat{b}}) \leq ab \leq \hat{a}\hat{b} + z_{\alpha/2}(s_{\hat{a}\hat{b}}) \quad (4)$$

Where $\hat{a}\hat{b}$ is the sample estimate of the mediated effect, $z_{\alpha/2}$ is the critical value from the normal distribution, and $s_{\hat{a}\hat{b}}$ is the product of coefficients sample standard error of the mediated effect (MacKinnon, 2008).

The most commonly used formula for the standard error for the mediated effect is called the multivariate delta method standard error (Sobel 1982):

$$s_{\hat{a}\hat{b}} = \sqrt{\hat{a}^2 s_{\hat{b}}^2 + \hat{b}^2 s_{\hat{a}}^2} \tag{5}$$

Two alternative ways of computing the standard error of the mediated effect are (MacKinnon & Dwyer, 1993; MacKinnon, Warsi & Dwyer, 1995):

$$s_{\sec ond} = \sqrt{\hat{a}^2 s_{\hat{b}}^2 + \hat{b}^2 s_{\hat{a}}^2 + s_{\hat{a}}^2 s_{\hat{b}}^2} \tag{6}$$

and

$$s_{unbiased} = \sqrt{\hat{a}^2 s_{\hat{b}}^2 + \hat{b}^2 s_{\hat{a}}^2 - s_{\hat{a}}^2 s_{\hat{b}}^2}\,. \tag{7}$$

In simulation studies, the multivariate delta method standard error of the mediated effect performs better than standard errors calculated from different formulas, and the multivariate delta method is also used in many covariance structure computer programs to compute the standard error of the mediated effect (MacKinnon, 2008). Thus, it is the only standard error formula considered for this project.

The product of two normal distributions is not normal (Lomnicki, 1967; Springer & Thompson, 1966); instead it is symmetric with a kurtosis of six when the two variables have means equal to zero, and skewed with excess kurtosis when the two variables have means different from zero (Craig, 1936). This is the reason why basing the confidence interval of the mediated effect *ab* on the critical values from the normal distribution produces inaccurate estimates. Another way of constructing the confidence limits would be to substitute the critical values from the normal distribution with critical values from the distribution of the product (MacKinnon, Lockwood, Hoffman, West & Sheets, 2002). Unlike the normal distribution, the distribution of the product is not symmetric, and thus the critical values used to compute the upper and lower confidence limits often have different absolute values. The calculation of the confidence intervals for the mediated effect based on the distribution of the product has been simplified with programs called PRODCLIN and RMediation (MacKinnon, Fritz, Williams, & Lockwood, 2007; Tofighi & MacKinnon, 2011). In a simulation study of confidence limits for the mediated effect MacKinnon, Lockwood, and Williams (2004) compared normal theory confidence limits with confidence limits obtained using the distribution of the product, and they found that the distribution of the product critical values lead to fewer problems with coverage and confidence interval imbalance than the normal theory critical values. In other words, the distribution of the product confidence limits had Type I error rates closer to the nominal level as well as more balanced confidence intervals, meaning that the Type I error rate was equally distributed between the left and right sides of the distribution of estimates.

The third way of constructing confidence limits for the mediated effect is by using bootstrap methods (Manly, 1997; Shrout & Bolger, 2002; MacKinnon, Lockwood & Williams, 2004; MacKinnon, 2008). Bootstrap methods consist of rearranging the observed data in order to construct a distribution of the estimate of interest. Once the estimate of interest has been calculated from the desired number of samples, the value of the observed estimate can be compared to the resampling distribution in order to calculate a *p*-value, or a confidence interval for the estimate can be formed from the $\alpha/2$ and $(1-\alpha/2)$ points of the distribution. In the case of the mediated effect, bootstrapping would consist of sampling N observations with replacement from the original sample of size N, calculating the mediated effect, and repeating this procedure a large number of times; after a distribution of the mediated effect has been formed, one would form a 95%

confidence interval from the 2.5% and 97.5% quantiles of the distribution. This method is called the percentile bootstrap or Efron's percentile method. Assume θ is the parameter of interest, and $\hat{\theta}$ is an estimate of θ. The percentile method assumes the existence of a transformation $\hat{\phi} = t(\hat{\theta})$ that follows the distribution $\hat{\phi} \sim N(\phi, sd^2)$ for some standard deviation *sd* and that perfectly normalizes the distribution of $\hat{\theta}$; then the percentile interval of $\hat{\theta}$ equals

$$[t^{-1}(\hat{\phi} - z^{(1-\alpha/2)} sd), t^{-1}(\hat{\phi} - z^{\alpha/2} sd)]. \tag{8}$$

The percentile bootstrap is *transformation-respecting*, meaning that the percentile interval for any monotone (order-preserving) parameter transformation $\phi = t(\theta)$ is the percentile interval for θ mapped by $t(\theta)$:

$$[\hat{\phi}\%_{lower\lim it}, \hat{\phi}\%_{upper\lim it}] = [t(\hat{\theta}\%_{lower\lim it}), t(\hat{\theta}\%_{upper\lim it})]. \tag{9}$$

In other words, the transformation step of the percentile bootstrap procedure does not change the order of data points from the order in the original sample. Thus, after computing the 95% interval estimate on the transformed (normal) distribution, it suffices to transform the estimates of the 2.5% and 97.5% percentiles back into the original 'metric' in order to get the lower and upper limits of the 95% percentile bootstrap interval. The user does not need to know which function is used as the transformation since the percentile bootstrap makes this transformation automatically (Efron & Tibshirani, 1993). However, such a transformation may not always exist, and bias arises when the true value of the parameter does not correspond to the median of the distribution of estimates (Manly, 1997). Bias is handled by finding the proportion of times *p* that the bootstrapped estimates exceed the sample (observed) value of the estimate, and z_0 which is the z value that corresponds to this *p*-value. This method is called the bias-corrected percentile bootstrap. The lower confidence limit is then the estimate that just exceeds the proportion $\phi(2z_0 - z_{\alpha/2})$ of all values in the bootstrap distribution of estimates. The upper confidence limit for the estimate is the value that exceeds a proportion $\phi(2z_0 + z_{\alpha/2})$ in that same distribution (Manly, 1997). Simulation studies of the performance of various interval estimates of the mediated effect showed that overall the bias-corrected bootstrap method has the most empirical power, though, it also has excessive Type I error in some parameter combinations. On the other hand, the percentile bootstrap has the best coverage and slightly less empirical power than the bias-corrected bootstrap, while the distribution of the product confidence limits had more power and narrower confidence intervals than the percentile bootstrap (MacKinnon, Lockwood, & Williams, 2004).

Bayesian Methods and Application to Mediation

In a frequentist framework, power and Type I error rate vary together: as one increases, so does the other. With Bayesian methods, one can still obtain meaningful information from a study without having to consider Type I error rate and power (Van de Schoot, Hoijtink, Mulder, Van Aken, Orobio de Castro, Meeus et al., 2011). The primary distinction between Bayesian and frequentist philosophies lies in their respective applications of the probability concept. In the Bayesian school of thought, probability is a measure of uncertainty (Gelman, Carlin, Stern, & Rubin, 2004), and is thus subjective. For frequentists, however, probability is defined as the long-run frequency of the occurrence of an event E, and is thus a property of the external world. In order to reflect the uncertainty about parameters, the Bayesian framework places distributions around parameters. Thus, in the Bayesian framework, the prior information, the data, and the final estimate are all in distribution form. The Bayes Theorem is expressed with the following formula:

$$p(\theta \mid data) = \frac{p(\theta, data)}{p(data)} = \frac{p(\theta)\, p(data \mid \theta)}{p(data)}, \tag{10}$$

where $p(\theta \mid data)$ represents the posterior distribution, $p(\theta)$ is the prior distribution placed on unknown parameters in the model, $p(data \mid \theta)$ is the sampling distribution of the data given the parameter, and $p(data)$ is a constant with respect to the parameter of interest, and can thus be omitted in order to produce a simpler way to compute a quantity proportional to the posterior distribution:

$$p(\theta \mid data) \propto p(\theta)\, p(data \mid \theta). \tag{11}$$

The most common criticism of Bayesian methods states that the inclusion of a prior distribution in the statistical analysis introduces subjectivity that might lead the results away from reality, in the researcher's desired direction. However, Greenland (2006) points out that carefully chosen prior distributions do not introduce any assumptions that are more questionable than the assumptions made by frequentist models (and some are even less questionable than those of frequentist models, according to Greenland). Greenland rejects the idea of data being able to "speak for themselves", as frequentists would want them to. Even in a frequentist framework, the results are compared to probability distributions with the (sometimes unrealistic) assumption of no bias. Finally, Bayesian prior distributions are more transparent in the assumptions that are made than are frequentist analyses, which makes them easier to criticize. As Little (2006) pointed out, both Bayesian and frequentist methods have their strengths and weaknesses, and the choice of method should be tailored to the type of study, characteristics of data, and questions that are being answered by the data analysis.

Just like frequentist methods, Bayesian methods offer both point and interval estimates of parameters. One form of interval estimates in the Bayesian framework is called credibility intervals. As opposed to frequentist confidence intervals for which one can say that in the long run (1-α)% confidence intervals will contain the true value of the parameter being estimated, the Bayesian credibility intervals can be interpreted in terms of probability. In other words, as long as the assumptions of the model hold, a Bayesian 95% credibility interval means that there is 95% probability that the parameter value is included in the interval, as opposed to “95% confidence”.

There are two ways to perform a Bayesian mediation analysis and obtain credible intervals described in the literature on Bayesian mediation analysis (Yuan & MacKinnon, 2009; Enders, Fairchild, & MacKinnon, 2013). In this thesis, they are referred to as the method of coefficients and the method of covariances. The method of coefficients is inspired by the Yuan and MacKinnon (2009) approach, and its implementation starts with the assignment of prior distributions to the parameters in Equations 2 and 3 for the single mediator model, which are *a*, *b*, *c'*, σ_2^2 and σ_3^2 (the regression coefficients and the error variances of the mediator and dependent variable, respectively). Yuan and MacKinnon (2009) recommend normal distributions as priors for the regression coefficients *a* and *b*, and an inverse gamma distribution (IG) with parameters *m* (for shape) and *n* (for scale) as a prior distribution for the variance. Assigning small values to *m* and *n* gives the prior distribution a large variance, thus reducing its impact on the shape of the posterior distribution. Assuming priors are independent *a priori*, the prior distributions for the five parameters can be combined into a joint prior distribution expressed as

$$p(a,b,c',\sigma_2^2,\sigma_3^2) = N(\mu_a,\sigma_a^2)N(\mu_b,\sigma_b^2)N(\mu_{c'},\sigma_{c'}^2)IG(m_M,n_M)IG(m_Y,n_Y). \tag{12}$$

Using Gibbs sampling (Geman & Geman, 1984) and the above joint prior distribution with the observed data, one can obtain the posterior distributions for the parameters of interest, as well the posterior distribution of any function of those parameters. In this case, the function of interest is the product *ab.* The 95% central credibility intervals are then obtained by taking the 0.025 and 0.975 quantiles from the posterior distribution of *ab.*

The method of covariances uses the covariance matrix of the variables X, M, and Y (Enders, Fairchild & MacKinnon, 2012). A prior distribution for the covariance matrix is modeled as an inverse Wishart distribution (a multivariate generalization of the chi-squared distribution) with hyperparameters df_p (the degrees of freedom) and Λ_p (the sum of squares and cross products matrix) that define the center and spread of the distribution (respectively). The prior distribution for the covariance matrix is then written as:

$$p(\Sigma) \sim W^{-1}(df_p, \Lambda_p^{-1}). \tag{13}$$

If a covariance matrix from a prior study is available, the following conversion can be applied to it in order to obtain a prior distribution for the model:

$$\Lambda_p = (N_p - 1)\Sigma_p. \tag{14}$$

In the above formula, Σ_p is the covariance matrix and N_p is the sample size from the previous study. The df_p value determines the influence of the prior distribution on the posterior distribution. The larger the degrees of freedom value, the more weight is given to the prior distribution. Assigning a value of for example 20 to df_p would be the same as saying that the prior distribution contributes 20 data points to the analysis (Enders, Fairchild & MacKinnon, 2013). The posterior distribution of a covariance matrix is also an inverse Wishart denoted as:

$$p(\Sigma \mid data) \sim W^{-1}(df, \Lambda^{-1}) \tag{15}$$

where the degrees of freedom are a sum of the degrees of freedom from the prior distribution and the data, and the sum of squares and cross product matrix is the sum of the inverse lambda matrices of the prior distribution and the data. The regression coefficients *a* and *b* can be obtained from the covariance matrix:

$$a = \frac{s_{XM}}{s_X^2} \tag{16}$$

$$b = \frac{(s_{MY}s_X^2 - s_{XM}s_{XY})}{(s_X^2 s_M^2 - s_{XM}{}^2)} \tag{17}$$

and their product can be computed at each draw from the posterior distribution thus creating a posterior distribution for the mediated effect from which the credible interval is estimated.

The two Bayesian approaches described above are expected to produce nearly identical estimates because there is a one-to-one relationship between the linear regression model and a saturated covariance matrix for multivariate normal outcomes (Enders, Fairchild, & MacKinnon, 2013).

Properties of Interval Estimates

Confidence intervals are more informative than significance tests because they offer a range of plausible values of the estimate that is useful in judging the practical significance of results (Krantz, 1999; Stevens, 2007). Confidence intervals are also more informative than standard error bars because they provide inferential information, whereas standard error bars merely describe the data (Cumming, 2012). There are certain characteristics that make some confidence intervals more informative than others.

The width of the confidence interval inherently reflects the uncertainty of the point estimate (Ramachandran & Tsokos, 2009). It follows that a narrower interval with higher probability of enclosing the true value of the parameter of interest is more desirable than a wide interval with a lower probability of enclosing the parameter of interest. The 'arm' of a confidence interval is called a margin of error (Cumming, 2012), and it is defined as a common summary of sampling error that quantifies uncertainty about an estimate (Ramachandran & Tsokos, 2009). Margins of error decrease as sample size increases, and it is possible to calculate the sample size necessary in order to obtain a desired value of the margin of error (Cumming, 2012). Such a calculation is useful in increasing the precision of the estimate by making the confidence interval narrower, however, this procedure is not implemented in this project.

The probability that the confidence interval contains the parameter of interest is called the confidence coefficient, and it quantifies the fraction of time the constructed interval contains the true parameter, under repeated sampling. As already stated, short arms and large confidence coefficients are two desirable characteristics of a confidence interval. A common choice of confidence coefficient is 95%. The length of the margin of error depends on the standard error of the estimate, and the distribution from which the critical value for the computation was obtained. Normal theory confidence intervals for the mediated effect are symmetric, meaning that the distance between the estimate and the lower limit of the interval is the same as the distance between the estimate and the upper limit. This is not necessarily the case with confidence intervals formed using the distribution of the product, resampling methods, and Bayesian estimation. Thus, when comparing normal theory interval estimates to interval estimates obtained from other methods, one should not define precision as the length of a single arm of the confidence interval. The width of the entire interval is a more accurate way of evaluating the precision of an interval estimate. A narrower interval estimate is more informative than a wider one.

There are other criteria to be considered when choosing a method of interval estimation. An interval method that contains the parameter with the exact frequency the confidence coefficient indicates it would (nominal coverage) is more desirable than one that doesn't. Thus, interval estimates with empirical coverage equal to nominal coverage are more accurate.

Another important consideration that relates to coverage is target miscoverage, also called imbalance. Ideally, an interval with the confidence coefficient of 95% will have a lower limit higher than the parameter value 2.5% of the time, and an upper limit lower than the parameter value 2.5% under repeated sampling (Efron & Tibshirani, 1993). Two interval estimates can have the same level of coverage but different levels of imbalance. An interval that has balanced miscoverage is more desirable than an interval that does not.

The significance testing aspects of interval estimation are also an important consideration. An interval with an empirical Type I error rate equal to nominal levels of Type I error rate is more desirable than an interval with empirical levels of Type I error rate different than nominal. Empirical Type I error rate higher than nominal Type I error rate can be especially problematic.

Another consideration regarding the use of interval estimates for significance testing is empirical power. If an interval estimate for the mediated effect contains the value of zero, then the mediated effect is not statistically significant. However, if the mediated effect in the population is different from zero, this would be a Type II error. Power is defined as $(1-\beta)$ with β being the Type II error rate. A conventionally chosen level of power to aim for is 0.80. Empirical power refers to the proportion of times the interval does not contain the value of zero, over repeated sampling, when a true effect exists in the population. Simulation studies are often used to find out the empirical power of different methods. An interval estimation method with higher empirical power is more desirable than an interval estimation method with lower empirical power.

Findings from previous studies

Coverage and Type I error rate. In a simulation study with normally distributed data and no prior information, the distribution of the product and normal theory estimates of the mediated effect had Type I error rates lower than the nominal value, meaning their coverage was above the nominal 95% level (MacKinnon, Lockwood & Williams, 2004;

MacKinnon, Lockwood, et al., 2002). The bias-corrected bootstrap had average coverage closest to the nominal level with N≤200, while all other methods had Type I error rates lower than the nominal level of 0.05 for these values of sample size. However, for some combinations the bias-corrected bootstrap had empirical Type I error rates above the nominal level meaning that its empirical coverage was lower than the nominal 0.95 level, but on average this method performed well. The value of coverage for all methods approached the nominal level as sample size increased from 25 to 200 (MacKinnon, Lockwood & Williams, 2004). Biesanz, Falk, and Savalei (2010) compared coverage of different methods for the mediated effect in four different situations in a simulation: complete normally distributed data, MCAR normally distributed data, complete non-normally distributed data, and MCAR non-normally distributed data. None of the four combinations incorporated prior information, and for the purposes of this project, only the findings with complete normally distributed data are discussed. Out of the methods compared in the above simulation that are also tested in this study, the distribution of the product and percentile bootstrap confidence limits maintained alpha levels close to the nominal value, meaning that the coverage was also close to 0.95, while the accelerated bias-corrected bootstrap (the bias-corrected bootstrap with an added acceleration constant, denoted BCa in the article) had inflated Type I error rates, and consequently lower than nominal coverage for zero effects, for N<200. All methods had lower than nominal Type I error rates (and consequently excessively high coverage rates) for N<200, especially for small effect sizes. Coverage improved with increases in effect sizes, and overall the distribution of the product and normal theory confidence limits using the Sobel (1982) standard error had better coverage than the accelerated bias-corrected bootstrap. Preacher and Selig (2012) replicated MacKinnon, Lockwood, and Williams (2004) and found that for complete normally distributed data with no prior information, percentile bootstrap and distribution of the product confidence limits had comparable levels of coverage that were better than coverage for the normal theory confidence limits using the Sobel (1982) standard error. From the findings of the four studies it can be expected that in the case with no prior information, all methods except the bias-corrected bootstrap that were tested in previous studies (meaning all except Bayesian credibility intervals) will have close to nominal coverage at N = 500, and excessive coverage as N is reduced. None of the four studies above tested the performance of the credibility intervals obtained using the method in Yuan and MacKinnon (2009) nor the method proposed by Enders, Fairchild, and MacKinnon (2012). Therefore, no predictions can be made about the coverage of Bayesian credibility intervals obtained from these two methods relative to others with or without prior information available.

Imbalance. In Study 1 by MacKinnon, Lockwood, and Williams (2004) the liberal robustness criterion proposed by Bradley (1978) was used for evaluating imbalance. More specifically, instead of calling any interval estimate that does not produce 2.5% of Type I errors of each side imbalanced, the range of permitted deviation from 2.5% was expanded to $(.5\alpha/2, 1.5\alpha/2) = (0.0125, 0.0375)$. The results of this study indicated that the bias-corrected bootstrap was the only method to have imbalance that satisfied Bradley's liberal robustness criterion. The comparison of the remaining method showed that the distribution of the product confidence limits had less imbalance than normal theory confidence limits. Preacher and Selig (2012) computed imbalance as the ratio of times the true value fell above the upper confidence limit over the number of times the true value of the parameter was lower than the lower confidence limit; the further this ratio was from one, the more imbalanced the confidence interval. They found that the normal theory confidence limits using the Sobel (1982) standard error had the highest ratio of misses (meaning imbalance), and that the distribution of the product confidence limits had slightly more imbalance than the percentile and bias-corrected bootstrap confidence limits. For the case with no prior information, it can be expected that the normal theory confidence limits will have the most imbalance, followed by the distribution of the product confidence limits, and that the bias-corrected and percentile bootstrap will produce the least imbalanced confidence intervals. None of the studies comparing methods examined Bayesian credibility intervals, nor did any of them include prior information, thus no predictions can be made about the performance of this method compared to others.

Empirical Power. MacKinnon, Lockwood, and Williams (2004) found that for sample sizes between 25 and 200 the bias-corrected bootstrap had slightly more empirical power than the distribution of product and percentile bootstrap methods. A previous study also found that the distribution of product method has power above 0.80 to detect small effects at N=500 (MacKinnon, Lockwood, et al., 2002). Consistently with their findings about Type I error rates (power and Type I error rate have a positive relationship), Biesanz, Falk, and Savalei (2010) found that the normal theory confidence limits had fairly low empirical power whereas the accelerated bias-corrected bootstrap had the highest value of empirical power followed by the distribution of the product. All methods had low empirical power for sample sizes smaller than or equal to 200 and small effect sizes. Also, the empirical power was lower for the parameter combination where *a*=0.59 and *b*=0.14 regardless of sample size and method (Fritz, Taylor & MacKinnon, 2012). An increase in effect sizes leads to an increase in power, however, in addition to this, the authors concluded that the value of *b* was more related to increases in power than was the value of *a*. This means that power for *a*=0.59 and *b*=0.14 was lower than for *a*=0.14 and *b*=0.59 even though the value of the mediated effect was the same for the two combinations of parameter values. At N = 500 all methods had power of above 0.80 for most parameter combinations except for the combination with *a*=0.59 and *b*=0.14. It is expected that the bias-corrected bootstrap will have the highest empirical power, followed closely by the distribution of the product, and the percentile bootstrap. Normal theory confidence limits are expected to have very low empirical power. Yuan and MacKinnon (2009) found that an informative prior can increase power. Since the Enders, Fairchild, and MacKinnon (2012) approach is closely related to the Yuan and MacKinnon (2009) approach with complete data, the two are expected to perform similarly to each other with and without prior information, and better than the normal theory, distribution of the product, percentile bootstrap, and bias-corrected bootstrap confidence limits when prior information is used.

Interval Width. Preacher and Selig (2012) found that the normal theory confidence limits had the narrowest confidence intervals followed by the distribution of the product confidence intervals. It is expected that these results will be replicated when there is no prior information available. Yuan and MacKinnon (2009) found that an informative prior can reduce interval width. Given the similarity of this approach to the one proposed by Enders, Fairchild, and MacKinnon (2012), the two methods are expected to perform in a similar way in terms of interval width.

Improvements in interval estimation with the inclusion of prior information

Frequentist statistical analyses are often performed on one sample at a time without taking into consideration the findings from previous studies. Even though frequentists use prior knowledge in calculations of *a priori* power and the selection of sample size for studies, the knowledge about the relationships being studied is not included in the statistical analysis. Bayesian methods incorporate prior knowledge in the statistical analysis in the form of a prior distribution. Little (2006) advocated the use of frequentist methods in model development and assessment, and Bayesian methods in statistical inference under the assumed model. Given the available statistical tools, the statistical approach to examining a phenomenon for the first time could differ from the statistical approach to studying a somewhat familiar relationship. However, most analyses are performed as if there was no prior information available. Recently, researchers have suggested that augmenting data with existing prior information produces more accurate estimates than the data set from a given experiment alone (Leeuw & Klugkist, 2012). The goal of this project is to examine the benefits of the inclusion of prior knowledge into the statistical analysis by evaluating several interval estimates for the mediated effect and the smallest sample size necessary to produce accurate estimates of the mediated effect. The project consists of three studies.

The goal of the first study is to examine which interval estimate has coverage closest to the nominal level of 0.95, Type I error rate closest to the nominal 0.05 level, the lowest imbalance, smallest width, and highest empirical power. Normal theory confidence limits using the multivariate delta standard error, percentile bootstrap confidence limits, bias-corrected bootstrap confidence limits, and the Bayesian credibility intervals are evaluated on the following criteria:

I. The closer coverage is to the nominal 0.95 level, the better. Values in the range of Bradley's robustness criterion (1978), between 0.925 and 0.975, are considered adequate. Coverage above 0.975 is less problematic than coverage below 0.925.

II. The closer the empirical Type I error is to the nominal Type I error, the better. Values in the range of Bradley's robustness criterion (1978), between 0.025 and 0.075, are considered adequate. Type I error rate above 0.075 is considered more problematic than Type I error rate below 0.025.

III. Imbalance is defined as the difference between the proportion of true values that fall to the right versus to the left of the interval, and the closer imbalance is to zero, the better.

IV. Empirical power is defined as the number of intervals out of the number of replications that do not contain the value of zero when a true effect exists in the simulated population. In most studies empirical power of at least 0.80 is considered adequate, however, power will be evaluated as a continuous variable in this study.

V. The smaller the interval width, the better.

The goal of the second study was to examine how informative (narrow) the prior distributions have to be in order for the Bayesian methods to outperform frequentist methods in small samples. The two Bayesian methods were tested with two different amounts of prior information, and compared to normal theory, distribution of the product, percentile bootstrap, and bias-corrected bootstrap confidence intervals in order to answer this question.

The third study examined how power changes with the precision (inverse variance) of the prior distributions for regression coefficients *a* and *b* and evaluated the extent to which more precision can increase power.

Hypotheses

I. Bayesian methods with informative prior distributions will have coverage closer to the nominal level of 0.95, Type I error rates closer to the nominal level of 0.05, imbalance closer to 0, higher empirical power, and lower interval width than Bayesian methods with diffuse prior distributions, normal theory confidence limits, distribution of the product confidence limits, percentile bootstrap confidence limits, and bias-corrected bootstrap confidence limits.

II. The power increase for the Bayesian method of coefficients differ depending on whether the precision parameter was increased for the prior distribution of the *a* path versus the prior distribution of the *b* path.

CHAPTER 2

GENERAL METHOD

A Monte Carlo simulation was used to compare interval estimates for the mediated effect on five criteria. Appendix A contains explanations for document notation used in the subsequent studies. The data were simulated based on Equations 2 and 3. In the simulation the residual variances of M and Y were set to 1, and X was simulated as a random variable. There are four values of effect sizes for regression coefficients (0, 0.14, 0.39, 0.59) that correspond approximately to zero, small (2% of the variance), medium (13% of variance), and large (26% of the variance) effect sizes as described by Cohen (1988). SAS syntax (Version 9.3 of the SAS System for Windows) was written to simulate every possible combination of these values parameter *a*, *b*, and *c'*.

It has been shown that tests of mediation, except the causal steps test (which is not studied in this project), are unaffected by the values of the parameter *c'* (Fritz & MacKinnon, 2007; MacKinnon, Lockwood, et al., 2002). However, the relationship between the value of *c'* and interval width has not been documented, thus all four effect sizes of this coefficient are included in the simulation for Study 1. Furthermore, Study 1 examined all 64 possible combinations of parameter values of *a*, *b*, and *c'*. Due to computation time required for all combinations of parameter values, studies 2 and 3 examined smaller subsets of the 64 combinations of parameter values.

CHAPTER 3

STUDY 1

Methods

Random samples of size N = 20, 40, 60, 100, and 500 were drawn from each population and 95% confidence limits/credibility intervals for the mediated effect were calculated using (1) normal theory, (2) the distribution of the product method, (3) Bayesian method with diffuse normal prior distributions for regression coefficients centered at $\mu_a = \mu_b = \mu_{c'} = 0$ with variance equal to 10^6, and σ^2_M and σ^2_Y modeled as inverse-gamma distributions with shape and inverse-scale parameters equal to .01 (so that the expectation of this distribution is 1), (4) Bayesian method with informative normal prior distributions for regression coefficients centered at the true value of the regression coefficients with variance equal to 10^6, and σ^2_M and σ^2_Y modeled as inverse-gamma distributions with shape and inverse-scale parameters equal to .01 (so that the expectation of this distribution is 1), (5) Bayesian method with a multivariate normal distribution for the means of variables X, M, and Y (the means in the prior distribution were set to 0, the variance of the variables to 1000, and the covariances to 0) and a diffuse inverse Wishart prior distributions for the covariation matrix of variables X, M, and Y constructed so that the expected covariance between variables is zero and the expected variance of each variable was 1 and the degrees of freedom parameter was equal to the observed sample size, (6) Bayesian method with the same prior distribution for the matrix of means as in method (5) and an informative inverse Wishart prior distributions for the covariation matrix of variables X, M, and Y constructed so that the expected variances and covariances equal the true variances and covariances and the degrees of freedom parameter was equal to the observed sample size, (7) percentile bootstrap, and (8) bias-corrected bootstrap. The highest sample size tested in this project was N = 500 because this is the smallest sample size required for bias-corrected bootstrap to have adequate Type I error rate and empirical power (Fritz, Taylor & MacKinnon, 2012).

For each combination of effect size and sample size 1000 replications were generated. Average coverage, Type I error rate for cases where either *a* or *b* equal zero, confidence interval imbalance (the number of times the estimate is to the right of the upper confidence/credibility limit (UCL) minus the number of times the estimate is to the left of the lower confidence/ credibility limit (LCL)), empirical power for cases where $a>0$ and $b>0$, and interval width were obtained for all eight interval methods in the study.

Type I error rates were evaluated according to Bradley's robustness criterion (1978). The Type I error rate of a statistical method is considered adequate if it falls within 0.025 (one half of the nominal level of Type I error rate) of the nominal 0.05 level. Values of Type I error rate above 0.075 are considered excessive. Values of Type I error rates below 0.025 are highlighted in the results below, but are not a reason to avoid using the method as long as the Type I error rate is less than 0.05. According to Bradley's robustness criterion (1978), the coverage of a statistical method is considered adequate if it falls within 0.025 (one half of the nominal level of Type I error rate) of the nominal 0.95 level. Values of coverage below 0.925 are considered low and problematic. Values of coverage above 0.975 are highlighted in the results below, but are not a reason to avoid using the method with these coverage rates.

ANOVAs were conducted on each of the five outcomes in Study 1 with *a*, *b*, and N as between factors, method as a within factor, and *c'* as a covariate in order to guide the creation of the plots and the summary of the findings. In all of the ANOVAs *c'* was always a covariate, and interactions between *a*, *b*, N, and method were estimated when possible. For Type I error rate it was not possible to estimate all interactions. The results of the analyses are presented below, along with explanations of the findings.

Results

After the simulations for Study 1 were complete, errors were caught in the code for the following methods: (2) distribution of the product, (5) Bayesian method with a diffuse prior distribution for the covariance matrix, and (6) Bayesian method with an informative prior distribution for the covariance matrix, rendering the findings for these methods unusable. Also, there was an error in the calculation for the imbalance of the bias-corrected bootstrap. Thus, Study 1 results consist of Type I error rate, power, coverage, and interval width for normal theory confidence limits, Bayesian method of coefficients credibility intervals with diffuse priors, Bayesian method of coefficients credibility intervals with informative priors, percentile bootstrap, and bias corrected bootstrap. Imbalance is available for normal theory confidence limits, Bayesian method of coefficients credibility intervals with diffuse priors, Bayesian method of coefficients credibility intervals with informative priors, and percentile bootstrap. Results for all combinations of parameters *a*, *b*, and *c'* from Study 1 are summarized in Tables 1-5. Results for all possible combinations of parameter values and sample size in Study 1 can be found in Appendix C, in Tables 18-42. Coefficients *a* and *b* are referred to as the mediation paths in the subsequent descriptions of the findings.

Significance Testing: Type I error rate and Power

Type I error rate

Table 1 contains average Type I error rates for all the methods and sample sizes in study 1 across all 64 combinations of parameter values for *a, b,* and *c'*. Normal theory and percentile bootstrap confidence limits had average Type I error rates lower than the nominal 0.05 level at all sample sizes in the study. The average Type I error rates for the Bayesian method of coefficients were either below or within the limits of the robustness criterion at all sample sizes in the study. The average Type I error rates for the bias-corrected bootstrap were the highest out of all methods at each sample size, became higher as sample size increased, but still remained within the limits of the robustness criterion on average.

Insert Table 1 about here

The results of the ANOVA for Type I error rate indicate that the method factor had statistically significant interactions with *a*, sample size, *a* and sample size, *b*, and *b* and sample size. Following up these significant interactions in a detailed examination of the tables revealed that at N = 20 all methods had Type I error rates below 0.025 when both mediation paths are zero (Table 18). The larger the non-zero mediation path, the higher the Type I error rates for all methods. At N = 20, normal theory confidence limits had Type I error rates consistently lower than 0.025. The Bayesian method of coefficients with diffuse and informative prior distributions had Type I error rates in the range of the robustness criterion (0.025-0.075) only when the non-zero path was large (when *c'*=0, this is true when *a*=0.59 and *b*=0, but not when *b*=0.59 and *a*=0). In all other situations Type I error rates for the Bayesian method of coefficients were below 0.025. At N = 20, the percentile bootstrap had values of Type I error rates in the robustness criterion only when the non-zero path is at least medium, and did not have Type I error rates above 0.075. Type I error rates for the bias-corrected bootstrap were higher than 0.075 whenever *c'*>0 and the non-zero mediation path was 0.59; otherwise they were within the bounds of the robustness criterion, or lower. The bias-corrected bootstrap had the highest Type I error rates of all methods at each combination of effect sizes, and was the only method to have parameter combinations with Type I error rates above 0.075 at N = 20. At N = 40 normal theory confidence limits still had Type I error rates below 0.025 except in three cases when the non-zero mediation path is 0.59 and the Type I error rates were between 0.025 and 0.075 (Table 19). The Bayesian method of coefficients with diffuse and informative prior distributions had Type I error rates in the range of the robustness criterion (0.025-0.075) only when the non-zero path was large; in all other situations Type I error rates for the Bayesian method of coefficients were below 0.025. As with N = 20, at N = 40 the percentile bootstrap had a Type I error rates between 0.025 and 0.075 only if the non-zero mediation path is at least 0.39, otherwise Type I error rates were below 0.025. At N = 40 the bias-corrected bootstrap had Type I error rates between 0.025 and 0.075 when the non-zero mediation path is medium, and was the only method to have Type I error rates higher than 0.075 for large non-zero mediation paths.

At N = 60 normal theory confidence limits still had Type I error rates below 0.025 for all parameter combinations except for a few instances where the non-zero mediation path was large (Table 20). At N = 60 the Bayesian method of coefficients with diffuse and informative prior distributions had Type I error rates in the range of the robustness criterion in the majority of combinations where the zero mediation path is at least 0.39; otherwise, the Type I error rates for the Bayesian method of coefficients with diffuse and informative prior distributions were lower than 0.025. The percentile bootstrap had values of Type I rate in the range of the robustness criterion whenever the non-zero mediation path was at least 0.39, and the only instance of Type I error rate above 0.075 occurred when *c'*=0, *a*=0.59, and *b*=0. At N = 60 the bias-corrected bootstrap had Type I error rates below 0.025 when both mediation paths were 0 or the non-zero mediation path was small, and Type I error rates in the upper range or above the robustness criterion when the non-zero mediation path was medium or large; the bias-corrected bootstrap had Type I error rates in the range of the robustness criterion in only five out of the twenty eight combinations of parameter values.

At N = 100 normal theory confidence limits had Type I error rate below 0.025 whenever both mediation paths were 0, or when the non-zero mediation path was small (Table 21). Type I error rates for normal theory confidence limits were always in the range of the robustness criterion when the non-zero mediation path was large, and in half of the instances when *a*=0.39, but not when *b*=0.39. At N = 100 the Bayesian method of coefficients with diffuse and informative prior distributions always had Type I error rates in the range of the robustness criterion when *a*=0 and *b*=0.39, 0.59, and in half of the cases when *a*=0.14 and *b*=0. Excessive Type I error rates (above 0.075) occurred whenever *b*=0, and *a*=0.39 and 0.59. Thus, at N=100 a larger *a* coefficient signified higher Type I error rates for the Bayesian method of coefficients with diffuse and informative prior distributions. The Type I error rate for the percentile bootstrap was never above 0.075, and was in the range of the robustness criterion whenever the non-zero mediation path was at least 0.39. In all other situations Type I error rates for the percentile bootstrap at N = 100 were below 0.025. The bias-corrected bootstrap had Type I error rates in the range of the robustness criterion for most combinations of *a*, *b*, and *c'*; values of Type I error rate below 0.025 occurred whenever both mediation paths were 0, and once for *a*=0.14 and *b*=*c'*=0. Excessive Type I error rates for the bias-corrected bootstrap occurred only in situations when either *a* or *b* was at least 0.39, however, this was not as consistent of an occurrence as it was for the Bayesian method of coefficients at N = 100.

At N = 500 normal theory confidence limits had Type I error rates below 0.025 when both mediation paths were zero or when the non-zero path was small, and values of Type I error rate in the range of the robustness criterion whenever the non-zero mediation path was at least 0.39. At N = 500 normal theory confidence limits did not have Type I error rates above 0.075 for any of the parameter combinations (Table 22). The Bayesian method of coefficients with diffuse and informative prior distributions and the percentile bootstrap had Type I error rates below 0.025 whenever both *a* and *b* were zero, and values of Type I error rate in the range of the robustness criterion whenever at least one of the mediation paths was larger than zero. Neither the Bayesian method of coefficients nor the percentile bootstrap had Type I error rates above 0.075 for any of the parameter combinations. The bias-corrected bootstrap had Type I error rates in the range of

Bradley's robustness criterion for all combinations of effect sizes, except *a*=*b*=0 when Type I error rates were below 0.025, and for some instances in which the non-zero mediation path is 0.14 and Type I error rates were above 0.075.

Overall, Type I error rates increased with sample size for normal theory confidence limits, and Bayesian credibility limits. Percentile bootstrap was the method that had Type I error rates in the range of Bradley's robustness criterion the most often across all sample sizes, and at N = 500 the Bayesian methods were performing identically to the percentile bootstrap in terms of Type I error rate. Normal theory confidence limits were the only method never to have Type I error rates above 0.075, followed by the percentile bootstrap that had only one instance of excessive Type I error rate across all parameter combinations and sample size. The Bayesian method of coefficients with diffuse and informative prior distributions came next in terms of the number of instances of excessive Type I error rate across all parameter combinations and sample size, and the bias-corrected bootstrap had the highest number of Type I error rates above 0.075 out of all methods tested in this study.

The Type I error rate for the Bayesian method with informative prior distributions did not differ much from the Type I error rate of the Bayesian method with diffuse prior distributions suggesting either that prior information cannot make Type I error rate closer to the nominal level, or that the informative prior distributions used in this study were still too diffuse to induce such a change in the interval estimate.

