Preliminary Structural Design Optimization of Tall Buildings Using GS-USA© Frame3D

by

Mamatha Sirigiri

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Graduate Supervisory Committee:

Subramaniam Rajan, Chair
Narayanan Neithalath
Barzin Mobasher

ARIZONA STATE UNIVERSITY

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ABSTRACT

Tall buildings are spreading across the globe at an ever-increasing rate (www.ctbuh.org). The global number of buildings 200m or more in height has risen from 286 to 602 in the last decade alone. The increasing complexity of building architecture poses unique challenges in the structural design of modern tall buildings. Hence, innovative structural systems need to be evaluated to create an economical design that satisfies multiple design criteria. Design using traditional trial-and-error approach can be extremely time-consuming and the resultant design uneconomical. Thus, there is a need for an efficient numerical optimization tool that can explore and generate several design alternatives in the preliminary design phase which can lead to a more desirable final design.

In this study, we present the details of a tool that can be very useful in preliminary design optimization - finite element modeling, design optimization, translating design code requirements into components of the FE and design optimization models, and pre-and post-processing to verify the veracity of the model. Emphasis is placed on development and deployment of various FE models (static, modal and dynamic analyses; linear, beam and plate/shell finite elements), design optimization problem formulation (sizing, shape, topology and material selection optimization) and numerical optimization tools (gradient-based and evolutionary optimization methods) [Rajan, 2001]. The design optimization results of full scale three dimensional buildings subject to multiple design criteria including stress, serviceability and dynamic response are discussed.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor, Dr. Rajan for his continuous support and help throughout my studies. The mentoring, encouragement and supervision of my advisor allowed me to learn the conceptual aspects of research and develop invaluable skills. I would also like to extend my gratitude to Dr. Neithalath and Dr. Mobasher for reviewing my research work.

I thank my family and friends for all their support and motivation. I would like to acknowledge all my classmates and colleagues at ASU for their support and assistance.

I would like to dedicate this thesis to my parents Malleswari and S.M.Reddy.

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LIST OF SYMBOLS

ABBREVIATIONS

GA	Genetic algorithm
MFD	Method of feasible directions
DO	Design optimization
DDV	Discrete design variables
CDV	Continuous design variables
BDV	Boolean design variables
FEA	Finite element analysis

NOMENCLATURE

$\delta_{\max}^{L}$	-Maximum nodal displacement in the longitudinal direction (in)
$\delta_{\max}^{T}$	-Maximum nodal displacement in the transverse direction (in)
$\delta_{\max}^{V}$	-Maximum nodal displacement in the vertical direction (in)
δ_{a}^{L}	-Allowable nodal displacement in the longitudinal direction (in)
δ_{a}^{T}	-Allowable nodal displacement in the transverse direction (in)
δ_{a}^{V}	-Allowable nodal displacement in the vertical direction (in)
f_{lowest}	-Lowest frequency of the structure (Hz)
h_w	-Web height of the wide flange section (in)
b_f	-Flange width of the wide flange section (in)
t_w	-Web thickness of the wide flange section (in)
t_f	-Flange thickness of the wide flange section (in)
$\sigma_{\max,i}^{t}$	-Maximum tensile stress in the i^{th} member (psi)
$\sigma_{\max,i}^{c}$	-Maximum compressive stress in the i^{th} member (psi)
$\sigma_{\max,i}^{s}$	-Maximum shear stress in the i^{th} member (psi)
σ_{a}^{t}	-Allowable tensile stress (psi)
σ_{a}^{c}	-Allowable compressive stress (psi)
σ_{a}^{s}	-Allowable shear stress (psi)
f_y	- Yield stress in steel (psi)
$\mathbf{x}$	-Vector of design variables
$\mathbf{S_n}$	-Vector of 267 wide flange sections from AISC steel database.

