

Optimal Utilization of Distributed Resources with an Iterative Transmission and
Distribution Framework

by

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A Thesis Presented in Partial Fulfillment
of the Requirements for the Degree
Master of Science

Approved June 2014 by the
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ARIZONA STATE UNIVERSITY

August 2014

ABSTRACT

The distribution grid is expected to change in the near future as a result of recent advancements in the field of smart grids. The future grid will accommodate generation and storage options, active consumer participation through demand response schemes, and the widespread installation of smart energy management systems. With more demand side participation, distributed generators, and (potentially) meshed distribution system networks, there is a push to integrate transmission and distribution (T&D) systems models together. Ideally, the T&D systems should be modeled by an integrated optimal power flow (OPF) framework and solved simultaneously to schedule the generation and demand in the entire system. In comparison, existing practices do not include the distribution system when solving the OPF for the transmission system; instead, the load is estimated and placed at the connection point at the sub-transmission level. However, integrating T&D system models together is a challenge for OPF due to the size of the system, which makes these problems computationally intractable with existing technologies.

The objective of this research is to develop an integrated T&D framework that couples the two sub-systems together with due consideration to conventional demand flexibility. The proposed framework ensures accurate representation of the system resources and the network conditions when modeling the distribution system in the transmission OPF and vice-versa. It is further used to develop an accurate pricing mechanism (Distribution-based Location Marginal Pricing,

DLMP), which is reflective of the moment-to-moment costs of generating and delivering electrical energy, for the distribution system. By accurately modeling the two sub-systems, we can improve the economic efficiency and the system reliability, as the price sensitive resources (PSR) can be controlled to behave in a way that benefits the power system as a whole.

The proposed framework decomposes the integrated OPF framework into two subsequent OPF problems: the transmission OPF and the distribution OPF. The decomposition requires iterations between the two sub-problems to ensure adequate representation of one sub-system when solving the other sub-system. Instead of using a one-shot approach where the transmission system modeled is solved only once, the proposed approach requires resolving the transmission OPF with an updated residual demand curve. The distribution system is modeled by its aggregate residual demand curve in the transmission OPF while the transmission system is modeled by a transmission-constrained residual supply curve in the distribution OPF. The iterative framework is further used to demonstrate the application and potential benefits of DLMP.

ACKNOWLEDGMENTS

I would like to express my sincere appreciation and gratitude to my advisor, Dr. Kory W. Hedman, for his invaluable guidance and support throughout my graduate experience. He has always been a great source of encouragement and inspiration. I appreciate his patience and dedication to helping his students. I would also like to thank my committee members, Dr. Lalitha Sankar and Dr. Daniel J. Tylavsky, for their valuable time and suggestions. I appreciate their feedback and assistance with my thesis.

I am also grateful to my parents for their endless love and support through all these years. This thesis work is sponsored by the Power Systems Engineering Research Center (PSERC) under project M-25.

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NOMENCLATURE

ACOPF	Alternating current optimal power flow
AMPL	A mathematical programming language
b_d	Price associated with load bid d
B_k	Susceptance of line k
c_g	Linear operating cost of generator g
$C_i(q_i)$	Net cost function or net benefit function at bus i
CV	Conjectural variations
CVE	Conjectural variation based equilibrium
D_d	Cleared demand associated with load bid d
$D_j(\cdot)$	Demand function at node j
D_n	Real power demand of inelastic load at bus n
DCOPF	Direct current optimal power flow
DER	Distributed energy resources
DG	Distributed generation
DLMP	Distribution-based Locational Marginal Price
DR	Demand response
DSR	Distributed system resources
EISA	Energy independence and security act
ESI	Energy services industry
EV	Electrical vehicles
ϵ	Price elasticity of demand
FR	Flat rate

G_k	Conductance of line k
G_n	Set of generators at bus n
IEEE	Institute of Electrical and Electronics Engineers
ISO	Independent system operator
$k(n, ;)$	Set of lines with n as the receiving bus
$k(, n)$	Set of lines with n as the sending bus
\mathcal{L}	Lagrangian
LMP	Locational marginal price
LP	Linear programming
LSE	Load serving entity
MCC	Marginal congestion component
MEC	Marginal energy component
MLC	Marginal loss component
OPF	Optimal power flow
PJM	Pennsylvania-New Jersey-Maryland
PRL	Price responsive loads
PSP	Power supply point
PSR	Price sensitive resources
PURPA	Public utility regulatory policy act
p_j	Market price at bus j
P_k	Power flow on line k
P_k^{max}	Maximum power flow limit of line k
P_k^{min}	Minimum power flow limit of line k

P_g	Real power output of generator g
P_g^{max}	Maximum real power capacity of generator g
P_g^{min}	Minimum real power capacity of generator g
P_n^R	Real power injected at bus n and withdrawn at reference bus R
$PTDF_{k,n}^R$	Power transfer distribution factor
q_j	Market clearing quantity at bus j
RBTS	Roy Billinton Test System
RTO	Regional transmission organization
RTP	Real-time price
$R_j(p_j)$	Residual supply at bus j
$S_j(\cdot)$	Supply function at bus j
x/r	Ratio of reactance to resistance
t	Time period index
TOU	Time-of-use rate
T&D	Transmission and distribution
V_m	Voltage magnitude of designated sending end bus m
V_n	Voltage magnitude of designated receiving end bus n
V_n^{max}	Maximum voltage magnitude at bus n
V_n^{min}	Minimum voltage magnitude at bus n
θ_m	Voltage angle of designated sending end bus m
θ_n	Voltage angle of designated receiving end bus n

CHAPTER 1

INTRODUCTION

1.1 Overview

Restructuring of the electric services industry (ESI) is accelerating worldwide [1]. The ESI has seen a fundamental transformation from one dominated by regulated vertically integrated monopolies (each within its own geographic area) to an industry where electricity is produced and traded as a commodity through competitive markets [2]. The Energy Policy Act of 1992 provided the framework for competition in wholesale generation markets, which have since flourished under open access transmission [1]. In the United States, this change was pioneered by a number of Independent System Operators (ISOs) or Regional Transmission Organizations (RTOs) such as California ISO, Pennsylvania-New Jersey-Maryland (PJM) Interchange, New York ISO, and ISO-New England, which established competitive markets for electricity. The LMP methodology has been predominantly implemented or is under consideration (by all ISOs) to price electricity and manage congestion in the transmission system networks [3]. References [4], [5] describe some of the successes obtained with this approach. LMPs have proven to be beneficial to both market and system operations. Despite its success, the LMP methodology is not used to price electricity in the distribution system. Instead, the utilities supply the distribution customers at a rate (flat rate, time-of-use rates, or real-time prices), which is independent of both individual consumer preferences and the cost of energy at a particular location in real-time. The impact of the inaccuracy of contemporary rate structures with

increased flexible resources is illustrated in [6].

Although considerable work has been done in the past on the application of some type of systematic approach to generation and transmission system planning, its application to distribution system planning, unfortunately, has been neglected [7]. In the future, there will be a need for an economic planning tool to evaluate the consequences of various proposed alternatives and their impact on the rest of the system to provide necessary economical, reliable, and safe electrical energy to consumers [7].

Furthermore, some of the proposed new advancements in the field of smart grids include: self-healing capability from the disturbances witnessed by the power grid, accommodation of all generation and storage options, enabling active customer participation through demand response schemes, enabling new products, services and markets and to optimize the assets and operate efficiently [8]. Thus, in order to meet the various visions of the smart grid initiative, there is a need to extend the existing power markets to incorporate the distribution system details, thereby, accounting for the flexibility and price elasticity of demand as well as distribution system resources (DSR). However, extending the transmission OPF to incorporate the distribution system is a challenge for OPF due to the size of the system, which makes these problems computationally intractable with existing technologies. Presently, there is no integrated framework for the T&D systems that provides a closed loop solution with due consideration to conventional demand elasticity [9]. Also, there is a need for a paradigm shift in the existent distribution pricing structures to increase flexibility and to achieve the desired

level of accuracy and efficiency.

1.2 Research Focus

This research focuses on developing an iterative approach to integrate the T&D systems together via a distribution engineering analog of transmission LMPs (DLMP). The DLMP is envisioned to be used for energy and power flow management in networked distribution systems as well as pricing. However, since this research proposes nodal pricing in the distribution system similar to LMPs in the transmission system, there is always the question of fairness of such a pricing scheme apart from the issue of consumer exposure to price volatility. It is important to note that the primary objective of the ISOs is to maximize social welfare and this can be assured by ensuring that the prices are proper economic signals. Contemporary rate structures result in distorted price signals, which cause market inefficiencies whereas a nodal pricing scheme is sure to incentivize economically efficient behavior from the market participants. Also, the consumers have no set entitlement to an inadequate or inaccurate price signal. It is also important to note that, while price volatility may be disregarded as undesirable, it is necessary to reflect the state of the system. Factors that may influence the volatility of the proposed distribution pricing scheme (DLMP) include price at the proxy node, congestion, and scarcity of resources in the distribution system and strategic bidding practices of DSR. The purpose of a price signal is to reflect the system conditions appropriately. For example, a high price as a result of scarcity or congestion shows that there may be a need for an upgrade at a particular location. Thus, price volatility in the distribution system is not necessarily bad.

The integrated OPF problem is decomposed into two subsequent OPF problems: the transmission OPF and the distribution OPF. Prior work [6] discusses an iterative framework for calculating the DLMP, which includes a two-stage optimization problem. In this prior work, the iterative framework was implemented to ensure accurate representation of the price sensitive DSR and the distribution network conditions in one of the stages while the other stage captures the transmission system and its resources. However, in this prior work, the transmission system is simply modeled as an infinite generator that sells at the resulting transmission LMP (at the interconnection point between the T&D systems), in the distribution OPF. This approach results in a perfectly elastic supply curve, which inaccurately approximates the sensitivity of the transmission system relative to a change in demand from the distribution system, thereby convergence issues are observed due to non-unique solutions. Also, in this prior work, the details of the distribution system are not modeled in the transmission OPF. Rather, the distribution system is represented by a highly inelastic aggregate demand curve (forecasted aggregate demand) in the transmission OPF. This inaccurately represents the price sensitivity of the DSR and the distribution system network conditions over a range of possible prices that may result from the transmission OPF. In this thesis, an iterative T&D framework is proposed, which improves the ability to determine accurate DLMPs and this thesis develops a technique to improve the convergence for this decomposed approach.

Both OPFs in the two-stage optimization process can be based on either the DCOPF or the ACOPF formulation. It is important to note that, the primary

motivation for this research is not to propose for an accurate OPF formulation to be used in solving the sub-systems. The primary motivation for this research is to show that there is a need to incorporate the distribution system details in contemporary market structures. In this research, the DCOPF formulation is used for both the transmission OPF and the distribution OPF to maintain consistency with existing market practices. Traditionally, the LMPs in the transmission system are calculated based on the DCOPF. In order to obtain the DCOPF formulation, the equations in the ACOPF formulation are greatly simplified by making a series of assumptions, which may not be valid for distribution systems. Most importantly, the percentage losses in the distribution system are greater when compared to the percentage losses in the transmission system due to lower voltage of operation and higher conductor resistances. Since the traditional DCOPF is valid only for high voltage systems and is less usable on the lower voltage circuits, a DCOPF formulation that endogenously captures the effect of real power losses is used for the distribution OPF. It is also important to note that the DCOPF assumes a balanced 3-phase operation. However, the distribution system is unbalanced. While this research uses a lossy DCOPF for the distribution system, which is a rough approximation, the primary motivation to demonstrate the importance in regards to distribution system pricing as well as the importance of accurate DLMPs. Future research will extend the proposed T&D framework in order to incorporate a more accurate OPF model (e.g., a warm-start DCOPF that is based on the AC operating state) for the distribution system.

In the proposed integrated T&D framework, the distribution system is

modeled by its aggregate residual demand curve in the transmission OPF while the transmission system is modeled by a transmission-constrained residual supply curve in the distribution OPF. This process is repeated until convergence is achieved between the transmission system and the distribution system models. This results in a better representation of the characteristics of the price sensitive DSR and the distribution system network conditions when solving the transmission OPF and vice-versa. By doing so, we can improve the economic efficiency and the system reliability as the PSR can be controlled to behave in a way that benefits the power system as a whole. The iterative framework is further used to demonstrate the application and potential benefits of the T&D iterative framework combined with DLMP settlements.

1.3 Summary of Chapters

This thesis is organized into six chapters. The goal of Chapter 2 is to provide the background to understand the motivation and premise of this thesis work. Chapter 2 introduces the concept of DLMP and provides a literature review of contemporary works on nodal pricing in the distribution system. Chapter 2 also briefly discusses the proposed iterative framework to integrate the T&D systems models, and provides a literature review of contemporary works on transmission-constrained residual curves.

The goal of Chapter 3 is to provide the readers with the knowledge of necessary fundamentals that are critical to understand the technical details in the subsequent chapters. Chapter 3 provides information on the economic dispatch problem and the various OPF formulations that are used in the industry. Chapter

3 also provides information on the price elasticity of demand and a brief discussion on linear optimization.

The focus of Chapter 4 is the proposed integrated T&D systems model. The chapter provides a comprehensive illustration of the mathematical optimization problem to integrate the T&D systems models. The chapter also provides the background to understand the motivation behind obtaining residual supply and demand curves. The derivation of the aggregate residual demand curve and the transmission-constrained residual supply curve is also presented in this chapter.

The integrated T&D systems model developed in Chapter 4 is used to accurately couple the T&D systems in Chapter 5. The model was tested on a traditional transmission system with no congestion and a radial distribution system with price responsive loads (PRL), a transmission system with congestion and a radial distribution system with PRL, a congested transmission system and a traditional distribution system with PRL, radial topology and no congestion, and a congested transmission system and an enhanced distribution system with PRL, meshed topology and congestion. The proposed framework was also compared against a conventional transmission OPF (ignoring distribution system details) and an integrated T&D model (T&D systems solved simultaneously). The results demonstrate the superiority of the proposed framework over existing procedures.

Chapter 6 discusses the conclusions of this research and the scope for future work. Appendix A consists of the tables of the data used in conducting the numerous studies in this thesis. The references are included at the end of this thesis.

CHAPTER 2

LITERATURE REVIEW

This chapter provides a discussion on the context for this research work in Section 2.1. The national directives are discussed in Section 2.2. A literature review of the contemporary work on the subject of nodal pricing in the distribution system is provided in Section 2.3 followed by a discussion on the proposed distribution-based location marginal prices in Section 2.4. A brief discussion on the proposed iterative framework is provided in Section 2.5. Finally, a literature review of the contemporary work on the subject of transmission-constrained residual curves is provided in Section 2.6.

2.1 Introduction

While the majority of the smart grid discussions are focused on new technologies, there has been limited discussion on the market structure in which these technologies will function. This thesis focuses on developing a market structure that integrates the T&D systems together. Contemporary market structures ignore the distribution system when solving the OPF for the transmission system; instead, the aggregate load (of the distribution system) is estimated, assumed to be perfectly inelastic, and placed at the proxy node (interconnection point between the T&D systems). Thus, the resulting prices are not directly related to the moment-to-moment costs of generation and delivery to their location. Furthermore, the historically held approach to model demand as perfectly inelastic creates market inefficiencies by having a one-sided market. However, recent and pending smart grid advancements facilitate customer choice

and market participation through demand response mechanisms and/or distributed energy resources (DER). This research proposes that these individual elements would function as a part of an interactive, integrated market framework that integrates the T&D systems together. The key element of the integrated framework is the design of the pricing scheme to be used in the distribution OPF, which will provide the necessary information for socially efficient consumption, valuation of renewables when and where delivered, and the trigger for charging and discharging of distributed storage. Such an integrated framework relies on the preferences of the market participants.

One of the important reasons for deregulation of electricity was that consumers wanted choice. According to numerous large-scale pilot programs [1], it is not just the large-scale consumers who show the willingness to respond to dynamic pricing signals, even the small-scale consumers (including small commercial and residential consumers) demonstrate a desire to have and use the information to regulate and manage their consumption. Discussion on retail competition is not new. Fred C. Scheweppe discussed his views on retail competition in the late 1970s in [10]. Although, Scheweppe et al. proposed the LMP methodology in the late 1980s, the basic message of that work is yet to be implemented in the distribution system. Today, LMPs reflect centralized generation and high-voltage transmission operational costs, which represent around 65 percent of the total delivered cost of electricity consumed in the United States. The LMP methodology, however, does not address the medium and low-voltage distribution costs. This, in part, is due to the fact that the conditions in the distribution system are still assumed to be

uncompetitive. However, with the advent of smart grids and (potentially) meshed distribution systems, the conditions are ripe to extend pricing in the distribution system, thereby, accounting for the remaining 35 percent of overall electricity costs [11]. In other words, there is a need to broaden the power markets to incorporate the PRL and the DSR that are connected to the distribution system. Consequently, there is a need for a genuine integrated market structure in which both the demand and supply sides interact in a fully functional market.