Power

Table 2 contains average values of power for five methods in Study 1. All methods had average power greater than 0.8 at N = 500, and the bias-corrected bootstrap had the highest average power for each sample size (but also had the highest value of Type I error rate as described above). Bayesian methods with diffuse and informative prior distributions for the regression coefficients consistently outperformed normal theory confidence limits, but had lower average power than the two bootstrap methods. This finding suggests that either prior information cannot increase power or that there was not enough prior information in the informative prior distributions to produce an increase in power in this study.

Insert Table 2 about here

The full-factorial ANOVA for power revealed that all interactions were statistically significant, including the four-way interaction between the factors method, *a*, *b*, and sample size. A careful examination of the tables of results revealed that at N = 20 and 40 for all combinations of parameter values the bias-corrected bootstrap had the highest value of power, followed by the percentile bootstrap (Tables 23 and 24). The Bayesian method of coefficients had almost identical power for diffuse or informative prior distributions, and had slightly lower power than the percentile bootstrap (0.008 versus 0.013 at the parameter combination with the least power for all methods and $a=b=0.14$, $c'=0$, and 0.331 versus 0.399 at the parameter combination with the most power for all methods and $a=b=c'=0.59$ at N = 20). Normal theory confidence limits had the least power out of all methods for all combinations of parameter values at N = 20, and 40. At N = 60 and at larger values of mediation paths, especially at larger values of *a*, normal theory confidence limits had power closer to all other methods except the bias-corrected bootstrap (Table 25). For N < 60 the percentile bootstrap had higher power than the Bayesian method of coefficients, but at N = 60 this was not the case for all parameter combinations. At N = 60, *c'*=0, 0.14, and *b*=0.14 the Bayesian method of coefficients had more power than the percentile bootstrap. At *c'*=0.39, 0.59 and *a*=0.59 the Bayesian method of coefficients and the percentile bootstrap had identical power. At N = 60 and for all other parameter combinations, the percentile bootstrap had slightly more power than the Bayesian method of coefficients with diffuse and informative prior distributions. At N = 100 the bias-corrected bootstrap was still the method that had the highest power for the majority of combinations of parameter values, however, some interesting patterns in the results also emerged (Table 26). The Bayesian method of coefficients and the percentile bootstrap would alternate as the method with the second-highest power after the bias-corrected bootstrap, and when *c'*>0, *a*=0.14, and *b*=0.59 the Bayesian method of coefficients had power identical to or higher than the bias-corrected bootstrap, and higher power than the percentile bootstrap. There were also parameter combinations when normal theory confidence limits had more power than the Bayesian method of coefficients; this occurred whenever *a*=0.59 and *b*=0.14. In these instances the bias-corrected bootstrap and the percentile bootstrap still had more power than normal theory confidence limits. At N = 100 and *a*=*b*=0.59 all methods had power equal to 1. At N = 500 all methods had power equal to 1 when one or both mediation paths were medium or large (Table 27). The bias-corrected bootstrap was still the method with the most power in the majority of parameter combinations, however, the power values for all the methods became more similar as effect size increased, and when one or both mediation paths were medium or large all methods had power of 1. Furthermore, for all parameter combinations except when a=b=0.14 all methods had power values in the range of 0.8-0.9, and when a=b=0.14 the bias-corrected bootstrap had the most power (above 0.8), followed by the percentile bootstrap (0.738-0.758), the Bayesian method of coefficients with diffuse and informative priors (0.721-0.746), and normal theory confidence limits (0.563-0.596).

When Type I error rates and Power are taken into account, the percentile bootstrap and the Bayesian method of coefficients with diffuse and informative prior distributions are the optimal choices because they offer high power relative to normal theory confidence limits without excessive Type I error rates of the bias-corrected bootstrap.

Interval Estimation: Coverage, Interval Width, and Imbalance

Coverage

Table 3 contains average values of coverage for five methods in Study 1. All methods had average coverage within .02 of the nominal level of 0.95, regardless of sample size. The coverage for the Bayesian method with informative prior distributions (third method in the table) is not closer to the nominal level than the coverage for the Bayesian method with diffuse prior distributions, suggesting that either coverage cannot be approximated to the nominal level by adding prior information or that the informative prior distributions used were still too diffuse.

Insert Table 3 about here

The full-factorial ANOVA for coverage revealed that all interactions were statistically significant, including the four-way interaction between the factors method, size of *a*, size of *b*, and sample size. A careful examination of the tables of results revealed that at N = 20 and 40 (Tables 28 and 29, respectively) normal theory confidence limits have coverage above the robustness criterion (0.925-0.975) when at least one of the mediation paths is zero. Coverage for normal theory confidence limits is within the range of the robustness criterion when at least one of the mediation paths is small and the other mediation path is not zero. Coverage equals or is below 0.925 in certain combinations of medium and large mediation paths. In other words, at N = 20 and 40 normal theory confidence limits occasionally had lower coverage for larger effects than for smaller and zero effects, and this phenomenon occurs less at N = 40 than at N = 20.

At N = 60 normal theory confidence limits had fewer instances of coverage below .925 than at N = 40, and overall had coverage either within the range of the robustness criterion or higher. As with smaller sample sizes, at N = 60 normal theory confidence limits had higher coverage for zero and smaller effects (Table 30).

The Bayesian method of coefficients with diffuse and informative prior distributions have identical levels of coverage at N = 20, 40, and 60, and never have coverage below 0.925 at N = 20 and 60. At N = 40 the parameter combination *c'*=0.39 and *a*=*b*=0.59 is the only case where the Bayesian method of coefficients with diffuse and informative prior distributions has coverage slightly below the robustness criterion. Levels of coverage for the Bayesian method are above 0.975 for zero and smaller effects, and fall within the robustness criterion when at least one of the mediation paths is equal to or greater than 0.39 and the other path is non-zero.

At N = 20 the percentile bootstrap has coverage in the range of the robustness criterion for the majority of combinations of values of *a* and *b*, coverage equal to or larger than 0.975 when both mediation paths are zero and when one mediation path is zero and the other is small. The only instances of the percentile bootstrap having coverage below 0.925 at N = 20 occur at *c'*=0.59, *a*=0.59, and non-zero values of *b*. At N = 40 the percentile bootstrap has coverage within the range of the robustness criterion or larger for all parameter combinations except when *a*=0.59 and *b* is larger than zero, which is when coverage is below 0.925. At N = 60 the percentile bootstrap has coverage above the upper bound of the robustness criterion when one or both paths are 0 or 0.14, and coverage within the range of the robustness criterion in all other situations except for the parameter combination *c'*=0, *a*=0.59, *b*=0 when coverage is 0.924, just below the lower bound of the robustness criterion.

At N = 20 the bias-corrected bootstrap had coverage below 0.925 for certain combinations of effect sizes where at least one path is medium or large. At N = 40 the bias-corrected bootstrap had coverage within the range of the robustness criterion or larger for all combination of parameter values except in certain combinations where one mediation path was larger than the other one (*a*=0.59 versus *b*=0, and even *b*=0.39 versus *a*=0.14). At N = 60 the bias-corrected bootstrap has the most instances of coverage below 0.925 out of all methods, and the lowest coverage for this method equals 0.904 and occurs when *c'*=0.59, *a*=0.14, and *b*=0.39. As with N = 40, at N = 60 the bias-corrected bootstrap seems to have coverage below 0.925 only in situations where one of the mediation paths is much smaller than the other one (0.59 versus 0.14), and also when *c'*=0.59 and one mediation path equals 0.39 while the other is 0.14. Overall, at N = 20, 40, and 60 for all methods, coverage would occasionally become lower with increases in one mediation coefficient when holding the other mediation coefficient constant, thus highlighting that the relation between effect size and coverage is not linear. There was no clear pattern when this occurred.

At N = 100 normal theory confidence limits have coverage above 0.975 for mediated effects equal to zero, and coverage within the range of the robustness criterion for all other combinations of parameter values, except when *a*=*b*=0.14, and *a*=*b*=*c'*=0.39 when coverage is below 0.925, but still equal to or above 0.90 (Table 31). At N = 100 the Bayesian method of coefficients still had almost identical levels of coverage with diffuse and informative prior distributions. Coverage at N = 100 was within the limits of the robustness criterion or higher for all combinations of parameter values except *a*=0.39 and *b*=0, 0.14, *a*=0.59 and *b*=0, 0.14, and at *c'*=0 and 0.14 when *a*=0.59 and *b*=0.39. Thus, coverage for the Bayesian method of coefficients at N = 100 was at least 0.925 whenever the *a* path was not larger than the *b* path. At N = 100 the percentile bootstrap has coverage within the bounds of the robustness criterion for almost all parameter combinations; coverage is above 0.975 when *a*=*b*=0 and for all parameter combinations where one mediation path is zero and the other equals 0.14. At N = 100 the bias-corrected bootstrap had coverage values within the bounds of the robustness criterion for the majority of parameter combinations. Coverage for the bias-corrected bootstrap at N =100 is consistently below 0.925 when *a*=*b*=0.14, and for a few other combinations of parameter values *a*, *b*, and *c'*, but never below 0.90.

At N = 500 all methods have coverage in the bounds of the robustness criterion for almost all parameter combinations. When both mediation paths are zero at N = 500, coverage is above 0.975 for all methods, and coverage is above 0.975 for normal theory confidence limits even when one of the mediation paths is 0.14 and the other is zero (Table 32). The bias-corrected bootstrap has coverage above 0.90 but below 0.925 when *a*=0 and *b*=0.14 regardless of value of *c'*, and when *a*=0.14, *b*=0, and *c'*=0.

In summary, coverage was higher for zero effects than for non-zero effects at all sample sizes and for all methods. Generally, coverage was close to 0.95 for all methods.

Interval Width

Table 4 contains average values of interval width for five methods in Study 1. Normal theory confidence limits had consistently lower interval width than the remaining four methods at each sample size, however, as sample size increased, the interval width differed less between the five methods.

Insert Table 4 about here

The full-factorial ANOVA for interval width revealed that the four-way interaction between the factors method, size of *a*, size of *b*, and sample size was statistically significant. A careful examination of the tables of results revealed that at N = 20, 40 normal theory confidence limits have the lowest interval width of all methods at all combinations of parameter values, followed by the percentile bootstrap (Tables 33 and 34, respectively). The values of interval width were often similar for the remaining three methods, and depending on the combination of parameter values, either the Bayesian method of coefficients with diffuse and informative priors (the two were almost identical) or the bias-corrected bootstrap had the highest interval width. At N = 60, 100 and 500 the trend of the bias-corrected bootstrap and Bayesian method of coefficients having the highest interval width, followed by the percentile bootstrap, and normal theory confidence limits continues, however, the differences in interval width between methods are less noticeable, especially when *a* and *b* are larger (Tables 35-37). Interval width got smaller for all methods as sample size increased, and values of interval width varied less between methods at larger sample sizes. Even though normal theory confidence limits had the narrowest interval width, this method has low power relative to others, so other methods have better performance.

Imbalance

Table 5 contains average values of imbalance for four methods in Study 1. All methods had positive imbalance that was within .03 of the ideal level of 0, regardless of sample size. The fact that imbalance was greater than zero indicates that there was more miscoverage to the right than to the left of the interval for all four methods in the table.

Insert Table 5 about here

Imbalance was evaluated for normal theory confidence limits, the Bayesian method of coefficients with diffuse and informative prior distributions, and the percentile bootstrap. The full-factorial ANOVA for imbalance did not converge, thus the reported results are based solely on a careful examination of Tables 38-42 that contain the values of imbalance for all parameter combinations in Study 1. Imbalance was fairly close to zero for zero effects, and increased in absolute value for all methods. A comparison between methods revealed that normal theory confidence limits had the highest absolute value of imbalance for the highest number of parameter combinations, followed by the percentile bootstrap. The Bayesian method of coefficients had almost identical values of imbalance for diffuse and informative prior distributions, and had the lowest absolute value of imbalance out of all methods for most parameter combinations. The absolute value of imbalance for all methods never exceeded 0.10, and became lower as sample size increased.

Discussion

The Bayesian method of coefficients and the percentile bootstrap were optimal methods given their performance on measures related to both significance testing and interval estimation. A notable finding from Study 1 is that when the variance of the prior distributions for the regression coefficients was specified to be 10^6 (the variances of regression coefficients calculated from the parameters in this simulation are usually smaller than 1; thus, the prior distributions for regression coefficients are extremely diffuse) the Bayesian credibility intervals did not show much change in simulation outcomes, such as an increase in power, a Type I error rate closer to 0.05, reduced interval width, imbalance closer to 0, and coverage closer to the nominal level of 0.95 compared to the remaining methods in the study. Due to the large variance of prior distributions, the difference in results from analyses with non-informative and informative priors was not substantial. An examination of plots of normal prior distributions with different means (0, 0.14, 0.39, and 0.59) and the variance parameters equal to 10^6 shows that the distributions are almost identical.

Thus, the Study 1 findings for N = 20, 40, 60, 100, and 500 lead to the question of how narrow does the prior distribution have to be in order to produce an increase in power, a Type I error rate closer to 0.05, reduced interval width, imbalance closer to 0, and coverage closer to the nominal level of 0.95 for the Bayesian methods compared to the remaining methods in the study. Given that for N = 500 all methods had adequate power, this sample size was not included in Study 2, however, the condition N = 200 was added instead.

CHAPTER 4

STUDY 2

Due to the duration of the simulation study for all possible parameter combinations (64), a smaller subset of parameter combinations (13) was chosen based on the findings in Study 1 and conditions where there is the most discrepancy between methods in confidence limit estimation in prior research (MacKinnon, Lockwood & Williams, 2004). The purpose of the study was to determine how informative a prior distribution would have to be in order to increase power, produce Type I error rate equal to the nominal rate of 0.05, reduce interval width, produce imbalance of 0, and coverage equal to the nominal level of 0.95 for Bayesian credibility intervals for N = 20, 40, 60, 100, and 200.

Methods

Populations with the following combinations of values for parameters were simulated (example SAS simulation code can be found in Appendix D): *a*=0 *b*=0 *c'*=0, *a*=0 *b*=0.14 *c'*=0, *a*=0 *b*=0.39 *c'*=0, *a*=0 *b*=0.59 *c'*=0, *a*=0.14 *b*=0.14 *c'*=0, *a*=0.39 *b*=0.39 *c'*=0, *a*=0.59 *b*=0.59 *c'*=0, *a*=0.14 *b*=0.39 *c'*=0, *a*=0.14 *b*=0.59 *c'*=0, *a*=0.39 *b*=0.59 *c'*=0, *a*=0.14 *b*=0.14 *c'*=0.39, *a*=0.39 *b*=0.39 *c'*=0.39, *a*=0.59 *b*=0.59 *c'*=0.39. Two values of the *c'* coefficient (0 and 0.39) were included in the simulation in order to determine whether performance of interval estimates differs for complete and incomplete mediation models.

Random samples of sizes N = 20, 40, 60, 100, and 200 were obtained from each population and 95% confidence limits/credibility intervals for the mediated effect were calculated using normal theory, the distribution of the product method, percentile bootstrap, and bias-corrected bootstrap. Also, four Bayesian methods were tested in order to determine whether prior information could improve the performance of credibility intervals in comparison to other methods in the study. The four Bayesian methods were the method of coefficients with diffuse prior distributions, the method of coefficients with informative prior distributions, the method of covariances with a diffuse prior distribution, and the method of covariances with an informative prior distribution. The informative prior distributions for the methods of coefficients and covariances consisted of simulated values and quantities calculated from simulated values; for simplicity, this specification of priors will be referred to as the "truth" about the parameters. For information on the arithmetic correspondence of the parameters in these two methods see Appendix B.

Method of coefficients with diffuse prior distributions. Diffuse normal prior distributions centered at the true values of coefficients *a*, *b*, and *c'* with variance equal to 10^3 were assigned to the regression coefficients, and σ^2_M and σ^2_Y were assigned inverse-gamma distributions with shape and inverse-scale parameters equal to .01 (so that the expectation of this distribution is 1).

Method of coefficients with informative prior distributions. Normal prior distributions centered at the true values of coefficients *a*, *b*, and *c'* with standard deviations equal to the standard errors of respective coefficients calculated using simulated values at a given sample size were assigned to the regression coefficients, and σ^2_M and σ^2_Y were assigned inverse-gamma distributions with shape and inverse-scale parameters equal to .01 (so that the expectation of this distribution is 1).

Method of covariances with a diffuse prior distribution. The vector of means of X, M, and Y was assigned a multivariate normal distribution, while the covariance matrix was assigned an inverse Wishart distribution with 3 as the degrees of freedom parameter. The degrees of freedom parameter has to equal at least the number of variables in the model so the inverse Wishart is not in a degenerate form (Gelman, Carlin, Stern, & Rubin, 2004) so the degrees of freedom were set to 3 for this method. The covariation matrix is the second parameter of the inverse Wishart prior, and it was specified so that the prior expectation for each covariance between variables is zero, and the variance of each variable is 1.

Method of covariances with an informative prior. The vector of means of X, M, and Y was assigned a multivariate normal distribution with means of 0, variances of 1000, and covariances of 0. The covariance matrix was assigned an inverse Wishart distribution. The degrees of freedom parameter of the inverse Wishart distribution was set to equal N_p, the size of the observed sample in the condition (e.g. if the sample size in the condition examined was 60, the degrees of freedom parameter of the inverse Wishart was also set to 60 so that the prior and observed sample are of the same size). The covariation matrix is the second parameter of the inverse Wishart prior, and it was specified so that the prior expectations for each variance and covariance equal their respective true values.

The simulation consisted of 1000 replications for each of the 13 combinations of effect size and sample size. Average coverage, Type I error rate for cases where either *a* or *b* equal zero, confidence interval imbalance (number of times the estimate is to the right of the UCL minus the number of times the estimate is to the left of the LCL), empirical power for cases where *a*>0 and *b*>0, and interval width were obtained for all eight methods being studied.

Results

Results for all combinations of parameters *a*, *b*, and *c'* from Study 2 are summarized in Tables 6-10. Results for all possible combinations of parameter values and sample size in Study 2 can be found in Appendix C, in Tables 43-67. Coverage, power, imbalance, and interval width for combinations with *a=b* with c'=0 and *a=b* with c'=0.39 were compared using t-tests in order to determine whether the size of the *c'* parameter has an impact on the performance of the method. None of the t-tests for any of the methods (normal theory, distribution of the product, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Bayesian method of covariances with diffuse prior distributions, Bayesian method of covariances with informative prior distributions, percentile bootstrap, and bias-corrected bootstrap) at any of the sample sizes (N = 20, 40, 60, 100, and 200) were statistically significant. Thus, the size of the *c'* coefficient did not have an impact on the statistical performance of the interval estimates for the mediated effect for the parameter and sample size combinations examined in this study.

There were four combinations where the mediated effect was zero. The average Type I error rates for these combinations are displayed in Table 6. All eight methods had Type I error rates lower than the nominal 0.05 level at all sample sizes. The bias-corrected bootstrap had the highest Type I error rates, whereas the Bayesian methods with prior information had almost no Type I error.

Insert Table 6 about here

Figure 1 shows plots of the Type I error rates when *a* is 0, and *b* ranges from 0 to 0.59. When both *a* and *b* are 0, all eight methods have comparable Type I error rates that are close to zero. However, when *b*=0.14, and 0.39 the Type I error rates for all methods, except the Bayesian methods with informative prior distributions, increases with sample size and is highest for the bias-corrected bootstrap. The Type I error rates remain close to 0 for Bayesian methods with informative prior distributions regardless of sample size or the size of the *b* coefficient. Bayesian methods with diffuse prior distributions had Type I error rates below 0.05 for all combinations of *b* and sample size except when *b*=0.39, and 0.59 and N = 200. For N≥40 and *b*=0.39 the Type I error rate for the bias-corrected bootstrap is above 0.05. When *b* is large, the Type I error rate for the bias-corrected bootstrap is above 0.05 for all sample sizes, and is above 0.05 for the percentile bootstrap for four out of the five sample sizes. Normal theory and distribution of the product confidence limits have Type I error rates consistently lower than 0.05 regardless of the size of *b* and N.

- Insert Figure 1 about here -

Table 7 displays average values of power for the 9 combinations of *a*, *b*, and *c'* with non-zero values of the mediated effect. Overall, normal theory confidence limits had the lowest power at each sample size, followed by the distribution of the product, Bayesian methods with diffuse priors, and the percentile bootstrap. The bias-corrected bootstrap had higher average power (but also had higher Type I error rate, which is important to take into consideration when selecting a statistical method) than normal theory, distribution of the product, percentile bootstrap confidence limits, and Bayesian methods with diffuse priors at each sample size. Bayesian methods with informative prior distributions had the highest power of all methods at all sample sizes.

Insert Table 7 about here

The only combination of method and sample size that attained power of above 0.80 was Bayesian credibility intervals at a sample size of 200 (see Figure 2). All other combinations of method and sample size had average power of less than 0.80. Power increased as a consequence of incorporating prior information in the analysis (Figure 2). Furthermore, given the same amount of prior information (diffuse or informative prior distributions), the Bayesian method of coefficients and the Bayesian method of covariances had almost identical power at all sample sizes below 100 (see Figure 2). At N = 100 the method of covariances with informative prior distributions had slightly more power than the method of coefficients with informative prior distributions. Figure 2 contains the plots of power for the nine different combinations of parameters *a*, *b,* and *c'* and all sample sizes. When *a* and *b* are both small, change in power for the eight methods is the same regardless of whether *c'* is 0 or .39. In both situations, all methods have power close to 0 for N = 20, 40, and 60, and with N = 100 the two Bayesian methods with informative prior distributions have at least twice the power of some of the other methods. When *a* is small and *b* is medium and large, the Bayesian methods with informative prior distributions still have the highest power of all methods. The bias-corrected bootstrap, Bayesian methods with diffuse prior distributions, and the distribution of the product have lower values of power than the Bayesian methods with informative prior distributions, but consistently higher power than normal theory confidence limits. With medium and large *a* and *b* the values of power for the Bayesian methods with informative prior distributions plateau at N = 40. The remaining six methods have power curves that start plateauing at N = 100 if at least one of the coefficients (*a* or *b*) is 0.59. The value of *c'* did not change the power profile of any of the methods.

- Insert Figure 2 about here -

As shown in Table 8, coverage was evaluated for each method at the five sample sizes and thirteen parameter combinations. Normal theory confidence limits and the two bootstrap methods had coverage levels closest to the nominal level of 0.95. The distribution of the product and the Bayesian methods with diffuse and informative priors exceeded the nominal level of coverage at all sample sizes.

Insert Table 8 about here

Figure 3 displays the trellis plot of coverage for thirteen combinations of effect sizes. With *a*= 0 and b>0 coverage is highest and equals 1 at N= 20, and decreases as sample size increases for all methods except for the Bayesian methods with informative prior distributions. This decrease in coverage as sample size increases is also observed when *a*= 0.14 and *b*= 0.14, and 0.39. A finding made clear by all the plots of coverage is that the Bayesian methods with informative prior distributions have coverage close to 1 regardless of sample size or effect size, and that the remaining six methods are comparable to each other in terms of coverage. The bias-corrected bootstrap has the lowest coverage for a=0 and b>0, which is consistent with the findings that the bias-corrected bootstrap has higher Type I error than other methods for a=0 and b>0.

- Insert Figure 3 about here -

Table 9 displays the average values of interval width for the 13 combinations of parameter values and for all four sample sizes. Bayesian methods with informative prior distributions have the lowest values of interval width regardless of sample size and effect size. All other methods in the study have higher interval widths than Bayesian methods with informative prior distributions.

Insert Table 9 about here

As can be seen from Figure 4, the value of the *c'* coefficient (0 versus 0.39) does not change the shape of the interval width curves for any of the methods.

- Insert Figure 4 about here –

Table 10 displays the average values of imbalance for the eight methods at all four sample sizes. Normal theory confidence limits had the highest imbalance at N = 20, however, the bias-corrected bootstrap had the highest absolute value of imbalance at N = 40 and N = 100. The Bayesian methods with informative prior distributions had the lowest imbalance of all eight methods.

Insert Table 10 about here

Figure 5 displays the trellis plot for imbalance of the eight methods for thirteen combinations of effect sizes. Imbalance of all eight methods is close to zero and comparable for all eight methods and at all effect sizes examined in this study except when a=b=0.14 (both with c'=0 and c'=0.39) and N≥60 which is when imbalance for normal theory confidence limits is positive and larger than imbalance of the other seven methods.

- Insert Figure 5 about here -

Discussion

Overall, the addition of prior information into the statistical analysis improved interval estimates of the mediated effect. Power was higher with prior information in the analysis, imbalance was closer to zero, and interval width was lower than for methods that do not incorporate prior information. The Bayesian method of coefficients and the Bayesian method of covariances had almost identical values of power both with and without prior information (Figure 6).

- Insert Figure 6 about here -

When prior information was included in the analysis, the Type I error rate was below the nominal level of 0.05, and coverage was above the nominal level of 0.95. Bayesian credibility intervals with informative prior distributions lower the risk of Type I error rates and have higher coverage. These properties of Bayesian credibility intervals are difficult to label as positive or negative. On one hand, having a very low Type I error rate and high coverage is beneficial to the researcher, as it translates to fewer false positives and fewer intervals that do not contain the true value of the parameter. On the other hand, when choosing a confidence coefficient of 0.95, the researcher is expecting a Type I error rate of 5% and coverage of 95%, and thus a method that does not display Type I error rates and coverage equal to nominal levels is performing differently from the expectation. Furthermore, the notions of Type I error rate and coverage are inherently frequentist, given that they rest on the assumption of repeated sampling. Thus, it is not in the nature of credibility intervals to conform to a value of Type I error rate and coverage selected prior to an experiment. For more on the difference in inference and criteria of the frequentist and Bayesian frameworks, see Gigerenzer (1993).

Study 2 examined how Bayesian credibility intervals perform when the prior information consists of the true values of parameters and quantities calculated from true values in the simulation. However, if one knew the truth about a phenomenon, they would not be studying it. Thus, it would be beneficial to examine how Bayesian methods perform with prior distributions that are more diffuse than the priors examined in Study 2, but less diffuse than normal prior distributions with a variance of 10^3. Furthermore, researchers might be more confident in the available information for the *a*-path (action theory) or in the information for the *b*-path (conceptual theory), and thus might assign different variance parameters to the prior distributions of the two coefficients. Knowing more about action theory versus conceptual theory or vice versa (and using this information to assign a less diffuse prior distribution for the path in question) might have different effects on power. It is unclear whether a narrower prior distribution for the *a* path or for the *b* path leads to a greater increase in power.

CHAPTER 5

STUDY 3

Study 3 examined if the observed increases in power with informative prior distributions are still present with precision parameters in the prior distributions for regression coefficients that are different than the true precisions (calculated from true parameters in the simulation) investigated in Study 2. A second goal of Study 3was to investigate whether more prior information (more precision) about action (*a*-path) or conceptual (*b*-path) theory led to greater increases in power for the mediated effect.

Six parameter combinations were selected in this study based on the prior studies to reduce the computation burden. Power of the Bayesian method of coefficients is observed as a function of the precision parameter, and precision equated across sample sizes.

Methods

Populations with the following combinations of values for parameters *a*, *b*, and *c'* were simulated (example SAS code is in Appendix E): *a*=*b*=0.14 with *c'*=0, *a*=*b*=0.39 with *c'*=0, *a*=*b*=0.59 with *c'*=0, *a*=*b*=0.14 with *c'*=0.39, *a*=*b*=0.39 with *c'*=0.39, and *a*=*b*=0.59 with *c'*=0.39.

Study 3 examined the effect of precision for regression coefficients *a* and *b* on changes in power to detect the mediated effect for the Bayesian method of coefficients. Random samples of sizes N = 20, 40, 60, 100, and 200 were generated from each population and 95% credibility intervals for the mediated effect were calculated using the method of coefficients with different priors for regression coefficients *a* and *b*. Ten conditions were evaluated in the study: five where the precision parameter in the prior for the coefficient *b* was set to 10^{-3} and the precision parameter in the prior for coefficient *a* was varied (10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}), and five where the precision parameter in the prior for the coefficient *a* was set to 10^{-3} and the precision parameter in the prior for coefficient *b* was varied (10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}).

The range (spread) of values of a coefficient in a normal prior distribution can be defined as variance, standard deviation, or precision. The standard deviation is the square root of the variance, and the precision is the reciprocal of variance. In Study 3, normal prior distributions for regression coefficients were used with the precision parameterization, so the precision equaled 1 divided by the variance of the coefficient.

The true precisions for *a* and *b* are equal to 1 divided by the true derived variance of the respective parameter in the equation at each sample size. The true variance (and precision) of the coefficient depends on effect sizes and sample size. Thus, a fixed value of the precision parameter in the prior for *a* or *b* (10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}) is a different fraction of the true precision of the coefficient at different values of effect size and sample size. Table 11 shows how the values of precision are related to the true precision values of parameters *a* and *b* at each sample size and for each precision parameter. The entries in Table 11 are the fractions of the true precision values for values of regression coefficients examined in Study 3. For example, when the precision parameter of the prior distribution for regression coefficient *a* is set to 10 and the sample size is 100, the precision parameter of 10 is 10.2% of the true precision of *a* (the fraction entry in the table is 0.102). The values of precision examined in Study 3 (10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}) are generally lower than the true precisions of each parameter, except with the precision parameter in the prior equals 100, and sample size is between 20 and 100. When the precision parameter in the prior distribution for the regression coefficient is smaller than the true precision of the coefficient, the fraction is smaller than 1. For a precision parameter in the prior that is larger than the true precision of the regression coefficient, the fraction is larger than 1. If the precision parameter in the prior distribution for the regression coefficient is equal to the true precision of the coefficient, then the fraction equals 1, and this was the way prior distributions were specified in Study 2 for the Bayesian methods with informative prior distributions.

In Study 3 the mean parameters in the prior distributions for the regression coefficients *a* and *b* were always set to equal the true value. In all ten conditions, the coefficient *c'* was assigned a normal prior distribution with the true value as the mean parameter of the prior distribution, and a variance parameter of 10^3 (which is the same as setting the precision parameter to 10^{-3}). The intercepts in the regression equations predicting M and Y both received normal prior distributions with the mean parameter equal to 0 and the variance parameter equal to 10^3. The priors for the precision of M and Y were the same for all conditions (gamma distributions with shape and inverse scale parameters equal to .01 so that the expectation of the distribution equals to 1), as was done in studies 1 and 2. The simulation consisted of 1000 iterations per combination of effect sizes and sample size. Average empirical power for all six conditions and sample sizes was computed for N = 20, 40, 60, 100, and 200.

Results

Study 3 examined the extent to which the observed increases in power are still present with precision parameters in the prior distributions for regression coefficients equal to powers of 10, which made some precision parameters smaller and some precision parameters larger than the true precisions of *a* and *b*. Another research question was whether more prior information (more precision) about action (*a* path) or conceptual (*b* path) theory lead to greater increases in power to detect the mediated effect. There was no consistent pattern that could answer the question of whether more precision in the prior for *a* or more precision in the prior for *b* lead to greater power. In Figures 7-11, the plots of power as a function of the prior precision of *a* are almost identical to the plots of power as a function of prior precision of *b* for each sample size. Thus, there is no differential increase in power depending on whether there is more precision in the prior for the action (*a* path) versus the prior for the conceptual theory (*b* path).

Results from Study 3 are summarized in Tables 12-16 and Figures 7-13. In addition to graphical displays in Figures 7-11, the differences in power between combinations where *a=b* with *c'*=0 and *a=b* with *c'*=0.39 were also assessed using t-tests. None of the t-tests for any of the prior distribution specifications at any of the sample sizes (N = 20, 40, 60, 100, and 200) were statistically significant. Thus, having a medium size of the *c'* parameter versus complete mediation has no impact on the power to detect the mediated effect using the Bayesian method of coefficients for the parameter, precision, and sample size combinations examined in this study.

The question of how much increases in precision increase power will be answered in two parts. First, increases in power will be observed as a function of the values of the precision parameter for *a* and *b*, which were 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, and 10^{2}. Then, increases in power will be assessed as a function of increases in precision equated across sample size; that is, increases in power will be evaluated as a function of the fraction of the true precision that was encoded in the prior distribution.

The findings indicate that increasing precision tenfold affects power differently depending on the sample size and effect size. At N = 20, the increase in power is most notable for medium values of *a* and *b*, slightly less pronounced for high values of *a* and *b*, and non-existent when *a*=*b*=0.14 (Figure 7).

- Insert Figure 7 about here -

As can be seen in Table 12, there are notable increases in power when *a*=*b*=0.59 when the precision parameter in the prior has been increased from 10^0 to 10^1 and to 10^2 (this is equivalent to saying that the variance parameter in the prior for *a* is decreased from 10^0 to 10^{-1} and to 10^{-2}), however, power at N=20 remains below 0.80 regardless of the precision of the priors for regression coefficients *a* and *b*.

Insert Table 12 about here

At N = 40, for *a*=*b*=0.59 the power is already above 0.80, regardless of the precision parameter in the prior distribution for the regression coefficients (Table 13).

Insert Table 13 about here

The pattern of findings is similar to those at N = 20, indicating that increases in the precision parameter (decreases in the variance parameter) produce the steepest increases in power for medium effects, some increases in power for large effects, and no increases in power for small effects (Figure 8).

- Insert Figure 8 about here -

At N=60 the increases in precision increase power only for medium effects (Table 14; Figure 9). Large effects already had power close to 1 at the lowest value of precision, and thus there was a ceiling effect of increases in precision. Small effects show a very slight increase in power when the precision parameter is increased from 10 to 100.

Insert Table 13 about here

- Insert Figure 9 about here -

At N = 100 the ceiling effect of increases in precision on power is still present for large effects, and is starting to occur for medium effects as well, although at a=b=0.39 power still increases when the precision parameter is raised from 1 to 10 to 100 (Figure 10).

At N = 200 the ceiling effect of increases in precision on power occurs for both medium and large effects (Figure 11). At N = 200 the only conditions where power increases are ones where a=b=0.14, and this only occurs when the precision parameter is increased from 10 to 100. Figures 10-11 and Tables 15-16 show that power for medium and large effects is almost 1 when N = 100, and 200. Thus, the benefits of increasing precision for N=100, and 200 are not notable given that power is either already satisfactory (for medium and large effect sizes) or cannot be increased above 0.40 with the changes in the precision of the prior distribution for either a or b.

- Insert Figures 10 and 11 about here –

Insert Tables 15 and 16 about here

Figures 12 and 13 shows the relationship of power to the fraction of the true precision of *a* and *b* in the prior distribution, and are useful for examining how power increases with increases in precision equated across sample sizes. The plots for the two coefficients are identical for each effect size indicating that there are no differential changes in power depending on whether the increase in precision occurred for coefficient *a* or *b*. In the discussion of the findings the term "the coefficient" refers to both *a* and *b* given that everything that is pointed out in the figure is true for both coefficients.

- Insert Figures 12 and 13 about here –

When N = 20 for a=b=0.14 a precision parameter that is more than 5 times the true precision of the coefficient still produces no increases in power. When N = 40 and 60 the increases in power for small effect sizes (a=b=0.14) are very small even when the precision parameter is greater than the true precision of the coefficient. However, for small effects and N = 100 and 200 an increase in fraction of true precision does lead to notable increases in power. Thus, depending on the pseudo-sample size in the prior (which with the observed sample size in the data adds up to the pseudo-sample size of the posterior) increasing the fraction of true precision of the coefficient may not lead to substantial increases in power for small effects unless the sample size is at least 100.

At medium effect sizes (*a*=*b*=0.39) power noticeably increases with increases in the fraction of true precision. At N = 100 and 200 and *a*=*b*=0.39 power is already high, and there is no room for it to increase drastically.

At large effect sizes (*a*=*b*=0.59) increases in fraction of true precision produce the greatest benefits at N = 20. The changes in power for large effects become smaller and smaller as sample size increases, reaching a ceiling effect at N = 100.

Discussion

The most notable finding in Study 3 is that for certain combinations of parameter values and sample size, increasing the precision parameter of the prior distribution by a factor of 10 of either *a* or *b* can have almost no impact on power. For small effects and small sample sizes, and for large effects and sample sizes of at least 100 there are almost no changes in power as a consequence of increasing the precision parameter.

When looking at how the precision in the prior is related to the true precision of the parameter (which is unknown to the researcher), and the extent to which prior precisions larger than the true precision of the coefficient lead to more power, the conclusion is similar: increases in precision relative to the true precision may not always lead to noticeable increases in power. The findings for the combinations of parameter values, precision values, and sample sizes in this study indicate that increases in precision lead to greater power only when the sample size is at least 100 and the effects are small, when sample size is below 60 and the effects are large, and the most notable increases in power were when N<200 and the effects are medium.

CHAPTER 6

EMPIRICAL EXAMPLE

The data for the empirical example come from a study of memory for words conducted at a large university in the southwestern United States. The same experiment was conducted twice, and the data from the two experiments are referred to as Year 1 and Year 2. In both experiments, students were instructed to either make images for words or repeat them (X), and were subsequently asked about the extent to which they made images for the words (M). The dependent variable was number of words recalled (Y). There were 44 participants in the Year 1 experiment, and 42 participants in the Year 2 experiment.

For the empirical example, interval estimates of the mediated effect for the Year 2 data were computed using the eight methods evaluated in Study 2: normal theory confidence limits, distribution of the product confidence limits, Bayesian method with diffuse normal priors for the regression coefficients with a mean of 0 and variance of 10^6, Bayesian method with informative prior distributions for the regression coefficients (using the information from the Year 1 experiment), Bayesian method with a diffuse prior distribution for the covariance matrix of X, M, and Y, Bayesian method with an informative prior distribution for the covariance matrix of X, M, and Y (using the information from the Year 1 experiment), percentile bootstrap, and bias-corrected bootstrap. The informative prior distributions for the Bayesian method of coefficients were based on the observed regression coefficients and their standard errors. Each regression coefficient was assigned a normal prior distribution with the observed coefficient from the Year 1 data as the mean parameter (a= 3.558, b= 0.614, c'= 0.332) and their standard errors as the standard deviation parameters of the normal prior distributions (0.689, 0.614, 1.292, respectively). Precision parameters σ^2_M and σ^2_Y were modeled as inverse-gamma distributions with shape and inverse-scale parameters equal to .01 (so that the expectation of this distribution is 1). The informative prior distributions for the method of covariances were a multivariate normal prior with observed means for X, M, and Y (0.454, 6.159, 13.227, respectively) as the mean parameter, and the observed covariance matrix (s^2_X= 10.909, s_{XY}=, s_{XM}= 38.818, s^2_M= 355.886, s_{MY}= 231.409, s^2_Y= 607.727) as the covariance parameter. The covariance matrix was assigned an inverse Wishart prior distribution with 44 degrees of freedom and the observed sums of squares and cross-products as the scale matrix parameter. Estimates for all methods except Bayesian methods with informative prior distributions have then been computed using the combined data from Year 1 and Year 2. The prior distributions for the Bayesian methods with diffuse priors were identical in the first and in the second analysis. Table 17 contains the results for all the methods and for both samples.