A -Cross sectional area

J -Torsional constant

S_y -Section modulus about the local y axis

S_z -Section modulus about the local z axis

T_J -"Torsional factor" for computing the shear stress due to torsional moment

SF_y -"Shear factor" for computing shear stress (shear force acting in the local y direction)

SF_z -"Shear factor" for computing shear stress (shear force acting in the local z direction)

N_x -Axial force in the local x direction

V_y -Shear force in the local y direction

V_z -Shear force in the local z direction

M_y -Moment about the local y axis

M_z -Moment about the local z axis

T_x -Torsional moment about the local x axis

1 INTRODUCTION

1.1 Tall Buildings

Tall buildings are spreading across the globe at an ever-increasing rate. The global number of buildings 200m or more in height has risen from 286 to 602 in the last decade alone. The recent increase in the number of tall buildings has been fueled by a large variety of local and global motivations and therefore cannot be directly related to any single factor. Rapid economic growth in cities, scarcity of land and increasing population density demanded the need for tall buildings.

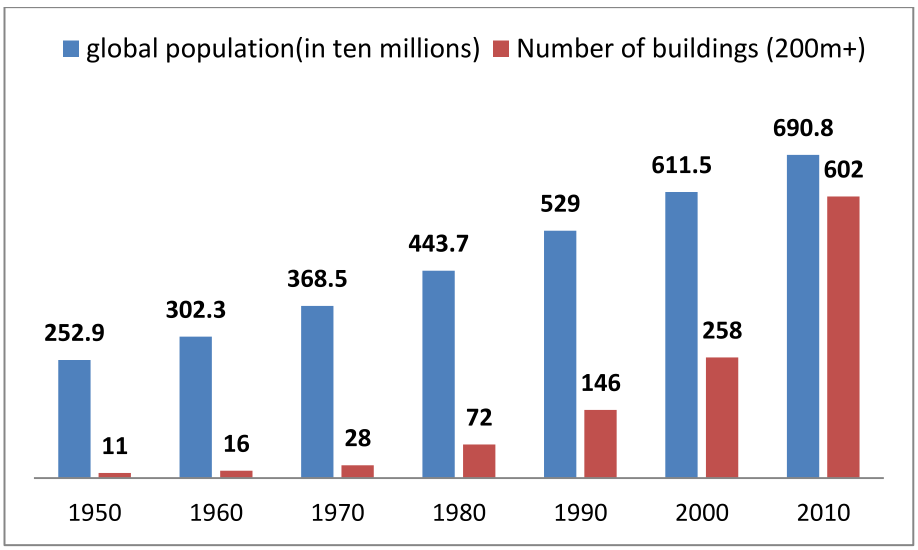


Figure 1-1 Global population and increase in number of buildings 200+m height, 1950-2011(Data source: http://www.ctbuh.org)

A building can be attributed as "tall" if it contains the specific vertical transport technologies, structural wind bracings etc. [http://www.ctbuh.org]. The number of floors in a building is a poor indicator of defining a tall building due to the changing floor to floor height between differing buildings and functions (e.g., office versus residential usage), a building of 14 or more stories or over 50 meters (165 feet) in height can be used as a threshold for considering it a “tall building”.

1.2 Structural Systems in Steel Buildings

Structural systems for tall buildings have undergone a dramatic evolution throughout the previous two decades (Bungale, 2012). Initially, the role of steel members in a structure was to carry the gravity loads. Gradually, it was enhanced to include wind and seismic resistance using innovative systems. Today there are various lateral load resisting systems to choose for a tall building depending on the height of the building.

i. Rigid Frame

ii. Concentrically braced frame

iii. Eccentric braced frame

iv. Outrigger and belt truss system

v. Framed tube

A rigid frame is a vertical support system that also serves as a lateral load resisting system. It derives its unique strength to resist lateral loads from the moment interaction between beams and columns. A rigid frame is efficient for buildings that have less than 20 stories. A braced frame improves upon the efficiency of a rigid frame by eliminating the bending of columns and girders. The diagonal bracing members carry the lateral shear predominantly by axial forces and thus minimize the bending of beams and columns. A braced frame can be concentric or eccentric with respect to the beams and columns at the intersection joint. In concentric braced frames the member forces in bracing members are axial without significant moments. In eccentric braced frames, the axis is at an offset to deliberately introduce flexure and shear in beam segments to increase ductility.