2.2 National Directives

The recent government stimulus plans from both States and the Federal government includes several billion dollars in order to facilitate the development and deployment of a smart grid [12]. This thesis is motivated by the national push to create a smarter, more reliable, robust system. The US Congress passed the Energy Independence and Security Act of 2007 (EISA) in order to move towards greater energy independence and security, to increase production of clean renewable fuels, to promote the deployment of storage options, and to increase the efficiency of products among the other purposes [13]. This act required the utilities to translate their real-time costs to consumer prices. It further mandates the unbundling of the contemporary rate structures (i.e., to unbundle the relationship between kWh and revenue) in order to allow the utilities to recover their costs and, most importantly, the co-ordination of the wholesale and the retail markets [12]. It is also important to note that Section 1307 of the EISA includes a real-time pricing requirement, amended to Section 111(d) of the Public Utility Regulatory Policies Act of 1978 (PURPA). The amendment requires that

purchasers and other interested persons shall be provided with information on: 1) time-based electricity prices in the wholesale electricity market, and 2) time-based electricity retail prices or rates that are available to the purchasers. Updates of information on prices and usage shall be offered on not less than a daily basis, shall include hourly price and use information, where available, and shall include a day-ahead projection of such price information to the extent available [13].

The real-time pricing requirement is a key element of the smart grid initiative. However, there is no market (currently) that assures real-time pricing on a nodal basis in the distribution system. This research aims to address these national directives by developing an integrated market structure with the DLMP methodology. This research proposes an integrated market structure that will define and provide information like marginal price, marginal quantity, and control signals at the distribution system level. Thus, using the information both the consumer and the utility will find economically efficient solutions based up their individual welfare.

2.3 Literature Review: Distribution Network Marginal Pricing

For the larger benefit of the society it is incumbent that the market be designed based on the true fundamentals of economic theories and this requires the introduction of Distribution-based LMPs in the distribution system analogous to LMPs in the transmission system. A market structure that allows for adjustments on both the supply and demand side is sure to improve efficiency, reduce costs, and benefits society. Prior work [14]-[19] has examined applying the concept of locational marginal pricing to the distribution system, which is referred

to as a DLMP. The goal of developing and applying a DLMP is to incentivize the loads and distributed resources in the distribution system to schedule their assets efficiently.

In [14], Sotkiewicz and Vignolo propose the use of nodal prices in distribution networks to send the right price signals to locate distributed generation (DG) resources and to appropriately reward DG resources for reduction in line losses and line loading. The authors' further show that DG resources have significantly greater revenue under nodal pricing, thereby reflecting their contribution to reduced line losses and loading. The nodal prices proposed in [14] are similar to the LMPs employed in the transmission system consisting of three constituent components: the marginal energy component (MEC), the marginal loss component (MLC) and the marginal congestion component (MCC). However, since a radial distribution network model is considered in [14], the MCC is absent from the nodal prices. The MEC of the nodal prices is equated to the LMP at the power supply point, PSP (interface between the T&D systems) and the MLC of the nodal prices is calculated based on the corresponding DG's contribution towards the reduction in distribution system losses. Consequently, the distribution based nodal prices incentivize DG resources to locate and operate so that they can provide system benefits.

In [15], Murphy et al. derive an expression for spot prices in radial distribution systems, in terms of system quantities such as line flows. Reference [15], also gives an outline of the algorithm used to derive the expression. In [15], the integrated OPF problem is decomposed into two subsequent OPF problems:

the transmission OPF and the distribution OPF. Each distribution substation is represented by its aggregate demand in the transmission system OPF. The transmission system operator then transmits the value of the bulk system lambda to all distribution substations. The substation at each distribution connection point in [15] is assumed to have its own independent operator or processor and is required to be responsible for the operation and control of its own distribution subsystem, including its interface with the bulk power system. Thus, the pricing algorithm proposed in [15] addresses the issue of decentralized knowledge by using a distributed processing scheme that emphasizes on local computations. Each distribution substation operator controls its own processor known as the root processor, which solves an OPF problem for the corresponding distribution system. The loads in the distribution system are associated with a utility-owned processor, such a smart meter. Customers are assumed to be solving individual benefit maximization problems to decide their consumption level. The pricing algorithm is iterative within the second stage of the decomposed OPF problem. The spot prices presented to the consumers are changed from iteration to iteration until the individual benefit maximization by the consumers coincide with global welfare maximization. The spot prices in [15] have two constituent components: the MEC, which is equated to the value of the bulk system lambda, and the MLC. The losses are assumed to be quadratic functions of the power injected into the corresponding bus.

Reference [16] presents a novel LMP policy for uncongested distribution systems with significant DG penetration. The proposed LMP is composed of two

main components: energy price, which refers to the wholesale market price at the PSP (interconnection point between T&D systems), and the cost of distribution losses. The pricing methodology is based on compensating DG units for their participation in reduced amount of distribution system losses. A cooperative game theory approach is applied for calculating the participation factor of each DG unit in loss reduction allocation. In other words, the LMP policy is based on loss reduction allocation rather than loss allocation or marginal loss. In distribution systems with private agents, there is no way of knowing net generation (particularly, from DG) and consumption of each system bus before real-time operation. Thus, the authors in [16] introduce an iterative method to estimate the production of DG units, thereby aiding the distribution companies to obtain the state of the system in the subsequent intervals. The convergence criterion for the iterative method for LMP calculation is defined by the production of DG units. Finally, in order to overcome the error and uncertainty involved in predicting demand and market price, scenario analysis is employed. Also, an ANN forecasting tool is used to generate the market price and demand scenarios. It is important to note that this work considers nodal pricing only at DG connected buses.

Reference [17] proposes the use of sensory information to upgrade and automate power distribution systems. This includes the utilization of a distribution class LMP index (D-LMP) to drive energy management related controls. In [17], the concept of LMP in transmission engineering is modified for use in distribution engineering by including conceptualized objectives like renewable resource

encouragement. The D-LMP is envisioned to be a price signal with weighted terms to achieve a given set of distribution system objectives. The proposed D-LMP is composed of four main components: the energy component obtained by multiplying the transmission LMP at the corresponding supply substation by the generation participation factor at the connecting bus, the loss component which captures the incremental line active power losses in the distribution system (in each line), the congestion component, which relates to the circuit loading represented as a fraction of the circuit rating, and a component that captures the objective of encouraging the use of renewables at the corresponding load point. The author in [17] stresses that the definition of the D-LMP could be altered based on the distribution engineering application envisioned. However, it is important to note that the proposed D-LMP is not a byproduct of an optimization problem; rather, it is calculated from system-wide measurements or estimates.

Three alternative formulations of the distribution-class LMP signal are discussed in [18]. The D-LMP formulations are developed by adopting the fundamental concept of cost of energy at the substation, i.e., the transmission LMP, and inclusion of distribution system costs and objectives. In each of the formulations weighted terms are added to capture the envisioned distribution system objectives like renewable resource encouragement. The choice of the weighted terms is heuristic and requires experience with the distribution system and the envisioned applications. The possibility of calculating the DLMPs without the presence of a centralized entity to oversee the day to day transactions is also proposed in [18]. Reference [19] proposes the application of DLMP as an

economic signal for power dispatch and system control in distribution microgrids. The authors in [19] present a new methodology to evaluate the marginal cost of energy, losses, and congestion.

2.4 Distribution-Class LMP Index

In this research, the DLMP is proposed to be determined in a similar fashion as the LMP, using a linear programming (LP) problem. It is assumed that the ISO would determine the DLMPs by solving a distribution OPF that endogenously captures the effect of real power losses and congestion in the distribution system. The transmission nodes are represented by generators with linear cost functions (transmission-constrained residual supply curves) in the distribution OPF. The linear cost functions are obtained for the corresponding transmission nodes around the market operating point, i.e., the corresponding transmission node LMPs and cleared market quantities. These linear cost functions reflect the generator availability and the network conditions in the transmission system. Thus, both the T&D system states are considered in calculating the DLMP. The DLMPs are defined as the Lagrange multipliers (shadow prices) resulting from the power balance constraints at each node in the distribution OPF. In other words, the DLMP reflects the marginal cost to supply one additional (or one less) MW to a bus/node in the distribution OPF. The DLMP is proposed to have the same properties as the LMP; therefore, it may lead to less operator interaction and a higher degree of automation in the area of distributed operations. In fact, the DLMP is envisioned to be used in distribution operation for congestion management and pricing because it reflects the true cost of generating and

delivering electrical energy.

The advantages of nodal pricing in the distribution system are manifold. Dynamic prices have the added capability of reflecting the marginal value of distribution line losses and congestion apart from participant actions that tend to hasten or delay distribution transformer replacement. Up until now, due to the assumption of the inelasticity of the distribution resources, creation of a competitive market structure at the distribution level was not warranted. Establishing a competitive market will provide strong incentives to larger sets of distributed resources to act efficiently in a manner that benefits the power system as a whole, thereby improving the economic efficiency and the system reliability. The DLMP has the operational benefits of providing real-time control signals and supporting cost effective operating strategies for energy utilization from DSR and for the deployment of local and external resources. In the case of networked distribution systems, power flow control and management is possible. The DLMP concept could be a driver for networking distribution systems when it is warranted and to provide a cost effective way for implementing and operating DSR. In legacy distribution systems, the DLMP could be used to recover costs for upgrades and to guide investment or planning decisions. Additionally, the DLMP would help to identify consumers who have been subsidizing the ones with greater consumption during the peak hours, thus, resulting in an end to historical cross subsidy.

Another added advantage of the DLMP is in its ability to reduce market power. Since, both the T&D system states are considered in calculating the

DLMP, the DLMP would reflect the changes in the wholesale prices. Therefore, if a market participant was to exercise market power by withholding capacity to increase the wholesale price, the DLMP would reflect this increase, consequently, forcing the end users to reduce their consumption. It is important to note that the spot pricing of electricity in the distribution system is based upon the assumption of economically rational consumer response. Thus, price fluctuation could be used as a potential tool to incentivize customers to reduce their consumption when the grid is stressed or short of capacity. Demand response to real-time prices that reflect the true supply and demand situation in the market will increase market efficiency. Consequently, in the short run, the total payment to the generators in the wholesale electricity markets would be reduced due to reduced peak demands, while in the long run, reduced peak demands would reduce the cost to build new capacity, which in turn would reduce the cost passed onto the consumers. It is the combined efforts of all the consumers and not an individual's effort to lower their peak demands that drives the system. It is certain that flexible demand would respond to price spikes either by conserving or shifting their demand. This would lower the price spike and prevent blackouts. Demand response from the flexible loads would certainly aid in meeting capacity reserve requirements. Contemporary standards require the reserves to be in the range of 15-20 percent of the forecasted system peak. Introduction of DLMPs will reduce the forecasted peak and, therefore, reduce the reserve requirements because an unexpected emergency situation could be addressed through demand response. With increased penetration of intermittent energy sources, like wind and solar, there will be a

need for flexible reserves with fast ramp rates. It has been estimated that a 30% increase in renewable generation will require a three to four-fold increase in flexible reserves [11]. Since, ancillary services clear today at prices that are comparable to energy clearing prices, increasing the requirement of flexible reserves will pose significant hindrance to renewable generation expansion. However, these extra costs can be avoided if the flexible reserves can be obtained from price responsive loads with storage capabilities (e.g., flexible building loads, and EV battery charging) by using appropriate price incentives.

With the deployment of DLMPs, the retail electricity markets would bear a striking resemblance with the wholesale electricity markets due to increased retail competition. The DLMPs could be set either in the day-ahead markets or the real-time markets.

2.5 Proposed Iterative Framework

As seen in Section 2.3, all of the prior work on DLMPs has primarily focused on a one-shot approach: first, the traditional transmission OPF is solved (where each distribution system is modeled as a single, equivalent bus with one perfectly inelastic load), and then a distribution OPF is solved (without the transmission system being modeled) to produce the DLMPs or the DLMPs are calculated based on the resulting LMPs from the transmission OPF. However, this approach does not reflect the interaction between the T&D systems nor does it capture the effect of true price responsive behavior of the flexible loads and DSR on the transmission system operations. Presently, there is no integrated framework for the T&D systems that provides a closed loop solution with due consideration to

conventional demand elasticity.

This research proposes an iterative framework to integrate the T&D systems together. The integrated OPF problem is decomposed into two subsequent OPF problems: the transmission OPF and the distribution OPF. The distribution OPF incorporates characteristics of the DSR and determines the appropriate DLMP in order to incentivize efficient scheduling of the resources. Also, in order to ensure an accurate representation of one sub-system when modeling the other sub-system, aggregate residual demand curve (represents the distribution system in the transmission OPF) and transmission-constrained residual supply curves (represents the transmission system in the distribution OPF) are proposed. Instead of using a one-shot approach where the transmission system modeled is solved only once, the proposed iterative framework resolves the transmission OPF with updated residual demand curves. In order to ensure accurate coupling between the two sub-systems, the proposed framework is iterated between two OPF models until convergence is achieved. The convergence criterion is expressed in terms of either the LMP or the cleared aggregate demand at the proxy buses (interconnection points between the T&D systems). The iterative framework stops when the maximum LMP error or the cleared aggregate demand error between two consecutive iterations is lower than a certain threshold. Furthermore, the iterative framework enables accurate coupling between the two sub-systems by ensuring: 1) the appropriate representation of the flexibility of the DSR and the distribution system network conditions in the transmission OPF and 2) the appropriate representation of the generator availability and transmission system

network conditions in the distribution OPF. Thus, the iterative framework is successful in capturing the interaction between the local allocation of resources and the DLMPs with the transmission OPF and the transmission LMPs. Also, the proposed iterative framework is successful in extracting the flexibility of the DSR to benefit the transmission system operations, incentivizing optimal DSR decisions and improving market efficiency and system reliability. Hence, the DLMP can be utilized as a control signal to align the operation of the DSR with the objectives of the bulk energy system.

2.6 Literature Review: Transmission-Constrained Residual Curves

In [20], Xu presents the concept of transmission-constrained residual demand, and the analytical calculation of its derivative. The residual demand derivative is used in formulating the generator's profit maximization problem, which in turn is useful in constructing optimal bidding strategies (profit maximizing offers) for the generator under consideration. It plays a vital role in constructing the generator's best response to competitors' strategies in transmission-constrained networks. The derivative reveals the sensitivities of the generation dispatch to the incremental market price changes. In [21], Xu and Baldick further use the residual demand derivative characterization to analyze the strategic behavior in widely used strategic models such as the Cournot model and the supply function model with transmission constraints. The authors use the DCOPF model to characterize the residual demand derivative analytically, which in turn aids in characterizing the market Nash equilibrium. Here, the residual demand of a generator is defined as the system demand minus the aggregated supply of all the other suppliers in the

market. Reference [22] discusses numerous applications of the transmission-constrained residual demand derivative. One important feature of the transmission-constrained residual demand derivative is that it avoids full network representation in optimization models. This property of the transmission-constrained residual demand derivative was used to derive the transmission-constrained residual supply curve, thereby avoiding full network representation of the transmission system in the distribution OPF.

Reference [23] proposes a conjectural variation-based equilibrium (CVE) model to estimate the agent's behavior (firms' bidding strategies) in power market models. Such an equilibrium model relies on user supplied parameters, conjectural variations (CV), which allow for a more flexible representation of the agents' behavior in competitive market structures. The equilibrium model also provides an insight on the sensitivity of the market equilibrium to the agent's bidding strategies. Here, CV is defined as the belief that one agent has regarding the manner in which its competitor(s) would react if it were to vary its price or output. The authors in [23] discuss several methodologies for estimating this parameter. They also propose a time-series model for estimating the future values of the CV. This concept of CV is utilized in deriving the residual supply curve for the proposed iterative framework. The residual supply curve reflects the transmission system's response relative to a change in consumption from the corresponding distribution system.

CHAPTER 3

NECESSARY FUNDAMENTALS

This chapter provides the fundamental concepts needed to understand the technical details in the subsequent chapters. The chapter describes the unconstrained economic dispatch problem in Section 3.1 and the optimal power flow problem in Section 3.2. Section 3.3 describes the concept of price elasticity of demand from an economics point of view. Finally, a brief introduction to linear optimization including Lagrange relaxation and primal-dual relationships is provided in Section 3.4.