Insert Table 17 about here

According to the normal theory confidence limits, with infinite samples, 95% of the confidence intervals will contain the true value of the mediated effect. Normal theory confidence limits using the Year 2 sample alone are 2.70 and 10.39, and with both the Year 1 and 2 samples the limits are 2.36 and 5.81. Using the distribution of the product, it was found that the mediated effect for the Year 2 sample lies between 2.78 and 10.50, while the mediated effect for both Year 1 and 2 samples lies between 2.44 and 5.90, with

95% confidence. The results of the Bayesian method of coefficients with diffuse prior distributions for regression coefficients yield a mediated effect between 2.78 and 10.55 with 95% probability for the Year 2 sample, and a mediated effect between 2.42 and 6.22 for the combined sample. For the Bayesian method with informative prior distributions for regression coefficients and intercepts one can conclude that the mediated effect lies between 1.81 and 4.30 with 95% probability. According to the Bayesian method with a diffuse prior distribution for the covariance matrix, there is 95% probability that the mediated effect lies between 2.56 and 7.70 for the Year 2 sample, and 95% probability that the mediated effect lies between 2.21 and 5.28 for the combined sample. With prior information from Year 1, the findings using the Bayesian method of covariances indicate that the mediated effect lies between 2.08 and 5.55 with 95% probability. The percentile bootstrap results show that with 95% confidence, the mediated effect for the Year 2 sample alone lies between 4.08 and 10.24, while the mediated effect for the combined sample lies between 0.64 and 3.78. The 95% bias-corrected bootstrap limits for the Year 2 sample are 4.14 and 10.40, and for the combined sample the limits are 0.68 and 3.98. None of the intervals contain the value of 0, thus the mediated effect is statistically significant according to all methods. In the analyses of the Year 2 sample alone, the Bayesian methods with informative priors for both regression coefficients and the covariance matrix have the smallest interval width out of all methods. Combining data from the two years produced narrower interval estimates for all methods that did not incorporate prior information from Year 1 into the analysis.

CHAPTER 7

SUMMARY AND CONCLUDING DISCUSSION

The general goal of the project was to evaluate the benefits of using prior information in the interval estimation of the mediated effect. The discussion of the project will begin with a summary of the findings from the three studies, followed by the fit with earlier literature, and the limitations of the project. The section will conclude with future directions for research in interval estimation using Bayesian mediation.

Summary of Results

It was hypothesized that Bayesian methods with informative prior distributions would have coverage closer to the nominal level of 0.95, a Type I error rate closer to the nominal level of 0.05, imbalance closer to 0, higher empirical power, and lower interval width than Bayesian methods with diffuse prior distributions, normal theory confidence limits, distribution of the product confidence limits, percentile bootstrap confidence limits, and bias-corrected bootstrap confidence limits. Study 1 was designed to test this hypothesis, however, due to the errors in the simulation, only a partial answer was available. The most important finding from Study 1 is that the variance of the prior distributions for regression coefficients *a* and *b* plays a big role in the performance of the Bayesian method of coefficients. If the variance of the prior distributions is too large and thus the prior distribution too diffuse, then Bayesian methods with mean parameters in the prior distributions equal to the true value of the regression coefficient do not have coverage closer to the nominal level of 0.95, Type I error rate closer to the nominal level of 0.05, imbalance closer to 0, higher empirical power, and lower interval width than

Bayesian methods with diffuse prior distributions, normal theory confidence limits, distribution of the product confidence limits, percentile bootstrap confidence limits, and bias-corrected bootstrap confidence limits.

Study 2 was designed to examine additional methods that were absent from Study 1, and to examine the effect of a lower variance parameter in the prior distribution for regression coefficients *a* and *b*. Bayesian methods with informative prior distributions in Study 2 examined how Bayesian credibility intervals perform with prior information that was the true parameter value and standard error about the parameters of interest. The findings indicate that with true values for the parameters in the prior distribution, Bayesian methods with informative prior distributions have higher power, lower interval width, and less imbalance than Bayesian methods with diffuse priors and frequentist methods.

Study 3 was designed to evaluate whether using values other than the true variance in the prior distribution for regression coefficients in the Bayesian methods of coefficients still leads to increases in power compared to using a diffuse prior distribution. It was also hypothesized that increasing the precision parameter in the prior distribution of coefficient *a* and increasing the precision parameter in the prior distribution of the coefficient *b* can have different effects on power. The findings from Study 3 indicate that power of the Bayesian credibility interval increases with the increase in precision of the prior distribution, but that the extent to which power increases depends on the size of coefficients *a* and *b*, and the sample size. It was found that Bayesian methods with informative prior distributions are a promising way to increase power in samples smaller than 100, but the increase in power for a tenfold increase in the prior precision parameter of either *a* or *b* may not always be substantial.

Overall, the findings from the three studies indicate that Bayesian methods are a promising way of improving interval estimates of the mediated effect when pertinent and accurate prior information is available. With no prior information and diffuse prior distributions, Bayesian methods perform as well as the percentile bootstrap and the distribution of the product.

Fit with Earlier Literature

The findings of this study replicate prior findings that the bias-correct bootstrap has excessive Type I error rates in some conditions and that the distribution of the product can have coverage above the nominal 0.95 level (Fritz, Taylor & MacKinnon, 2012; MacKinnon, Lockwood & Williams, 2004; MacKinnon, Lockwood, et al., 2002). The findings that the bias-corrected bootstrap has slightly higher power than the distribution of the product and the percentile bootstrap are also consistent with earlier work (MacKinnon, Lockwood & Williams, 2004), and so is the finding that normal theory confidence limits have smaller interval width than other methods (Preacher & Selig, 2012). The new insights gained from this study are that given enough prior information, the Bayesian method of coefficients (Yuan and MacKinnon, 2009) and the Bayesian method of covariances (Enders, Fairchild & MacKinnon, 2013) can have power as high or greater than the bias-corrected bootstrap without having inflated Type I error rates in some conditions. Furthermore, it was found that with no available prior information, the Bayesian methods still perform as well as the distribution of the product and the percentile bootstrap, and that Bayesian methods are always a better choice in terms of power than the normal theory confidence limits.

Limitations

The interval estimates obtained using Bayesian methods in this study were central credibility intervals, not highest posterior density intervals. Central credibility intervals are formed by taking the $\alpha/2^{th}$ and $(1-\alpha/2)^{th}$ percentiles of the posterior distribution that will produce the $(1-\alpha)\%$ credibility interval. Highest posterior density intervals are obtained by taking the $(1-\alpha)\%$ area of the posterior distribution that has the highest probability density and subsequently determining the values of the parameter that correspond to the lower and upper limit. Highest posterior density intervals have the characteristic that the density within the interval is never lower than the density outside the interval (Gelman, Carlin, Stern, & Rubin, 2004). Central credibility interval limits and highest posterior distribution credibility interval limits are identical for symmetrical distributions. However, the mediated effect follows the distribution of the product, which is skewed and kurtotic (Lomnicki, 1967; Springer & Thompson, 1966; Craig, 1936). This would cause the central credibility limits for *ab* to be different from the highest posterior density limits for *ab* (Gelman, Carlin, Stern, & Rubin, 2004). The performance of Bayesian highest credibility intervals relative to frequentist methods might be different than the performance of central credibility intervals relative to frequentist methods. Preliminary work comparing the high posterior density intervals to the central credibility intervals for the mediated effect for a limited number of conditions suggest that they are similar.

Another limitation of Study 2 is that the prior distributions for error precisions of variables M and Y in the informative case of the Bayesian method of coefficients were non-informative as in the noninformative case. That is, the specification of the gamma distributions for the error precisions of M and Y were identical for the diffuse and informative cases, and consisted of a gamma distribution with the expectation of 1. It is possible that encoding prior information for the error precision parameters of M and Y could have increased power and coverage, and decreased Type I error rate, interval width, and imbalance of the Bayesian method of coefficients with informative prior distributions.

Future Directions

Sometimes the necessary prior information for the Bayesian method of coefficients is not available even though the phenomenon has already been studied. This study assumed prior information about the *a* and *b* paths was available, but this may not always be the case. The articles in the literature may only report estimates of the mediated effect and one (or neither) of the paths. In that case, a researcher would want to incorporate the knowledge/intuition about mediated effect by specifying a prior distribution for the product *ab.* The method for specifying a prior for *ab* has not yet been developed. Even if specifying prior information about *ab*, the researcher would have to specify a prior for one of the regression coefficients as well (even if no intuition or information was available and the only choice was a uniform prior ranging from $-\infty$ to ∞), which would determine the prior of the remaining regression coefficient as the prior for the mediated effect divided by the prior for the other coefficient. For example, for the mediated effect *ab* the researcher might specify a prior distribution for which the range of the reported mediated effects in the literature has the highest probability, and a uniform prior with a wide range may be chosen for the *a* path; these two specifications automatically constrain the prior for the *b* path because $b = \frac{ab}{a}$. This method is more complex than the one proposed by Yuan and MacKinnon (2009). Alternatively, the literature on a topic may contain the covariance matrix and sample size from previous studies, but not the values of *a* or *b*, in which case the researchers can use the method suggested by Enders, Fairchild, and MacKinnon (2013).

The studies in this project mark a beginning of exploring how Bayesian methods can be used to improve interval estimation in mediation models. Findings thus far indicate that Bayesian methods are a promising way to increase power and reduce interval width in the single mediator model, however, these benefits are highly dependent on the value of the variance (or precision) parameter of the prior distribution. Future studies will examine ways to optimize Bayesian mediation for the single mediator model and will extend the application of Bayesian statistics to models with multiple mediators.

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Table 1

Average Type I error rate for N=20, 40, 60, 100, and 500 for normal theory, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 1 (note: Type I error is not observable when both a and b are non-zero).

	Method				
Sample Size	normal	YMdiff	YMinfo	PercBoot	BCBoot
n=20	0.007	0.016	0.016	0.024	0.044
n=40	0.008	0.017	0.017	0.027	0.050
n=60	0.011	0.026	0.026	0.030	0.051
n=100	0.019	0.052	0.053	0.036	0.054
n=500	0.030	0.044	0.044	0.041	0.054

Table 2

Average Power for N=20, 40, 60, 100, and 500 for normal theory, method of coefficients with diffuse priors, method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 1 (note: Power is not observable when either a or b is zero).

	Method				
Sample Size	normal	YMdiff	YMinfo	PercBoot	BCBoot
n=20	0.076	0.111	0.111	0.137	0.207
n=40	0.235	0.285	0.285	0.320	0.384
n=60	0.371	0.433	0.432	0.441	0.494
n=100	0.512	0.544	0.544	0.562	0.597
n=500	0.895	0.909	0.910	0.916	0.929

Table 3

Average Coverage for N=20, 40, 60, 100, and 500 for normal theory, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 1.

	Method				
Sample Size	normal	YMdiff	YMinfo	PercBoot	BCBoot
n=20	0.962	0.971	0.970	0.958	0.939
n=40	0.963	0.966	0.966	0.957	0.940
n=60	0.962	0.957	0.958	0.956	0.942
n=100	0.958	0.940	0.940	0.954	0.942
n=500	0.957	0.948	0.948	0.953	0.947

Table 4

Average Interval Width for N=20, 40, 60, 100, and 500 for normal theory, method of coefficients with diffuse priors, method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 1.

	Method				
Sample Size	normal	YMdiff	YMinfo	PercBoot	BCBoot
n=20	0.461	0.546	0.546	0.533	0.551
n=40	0.325	0.366	0.366	0.352	0.362
n=60	0.265	0.290	0.290	0.282	0.289
n=100	0.190	0.193	0.193	0.197	0.200
n=500	0.081	0.083	0.083	0.082	0.082

Table 5

Average Imbalance for N=20, 40, 60, 100, and 500 for normal theory, method of coefficients with diffuse priors, method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 1.

	Method			
Sample Size	normal	YMdiff	YMinfo	PercBoot
n=20	0.029	0.012	0.012	0.017
n=40	0.026	0.013	0.014	0.016
n=60	0.026	0.014	0.014	0.014
n=100	0.022	0.028	0.028	0.010
n=500	0.011	0.005	0.004	0.005

Table 6

Average Type I error rate for N=20, 40, 60, 100, and 200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates for 13 combinations of a, b, and c' in Study 2.

	Method							
Sample Size	norm	prod	YMd	YMi	EFMd	EFMi	PercBoot	BCBoot
n=20	0.005	0.017	0.009	0.004	0.008	0.002	0.020	0.037
n=40	0.007	0.025	0.020	0.005	0.015	0.002	0.028	0.048
n=60	0.008	0.019	0.018	0.003	0.014	0.002	0.021	0.042
n=100	0.014	0.027	0.026	0.002	0.027	0.003	0.032	0.046
n=200	0.024	0.037	0.038	0.005	0.040	0.006	0.039	0.052

Table 7

Average Power for N=20, 40, 60, 100, and 200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates for 13 combinations of a, b, and c'in Study 2.

	Method

Sample Size	norm	prod	YMd	YMi	EFMd	EFMi	PercBoot	BCBoot
n=20	0.090	0.162	0.113	0.390	0.123	0.383	0.149	0.216
n=40	0.289	0.379	0.355	0.577	0.353	0.580	0.371	0.436
n=60	0.424	0.501	0.486	0.621	0.478	0.611	0.497	0.548
n=100	0.571	0.613	0.604	0.709	0.605	0.742	0.615	0.644
n=200	0.676	0.717	0.717	0.914	0.739	0.919	0.717	0.753

Table 8
Average Coverage for N=20, 40, 60, 100, and 200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates for 13 combinations of a, b, and c'in Study 2.

	Method							
Sample Size	norm	prod	YMd	YMi	EFMd	EFMi	PercBoot	BCBoot
n=20	0.953	0.965	0.973	0.994	0.965	0.997	0.956	0.945
n=40	0.957	0.963	0.971	0.994	0.968	0.996	0.955	0.944
n=60	0.952	0.964	0.965	0.995	0.967	0.996	0.956	0.941
n=100	0.950	0.960	0.963	0.994	0.957	0.995	0.954	0.943
n=200	0.949	0.952	0.953	0.993	0.948	0.993	0.951	0.947

Table 9
Average Interval Width for N=20, 40, 60, 100, and 200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates for 13 combinations of a, b, and c' in Study 2.

	Method							
Sample Size	norm	prod	YMd	YMi	EFMd	EFMi	PercBoot	BCBoot
n=20	0.527	0.598	0.659	0.350	0.513	0.357	0.621	0.645
n=40	0.332	0.359	0.380	0.227	0.342	0.227	0.358	0.370
n=60	0.259	0.274	0.280	0.179	0.265	0.178	0.272	0.279
n=100	0.193	0.201	0.205	0.135	0.191	0.130	0.199	0.203
n=200	0.133	0.136	0.137	0.093	0.130	0.094	0.135	0.137

Table 10
Average Imbalance for N=20, 40, 60, 100, and 200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates for 13 combinations of a, b, and c'in Study 2.

	Method

Sample Size	norm	prod	YMd	YMi	EFMd	EFMi	PercBoot	BCBoot
n=20	0.038	0.014	0.015	0.002	0.028	0.001	0.020	0.016
n=40	0.032	0.009	0.008	0.002	0.019	0.002	0.015	0.010
n=60	0.037	0.013	0.014	0.003	0.022	0.002	0.017	0.012
n=100	0.035	0.011	0.011	0.002	0.023	0.001	0.012	0.009
n=200	0.027	0.010	0.012	0.002	0.012	0.004	0.011	0.004

Table 11

The fractions of true precision values of a and b to obtain the simulated values of precision parameters in prior distributions for a and b.

	prec(*a*)					prec(*b*)				
N	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
20	0.001	0.006	0.056	0.556	5.556	0.000	0.006	0.059	0.588	5.882
40	0.000	0.003	0.026	0.263	2.632	0.000	0.003	0.027	0.270	2.703
60	0.000	0.002	0.017	0.172	1.724	0.000	0.002	0.017	0.175	1.754
100	0.000	0.001	0.010	0.102	1.020	0.000	0.001	0.010	0.103	1.031
200	0.000	0.000	0.005	0.050	0.505	0.000	0.000	0.005	0.051	0.508

Table 12

Power for N=20 for the Bayesian method of coefficients for conditions in which the precision in the prior for b set to 10^{-3}, and the precision in the prior for a equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, followed by the conditions in which the precision in the prior for a was set to 10^{-3}, and the variance in the prior for b equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} in Study 3.

			Condition									
			Prior Prec(*a*)					Prior Prec(*b*)				
a	*b*	*c'*	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
.14	.14	0	0.003	0.003	0.003	0.002	0.005	0.003	0.003	0.003	0.002	0.002
.39	.39	0	0.078	0.081	0.113	0.134	0.32	0.079	0.078	0.102	0.139	0.338
.59	.59	0	0.354	0.374	0.428	0.564	0.631	0.357	0.353	0.42	0.564	0.663
.14	.14	.39	0.003	0.004	0.002	0.001	0.006	0.003	0.003	0.004	0.003	0.004
.39	.39	.39	0.083	0.086	0.097	0.147	0.369	0.088	0.081	0.099	0.152	0.345
.59	.59	.39	0.348	0.363	0.394	0.561	0.627	0.349	0.35	0.383	0.592	0.665

Table 13

Power for N=40 for the Bayesian method of coefficients for conditions in which the precision in the prior for b set to 10^{-3}, and the precision in the prior for a equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, followed by the conditions in which the precision in the prior for a was set to 10^{-3}, and the variance in the prior for b equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} in Study 3.

			Condition									
			Prior Prec(*a*)					Prior Prec(*b*)				
a	*b*	*c'*	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
.14	.14	0	0.011	0.010	0.010	0.009	0.022	0.011	0.013	0.007	0.010	0.017
.39	.39	0	0.357	0.353	0.378	0.469	0.625	0.350	0.345	0.379	0.446	0.633
.59	.59	0	0.860	0.870	0.860	0.914	0.926	0.860	0.845	0.871	0.912	0.917
.14	.14	.39	0.007	0.008	0.006	0.011	0.020	0.007	0.008	0.009	0.008	0.018
.39	.39	.39	0.369	0.371	0.383	0.464	0.606	0.368	0.359	0.389	0.461	0.635
.59	.59	.39	0.857	0.859	0.870	0.917	0.915	0.856	0.844	0.873	0.914	0.918

Table 14

Power for N=60 for the Bayesian method of coefficients for conditions in which the precision in the prior for b set to 10^{-3}, and the precision in the prior for a equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, followed by the conditions in which the precision in the prior for a was set to 10^{-3}, and the variance in the prior for b equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} in Study 3.

			Condition									
			Prior Prec(*a*)					Prior Prec(*b*)				
a	*b*	*c'*	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{-2}	10^{-1}	10^{0}	10^{-1}	10^{2}
.14	.14	0	0.029	0.029	0.032	0.025	0.036	0.029	0.028	0.034	0.014	0.036
.39	.39	0	0.675	0.676	0.660	0.712	0.828	0.673	0.676	0.700	0.718	0.789
.59	.59	0	0.970	0.974	0.972	0.986	0.982	0.969	0.971	0.965	0.992	0.986
.14	.14	.39	0.016	0.018	0.017	0.016	0.044	0.018	0.018	0.020	0.026	0.042
.39	.39	.39	0.679	0.673	0.664	0.713	0.817	0.670	0.670	0.698	0.728	0.817
.59	.59	.39	0.969	0.970	0.970	0.982	0.983	0.970	0.967	0.963	0.980	0.993

Table 15

Power for N=100 for the Bayesian method of coefficients for conditions in which the precision in the prior for b set to 10^{-3}, and the precision in the prior for a equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, followed by the conditions in which the precision in the prior for a was set to 10^{-3}, and the variance in the prior for b equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} in Study 3.

			Condition									
			Prior Prec(*a*)					Prior Prec(*b*)				
a	*b*	*c'*	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
.14	.14	0	0.046	0.048	0.052	0.058	0.117	0.046	0.046	0.051	0.055	0.098
.39	.39	0	0.917	0.916	0.916	0.939	0.967	0.918	0.915	0.927	0.960	0.964
.59	.59	0	0.999	0.999	0.999	0.999	1.000	0.999	0.999	0.999	1.000	1.000
.14	.14	.39	0.047	0.046	0.045	0.061	0.102	0.049	0.047	0.047	0.048	0.122

.39	.39	.39	0.911	0.911	0.913	0.922	0.946	0.910	0.913	0.923	0.944	0.961
.59	.59	.39	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 16

Power for N=200 for the Bayesian method of coefficients for conditions in which the precision in the prior for b set to 10^{-3}, and the precision in the prior for a equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, followed by the conditions in which the precision in the prior for a was set to 10^{-3}, and the variance in the prior for b equal to 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} in Study 3.

			Condition									
			Prior Prec(*a*)					Prior Prec(*b*)				
a	*b*	*c'*	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}	10^{-2}	10^{-1}	10^{0}	10^{1}	10^{2}
.14	.14	0	0.237	0.24	0.234	0.249	0.323	0.238	0.239	0.231	0.231	0.321
.39	.39	0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000
.59	.59	0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.14	.14	.39	0.217	0.216	0.217	0.214	0.290	0.217	0.220	0.219	0.204	0.300
.39	.39	.39	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.998
.59	.59	.39	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 17

Interval estimates for the indirect effect of making images for words versus repeating words(X) on the number of words recalled (Y) through imagery (M) computed using the eight methods evaluated in Study 2.

Method	95% CI Year 2 data	95% CI Years 1 and 2 data
Normal theory	[2.70, 10.39]	[2.36, 5.81]
Distribution of the product	[2.78, 10.50]	[2.44, 5.90]
Bayesian method of coefficients with diffuse priors	[2.78, 10.55]	[2.42, 6.22]
Bayesian method of coefficients with informative priors	[1.81, 4.30]	-
Bayesian method of covariances with diffuse priors	[2.65, 7.70]	[2.21, 5.28]
Bayesian method of covariances with informative priors	[2.08, 5.55]	-
Percentile bootstrap	[4.08, 10.24]	[0.64, 3.78]
Bias-corrected bootstrap	[4.14, 10.40]	[0.68, 3.98]

Figure 1

Type I error rate

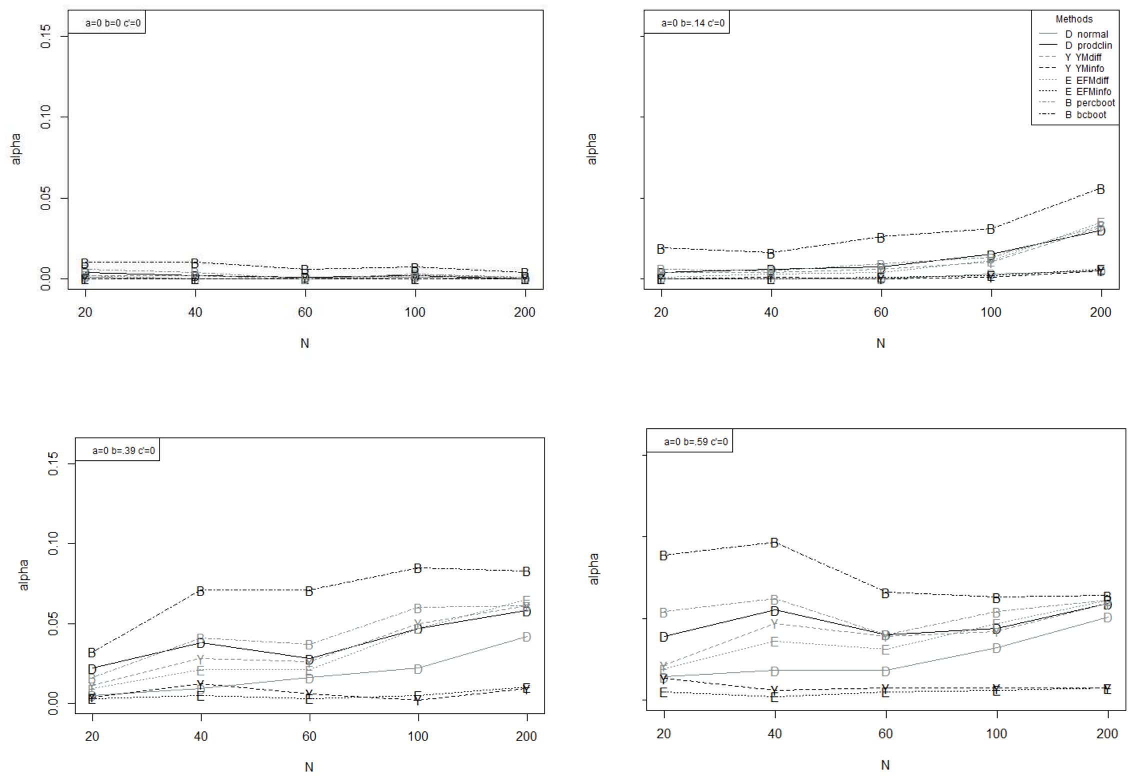
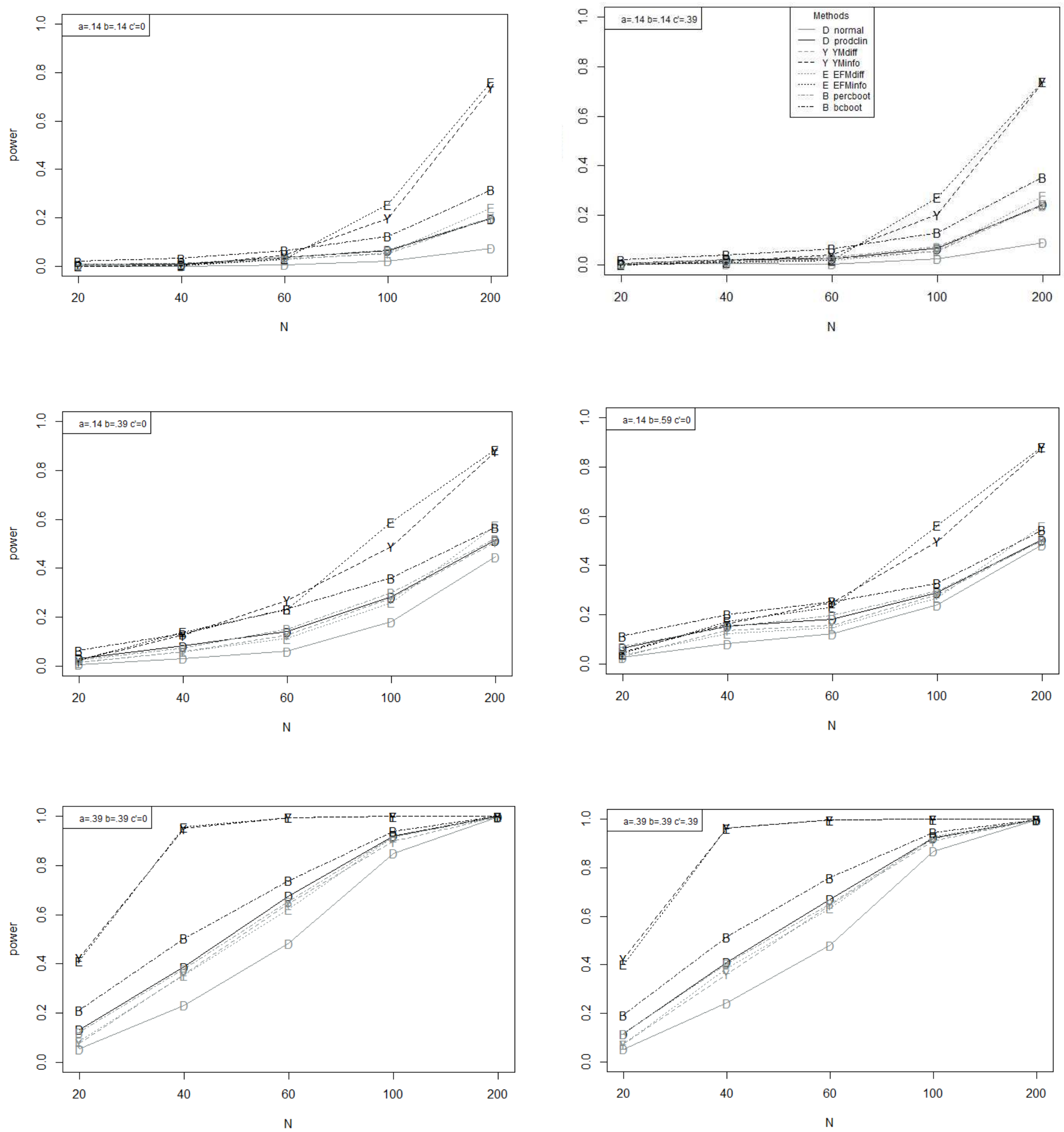


Figure 1. Trellis plot of Type I error rate for all methods and all parameter combinations as a function of sample size in Study 2. The letter markers indicate the following: D (distribution) codes normal theory (gray solid line) and distribution of the product (black solid line) confidence limits, Y (Yuan & MacKinnon, 2009) codes credibility intervals formed using the Bayesian method of coefficients with diffuse (gray dashed line) and informative (black dashed line) prior distributions, E (Enders, Fairchild, and MacKinnon, 2013) codes credibility intervals formed using the Bayesian method of covariances with diffuse (gray dotted line) and informative (black dotted line) distributions, and B (bootstrap) codes the percentile (gray dot-dash line) and bias-corrected (black dot-dash line) bootstrap confidence limits.

Figure 2

Power



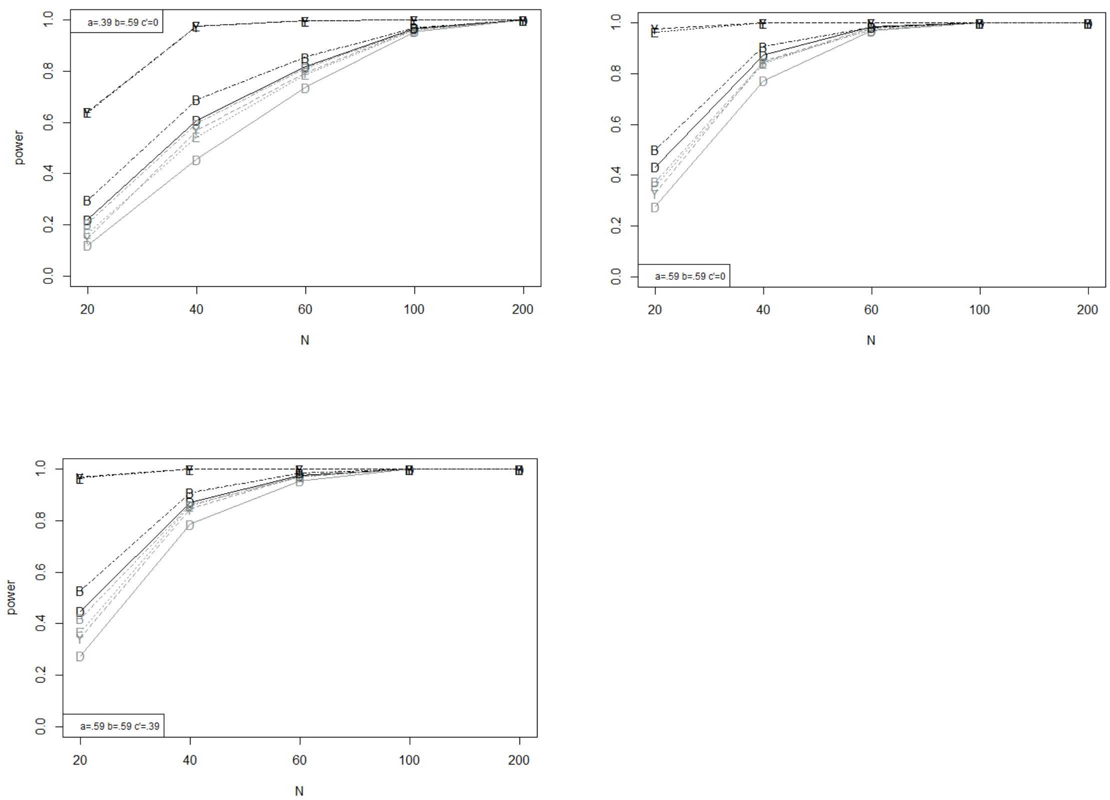
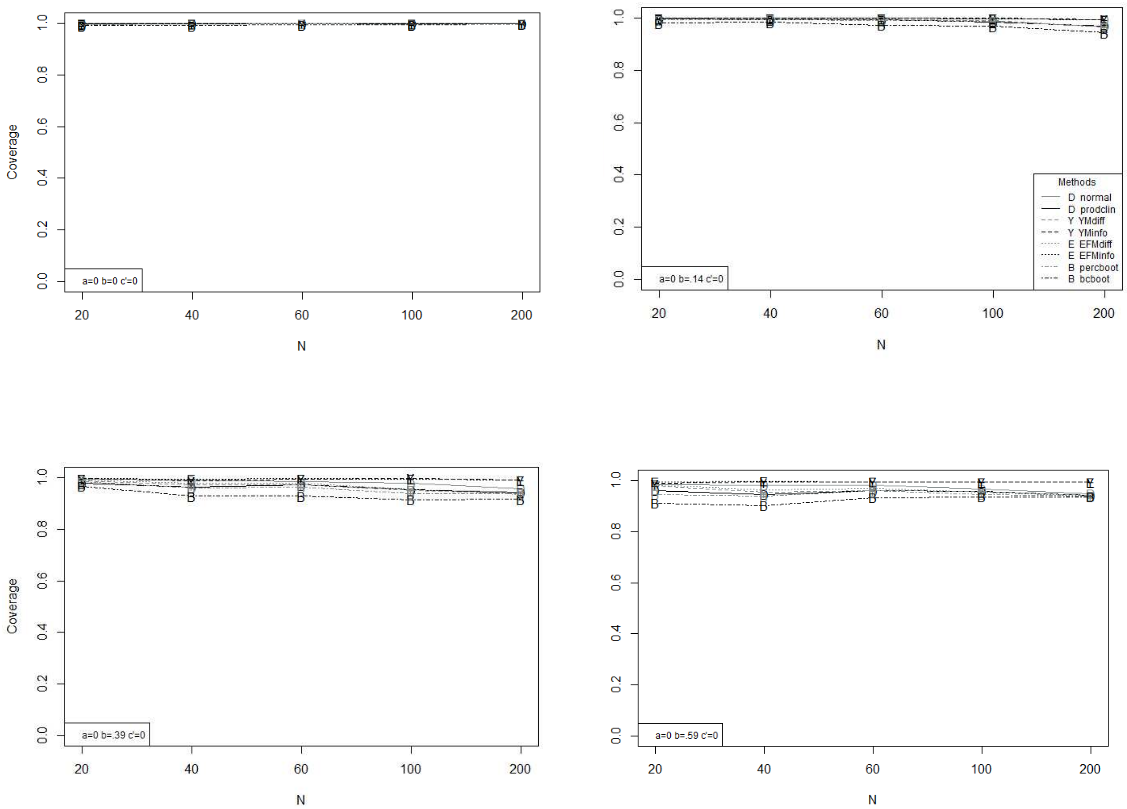
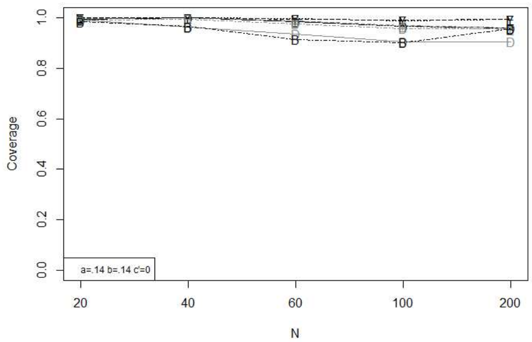


Figure 2. Trellis plot of Power for all methods and all parameter combinations as a function of sample size in Study 2. The letter markers indicate the following: D (distribution) codes normal theory (gray solid line) and distribution of the product (black solid line) confidence limits, Y (Yuan & MacKinnon, 2009) codes credibility intervals formed using the Bayesian method of coefficients with diffuse (gray dashed line) and informative (black dashed line) prior distributions, E (Enders, Fairchild, and MacKinnon, 2013) codes credibility intervals formed using the Bayesian method of covariances with diffuse (gray dotted line) and informative (black dotted line) distributions, and B (bootstrap) codes the percentile (gray dot-dash line) and bias-corrected (black dot-dash line) bootstrap confidence limits.