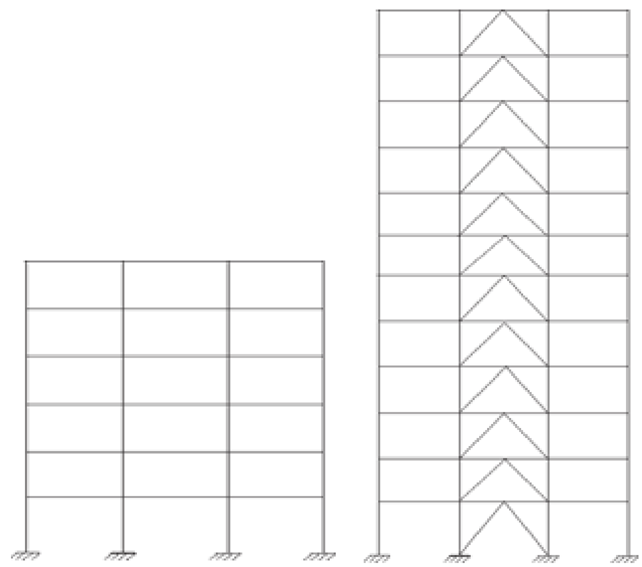


Figure 1-2 Rigid frames and braced frames

Modern high-rise buildings always incorporate central elevator core. The core is very effective in resisting the lateral forces and shear deformations but it is ineffective in resisting the overturning component of the drift. The core and out rigger system's unique feature is to invoke the axial stiffness of the perimeter columns for increasing the resistance of overturning moments. Structural form consists of central core with horizontal outrigger trusses connecting the core to the outer columns. Under lateral loads the column restrained outriggers resist the rotation of core causing smaller lateral deflections of the building than if the free standing core alone resisted the loading. The result is to increase the effective depth of the structure when it flexes as a vertical cantilever by inducing tension in the wind ward columns and compression in the lee ward columns.

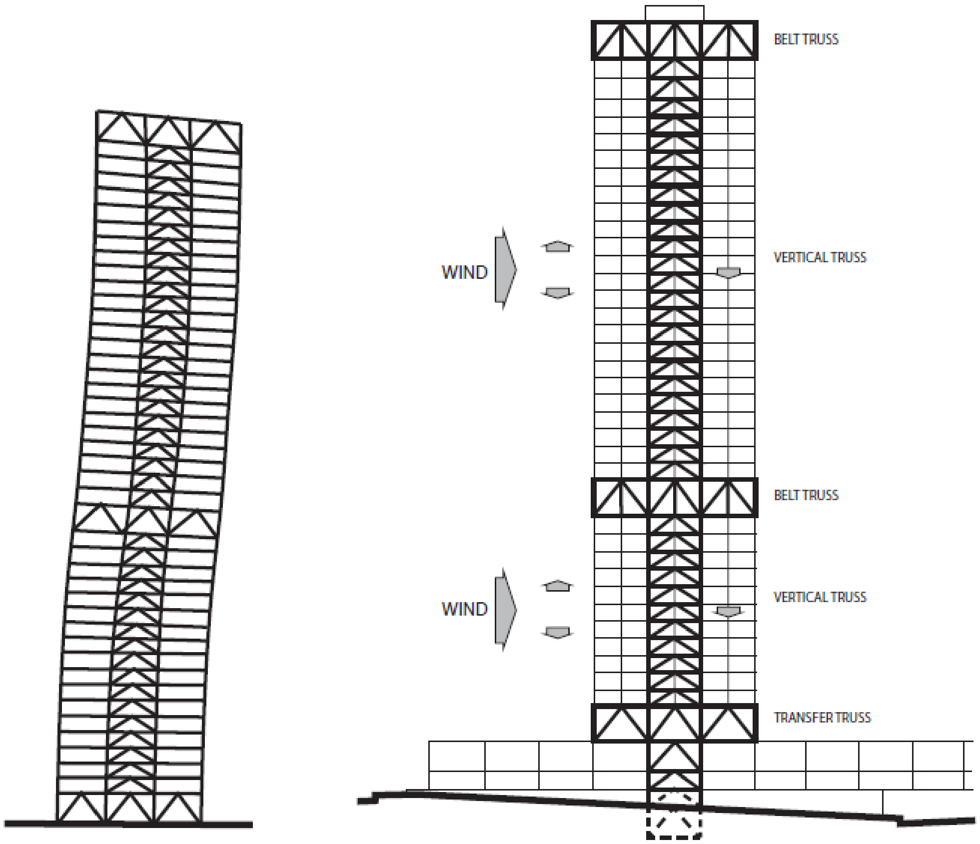


Figure 1-3 Outrigger braced system (Source: Beedle & Iyengar 1982)

A framed tube system can be defined as a three dimensional system that utilizes the entire building perimeter to resist lateral loads. The exterior columns are placed close to each other and deep spandrels are rigidly connected to the columns. The structural optimization reduces to examining the column spacing and member proportions. Typically the column spacing is 10ft. to 20 ft. and spandrels depth varies from 3ft. to5 ft.