3.1 Economic Dispatch

The economic dispatch problem is a standard dispatch optimization problem, which considers only the generator capacity constraints (limits the generator outputs) and the system wide energy balance constraint (ensures the fact that the total supply must equal the total demand). It does not incorporate the network constraints, thereby producing a lower bound on OPF problems. Also, most economic dispatch problems ignore reactive power. In other words, the economic dispatch problem optimally determines the generator dispatch values to satisfy demand while meeting the generator operating limits. Thus, the objective of the economic dispatch problem is to minimize the total dispatch cost to meet demand.

Production cost curves for generators are typically quadratic (and convex) and are often approximated with piecewise linear cost curves. A piecewise linear cost curve creates a block marginal cost curve, which is also referred to as a staircase offer curve. The assumption that generators have linear cost functions makes the

economic dispatch problem an LP problem.

3.2 Optimal Power Flow (OPF) Problem

An OPF problem is an economic dispatch problem that incorporates the network constraints such as branch thermal limits. The objective of an OPF problem is to minimize the total dispatch cost to satisfy the demand in the system while ensuring reliability. For an energy market setting, the objective of an OPF problem is either to minimize the total dispatch cost (perfectly inelastic demand) or to maximize the social welfare (elastic demand), while ensuring reliability. Social welfare (or market surplus) refers to the overall welfare of society and is a measure of the benefits to both suppliers and consumers for participating in the market [24]. Social welfare is usually calculated based on the supply offers and demand bids submitted by the generators and loads respectively. Equation (3.1) represents the objective function of an OPF problem for the case when the demand is perfectly inelastic. Such an OPF problem is sometimes referred to as a generation cost minimization problem. The objective is the sum, for all generators g , of the product of the marginal cost (c_g) of generator and the real power output (P_g) of the generator. Equation (3.2) represents the objective function of an OPF problem for the case when the demand bids into the energy market (elastic demand). Such an OPF problem is sometimes referred to as the bid cost maximization problem. The first term of (3.2) is the total consumer bid value. It is the sum, for all load bids d , of the product of the bid price (b_d) and the cleared demand (D_d) associated with a load bid (d). The second term represents the total generation cost as seen in (3.1).

$$\min: \sum_g c_g P_g \quad (3.1)$$

$$\max: \sum_d d_d D_d - \sum_g c_g P_g \quad (3.2)$$

In order to ensure reliability, the power system is usually operated within certain limits. In the case of OPF problems, these limits are modeled by the node balance constraints, the network constraints, which are used to impose limits on network parameters such as bus voltage magnitudes and angles and transmission line flows, and the generator constraints which define the reliable operating limits of the generators in the system. The flow of electricity obeys the Kirchhoff's laws. OPF problems are classified into two types: the Alternating Current Optimal Power Flow (ACOPF) problem and the DCOPF problem. While both the problems have similar objective functions and category of constraints (node balance constraints, network constraints and generator constraints), they use different power flow equations in the constraints. Equations (3.3) and (3.4) represent the power flow equations for the flow of electric power into bus n from transmission line k (line k is connected from bus m to bus n) used in the ACOPF problem.

$$P_k = |V_m|^2 G_k - |V_m||V_n|(G_k \cos(\theta_m - \theta_n) + B_k \sin(\theta_m - \theta_n)), \forall k \quad (3.3)$$

$$Q_k = -|V_m||V_n|(G_k \sin(\theta_m - \theta_n) - B_k \cos(\theta_m - \theta_n)) - |V_m|^2 B_k, \forall k \quad (3.4)$$

P_k is the real power flow and Q_k is the reactive power flow. V_n , V_m , θ_n , and θ_m represent the bus voltages and phase angles for buses m and n respectively. Consequently, the ACOPF optimization problem is a very difficult problem to solve since it is a non-convex optimization problem, which contains trigonometric

and quadratic functions as seen in (3.3) and (3.4). The non-linearity in these equations complicate the optimization problem significantly.

It is common to use a linearized approximation of the ACOPF problem. The first approximation concerns the voltage variables. Since, the voltage levels are generally very close to one (on a per unit basis), all voltage variables are assumed to have a value of one in the DCOPF problem. This removes the non-linearity associated with the quadratic voltage terms in (3.3) and (3.4). The second approximation comes from the fact that the bus angle difference between two buses is usually very small. Thus, the Sine and the Cosine of a small angle difference can be approximated by the angle difference itself and one respectively. These two approximations cause the G_k terms to cancel in (3.3) and the B_k terms to cancel in (3.4). The third approximation is to ignore the remaining reactive power term in (3.4) and the last approximation is to assume that the resistance of a line is very small in comparison to its reactance thereby making the susceptance equal to the negative inverse of the reactance. This assumption makes the conventional DCOPF problem lossless; however, the conventional DCOPF problem can be modified to account for losses. In this research, the lossy DCOPF formulation [25] is used to solve the distribution system. In this formulation, the loss equation is approximated by a piecewise linear approximation. The preceding assumptions are used to develop a linearized approximation of (3.3) and the DCOPF optimization problem as shown below in (3.7) and (3.5)-(3.10) respectively.

$$\min: \sum_g c_g P_g \quad (3.5)$$

subject to:

$$\sum_{\forall k(n,:)} P_k - \sum_{\forall k(:,n)} P_k - D_n + \sum_{g \in G_n} P_g = 0 \quad (3.6)$$

$$B_k(\theta_n - \theta_m) - P_k = 0 \quad (3.7)$$

$$P_k^{min} \leq P_k \leq P_k^{max} \quad (3.8)$$

$$\theta_{nm}^{min} \leq (\theta_n - \theta_m) \leq \theta_{nm}^{max} \quad (3.9)$$

$$P_g^{min} \leq P_g \leq P_g^{max} \quad (3.10)$$

Equation (3.5) represents the objective function. Equation (3.6) represents the node balance constraint that specifies that the power flow into a bus must equal the power flow out of a bus. The first two terms in (3.6) represent the power flowing into and out of the bus respectively. Equation (3.7) is the linear approximation of the power flow equation. Equation (3.8) represents the capacity constraint on transmission line k . Equation (3.9) restricts the bus voltage angles for any two buses that are connected by a transmission element and is the transient stability proxy limit. Equation (3.10) represents the generator real power output limit.

Another formulation commonly used for the DCOPF is the formulation obtained by using Power Transfer Distribution Factors (PTDFs) as shown below in (3.11)-(3.15). In this formulation the linear approximation of the power flow equation is further approximated using PTDFs. A PTDF ($PTDF_{k,i}^R$) is the proportion of flow on line k resulting from an injection (positive) or withdrawal (negative) of one MW at a node i and corresponding to a one MW withdrawal or

injection at a reference node R .

$$\min: \sum_g c_g P_g \quad (3.11)$$

subject to:

$$P_n^R + D_n - \sum_{g \in G_n} P_g = 0 \quad (3.12)$$

$$\sum_n P_n^R = 0 \quad (3.13)$$

$$P_k^{min} \leq \sum_n PTDF_{k,n}^R \cdot P_n^R \leq P_k^{max} \quad (3.14)$$

$$P_g^{min} \leq P_g \leq P_g^{max} \quad (3.15)$$

Equation (3.13) represents the system wide node balance constraint that specifies that the net power injection in the system must equal zero. The generator supplies at a node are injections while the load is a withdrawal. In this research, the PTDF formulation is used for transmission system.

3.3 Price Elasticity

It is usually of interest to have a measure of how responsive the demand is to a change in price. The slope of a demand function could be used as an indicator of elasticity or a measure of responsiveness. After all, by definition the slope of a demand function is change in quantity demanded by change in price. However, the slope of a demand function is dependent on the units in which price and quantity are measured. It is preferable to use a unit free measure of responsiveness or elasticity [26]. Thus, price elasticity of demand, ϵ , is used instead. The price elasticity of demand is defined as the percent change in quantity divided by the percent change in price or the ratio of price to quantity multiplied by the slope of the demand function as shown in (3.16).

$$\epsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} = \frac{p\Delta q}{q\Delta p} \quad (3.16)$$

The demand for goods can be treated as highly elastic or less elastic. The goods with a high elasticity are non-essential and are easy to substitute whereas the goods with a low elasticity are essential and have very few close substitutes. An elastic good is one for which the quantity demanded is very responsive to price, for example, restaurant meals and candy bars since they can be easily substituted. While an inelastic good is one for which the quantity demanded is not very responsive to price, for example, gas, petrol and water since they have very few alternatives. The demand for electrical energy is usually assumed to be perfectly inelastic ($\epsilon = 0$). The sign of elasticity of demand is usually negative because demand curves invariably have a negative slope. An inelastic good has a price elasticity of ($-1 < \epsilon < 0$) while an elastic good has a price elasticity of ($-\infty < \epsilon < -1$). A perfectly elastic good has a price elasticity of $-\infty$. If a good has an elasticity of exactly -1 , it is said to have unit elasticity.

3.4 Introduction to Linear Optimization

An LP consists of a linear objective, linear equality and inequality constraints, and continuous variables. Consider a general linear programming problem [27] with N variables. Let N_1 be the subset of variables with a non-negativity constraint and N_2 be the subset of variables with a non-positive constraint. If a variable is not constrained to be non-negative or non-positive, the variable is said to be a free or unrestricted variable. Let N_3 be the subset of free variables. Let the

LP have M constraints with M_1 greater than or equal to inequalities, M_2 lesser than or equal to inequalities and M_3 equalities.

$$\min_x c^T x \quad (3.17)$$

subject to:

$$a_i^T x \geq b_i \quad i \in M_1 \quad (3.18)$$

$$a_i^T x \leq b_i \quad i \in M_2 \quad (3.19)$$

$$a_i^T x = b_i \quad i \in M_3 \quad (3.20)$$

$$x_j \geq 0 \quad j \in N_1 \quad (3.21)$$

$$x_j \leq 0 \quad j \in N_2 \quad (3.22)$$

$$x_j \text{Free} \quad j \in N_3 \quad (3.23)$$

Here, x is a vector of n variables and c is the cost vector with n entries. Also, each constraint has an n -dimensional vector a_i and a scalar b_i . All linear programming problems can be converted into standard form LPs by 1) eliminating the free variables (any free variable can be written as two non-negative variables), and 2) eliminating the inequality constraints (by adding slack or surplus variables, s). A standard form LP is given by (3.24)-(3.26) as follows:

$$\min_x c^T x \quad (3.24)$$

subject to:

$$Ax = b \quad (3.25)$$

$$x \geq 0 \quad (3.26)$$

The standard form LP for the general linear programming problem given by (3.17)-(3.23) is as follows:

$$\min_x c^T x \quad (3.27)$$

subject to:

$$a_i^T x - s_i = b_i \quad i \in M_1 \quad (3.28)$$

$$a_i^T x + s_i = b_i \quad i \in M_2 \quad (3.29)$$

$$a_i^T x = b_i \quad i \in M_3 \quad (3.30)$$

$$x_j \geq 0 \quad j \in N_1 \quad (3.31)$$

$$-x_j \geq 0 \quad j \in N_2 \quad (3.32)$$

$$x_j = x_j^+ - x_j^- \quad j \in N_3 \quad (3.33)$$

$$x_j^+ \geq 0, x_j^- \geq 0 \quad j \in N_3 \quad (3.34)$$

$$s_i \geq 0 \quad (3.35)$$

Each constraint in a LP has an associated price known as the shadow price or the dual variable (Lagrange multiplier). Lagrange multipliers can be viewed as the price to violate the respective constraint. In order to derive the dual of a LP, the goal is to find the price that will not affect the objective (optimal cost) no matter if the constraint is treated as a hard constraint or relaxed. The following steps are taken in order to derive the dual of the standard form LP (referred to as the primal problem from here on) given in (3.24)-(3.26):

- i. Formulate a relaxed problem (Lagrangian) using the Lagrangian relaxation. The Lagrangian allows the constraint to be violated.

$$\min_{x \geq 0} c^T x + p^T (b - Ax) \quad (3.36)$$

Here, p is the shadow price to violate constraint (3.25).

- ii. Let $g(p)$ represent the optimal cost to the relaxed problem such that:

$$g(p) = \min_{x \geq 0} \{c^T x + p^T (b - Ax)\} \quad (3.37)$$

Since the original standard form LP (minimization) was relaxed, $g(p)$ gives the lower bound to the original problem. In other words, $g(p) \leq c^T x^*$, $\forall p$. Here, $c^T x^*$ represents the original problem's optimal cost. Each p gives a different lower bound $g(p)$ and the objective is to find the tightest lower bound. In other words, the objective is to maximize $g(p)$. This, in turn, results in what is called the dual problem, which is to max: $g(p)$, subject to no constraints. Since $p^T b$ is a constant in the minimization problem, pulling it out of the minimization results in $g(p) = p^T b + \min_{x \geq 0} \{c^T x - p^T Ax\}$.

It is important to note that,

$$\min\{(c^T - p^T A)x \mid x \geq 0\} = 0 \quad \text{if } (c^T - p^T A) \geq 0^T$$

$$\min\{(c^T - p^T A)x \mid x \geq 0\} = -\infty \quad \text{otherwise}$$

Here, the obvious choice would be to choose p^T such that $p^T A \leq c^T$, otherwise, the lower bound would indeterminate. Therefore, the dual of the standard form LP is given by:

$$\max p^T b \quad (3.38)$$

subject to:

$$p^T A \leq c^T \quad (3.39)$$

It is important to note that, in this case, p is unconstrained.

There are numerous applications of the dual of LPs, for instance: 1) it provides the basis for sensitivity analysis, 2) economic application like shadow price, and 3) it provides insights to LP theory. The dual of the general linear programming

problem given in (3.17)-(3.23) can be derived from the duality theory as explained above. The resultant primal-dual pair is as follows:

$$\begin{array}{ll}
 \min_x c^T x & \max_p p^T b \\
 \text{subject to:} & \text{subject to:} \\
 a_i^T x \geq b_i & i \in M_1 & p_i \geq 0 & i \in M_1 \\
 a_i^T x \leq b_i & i \in M_2 & p_i \leq 0 & i \in M_2 \\
 a_i^T x = b_i & i \in M_3 & p_i \text{Free} & i \in M_3 \\
 x_j \geq 0 & j \in N_1 & p^T A_j \leq c_j & j \in N_1 \\
 x_j \leq 0 & j \in N_2 & p^T A_j \geq c_j & j \in N_2 \\
 x_j \text{Free} & j \in N_3 & p^T A_j = c_j & j \in N_3
 \end{array}$$

When comparing these primal-dual pairs it can be observed that there is a relationship between the (sign of) variables in one problem and the constraints (inequalities or equalities) in its dual. Table 3.1 summarizes the relationships between the primal-dual pairs [27].

TABLE 3.1.
PRIMAL-DUAL RELATIONSHIPS

Primal	Minimization	Maximization	Dual
Constraints	$\geq b_i$ $\leq b_i$ $= b_i$	≥ 0 ≤ 0 <i>Free</i>	Variables
Variables	≥ 0 ≤ 0 <i>Free</i>	$\leq c_j$ $\geq c_j$ $= c_j$	Constraints

It is important to note that if we take the dual of the standard form LP and then

take the dual of this problem, we get back the primal, which is the standard form LP. In other words, the dual of a dual is the primal problem. Thus, the proposition for the corresponding primal-dual forms is that, for any primal problem and its dual problem, all relationships between them must be symmetric because the dual of the dual problem is the primal problem.

CHAPTER 4

INTEGRATED TRANSMISSION AND DISTRIBUTION SYSTEMS MODEL

This chapter gives an outline of the proposed iterative approach to couple the T&D systems together. Section 4.1 gives a comprehensive illustration of the mathematical optimization problem for integrating the T&D systems models. Section 4.2 discusses the need for residual demand and supply curves. The derivation of the aggregate residual demand curve and the transmission-constrained residual supply curve is described in Section 4.3 and Section 4.4 respectively.

4.1 Mathematical Optimization Problem for Integrating the T&D Systems Models

Ideally, the T&D systems should be modeled by an integrated OPF problem and solved simultaneously to schedule the generation and demand in the entire system. However, integrating T&D system models together is a challenge for OPF due to the size of the system, which makes it computationally intractable with existing technologies. Consequently, a two-stage optimization process is proposed, with the T&D systems modeled separately. This section presents an iterative approach to couple the T&D systems together, which involves two stages.

Fig. 4.1 gives a comprehensive illustration of the mathematical optimization problem for integrating the T&D systems models [28]. The first stage of the two-stage optimization problem is the transmission OPF. The distribution system details are not modeled in the transmission OPF; instead, the distribution system is modeled by its aggregate residual demand curve in the transmission OPF [9].