Figure 3

Coverage

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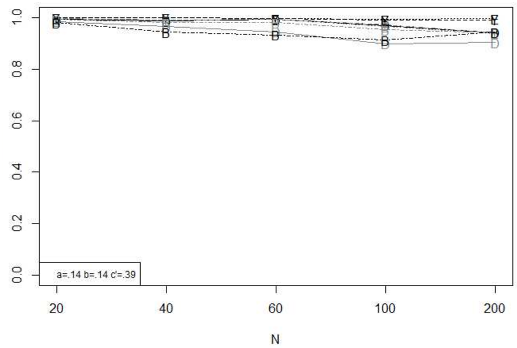
0.4
0.6
0.8
1.0
20
40
60
100
200
N
a=.14 b=.14 c'=0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1.0
20
40
60
100
200
N
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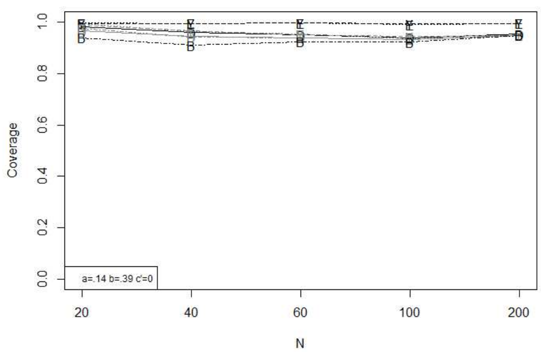
Coverage
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0.4
0.6
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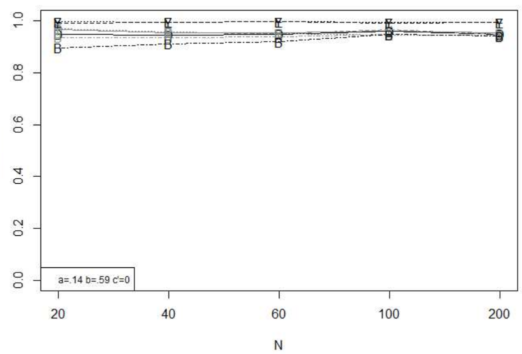
20
40
60
100
200
N
a=.14 b=.39 c'=0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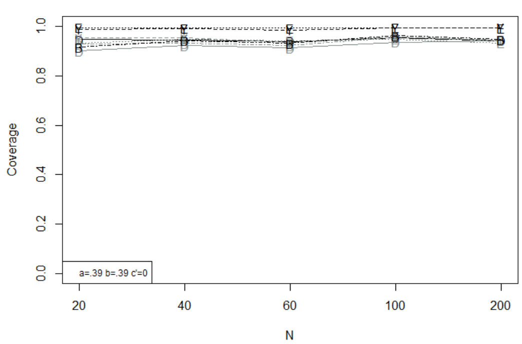
0.6
0.8
1.0
20
40
60
100
200
N
a=.14 b=.59 c'=0


Coverage
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0.2
0.4
0.6
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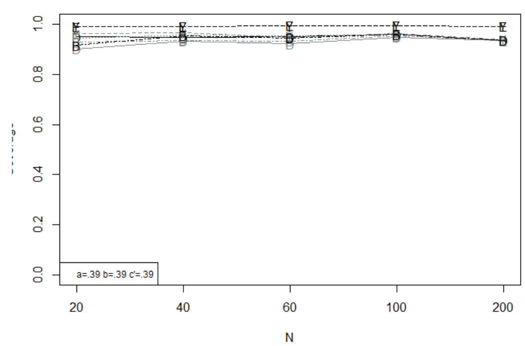
1.0
20
40
60
100
200
N
a=.39 b=.39 c'=0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0.2
0.4
0.6
0.8
1.0
20
40
60
100
200
N
a=.39 b=.39 c'=.39

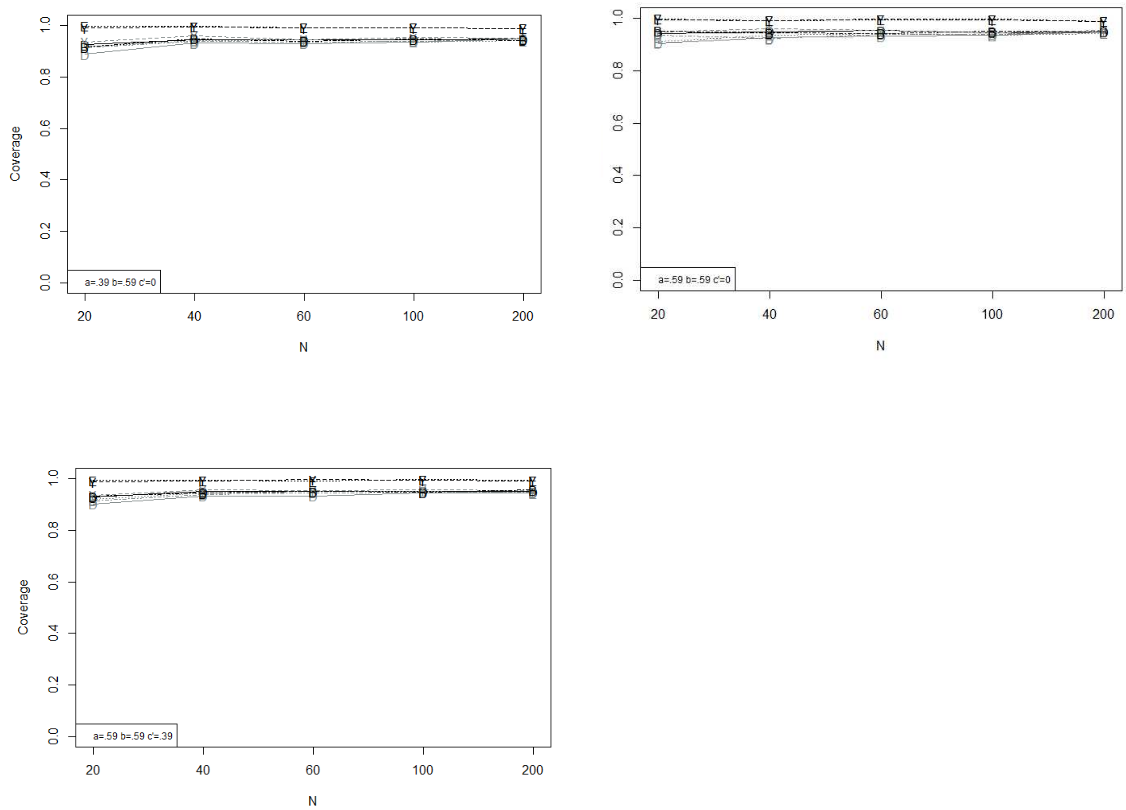
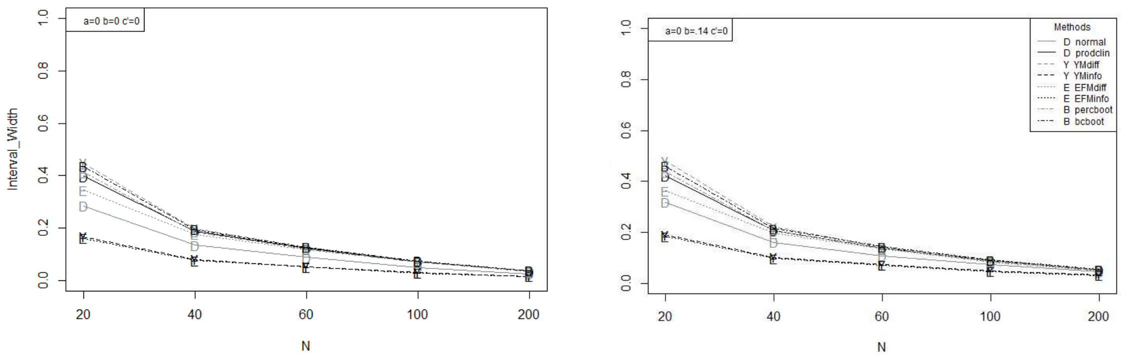
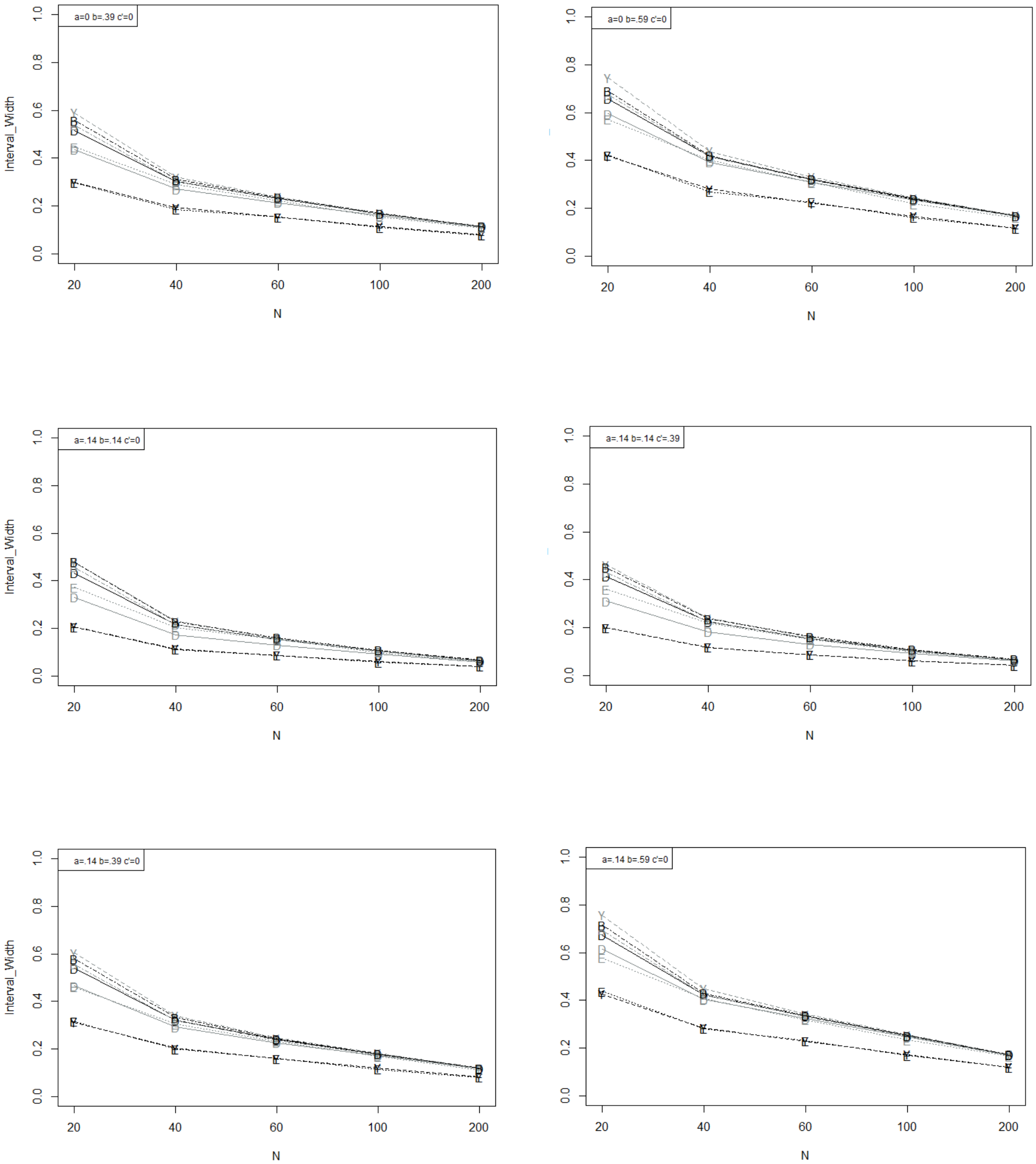


Figure 3. Trellis plot of coverage for all methods and all parameter combinations as a function of sample size in Study 2. The letter markers indicate the following: D (distribution) codes normal theory (gray solid line) and distribution of the product (black solid line) confidence limits, Y (Yuan & MacKinnon, 2009) codes credibility intervals formed using the Bayesian method of coefficients with diffuse (gray dashed line) and informative (black dashed line) prior distributions, E (Enders, Fairchild, and MacKinnon, 2013) codes credibility intervals formed using the Bayesian method of covariances with diffuse (gray dotted line) and informative (black dotted line) distributions, and B (bootstrap) codes the percentile (gray dot-dash line) and bias-corrected (black dot-dash line) bootstrap confidence limits.

Figure 4

Interval Width




a=0 b=.39 c'=0
a=0 b=.59 c'=0
a=.14 b=.14 c'=0
a=.14 b=.14 c'=.39
a=.14 b=.39 c'=0
a=.14 b=.59 c'=0
Interval_Width
N

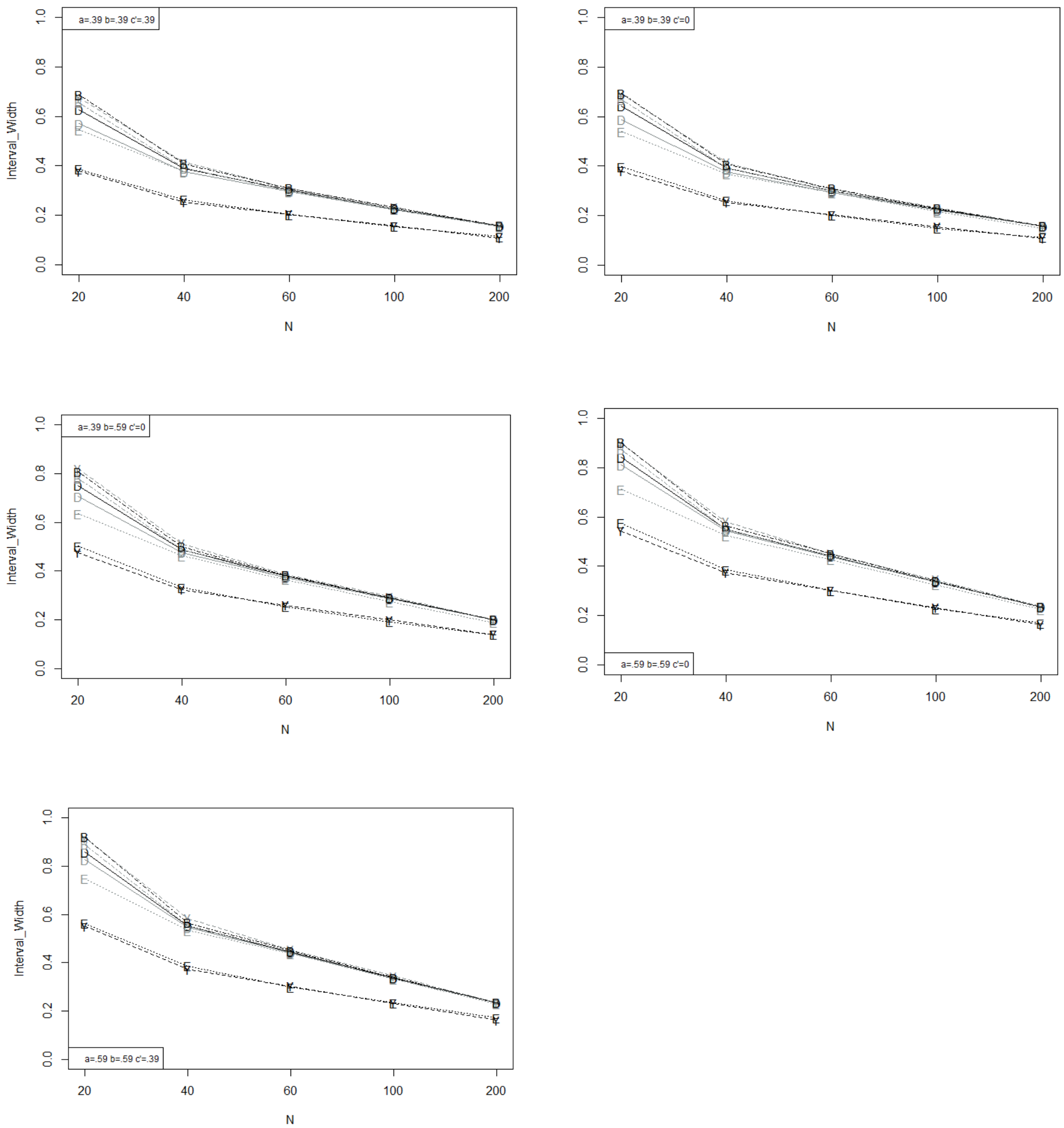
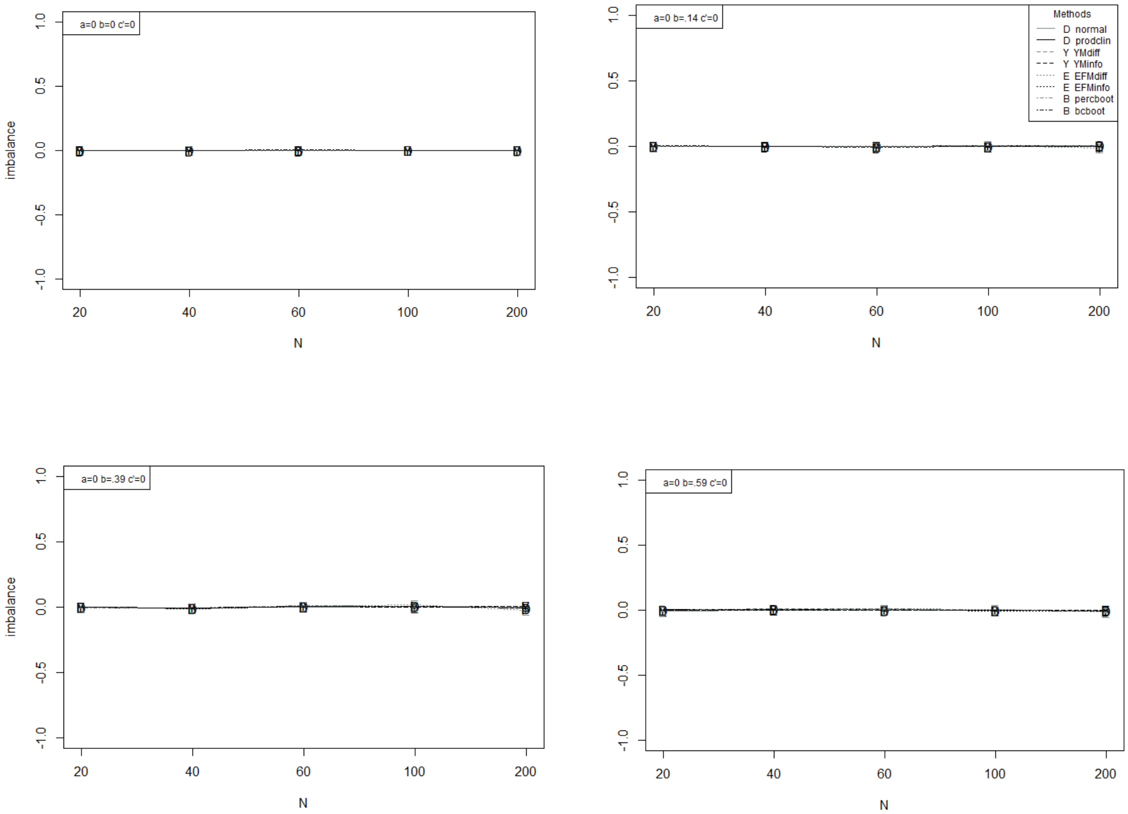
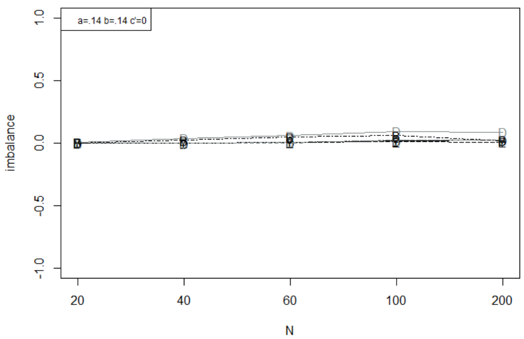


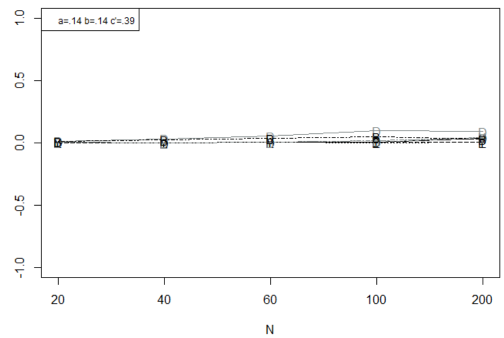
Figure 4. Trellis plot of interval width for all methods and all parameter combinations as a function of sample size in Study 2. The letter markers indicate the following: D (distribution) codes normal theory (gray solid line) and distribution of the product (black solid line) confidence limits, Y (Yuan & MacKinnon, 2009) codes credibility intervals formed using the Bayesian method of coefficients with diffuse (gray dashed line) and informative (black dashed line) prior distributions, E (Enders, Fairchild, and MacKinnon, 2013) codes credibility intervals formed using the Bayesian method of covariances with diffuse (gray dotted line) and informative (black dotted line) distributions, and B (bootstrap) codes the percentile (gray dot-dash line) and bias-corrected (black dot-dash line) bootstrap confidence limits.

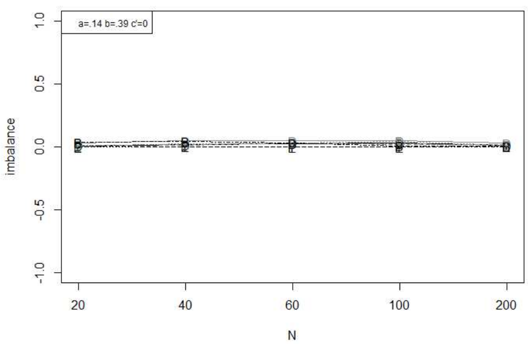
Figure 5

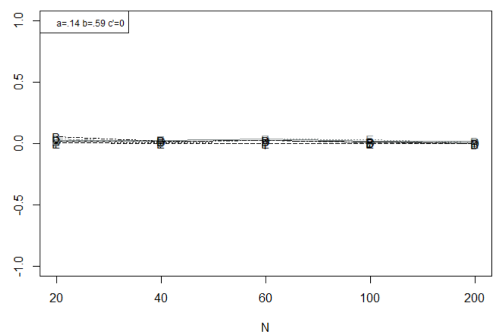
Imbalance

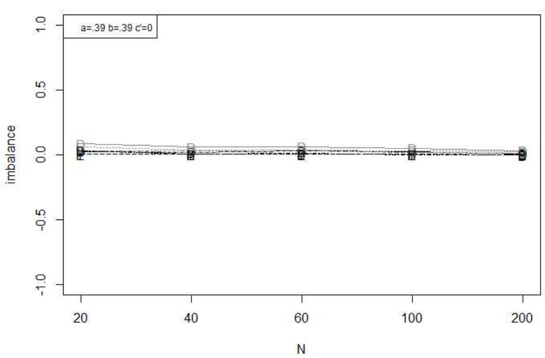


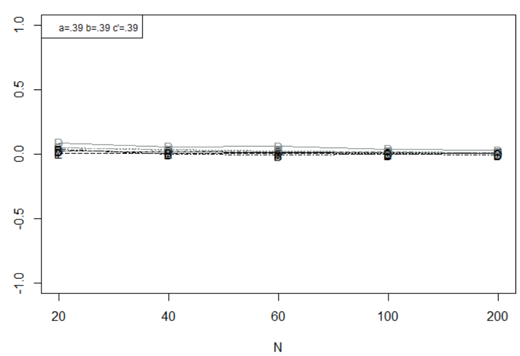

a=.14 b=.14 c'=0
imbalance
N


a=.14 b=.14 c'=.39
N


a=.14 b=.39 c'=0
imbalance
N


a=.14 b=.59 c'=0
N


a=.39 b=.39 c'=0
imbalance
N


a=.39 b=.39 c'=.39
N

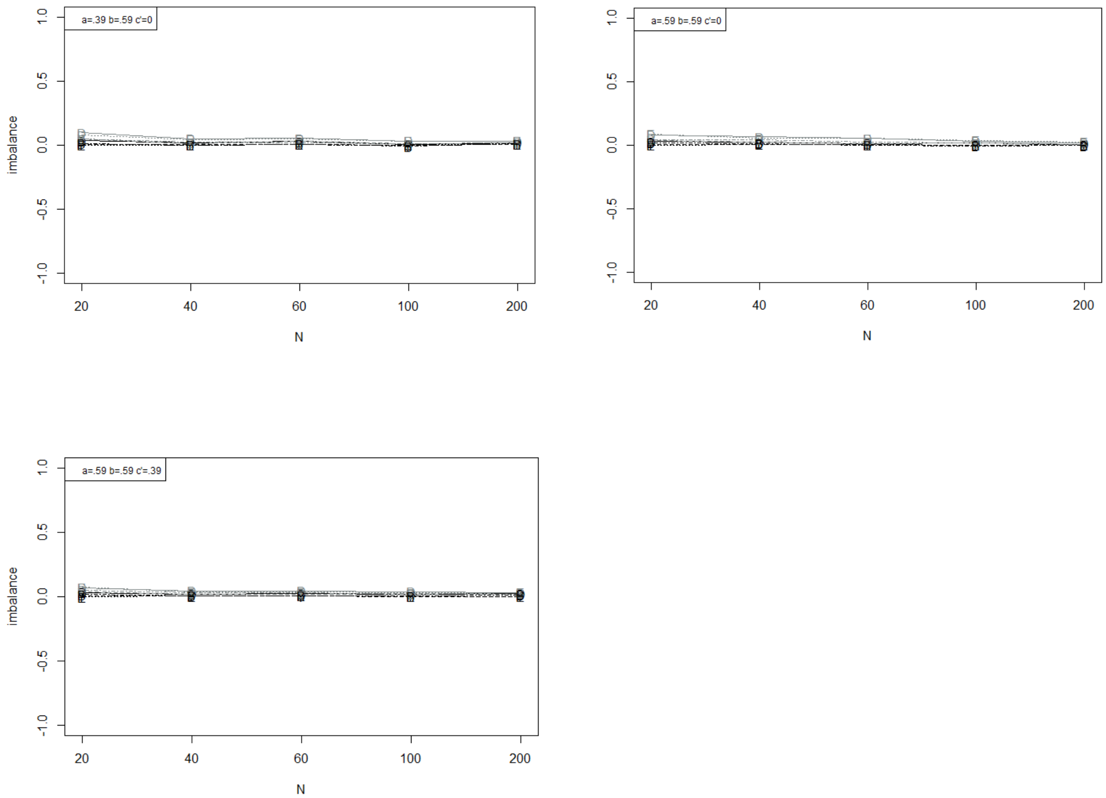


Figure 5. Trellis plot of imbalance for all methods and all parameter combinations as a function of sample size in Study 2. The letter markers indicate the following: D (distribution) codes normal theory (gray solid line) and distribution of the product (black solid line) confidence limits, Y (Yuan & MacKinnon, 2009) codes credibility intervals formed using the Bayesian method of coefficients with diffuse (gray dashed line) and informative (black dashed line) prior distributions, E (Enders, Fairchild, and MacKinnon, 2013) codes credibility intervals formed using the Bayesian method of covariances with diffuse (gray dotted line) and informative (black dotted line) distributions, and B (bootstrap) codes the percentile (gray dot-dash line) and bias-corrected (black dot-dash line) bootstrap confidence limits.

Figure 6

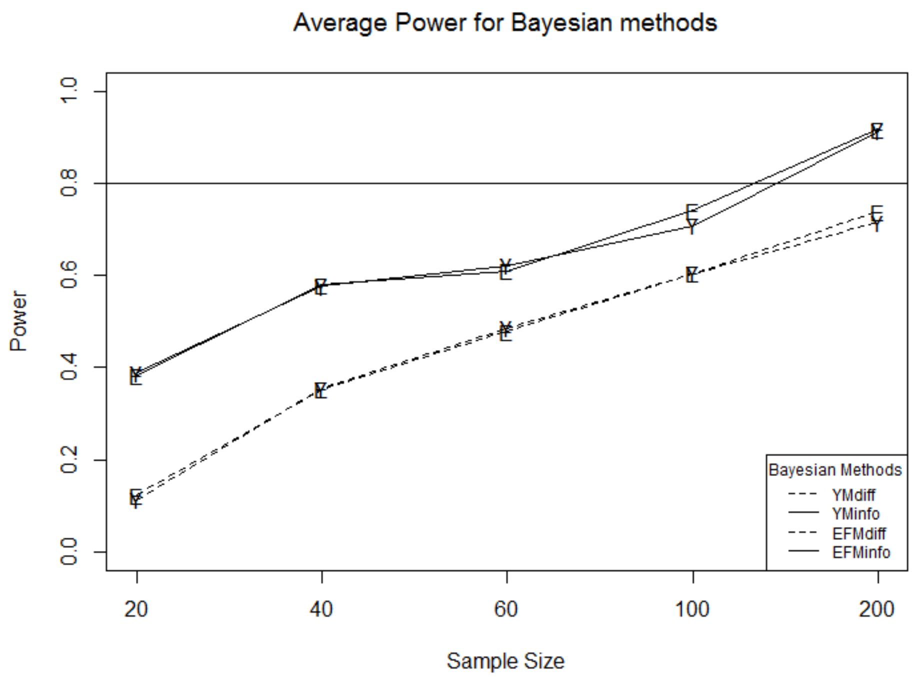


Figure 6. Plot of power for the four Bayesian methods as a function of sample size in Study 2; the dashed line with the letter marker Y (Yuan and MacKinnon, 2009) represents the Bayesian method of coefficients with diffuse prior distributions, the solid line with the letter marker Y (Yuan and MacKinnon, 2009) represents the Bayesian method of coefficients with informative prior distributions, the dashed line with the letter marker E (Enders, Fairchild, and MacKinnon, 2013) represents the Bayesian method of covariances with diffuse prior distributions, and the solid line with the letter marker E (Enders, Fairchild, and MacKinnon, 2013) represents the Bayesian method of covariances with informative prior distributions.

Figure 7

Power as a function of precision of *a* and *b*

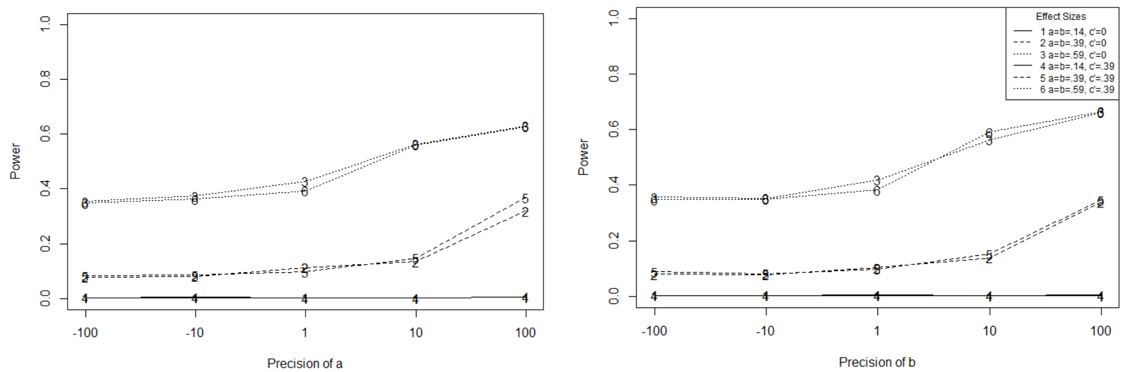


Figure 7. Plot of power for the method of coefficients as a function of the precision parameter in the prior distributions for *a* and *b* for N=20 in Study 3; the six lines represent six combinations of effect size. The number markers represent the different combinations of parameter values for *a, b,* and *c'*, and the three line types code for the different magnitudes of the mediation paths *a* and *b*: solid (small), dash (medium), and dot (large).

Figure 8

Power as a function of precision of *a* and *b*

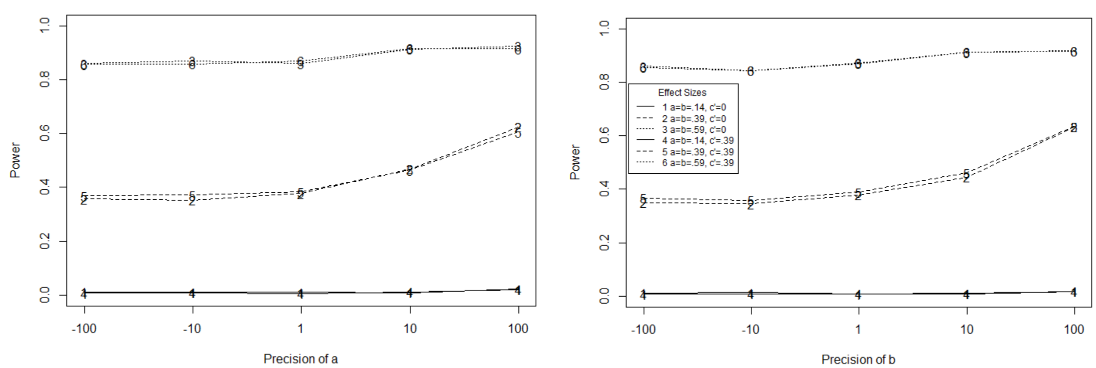


Figure 8. Plot of power for the method of coefficients as a function of the precision parameter in the prior distributions for *a* and *b* for N=40 in Study 3; the six lines represent six combinations of effect size. The number markers represent the different combinations of parameter values for *a, b,* and *c'*, and the three line types code for the different magnitudes of the mediation paths *a* and *b*: solid (small), dash (medium), and dot (large).

Figure 9

Power as a function of precision of *a* and *b*

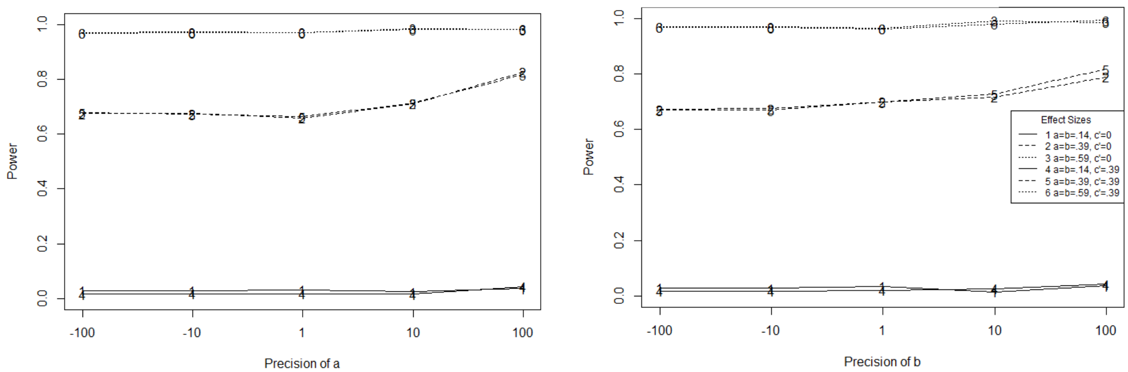


Figure 9. Plot of power for the method of coefficients as a function of the precision parameter in the prior distributions for *a* and *b* for N=60 in Study 3; the six lines represent six combinations of effect size. The number markers represent the different combinations of parameter values for *a*, *b*, and *c'*, and the three line types code for the different magnitudes of the mediation paths *a* and *b*: solid (small), dash (medium), and dot (large).

Figure 10

Power as a function of precision of *a* and *b*

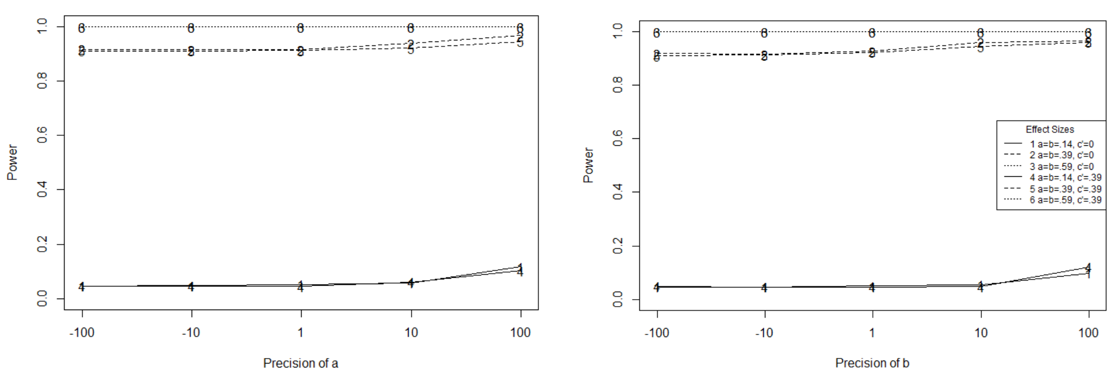


Figure 10. Plot of power for the method of coefficients as a function of the precision parameter in the prior distributions for *a* and *b* for N=100 in Study 3; the six lines represent six combinations of effect size. The number markers represent the different combinations of parameter values for *a*, *b*, and *c'*, and the three line types code for the different magnitudes of the mediation paths *a* and *b*: solid (small), dash (medium), and dot (large).

Figure 11

Power as a function of precision of *a* and *b*

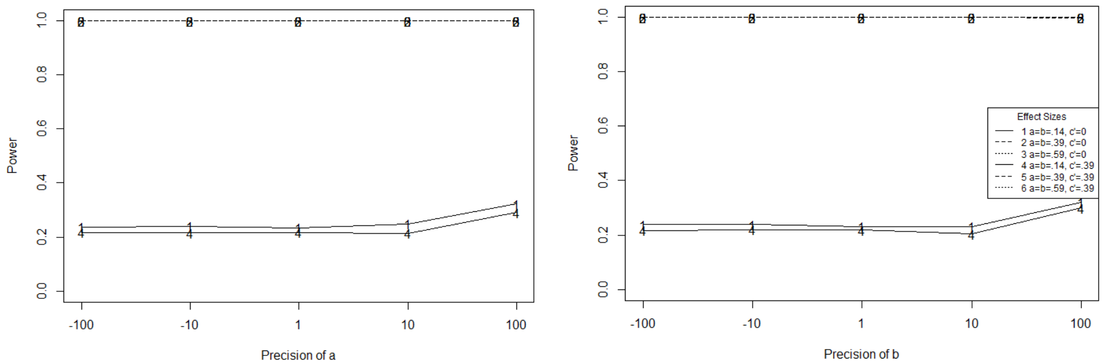
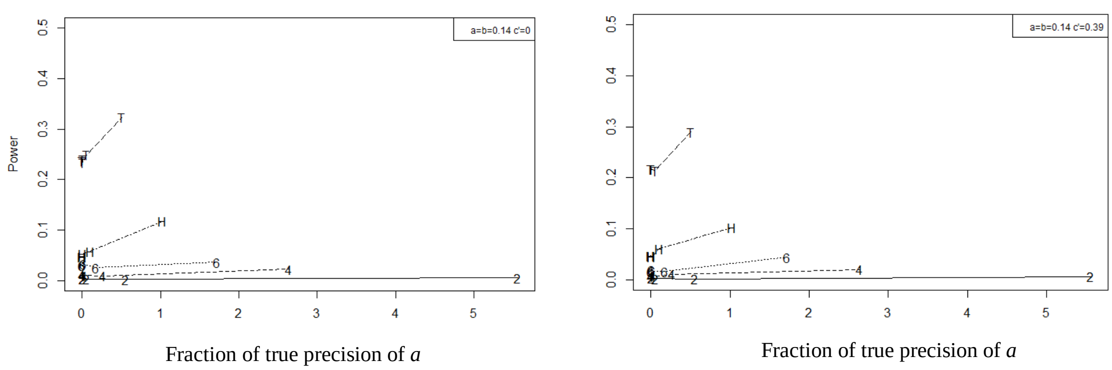


Figure 11. Plot of power for the method of coefficients as a function of the precision parameter in the prior distribution for *a* and *b* for N=200 in Study 3; the six lines represent six combinations of effect size. The number markers represent the different combinations of parameter values for *a*, *b*, and *c'*, and the three line types code for the different magnitudes of the mediation paths *a* and *b*: solid (small), dash (medium), and dot (large).