In order to allow larger column spacing diagonal bracings can be used at the exterior and such a system is called braced tube system.

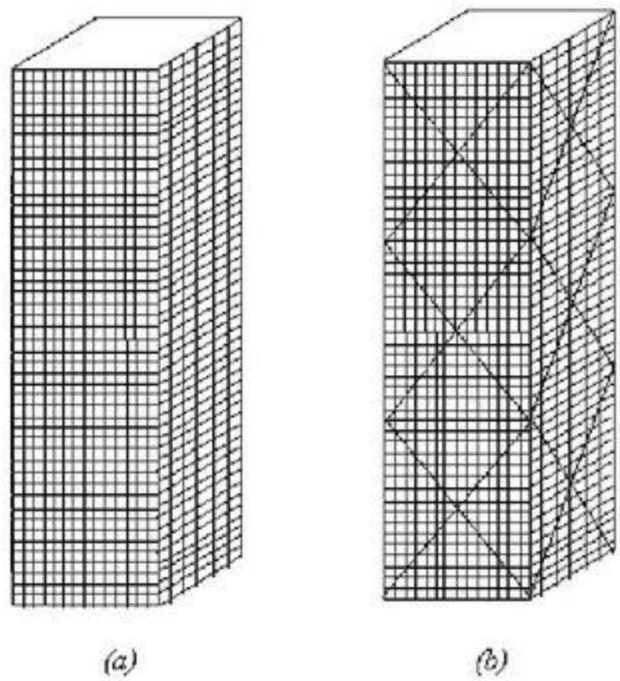


Figure 1-4 (a) Framed tube (b) Braced tube (Source: http://www.yousaytoo.com)

The structural design of modern tall buildings involves the challenging task of finding the most economical option that satisfies the safety and serviceability performance requirements. The structural design of a tall building involves conceptual design, approximate analysis, and preliminary design optimization, followed by detailed and final design (Jayachandran, 2009). With the rapid advancement of computing technology, structural analysis of a complex building model with thousands of members can be accurately performed within few seconds. But structural design of such buildings following the traditional trial-and-error approach is highly iterative, extremely time-consuming and the resultant design could be uneconomical (Ng and Lam, 2005). An efficient optimization tool for conceptual and numerical design of tall buildings can provide a good preliminary design which will in turn lead to an efficient structural design in terms of lower fabrication and erection costs, and better construction (Jayachandran, 2009). Several topology and sizing optimization techniques are currently available in the literature.

2 LITERATURE REVIEW

2.1 Sizing Optimization

Chan et al. developed a optimization algorithm for the minimum weight design of lateral load-resisting steel frameworks subjected to multiple inter-story drift and member strength and sizing constraints in accordance with building code and fabrication requirements (Chan, Grierson, & Sherbourne, 1995). The most economical standard steel sections to use for the structural members were automatically selected from commercially available standard section databases. A full-scale 50-story three-dimensional (3D) asymmetrical building framework example was discussed to illustrate the effectiveness, efficiency, and practicality of the automatic resizing technique.

An optimum bracing design method using GA optimization technique was developed for multistory non-swaying steel frames in order to obtain the least weight design by selecting appropriate sections for beams, columns and bracing members from the standard set of steel sections (Kameshki and Saka, 2001). The algorithm accounts for serviceability and strength constraints as specified in BS 5950. A planar frame with three-bays and fifteen stories was considered with different bracing types - X bracing, V bracing, Z bracing, X bracing (outrigger), Rigid frame (fixed base) with no bracings were designed using the optimizer and the normalized weights were used to choose the optimum bracing design.