The aggregate residual demand curve is proposed to represent the distribution system when solving the transmission OPF. It reflects the distribution system’s response (change in consumption) relative to a change in the transmission system LMP (price at the proxy node). The transmission OPF incorporates characteristics of the transmission system resources and transmission system network conditions and determines the appropriate LMP in order to incentivize the efficient scheduling of resources. Thus, the transmission OPF solution determines the current market operating point for the transmission system.

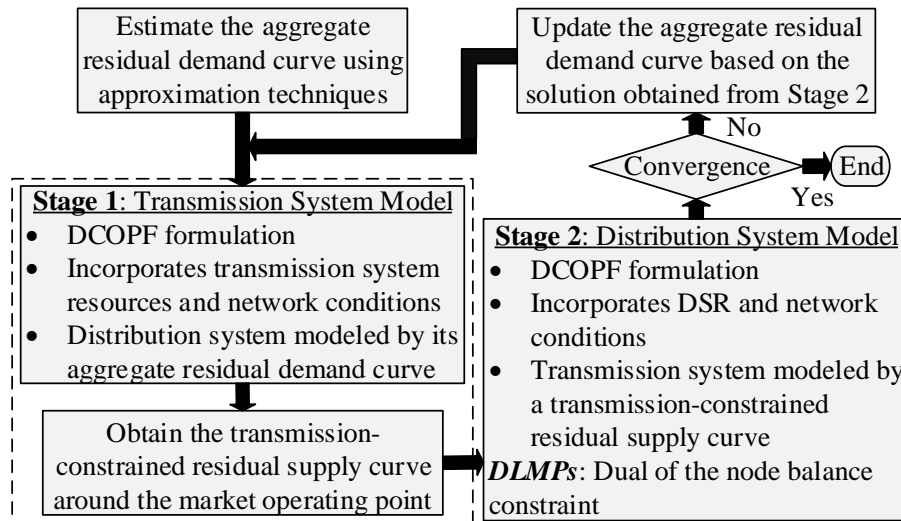


Fig. 4.1 A Mathematical Optimization Problem for Integrating the T&D systems Models

The second stage of the two-stage optimization problem is the distribution OPF. The transmission system details are not modeled in the distribution OPF; instead, the transmission system is modeled by a transmission-constrained residual supply curve in the distribution OPF [9]. The transmission-constrained residual supply curve reflects the transmission system’s response (change in price) relative to a change in the distribution system consumption and is

calculated for the transmission system around the market operating point. The distribution OPF incorporates characteristics of the distribution system resources and distribution system network conditions and determines the appropriate DLMP in order to incentivize efficient scheduling of the resources. The demand curves for the various load points (PRL with assumed price elasticity of demand) in the distribution OPF are derived using the technique presented in [6]. The proposed aggregate residual demand curve and transmission-constrained residual supply curve ensure accurate representation of one sub-system when modeling the other sub-system. Instead of using a one-shot approach where the transmission system is solved only once, the process is continued until convergence is achieved. Convergence can be defined in terms of either the LMP or the cleared aggregate demand at the proxy nodes.

For the sake of simplicity, the proposed iterative framework is derived under several assumptions. First, the market is assumed to be cleared by solving the traditional DCOPF formulation for both T&D system models. This was done to maintain consistency with existing practices. Contemporary markets solve the traditional DCOPF formulation for the transmission system in order to calculate the LMPs. The DCOPF is a widely used formulation due to its natural fit into the LP model. Its solutions are non-iterative, reliable and unique [29]. Moreover, various third-party LP solvers are readily available to plug into the DCOPF model [30]. However, both the OPFs in the proposed calculation framework could be based on the ACOPF. The assumptions used to obtain the DCOPF formulation from the ACOPF formulation can become more inaccurate for lower voltage, sub-

transmission and distribution lines and as line loading increases [10]. Most importantly, the distribution systems have a lower X/R ratio when compared to the transmission systems. This stems from the fact that the voltage on the distribution system primary feeders is particularly lower, thereby reducing the phase to phase conductor spacing, which in turn reduces the inductive reactance. Also, conductor and transformer winding resistances are higher in the distribution systems. Thus, the assumption postulating the resistance of a line being very small when compared to the reactance of a line does not always hold true for the distribution system. In fact, the percentage losses in the distribution systems are greater when compared to the percentage losses in the transmission systems. The distribution system energy losses are typically around 5% - 8% of the energy delivered [17], [31]. Table 4.1 shows the typical ranges of percentage energy losses in the distribution systems, with data taken from various sources, taken at 75% loading operation. Since the traditional DCOPF is valid only for high voltage systems and is less usable on the lower voltage circuits, a lossy DCOPF formulation [25] is used instead for the distribution OPF. The lossy DCOPF formulation accounts for the losses in the distribution system and the resulting LMPs inherently capture the marginal impact of losses.

TABLE 4.1.
TYPICAL RANGES OF PERCENTAGE ENERGY LOSSES IN THE DISTRIBUTION SYSTEM

Loss Component	Losses (%)
Distribution Substation Losses	0.46 - 2.15
Distribution Primaries	0.46 - 3.15
Distribution Secondaries	0.1 - 1.9
Secondary Conductor Losses	0.85 - 0.2
Revenue Meter Losses	0.19 - 0.4

Second, both the T&D OPFs in the proposed iterative framework assume a balanced 3-phase operation. The lossy DCOPF formulation used for the distribution OPF does not account for the unbalance in the distribution system networks because this will require a 3-phase unbalanced power flow study with potentially different prices on each phase, which is outside the scope of this research. A more accurate OPF for the distribution system can be used instead. Last, the DLMPs are calculated only on the primary distribution system feeders or at the secondary terminals of the distribution transformer [6].

4.2 Need for Residual Demand and Supply Curves

As seen in Section 4.1, the DLMP comes from a two-stage optimization problem and is proposed to be used for settlement purposes in the distribution system. The DLMP improves upon existing distribution pricing strategies and with improvements in the economic price signal, market efficiency is enhanced. However, separating the T&D systems creates issues regarding the accurate representation of one sub-system when solving the OPF for the other sub-system. Thus, an iterative approach has been proposed by [6], which includes a two-stage optimization process. However, in this prior work, the details of the transmission system are not modeled in the distribution OPF. Instead, the transmission system is simply modeled as an infinite generator (at the proxy node) that has a marginal cost equal to the resulting transmission LMP, in the distribution OPF. This inaccurately approximates the sensitivity of the transmission system relative to a change in the demand from the distribution system. Such an approach, however, results in a perfectly elastic supply curve and convergence issues are observed due

to a non-unique range of solutions. The iterative framework in [6] fails to converge in instances when the generator sets the market clearing price in the transmission OPF. Fig. 4.2 gives an illustration of an instance when the generator sets the market clearing price. Fig. 4.3 gives an illustration of the possible range of solutions, with a perfectly elastic supply curve representing the transmission system's supply curve in the distribution OPF. The interaction of the load bid curve and the perfectly elastic supply curve representation of the transmission system results in a non-unique range of market clearing quantities in the distribution OPF, as illustrated in Fig. 4.3. While the market clearing quantity in the transmission OPF is a specific quantity Q^* between Q_L and Q_H , the market clearing quantity in the distribution OPF could be any quantity between Q_L and Q_H . In such a situation, the market clearing point is randomly selected by the optimization solver depending on the solution algorithm in question. A perfectly elastic supply curve sends the signal that the marginal cost to serve an increment or decrement of 1 MW (at the interconnection point between the T&D systems) costs the same regardless of the consumption from the distribution system. In other words, a perfectly elastic supply curve implies that the cost to consume any quantity between Q_L and Q_H is the same, which is inaccurate. Non-convergence due to the infinite generator model is handled by replacing it with a transmission-constrained residual supply curve which will be discussed in detail in Section 4.4.

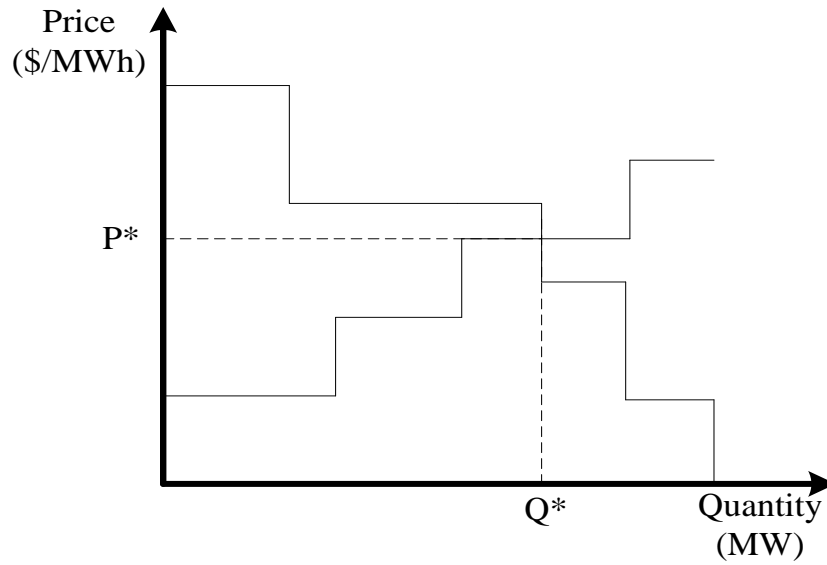


Fig. 4.2 Market Equilibrium with Generator Setting the Market Clearing Price

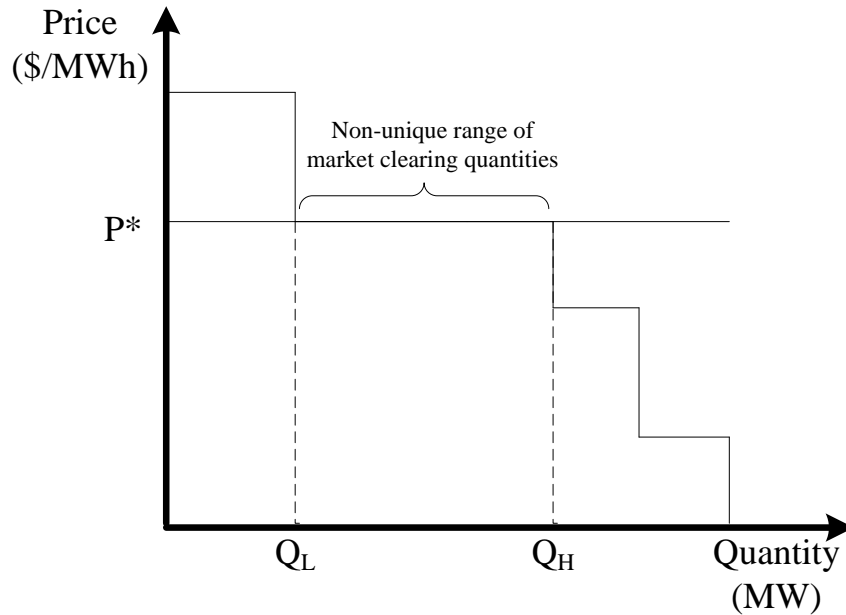


Fig. 4.3 Multiple Market Clearing Quantities due to the Interaction of Perfectly Elastic Supply Curve and Load Bid Curve in the Distribution OPF

Also, this prior work [6] ignores the distribution system details when modeling the distribution system in the transmission OPF. Rather, the distribution system is represented by a perfectly inelastic demand curve in the transmission

OPF. In other words, the distribution system's aggregate demand is forecasted and then used in the transmission OPF. This approach, however, can lead to non-convergence of the iterative framework due to the inaccurate representation of the distribution system in the transmission OPF. The iterative framework in [6] will fail to converge in instances when the load (assumed to be perfectly inelastic) sets the market clearing price in the distribution OPF. A perfectly inelastic demand curve represents a situation in which the demand for a good (the good in this context is electricity) is unaffected when the price of that good changes. Consequently, a perfectly inelastic demand curve model of the distribution system sends the signal that the loads in the distribution system are unaffected by the price at the proxy bus (interconnection point between the T&D systems). The interaction of the generator offers (supply curve) and the perfectly inelastic demand curve representation of the distribution system results in a non-unique range of market clearing prices in the transmission OPF, as illustrated in Fig. 4.4. While the market clearing price in the distribution OPF is a specific price P^* between P_L and P_H , the market clearing price from the transmission OPF could be any price between P_L and P_H . In such situations, the market clearing point is randomly selected by the optimization solver depending on the solution algorithm in question. This is an inadequate representation of the sensitivity of the DSR over a range of possible prices that may result from the transmission OPF. The perfectly inelastic demand curve representation of the distribution system gives an indication that the loads in the distribution system are willing to consume a fixed quantity, Q^* , for any price between P_L and P_H (at the proxy bus). Fig. 4.5 gives an

illustration of a price responsive load setting the market clearing price as opposed to a perfectly inelastic load setting the market clearing price. The interaction of the generator offers (supply curve) and the price responsive load bids (demand curve) in Fig. 4.5 results in a unique market clearing point denoted by the price-quantity pair, (P^*, Q^*) . Therefore, there is a need for a more accurate representation of the distribution system when solving the transmission OPF and vice-versa. Non-convergence due to the perfectly inelastic demand curve model is handled by replacing it with an aggregate residual demand curve, discussed in Section 4.3. The iterative framework for calculating the DLMPs is further investigated in Sections 4.3 and 4.4 respectively, to develop solutions to its convergence issues.

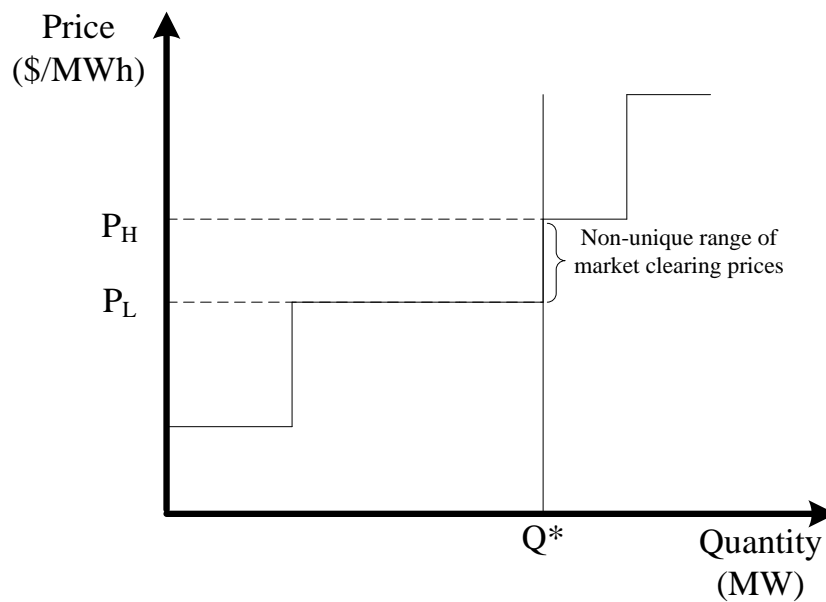


Fig. 4.4 Multiple Market Clearing Prices due to the Interaction of Generator Offer Curve and Perfectly Inelastic Load Bid Curve in the Transmission OPF

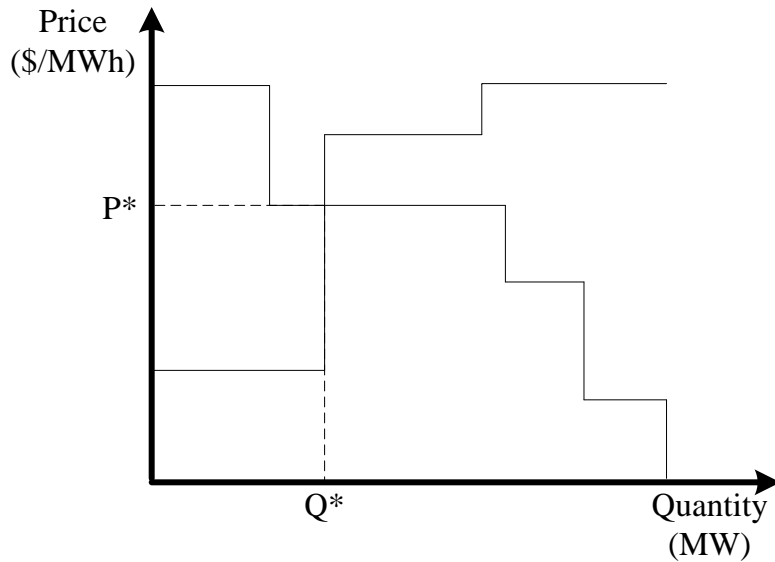


Fig. 4.5 Market Equilibrium with Price Responsive Load Setting the Market Clearing Price

4.3 Derivation of the Aggregate Residual Demand Curve

The aggregate residual demand curve is proposed to represent the distribution system when solving the transmission OPF. It reflects the distribution system's response relative to a change in the LMP at the proxy bus. Historical information and load forecasts give an approximate estimation of the demand of the distribution feeders. Today, utilities use these approximations to estimate the demand of the distribution system and apply the same at the proxy bus. The smart grid initiative is expected to result in a substantial presence of distributed generation, energy storage, and price responsive, flexible demand at the distribution level with the potential need for active power management and congestion management. In order to obtain a better representation of the DSR and to overcome the issues related to convergence in [6], it is preferable to derive an approximate demand curve to better represent the distribution system instead of

using a single forecast to represent the distribution system. Approximate demand curves can be generated by the Load Serving Entities (LSEs) over time due to the availability of historical information regarding the conditions in the distribution system. Due to the unavailability of historical information for this particular case study, a sampling approach is used to produce the demand curves that would otherwise be generated by the LSEs based on historical information. The sampling approach is not proposed for actual implementation since it is not the focus of this research.