Figure 12

Power regressed on the fraction of true precision of *a*



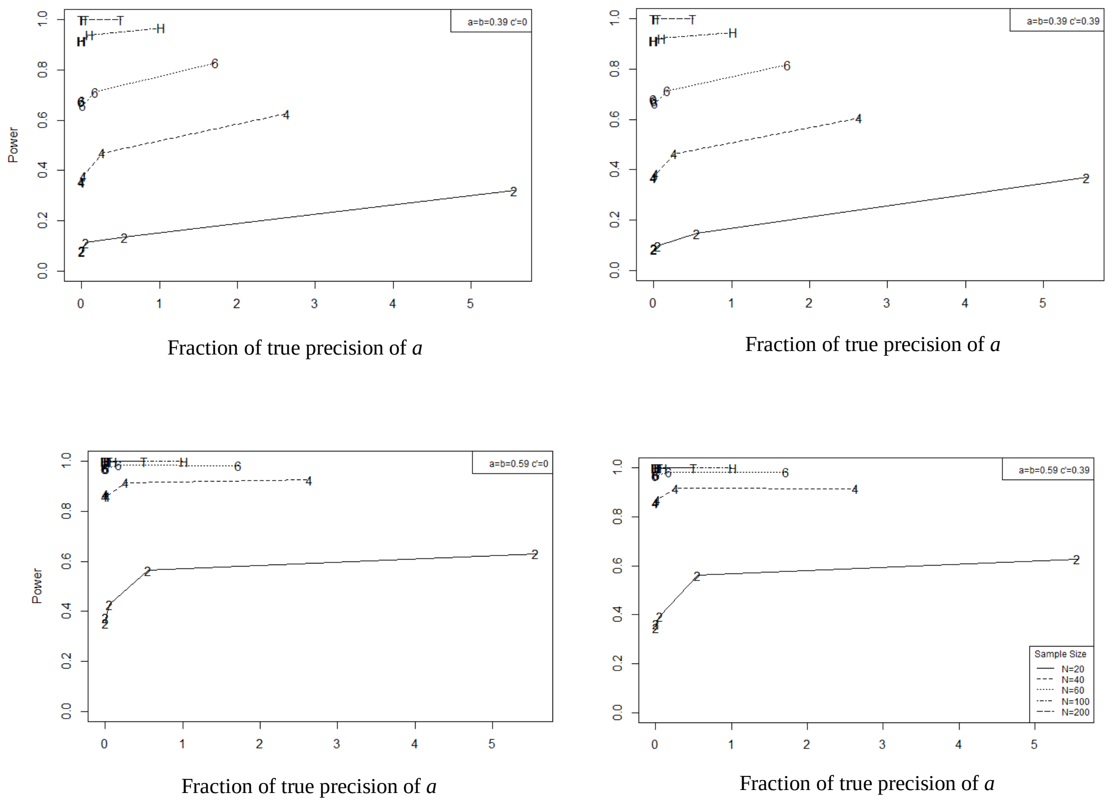
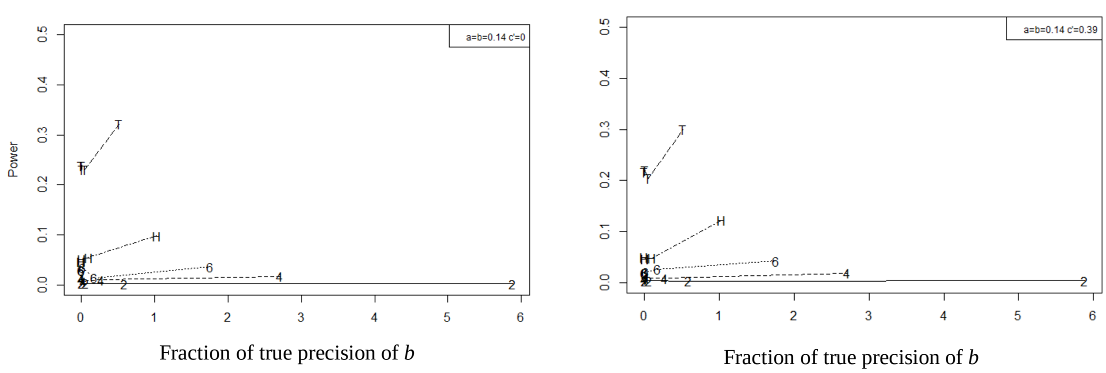


Figure 12. Trellis plot of empirical power for the Bayesian method of coefficients for all sample sizes and all parameter combinations as a function of fraction of true precision of parameter *a* in Study 3. The line types and number and letter markers indicate the sample size: the number 2 and the solid line stand for N = 20, the number 4 and the dashed line stand for N = 40, the number 6 and the dotted line stand for N = 60, the letter H and the dot-dash line stand for N = 100, and the letter T and the long-dash line stand for N = 200.

Figure 13

Power regressed on the fraction of true precision of *b*



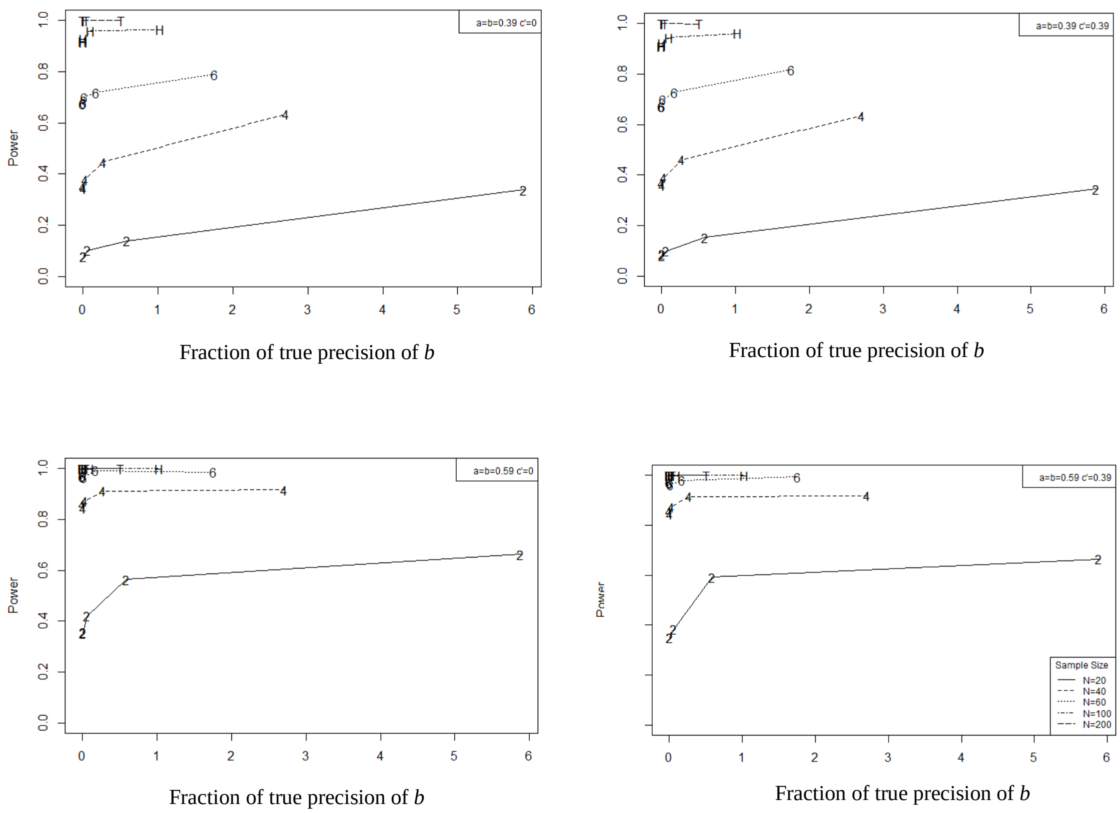


Figure 13. Trellis plot of empirical power for the Bayesian method of coefficients for all sample sizes and all parameter combinations as a function of the fraction of the true precision of parameter *b* in Study 3. The line types and number and letter markers indicate the sample size: the number 2 and the solid line stand for N = 20, the number 4 and the dashed line stand for N = 40, the number 6 and the dotted line stand for N = 60, the letter H and the dot-dash line stand for N = 100, and the letter T and the long-dash line stand for N = 200.

APPENDIX A

DOCUMENT NOTATION

α	Type I error, the rate at which a test incorrectly identifies the presence of a significant effect when no effect is present.
β	Type II error, the rate at which a test fails to find an effect that is truly present.
Λ_p	Sums of squares and cross-products of the inverse Wishart prior distribution.
Σ	Covariance matrix.
Σ_p	Covariance matrix from a prior study.
θ	Population value of the parameter of interest.
$\hat{\theta}$	Sample estimate of the parameter of interest.
$\hat{\phi}$	Estimate of the parameter of interest following a transformation in percentile bootstrap.
μ_a	Mean of the prior distribution for coefficient *a*.
μ_b	Mean of the prior distribution for coefficient *b*.
$\mu_{c'}$	Mean of the prior distribution for coefficient *c'*.
σ^2_{M}	True variance of M.
σ^2_{Y}	True variance of Y.
σ^2_1	Population error variance from the equation with X predicting Y.
σ^2_2	Population error variance from the equation with X predicting M for the single mediator model.
σ^2_3	Population error variance from the equation with X and M predicting Y for the single mediator model.
$1 - \beta$	Power, a test's ability to detect an effect when an effect is truly present.

a	Population path coefficient representing relationship between X and M.
$\hat{a}$	Sample path coefficient representing relationship between X and M.
b	Population path coefficient representing relationship between M and Y.
$\hat{b}$	Sample path coefficient representing relationship between M and Y.
c	Population path coefficient representing relationship between X and Y.
c'	Population path coefficient representing relationship between X and Y controlling for M in the single mediator model.
df_p	Degrees of freedom parameter of the inverse Wishart prior distribution.
e_j	Sample error variability in the mediation regression equations.
i_j	Population intercept in the mediation regression equations.
M	Mediator in the single mediator model.
m	Shape parameter of the Inverse-gamma distribution.
N	Sample size.
N_p	Sample size from a prior study.
n	Scale parameter of the Inverse-gamma distribution.
p-value	Significance level of a statistical test.
$p(\theta)$	Prior distribution for a parameter of interest.
$p(data \mid \theta)$	Sampling distribution of the data given the parameter.
$p(\theta \mid data)$	Posterior distribution of a parameter of interest.
$p(data)$	Normalizing constant in Bayes' theorem.
s_{ab}	Multivariate delta standard error for the single mediator model.
$s_{\hat{a}\hat{b}}$	Sample multivariate delta standard error for the single mediator model.

sd	Standard deviation of a distribution.
s_M^2	Population variance of M.
s_{MY}	Population covariance of M and Y.
s_{XM}	Population covariance of X and M.
s_{XY}	Population covariance of X and Y.
s_X^2	Population variance of X.
t	Normalizing transformation of a sample estimate used for percentile bootstrap.
W^{-1}	Inverse Wishart distribution.
X	Independent variable.
Y	Dependent variable.
z_0	The *z* value that corresponds to the proportion of times the bootstrap estimate exceeds the observed sample estimate of the parameter of interest.
$z_{\alpha/2}$	Value of z corresponding to the 2.5th percentile point of the standard normal distribution when = 0.05, with a value of -1.96.
$z_{1-\alpha/2}$	Value of *z* corresponding to the 97.5th percentile point of the standard normal distribution when α = 0.05, with a value of 1.96.

APPENDIX B

EQUATING PRIOR DISTRIBUTIONS FOR REGRESSION COEFFICIENTS WITH THE PRIOR DISTRIBUTIONS FOR THE COVARIANCE MATRIX

The purpose of this appendix is to show the direct arithmetic correspondence between the method of coefficients and the method of covariances. Given the information for the regression coefficients and error variances of M and Y, it is possible to calculate the values of the elements of the covariance matrix of X, M, and Y, and vice versa. The equations below outline the relationships between the parameters in the method of coefficients and the parameters in the method of covariances.

Equations in the method of coefficients:

$$X = X$$
$$M = aX + e_M$$
$$Y = bM + c'X + e_Y$$

Variance-Covariance Matrix of X, M, and Y:

$$\begin{bmatrix} Var(X) & & \\ Cov(X,M) & Var(M) & \\ Cov(X,Y) & Cov(M,Y) & Var(Y) \end{bmatrix}$$

Expressing the model parameters in variance and covariance terms from MacKinnon (2008) on pages 86-89:

$$a = \frac{Cov(X,M)}{Var(X)}$$

$$\sigma^2_{\hat{a}T} = \frac{Var(M) - \dfrac{Cov(X,M)^2}{Var(X)}}{(N-1)Var(X)}$$

$$b = \frac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}$$

$$\sigma^2_{\hat{b}T} = \frac{\begin{array}{l}[Var(Y) - \left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]Var(X) - \\ \left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]^2 Var(M) - \\ \left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]\left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]Cov(X,M)]\dfrac{1}{Var(M)}\end{array}}{(N-1)(1 - \dfrac{Cov(X,M)^2}{Var(X)Var(Y)})}$$

$$c' = \frac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}$$

$$\sigma^2_{\hat{c}'T} = \frac{\begin{array}{l}[Var(Y) - \left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]Var(X) - \\ \left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]^2 Var(M) - \\ \left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]\left[\dfrac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]Cov(X,M)]\dfrac{1}{Var(X)}\end{array}}{(N-1)(1 - \dfrac{Cov(X,M)^2}{Var(X)Var(Y)})}$$

$$\sigma^2_X = Var(X)$$

$$\sigma^2_{e_M} = Var(M) - a^2 Var(X) = Var(M) - \frac{Cov(X,M)^2}{Var(X)}$$

$$\sigma_{e_Y}^2 = Var(Y) - c'Var(X) - b^2 Var(M) - 2bc'Cov(X,M) =$$

$$Var(Y) - \left[\frac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]Var(X) -$$

$$\left[\frac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]^2 Var(M) -$$

$$\left[\frac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]\left[\frac{Cov(M,Y)Var(X) - Cov(X,M)Cov(X,Y)}{Var(X)Var(M) - Cov(X,M)^2}\right]Cov(X,M)$$

Expressing the variance and covariance terms in terms of model parameters:

$$Var(X) = \sigma_X^2$$

$$Var(M) = a^2 Var(X) + \sigma_{e_M}^2 + 2Cov(X, e_M) = a^2\sigma_X^2 + \sigma_{e_M}^2$$

$$Var(Y) = b^2 Var(M) + c'^2 Var(X) + \sigma_{e_Y}^2 + 2bc'Cov(M,X) =$$
$$b^2(a^2\sigma_X^2 + \sigma_{e_M}^2) + c'^2\sigma_X^2 + \sigma_{e_Y}^2 + 2bc'a\sigma_X^2$$

$$Cov(X,M) = a\sigma_X^2$$

$$Cov(M,Y) = Cov(aX + e_M, bM + c'X + e_Y) = abCov(X,M) + ac'Var(X) =$$
$$a^2 b\sigma_X^2 + ac'\sigma_X^2$$

$$Cov(X,Y) = Cov(X, bM + c'X + e_Y) = bCov(X,M) + c'Var(X) =$$
$$ba\sigma_X^2 + c'\sigma_X^2$$

The difference in the two specifications of prior distributions lies in how X is modeled. In the method of coefficients, X is not modeled as a random variable, and consequently does not have a prior or posterior distribution. In the method of covariances, X is treated as a random variable, and the variance of X and the covariances of X with M and Y have prior and posterior distributions. Thus, even though there is an arithmetic correspondence between the two methods, they may not be conceptually and numerically equivalent in the Bayesian framework.

In the case of prior distributions, it is possible to encode the same information under both frameworks by using the formulas above to numerically equate the regression coefficients and covariance matrix elements. In Study 2 of this project this was not done for the diffuse prior distribution case, but in theory it is possible to do it for all parameters in the two frameworks. The informative prior distributions in Study 2 were based on the true values of parameters, thus even though the above formulas were not used to equate prior distributions for the method of coefficients and the method of covariances, the prior distributions were constructed from the same data set and therefore represent the same prior assumptions. The only differences that may exist between the two informative prior distribution specifications are due to the fact that in the method of covariances X is modeled as a random variable, whereas in the method of coefficients it is not.

APPENDIX C

TABLES CONTAINING VALUES OF TYPE I ERROR RATE, POWER, COVERAGE, INTERVAL WIDTH, AND IMBALANCE FOR ALL PARAMETER COMBINATIONS IN STUDIES 1 AND 2

Table 18

Type I error rate for N=20 in Study 1 (note: Type I error is not observable when both a and b are non-zero). The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.001	0.002	0.002	0.004	0.013
0	.14	0.000	0.001	0.001	0.006	0.011
0	.39	0.005	0.012	0.012	0.026	0.039
0	.59	0.011	0.020	0.021	0.038	0.064
.14	0	0.000	0.000	0.000	0.008	0.018
.39	0	0.007	0.021	0.021	0.025	0.052
.59	0	0.009	0.030	0.029	0.034	0.067
				c'=.14		
0	0	0.001	0.002	0.002	0.006	0.013
0	.14	0.000	0.001	0.001	0.005	0.013
0	.39	0.002	0.005	0.005	0.021	0.043
0	.59	0.017	0.035	0.034	0.045	0.084
.14	0	0.005	0.008	0.007	0.013	0.028
.39	0	0.007	0.013	0.013	0.018	0.040
.59	0	0.024	0.051	0.051	0.063	0.088
				c'=.39		
0	0	0.000	0.002	0.002	0.002	0.008
0	.14	0.000	0.004	0.004	0.006	0.014
0	.39	0.007	0.018	0.019	0.036	0.064
0	.59	0.018	0.039	0.039	0.057	0.086
.14	0	0.001	0.003	0.004	0.006	0.020
.39	0	0.005	0.022	0.022	0.025	0.052
.59	0	0.015	0.047	0.047	0.053	0.088
				c'=.59		
0	0	0.001	0.002	0.002	0.004	0.010
0	.14	0.000	0.001	0.001	0.002	0.011
0	.39	0.005	0.018	0.018	0.029	0.060
0	.59	0.022	0.043	0.043	0.063	0.094
.14	0	0.002	0.005	0.005	0.010	0.025
.39	0	0.009	0.018	0.019	0.029	0.043
.59	0	0.017	0.035	0.038	0.048	0.081

Table 19

Type I error rate for N=40 in Study 1 (note: Type I error is not observable when both a and b are non-zero). The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*	*c'*=0				
0	0	0.000	0.002	0.002	0.005	0.014
0	.14	0.000	0.002	0.002	0.008	0.022
0	.39	0.007	0.021	0.021	0.038	0.067
0	.59	0.024	0.035	0.034	0.049	0.080
.14	0	0.002	0.005	0.005	0.006	0.019
.39	0	0.012	0.023	0.023	0.033	0.062
.59	0	0.026	0.036	0.036	0.062	0.092
		c'=.14				
0	0	0.000	0.000	0.000	0.003	0.010
0	.14	0.001	0.003	0.003	0.006	0.013
0	.39	0.009	0.020	0.020	0.029	0.058
0	.59	0.025	0.042	0.041	0.054	0.093
.14	0	0.000	0.002	0.002	0.002	0.016
.39	0	0.006	0.019	0.019	0.044	0.076
.59	0	0.018	0.028	0.028	0.044	0.073
		c'=.39				
0	0	0.000	0.004	0.004	0.002	0.009
0	.14	0.001	0.004	0.004	0.009	0.019
0	.39	0.004	0.017	0.017	0.023	0.057
0	.59	0.015	0.034	0.033	0.050	0.086
.14	0	0.001	0.002	0.002	0.006	0.017
.39	0	0.008	0.014	0.014	0.029	0.060
.59	0	0.016	0.033	0.034	0.056	0.093
		c'=.59				
0	0	0.001	0.004	0.004	0.008	0.014
0	.14	0.001	0.006	0.006	0.012	0.026
0	.39	0.004	0.020	0.020	0.029	0.058
0	.59	0.017	0.038	0.038	0.050	0.080
.14	0	0.000	0.003	0.002	0.007	0.018
.39	0	0.003	0.013	0.012	0.043	0.072
.59	0	0.025	0.039	0.039	0.063	0.088

Table 20

Type I error rate for N=60 in Study 1 (note: Type I error is not observable when both a and b are non-zero). The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.000	0.002	0.002	0.003	0.010
0	.14	0.001	0.014	0.010	0.010	0.022
0	.39	0.015	0.040	0.042	0.043	0.076
0	.59	0.022	0.048	0.045	0.059	0.080
.14	0	0.001	0.003	0.003	0.011	0.024
.39	0	0.010	0.051	0.050	0.045	0.078
.59	0	0.041	0.074	0.074	0.076	0.106
				c'=.14		
0	0	0.000	0.003	0.002	0.001	0.008
0	.14	0.001	0.004	0.004	0.005	0.013
0	.39	0.011	0.025	0.025	0.045	0.067
0	.59	0.017	0.033	0.033	0.047	0.070
.14	0	0.001	0.002	0.002	0.006	0.019
.39	0	0.018	0.058	0.055	0.055	0.089
.59	0	0.019	0.040	0.042	0.042	0.062
				c'=.39		
0	0	0.000	0.000	0.000	0.002	0.009
0	.14	0.000	0.001	0.001	0.005	0.015
0	.39	0.005	0.016	0.017	0.036	0.076
0	.59	0.026	0.041	0.041	0.053	0.081
.14	0	0.000	0.008	0.007	0.004	0.017
.39	0	0.016	0.049	0.052	0.044	0.075
.59	0	0.029	0.064	0.069	0.054	0.085
				c'=.59		
0	0	0.000	0.001	0.001	0.005	0.008
0	.14	0.000	0.003	0.003	0.005	0.014
0	.39	0.003	0.013	0.014	0.025	0.054
0	.59	0.020	0.040	0.040	0.059	0.090
.14	0	0.000	0.008	0.009	0.011	0.023
.39	0	0.012	0.037	0.038	0.038	0.073
.59	0	0.028	0.063	0.061	0.056	0.084

Table 21

Type I error rate for N=100 in Study 1 (note: Type I error is not observable when both a and b are non-zero). The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.000	0.005	0.005	0.002	0.010
0	.14	0.003	0.009	0.009	0.011	0.027
0	.39	0.022	0.073	0.073	0.062	0.088
0	.59	0.040	0.071	0.071	0.064	0.083
.14	0	0.002	0.026	0.026	0.013	0.024
.39	0	0.031	0.105	0.108	0.063	0.097
.59	0	0.043	0.093	0.095	0.055	0.072
				c'=.14		
0	0	0.000	0.003	0.003	0.001	0.005
0	.14	0.000	0.009	0.009	0.013	0.025
0	.39	0.022	0.052	0.051	0.048	0.072
0	.59	0.035	0.067	0.068	0.055	0.076
.14	0	0.000	0.024	0.024	0.010	0.033
.39	0	0.021	0.096	0.096	0.059	0.082
.59	0	0.041	0.099	0.100	0.067	0.077
				c'=.39		
0	0	0.000	0.004	0.004	0.002	0.005
0	.14	0.001	0.010	0.009	0.011	0.033
0	.39	0.020	0.053	0.054	0.047	0.069
0	.59	0.045	0.061	0.061	0.057	0.074
.14	0	0.001	0.030	0.030	0.011	0.026
.39	0	0.027	0.110	0.110	0.052	0.087
.59	0	0.037	0.100	0.100	0.050	0.068
				c'=.59		
0	0	0.001	0.004	0.004	0.005	0.009
0	.14	0.001	0.011	0.013	0.011	0.031
0	.39	0.021	0.063	0.061	0.050	0.079
0	.59	0.042	0.074	0.075	0.056	0.075
.14	0	0.004	0.023	0.024	0.015	0.036
.39	0	0.030	0.097	0.098	0.056	0.077
.59	0	0.033	0.097	0.098	0.057	0.072

Table 22

Type I error rate for N=500 in Study 1 (note: Type I error is not observable when both a and b are non-zero). The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.000	0.003	0.003	0.001	0.003
0	.14	0.016	0.054	0.056	0.053	0.090
0	.39	0.049	0.062	0.061	0.053	0.056
0	.59	0.044	0.054	0.056	0.055	0.061
.14	0	0.011	0.054	0.055	0.045	0.078
.39	0	0.047	0.057	0.059	0.048	0.055
.59	0	0.043	0.047	0.051	0.050	0.052
				c'=.14		
0	0	0.000	0.002	0.003	0.003	0.005
0	.14	0.012	0.036	0.036	0.032	0.064
0	.39	0.052	0.062	0.062	0.058	0.060
0	.59	0.036	0.045	0.041	0.037	0.042
.14	0	0.013	0.045	0.044	0.040	0.084
.39	0	0.042	0.054	0.051	0.051	0.055
.59	0	0.060	0.066	0.066	0.067	0.070
				c'=.39		
0	0	0.000	0.000	0.000	0.000	0.005
0	.14	0.016	0.045	0.046	0.047	0.079
0	.39	0.054	0.058	0.057	0.061	0.071
0	.59	0.039	0.047	0.043	0.043	0.046
.14	0	0.015	0.045	0.043	0.037	0.060
.39	0	0.039	0.054	0.052	0.046	0.051
.59	0	0.044	0.045	0.045	0.042	0.046
				c'=.59		
0	0	0.000	0.002	0.002	0.001	0.006
0	.14	0.018	0.053	0.055	0.049	0.082
0	.39	0.045	0.058	0.059	0.048	0.052
0	.59	0.044	0.055	0.055	0.049	0.057
.14	0	0.016	0.040	0.041	0.045	0.071
.39	0	0.043	0.051	0.049	0.049	0.058
.59	0	0.040	0.046	0.046	0.047	0.046

Table 23

Power for N=20 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
.14	.14	0.005	0.008	0.008	0.013	0.030
.14	.39	0.032	0.061	0.063	0.087	0.140
.14	.59	0.068	0.097	0.100	0.137	0.192
.39	.14	0.011	0.030	0.030	0.037	0.069
.39	.39	0.046	0.083	0.083	0.125	0.201
.39	.59	0.128	0.161	0.162	0.204	0.304
.59	.14	0.019	0.059	0.058	0.061	0.107
.59	.39	0.117	0.186	0.189	0.211	0.308
.59	.59	0.273	0.335	0.339	0.384	0.518
				c'=.14		
.14	.14	0.000	0.002	0.002	0.020	0.038
.14	.39	0.028	0.052	0.051	0.093	0.149
.14	.59	0.061	0.084	0.084	0.125	0.181
.39	.14	0.011	0.029	0.028	0.032	0.064
.39	.39	0.044	0.084	0.086	0.119	0.203
.39	.59	0.125	0.157	0.159	0.202	0.319
.59	.14	0.023	0.065	0.066	0.066	0.111
.59	.39	0.115	0.183	0.183	0.214	0.303
.59	.59	0.287	0.359	0.365	0.378	0.516
				c'=.39		
.14	.14	0.004	0.011	0.011	0.021	0.049
.14	.39	0.025	0.054	0.054	0.086	0.151
.14	.59	0.076	0.105	0.105	0.145	0.205
.39	.14	0.006	0.016	0.016	0.027	0.052
.39	.39	0.051	0.093	0.093	0.120	0.206
.39	.59	0.121	0.150	0.152	0.214	0.309
.59	.14	0.029	0.055	0.055	0.070	0.111
.59	.39	0.118	0.198	0.199	0.221	0.324
.59	.59	0.271	0.358	0.354	0.383	0.527
				c'=.59		
.14	.14	0.001	0.007	0.007	0.012	0.027
.14	.39	0.007	0.016	0.016	0.030	0.064
.14	.59	0.020	0.030	0.030	0.062	0.120
.39	.14	0.010	0.030	0.031	0.039	0.083
.39	.39	0.052	0.090	0.089	0.112	0.181
.39	.59	0.123	0.162	0.161	0.214	0.319
.59	.14	0.026	0.056	0.056	0.074	0.115
.59	.39	0.108	0.199	0.198	0.206	0.328
.59	.59	0.286	0.331	0.331	0.399	0.512

Table 24

Power for N=40 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
.14	.14	0.002	0.005	0.005	0.016	0.034
.14	.39	0.022	0.045	0.046	0.073	0.137
.14	.59	0.065	0.096	0.094	0.140	0.184
.39	.14	0.036	0.070	0.070	0.090	0.147
.39	.39	0.231	0.330	0.330	0.396	0.514
.39	.59	0.462	0.542	0.542	0.585	0.673
.59	.14	0.055	0.083	0.084	0.116	0.178
.59	.39	0.434	0.522	0.522	0.568	0.659
.59	.59	0.780	0.832	0.836	0.867	0.906
				c'=.14		
.14	.14	0.002	0.011	0.011	0.015	0.038
.14	.39	0.024	0.052	0.052	0.088	0.145
.14	.59	0.060	0.093	0.092	0.142	0.206
.39	.14	0.031	0.069	0.069	0.097	0.155
.39	.39	0.265	0.371	0.371	0.431	0.544
.39	.59	0.456	0.520	0.521	0.583	0.671
.59	.14	0.066	0.109	0.109	0.127	0.174
.59	.39	0.445	0.533	0.535	0.560	0.640
.59	.59	0.758	0.812	0.811	0.863	0.903
				c'=.39		
.14	.14	0.006	0.009	0.009	0.013	0.031
.14	.39	0.029	0.073	0.072	0.107	0.165
.14	.59	0.068	0.090	0.088	0.129	0.180
.39	.14	0.035	0.069	0.069	0.086	0.142
.39	.39	0.227	0.340	0.341	0.398	0.523
.39	.59	0.433	0.516	0.515	0.571	0.665
.59	.14	0.072	0.101	0.102	0.127	0.170
.59	.39	0.477	0.563	0.564	0.594	0.681
.59	.59	0.785	0.831	0.831	0.861	0.900
				c'=.59		
.14	.14	0.003	0.006	0.005	0.019	0.037
.14	.39	0.023	0.050	0.050	0.076	0.131
.14	.59	0.068	0.098	0.097	0.126	0.176
.39	.14	0.025	0.060	0.059	0.084	0.146
.39	.39	0.232	0.329	0.328	0.396	0.514
.39	.59	0.458	0.531	0.529	0.588	0.686
.59	.14	0.068	0.109	0.109	0.128	0.187
.59	.39	0.449	0.549	0.549	0.608	0.700
.59	.59	0.797	0.833	0.833	0.856	0.899

Table 25

Power for N=60 in Study 1. The column names refer to normal theory confidence limits, Bayesian *method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).*

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
.14	.14	0.006	0.032	0.035	0.029	0.065
.14	.39	0.070	0.144	0.145	0.159	0.222
.14	.59	0.119	0.156	0.156	0.192	0.237
.39	.14	0.067	0.167	0.163	0.141	0.226
.39	.39	0.490	0.641	0.644	0.650	0.758
.39	.59	0.766	0.792	0.795	0.821	0.865
.59	.14	0.095	0.169	0.172	0.162	0.216
.59	.39	0.747	0.802	0.794	0.798	0.838
.59	.59	0.964	0.968	0.970	0.973	0.978
				c'=.14		
.14	.14	0.005	0.039	0.035	0.027	0.056
.14	.39	0.071	0.150	0.150	0.154	0.246
.14	.59	0.142	0.200	0.197	0.217	0.268
.39	.14	0.071	0.159	0.155	0.151	0.221
.39	.39	0.487	0.630	0.629	0.646	0.741
.39	.59	0.746	0.796	0.801	0.821	0.868
.59	.14	0.136	0.201	0.198	0.194	0.244
.59	.39	0.742	0.793	0.796	0.813	0.854
.59	.59	0.967	0.979	0.978	0.981	0.986
				c'=.39		
.14	.14	0.002	0.031	0.033	0.028	0.054
.14	.39	0.053	0.114	0.121	0.125	0.197
.14	.59	0.147	0.189	0.193	0.211	0.261
.39	.14	0.069	0.149	0.145	0.155	0.219
.39	.39	0.498	0.625	0.628	0.658	0.739
.39	.59	0.736	0.771	0.772	0.800	0.847
.59	.14	0.119	0.176	0.179	0.178	0.231
.59	.39	0.736	0.800	0.795	0.799	0.834
.59	.59	0.972	0.976	0.976	0.978	0.987
				c'=.59		
.14	.14	0.011	0.038	0.036	0.036	0.062
.14	.39	0.065	0.135	0.127	0.163	0.231
.14	.59	0.132	0.183	0.176	0.192	0.255
.39	.14	0.071	0.170	0.170	0.186	0.261
.39	.39	0.506	0.648	0.653	0.682	0.787
.39	.59	0.726	0.787	0.785	0.797	0.848
.59	.14	0.130	0.200	0.201	0.198	0.239
.59	.39	0.745	0.796	0.797	0.806	0.853
.59	.59	0.961	0.969	0.970	0.973	0.982

Table 26

Power for N=100 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative *prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).*

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
.14	.14	0.019	0.061	0.060	0.066	0.124
.14	.39	0.141	0.293	0.294	0.240	0.317
.14	.59	0.234	0.316	0.319	0.289	0.334
.39	.14	0.166	0.182	0.184	0.272	0.331
.39	.39	0.866	0.899	0.900	0.924	0.942
.39	.59	0.966	0.972	0.972	0.969	0.977
.59	.14	0.233	0.190	0.189	0.277	0.315
.59	.39	0.943	0.916	0.917	0.956	0.960
.59	.59	0.999	0.999	0.999	0.999	0.999
				c'=.14		
.14	.14	0.013	0.058	0.057	0.067	0.125
.14	.39	0.151	0.313	0.314	0.258	0.339
.14	.59	0.240	0.338	0.342	0.289	0.337
.39	.14	0.182	0.196	0.192	0.280	0.337
.39	.39	0.884	0.916	0.915	0.933	0.953
.39	.59	0.955	0.974	0.974	0.963	0.976
.59	.14	0.262	0.217	0.219	0.314	0.348
.59	.39	0.956	0.934	0.934	0.962	0.971
.59	.59	1.000	1.000	1.000	1.000	1.000
				c'=.39		
.14	.14	0.014	0.064	0.064	0.061	0.119
.14	.39	0.164	0.308	0.310	0.272	0.360
.14	.59	0.233	0.344	0.345	0.306	0.341
.39	.14	0.187	0.212	0.211	0.281	0.356
.39	.39	0.847	0.893	0.892	0.918	0.946
.39	.59	0.955	0.975	0.976	0.962	0.973
.59	.14	0.230	0.187	0.188	0.282	0.320
.59	.39	0.948	0.917	0.917	0.955	0.969
.59	.59	1.000	0.997	0.997	1.000	1.000
				c'=.59		
.14	.14	0.016	0.056	0.056	0.077	0.136
.14	.39	0.185	0.312	0.311	0.288	0.362
.14	.59	0.241	0.351	0.350	0.290	0.333
.39	.14	0.178	0.203	0.202	0.284	0.348
.39	.39	0.839	0.881	0.883	0.922	0.943
.39	.59	0.960	0.975	0.975	0.970	0.976
.59	.14	0.260	0.214	0.214	0.330	0.360
.59	.39	0.960	0.929	0.929	0.965	0.972
.59	.59	1.000	1.000	1.000	1.000	1.000

Table 27

Power for N=500 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
.14	.14	0.596	0.733	0.736	0.751	0.826
.14	.39	0.877	0.887	0.884	0.886	0.902
.14	.59	0.880	0.879	0.879	0.875	0.878
.39	.14	0.847	0.842	0.844	0.860	0.867
.39	.39	1.000	1.000	1.000	1.000	1.000
.39	.59	1.000	1.000	1.000	1.000	1.000
.59	.14	0.857	0.836	0.836	0.856	0.871
.59	.39	1.000	1.000	1.000	1.000	1.000
.59	.59	1.000	1.000	1.000	1.000	1.000
				c'=.14		
.14	.14	0.584	0.746	0.743	0.758	0.845
.14	.39	0.877	0.876	0.875	0.887	0.892
.14	.59	0.873	0.869	0.873	0.869	0.878
.39	.14	0.855	0.850	0.852	0.863	0.873
.39	.39	1.000	1.000	1.000	1.000	1.000
.39	.59	1.000	1.000	1.000	1.000	1.000
.59	.14	0.890	0.867	0.871	0.895	0.899
.59	.39	1.000	1.000	1.000	1.000	1.000
.59	.59	1.000	1.000	1.000	1.000	1.000
				c'=.39		
.14	.14	0.563	0.721	0.726	0.743	0.826
.14	.39	0.874	0.872	0.874	0.884	0.894
.14	.59	0.879	0.881	0.882	0.888	0.894
.39	.14	0.851	0.833	0.835	0.859	0.873
.39	.39	1.000	1.000	1.000	1.000	1.000
.39	.59	1.000	1.000	1.000	1.000	1.000
.59	.14	0.876	0.845	0.853	0.868	0.876
.59	.39	1.000	1.000	1.000	1.000	1.000
.59	.59	1.000	1.000	1.000	1.000	1.000
				c'=.59		
.14	.14	0.587	0.742	0.739	0.738	0.826
.14	.39	0.853	0.859	0.861	0.869	0.884
.14	.59	0.869	0.874	0.875	0.875	0.881
.39	.14	0.859	0.853	0.855	0.865	0.880
.39	.39	1.000	1.000	1.000	1.000	1.000
.39	.59	1.000	1.000	1.000	1.000	1.000
.59	.14	0.876	0.853	0.853	0.876	0.882
.59	.39	1.000	1.000	1.000	1.000	1.000
.59	.59	1.000	1.000	1.000	1.000	1.000

Table 28
Coverage for N=20 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative

prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

a	*b*	normal	YMdiff	YMinfo	PercBoot	BCBoot
				c'=0		
0	0	0.999	0.998	0.998	0.996	0.987
0	.14	1.000	0.999	0.999	0.994	0.989
0	.39	0.995	0.988	0.988	0.974	0.961
0	.59	0.989	0.980	0.979	0.962	0.936
.14	0	1.000	1.000	1.000	0.992	0.982
.14	.14	0.963	0.990	0.990	0.989	0.954
.14	.39	0.947	0.949	0.948	0.947	0.902
.14	.59	0.970	0.958	0.957	0.944	0.920
.39	0	0.993	0.979	0.979	0.975	0.948
.39	.14	0.965	0.976	0.976	0.964	0.924
.39	.39	0.908	0.955	0.954	0.938	0.923
.39	.59	0.914	0.955	0.955	0.930	0.924
.59	0	0.991	0.970	0.971	0.966	0.933
.59	.14	0.962	0.955	0.955	0.950	0.910
.59	.39	0.914	0.947	0.947	0.939	0.939
.59	.59	0.925	0.958	0.958	0.937	0.945
				c'=.14		
0	0	0.999	0.998	0.998	0.994	0.987
0	.14	1.000	0.999	0.999	0.995	0.987
0	.39	0.998	0.995	0.995	0.979	0.957
0	.59	0.983	0.965	0.966	0.955	0.916
.14	0	0.995	0.992	0.993	0.987	0.972
.14	.14	0.968	0.996	0.996	0.986	0.955
.14	.39	0.946	0.955	0.956	0.945	0.907
.14	.59	0.950	0.949	0.948	0.943	0.920
.39	0	0.993	0.987	0.987	0.982	0.960
.39	.14	0.974	0.973	0.973	0.970	0.929
.39	.39	0.904	0.956	0.956	0.940	0.922
.39	.59	0.915	0.952	0.952	0.929	0.934
.59	0	0.976	0.949	0.949	0.937	0.912
.59	.14	0.957	0.950	0.952	0.943	0.899
.59	.39	0.904	0.940	0.940	0.923	0.920
.59	.59	0.904	0.938	0.938	0.906	0.922
				c'=.39		
0	0	1.000	0.998	0.998	0.998	0.992
0	.14	1.000	0.996	0.996	0.994	0.986
0	.39	0.993	0.982	0.981	0.964	0.936
0	.59	0.982	0.961	0.961	0.943	0.914
.14	0	0.999	0.997	0.996	0.994	0.980
.14	.14	0.962	0.994	0.994	0.984	0.951
.14	.39	0.959	0.960	0.961	0.960	0.918
.14	.59	0.946	0.939	0.939	0.928	0.905
.39	0	0.995	0.978	0.978	0.975	0.948
.39	.14	0.959	0.990	0.990	0.979	0.941
.39	.39	0.888	0.960	0.960	0.930	0.921