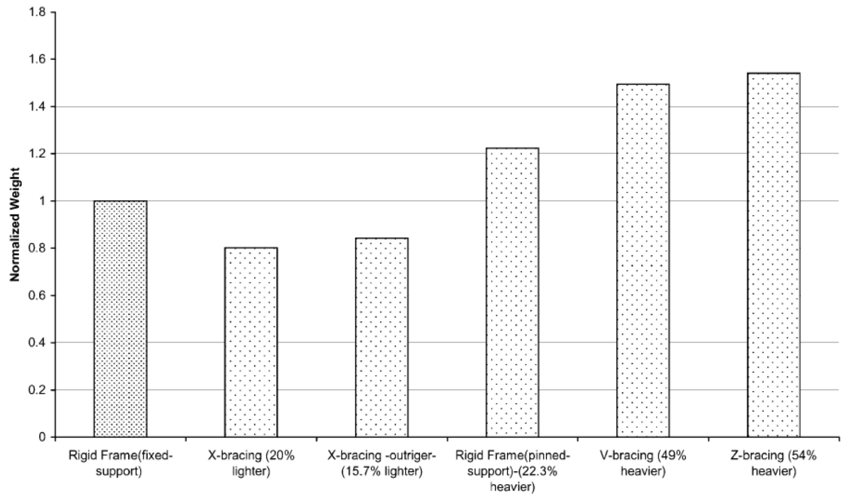


Figure 2-1 Comparison of normalized weights of tall frames with different bracing systems against fixed support frame (Source: Kameshki and Saka, 2001)

Park et.al presented three algorithms for sizing optimization by formulating the drift design process into an optimization problem in order to minimize lateral displacement of the system without changing the weight of a structure (Park et al., 2002). In his work, he reported the results for a 60-storey hybrid structure with reinforced concrete walls, steel frames and outrigger trusses. The lateral drift of 34.07 cm at the top of the structure and the maximum inter-story drift of 0.74 cm are reduced to 21.15 cm and 0.49 cm, respectively. Chan & Chui discussed optimality criteria method to minimize structural cost of steel buildings subjected to target frequency constraints (Chan and Chui, 2006). They presented an integrated wind-induced vibration analysis and optimal resizing technique for element stiffness design of tall steel building structures subject to occupant comfort serviceability design criteria.

2.2 Topology Optimization

Chan and Wong used optimality criteria (OC) design method for simultaneous sizing and topological optimization. The method explored topologies using GA and then refined the element sizes using OC algorithm. The examples used were 40 storied three bay planar frames shown below in Fig2-2. The topology optimization yielded over 35% material savings when compared to using only element sizing optimization.