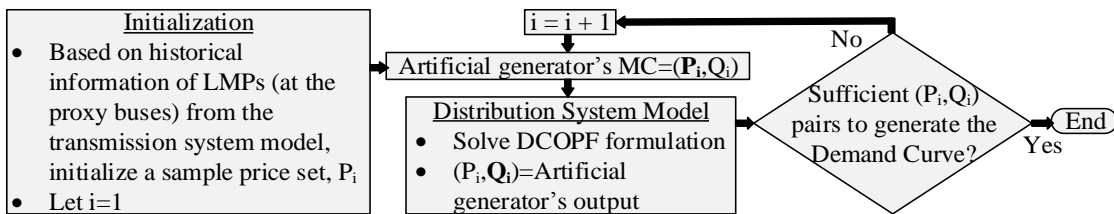


Fig. 4.6 An Approximate Technique to Generate an Initial Estimate of the Aggregate Residual Demand Curve

A sampling approach [28] is used to generate an initial estimate of the aggregate residual demand curve. Fig. 4.6 gives an outline of the process used to obtain the aggregate residual demand curve. The process involves using an artificial generator to represent the transmission system at the proxy bus and generating multiple price-quantity pairs by solving the lossy DCOPF formulation for the distribution system alone. The output of the artificial generator at the proxy bus gives an indication of the net response (aggregate demand) of the distribution system to a particular price at the proxy bus. The artificial generator's marginal cost is equated to different prices from a sample price set in order to simulate the different prices that may result from the transmission OPF and the

resulting quantities (output of the artificial generator) are used to create the necessary price-quantity pairs. This process is used to replicate historical information that would otherwise exist. This curve is also updated at each consecutive iteration of the iterative framework by adding a new segment (price-quantity pair) that is reflective of the new OPF solution at the proxy bus from the distribution OPF. This procedure is used to obtain the aggregate residual demand curve and it is better at accounting for the price elasticity of the individual load points in the distribution system, the distributed generation resources and the distribution system network conditions, e.g., congestion. Note that this method is more accurate than simply aggregating individual demand curves from the distribution system as this approach then accounts for losses and other network limitations in the distribution system. It is important to note that the aggregate residual demand curve is derived under the assumption that, in actual practice, information regarding the price sensitivity of the DSR and distribution system network conditions is available.

4.4 Derivation of the Transmission-Constrained Residual Supply Curve

Consider a system with n buses. For a system without congestion, the residual supply at node j , $R_j(p_j)$, can be derived from the law of conservation of energy (supply equals demand) as follows:

$$\sum_{j=1}^n (S_j(p_j) - D_j(p_j)) = 0 \quad (4.1)$$

where p_j is the market price at node j , $S_j(\cdot)$ is the supply function at node j and $D_j(\cdot)$ is the demand function at node j . Consider a case wherein the residual

supply at bus n is to be determined. Rearranging the terms in (3.1) to obtain $S_n(p_n) - D_n(p_n)$ on the left-hand side [21]

$$S_n(p_n) - D_n(p_n) = \sum_{j=1, j \neq n}^n (D_j(p_j) - S_j(p_j)) \quad (4.2)$$

The supply at each bus in the system can be treated as negative demand; therefore, the net injection at bus n is obtained as

$$-D_n(p_n) = \sum_{j=1, j \neq n}^n D_j(p_j) \quad (4.3)$$

The constraint for nodal power balance for the residual market at bus n necessitates the residual supply at bus n to be equal to the demand at bus n . Hence,

$$R_n(p_n) = -D_n(p_n) \quad (4.4)$$

The derivative of the residual supply in (4.4) reflects the price sensitivity of the transmission system relative to a change in the demand at bus n . Here, bus n is assumed to be the proxy bus. A DCOPF formulation is solved for the transmission system alone in order to obtain the transmission-constrained residual supply curve. The objective of the transmission OPF is to minimize the total generation cost to satisfy the demand in the system while ensuring reliability. The conventional DCOPF formulation in its simplest form is defined below

$$\min_{q_i} \sum_{i=1}^n C_i(q_i) \quad (4.5)$$

subject to:

$$1^T q = 0 \quad (4.6)$$

$$Hq \geq Z \quad (4.7)$$

$$q_n \geq q_n^{\min} \quad (4.8)$$

$$-q_n \geq -q_n^{max} \quad (4.9)$$

where n is defined as the reference bus or the slack bus, q is the nodal power injection quantity vector, $C_i(q_i)$ represents the net cost function or the net benefit function at bus i , 1 is a unit column vector with n entries and H is a $m \times n$ matrix that consists of a sub-matrix of power transfer distribution factors (PTDFs) that correspond to the transmission line power flow constraints and a sub-matrix of the power injection capacity constraints that correspond to the non-slack buses. Z is a column vector that consists of the capacity limits on the transmission line power flows and the power injections for the non-slack buses. Equation (4.5) represents the objective function. Equation (4.6) represents the global node balance constraint that specifies that the net power injection in the system must equal zero. Equation (4.7) includes the transmission line power flow constraints and the power injection capacity constraints for the non-slack buses. Equations (4.8) and (4.9) define the power injection capacity constraints at the slack bus n .

The transmission-constrained residual supply derivative is derived through a post-OPF analysis [28]. It is important to note that the derivation that follows considers only the active constraints from the DCOPF optimization problem presented by (4.5)-(4.9) because this is a post-OPF analysis. Thus, the optimization problem defined above transforms into a LP with only equality constraints and no sign restrictions on the signs of its dual variables. For the DCOPF formulation presented above, only one of the two power injection capacity constraints (either the minimum or the maximum) can be active (at the slack bus) at a given OPF solution. Thus, (4.10) represents the Lagrangian for the

case when (4.8) is active at the given OPF solution while (4.11) represents the Lagrangian for the case when (4.9) is active at the given OPF solution.

$$\mathcal{L} = \sum_{i=1}^n C_i(q_i) + \lambda(-\sum_{i=1}^n q_i) + \mu_b^T(Z_b - H_b q) + \alpha_n^{min}(q_n^{min} - q_n) \quad (4.10)$$

$$\mathcal{L} = \sum_{i=1}^n C_i(q_i) + \lambda(-\sum_{i=1}^n q_i) + \mu_b^T(Z_b - H_b q) + \alpha_n^{max}(-q_n^{max} + q_n) \quad (4.11)$$

The partial derivatives of the Lagrangian in (4.10) and (4.11) with respect to q_i , q_n , λ , μ_b^T , α_n^{min} , α_n^{max} are given by

$$\frac{\partial \mathcal{L}}{\partial q_i} = C'_i(q_i) - \lambda - \mu_b^T \bar{H}_{bi} = 0, \quad i = 1, \dots, (n-1) \quad (4.12)$$

The partial derivative of (4.10) with respect to q_n is given by,

$$\frac{\partial \mathcal{L}}{\partial q_n} = C'_n(q_n) - \lambda - \alpha_n^{min} = 0 \quad (4.13)$$

The partial derivative of (4.11) with respect to q_n is given by,

$$\frac{\partial \mathcal{L}}{\partial q_n} = C'_n(q_n) - \lambda + \alpha_n^{max} = 0 \quad (4.14)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -\sum_{i=1}^n q_i = 0 \quad (4.15)$$

Rearranging the terms in (4.15),

$$-q_n = \sum_{i=1}^{n-1} q_i \quad (4.16)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_b^T} = Z_b - \bar{H}_b \bar{q} = 0 \quad (4.17)$$

The partial derivative of (4.10) with respect to α_n^{min} is given by,

$$\frac{\partial \mathcal{L}}{\partial \alpha_n^{min}} = q_n^{min} - q_n = 0 \quad (4.18)$$

The partial derivative of (4.11) with respect to α_n^{max} is given by,

$$\frac{\partial \mathcal{L}}{\partial \alpha_n^{max}} = -q_n^{max} + q_n = 0 \quad (4.19)$$

Here, the binding constraints are denoted by the subscript "b", $\lambda, \mu, \alpha^{min}, \alpha^{max}$ are the Lagrange multipliers for constraints (4.6), (4.7), (4.8) and

(4.9) respectively, \bar{H}_b is a $m_b \times (n - 1)$ matrix obtained by eliminating the n^{th} column of matrix H_b . The n^{th} column of matrix H_b is eliminated because it is equivalent to a column vector with all zero entries. The n^{th} column of matrix H_b corresponds to the PTDFs for the active transmission line constraints with both injection and withdrawal at the reference node. \bar{H}_{bi} is the i^{th} column of \bar{H}_b , \bar{q} is a column vector generated by eliminating the n^{th} entry of q and Z_b is a column vector that consists of the capacity limits on the binding transmission line power flows and the binding power injections for the non-slack buses. Since the Lagrangian includes only the active or the binding constraints for the linear DCOPF problem presented in (4.5)-(4.9), (4.12)-(4.19) represent the necessary and sufficient conditions for optimality. The simultaneous solution of the necessary and sufficient conditions gives the optimal solution to the problem. The solution is denoted by

$$[\hat{q}_1 \dots \hat{q}_n \hat{\lambda} \hat{\mu}_{b1} \dots \hat{\mu}_{bm_b} \hat{\alpha}_n^{min} \hat{\alpha}_n^{max}]^T \quad (4.20)$$

The condition for nodal power balance for the residual market at bus n necessitates that the residual supply at bus n be equal to the demand at bus n . This condition was described in (4.4). Thus, the intersection point of the residual supply function and the demand bid function at bus n denotes the market clearing condition, $(\hat{\lambda}, \hat{q}_n)$, at bus n . If the demand at bus n was to change its bid function, then the residual market at bus n would operate at a new market clearing point that can be obtained from the interaction between the residual supply function and the modified demand bid function at bus n . Thus, the intersection points obtained from repeated interactions between the residual supply function

and the modified demand bid functions at the reference bus n gives nothing but the points on the residual supply curve at bus n . If the equations that consist of information on the demand bids at bus n are removed from the first order conditions, the remainder of the equations would implicitly characterize the residual supply function. Therefore, in order to obtain an implicit characterization of the residual supply function, (4.13) and (4.14), which depend on the benefit function of the demand located at the reference bus n , and (4.18) and (4.19), which specify the bounds on the demand bids, are removed from the first order conditions.

Sensitivity analysis is used to obtain the slope of the transmission-constrained residual supply curve. Let (4.12), (4.16), and (4.17) be parameterized by λ . By the implicit function theorem, the Lagrangian can be solved in the neighborhood of a point $\hat{\lambda}$ if (4.20) solves the Lagrangian and if the first order partial derivatives of the Lagrangian are continuous. Therefore, there exists a unique function

$$[\tilde{q}_1 \dots \tilde{q}_n \tilde{\lambda} \tilde{\mu}_{b1} \dots \tilde{\mu}_{bm_b} \tilde{\alpha}_n^{min} \tilde{\alpha}_n^{max}]^T \quad (4.21)$$

in the neighborhood of $\hat{\lambda}$ that solves (4.12), (2.16), and (4.17). Since the equations that consist of information on the demand bids placed at reference bus n have been removed from the first order conditions, the power injection q_n in (4.16) is representative of the residual supply function, R_n , at bus n and, hence,

$$R_n(\lambda) = \sum_{i=1}^{n-1} \tilde{q}_i(\lambda) \quad (4.22)$$

The slope of the residual supply function with respect to λ evaluated at $\hat{\lambda}$ is obtained using sensitivity analysis as follows:

$$\frac{dR_n(\hat{\lambda})}{d\lambda} = \sum_{i=1}^{n-1} \frac{d\tilde{q}_i(\hat{\lambda})}{d\lambda} \quad (4.23)$$

From (4.12) and (4.17),

$$C'_i(\tilde{q}_i(\lambda)) - \tilde{\mu}_b^T(\lambda)\bar{H}_{bi} = \lambda, \quad i = 1, \dots, (n-1) \quad (4.24)$$

$$\bar{H}_b \tilde{q}(\lambda) = Z_b \quad (4.25)$$

where $\tilde{q}(\lambda) = [\tilde{q}_1 \ \tilde{q}_2 \ \dots \ \tilde{q}_{n-1}]^T(\lambda)$. Differentiating (4.24) and (4.25) with respect to λ gives,

$$\begin{bmatrix} C''_1(\hat{q}_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C''_{n-1}(\hat{q}_{n-1}) \end{bmatrix} \frac{d\tilde{q}(\hat{\lambda})}{d\lambda} - \bar{H}_b^T \frac{d\tilde{\mu}_b(\hat{\lambda})}{d\lambda} = \bar{1} \quad (4.26)$$

$$\bar{H}_b \frac{d\tilde{q}(\hat{\lambda})}{d\lambda} = 0 \quad (4.27)$$

where $\bar{1}$ is a unit column vector with $(n-1)$ entries.

$$\text{Let } \Lambda = \begin{bmatrix} C''_1(\hat{q}_1) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & C''_{n-1}(\hat{q}_{n-1}) \end{bmatrix}^{-1}.$$

By solving (4.26) and (4.27),

$$\frac{d\tilde{\mu}_b(\hat{\lambda})}{d\lambda} = -(\bar{H}_b \Lambda \bar{H}_b^T)^{-1} \bar{H}_b \Lambda \bar{1} \quad (4.28)$$

$$\frac{d\tilde{q}(\hat{\lambda})}{d\lambda} = \Lambda \bar{1} - \Lambda \bar{H}_b^T [(\bar{H}_b \Lambda \bar{H}_b^T)^{-1} \bar{H}_b \Lambda \bar{1}] \quad (4.29)$$

Therefore, the slope of the transmission-constrained residual supply curve at bus n , evaluated at the market clearing point is given by

$$\frac{d\bar{R}_n(\hat{\lambda})}{d\lambda} = \bar{1}^T \Lambda \bar{1} - \bar{1}^T \Lambda \bar{H}_b^T [(\bar{H}_b \Lambda \bar{H}_b^T)^{-1} \bar{H}_b \Lambda \bar{1}] \quad (4.30)$$

Similarly, the slope of the transmission-constrained residual supply curve can be calculated at any arbitrary bus in the system by repeating the entire procedure with that particular bus as the reference bus. The slope of the transmission-

constrained residual supply curve is useful in deriving the inverse function of the residual supply curve, $P(q)$, which in turn is useful in the objective function of the distribution OPF. Consequently, the inverse function of the residual supply curve, $P(q)$, evaluated at the market clearing point, $(\hat{\lambda}, \hat{q}_n)$, can be expressed as

$$P(q) = \frac{1}{\frac{d\tilde{R}_n(\hat{\lambda})}{d\lambda}} (q - \hat{q}_n) + \hat{\lambda} \quad (4.31)$$

CHAPTER 5

RESULTS AND ANALYSIS

The integrated T&D systems model developed in Chapter 4 is used to accurately couple the T&D systems in this chapter. A network overview is provided in Section 5.1. The calculations are conducted for: (1) a traditional transmission system with no congestion and a traditional distribution system with PRL, radial topology and no congestion, (2) a transmission system with congestion and a traditional distribution system with PRL, radial topology and no congestion, (3) a transmission system with congestion and a traditional distribution system with PRL, radial topology and no congestion, and (4) a transmission system with congestion and an enhanced distribution system with PRL, meshed topology and congestion. In order to make a fair comparison with the proposed framework, a traditional transmission OPF was also solved without considering the distribution system details. The integrated T&D systems model proposed in Chapter 4 is also tested against an integrated T&D model wherein the two sub-systems are solved simultaneously in a single OPF as opposed to a two-stage optimization process.