.39	.59	0.915	0.956	0.955	0.932	0.937
.59	0	0.985	0.953	0.953	0.947	0.912
.59	.14	0.964	0.955	0.955	0.945	0.896
.59	.39	0.902	0.939	0.941	0.922	0.922
.59	.59	0.907	0.944	0.944	0.922	0.945
				c'=.59		
0	0	0.999	0.998	0.998	0.996	0.990
0	.14	1.000	0.999	0.999	0.998	0.989
0	.39	0.995	0.982	0.982	0.971	0.940
0	.59	0.978	0.957	0.957	0.937	0.906
.14	0	0.998	0.995	0.995	0.990	0.975
.14	.14	0.989	0.996	0.996	0.992	0.970
.14	.39	0.966	0.990	0.990	0.972	0.929
.14	.59	0.967	0.982	0.982	0.954	0.908
.39	0	0.991	0.982	0.981	0.971	0.957
.39	.14	0.961	0.975	0.975	0.963	0.920
.39	.39	0.915	0.956	0.956	0.924	0.920
.39	.59	0.905	0.952	0.952	0.927	0.934
.59	0	0.983	0.965	0.962	0.952	0.919
.59	.14	0.963	0.954	0.954	0.941	0.903
.59	.39	0.896	0.943	0.944	0.913	0.927
.59	.59	0.910	0.938	0.938	0.916	0.938

Table 29

Coverage for N=40 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	1.000	0.998	0.998	0.995	0.986
0	.14	1.000	0.998	0.998	0.992	0.978
0	.39	0.993	0.979	0.979	0.962	0.933
0	.59	0.976	0.965	0.966	0.951	0.920
.14	0	0.998	0.995	0.995	0.994	0.981
.14	.14	0.965	0.996	0.996	0.991	0.962
.14	.39	0.950	0.967	0.967	0.953	0.917
.14	.59	0.958	0.950	0.950	0.952	0.924
.39	0	0.988	0.977	0.977	0.967	0.938
.39	.14	0.944	0.954	0.956	0.945	0.906
.39	.39	0.910	0.943	0.943	0.933	0.945
.39	.59	0.929	0.944	0.944	0.936	0.947
.59	0	0.974	0.964	0.964	0.938	0.908
.59	.14	0.949	0.948	0.947	0.939	0.918
.59	.39	0.924	0.939	0.938	0.926	0.934
.59	.59	0.941	0.950	0.949	0.947	0.953
				c'=.14		
0	0	1.000	1.000	1.000	0.997	0.990

0	.14	0.999	0.997	0.997	0.994	0.987
0	.39	0.991	0.980	0.980	0.971	0.942
0	.59	0.975	0.958	0.959	0.946	0.907
.14	0	1.000	0.998	0.998	0.998	0.984
.14	.14	0.964	0.990	0.990	0.988	0.954
.14	.39	0.937	0.958	0.957	0.957	0.915
.14	.59	0.950	0.950	0.950	0.930	0.910
.39	0	0.994	0.981	0.981	0.956	0.924
.39	.14	0.941	0.945	0.946	0.931	0.892
.39	.39	0.908	0.926	0.926	0.926	0.938
.39	.59	0.934	0.936	0.938	0.932	0.938
.59	0	0.982	0.972	0.972	0.956	0.927
.59	.14	0.954	0.954	0.955	0.940	0.924
.59	.39	0.923	0.946	0.946	0.932	0.936
.59	.59	0.933	0.943	0.944	0.941	0.960
				c'=.39		
0	0	1.000	0.996	0.996	0.998	0.991
0	.14	0.999	0.996	0.996	0.991	0.981
0	.39	0.996	0.983	0.983	0.977	0.943
0	.59	0.985	0.966	0.967	0.950	0.914
.14	0	0.999	0.998	0.998	0.994	0.983
.14	.14	0.966	0.992	0.992	0.991	0.961
.14	.39	0.938	0.939	0.938	0.931	0.886
.14	.59	0.949	0.943	0.942	0.933	0.914
.39	0	0.992	0.986	0.986	0.971	0.940
.39	.14	0.935	0.953	0.953	0.938	0.896
.39	.39	0.920	0.949	0.950	0.932	0.943
.39	.59	0.935	0.945	0.945	0.943	0.956
.59	0	0.984	0.967	0.966	0.944	0.907
.59	.14	0.962	0.963	0.963	0.944	0.928
.59	.39	0.926	0.945	0.946	0.943	0.948
.59	.59	0.917	0.924	0.923	0.928	0.936
				c'=.59		
0	0	0.999	0.996	0.996	0.992	0.986
0	.14	0.999	0.994	0.994	0.988	0.974
0	.39	0.996	0.980	0.980	0.971	0.942
0	.59	0.983	0.962	0.962	0.950	0.920
.14	0	1.000	0.997	0.998	0.993	0.982
.14	.14	0.974	0.995	0.995	0.994	0.963
.14	.39	0.934	0.961	0.960	0.946	0.909
.14	.59	0.949	0.939	0.941	0.933	0.916
.39	0	0.997	0.987	0.988	0.957	0.928
.39	.14	0.943	0.953	0.953	0.948	0.897
.39	.39	0.904	0.935	0.936	0.929	0.947
.39	.59	0.938	0.941	0.941	0.936	0.942
.59	0	0.975	0.961	0.961	0.937	0.912
.59	.14	0.956	0.959	0.959	0.840	0.845
.59	.39	0.940	0.959	0.959	0.863	0.876

.59	.59	0.945	0.952	0.952	0.847	0.857

Table 30

Coverage for N=60 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	1.000	0.998	0.998	0.997	0.990
0	.14	0.999	0.986	0.990	0.990	0.978
0	.39	0.985	0.960	0.958	0.957	0.924
0	.59	0.978	0.952	0.955	0.941	0.920
.14	0	0.999	0.997	0.997	0.989	0.976
.14	.14	0.941	0.981	0.978	0.982	0.922
.14	.39	0.930	0.941	0.944	0.940	0.918
.14	.59	0.960	0.952	0.953	0.945	0.930
.39	0	0.990	0.949	0.950	0.955	0.922
.39	.14	0.946	0.946	0.943	0.946	0.935
.39	.39	0.930	0.946	0.946	0.938	0.952
.39	.59	0.934	0.937	0.942	0.942	0.941
.59	0	0.959	0.926	0.926	0.924	0.894
.59	.14	0.958	0.942	0.948	0.946	0.937
.59	.39	0.941	0.935	0.938	0.947	0.956
.59	.59	0.948	0.943	0.943	0.946	0.958
				c'=.14		
0	0	1.000	0.997	0.998	0.999	0.992
0	.14	0.999	0.996	0.996	0.995	0.987
0	.39	0.989	0.975	0.975	0.955	0.933
0	.59	0.983	0.967	0.967	0.953	0.930
.14	0	0.999	0.998	0.998	0.994	0.981
.14	.14	0.945	0.976	0.980	0.981	0.940
.14	.39	0.947	0.948	0.953	0.942	0.930
.14	.59	0.954	0.954	0.952	0.946	0.931
.39	0	0.982	0.942	0.945	0.945	0.911
.39	.14	0.946	0.942	0.944	0.949	0.928
.39	.39	0.918	0.937	0.943	0.937	0.952
.39	.59	0.934	0.940	0.940	0.942	0.955
.59	0	0.981	0.960	0.958	0.958	0.938
.59	.14	0.946	0.935	0.936	0.942	0.930
.59	.39	0.927	0.931	0.930	0.937	0.942
.59	.59	0.930	0.942	0.935	0.939	0.949
				c'=.39		
0	0	1.000	1.000	1.000	0.998	0.991
0	.14	1.000	0.999	0.999	0.995	0.985
0	.39	0.995	0.984	0.983	0.964	0.924
0	.59	0.974	0.959	0.959	0.947	0.919
.14	0	1.000	0.992	0.993	0.996	0.983

.14	.14	0.933	0.982	0.982	0.994	0.927
.14	.39	0.945	0.955	0.951	0.945	0.928
.14	.59	0.958	0.944	0.943	0.930	0.922
.39	0	0.984	0.951	0.948	0.956	0.925
.39	.14	0.948	0.935	0.946	0.944	0.926
.39	.39	0.914	0.927	0.925	0.926	0.935
.39	.59	0.928	0.938	0.935	0.943	0.939
.59	0	0.971	0.936	0.931	0.946	0.915
.59	.14	0.954	0.924	0.929	0.940	0.924
.59	.39	0.934	0.932	0.930	0.938	0.946
.59	.59	0.942	0.954	0.956	0.938	0.947
				c'=.59		
0	0	1.000	0.999	0.999	0.995	0.992
0	.14	1.000	0.997	0.997	0.995	0.986
0	.39	0.997	0.987	0.986	0.975	0.946
0	.59	0.980	0.960	0.960	0.941	0.910
.14	0	1.000	0.992	0.991	0.989	0.977
.14	.14	0.943	0.982	0.983	0.975	0.927
.14	.39	0.939	0.937	0.935	0.926	0.904
.14	.59	0.963	0.961	0.959	0.954	0.944
.39	0	0.988	0.963	0.962	0.962	0.927
.39	.14	0.940	0.94	0.939	0.944	0.922
.39	.39	0.939	0.934	0.935	0.938	0.951
.39	.59	0.935	0.940	0.942	0.956	0.961
.59	0	0.972	0.937	0.939	0.944	0.916
.59	.14	0.955	0.941	0.943	0.943	0.929
.59	.39	0.922	0.927	0.929	0.933	0.952
.59	.59	0.936	0.934	0.934	0.936	0.942

Table 31

Coverage for N=100 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	1	0.995	0.995	0.998	0.99
0	.14	0.997	0.991	0.991	0.989	0.973
0	.39	0.978	0.927	0.927	0.938	0.912
0	.59	0.960	0.929	0.929	0.936	0.917
.14	0	0.998	0.974	0.974	0.987	0.976
.14	.14	0.907	0.949	0.949	0.956	0.92
.14	.39	0.945	0.944	0.945	0.941	0.936
.14	.59	0.942	0.928	0.927	0.933	0.927
.39	0	0.969	0.895	0.892	0.937	0.903
.39	.14	0.947	0.908	0.908	0.950	0.927
.39	.39	0.930	0.939	0.938	0.935	0.951
.39	.59	0.953	0.964	0.964	0.963	0.966

.59	0	0.957	0.907	0.905	0.945	0.928
.59	.14	0.956	0.92	0.919	0.947	0.940
.59	.39	0.945	0.924	0.922	0.945	0.947
.59	.59	0.937	0.945	0.946	0.940	0.946
				c'=.14		
0	0	1.000	0.997	0.997	0.999	0.995
0	.14	1.000	0.991	0.991	0.987	0.975
0	.39	0.978	0.948	0.949	0.952	0.928
0	.59	0.965	0.933	0.932	0.945	0.924
.14	0	1.000	0.976	0.976	0.990	0.967
.14	.14	0.901	0.958	0.958	0.962	0.917
.14	.39	0.947	0.946	0.949	0.943	0.933
.14	.59	0.951	0.932	0.933	0.940	0.932
.39	0	0.979	0.904	0.904	0.941	0.918
.39	.14	0.941	0.887	0.888	0.931	0.917
.39	.39	0.941	0.953	0.949	0.956	0.964
.39	.59	0.950	0.959	0.959	0.947	0.950
.59	0	0.959	0.901	0.900	0.933	0.923
.59	.14	0.956	0.908	0.908	0.945	0.936
.59	.39	0.935	0.922	0.920	0.949	0.949
.59	.59	0.952	0.949	0.950	0.952	0.959
				c'=.39		
0	0	1.000	0.996	0.996	0.998	0.995
0	.14	0.999	0.990	0.991	0.989	0.967
0	.39	0.980	0.947	0.946	0.953	0.931
0	.59	0.955	0.939	0.939	0.943	0.926
.14	0	0.999	0.970	0.970	0.989	0.974
.14	.14	0.895	0.942	0.941	0.958	0.894
.14	.39	0.949	0.944	0.944	0.949	0.938
.14	.59	0.946	0.938	0.937	0.938	0.930
.39	0	0.973	0.890	0.890	0.948	0.913
.39	.14	0.947	0.900	0.902	0.937	0.923
.39	.39	0.917	0.931	0.931	0.936	0.949
.39	.59	0.952	0.962	0.962	0.955	0.959
.59	0	0.963	0.900	0.900	0.950	0.932
.59	.14	0.951	0.889	0.890	0.948	0.942
.59	.39	0.948	0.925	0.924	0.944	0.944
.59	.59	0.939	0.948	0.948	0.945	0.948
				c'=.59		
0	0	0.999	0.996	0.996	0.995	0.991
0	.14	0.999	0.989	0.987	0.989	0.969
0	.39	0.979	0.937	0.939	0.950	0.921
0	.59	0.958	0.926	0.925	0.944	0.925
.14	0	0.996	0.977	0.976	0.985	0.964
.14	.14	0.916	0.939	0.937	0.962	0.909
.14	.39	0.941	0.954	0.952	0.939	0.928
.14	.59	0.944	0.927	0.926	0.923	0.915
.39	0	0.970	0.903	0.902	0.944	0.923

.39	.14	0.948	0.911	0.911	0.940	0.932
.39	.39	0.925	0.935	0.937	0.940	0.960
.39	.59	0.942	0.957	0.958	0.953	0.958
.59	0	0.967	0.903	0.902	0.943	0.928
.59	.14	0.959	0.902	0.905	0.952	0.934
.59	.39	0.952	0.933	0.934	0.946	0.947
.59	.59	0.953	0.955	0.954	0.960	0.952

Table 32

Coverage for N=500 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	1.000	0.997	0.997	0.999	0.997
0	.14	0.984	0.946	0.944	0.947	0.910
0	.39	0.951	0.938	0.939	0.947	0.944
0	.59	0.956	0.946	0.944	0.945	0.939
.14	0	0.989	0.946	0.945	0.955	0.922
.14	.14	0.930	0.935	0.937	0.947	0.952
.14	.39	0.948	0.943	0.942	0.945	0.940
.14	.59	0.963	0.949	0.946	0.955	0.958
.39	0	0.953	0.943	0.941	0.952	0.945
.39	.14	0.936	0.934	0.938	0.940	0.942
.39	.39	0.952	0.941	0.938	0.954	0.959
.39	.59	0.950	0.943	0.947	0.950	0.948
.59	0	0.957	0.953	0.949	0.950	0.948
.59	.14	0.952	0.947	0.949	0.949	0.947
.59	.39	0.954	0.950	0.943	0.958	0.957
.59	.59	0.951	0.937	0.941	0.955	0.950
				c'=.14		
0	0	1.000	0.998	0.997	0.997	0.995
0	.14	0.988	0.964	0.964	0.968	0.936
0	.39	0.948	0.938	0.938	0.942	0.940
0	.59	0.964	0.955	0.959	0.963	0.958
.14	0	0.987	0.955	0.956	0.960	0.916
.14	.14	0.929	0.940	0.938	0.947	0.950
.14	.39	0.964	0.947	0.950	0.957	0.955
.14	.59	0.957	0.939	0.944	0.950	0.948
.39	0	0.958	0.946	0.949	0.949	0.945
.39	.14	0.940	0.942	0.943	0.946	0.943
.39	.39	0.952	0.935	0.931	0.954	0.950
.39	.59	0.948	0.942	0.948	0.941	0.942
.59	0	0.940	0.934	0.934	0.933	0.930
.59	.14	0.958	0.949	0.943	0.950	0.945
.59	.39	0.950	0.935	0.935	0.949	0.944
.59	.59	0.946	0.949	0.949	0.949	0.945

				c'=.39		
0	0	1.000	1.000	1.000	1.000	0.995
0	.14	0.984	0.955	0.954	0.953	0.921
0	.39	0.946	0.942	0.943	0.939	0.929
0	.59	0.961	0.953	0.957	0.957	0.954
.14	0	0.985	0.955	0.957	0.963	0.940
.14	.14	0.931	0.952	0.951	0.952	0.966
.14	.39	0.952	0.934	0.937	0.947	0.944
.14	.59	0.960	0.941	0.940	0.951	0.945
.39	0	0.961	0.946	0.948	0.954	0.949
.39	.14	0.947	0.941	0.939	0.949	0.949
.39	.39	0.945	0.940	0.940	0.945	0.949
.39	.59	0.931	0.929	0.932	0.940	0.939
.59	0	0.956	0.955	0.955	0.958	0.954
.59	.14	0.949	0.933	0.939	0.944	0.936
.59	.39	0.949	0.934	0.940	0.949	0.944
.59	.59	0.951	0.948	0.948	0.950	0.947
				c'=.59		
0	0	1.000	0.998	0.998	0.999	0.994
0	.14	0.982	0.947	0.945	0.951	0.918
0	.39	0.955	0.942	0.941	0.952	0.948
0	.59	0.956	0.945	0.945	0.951	0.943
.14	0	0.984	0.960	0.959	0.955	0.929
.14	.14	0.925	0.942	0.940	0.947	0.958
.14	.39	0.940	0.923	0.920	0.941	0.932
.14	.59	0.956	0.950	0.949	0.952	0.950
.39	0	0.957	0.949	0.951	0.951	0.942
.39	.14	0.950	0.952	0.954	0.949	0.950
.39	.39	0.937	0.939	0.943	0.933	0.942
.39	.59	0.953	0.943	0.943	0.959	0.961
.59	0	0.960	0.954	0.954	0.953	0.954
.59	.14	0.948	0.946	0.941	0.946	0.948
.59	.39	0.954	0.936	0.937	0.954	0.954
.59	.59	0.951	0.946	0.944	0.951	0.946

Table 33

Interval Width for N=20 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.287	0.424	0.424	0.416	0.437
0	.14	0.305	0.455	0.455	0.426	0.446
0	.39	0.442	0.602	0.602	0.544	0.565
0	.59	0.599	0.763	0.762	0.677	0.694
.14	0	0.156	0.209	0.210	0.203	0.213
.14	.14	0.171	0.227	0.227	0.212	0.222
.14	.39	0.286	0.335	0.335	0.311	0.321
.14	.59	0.404	0.452	0.452	0.421	0.428

.39	0	0.435	0.505	0.507	0.540	0.561
.39	.14	0.467	0.553	0.553	0.565	0.586
.39	.39	0.578	0.688	0.689	0.663	0.688
.39	.59	0.722	0.841	0.842	0.793	0.821
.59	0	0.617	0.658	0.660	0.718	0.733
.59	.14	0.628	0.684	0.683	0.723	0.743
.59	.39	0.717	0.791	0.791	0.796	0.821
.59	.59	0.836	0.935	0.934	0.900	0.929
				c'=.14		
0	0	0.282	0.426	0.425	0.406	0.428
0	.14	0.300	0.458	0.459	0.425	0.448
0	.39	0.443	0.600	0.601	0.540	0.562
0	.59	0.389	0.446	0.446	0.410	0.416
.14	0	0.157	0.211	0.211	0.203	0.212
.14	.14	0.176	0.231	0.231	0.222	0.232
.14	.39	0.291	0.341	0.340	0.316	0.328
.14	.59	0.403	0.451	0.451	0.422	0.429
.39	0	0.452	0.519	0.520	0.555	0.574
.39	.14	0.473	0.560	0.561	0.574	0.594
.39	.39	0.587	0.699	0.699	0.673	0.702
.39	.59	0.707	0.828	0.827	0.776	0.800
.59	0	0.611	0.652	0.652	0.703	0.721
.59	.14	0.621	0.678	0.678	0.710	0.727
.59	.39	0.705	0.782	0.781	0.784	0.812
.59	.59	0.825	0.923	0.923	0.892	0.922
				c'=.39		
0	0	0.132	0.196	0.196	0.183	0.193
0	.14	0.154	0.222	0.221	0.201	0.211
0	.39	0.271	0.330	0.330	0.298	0.306
0	.59	0.390	0.447	0.446	0.412	0.417
.14	0	0.152	0.208	0.208	0.201	0.216
.14	.14	0.176	0.232	0.232	0.219	0.230
.14	.39	0.293	0.340	0.340	0.317	0.327
.14	.59	0.404	0.452	0.452	0.422	0.429
.39	0	0.450	0.526	0.527	0.550	0.569
.39	.14	0.463	0.553	0.552	0.570	0.592
.39	.39	0.567	0.674	0.674	0.649	0.676
.39	.59	0.714	0.836	0.835	0.786	0.812
.59	0	0.620	0.664	0.663	0.718	0.735
.59	.14	0.626	0.689	0.688	0.711	0.728
.59	.39	0.716	0.796	0.797	0.783	0.808
.59	.59	0.827	0.925	0.925	0.893	0.926
				c'=.59		
0	0	0.130	0.192	0.192	0.182	0.191
0	.14	0.154	0.220	0.220	0.201	0.211
0	.39	0.274	0.332	0.332	0.301	0.309
0	.59	0.395	0.451	0.451	0.414	0.420
.14	0	0.152	0.208	0.208	0.201	0.209

.14	.14	0.330	0.468	0.469	0.445	0.467
.14	.39	0.467	0.603	0.604	0.554	0.576
.14	.59	0.610	0.766	0.766	0.696	0.715
.39	0	0.446	0.528	0.527	0.555	0.573
.39	.14	0.470	0.559	0.559	0.564	0.588
.39	.39	0.570	0.673	0.673	0.655	0.680
.39	.59	0.702	0.823	0.823	0.772	0.800
.59	0	0.629	0.676	0.676	0.722	0.743
.59	.14	0.626	0.683	0.682	0.719	0.741
.59	.39	0.720	0.798	0.798	0.795	0.822
.59	.59	0.820	0.917	0.916	0.888	0.916

Table 34

Interval Width for N=40 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.129	0.193	0.193	0.181	0.191
0	.14	0.154	0.221	0.221	0.201	0.212
0	.39	0.275	0.335	0.335	0.302	0.310
0	.59	0.388	0.445	0.444	0.407	0.412
.14	0	0.151	0.206	0.206	0.199	0.208
.14	.14	0.180	0.234	0.234	0.222	0.233
.14	.39	0.290	0.340	0.341	0.318	0.329
.14	.59	0.406	0.451	0.451	0.423	0.431
.39	0	0.275	0.314	0.314	0.306	0.315
.39	.14	0.292	0.326	0.327	0.322	0.332
.39	.39	0.370	0.393	0.393	0.385	0.399
.39	.59	0.470	0.491	0.491	0.483	0.494
.59	0	0.397	0.432	0.432	0.419	0.424
.59	.14	0.406	0.436	0.436	0.428	0.435
.59	.39	0.473	0.489	0.489	0.484	0.495
.59	.59	0.551	0.561	0.561	0.557	0.572
				c'=.14		
0	0	0.130	0.193	0.193	0.184	0.193
0	.14	0.153	0.220	0.220	0.201	0.210
0	.39	0.271	0.332	0.332	0.301	0.309
0	.59	0.391	0.446	0.446	0.412	0.417
.14	0	0.156	0.208	0.209	0.205	0.215
.14	.14	0.175	0.231	0.230	0.220	0.232
.14	.39	0.286	0.336	0.336	0.314	0.324
.14	.59	0.402	0.450	0.450	0.421	0.429
.39	0	0.279	0.318	0.318	0.308	0.317
.39	.14	0.296	0.329	0.328	0.324	0.335
.39	.39	0.377	0.400	0.400	0.394	0.409
.39	.59	0.468	0.490	0.491	0.478	0.491

.59	0	0.395	0.430	0.430	0.422	0.426
.59	.14	0.409	0.438	0.438	0.429	0.436
.59	.39	0.471	0.488	0.489	0.485	0.498
.59	.59	0.547	0.557	0.558	0.555	0.569
				c'=.39		
0	0	0.131	0.195	0.195	0.184	0.193
0	.14	0.154	0.221	0.221	0.201	0.211
0	.39	0.273	0.333	0.333	0.303	0.312
0	.59	0.392	0.448	0.448	0.412	0.418
.14	0	0.152	0.206	0.207	0.200	0.210
.14	.14	0.178	0.234	0.235	0.220	0.231
.14	.39	0.290	0.337	0.337	0.314	0.325
.14	.59	0.402	0.451	0.452	0.422	0.429
.39	0	0.275	0.312	0.312	0.304	0.312
.39	.14	0.286	0.319	0.319	0.312	0.322
.39	.39	0.377	0.402	0.402	0.392	0.407
.39	.59	0.465	0.486	0.486	0.474	0.487
.59	0	0.401	0.435	0.435	0.425	0.430
.59	.14	0.404	0.432	0.432	0.428	0.435
.59	.39	0.473	0.492	0.492	0.489	0.502
.59	.59	0.549	0.559	0.560	0.558	0.572
				c'=.59		
0	0	0.132	0.195	0.195	0.185	0.194
0	.14	0.158	0.223	0.223	0.204	0.214
0	.39	0.270	0.329	0.329	0.299	0.308
0	.59	0.378	0.434	0.434	0.399	0.405
.14	0	0.153	0.207	0.207	0.200	0.209
.14	.14	0.175	0.232	0.231	0.220	0.232
.14	.39	0.286	0.333	0.334	0.313	0.323
.14	.59	0.404	0.455	0.455	0.426	0.433
.39	0	0.273	0.311	0.311	0.304	0.312
.39	.14	0.288	0.321	0.321	0.314	0.324
.39	.39	0.371	0.396	0.396	0.387	0.402
.39	.59	0.467	0.486	0.487	0.477	0.490
.59	0	0.399	0.432	0.432	0.424	0.428
.59	.14	0.398	0.427	0.427	0.420	0.427
.59	.39	0.473	0.491	0.492	0.484	0.497
.59	.59	0.550	0.561	0.562	0.556	0.568

Table 35

Interval Width for N=60 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.087	0.127	0.127	0.120	0.127
0	.14	0.106	0.141	0.141	0.135	0.142

0	.39	0.212	0.239	0.238	0.228	0.233
0	.59	0.313	0.336	0.336	0.324	0.326
.14	0	0.159	0.212	0.212	0.203	0.213
.14	.14	0.128	0.158	0.158	0.151	0.159
.14	.39	0.223	0.242	0.243	0.237	0.243
.14	.59	0.320	0.339	0.338	0.327	0.330
.39	0	0.215	0.236	0.235	0.232	0.236
.39	.14	0.229	0.243	0.244	0.241	0.248
.39	.39	0.293	0.298	0.297	0.298	0.308
.39	.59	0.375	0.383	0.383	0.376	0.384
.59	0	0.312	0.328	0.330	0.323	0.325
.59	.14	0.322	0.335	0.336	0.334	0.337
.59	.39	0.376	0.375	0.376	0.380	0.388
.59	.59	0.443	0.445	0.445	0.446	0.455
				c'=.14		
0	0	0.086	0.126	0.125	0.120	0.127
0	.14	0.107	0.143	0.143	0.136	0.143
0	.39	0.271	0.333	0.333	0.302	0.311
0	.59	0.390	0.447	0.447	0.409	0.414
.14	0	0.157	0.211	0.211	0.205	0.215
.14	.14	0.128	0.157	0.157	0.154	0.162
.14	.39	0.228	0.248	0.248	0.241	0.247
.14	.59	0.321	0.337	0.338	0.329	0.333
.39	0	0.216	0.235	0.236	0.234	0.239
.39	.14	0.224	0.238	0.239	0.238	0.245
.39	.39	0.296	0.300	0.300	0.301	0.312
.39	.59	0.375	0.383	0.383	0.378	0.387
.59	0	0.313	0.330	0.329	0.325	0.327
.59	.14	0.325	0.337	0.337	0.334	0.338
.59	.39	0.374	0.379	0.378	0.376	0.384
.59	.59	0.443	0.447	0.447	0.446	0.455
				c'=.39		
0	0	0.134	0.196	0.196	0.186	0.196
0	.14	0.150	0.215	0.216	0.197	0.207
0	.39	0.273	0.333	0.333	0.302	0.311
0	.59	0.391	0.449	0.449	0.413	0.418
.14	0	0.106	0.139	0.138	0.135	0.142
.14	.14	0.125	0.155	0.156	0.150	0.158
.14	.39	0.226	0.247	0.247	0.238	0.245
.14	.59	0.319	0.337	0.337	0.325	0.329
.39	0	0.213	0.233	0.232	0.229	0.234
.39	.14	0.230	0.246	0.245	0.244	0.250
.39	.39	0.297	0.307	0.307	0.301	0.312
.39	.59	0.376	0.385	0.385	0.381	0.389
.59	0	0.317	0.334	0.335	0.328	0.330
.59	.14	0.321	0.332	0.330	0.331	0.335
.59	.39	0.374	0.379	0.379	0.378	0.387
.59	.59	0.441	0.448	0.448	0.442	0.451

				c'=.59		
0	0	0.132	0.195	0.195	0.185	0.194
0	.14	0.156	0.221	0.221	0.204	0.214
0	.39	0.273	0.334	0.333	0.305	0.313
0	.59	0.386	0.446	0.446	0.405	0.411
.14	0	0.108	0.143	0.143	0.137	0.144
.14	.14	0.131	0.162	0.161	0.155	0.163
.14	.39	0.227	0.246	0.247	0.238	0.245
.14	.59	0.323	0.341	0.341	0.329	0.334
.39	0	0.213	0.233	0.232	0.229	0.233
.39	.14	0.231	0.248	0.247	0.243	0.250
.39	.39	0.298	0.304	0.304	0.304	0.314
.39	.59	0.377	0.387	0.386	0.382	0.391
.59	0	0.316	0.333	0.333	0.330	0.333
.59	.14	0.331	0.343	0.345	0.341	0.345
.59	.39	0.372	0.378	0.378	0.377	0.385
.59	.59	0.438	0.443	0.443	0.439	0.447

Table 36

Interval Width for N=100 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.051	0.071	0.071	0.071	0.075
0	.14	0.070	0.086	0.086	0.086	0.091
0	.39	0.158	0.161	0.161	0.166	0.168
0	.59	0.237	0.234	0.234	0.242	0.243
.14	0	0.071	0.086	0.086	0.087	0.092
.14	.14	0.090	0.101	0.101	0.101	0.107
.14	.39	0.170	0.174	0.173	0.174	0.178
.14	.59	0.242	0.243	0.243	0.246	0.248
.39	0	0.159	0.158	0.158	0.165	0.167
.39	.14	0.171	0.171	0.171	0.176	0.180
.39	.39	0.223	0.225	0.225	0.224	0.230
.39	.59	0.286	0.288	0.288	0.286	0.291
.59	0	0.239	0.234	0.234	0.245	0.246
.59	.14	0.243	0.240	0.240	0.246	0.248
.59	.39	0.285	0.282	0.282	0.286	0.290
.59	.59	0.335	0.337	0.337	0.334	0.338
				c'=.14		
0	0	0.051	0.070	0.070	0.070	0.074
0	.14	0.073	0.088	0.087	0.089	0.093
0	.39	0.160	0.162	0.162	0.167	0.168
0	.59	0.237	0.234	0.234	0.243	0.243
.14	0	0.073	0.087	0.087	0.088	0.093
.14	.14	0.088	0.100	0.100	0.100	0.106

.14	.39	0.168	0.172	0.172	0.173	0.177
.14	.59	0.244	0.243	0.244	0.247	0.248
.39	0	0.162	0.160	0.160	0.169	0.171
.39	.14	0.172	0.172	0.172	0.177	0.181
.39	.39	0.225	0.227	0.227	0.225	0.231
.39	.59	0.285	0.288	0.289	0.286	0.290
.59	0	0.238	0.232	0.232	0.243	0.244
.59	.14	0.244	0.240	0.240	0.247	0.249
.59	.39	0.282	0.278	0.277	0.283	0.287
.59	.59	0.334	0.336	0.335	0.334	0.339
				c'=.39		
0	0	0.051	0.069	0.068	0.071	0.075
0	.14	0.072	0.086	0.086	0.088	0.092
0	.39	0.157	0.159	0.159	0.165	0.167
0	.59	0.238	0.233	0.233	0.244	0.244
.14	0	0.072	0.084	0.084	0.087	0.092
.14	.14	0.088	0.098	0.098	0.100	0.105
.14	.39	0.171	0.174	0.174	0.176	0.179
.14	.59	0.243	0.242	0.242	0.247	0.249
.39	0	0.162	0.158	0.158	0.168	0.170
.39	.14	0.173	0.171	0.171	0.177	0.181
.39	.39	0.223	0.226	0.226	0.225	0.231
.39	.59	0.285	0.287	0.287	0.285	0.290
.59	0	0.238	0.230	0.229	0.245	0.245
.59	.14	0.242	0.235	0.235	0.247	0.248
.59	.39	0.286	0.283	0.283	0.287	0.291
.59	.59	0.336	0.338	0.339	0.336	0.341
				c'=.59		
0	0	0.051	0.069	0.069	0.071	0.075
0	.14	0.074	0.088	0.088	0.089	0.093
0	.39	0.159	0.161	0.161	0.167	0.169
0	.59	0.240	0.234	0.234	0.245	0.245
.14	0	0.071	0.084	0.084	0.086	0.091
.14	.14	0.090	0.101	0.101	0.109	0.107
.14	.39	0.173	0.176	0.176	0.177	0.181
.14	.59	0.245	0.244	0.244	0.249	0.250
.39	0	0.160	0.158	0.158	0.168	0.169
.39	.14	0.172	0.172	0.172	0.177	0.181
.39	.39	0.223	0.224	0.224	0.222	0.228
.39	.59	0.284	0.287	0.286	0.284	0.288
.59	0	0.240	0.230	0.230	0.245	0.245
.59	.14	0.245	0.238	0.238	0.249	0.251
.59	.39	0.287	0.284	0.284	0.288	0.292
.59	.59	0.336	0.337	0.337	0.335	0.339

Table 37

Interval Width for N=500 in Study 1. The column names refer to normal theory confidence limits, *Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, Percentile bootstrap, and Bias-corrected bootstrap (respectively).*

		normal	YMdiff	YMinfo	PercBoot	BCBoot
a	*b*			*c'*=0		
0	0	0.010	0.014	0.014	0.014	0.015
0	.14	0.026	0.028	0.028	0.028	0.028
0	.39	0.068	0.070	0.070	0.069	0.069
0	.59	0.104	0.105	0.105	0.104	0.104
.14	0	0.026	0.029	0.029	0.028	0.029
.14	.14	0.036	0.037	0.037	0.037	0.038
.14	.39	0.073	0.074	0.074	0.074	0.074
.14	.59	0.107	0.107	0.107	0.107	0.108
.39	0	0.069	0.072	0.072	0.070	0.070
.39	.14	0.073	0.075	0.075	0.073	0.074
.39	.39	0.097	0.099	0.099	0.097	0.098
.39	.59	0.125	0.128	0.128	0.125	0.125
.59	0	0.104	0.108	0.108	0.104	0.104
.59	.14	0.107	0.110	0.110	0.107	0.107
.59	.39	0.125	0.127	0.127	0.126	0.126
.59	.59	0.147	0.150	0.150	0.146	0.147
				c'=.14		
0	0	0.010	0.014	0.014	0.014	0.015
0	.14	0.025	0.028	0.028	0.027	0.028
0	.39	0.069	0.070	0.070	0.070	0.070
0	.59	0.103	0.104	0.104	0.104	0.104
.14	0	0.026	0.029	0.029	0.028	0.029
.14	.14	0.036	0.037	0.037	0.036	0.038
.14	.39	0.073	0.074	0.074	0.074	0.074
.14	.59	0.106	0.107	0.107	0.106	0.107
.39	0	0.069	0.071	0.072	0.069	0.070
.39	.14	0.073	0.075	0.075	0.074	0.074
.39	.39	0.097	0.098	0.098	0.097	0.098
.39	.59	0.125	0.129	0.128	0.124	0.125
.59	0	0.104	0.106	0.106	0.104	0.104
.59	.14	0.107	0.109	0.109	0.107	0.107
.59	.39	0.125	0.127	0.126	0.124	0.125
.59	.59	0.147	0.151	0.151	0.147	0.148
				c'=.39		
0	0	0.010	0.014	0.014	0.014	0.015
0	.14	0.026	0.028	0.028	0.028	0.028
0	.39	0.069	0.070	0.070	0.069	0.070
0	.59	0.104	0.105	0.105	0.104	0.104
.14	0	0.026	0.029	0.029	0.028	0.029
.14	.14	0.036	0.037	0.037	0.036	0.038
.14	.39	0.073	0.074	0.073	0.073	0.074
.14	.59	0.106	0.107	0.107	0.106	0.106
.39	0	0.069	0.072	0.072	0.070	0.070
.39	.14	0.073	0.074	0.074	0.073	0.073

.39	.39	0.097	0.099	0.099	0.097	0.098
.39	.59	0.124	0.127	0.127	0.124	0.124
.59	0	0.104	0.107	0.107	0.104	0.104
.59	.14	0.107	0.108	0.108	0.107	0.108
.59	.39	0.125	0.126	0.126	0.125	0.125
.59	.59	0.147	0.150	0.150	0.147	0.148
				c'=.59		
0	0	0.010	0.014	0.014	0.014	0.015
0	.14	0.026	0.0281	0.028	0.028	0.028
0	.39	0.069	0.071	0.071	0.070	0.070
0	.59	0.104	0.105	0.105	0.104	0.104
.14	0	0.026	0.029	0.029	0.028	0.028
.14	.14	0.036	0.037	0.037	0.036	0.038
.14	.39	0.073	0.073	0.073	0.073	0.073
.14	.59	0.107	0.108	0.108	0.107	0.107
.39	0	0.069	0.072	0.072	0.070	0.070
.39	.14	0.073	0.076	0.076	0.073	0.074
.39	.39	0.097	0.100	0.100	0.097	0.098
.39	.59	0.125	0.126	0.126	0.125	0.125
.59	0	0.104	0.107	0.107	0.105	0.105
.59	.14	0.107	0.108	0.108	0.108	0.108
.59	.39	0.124	0.128	0.128	0.124	0.125
.59	.59	0.147	0.148	0.148	0.147	0.148