5.1 Network Overview

The proposed iterative framework was tested using the Roy Billinton Test System (RBTS) [32]-[35] and the IEEE 118-bus test case, see UW (2012) [36]. Two case studies were conducted. In the first case study, as shown in Section 5.2, the algorithm was first implemented on the RBTS. The RBTS is a six bus test system with five load buses that represent the distribution system connected to the

transmission system, nine transmission lines, and eleven generating units. Four of the generators are located at bus 1 and the remaining seven are located at bus 2. The voltage of the transmission system is 230 kV. The total installed generation capacity is 240 MW and the peak load of the system is 185 MW. It has been assumed that the power factor at each bus is unity. Distribution system networks were incorporated at each of the load bus-bars down to a voltage level of 11 kV. The feeders in the distribution system are operated as radial feeders; however, in another instance, in order to create a meshed distribution system some of the normally opened switches were closed to obtain a meshed structure. A single-line diagram of the RBTS (transmission system) is shown in Fig. 5.1 and the test system data is listed in Table 5.1. It is important to note that, the original RBTS generator cost data was modified for the purpose of this study. The transmission system's branch data is provided in Table A.1 in Appendix A.

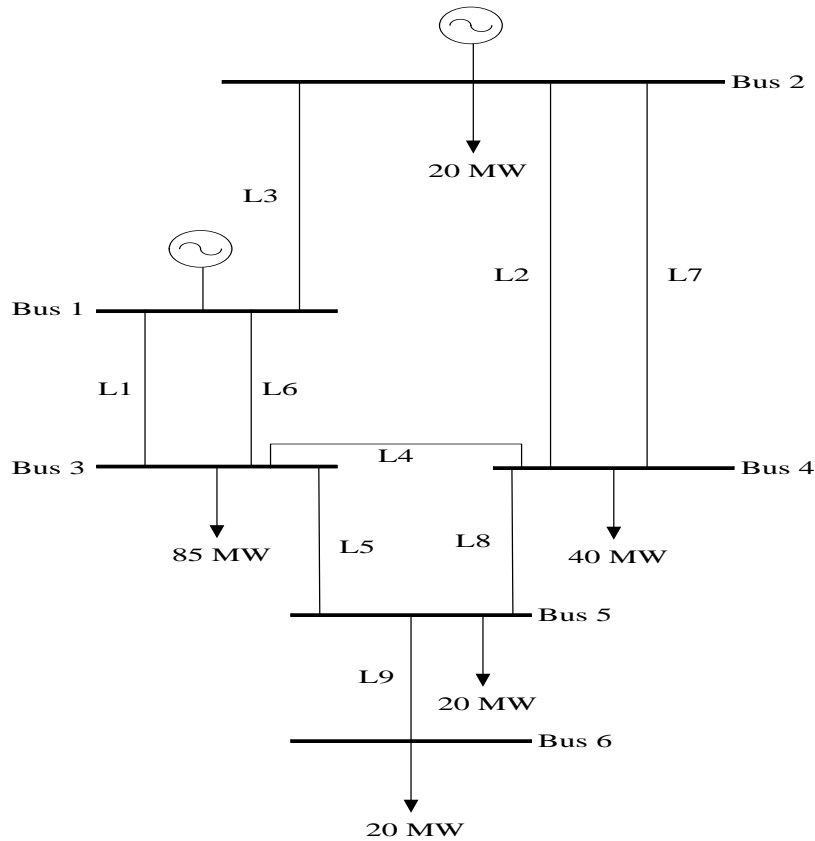


Fig. 5.1 RBTS Transmission System with Aggregate Representation of the Distribution System Networks

TABLE 5.1.
ROY BILLINTON TEST SYSTEM DATA

Bus	Generator Capacity (MW)	Load Specifications (MW)
1	$P_{g1}=40, P_{g2}=40, P_{g3}=10, P_{g4}=20$	-
2	$P_{g5}=5, P_{g6}=5, P_{g7}=40, P_{g8}=20,$ $P_{g9}=20, P_{g10}=20, P_{g11}=20$	$P_{d1} =20$ (Elastic)
3	-	$P_{d2} =85$ (Elastic)
4	-	$P_{d3} =40$ (Elastic)
5	-	$P_{d4} =20$ (Elastic)
6	-	$P_{d5} =20$ (Elastic)

In the second case study, as shown in Section 5.3, the IEEE 118-bus test case was used to represent the transmission system while the distribution networks at buses 2, 3, 4, 5, and 6 of the RBTS were used to represent the distribution system.

Distribution networks were incorporated at half of the load bus-bars in the IEEE 118-bus test case down to a voltage level of 11 kV. It is important to note that, the standard IEEE 118- bus test system was modified for the purpose of this study. The IEEE 118-bus system consists of 118 buses, 186 transmission elements, 19 generators with a total capacity of 5859 MW, and 99 load buses with a total load of 4519 MW. Buses 2, 3, 4, 5, and 6 of the RBTS consist of 22, 44, 38, 26, and 40 load buses with total peak loads of 20 MW, 85 MW, 40 MW, 20 MW, and 20 MW respectively. The distribution network at buses 2 and 4 of the RBTS represent a typical residential and small user distribution system, the distribution network at bus 5 of the RBTS represents a typical residential, government, and commercial user distribution system, the distribution network at bus 6 of the RBTS represents a typical agricultural, small industrial, commercial and residential user distribution system, and, the distribution network at bus 3 of the RBTS represents a typical industrial and large user distribution system [32]-[35]. A single line diagram of the IEEE 118-bus test case is shown in Fig. 5.2 and a summary of the generator data for the test system is presented in Table 5.2. Generator information from the reliability test system-1996 [36] and [37] was used to create the generator data for the test system. The branch details of the test system are listed in Table A.2 in Appendix A.

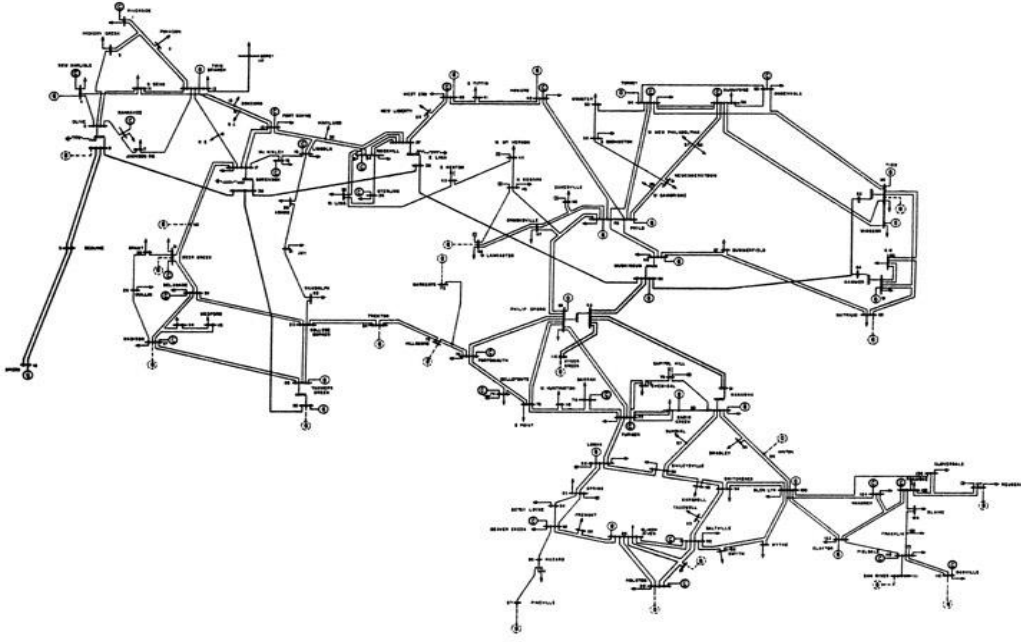


Fig. 5.2 Single Line Diagram of the IEEE 118-Bus Test System

TABLE 5.2.
GENERATOR DATA FOR THE IEEE-118 BUS TEST SYSTEM

Gen No.	Bus No.	Min. Output (MW)	Max. Output (MW)	A_i (\$/h)	B_i (\$/MWh)	C_i (\$/MW/MWh)
1	10	0	550	395.37	4.42	0.00021
2	12	0	185	832.76	48.58	0.00717
3	25	0	320	665.11	11.85	0.00490
4	26	0	414	395.37	4.42	0.00021
5	31	0	107	781.52	43.66	0.05267
6	46	0	119	781.52	43.66	0.05267
7	49	0	304	665.11	11.85	0.00490
8	54	0	148	382.24	12.39	0.00834
9	59	0	255	832.76	48.58	0.00717
10	61	0	260	832.76	48.58	0.00717
11	65	0	491	395.37	4.42	0.00021
12	66	0	492	395.37	4.42	0.00021
13	69	0	805	395.37	4.42	0.00021
14	80	0	577	395.37	4.42	0.00021
15	87	0	104	781.52	43.66	0.05267
16	92	0	100	781.52	43.66	0.05267
17	100	0	352	665.11	11.85	0.00490
18	103	0	140	382.24	12.39	0.00834
19	111	0	136	382.24	12.39	0.00834

Also, a complete single line diagram of the RBTS including both the T&D systems is shown in Fig. 5.3. The load details of the test distribution systems, buses 2, 3, 4, 5, and 6 of the RBTS are listed in Table. 5.3. As seen in Table 5.3, the load points are aggregates of multiple customers with similar service requirements: residential users, large industrial users, small industrial users, commercial users, government and institution users, farms, and office buildings. The primary feeders in the test distribution systems have section types that are listed in Table 5.4. The impedance and the peak loading data for each of the feeders in the test distribution systems are listed in Table 5.5. Each of the loads points in the distribution networks were modeled as PSR with assumed price elasticity of demand based on the technique presented in [6]. The model was written in AMPL and solved with CPLEX version 12.3.0.1.

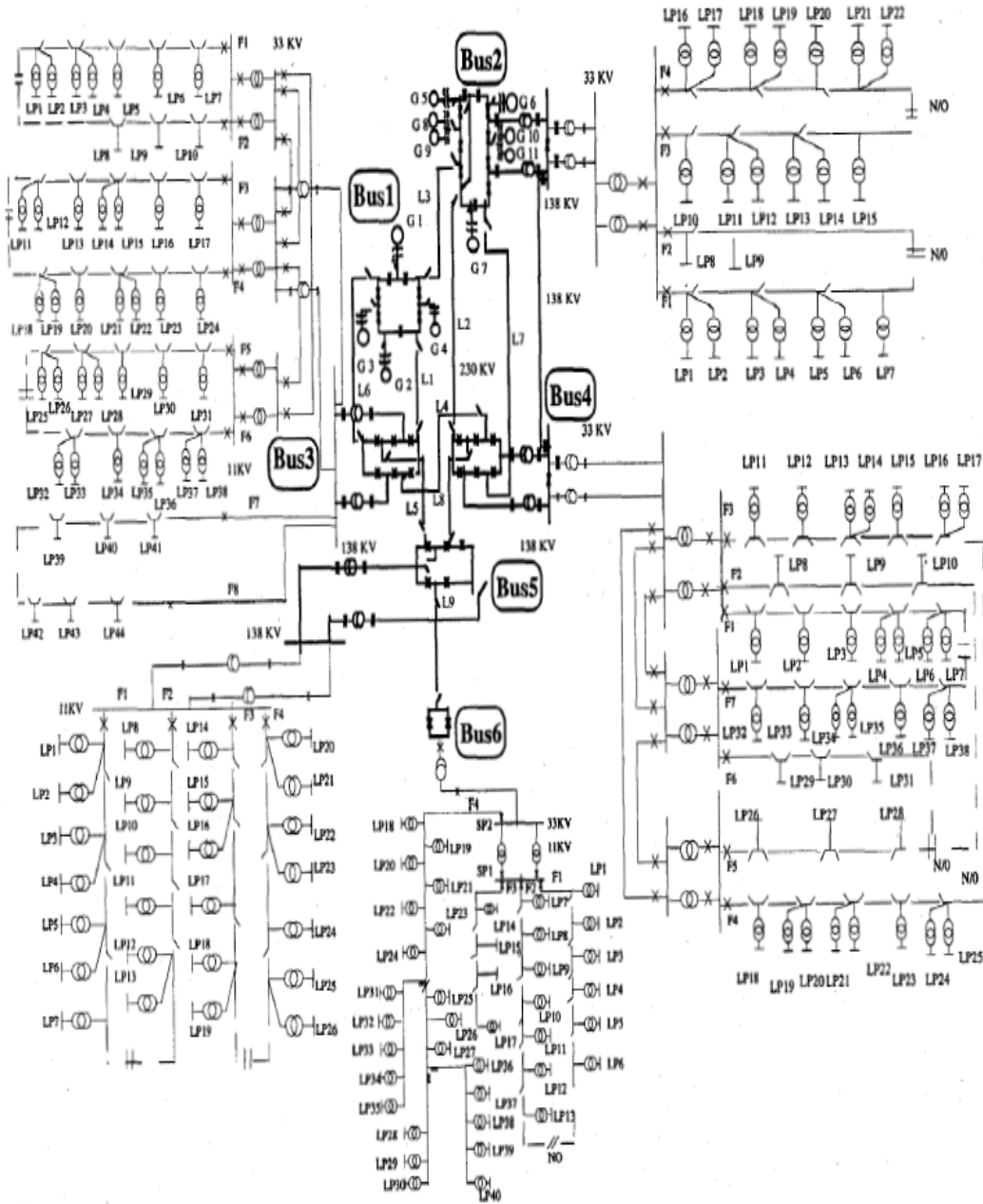


Fig. 5.3 Complete Single Line Diagram of the RBTS

TABLE 5.3.
LOAD DETAILS OF THE TEST DISTRIBUTION SYSTEMS

Bus	Customer Type	Peak Load (MW)	Load Points
2	Residential	0.8668	1-3, 10, 11
	Residential	0.7291	12, 17-19
	Small user	1.6279	8
	Small user	1.8721	9
	Govt/Inst.	0.9167	4, 5, 13, 14, 20, 21
	Commercial	0.7500	6, 7, 15, 16, 22
3	Residential	0.8367	1, 4-7, 20, 24, 32, 36
		0.8500	11, 12, 13, 18, 25
		0.7750	2, 15, 26, 30
	Large users	6.9167	39, 40, 44
		11.5833	41-43
	Small industrial	1.0167	8, 9, 10
	Commercial	0.5222	3, 16, 17, 19, 28, 29, 31, 37, 38
	Office buildings	0.9250	14, 27
4	Residential	0.8869	1-4, 11-13, 18-21, 32-35
	Residential	0.8137	5, 14, 15, 22, 23, 36, 37
	Small user	1.6300	8, 10, 26-30
	Small user	2.4450	9, 31
	Commercial	0.6714	6, 7, 16, 17, 24, 25, 38
5	Residential	0.7625	1-2, 20, 21
		0.7450	4, 6, 15, 25
		0.5740	26, 9-11, 13
	Govt/Inst.	1.1100	3, 5, 8, 17, 23
	Commercial	0.7400	7, 14, 18, 22, 24
	Office building	0.6167	12, 16, 19
6	Residential	0.3171	1, 3, 9
		0.3229	2, 4, 11, 19
		0.3864	5, 6
		0.2964	7, 8, 10, 18, 23
		0.3698	12, 13, 22
		0.2776	25, 28, 31, 36
		0.2831	27, 29, 33, 39
	Commercial	0.8500	14, 17
	Small	1.9637	15
		1.0830	16
	Farm	0.5025	32, 37
		0.6517	20, 30, 34
		0.6860	21, 35
		0.7965	24, 40
0.7375		26, 38	

TABLE 5.4.
FEEDER SECTION TYPES AND LENGTHS OF THE TEST DISTRIBUTION SYSTEMS

Bus	Section Type	Length (mi)	Section Number
2	1	0.3728	2, 6, 10, 14, 17, 21, 25, 28, 30, 34
	2	0.4660	1, 4, 7, 9, 12, 16, 19, 22, 24, 27, 29, 32, 35
	3	0.4971	3, 5, 8, 11, 13, 15, 18, 20, 23, 26, 31, 33, 36
3	1	0.3728	1, 2, 3, 7, 11, 12, 15, 21, 22, 29, 30, 31, 36, 40, 42, 43, 48, 49, 50, 56, 58, 61, 64, 67, 70, 72, 76
	2	0.4971	4, 8, 9, 13, 16, 19, 20, 25, 26, 32, 35, 37, 41, 46, 47, 51, 53, 57, 60, 62, 65, 68, 71, 75, 77
	3	0.5592	5, 6, 10, 14, 17, 18, 23, 24, 27, 28, 33, 34, 38, 39, 44, 45, 52, 54, 55, 59, 63, 66, 69, 73, 74
4	1	0.3728	2, 6, 10, 14, 17, 21, 25, 28, 30, 34, 38, 41, 43, 46, 49, 51, 55, 58, 61, 64, 67
	2	0.4660	1, 4, 7, 9, 12, 16, 19, 22, 24, 27, 29, 32, 35, 37, 40, 42, 45, 48, 50, 53, 56, 60, 63, 65
	3	0.4971	3, 5, 8, 11, 13, 15, 18, 20, 23, 26, 31, 33, 36, 39, 44, 47, 52, 54, 57, 59, 62, 66
5	1	0.3107	1, 6, 9, 13, 14, 18, 21, 25, 27, 31, 35, 36, 39, 42
	2	0.4039	4, 7, 8, 12, 15, 16, 19, 22, 26, 28, 30, 33, 37, 40
	3	0.4971	2, 3, 5, 10, 11, 17, 20, 23, 24, 29, 32, 34, 38, 41, 43
6	1	0.3728	2, 3, 8, 9, 12, 13, 17, 19, 20, 24, 25, 28, 31, 34, 41, 47
	2	0.4660	1, 5, 6, 7, 10, 14, 15, 22, 23, 26, 27, 30, 33, 43, 61
	3	0.4971	4, 11, 16, 18, 21, 29, 32, 35, 55
	4	0.5592	38, 44
	5	0.9942	37, 39, 42, 49, 54, 62
	6	1.5534	36, 40, 52, 57, 60
	7	1.7398	35, 46, 50, 56, 59, 64
	8	1.9884	45, 51, 53, 58, 63
	9	2.1748	48

TABLE 5.5.
FEEDER DETAILS FOR THE TEST DISTRIBUTION SYSTEMS

Bus	Voltage Level (kV)	Feeder	Peak Load (MW)	Length (mi)	R (Ω /mi)	X (Ω /mi)		
2	11	1	5.934	4.971	0.3071	0.6296		
		2	3.500	1.8330				
		3	5.057	4.4739				
		4	5.509	4.971				
3	11	1	5.4807	5.4057	0.3071	0.6296		
		2	3.0501	3.0446				
		3	5.2944	5.7164	0.1877	0.6001		
		4	5.5557					
		5	4.8916					
		6	5.2279	5.1572				
	138	7	25.4167	2.8582			0.5926	0.7628
		8	30.0833					
4	11	1	5.704	5.4370	0.3071	0.6296		
		2	5.705	2.7030				
		3	5.631	5.3127				
		4	6.518	5.8098				
		5	4.890	2.6719				
		6	5.705	2.6719				
		7	5.847	5.3438				
5	11	1	5.975	5.6292	0.3071	0.6296		
		2	4.0227	4.2564	0.5926	0.7628		
		3	4.5684	4.1321				
		4	5.434	4.4428	0.3071	0.6296		
6	11	1	2.0528	5.1884	0.5926	0.7628		
		2	2.2688	6.0583				
		3	4.7500	3.5107				
	33	4	10.9284	39.5813			0.1877	0.6001

5.2 Case Study One: Roy Billinton Test System

In this case study, the proposed algorithm was tested on the six-bus test system shown in Fig. 5.1. Buses 2, 3, 4, 5, and 6 represent the proxy nodes (interconnection points between the T&D systems). All the loads in the distribution systems are assumed to be price responsive. The proposed framework was also tested against an integrated T&D framework wherein the T&D systems

were modeled simultaneously in a single OPF. This was done to validate the authenticity of the results obtained from the proposed framework.

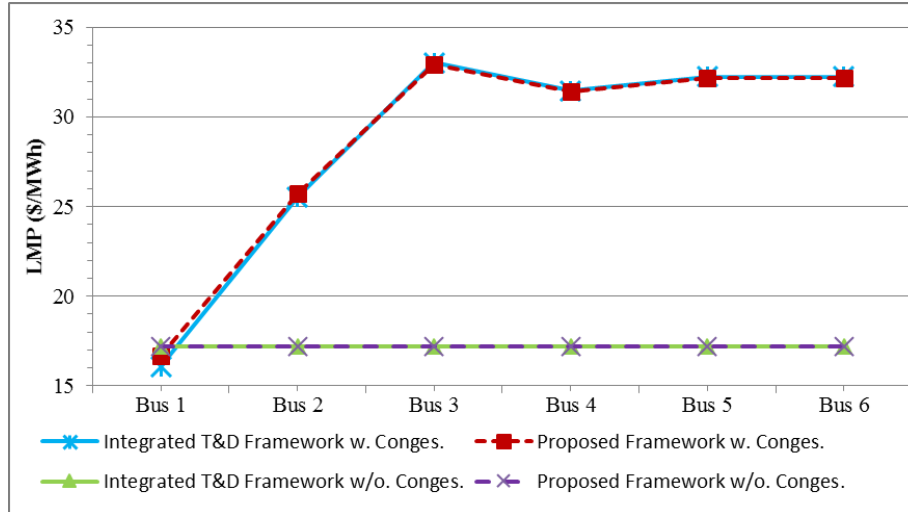


Fig. 5.4 Comparison of the LMPs Obtained from the Integrated T&D and the Proposed Iterative Frameworks

Two cases were studied. In the first case, the transmission system was modeled without modifying the original branch data. As seen earlier, the traditional DCOPF formulation does not account for the marginal loss component of the LMP. Also, for this result, there was no congestion within the transmission system, therefore, the resulting LMPs were uniform throughout the test system as evident in Fig. 5.4. In the second case, the line flow limits of lines one and six, which have the maximum flow in the transmission OPF, between buses one and three were arbitrarily reduced to create artificial congestion in the transmission system. It can be seen from Fig. 5.4 that congestion causes price separation in the transmission OPF. In this case, due to congestion, the power injection (positive) at bus one reduces, thereby forcing the expensive generator at bus two to pick up the extra demand in the system. Here, the total generation at bus one (cheaper)

reduced from 99.43 MW to 61.83 MW while the total generation at bus two (costlier) increased from 0 MW to 28 MW with congestion. Congestion also causes a reduction in the demand satisfied at bus three as evident from Fig. 5.5. Thus, it can be deduced that congestion causes a reduction in social welfare, which can also be verified from the results. The social welfare for the case when the transmission system was modeled without congestion was \$5356.68, whereas the social welfare for the case when the transmission system was modeled with congestion was \$5170.95. Also note that, the results obtained from the proposed iterative framework for both the cases of uncongested and congested transmission system are comparable (minimal difference) to the optimal solution obtained from the integrated T&D framework; such results indicate that the proposed framework is a practical approach that is able to obtain near-optimal solutions. Fig. 5.5 shows the results obtained, in terms of cleared quantities, from the proposed iterative framework and the integrated T&D framework for both the cases of an uncongested and a congested transmission system. While such results are not guaranteed as there can still be convergence problems, it shows that it is an improvement upon existing procedures that take a one shot approach, which do not obtain the true optimal solution.

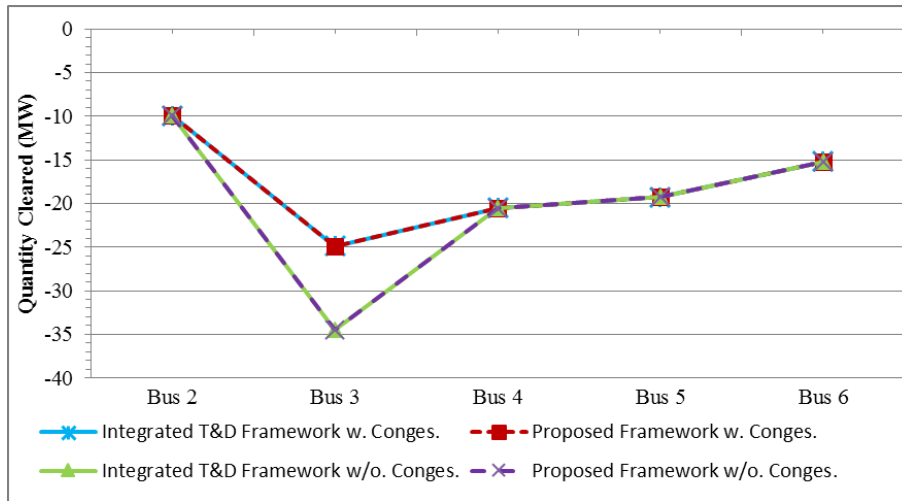


Fig. 5.5 Comparison of the Cleared Quantities (at the Proxy Buses) from the Integrated T&D and the Proposed Iterative Frameworks

5.3 Case Study Two: IEEE 118-bus Test Case and the RBTS

In this case study the IEEE-118 bus test system was used to represent the transmission system whereas the test distribution systems at buses 2, 3, 4, 5, and 6 of the RBTS were used to represent the distribution system. In order to make a fair comparison with the proposed framework, the conventional transmission OPF was solved without considering the distribution system details, wherein, the distribution system was represented by aggregate demand curves at the proxy buses. The aggregate demand curve in this case was obtained by simply aggregating the demand bids at various load points in the test distribution system. The proposed iterative framework was tested against both radial and meshed distribution networks to show the potential benefits from accurately modeling the T&D systems. It is important to note that the proposed framework does not always result in optimal coupling between the two sub-systems due to the approximations in the proposed curves. The technique used to obtain the approximate curves was merely done to generate an appropriate representation of

the system under consideration due to the unavailability of historical information. However, the proposed framework is also tested against an integrated T&D model wherein the two sub-systems are solved simultaneously in a single OPF as opposed to a two-stage optimization process. The integrated T&D framework will give the optimal solution because both the T&D systems are modeled simultaneously in one OPF framework. Ideally, the T&D systems should be modeled by the integrated T&D framework; however, this is computationally intractable with existing technologies. For the purpose of this study, the size of the T&D systems was restricted to ensure computational tractability of the integrated T&D framework. This was done to have a means to ensure that the results obtained from the proposed framework could be tested and verified in regards to the accuracy of the proposed approach. Here, the motive is not to prove optimality. For this research, the motivation is to develop an integrated T&D model that appropriately couples the two sub-systems together, improves systems operations (market efficiency), and improves the corresponding economic signals (prices) for the distribution system.

Table 5.6 lists information regarding the generators including the system-wide generation, generation cost, average generation cost, and generation revenue obtained with the following frameworks: 1) conventional transmission OPF neglecting distribution system details, 2) the proposed iterative framework with DLMP pricing and 3) the integrated T&D framework with both the T&D systems solved simultaneously.

TABLE 5.6.
COMPARISON OF GENERATOR INFORMATION

		Generation (MW)	Generation Cost (\$/h)	Average Generation Cost (\$/MWh)	Generation Revenue (\$/h)
Conventional Transmission OPF		4,261	28,502	6.69	59,590
Proposed T&D Framework	Radial Distribution System	3,833	21,509	5.61	48,974
	Meshed Distribution System	3,825	21,397	5.59	48,795
Integrated T&D Framework	Radial Distribution System	3,833	21,574	5.63	48,368
	Meshed Distribution System	3,825	21,462	5.61	48,190

It can be observed from Table 5.6 that the conventional transmission OPF has a generation cost of \$28,502/h, average generation cost of \$6.69/MWh, and generation revenue of \$59,590/h. Note that the generation cost and the generation revenue are the highest in this case. Conventional transmission OPF inaccurately models the distribution system by demand curves, which are obtained by simply aggregating the demand bids at various load points in the distribution system without considering the losses and congestion in the distribution system. Therefore, the resultant demand curves fail to reflect the true system conditions. The optimal solution obtained from the integrated T&D framework reduces generation to 3,833 MW, generation cost to \$21,574/h, average generation cost to \$5.63/MWh, and generation revenue to \$48,368/h in the case of radial distribution system. Note that the results obtained from the proposed framework for both the

cases of radial and meshed distribution system are comparable (minimal difference) to the optimal solution obtained from the integrated T&D framework; such results indicate that the proposed approach is a practical approach that is able to obtain near-optimal solutions without the computational burden that would be required by a simultaneous T&D model.

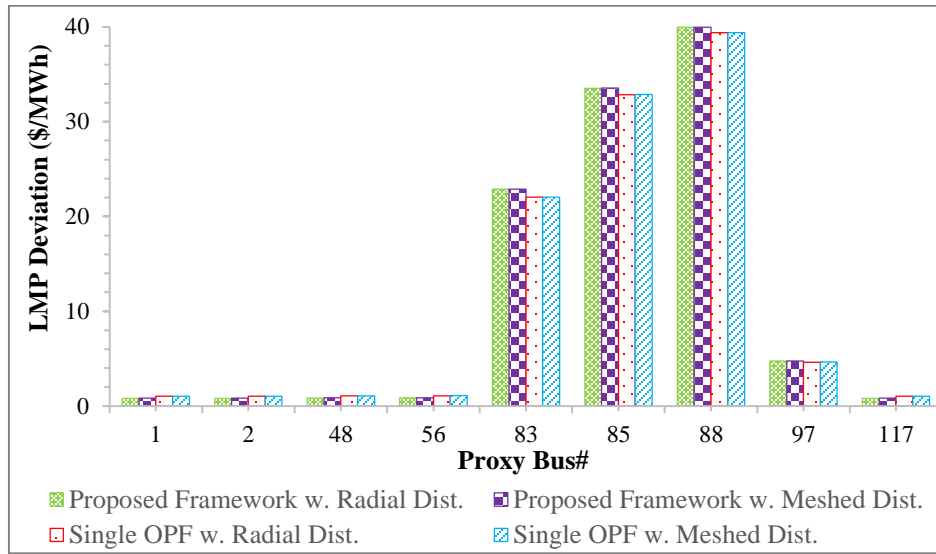


Fig. 5.6 Deviations in LMPs Comparing the Integrated T&D and the Proposed Iterative Framework Solutions to the Solutions Obtained from the Conventional Transmission OPF

Fig. 5.6 presents the change in LMPs (at selected proxy buses) and Fig. 5.7 presents the deviations in net demands (from the distribution system connected to selected proxy buses) comparing the integrated T&D and the proposed iterative frameworks to the conventional transmission OPF. It can be seen that, all the proxy buses saw a decrease in LMPs and loads. The largest decrement in LMP for a proxy bus was \$39.97/MWh. The largest decrement in the net demand from a distribution system connected to a proxy bus was 15.26 MW. The proposed iterative framework gave almost the same deviations in LMPs and loads as the

integrated T&D framework. Another interesting result is the load deviation observed at proxy buses 97 and 117. It is important to note that, for simulations with meshed distribution system, proxy buses 83, 97, 101, 115, and 117 were considered to have meshed distribution networks (with congestion), while the remaining proxy buses had a radial distribution system. The load deviations observed at proxy bus 97 for the case of radial and meshed distribution system were 9.19 MW and 11.33 MW respectively. This suggests that distribution system consumption was less (by 2.14 MW) in the case of meshed distribution system when compared to the case of radial distribution system due to congestion. This example demonstrates that the DLMPs can be used as an efficient pricing tool for congestion management and pricing in distribution operations. Likewise, there is a significant load deviation at proxy bus 117. However, this effect was not observed at proxy bus 83 because the distribution system connected at this location did not have sufficient congestion to cause a considerable separation in DLMPs.

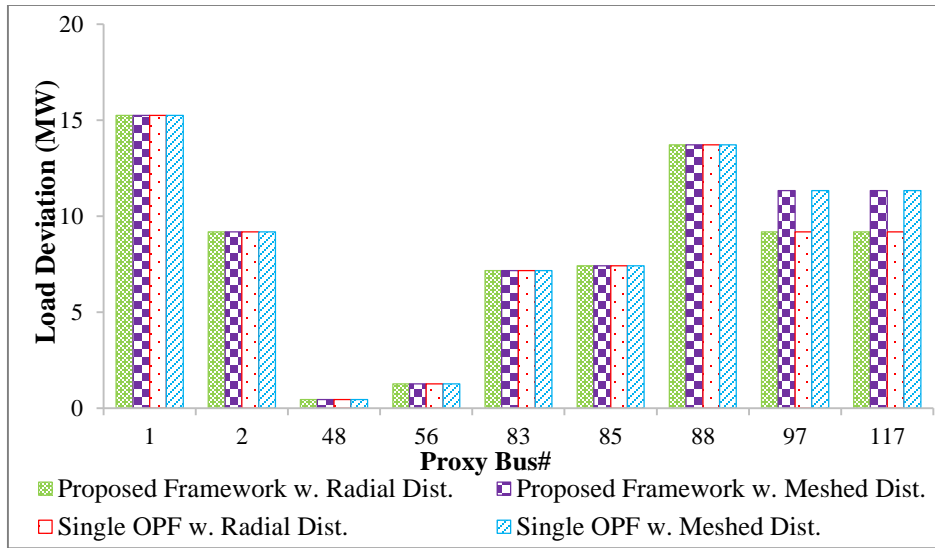


Fig. 5.7 Deviations in Net Demands Comparing the Integrated T&D and the Proposed Iterative Framework Solutions to the Solutions Obtained from the Conventional Transmission OPF

Therefore, with the proposed framework the PSR in the distribution system can be controlled to behave in a manner that benefits the power system as a whole.