Table 38

Imbalance for N=20 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, and Percentile bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot
a	*b*		*c'*=0		
0	0	-0.001	0.000	0.000	0.002
0	.14	0.000	-0.001	-0.001	-0.002
0	.39	-0.001	-0.006	-0.006	0.002
0	.59	-0.003	-0.002	-0.001	-0.002
.14	0	0.000	0.000	0.000	0.000
.14	.14	0.033	-0.002	-0.002	0.001
.14	.39	0.043	0.025	0.024	0.027
.14	.59	0.020	0.014	0.015	0.016
.39	0	-0.001	0.003	0.003	-0.001
.39	.14	0.031	0.004	0.004	0.010
.39	.39	0.086	0.027	0.028	0.038
.39	.59	0.074	0.025	0.025	0.044
.59	0	-0.001	-0.004	-0.003	-0.004
.59	.14	0.026	0.021	0.021	0.032
.59	.39	0.080	0.035	0.035	0.043
.59	.59	0.059	0.020	0.020	0.031

			c'=.14		
0	0	-0.001	0.000	0.000	-0.002
0	.14	0.000	-0.001	-0.001	-0.003
0	.39	0.000	-0.001	-0.001	0.001
0	.59	0.007	0.021	0.020	0.003
.14	0	0.001	-0.002	-0.001	0.001
.14	.14	0.032	0.004	0.004	0.006
.14	.39	0.042	0.021	0.020	0.033
.14	.59	0.034	0.029	0.030	0.033
.39	0	-0.001	-0.007	-0.007	-0.002
.39	.14	0.024	0.005	0.005	0.012
.39	.39	0.094	0.036	0.036	0.042
.39	.59	0.075	0.030	0.030	0.047
.59	0	-0.002	-0.011	-0.011	0.003
.59	.14	0.029	0.026	0.026	0.037
.59	.39	0.084	0.046	0.048	0.053
.59	.59	0.084	0.046	0.046	0.060
			c'=.39		
0	0	0.000	0.002	0.002	0.002
0	.14	0.000	0.002	0.002	0.000
0	.39	0.007	0.008	0.009	-0.002
0	.59	0.002	0.009	0.009	-0.007
.14	0	0.001	0.003	0.002	0.000
.14	.14	0.034	-0.002	-0.002	0.002
.14	.39	0.039	0.022	0.021	0.024
.14	.59	0.034	0.023	0.023	0.020
.39	0	-0.005	-0.002	-0.002	0.009
.39	.14	0.037	0.000	0.000	0.009
.39	.39	0.098	0.018	0.018	0.048
.39	.59	0.077	0.034	0.035	0.040
.59	0	0.007	0.011	0.011	0.009
.59	.14	0.020	0.021	0.021	0.015
.59	.39	0.084	0.033	0.033	0.044
.59	.59	0.071	0.028	0.028	0.048
			c'=.59		
0	0	-0.001	-0.002	-0.002	-0.004
0	.14	0.000	0.001	0.001	0.000
0	.39	-0.001	0.008	0.008	-0.003
0	.59	-0.004	0.009	0.009	-0.001
.14	0	-0.002	-0.003	-0.003	0.000
.14	.14	0.009	-0.002	-0.002	-0.006
.14	.39	0.026	-0.002	-0.002	0.004
.14	.59	0.027	0.000	0.000	0.026
.39	0	0.003	0.004	0.003	0.013
.39	.14	0.035	0.005	0.005	0.017
.39	.39	0.081	0.040	0.040	0.050
.39	.59	0.077	0.028	0.028	0.045
.59	0	0.007	0.007	0.006	0.008

.59	.14	0.021	0.020	0.020	0.025
.59	.39	0.086	0.043	0.042	0.061
.59	.59	0.072	0.032	0.032	0.054

Table 39

Imbalance for N=40 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, and Percentile bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot
a	*b*		*c'*=0		
0	0	0.000	0.002	0.002	0.003
0	.14	0.000	0.002	0.002	0.004
0	.39	-0.001	0.009	0.009	-0.002
0	.59	-0.002	0.007	0.008	0.009
.14	0	-0.002	-0.001	-0.001	-0.002
.14	.14	0.031	-0.002	-0.002	-0.001
.14	.39	0.046	0.017	0.017	0.025
.14	.59	0.036	0.026	0.026	0.022
.39	0	0.000	-0.003	-0.003	0.001
.39	.14	0.050	0.020	0.022	0.023
.39	.39	0.082	0.041	0.041	0.045
.39	.59	0.055	0.030	0.030	0.036
.59	0	0.006	-0.006	-0.004	0.012
.59	.14	0.029	0.010	0.011	0.023
.59	.39	0.056	0.025	0.026	0.042
.59	.59	0.039	0.016	0.017	0.017
			c'=.14		
0	0	0.000	0.000	0.000	-0.001
0	.14	-0.001	0.001	0.001	0.002
0	.39	0.001	0.010	0.010	0.003
0	.59	-0.007	0.006	0.007	-0.008
.14	0	0.000	0.000	0.000	0.000
.14	.14	0.036	0.004	0.004	0.006
.14	.39	0.055	0.028	0.027	0.021
.14	.59	0.028	0.018	0.018	0.036
.39	0	-0.002	-0.005	-0.005	0.012
.39	.14	0.051	0.037	0.036	0.041
.39	.39	0.072	0.030	0.030	0.034
.39	.59	0.048	0.026	0.024	0.030
.59	0	-0.002	-0.002	-0.002	0.006
.59	.14	0.024	0.014	0.013	0.012
.59	.39	0.061	0.030	0.030	0.040
.59	.59	0.051	0.029	0.030	0.031
			c'=.39		
0	0	0.000	0.002	0.002	0.002
0	.14	-0.001	0.004	0.004	0.003

0	.39	-0.002	0.005	0.005	0.001
0	.59	-0.003	0.008	0.009	-0.004
.14	0	-0.001	0.000	0.000	0.000
.14	.14	0.032	0.000	0.000	0.005
.14	.39	0.052	0.045	0.046	0.043
.14	.59	0.031	0.025	0.026	0.033
.39	0	-0.004	-0.008	-0.008	-0.005
.39	.14	0.057	0.025	0.025	0.026
.39	.39	0.064	0.027	0.028	0.026
.39	.59	0.057	0.031	0.031	0.037
.59	0	0.000	-0.007	-0.008	0.004
.59	.14	0.022	0.013	0.013	0.018
.59	.39	0.060	0.029	0.028	0.017
.59	.59	0.063	0.028	0.031	0.036
			c'=.59		
0	0	-0.001	-0.002	-0.002	-0.004
0	.14	-0.001	-0.002	-0.002	-0.006
0	.39	0.000	0.002	0.002	0.003
0	.59	0.001	0.012	0.012	0.010
.14	0	0.000	0.003	0.002	0.003
.14	.14	0.024	-0.001	-0.001	-0.002
.14	.39	0.062	0.027	0.028	0.036
.14	.59	0.029	0.023	0.023	0.023
.39	0	-0.001	-0.003	-0.002	0.003
.39	.14	0.051	0.027	0.027	0.036
.39	.39	0.080	0.033	0.032	0.033
.39	.59	0.052	0.029	0.027	0.034
.59	0	0.007	0.003	0.003	0.005
.59	.14	0.032	0.021	0.021	0.025
.59	.39	0.048	0.021	0.021	0.022
.59	.59	0.047	0.022	0.022	0.037

Table 40

Imbalance for N=60 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, and Percentile bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot
a	*b*		*c'*=0		
0	0	0.000	0.000	0.000	0.001
0	.14	-0.001	0.002	0.002	0.002
0	.39	0.007	0.006	0.006	0.001
0	.59	0.006	0.010	0.011	0.007
.14	0	0.001	0.001	0.001	0.005
.14	.14	0.059	0.001	0.004	0.002
.14	.39	0.060	0.029	0.030	0.044
.14	.59	0.024	0.016	0.013	0.019
.39	0	0.002	0.001	0.004	0.005

.39	.14	0.050	0.024	0.023	0.036
.39	.39	0.062	0.024	0.022	0.036
.39	.59	0.052	0.037	0.032	0.026
.59	0	0.007	0.010	0.014	0.006
.59	.14	0.026	0.030	0.026	0.020
.59	.39	0.051	0.033	0.028	0.031
.59	.59	0.042	0.021	0.019	0.024
			c'=.14		
0	0	0.000	0.001	0.000	-0.001
0	.14	-0.001	0.000	0.000	-0.001
0	.39	-0.001	0.005	0.005	0.009
0	.59	-0.003	0.001	0.001	-0.005
.14	0	0.001	0.002	0.002	0.004
.14	.14	0.053	0.004	0.002	0.005
.14	.39	0.039	0.014	0.013	0.024
.14	.59	0.026	0.008	0.008	0.014
.39	0	0.002	0.010	0.007	0.013
.39	.14	0.054	0.036	0.036	0.033
.39	.39	0.068	0.035	0.035	0.025
.39	.59	0.048	0.026	0.024	0.028
.59	0	0.001	0.004	0.006	0.006
.59	.14	0.028	0.029	0.026	0.024
.59	.39	0.047	0.021	0.022	0.019
.59	.59	0.038	0.026	0.033	0.019
			c'=.39		
0	0	0.000	0.000	0.000	0.000
0	.14	0.000	-0.001	-0.001	-0.001
0	.39	0.001	0.006	0.005	0.004
0	.59	0.002	0.017	0.017	0.013
.14	0	0.000	-0.006	-0.007	-0.002
.14	.14	0.067	0.006	0.000	0.000
.14	.39	0.045	0.023	0.023	0.033
.14	.59	0.022	0.004	0.005	0.010
.39	0	-0.004	-0.005	-0.004	-0.006
.39	.14	0.040	0.027	0.028	0.030
.39	.39	0.072	0.041	0.039	0.042
.39	.59	0.050	0.020	0.025	0.015
.59	0	-0.003	-0.006	-0.003	-0.006
.59	.14	0.022	0.026	0.025	0.018
.59	.39	0.042	0.024	0.026	0.032
.59	.59	0.040	0.014	0.014	0.020
			c'=.59		
0	0	0.000	-0.001	-0.001	-0.001
0	.14	0.000	0.001	0.001	0.001
0	.39	0.001	0.009	0.010	0.003
0	.59	0.014	0.018	0.018	0.017
.14	0	0.000	0.004	0.005	0.003
.14	.14	0.051	0.000	-0.003	0.003

.14	.39	0.049	0.039	0.037	0.040
.14	.59	0.017	0.005	0.007	0.010
.39	0	0.002	-0.003	-0.006	0.004
.39	.14	0.046	0.022	0.021	0.022
.39	.39	0.049	0.016	0.019	0.024
.39	.59	0.051	0.024	0.022	0.020
.59	0	-0.004	0.003	0.001	0.004
.59	.14	0.021	0.015	0.013	0.005
.59	.39	0.066	0.055	0.053	0.043
.59	.59	0.044	0.032	0.032	0.022

Table 41

Imbalance for N=100 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, and Percentile bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot
a	*b*	*c'*=0			
0	0	0.000	-0.001	-0.001	0.000
0	.14	0.001	0.003	0.003	0.003
0	.39	0.002	-0.015	-0.015	-0.006
0	.59	-0.004	-0.005	-0.005	0.008
.14	0	-0.002	0.012	0.012	-0.001
.14	.14	0.089	0.035	0.035	0.026
.14	.39	0.037	0.026	0.027	0.023
.14	.59	0.016	0.004	0.003	0.013
.39	0	-0.001	0.063	0.064	0.011
.39	.14	0.043	0.068	0.068	0.028
.39	.39	0.056	0.031	0.032	0.031
.39	.59	0.027	0.012	0.012	0.015
.59	0	0.001	0.057	0.059	0.003
.59	.14	0.016	0.050	0.051	0.005
.59	.39	0.037	0.052	0.052	0.027
.59	.59	0.033	0.021	0.020	0.012
		c'=.14			
0	0	0.000	0.001	0.001	-0.001
0	.14	0.000	-0.001	-0.001	0.001
0	.39	-0.002	-0.008	-0.009	-0.002
0	.59	-0.005	-0.013	-0.012	0.001
.14	0	0.000	0.018	0.018	0.002
.14	.14	0.097	0.034	0.034	0.028
.14	.39	0.035	0.010	0.013	0.011
.14	.59	0.017	0.004	0.003	0.008
.39	0	0.007	0.074	0.074	0.009
.39	.14	0.033	0.081	0.080	0.023
.39	.39	0.045	0.023	0.025	0.020
.39	.59	0.036	0.021	0.021	0.023
.59	0	-0.005	0.057	0.054	-0.011

.59	.14	0.022	0.068	0.068	0.011
.59	.39	0.049	0.062	0.062	0.023
.59	.59	0.028	0.015	0.016	0.012
			c'=.39		
0	0	0.000	-0.002	-0.002	-0.002
0	.14	-0.001	-0.008	-0.007	-0.007
0	.39	-0.004	-0.009	-0.010	0.003
0	.59	-0.009	-0.017	-0.017	-0.007
.14	0	0.001	0.024	0.024	0.001
.14	.14	0.101	0.046	0.047	0.030
.14	.39	0.039	0.020	0.020	0.023
.14	.59	0.030	0.012	0.013	0.018
.39	0	0.005	0.094	0.094	0.014
.39	.14	0.045	0.076	0.076	0.019
.39	.39	0.053	0.029	0.029	0.022
.39	.59	0.034	0.014	0.014	0.015
.59	0	-0.003	0.070	0.070	0.000
.59	.14	0.025	0.079	0.078	0.016
.59	.39	0.024	0.035	0.036	0.004
.59	.59	0.033	0.018	0.018	0.021
			c'=.59		
0	0	0.001	0.002	0.002	0.001
0	.14	-0.001	-0.005	-0.005	0.001
0	.39	-0.001	-0.001	-0.001	0.002
0	.59	-0.008	-0.020	-0.021	-0.010
.14	0	0.000	0.015	0.016	-0.003
.14	.14	0.082	0.047	0.049	0.022
.14	.39	0.045	0.024	0.026	0.029
.14	.59	0.018	0.005	0.004	0.015
.39	0	-0.004	0.055	0.056	-0.010
.39	.14	0.038	0.067	0.065	0.018
.39	.39	0.057	0.021	0.021	0.026
.39	.59	0.038	0.011	0.010	0.017
.59	0	-0.009	0.073	0.074	-0.007
.59	.14	0.021	0.072	0.067	0.014
.59	.39	0.032	0.043	0.040	0.006
.59	.59	0.027	0.021	0.020	0.016

Table 42

Imbalance for N=500 in Study 1. The column names refer to normal theory confidence limits, Bayesian method of coefficients with diffuse prior distributions, Bayesian method of coefficients with informative prior distributions, and Percentile bootstrap (respectively).

		normal	YMdiff	YMinfo	PercBoot
a	*b*		*c'*=0		
0	0	0.000	0.003	0.003	0.001
0	.14	0.008	0.006	0.006	0.009

0	.39	0.009	0.006	0.005	0.005
0	.59	0.004	0.008	0.006	0.009
.14	0	-0.005	-0.016	-0.013	-0.007
.14	.14	0.044	0.007	0.009	0.007
.14	.39	0.008	0.011	0.006	0.003
.14	.59	0.015	0.005	0.002	0.013
.39	0	-0.003	-0.007	-0.007	-0.002
.39	.14	0.028	0.024	0.018	0.014
.39	.39	0.016	0.019	0.022	0.002
.39	.59	0.012	-0.003	-0.003	-0.002
.59	0	-0.001	0.001	-0.003	0.000
.59	.14	0.016	0.011	0.013	0.011
.59	.39	0.024	0.012	0.013	0.020
.59	.59	0.009	0.003	-0.001	0.003
			c'=.14		
0	0	0.000	0.000	0.001	0.001
0	.14	0.002	-0.002	0.000	0.004
0	.39	0.008	0.008	0.008	0.010
0	.59	-0.006	-0.005	-0.011	-0.003
.14	0	0.001	0.001	0.000	0.002
.14	.14	0.055	0.020	0.024	0.021
.14	.39	0.008	-0.003	-0.006	0.001
.14	.59	0.009	0.001	0.002	0.002
.39	0	-0.004	0.008	0.005	-0.001
.39	.14	0.020	0.008	0.007	0.006
.39	.39	0.014	0.011	0.017	0.008
.39	.59	0.014	0.004	-0.002	0.005
.59	0	0.006	0.010	0.008	0.003
.59	.14	0.004	0.003	0.003	-0.002
.59	.39	0.020	0.011	0.011	0.007
.59	.59	0.018	0.005	0.005	0.013
			c'=.39		
0	0	0.000	0.000	0.000	0.000
0	.14	-0.006	-0.007	-0.004	-0.003
0	.39	0.000	0.002	0.003	0.005
0	.59	-0.003	0.001	-0.001	0.001
.14	0	-0.007	-0.011	-0.013	-0.009
.14	.14	0.055	0.024	0.023	0.024
.14	.39	0.016	0.004	0.009	0.007
.14	.59	0.004	-0.003	0.000	-0.003
.39	0	-0.003	0.012	0.006	0.000
.39	.14	0.037	0.025	0.029	0.019
.39	.39	0.027	0.014	0.012	0.017
.39	.59	0.025	0.001	0.002	0.012
.59	0	-0.002	0.001	0.001	0.002
.59	.14	0.007	0.009	0.005	0.004
.59	.39	0.017	0.014	0.014	0.003
.59	.59	-0.001	-0.006	-0.006	-0.012

			c'=.59		
0	0	0.000	0.000	0.000	-0.001
0	.14	-0.006	-0.007	-0.005	-0.003
0	.39	0.001	0.006	0.005	0.000
0	.59	-0.006	-0.003	-0.009	-0.001
.14	0	-0.004	-0.012	-0.011	-0.007
.14	.14	0.059	0.018	0.016	0.025
.14	.39	0.020	0.007	0.012	0.013
.14	.59	0.012	0.014	0.011	0.012
.39	0	-0.007	-0.003	-0.005	-0.005
.39	.14	0.022	0.006	0.006	0.013
.39	.39	0.033	0.021	0.023	0.021
.39	.59	0.015	0.001	0.001	0.011
.59	0	0.000	0.004	0.008	-0.003
.59	.14	0.014	0.002	0.007	0.010
.59	.39	0.016	-0.008	-0.007	0.004
.59	.59	0.007	-0.002	-0.002	0.003

Table 43

Type I error rate for N=20 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2 (c'=0).

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
0	0	0.001	0.004	0.002	0.000	0.002	0.000	0.006	0.010
0	.14	0.000	0.004	0.004	0.000	0.001	0.000	0.006	0.019
0	.39	0.005	0.022	0.011	0.004	0.009	0.003	0.016	0.032
0	.59	0.014	0.039	0.021	0.013	0.019	0.005	0.054	0.089

Table 44

Type I error rate for N=40 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2 (c'=0).

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
0	0	0	0.002	0.002	0	0	0	0.004	0.01
0	.14	0	0.006	0.004	0.001	0.003	0	0.005	0.016
0	.39	0.009	0.038	0.028	0.012	0.021	0.005	0.041	0.071
0	.59	0.018	0.055	0.047	0.006	0.036	0.002	0.062	0.097

Table 45

Type I error rate for N=60 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2 (c'=0).

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
0	0	0	0.001	0	0	0	0	0	0.006
0	.14	0	0.007	0.006	0	0.004	0.001	0.009	0.026
0	.39	0.016	0.028	0.026	0.006	0.021	0.003	0.037	0.071
0	.59	0.018	0.04	0.039	0.007	0.031	0.005	0.04	0.066

Table 46

Type I error rate for N=100 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2 (c'=0).

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
0	0	0.001	0.002	0.003	0	0.002	0	0.003	0.007
0	.14	0.003	0.015	0.01	0.001	0.011	0.002	0.013	0.031
0	.39	0.022	0.047	0.05	0.002	0.047	0.005	0.06	0.085
0	.59	0.032	0.044	0.042	0.007	0.047	0.006	0.054	0.063

Table 47

Type I error rate for N=200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2 (c'=0).

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
0	0	0	0	0	0	0.001	0	0.001	0.004
0	.14	0.005	0.03	0.032	0.005	0.035	0.006	0.033	0.056
0	.39	0.042	0.058	0.061	0.009	0.065	0.01	0.061	0.083
0	.59	0.051	0.059	0.059	0.007	0.061	0.007	0.061	0.064

Table 48

Power for N=20 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
		c'=0							

.14	.14	0.002	0.007	0.005	0	0.003	0	0.008	0.019
.14	.39	0.006	0.03	0.013	0.023	0.014	0.024	0.025	0.063
.14	.59	0.026	0.065	0.029	0.048	0.036	0.042	0.069	0.112
.39	.39	0.053	0.133	0.077	0.424	0.084	0.413	0.122	0.211
.39	.59	0.12	0.221	0.147	0.641	0.167	0.64	0.205	0.296
.59	.59	0.277	0.433	0.329	0.976	0.359	0.964	0.374	0.501
					c'=.39				
.14	.14	0.001	0.006	0.003	0	0.002	0	0.006	0.019
.39	.39	0.052	0.115	0.072	0.425	0.072	0.401	0.115	0.193
.59	.59	0.275	0.448	0.344	0.969	0.368	0.966	0.42	0.528

Table 49

Power for N=40 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
.14	.14	0	0.007	0.005	0.003	0.003	0.002	0.01	0.032
.14	.39	0.03	0.081	0.059	0.125	0.058	0.139	0.073	0.131
.14	.59	0.081	0.154	0.135	0.162	0.123	0.173	0.151	0.201
.39	.39	0.231	0.387	0.356	0.951	0.353	0.957	0.379	0.504
.39	.59	0.455	0.609	0.569	0.975	0.544	0.977	0.594	0.689
.59	.59	0.773	0.873	0.849	0.999	0.841	1	0.845	0.907
					c'=.39				
.14	.14	0.004	0.021	0.012	0.015	0.016	0.009	0.02	0.04
.39	.39	0.243	0.411	0.361	0.964	0.385	0.962	0.405	0.514
.59	.59	0.786	0.872	0.845	1	0.857	1	0.861	0.908

Table 50

Power for N=60 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
.14	.14	0.006	0.037	0.03	0.045	0.028	0.03	0.036	0.065
.14	.39	0.06	0.141	0.127	0.269	0.113	0.232	0.151	0.233
.14	.59	0.122	0.182	0.158	0.25	0.148	0.23	0.198	0.253
.39	.39	0.484	0.677	0.646	0.995	0.622	0.995	0.653	0.739

.39	.59	0.737	0.819	0.794	0.996	0.788	0.997	0.813	0.855
.59	.59	0.969	0.985	0.98	1	0.983	1	0.972	0.982
					c'=.39				
.14	.14	0.002	0.025	0.024	0.039	0.016	0.018	0.03	0.064
.39	.39	0.48	0.671	0.645	0.997	0.633	0.996	0.652	0.758
.59	.59	0.955	0.976	0.972	1	0.972	1	0.971	0.986

Table 51

Power for N=100 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
.14	.14	0.022	0.064	0.052	0.197	0.055	0.254	0.068	0.123
.14	.39	0.18	0.283	0.277	0.488	0.26	0.589	0.299	0.36
.14	.59	0.239	0.29	0.281	0.497	0.269	0.563	0.296	0.328
.39	.39	0.85	0.921	0.898	1	0.916	1	0.918	0.939
.39	.59	0.955	0.967	0.964	1	0.96	1	0.964	0.971
.59	.59	1	1	1	1	1	1	1	1
					c'=.39				
.14	.14	0.025	0.066	0.055	0.202	0.056	0.271	0.073	0.129
.39	.39	0.868	0.924	0.91	1	0.929	1	0.921	0.945
.59	.59	1	1	0.999	1	1	1	1	1

Table 52

Power for N=200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
.14	.14	0.074	0.196	0.201	0.735	0.240	0.761	0.199	0.315
.14	.39	0.447	0.515	0.506	0.878	0.574	0.885	0.521	0.567
.14	.59	0.483	0.505	0.502	0.878	0.561	0.883	0.501	0.543
.39	.39	0.996	0.999	0.998	1	0.999	1	0.996	0.999
.39	.59	0.999	0.999	1	1	1	1	1	1
.59	.59	1	1	1	1	1	1	1	1
					c'=.39				
.14	.14	0.09	0.244	0.244	0.739	0.278	0.74	0.238	0.353

.39	.39	0.996	0.997	0.998	1	0.998	1	0.996	0.998
.59	.59	1	1	1	1	1	1	1	1

Table 53

Coverage for N=20 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0.999	0.996	0.998	1	0.998	1	0.994	0.99
0	.14	1	0.996	0.996	1	0.999	1	0.994	0.981
0	.39	0.995	0.978	0.989	0.996	0.991	0.997	0.984	0.968
0	.59	0.986	0.961	0.979	0.987	0.981	0.995	0.946	0.911
.14	.14	0.992	0.993	0.996	1	0.997	1	0.991	0.984
.14	.39	0.966	0.982	0.989	0.993	0.993	0.997	0.975	0.94
.14	.59	0.967	0.948	0.965	0.992	0.97	0.996	0.936	0.896
.39	.39	0.901	0.947	0.953	0.989	0.933	0.995	0.928	0.917
.39	.59	0.889	0.926	0.937	0.99	0.917	0.998	0.914	0.916
.59	.59	0.905	0.943	0.948	0.995	0.91	0.998	0.935	0.952
					c'=.39				
.14	.14	0.986	0.993	0.996	1	0.998	1	0.994	0.981
.39	.39	0.903	0.951	0.962	0.992	0.946	0.991	0.929	0.916
.59	.59	0.902	0.928	0.937	0.988	0.919	0.993	0.915	0.931

Table 54

Coverage for N=40 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	1	0.998	0.998	1	1	1	0.996	0.99
0	.14	1	0.994	0.996	0.999	0.997	1	0.995	0.984
0	.39	0.991	0.962	0.972	0.988	0.979	0.995	0.959	0.929
0	.59	0.982	0.945	0.953	0.994	0.964	0.998	0.938	0.903
.14	.14	0.962	0.999	0.999	1	1	1	0.995	0.966
.14	.39	0.944	0.959	0.966	0.994	0.964	0.995	0.943	0.91
.14	.59	0.955	0.945	0.955	0.993	0.958	0.995	0.936	0.911
.39	.39	0.923	0.944	0.953	0.99	0.948	0.993	0.932	0.943

.39	.59	0.932	0.944	0.96	0.995	0.942	0.997	0.938	0.948
.59	.59	0.925	0.948	0.96	0.99	0.937	0.992	0.923	0.944
					c'=.39				
.14	.14	0.966	0.987	0.991	0.999	0.991	1	0.981	0.944
.39	.39	0.934	0.947	0.965	0.991	0.957	0.992	0.937	0.955
.59	.59	0.933	0.95	0.957	0.992	0.946	0.993	0.94	0.944

Table 55

Coverage for N=60 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	1	0.999	1	1	1	1	1	0.994
0	.14	1	0.993	0.994	1	0.996	0.999	0.991	0.974
0	.39	0.984	0.972	0.974	0.994	0.979	0.997	0.963	0.929
0	.59	0.982	0.96	0.961	0.993	0.969	0.995	0.96	0.934
.14	.14	0.937	0.985	0.989	0.994	0.988	0.997	0.976	0.914
.14	.39	0.94	0.951	0.952	0.997	0.954	0.997	0.94	0.924
.14	.59	0.954	0.948	0.951	0.997	0.952	0.997	0.938	0.919
.39	.39	0.913	0.94	0.935	0.986	0.942	0.993	0.922	0.932
.39	.59	0.93	0.944	0.945	0.99	0.944	0.992	0.936	0.94
.59	.59	0.932	0.953	0.954	0.996	0.942	0.995	0.939	0.943
					c'=.39				
.14	.14	0.945	0.993	0.993	0.996	0.995	0.997	0.981	0.934
.39	.39	0.923	0.95	0.949	0.993	0.955	0.995	0.933	0.945
.59	.59	0.934	0.95	0.953	0.996	0.951	0.991	0.945	0.951

Table 56

Coverage for N=100 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0.999	0.998	0.997	1	0.998	1	0.997	0.993
0	.14	0.997	0.985	0.99	0.999	0.989	0.998	0.987	0.969
0	.39	0.978	0.953	0.95	0.998	0.953	0.995	0.94	0.915
0	.59	0.968	0.956	0.958	0.993	0.953	0.994	0.946	0.937

.14	.14	0.906	0.965	0.97	0.991	0.966	0.989	0.958	0.903
.14	.39	0.934	0.94	0.944	0.993	0.936	0.991	0.934	0.923
.14	.59	0.961	0.96	0.966	0.992	0.945	0.994	0.949	0.947
.39	.39	0.937	0.955	0.957	0.994	0.949	0.995	0.958	0.962
.39	.59	0.937	0.946	0.954	0.991	0.937	0.992	0.94	0.947
.59	.59	0.935	0.945	0.946	0.993	0.935	0.996	0.939	0.95
					c'=.39				
.14	.14	0.9	0.97	0.972	0.99	0.965	0.994	0.953	0.913
.39	.39	0.948	0.96	0.963	0.995	0.959	0.995	0.954	0.96
.59	.59	0.946	0.952	0.956	0.994	0.952	0.997	0.947	0.947

Table 57

Coverage for N=200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	1	1	1	1	0.999	1	0.999	0.996
0	.14	0.995	0.97	0.968	0.995	0.965	0.994	0.967	0.944
0	.39	0.958	0.942	0.939	0.991	0.935	0.99	0.939	0.917
0	.59	0.949	0.941	0.941	0.993	0.939	0.993	0.939	0.936
.14	.14	0.905	0.96	0.96	0.994	0.953	0.993	0.957	0.957
.14	.39	0.949	0.954	0.949	0.995	0.949	0.995	0.95	0.948
.14	.59	0.953	0.948	0.949	0.994	0.945	0.994	0.943	0.941
.39	.39	0.943	0.942	0.945	0.995	0.934	0.993	0.942	0.947
.39	.59	0.947	0.947	0.951	0.988	0.941	0.989	0.948	0.943
.59	.59	0.948	0.948	0.953	0.987	0.943	0.989	0.95	0.949
					c'=.39				
.14	.14	0.904	0.943	0.941	0.992	0.94	0.996	0.942	0.944
.39	.39	0.935	0.936	0.94	0.992	0.935	0.992	0.937	0.937
.59	.59	0.946	0.95	0.954	0.992	0.944	0.994	0.949	0.956

Table 58

Interval Width for N=20 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				

0	0	0.285	0.397	0.449	0.166	0.344	0.161	0.412	0.435
0	.14	0.318	0.421	0.480	0.190	0.363	0.184	0.437	0.460
0	.39	0.435	0.516	0.590	0.299	0.447	0.297	0.538	0.557
0	.59	0.594	0.658	0.746	0.418	0.571	0.422	0.675	0.691
.14	.14	0.331	0.433	0.480	0.207	0.372	0.206	0.457	0.479
.14	.39	0.465	0.539	0.604	0.312	0.460	0.316	0.557	0.579
.14	.59	0.615	0.674	0.757	0.427	0.578	0.437	0.695	0.716
.39	.39	0.588	0.642	0.691	0.380	0.539	0.397	0.669	0.694
.39	.59	0.706	0.751	0.820	0.476	0.634	0.502	0.780	0.808
.59	.59	0.812	0.842	0.900	0.544	0.712	0.574	0.874	0.902
					c'=.39				
.14	.14	0.310	0.413	0.462	0.200	0.360	0.199	0.432	0.451
.39	.39	0.570	0.627	0.677	0.378	0.545	0.387	0.656	0.687
.59	.59	0.827	0.858	0.916	0.5518	0.748	0.563	0.891	0.922

Table 59

Interval Width for N=40 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0.133	0.186	0.197	0.0794	0.175	0.0758	0.185	0.194
0	.14	0.160	0.207	0.220	0.101	0.195	0.096	0.206	0.215
0	.39	0.271	0.301	0.319	0.192	0.288	0.185	0.301	0.310
0	.59	0.392	0.415	0.438	0.279	0.400	0.268	0.415	0.421
.14	.14	0.172	0.216	0.231	0.114	0.203	0.111	0.216	0.227
.14	.39	0.292	0.319	0.339	0.202	0.304	0.200	0.321	0.331
.14	.59	0.404	0.424	0.447	0.284	0.406	0.282	0.423	0.430
.39	.39	0.375	0.390	0.414	0.253	0.367	0.259	0.390	0.406
.39	.59	0.475	0.486	0.513	0.325	0.462	0.332	0.487	0.501
.59	.59	0.543	0.550	0.580	0.373	0.523	0.385	0.549	0.564
					c'=.39				
.14	.14	0.182	0.224	0.239	0.117	0.218	0.117	0.225	0.236
.39	.39	0.375	0.390	0.412	0.253	0.378	0.262	0.391	0.408
.59	.59	0.546	0.553	0.584	0.372	0.533	0.385	0.551	0.565

Table 60

Interval Width for N=60 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and *bias-corrected bootstrap interval estimates in Study 2.*

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0.088	0.122	0.126	0.052	0.118	0.051	0.121	0.127
0	.14	0.108	0.138	0.143	0.072	0.134	0.071	0.137	0.144
0	.39	0.213	0.230	0.237	0.152	0.221	0.152	0.229	0.234
0	.59	0.309	0.321	0.331	0.221	0.306	0.225	0.318	0.320
.14	.14	0.129	0.154	0.158	0.086	0.152	0.087	0.152	0.161
.14	.39	0.225	0.239	0.246	0.158	0.231	0.159	0.236	0.242
.14	.59	0.323	0.333	0.341	0.228	0.318	0.230	0.332	0.336
.39	.39	0.293	0.300	0.306	0.203	0.292	0.200	0.298	0.309
.39	.59	0.372	0.378	0.385	0.259	0.365	0.253	0.374	0.382
.59	.59	0.438	0.442	0.451	0.302	0.426	0.301	0.441	0.449
					c'=.39				
.14	.14	0.182	0.224	0.239	0.117	0.218	0.117	0.225	0.236
.39	.39	0.375	0.390	0.412	0.253	0.378	0.262	0.391	0.408
.59	.59	0.546	0.553	0.584	0.372	0.533	0.385	0.551	0.565

Table 61

Interval Width for N=100 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0.049	0.070	0.072	0.030	0.069	0.028	0.069	0.074
0	.14	0.072	0.088	0.090	0.050	0.082	0.046	0.087	0.091
0	.39	0.159	0.167	0.169	0.117	0.155	0.109	0.166	0.168
0	.59	0.232	0.239	0.242	0.167	0.217	0.160	0.237	0.238
.14	.14	0.091	0.103	0.105	0.061	0.098	0.058	0.103	0.108
.14	.39	0.171	0.177	0.181	0.120	0.167	0.114	0.176	0.179
.14	.59	0.246	0.251	0.254	0.173	0.233	0.169	0.249	0.251
.39	.39	0.221	0.223	0.230	0.155	0.214	0.148	0.222	0.228
.39	.59	0.286	0.288	0.295	0.199	0.274	0.191	0.286	0.290
.59	.59	0.336	0.337	0.346	0.232	0.324	0.226	0.336	0.340
					c'=.39				
.14	.14	0.090	0.103	0.104	0.061	0.099	0.060	0.102	0.108
.39	.39	0.223	0.225	0.231	0.155	0.222	0.154	0.223	0.229
.59	.59	0.335	0.337	0.344	0.231	0.332	0.233	0.334	0.339

Table 62

Interval Width for N=200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
		c'=0							
0	0	0.025	0.035	0.035	0.015	0.033	0.015	0.035	0.037
0	.14	0.045	0.052	0.052	0.032	0.048	0.031	0.052	0.056
0	.39	0.111	0.114	0.114	0.078	0.108	0.076	0.113	0.114
0	.59	0.166	0.168	0.169	0.117	0.161	0.115	0.167	0.168
.14	.14	0.058	0.062	0.063	0.040	0.058	0.040	0.062	0.065
.14	.39	0.116	0.118	0.118	0.082	0.111	0.079	0.117	0.119
.14	.59	0.170	0.172	0.173	0.120	0.165	0.118	0.171	0.172
.39	.39	0.156	0.156	0.157	0.108	0.148	0.111	0.156	0.158
.39	.59	0.199	0.200	0.201	0.139	0.188	0.139	0.199	0.200
.59	.59	0.234	0.234	0.236	0.162	0.223	0.168	0.233	0.235
		c'=.39							
.14	.14	0.060	0.064	0.064	0.041	0.062	0.041	0.064	0.067
.39	.39	0.155	0.156	0.157	0.108	0.152	0.112	0.155	0.157
.59	.59	0.232	0.233	0.235	0.162	0.229	0.171	0.232	0.234

Table 63

Imbalance for N=20 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
		c'=0							
0	0	0.001	-0.002	0	0	0	0	0	0
0	.14	0	0	0	0	-0.001	0	0	0.005
0	.39	0.001	-0.002	0.001	0	-0.003	0.001	-0.004	0.002
0	.59	-0.004	0.003	-0.001	-0.003	-0.009	-0.001	-0.002	0.001
.14	.14	0.008	-0.001	0	0	0.001	0	-0.001	0
.14	.39	0.028	0.006	0.001	0.005	0.003	-0.001	0.013	0.034
.14	.59	0.027	0.02	0.021	0.006	0.024	0.004	0.034	0.052
.39	.39	0.083	0.025	0.029	0.007	0.061	0.003	0.03	0.031
.39	.59	0.097	0.036	0.049	0.004	0.079	0	0.046	0.014
.59	.59	0.081	0.029	0.038	0.003	0.088	0	0.043	0.018

					c'=.39				
.14	.14	0.014	0.005	0.004	0	0.002	0	0.004	0.009
.39	.39	0.087	0.027	0.024	0.006	0.05	0.007	0.047	0.034
.59	.59	0.07	0.032	0.035	0.004	0.071	-0.001	0.051	0.015

Table 64

Imbalance for N=40 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0	-0.002	-0.002	0	0	0	-0.002	-0.994
0	.14	0	0	-0.002	-0.001	0.001	0	-0.001	-0.002
0	.39	-0.007	-0.012	-0.008	-0.004	-0.011	-0.005	-0.005	-0.007
0	.59	0.002	0.007	0.009	0.002	0.006	0.002	0.006	0.007
.14	.14	0.038	-0.001	-0.001	0	0	0	0.001	0.024
.14	.39	0.046	0.015	0.01	0.002	0.022	0.001	0.019	0.04
.14	.59	0.025	0.015	0.015	0.001	0.026	0.005	0.014	0.021
.39	.39	0.061	0.02	0.017	0.006	0.036	0.003	0.032	0.005
.39	.59	0.05	0.02	0.014	0.003	0.044	0.001	0.022	0
.59	.59	0.065	0.022	0.024	0.006	0.053	0.006	0.045	0.014
					c'=.39				
.14	.14	0.032	-0.001	-0.001	-0.001	-0.001	0	0.001	0.022
.39	.39	0.056	0.021	0.019	0.005	0.035	0.006	0.031	0.001
.59	.59	0.045	0.018	0.015	0.006	0.036	0.003	0.028	0.008

Table 65

Imbalance for N=60 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0	-0.001	0	0	0	0	0	0.004
0	.14	0	0.001	0.002	0	0.002	0.001	-0.001	-0.008
0	.39	0.002	0.006	0.006	0.006	0.007	0.003	0.009	0.003
0	.59	0.004	0.004	0.009	0.001	0.009	0.003	0.004	-0.002
.14	.14	0.059	0.005	0.001	0.006	0.006	0.003	0.008	0.046
.14	.39	0.048	0.027	0.03	-0.003	0.032	-0.003	0.032	0.03

.14	.59	0.034	0.024	0.023	-0.001	0.034	0.001	0.024	0.023
.39	.39	0.063	0.028	0.029	0.004	0.036	0.003	0.038	0.01
.39	.59	0.054	0.03	0.029	0.006	0.046	0.006	0.02	0.008
.59	.59	0.054	0.013	0.014	0.004	0.052	0.003	0.027	0.001
					c'=.39				
.14	.14	0.055	0.003	0.005	0.004	0.003	0.003	0.005	0.036
.39	.39	0.061	0.012	0.021	0.003	0.025	0.003	0.021	-0.007
.59	.59	0.044	0.022	0.019	0.004	0.035	0.007	0.029	0.007

Table 66

Imbalance for N=100 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	-0.001	-0.002	-0.001	0	-0.002	0	-0.001	-0.994
0	.14	-0.001	0.003	0	0.001	0.007	-0.002	0.005	-0.001
0	.39	0.004	0.007	0.008	0.002	0.019	-0.001	0.004	0.007
0	.59	0	-0.002	-0.002	-0.001	0.007	-0.004	0	-0.001
.14	.14	0.094	0.017	0.022	0.009	0.024	0.011	0.024	0.059
.14	.39	0.046	0.028	0.02	-0.001	0.04	-0.003	0.024	0.007
.14	.59	0.017	0.01	0.004	0.002	0.033	0	0.011	0.011
.39	.39	0.051	0.023	0.021	0.004	0.033	0.003	0.018	0.002
.39	.59	0.033	0.006	0.01	-0.001	0.033	0	0.01	-0.009
.59	.59	0.033	0.015	0.02	-0.003	0.037	0	0.015	-0.006
					c'=.39				
.14	.14	0.096	0.006	0.006	0.008	0.017	0.002	0.017	0.047
.39	.39	0.038	0.01	0.015	0.001	0.021	0.005	0.012	-0.002
.59	.59	0.038	0.02	0.024	0	0.03	0.003	0.021	0.003

Table 67

Imbalance for N=200 for normal theory, distribution of the product, Bayesian method of coefficients with diffuse priors, Bayesian method of coefficients with informative priors, percentile bootstrap, and bias-corrected bootstrap interval estimates in Study 2.