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

A mathematical optimization problem for integrating the T&D systems models through a two-stage optimization process is developed in this thesis. The model improves upon the corresponding economic signals (prices) for the distribution system apart from ensuring accurate representation of the transmission system in the distribution OPF and vice-versa. The economic price signals are referred to as the DLMP in this thesis. The proposed DLMP is an extension of the LMP concept in the transmission system to the distribution system and has similar properties to the LMP. The objective of this thesis is to accurately couple the T&D systems via DLMP in order to improve market efficiency as well as enable DSR to provide ancillary services to facilitate renewable integration.

In order to integrate the two sub-systems, a transmission-constrained residual supply and an aggregate residual demand curve has been proposed and derived. The transmission-constrained residual supply curve avoids full network representation of the transmission system in the distribution OPF while the aggregate residual demand curve avoids full network representation of the distribution system in the transmission OPF, thereby overcoming the computational limitations with modeling both the T&D systems in a single OPF.

The proposed algorithm was tested on the RBTS and a combination of the IEEE 118-bus test system and the RBTS. For these specific test cases, the

proposed iterative framework achieved near-optimal coupling between the two sub-systems by ensuring that the PRL and DSR are appropriately represented in the transmission OPF through residual demand curves, which are updated based on the utilization of a DLMP pricing structure. However, the motivation is not to prove optimality. The motivation is to develop an integrated T&D model that appropriately couples the two sub-systems together, improves system operations, and improves the corresponding price signals for the distribution system.

The performance of the proposed iterative framework is compared against the conventional methods of solving the transmission OPF. The proposed framework is also tested against an integrated T&D model wherein the two sub-systems are solved simultaneously in a single OPF as opposed to a two-stage optimization process. The comparison results demonstrate that the proposed framework is an improvement upon contemporary methods of solving the transmission OPF. It is important to note that, the results obtained from the proposed framework are comparable to the optimal solution obtained from the single OPF. Such results show that the proposed technique is a practical technique that is able to produce near-optimal solutions without the computational burden that would be required by a single T&D OPF. However, due to non-convexities resulting from the staircase aggregate residual demand curve and the transmission-constrained residual supply curve, there is no guarantee that the proposed iterative framework will converge or converge to a globally optimal solution.

The results show that the proposed framework is successful in extracting the flexibility of the DSR to benefit transmission system operations, appropriately

reflecting the interaction between the T&D systems, incentivizing optimal DSR decisions, and improving market efficiency and system reliability. The proposed framework is further used to implement the concepts of spot pricing in the distribution system via DLMPs, thereby reflecting the true costs to delivery electrical energy to the distribution system. Simulations also show that, with increased flexible resources and congestion in the distribution system the advantages of the DLMP will be more prominent. It is evident that the use of contemporary pricing schemes result in sub-optimal behavior of the PRL.

6.2 Future Work

While this thesis studied the effect of price responsive loads (in the distribution system) on the transmission system, there is also a need to study the impact of renewable integration, load curtailments schemes, and distributed generation with the proposed iterative framework. Another aspect to be studied is the impact of the DLMP on transmission system operations and congestion management. Part of the benefit of nodal pricing in the distribution system is to enable DSR to provide ancillary services to facilitate renewable integration; these benefits should be studied and demonstrated.

The proposed iterative framework must be tested on larger sized test systems to further investigate its convergence issues. The solution may include a more appropriate representation of the distribution system in the transmission OPF and vice-versa. It is also important to verify the suitability of the lossy DCOPF formulation for the distribution system in the two-stage optimization model. A more accurate OPF for the distribution system can be used instead. Also, there is

scope for further investigation on unbalanced distribution operation.

Although, this thesis was focused on developing a model to accurately reflect the interactions between the T&D systems, the proposed model is transportable to deal with the “seams issue” of accurately modeling the interactions between neighboring transmission systems or control area boundaries. Here, the seams issue refers to a barrier or inefficiency resulting from either: 1) differences in market rules and designs, or 2) practices that inhibit the ability to trade energy products across neighboring wholesale electricity markets economically [38]. In other words, the proposed model could be extended to reflect the interactions between neighboring transmission systems.

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APPENDIX A

SIMULATION DATA AND RESULTS DETAILS

Appendix A. Simulation Data and Results Details

TABLE A.1.
BRANCH DATA FOR THE RBTS TRANSMISSION SYSTEM ON A 100 MVA BASE

Branch No.	From Bus	To Bus	Length (mi)	R (p.u.)	X (p.u.)	B/2 (p.u.)	Current Rating (p.u.)
1	1	3	46.60	0.0342	0.18	0.0106	0.85
2	2	4	155.34	0.1140	0.60	0.0352	0.71
3	1	2	124.27	0.0912	0.48	0.0282	0.71
4	3	4	31.07	0.0228	0.12	0.0071	0.71
5	3	5	31.07	0.0228	0.12	0.0071	0.71
6	1	3	46.60	0.0342	0.18	0.0106	0.85
7	2	4	155.34	0.1140	0.60	0.0352	0.71
8	4	5	31.07	0.0228	0.12	0.0071	0.71
9	5	6	31.07	0.0228	0.12	0.0071	0.71

TABLE A.2.
IEEE 118-BUS TEST SYSTEM BRANCH DATA ON A 100 MVA BASE

Branch No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)	Line Limit (p.u.)
1	1	2	0.0303	0.0999	-10.010	220
2	1	3	0.0129	0.0424	-23.585	220
3	2	12	0.0187	0.0616	-16.234	220
4	3	5	0.0241	0.1080	-9.259	220
5	3	12	0.0484	0.1600	-6.250	220
6	4	5	0.0018	0.0080	-125.000	440
7	4	11	0.0209	0.0688	-14.535	220
8	5	6	0.0119	0.0540	-18.519	220
9	5	11	0.0203	0.0682	-14.663	220
10	6	7	0.0046	0.0208	-48.077	220
11	7	12	0.0086	0.0340	-29.412	220
12	8	5	0.0024	0.0267	-37.453	880
13	8	9	0.0000	0.0305	-32.787	1100
14	8	30	0.0043	0.0504	-19.841	220
15	9	10	0.0026	0.0322	-31.056	1100
16	11	12	0.0060	0.0196	-51.020	220
17	11	13	0.0223	0.0731	-13.680	220
18	12	15	0.0215	0.0707	-14.144	220
19	12	17	0.0212	0.0834	-11.990	220

Branch No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)	Line Limit (p.u.)
20	12	117	0.0329	0.1400	-7.143	220
21	13	15	0.0744	0.2444	-4.092	220
22	14	15	0.0595	0.1950	-5.128	220
23	15	17	0.0132	0.0437	-22.883	440
24	15	19	0.0120	0.0394	-25.381	220
25	15	33	0.0380	0.1244	-8.039	220
26	16	17	0.0454	0.1801	-5.552	220
27	17	19	0.0123	0.0505	-19.802	220
28	17	31	0.0474	0.1563	-6.398	220
29	17	113	0.0091	0.0301	-33.223	220
30	18	19	0.0112	0.0493	-20.284	220
31	19	20	0.0252	0.1170	-8.547	220
32	19	34	0.0752	0.2470	-4.049	220
33	20	21	0.0183	0.0849	-11.779	220
34	21	22	0.0209	0.0970	-10.309	220
35	22	23	0.0342	0.1590	-6.289	220
36	23	24	0.0135	0.0492	-20.325	220
37	23	25	0.0156	0.0800	-12.500	440
38	23	32	0.0317	0.1153	-8.673	220
39	24	70	0.0022	0.4115	-2.430	220
40	24	72	0.0488	0.1960	-5.102	220
41	25	27	0.0318	0.1630	-6.135	440
42	26	25	0.0000	0.0382	-26.178	220
43	26	30	0.0080	0.0860	-11.628	660
44	27	28	0.0191	0.0855	-11.696	220
45	27	32	0.0229	0.0755	-13.245	220
46	27	115	0.0164	0.0741	-13.495	220
47	28	31	0.0237	0.0943	-10.604	220
48	28	32	0.0108	0.1261	-7.930	220
49	29	31	0.0000	0.0331	-30.211	220
50	30	17	0.0046	0.0388	-25.773	660
51	30	38	0.0298	0.0540	-18.519	220
52	31	32	0.0615	0.0985	-10.152	220
53	32	113	0.0135	0.2030	-4.926	220
54	32	114	0.0415	0.0612	-16.340	220
55	33	37	0.0087	0.1420	-7.042	220
56	34	36	0.0026	0.0268	-37.313	220
57	34	37	0.0413	0.0094	-106.383	440
58	34	43	0.0022	0.1681	-5.949	220
59	35	36	0.0110	0.0102	-98.039	220
60	35	37	0.0321	0.0497	-20.121	220
61	37	39	0.0593	0.1060	-9.434	220
62	37	40	0.0000	0.1680	-5.952	220

Branch No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)	Line Limit (p.u.)
63	38	37	0.0090	0.0375	-26.667	660
64	38	65	0.0184	0.0986	-10.142	440
65	39	40	0.0145	0.0605	-16.529	220
66	40	41	0.0555	0.0487	-20.534	220
67	40	42	0.0410	0.1830	-5.464	220
68	41	42	0.0715	0.1350	-7.407	220
69	42	49	0.0715	0.3230	-3.096	220
70	42	49	0.0608	0.3230	-3.096	220
71	43	44	0.0224	0.2454	-4.075	220
72	44	45	0.0400	0.0901	-11.099	220
73	45	46	0.0684	0.1356	-7.375	220
74	45	49	0.0380	0.1860	-5.376	220
75	46	47	0.0601	0.1270	-7.874	220
76	46	48	0.0191	0.1890	-5.291	220
77	47	49	0.0844	0.0625	-16.000	220
78	47	69	0.0179	0.2778	-3.600	220
79	48	49	0.0267	0.0505	-19.802	220
80	49	50	0.0486	0.0752	-13.298	220
81	49	51	0.0730	0.1370	-7.299	220
82	49	54	0.0869	0.2890	-3.460	220
83	49	54	0.0180	0.2910	-3.436	220
84	49	66	0.0180	0.0919	-10.881	440
85	49	66	0.0985	0.0919	-10.881	440
86	49	69	0.0474	0.3240	-3.086	220
87	50	57	0.0203	0.1340	-7.463	220
88	51	52	0.0255	0.0588	-17.007	220
89	51	58	0.0405	0.0719	-13.908	220
90	52	53	0.0263	0.1635	-6.116	220
91	53	54	0.0169	0.1220	-8.197	220
92	54	55	0.0028	0.0707	-14.144	220
93	54	56	0.0503	0.0096	-104.167	220
94	54	59	0.0049	0.2293	-4.361	220
95	55	56	0.0474	0.0151	-66.225	220
96	55	59	0.0343	0.2158	-4.634	220
97	56	57	0.0343	0.0966	-10.352	220
98	56	58	0.0825	0.0966	-10.352	220
99	56	59	0.0803	0.2510	-3.984	220
100	56	59	0.0317	0.2390	-4.184	220
101	59	60	0.0328	0.1450	-6.897	220
102	59	61	0.0026	0.1500	-6.667	220
103	60	61	0.0123	0.0135	-74.074	440
104	60	62	0.0082	0.0561	-17.825	220
105	61	62	0.0482	0.0376	-26.596	220

Branch No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)	Line Limit (p.u.)
106	62	66	0.0258	0.2180	-4.587	220
107	62	67	0.0000	0.1170	-8.547	220
108	63	59	0.0017	0.0386	-25.907	440
109	63	64	0.0000	0.0200	-50.000	440
110	64	61	0.0027	0.0268	-37.313	220
111	64	65	0.0000	0.0302	-33.113	440
112	65	66	0.0014	0.0370	-27.027	220
113	65	68	0.0224	0.0160	-62.500	220
114	66	67	0.0000	0.1015	-9.852	220
115	68	69	0.0018	0.0370	-27.027	440
116	68	81	0.0003	0.0202	-49.505	220
117	68	116	0.0300	0.0040	-250.000	440
118	69	70	0.0405	0.1270	-7.874	440
119	69	75	0.0309	0.1220	-8.197	440
120	69	77	0.0088	0.1410	-7.092	220
121	70	71	0.0401	0.0355	-28.169	220
122	70	74	0.0428	0.1323	-7.559	220
123	70	75	0.0446	0.1410	-7.092	220
124	71	72	0.0087	0.1800	-5.556	220
125	71	73	0.0123	0.0454	-22.026	220
126	74	75	0.0601	0.0406	-24.631	220
127	75	77	0.0145	0.1270	-7.874	220
128	75	118	0.0444	0.0481	-20.790	220
129	76	77	0.0164	0.1480	-6.757	220
130	76	118	0.0038	0.0544	-18.382	220
131	77	78	0.0170	0.0124	-80.645	220
132	77	80	0.0294	0.0485	-20.619	440
133	77	80	0.0298	0.1050	-9.524	220
134	77	82	0.0055	0.0853	-11.723	220
135	78	79	0.0156	0.0244	-40.984	220
136	79	80	0.0356	0.0704	-14.205	220
137	80	96	0.0183	0.1820	-5.495	220
138	80	97	0.0238	0.0934	-10.707	220
139	80	98	0.0454	0.1080	-9.259	220
140	80	99	0.0000	0.2060	-4.854	220
141	81	80	0.0112	0.0370	-27.027	220
142	82	83	0.0162	0.0367	-27.248	220
143	82	96	0.0625	0.0530	-18.868	220
144	83	84	0.0430	0.1320	-7.576	220
145	83	85	0.0302	0.1480	-6.757	220
146	84	85	0.0350	0.0641	-15.601	220
147	85	86	0.0200	0.1230	-8.130	220
148	85	88	0.0239	0.1020	-9.804	220

Branch No.	From Bus	To Bus	R (p.u.)	X (p.u.)	B (p.u.)	Line Limit (p.u.)
149	85	89	0.0283	0.1730	-5.780	220
150	86	87	0.0139	0.2074	-4.822	220
151	88	89	0.0518	0.0712	-14.045	440
152	89	90	0.0238	0.0320	-31.250	660
153	89	91	0.0099	0.0320	-31.250	220
154	89	92	0.0393	0.0505	-19.802	220
155	90	91	0.0254	0.0505	-19.802	660
156	91	92	0.0387	0.1272	-7.862	220
157	92	93	0.0258	0.0320	-31.250	220
158	92	94	0.0481	0.1580	-6.329	220
159	92	100	0.0648	0.2950	-3.390	220
160	92	102	0.0123	0.0559	-17.889	220
161	93	94	0.0223	0.0732	-13.661	220
162	94	95	0.0132	0.0434	-23.041	220
163	94	96	0.0269	0.0869	-11.507	220
164	94	100	0.0178	0.0580	-17.241	220
165	95	96	0.0171	0.0547	-18.282	220
166	96	97	0.0173	0.0885	-11.299	220
167	98	100	0.0397	0.1790	-5.587	220
168	99	100	0.0180	0.0813	-12.300	220
169	100	101	0.0277	0.1262	-7.924	220
170	100	103	0.0160	0.0525	-19.048	440
171	100	104	0.0451	0.2040	-4.902	220
172	100	106	0.0605	0.2290	-4.367	220
173	101	102	0.0246	0.1120	-8.929	220
174	103	104	0.0466	0.1584	-6.313	220
175	103	105	0.0535	0.1625	-6.154	220
176	103	110	0.0391	0.1813	-5.516	220
177	104	105	0.0099	0.0378	-26.455	220
178	105	106	0.0140	0.0547	-18.282	220
179	105	107	0.0530	0.1830	-5.464	220
180	105	108	0.0261	0.0703	-14.225	220
181	106	107	0.0530	0.1830	-5.464	220
182	108	109	0.0105	0.0288	-34.722	220
183	109	110	0.0278	0.0762	-13.123	220
184	110	111	0.0220	0.0755	-13.245	220
185	110	112	0.0247	0.0640	-15.625	220
186	114	115	0.0023	0.0104	-96.154	220