		Method							
a	*b*	normal	prodclin	YMdiff	YMinfo	EFMdiff	EFMinfo	PercBoot	BCBoot
					c'=0				
0	0	0	0	0	0	0.001	0	-0.001	0
0	.14	0.001	0.006	0.008	-0.001	-0.011	0.002	0.005	0

0	.39	-0.002	-0.008	-0.007	0.003	-0.023	0.006	-0.007	-0.007
0	.59	-0.005	-0.005	-0.005	0.001	-0.015	0.001	-0.007	-0.004
.14	.14	0.089	0.022	0.024	0.006	0.027	0.007	0.023	0.019
.14	.39	0.029	0.01	0.017	0.003	0.011	0.005	0.014	0.002
.14	.59	0.015	0.004	0.007	0.002	0.003	0.002	0.003	-0.001
.39	.39	0.033	0.006	0.011	-0.003	0.022	0.001	0.012	-0.001
.39	.59	0.031	0.021	0.019	0.008	0.033	0.009	0.02	0.011
.59	.59	0.018	0.008	0.007	0.003	0.027	0.003	0.006	-0.003
					c'=.39				
.14	.14	0.09	0.039	0.045	0.004	0.042	0.004	0.04	0.03
.39	.39	0.029	0.004	0.008	0.004	0.013	0.006	0.011	-0.005
.59	.59	0.03	0.022	0.02	0.002	0.032	0.002	0.029	0.014

APPENDIX D

SIMULATIONS PROGRAM TO EVALUATE TYPE I ERROR, POWER, COVERAGE, IMBALANCE, AND INTERVAL WIDTH FOR ONE PARAMETER COMBINATION IN STUDY 2

```
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

PROC DATASETS LIBRARY=WORK KILL NOLIST; RUN;
LIBNAME N40 "C:\MILICA\n20output";


%MACRO SIMULATE(NSIM,NOBS,BMX,BYX,BYM,FILE,TYPE,ERROR);

DATA SUMMARY; SET _NULL_;
%DO I=1 %TO &NSIM;
TITLE 'SIMULATION OF MEDIATION';
DATA SIM;
DO I=1 TO &NOBS;
X=(&error)*RANNOR(0);
M=&BMX*X+(&error)*RANNOR(0);
Y=&BYX*X+&BYM*M+(&error)*RANNOR(0);
X2=X*X;
OUTPUT;
END;


*This code obtains estimates of the mediation regression
equations for the sample generated in the macro program;

*Estimating the (Y=X) regression and saving the value of bm1 or c in the
text;
PROC REG OUTEST=FILE COVOUT noprint; MODEL Y=X/;
DATA B; SET FILE;
IF _TYPE_='PARMS'; BM1=X;MSE1=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y;
KEEP BM1 MSE1;
DATA C; SET FILE;IF _NAME_='X'; SEBM1=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y;
KEEP SEBM1;
DATA MODEL1; MERGE B C;

*Estimating the (Y=X M) regression and saving the values of
c prime and b;
PROC REG DATA=SIM OUTEST=FILE COVOUT NOPRINT;
MODEL Y=X M/;
DATA B; SET FILE;
IF _TYPE_='PARMS'; C=X;
BM2=X; MSE2=_RMSE_*_RMSE_;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y M;
KEEP MSE2 BM2 C;
DATA C; SET FILE;
IF _NAME_='X'; SEBM2=SQRT(X); SEC=SQRT(X);
KEEP SEBM2 SEC;
DATA D; SET FILE; IF _NAME_= 'M'; SEB=SQRT(M);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y M;
KEEP SEB;
DATA E; SET FILE; B=M; IF _TYPE_='PARMS';
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y M;
```

```
KEEP B;
DATA F; SET FILE; IF _NAME_='M'; CBC=X;
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X Y M;
KEEP CBC;
DATA MODEL2; MERGE B C D E F;

*Estimating the (M=X) regression and saving the value of a;
PROC REG DATA=SIM OUTEST=FILE COVOUT NOPRINT;
MODEL M=X;
DATA BB; SET FILE;
IF _TYPE_='PARMS'; M=X;
BM2=X; MSE3=_RMSE_*_RMSE_;
KEEP MSE3;

DATA B; SET FILE; A=X; IF _TYPE_='PARMS';
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M;
KEEP A;
DATA C; SET FILE; IF _NAME_='X'; SEA=SQRT(X);
DROP _MODEL_ _NAME_ _TYPE_ _DEPVAR_ _RMSE_ INTERCEP X M;
KEEP SEA;
DATA MODEL3; MERGE BB B C;

*This code saves  value of X squared in the sample;
PROC MEANS DATA=SIM SUM NOPRINT; VAR X2;
OUTPUT OUT=OUT SUM=SUMX;

*This code saves the variance of X in the sample;
PROC MEANS DATA=SIM STD NOPRINT; VAR X;
OUPUT OUT=OUTA STD=VARX;
DATA VARS; SET OUTA;
VARXX=VARX*VARX;
KEEP VARXX;

*This code saves the variance of M in the sample;
PROC MEANS DATA=SIM STD NOPRINT; VAR M;
OUPUT OUT=OUTA STD=VARM;
DATA VARSM; SET OUTA;
VARMM=VARM*VARM;
KEEP VARMM;

*This code saves the correlation between X and M in the sample;
PROC CORR DATA=SIM OUTPUT=COV NOPRINT;
DATA FR; SET COV; IF _NAME_='M'; CORRXM=X;
KEEP CORRXM;

*This code merges the different datasets that contain
estimates from the simulation replication;
DATA ALL; MERGE OUT MODEL1 MODEL2 MODEL3 VARS VARSM FR ;
RUN;

*This code calculates the population values for empirical
values estimated in the simulation;
DATA TEST;SET ALL;
TYPE=&TYPE;
```

```
ERROR=&ERROR;
DOF=_FREQ_-2;
NOBS=&NOBS;
BMX=&BMX;BYX=&BYX;BYM=&BYM;

*This section computes true variances and covariances as in Section 4.10
based
on residual error variance equal to 1. Note that VX1X1, VX2X2, and VX3X3 are
the residual variance in equations 3.1, 3.2, and 3.3, respectively;
EMOD1=(&ERROR)**2;
EMOD2=(&ERROR)**2;
EMOD3=(&ERROR)**2;
VX1X1=EMOD1;
CY1X1=BMX*EMOD1;
CY2X1=BYM*BMX*VX1X1+BYX*EMOD1;
CY1Y1=BMX*BMX*VX1X1+EMOD2;
CY2Y1=BMX*BMX*BYM*VX1X1+BMX*BYX*EMOD1+BYM*EMOD2;
CY2Y2=BYM*BYM*(BMX*BMX*EMOD1+EMOD2)+2*BMX*BYM*BYX*VX1X1+BYX*BYX*EMOD1
+EMOD3;

*The following code computes standard errors for
product of coefficients methods;
AB=A*B;
SOBEL=SQRT(A*A*SEB*SEB+B*B*SEA*SEA);

TRUEAB=&BMX*&BYM;
/*attempt*/
zaobs=A/SEA;
RZAOBS=ROUND(zaobs, .1);
za=RZAOBS;
zbobs=B/SEB;
RZBOBS=ROUND(zbobs, .1);
zb=RZBOBS;
/*may need to move the chunk above*/

data TESTsobel; set TEST;

*The following code computes two simulation outcome measures (confidence
limits and cases where the true value is outside the confidence limits)
for methods to test mediation;
LSOBEL=AB-1.96*SOBEL; USOBEL=AB+1.96*SOBEL;
RGSOBEL=0; LFSOBEL=0;
IF TRUEAB=0 && LSOBEL GT 0 && USOBEL GT 0 THEN LFSOBEL=1;
IF TRUEAB=0 && LSOBEL LT 0 && USOBEL LT 0 THEN RGSOBEL=1;

TYPEIERRORRATE=RGSOBEL+LFSOBEL;
CIWIDTH=USOBEL-LSOBEL;
IF TRUEAB GT 0 && LSOBEL LT 0 && USOBEL GT 0 THEN empbeta=1;
ELSE empbeta=0;
POWER=1-empbeta;
IMBRSOBEL=0; IMBLSOBEL=0;
IF TRUEAB GT USOBEL THEN IMBRSOBEL=1;
IF TRUEAB LT LSOBEL THEN IMBLSOBEL=1;
IMBALANCE=IMBRSOBEL-IMBLSOBEL;
```

```
IF TRUEAB GT LSOBEL && TRUEAB LT USOBEL THEN COVERAGE=1;
ELSE COVERAGE=0;

/*ADDING PRODCLIN */
options noxwait ;
*Designate location of prdclinforSAS.sas and prodclinsas2.exe;
libname save "C:\Users\psyripl\Desktop\";

%macro prodclin(a, sea, b, seb, rho, alpha);

data proddata1;
*Change file address to match the location of the file prodclin.exe;
file "C:\Users\psyripl\Desktop\raw.txt";
a=&a; sea=&sea; b=&b; seb=&seb; rho=&rho; alpha=&alpha;
put a @; put sea @; put b @; put seb @; put rho @; put alpha @;

*Change file address to match the location of the file prodclin.exe;
X cd C:\Users\psyripl\Desktop;
*Change file address to match the location of the file prodclin.exe;
X call "C:\Users\psyripl\Desktop\ProdClin2_Sas.exe";

data proddata2;
do;
rc=system("call C:\Users\psyripl\Desktop\ProdClin2_Sas.exe");
end;
run;

data proddata2;
infile "C:\Users\psyripl\Desktop\critval.txt";
input lowz highz;
a=&a; sea=&sea; b=&b; seb=&seb; rho=&rho; alpha=&alpha;
r=rho;
da=a/sea;
db=b/seb;
sedadb=sqrt(da*da+db*db+1);
dadb=da*db;
ab=a*b;
sobelse=sqrt(a*a*seb*seb+b*b*sea*sea);
MVDSE = sqrt(a*a*seb*seb+b*b*sea*sea);

if dadb gt 0 then prodlow=lowz;
if dadb gt 0 then produp=highz;
if dadb=0 then prodlow=lowz;
if dadb=0 then produp=highz;
if dadb lt 0 then prodlow=lowz;
if dadb lt 0 then produp=highz;


%mend;

DATA ALL;
SET ALL;
call symput("A",A);
```

```
call symput("SEA",SEA);
call symput("B",B);
call symput("SEB",SEB);

RUN;

%prodclin(a=&A, sea=&SEA, b=&B, seb=&SEB, rho=0, alpha = .05);
run;
quit;

data prodquant; merge proddata2 TEST;

RGprod=0; LFprod=0;

IF TRUEAB=0 && prodlow GT 0 && produp GT 0 THEN LFprod=1;
IF TRUEAB=0 && prodlow LT 0 && produp LT 0 THEN RGprod=1;

TYPEIERRORRATEprod=RGprod+LFprod;
if dadb gt 0 then CIWIDTHprod=produp-prodlow;
if dadb=0 then CIWIDTHprod=produp-prodlow;
if dadb lt 0 then CIWIDTHprod=produp-prodlow;
IF TRUEAB GT 0 && prodlow LT 0 && produp GT 0 THEN empbetaprod=1;
ELSE empbetaprod=0;
POWERprod=1-empbetaprod;
IMBLPROD=0; IMBRPROD=0;
IF TRUEAB GT produp THEN IMBRPROD=1;
IF TRUEAB LT prodlow THEN IMBLPROD=1;
IMBALANCEprod=IMBRPROD-IMBLPROD;
IF TRUEAB GT prodlow && TRUEAB LT produp THEN COVERAGEprod=1;
else COVERAGEprod=0;

upperlimit=produp;
lowerlimit=prodlow;

/*YUAN AND MACKINNON */
data priors1; set SIM;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=&BMX*&BYM;
proc mcmc data=priors1 outpost=out nmc=5000 thin=5 seed=2 stats=none
diag=none;
parms a 0 b 0 cpr 0 i2 0 i3 0;
parms sigmaem 1 sigmaey 1;
prior a ~ normal (mean=BMX, prec=1e-3);
prior b ~ normal (mean=BYM, prec=1e-3);
prior cpr ~ normal (mean=BYX, prec=1e-3);
prior i2 i3 ~ normal (mean=0, prec=1e-3);
prior sigmaem sigmaey ~ gamma(shape=0.01, iscale=0.01);
mum=i2+a*X;
muy=i3+b*M+cpr*X;
model M~ n(mum, prec=sigmaem);
model Y~ n(muy, prec=sigmaey);
```

```
run;

data ab; set out;
ab=a*b;
run;

proc sort data=ab out=percentiles;
by ab;
run;

proc univariate data=percentiles noprint;
var ab;
output out=p pctlpre=P_ pctlpts=2.5 97.5;
run;

data cl; set p;
rename P_2_5=LCL;
rename P_97_5=UCL;
run;

/*Computing the quantities of interest*/
data quant; set cl;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=&BMX*&BYM;

LYM=LCL;
UYM=UCL;
RGYM=0;
LFYM=0;
IF TRUEAB=0 && LYM GT 0 && UYM GT 0 THEN LFYM=1;
IF TRUEAB=0 && LYM LT 0 && UYM LT 0 THEN RGYM=1;
TYPEIERRORRATEymd=RGYM+LFYM;
CIWIDTHymd=UYM-LYM;
IF TRUEAB GT 0 && LYM LT 0 && UYM GT 0 THEN empbetaymd=1;
ELSE empbetaymd=0;
POWERymd=1-empbetaymd;

IMBLYMD=0; IMBRYMD=0;
IF TRUEAB GT UYM THEN IMBRYMD=1;
IF TRUEAB LT LYM THEN IMBLYMD=1;
IMBALANCEymd=IMBRYMD-IMBLYMD;

IF TRUEAB GT LYM && TRUEAB LT UYM THEN COVERAGEymd=1;
ELSE COVERAGEymd=0;

data priors2; set SIM;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=&BMX*&BYM;
```

```
proc mcmc data=priors2 outpost=outi nmc=5000 thin=5 seed=2 stats=none
diag=none;
parms ai 0 bi 0 cpri 0 i2i 0 i3i 0;
parms sigmaemi 1 sigmaeyi 1;
prior ai ~ normal (mean=BMX, sd=0.23570226);
prior bi ~ normal (mean=BYM, sd=0.242535625);
prior cpri ~ normal (mean=BYX, sd=0.260327848);
prior i2i i3i ~ normal (mean=0, prec=1e-3);
prior sigmaemi sigmaeyi ~ gamma(shape=0.01, iscale=0.01);
mumi=i2i+ai*X;
muyi=i3i+bi*M+cpri*X;
model M~ n(mumi, prec=sigmaemi);
model Y~ n(muyi, prec=sigmaeyi);
run;

data abi; set outi;
abi=ai*bi;

TRUEAB=&BMX*&BYM;
run;

proc sort data=abi out=percentilesi;
by abi;
run;

proc univariate data=percentilesi noprint;
var abi;
output out=pi pctlpre=P_ pctlpts=2.5 97.5;
run;

data cli; set pi;
rename P_2_5=LCLi;
rename P_97_5=UCLi;
run;

/*Computing the quantities of interest*/
data quanti; set cli;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=BMX*BYM;

LYMi=LCLi;
UYMi=UCLi;
RGYMi=0;
LFYMi=0;
IF TRUEAB=0 && LYMi GT 0 && UYMi GT 0 THEN RGYMi=1;
IF TRUEAB=0 && LYMi LT 0 && UYMi LT 0 THEN LFYMi=1;
TYPEIERRORRATEi=RGYMi+LFYMi;
CIWIDTHi=UYMi-LYMi;
IF TRUEAB GT 0 && LYMi LT 0 && UYMi GT 0 THEN empbetai=1;
ELSE empbetai=0;
POWERi=1-empbetai;
```

```
IMBRYMI=0; IMBLYMI=0;
IF TRUEAB GT UYMi THEN IMBRYMI=1;
IF TRUEAB LT LYMi THEN IMBLYMI=1;
IMBALANCEi=IMBRYMI-IMBLYMI;

IF TRUEAB GT LYMi && TRUEAB LT UYMi THEN COVERAGEi=1;
ELSE COVERAGEi=0;
COVERAGECHECK=1-IMBRYMI-IMBLYMI;
/*END OF YUAN AND MACKINNON */

/*calculating elements from the SS matrix for SDIFFUSE*/
/*the inverse-Wishart SS matrix is constructed so that it is based on as many
prior
observations as there are in the current sample*/
data SIMcov; set SIM;
NOBS=&NOBS;
SVX1X1=2;
SCY1X1=0;
SCY2X1=0;
SCY1Y1=2;
SCY2Y1=0;
SCY2Y2=2;
run;


proc mcmc data=SIMcov outpost=efmD thin=5 seed=2 nmc=5000 diag=none;
   array data[3] X M Y;
   array mu[3];
   array Sigma[3,3];
   array mu0[3] (0 0 0);
   array Sigma0[3,3] (1000 0 0 0 1000 0 0 0 1000);
   array SDIFFUSE[3,3] SVX1X1 SCY1X1 SCY2X1 SCY1X1 SCY1Y1 SCY2Y1 SCY2X1
SCY2Y1 SCY2Y2;
   parm mu Sigma;
   prior mu ~ mvn(mu0, Sigma0);
   prior Sigma ~ iwish(3, SDIFFUSE);
   model data ~ mvn(mu, Sigma);
   run;

data abefmD; set efmD;
abefmD=(Sigma4/Sigma1)*(Sigma1*Sigma6-Sigma4*Sigma3)/(Sigma1*Sigma5-
Sigma4**2);
RUN;

proc sort data=abefmD out=percentilesefmD;
by abefmD;
run;

proc univariate data=percentilesefmD noprint;
var abefmD;
output out=pefmD pctlpre=P_ pctlpts=2.5 97.5;
run;
```

```
data clefmD; set pefmD;
rename P_2_5=LCLefmD;
rename P_97_5=UCLefmD;
run;

data QUANTEFMD; set clefmD;
ERROR=&ERROR;
NOBS=&NOBS;
BMX=&BMX;BYX=&BYX;BYM=&BYM;
TRUEAB=&BMX*&BYM;
RGEFMDIFF=0;
LFEFMDIFF=0;
IF TRUEAB=0 && LCLefmD GT 0 && UCLefmD GT 0 THEN LFEFMDIFF=1;
IF TRUEAB=0 && UCLefmD LT 0 && LCLefmD LT 0 THEN RGEFMDIFF=1;
TYPEIERRORRATEEFMDIFF=RGEFMDIFF+LFEFMDIFF;
CIWIDTHEFMDIFF=UCLefmD-LCLefmD;
IF TRUEAB GT 0 && LCLefmD LT 0 && UCLefmD GT 0 THEN empbetaEFMDIFF=1;
ELSE empbetaEFMDIFF=0;
POWEREFMDIFF=1-empbetaEFMDIFF;
IMBREFMDIFF=0; IMBLEFMDIFF=0;
IF TRUEAB GT UCLefmD THEN IMBREFMDIFF=1;
IF TRUEAB LT LCLefmD THEN IMBLEFMDIFF=1;
IMBALANCEEFMDIFF=IMBREFMDIFF-IMBLEFMDIFF;

IF TRUEAB GT LCLefmD && TRUEAB LT UCLefmD THEN COVERAGEEFMDIFF=1;
ELSE COVERAGEEFMDIFF=0;

/* EFM informative */
data info; set SIM;
ERROR=&ERROR;
DOF=_FREQ_-2;
NOBS=&NOBS;
BMX=&BMX;BYX=&BYX;BYM=&BYM;

*This section computes true variances and covariances as in Section 4.10
based
on residual error variance equal to 1. Note that VX1X1, VX2X2, and VX3X3 are
the residual variance in equations 3.1, 3.2, and 3.3, respectively;
EMOD1=(&ERROR)**2;
EMOD2=(&ERROR)**2;
EMOD3=(&ERROR)**2;
VX1X1=EMOD1;
CY1X1=BMX*EMOD1;
CY2X1=BYM*BMX*VX1X1+BYX*EMOD1;
CY1Y1=BMX*BMX*VX1X1+EMOD2;
CY2Y1=BMX*BMX*BYM*VX1X1+BMX*BYX*EMOD1+BYM*EMOD2;
CY2Y2=BYM*BYM*(BMX*BMX*EMOD1+EMOD2)+2*BMX*BYM*BYX*VX1X1+BYX*BYX*EMOD1
+EMOD3;
SVX1X1=(NOBS-1)*VX1X1;
SCY1X1=(NOBS-1)*CY1X1;
SCY2X1=(NOBS-1)*CY2X1;
SCY1Y1=(NOBS-1)*CY1Y1;
SCY2Y1=(NOBS-1)*CY2Y1;
SCY2Y2=(NOBS-1)*CY2Y2;
```

```
run;

proc mcmc data=info outpost=efmi thin=5 seed=2 nmc=5000 diag=none;
   ods select PostSummaries PostIntervals;
   array datainfo[3] X M Y;
   array muI[3];
   array SigmaI[3,3];
   array mu0I[3] (0 0 0);
   array Sigma0I[3,3] (1000 0 0 0 1000 0 0 0 1000);
   array SINFO[3,3] SVX1X1 SCY1X1 SCY2X1 SCY1X1 SCY1Y1 SCY2Y1 SCY2X1 SCY2Y1
SCY2Y2;
   parm muI SigmaI;
   prior muI ~ mvn(mu0I, Sigma0I);
   prior SigmaI ~ iwish(&NOBS, SINFO);
   model datainfo ~ mvn(muI, SigmaI);
   run;
data abefmi; set efmi;
abefmi=(SigmaI4/SigmaI1)*(SigmaI1*SigmaI6-SigmaI4*SigmaI3)/(SigmaI1*SigmaI5-
SigmaI4**2);
RUN;

proc sort data=abefmi out=percentilesefmi;
by abefmi;
run;

proc univariate data=percentilesefmi noprint;
var abefmi;
output out=pefmi pctlpre=P_ pctlpts=2.5 97.5;
run;

data clefmi; set pefmi;
rename P_2_5=LCLefmi;
rename P_97_5=UCLefmi;
run;

data QUANTEFMi; set clefmi;
ERROR=&ERROR;
NOBS=&NOBS;
BMX=&BMX;BYX=&BYX;BYM=&BYM;
TRUEAB=&BMX*&BYM;
RGEFMinfo=0;
LFEFMinfo=0;
IF TRUEAB=0 && LCLefmi GT 0 && UCLefmi GT 0 THEN LFEFMinfo=1;
IF TRUEAB=0 && LCLefmi LT 0 && UCLefmi LT 0 THEN RGEFMinfo=1;
TYPEIERRORRATEEFMinfo=RGEFMinfo+LFEFMinfo;
CIWIDTHEFMinfo=UCLefmi-LCLefmi;
IF TRUEAB GT 0 && LCLefmi LT 0 && UCLefmi GT 0 THEN empbetaEFMinfo=1;
ELSE empbetaEFMinfo=0;
POWEREFMinfo=1-empbetaEFMinfo;
IMBREFMINFO=0; IMBLEFMINFO=0;
IF TRUEAB GT UCLefmi THEN IMBREFMINFO=1;
IF TRUEAB LT LCLefmi THEN IMBLEFMINFO=1;
IMBALANCEEFMinfo=IMBREFMINFO-IMBLEFMINFO;
```

```
IF TRUEAB GT LCLefmi && TRUEAB LT UCLefmi THEN COVERAGEEFMinfo=1;
ELSE COVERAGEEFMinfo=0;



/*Bootstrap*/
*This is where the bootstrap samples are made.;
*sampsize should be equal to the number of observations in the dataset;
*rep is the number of bootstrap samples you want;

proc reg data=SIM outest=out1 noprint;
model y = m x;
model m = x;
run;
quit;
data out1; set out1;
if _MODEL_='MODEL1' then call symput ("b", m);
if _MODEL_='MODEL2' then call symput ("a", x);
run;
quit;
%let nboot=1000;

proc surveyselect data=SIM noprint out=out2 method=urs sampsize=&NOBS
rep=&NBOOT outhits;
run;


proc reg data=out2 outest=out3 noprint;
by Replicate;
model y = m x;
model m = x;

data b; set out3;
if _MODEL_^='MODEL1' then delete;
b=m;
keep Replicate b;

data c; set out3;
if _MODEL_^='MODEL2' then delete;
a=x;
keep Replicate a;

data d; merge b c; by Replicate;
ab=a*b;
if ab<=&a*&b then z=1; else z=0;

proc means data=d noprint;
var z;
output out=out4 mean(z)=meanz;

data out4; set out4;
call symput("meanz", meanz);
```

```
proc sort data=d;
by ab;
proc univariate data=d NOPRINT;
var ab;
*Percentile Bootstrap and Bias-Corrected Bootstrap;
data e; set d;
z0=probit(&meanz);
if _N_=(ceil(.025*&nboot)) then call symput("LCL95", ab);
if _N_=(ceil(.975*&nboot)) then call symput("UCL95", ab);
if _N_=(ceil(&nboot*probnorm((2*z0)+probit(.025)))) then call
symput("BCLCL95", ab);
if _N_=(ceil(&nboot*probnorm((2*z0)+probit(.975)))) then call
symput("BCUCL95", ab);
run;


data f; SET e;
LCL95=&LCL95;
UCL95=&UCL95;
BCLCL95=&BCLCL95;
BCUCL95=&BCUCL95;
NOBS=&NOBS;
BMX=&BMX;BYX=&BYX;BYM=&BYM;

TRUEAB=BMX*BYM;

/*QUANTITIES OF INTEREST*/
/*PERCENTILE BOOTSTRAP*/
RGBOOT=0; LFBOOT=0;
IF TRUEAB=0 && LCL95 GT 0 && UCL95 GT 0 THEN LFBOOT=1;
IF TRUEAB=0 && UCL95 LT 0 && LCL95 LT 0 THEN RGBOOT=1;
TYPEIERRORRATEBOOT=RGBOOT+LFBOOT;
CIWIDTHBOOT=UCL95-LCL95;
IF TRUEAB GT 0 && LCL95 LT 0 && UCL95 GT 0 THEN empbetaB=1;
ELSE empbetaB=0;
POWERBOOT=1-empbetaB;
IMBRPERC=0; IMBLPERC=0;
IF TRUEAB GT UCL95 THEN IMBRPERC=1;
IF TRUEAB LT LCL95 THEN IMBLPERC=1;
IMBALANCEBOOT=IMBRPERC-IMBLPERC;
IF TRUEAB GT LCL95 && TRUEAB LT UCL95 THEN COVERAGEBOOT=1;
ELSE COVERAGEBOOT=0;
/*BC BOOTSTRAP */
RGBCBOOT=0; LFBCBOOT=0;
IF TRUEAB=0 && BCLCL95 GT 0 && BCUCL95 GT 0 THEN LFBCBOOT=1;
IF TRUEAB=0 && BCUCL95 LT 0 && BCLCL95 LT 0 THEN RGBCBOOT=1;
TYPEIERRORRATEBC=RGBCBOOT+LFBCBOOT;
CIWIDTHBC=BCUCL95-BCLCL95;
IF TRUEAB GT 0 && BCLCL95 LT 0 && BCUCL95 GT 0 THEN empbetaBC=1;
ELSE empbetaBC=0;
POWERBC=1-empbetaBC;
IMBRBC=0; IMBLBC=0;
IF TRUEAB GT BCUCL95 THEN IMBRBC=1;
IF TRUEAB LT BCLCL95 THEN IMBLBC=1;
```

```
IMBALANCEBC=IMBRBC-IMBLBC;
IF TRUEAB GT BCLCL95 && TRUEAB LT BCUCL95 THEN COVERAGEBC=1;
ELSE COVERAGEBC=0;

PROC MEANS DATA=F MEAN NOPRINT;
VAR AB
NOBS BMX BYX BYM TRUEAB
LCL95 UCL95 BCLCL95 BCUCL95
TYPEIERRORRATEBOOT CIWIDTHBOOT POWERBOOT IMBALANCEBOOT COVERAGEBOOT
TYPEIERRORRATEBC CIWIDTHBC POWERBC IMBALANCEBC COVERAGEBC
;
OUTPUT OUT=OUTA MEAN=MAB
MNOBS MBMX MBYX MBYM MTRUEAB
MLCL95 MUCL95 MBCLCL95 MBCUCL95
MTYPEIERRORRATEBOOT MCIWIDTHBOOT MPOWERBOOT MIMBALANCEBOOT MCOVERAGEBOOT
MTYPEIERRORRATEBC MCIWIDTHBC MPOWERBC MIMBALANCEBC MCOVERAGEBC;
/*PROC PRINT DATA=OUTA;*/

DATA NEW; SET SUMMARY;
DATA SUMMARY; SET NEW TESTsobel prodquant quant quanti quantefmd quantefmi
OUTA ;

proc datasets;
delete ab abefmd abefmi abi all b bb c cl clefmd clefmi cli cov d e efmd efmi
f file
fr info model1 model2 model3 out out1 out2 out3 out4 outi p pefmd pefmi
percentiles
percentilesefmd percentilesefmi percentilesi pi priors proddata1 proddata2
test
vars varsm;
run;

proc means data=summary noprint;
VAR NOBS TRUEAB
TYPEIERRORRATE TYPEIERRORRATEPROD TYPEIERRORRATEymd TYPEIERRORRATEi
TYPEIERRORRATEefmdiff TYPEIERRORRATEefminfo  MTYPEIERRORRATEBOOT
MTYPEIERRORRATEBC
COVERAGE COVERAGEPROD COVERAGEymd COVERAGEi COVERAGECHECK COVERAGEefmdiff
COVERAGEefminfo MCOVERAGEBOOT MCOVERAGEBC
CIWIDTH CIWIDTHPROD CIWIDTHymd CIWIDTHi CIWIDTHefmdiff CIWIDTHefminfo
MCIWIDTHBOOT MCIWIDTHBC
POWER POWERPROD POWERymd POWERi powerefmdiff powerefminfo MPOWERBOOT MPOWERBC
IMBALANCE IMBALANCEPROD IMBALANCEymd IMBALANCEi imbalanceefmdiff
imbalanceefminfo MIMBALANCEBOOT MIMBALANCEBC;
output out= MSUMMARY;
%END;


DATA N40.&FILE;
SET MSUMMARY;
RUN;


RUN;
```

```
%MEND;
%SIMULATE(NSIM=1000,NOBS=20,BMX=.39,BYX=.39,BYM=.39,
FILE=n20a39b39cp39,TYPE='CCC',ERROR=1);

RUN;
quit;
data final;
      set
      n20a39b39cp39
      ;
run;
```

APPENDIX E

SIMULATION PROGRAM TO EVALUATE CHANGES IN POWER AS A FUNCTION OF PRECISION IN STUDY 3

```
FILENAME NULLOG DUMMY 'C:\NULL';
PROC PRINTTO LOG=NULLOG;

PROC DATASETS LIBRARY=WORK KILL NOLIST; RUN;

LIBNAME N40 "C:\MILICA\";

%MACRO SIMULATE(NSIM,NOBS,BMX,BYX,BYM,FILE,TYPE,ERROR);

DATA SUMMARY; SET _NULL_;
%DO I=1 %TO &NSIM;
TITLE 'SIMULATION OF MEDIATION';
DATA SIM;
DO I=1 TO &NOBS;
X=(&error)*RANNOR(0);
M=&BMX*X+(&error)*RANNOR(0);
Y=&BYX*X+&BYM*M+(&error)*RANNOR(0);
X2=X*X;
OUTPUT;
END;

data priors; set SIM;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=&BMX*&BYM;

/*YUAN AND MACKINNON */
proc mcmc data=priors outpost=outan1 nmc=5000 thin=1 seed=2 stats=none
diag=none;
parms a 0 b 0 cpr 0 i2 0 i3 0;
parms sigmaem 1 sigmaey 1;
prior a ~ normal (mean=BMX, prec=1e1);
prior b ~ normal (mean=BYM, prec=1e-3);
prior cpr ~ normal (mean=BYX, prec=1e-3);
prior i2 i3 ~ normal (mean=0, prec=1e-3);
prior sigmaem sigmaey ~ gamma(shape=0.01, iscale=0.01);
mum=i2+a*X;
muy=i3+b*M+cpr*X;
model M~ n(mum, prec=sigmaem);
model Y~ n(muy, prec=sigmaey);
run;

data aban1; set outan1;
ab=a*b;
run;

proc sort data=aban1 out=percentilesan1;
by ab;
run;

proc univariate data=percentilesan1 noprint;
var ab;
```

```
output out=pan1 pctlpre=P_ pctlpts=2.5 97.5;
run;

data clan1; set pan1;
rename P_2_5=LCL;
rename P_97_5=UCL;
run;

/*Computing the quantities of interest*/
data quantan1; set clan1;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=&BMX*&BYM;

LYM=LCL;
UYM=UCL;
RGYM=0;
LFYM=0;
IF TRUEAB=0 && LYM GT 0 && UYM GT 0 THEN LFYM=1;
IF TRUEAB=0 && LYM LT 0 && UYM LT 0 THEN RGYM=1;
TYPEIERRORRATEan1=RGYM+LFYM;
CIWIDTHan1=UYM-LYM;
IF TRUEAB GT 0 && LYM LT 0 && UYM GT 0 THEN empbetaymd=1;
ELSE empbetaymd=0;
POWERan1=1-empbetaymd;

IMBLYMD=0; IMBRYMD=0;
IF TRUEAB GT UYM THEN IMBRYMD=1;
IF TRUEAB LT LYM THEN IMBLYMD=1;
IMBALANCEan1=IMBRYMD-IMBLYMD;

IF TRUEAB GT LYM && TRUEAB LT UYM THEN COVERAGEan1=1;
ELSE COVERAGEan1=0;


/*b2*/
proc mcmc data=priors outpost=outbn1 nmc=5000 thin=1 seed=2 stats=none
diag=none;
parms a 0 b 0 cpr 0 i2 0 i3 0;
parms sigmaem 1 sigmaey 1;
prior a ~ normal (mean=BMX, prec=1e-3);
prior b ~ normal (mean=BYM, prec=1e-2);
prior cpr ~ normal (mean=BYX, prec=1e-3);
prior i2 i3 ~ normal (mean=0, prec=1e-3);
prior sigmaem sigmaey ~ gamma(shape=0.01, iscale=0.01);
mum=i2+a*X;
muy=i3+b*M+cpr*X;
model M~ n(mum, prec=sigmaem);
model Y~ n(muy, prec=sigmaey);
run;

data abbn1; set outbn1;
```

```
ab=a*b;
run;

proc sort data=abbn1 out=percentilesbn1;
by ab;
run;

proc univariate data=percentilesbn1 noprint;
var ab;
output out=pbn1 pctlpre=P_ pctlpts=2.5 97.5;
run;

data clbn1; set pbn1;
rename P_2_5=LCL;
rename P_97_5=UCL;
run;

/*Computing the quantities of interest*/
data quantbn1; set clbn1;
BMX=&BMX;
BYM=&BYM;
BYX=&BYX;
NOBS=&NOBS;
TRUEAB=&BMX*&BYM;

LYM=LCL;
UYM=UCL;
RGYM=0;
LFYM=0;
IF TRUEAB=0 && LYM GT 0 && UYM GT 0 THEN LFYM=1;
IF TRUEAB=0 && LYM LT 0 && UYM LT 0 THEN RGYM=1;
TYPEIERRORRATEbn1=RGYM+LFYM;
CIWIDTHbn1=UYM-LYM;
IF TRUEAB GT 0 && LYM LT 0 && UYM GT 0 THEN empbetaymd=1;
ELSE empbetaymd=0;
POWERbn1=1-empbetaymd;

IMBLYMD=0; IMBRYMD=0;
IF TRUEAB GT UYM THEN IMBRYMD=1;
IF TRUEAB LT LYM THEN IMBLYMD=1;
IMBALANCEbn1=IMBRYMD-IMBLYMD;

IF TRUEAB GT LYM && TRUEAB LT UYM THEN COVERAGEbn1=1;
ELSE COVERAGEbn1=0;



DATA NEW; SET SUMMARY;
DATA SUMMARY; SET NEW quantan1 quantbn1 ;


%END;
```

```
DATA N40.&FILE;
SET SUMMARY;
RUN;



RUN;
%MEND;

%SIMULATE(NSIM=1000,NOBS=100,BMX=.14,BYX=.0,BYM=.14,
FILE=abc140,TYPE='CCC',ERROR=1);
%SIMULATE(NSIM=1000,NOBS=100,BMX=.39,BYX=.0,BYM=.39,
FILE=abc390,TYPE='CCC',ERROR=1);
%SIMULATE(NSIM=1000,NOBS=100,BMX=.59,BYX=.0,BYM=.59,
FILE=abc590,TYPE='CCC',ERROR=1);
%SIMULATE(NSIM=1000,NOBS=100,BMX=.14,BYX=.39,BYM=.14,
FILE=abc1439,TYPE='CCC',ERROR=1);
%SIMULATE(NSIM=1000,NOBS=100,BMX=.39,BYX=.39,BYM=.39,
FILE=abc3939,TYPE='CCC',ERROR=1);
%SIMULATE(NSIM=1000,NOBS=100,BMX=.59,BYX=.39,BYM=.59,
FILE=abc5939,TYPE='CCC',ERROR=1);

RUN;
quit;
data final;
      set
abc140
abc390
abc590
abc1439
abc3939
abc5939
      ;
run;
```