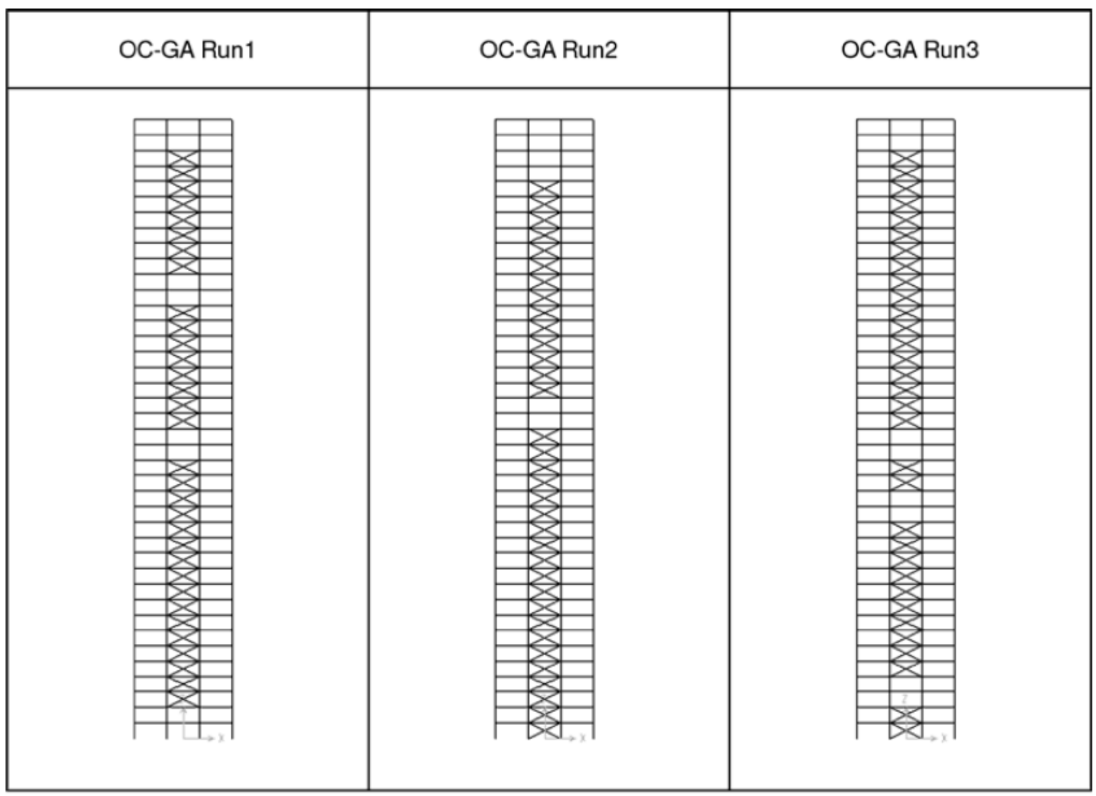


Figure 2-2 Final topologies for braced frame work (Source: Chan& Wong 2008)

Liang et al enumerated a performance-based optimization method for the minimum-weight and maximum-stiffness topology design of bracing systems. This method systematically removes inefficient materials from a continuum design domain that is used to stiffen the framework subjected to performance-based optimization criterion (Liang et al., 2000). Planar frame examples showed that the design method presented can generate efficient topologies, for multistory steel building frameworks. It did not however include any sizing optimization for bracing members. Another continuum topology optimization formulation using Voigt and Reuss rules for mixing materials is applied to the concept design optimization of structural bracing systems needed to stiffen tall structures against side sway under lateral-wind and seismic-type loading (Mijar et al., 1998).

Fawzia & Fatima (2010) studied the effect of belt truss and outrigger system on laeral deflection control of a composite building. Asymmetric building model was analyzed for DL, LL, WL load combinations in two stages. Stage 1 to obtain desired belt truss layout and location that can control lateral deflections and stage-2 included resizing based on strength requirements using OC method. They created the different models placing the belt truss at various locations to obtain the optimal location in stage-1.

An effective optimization tool that can provide optimal location for belt trusses and perform sizing optimization for strength and lateral deflection constraints is desirable in such studies to compare and evaluate various design alternatives.

2.3 Optimization of RC Structures

Sarma et al. concluded that for concrete structures, the goal should be minimization of cost and not weight because these structures are made of more than one material and include additional formwork cost (Sarma and Adeli, 1998). He emphasized the need to perform research on cost optimization of realistic three-dimensional structures, especially large structures with hundreds of members where optimization can result in substantial savings. Another simplified algorithm for RC member design was enumerated by Choi and Kwak with a ten story RC building example (Choi & Kwak, 1990). Chan & Wong presented a hybrid optimization method using optimality criteria (OC) technique and genetic algorithm (GA) (Chan and Wong, 2008). It can be applied for optimal structural design of tall hybrid steel and concrete buildings subject to multiple serviceability wind drift and acceleration constraints and practical element sizing requirements. The effectiveness and practicality of the optimization technique is illustrated through an actual application to the preliminary design of an 88-storey building in Hong Kong.

2.4 Motivation and Research Objectives

Most of the research (Chan and Grierson 1993; Kameshki & Saka, 2001; Park et al., 2002) considered sizing optimization with strength and drift constraints as design requirements. Some researchers addressed the topology optimization (Liang, Xie, & Steven, 2000, Li *et al.* 1999, Mijar, Swan, Arora, & Kosaka, 1998), but they considered planar frames and not full scale three dimensional buildings.

This study presents the results for structural design optimization of –six, sixteen and forty story buildings subject to multiple design criteria, such as the stress, lateral drift, and frequency, requirements as specified in the ACI and ASCE codes of practice. The structural analysis and design optimization of the building models are accomplished using the Frame3D program (Sirigiri and Rajan, 2013).

3 FINITE ELEMENT MODEL

3.1 Frame3D

The GS-USA Frame3D© computer program can be used for the linear analysis and optimal design of three dimensional structural systems using beam and thin plate/shell elements. The different steps in the program are shown in Fig 3-1.

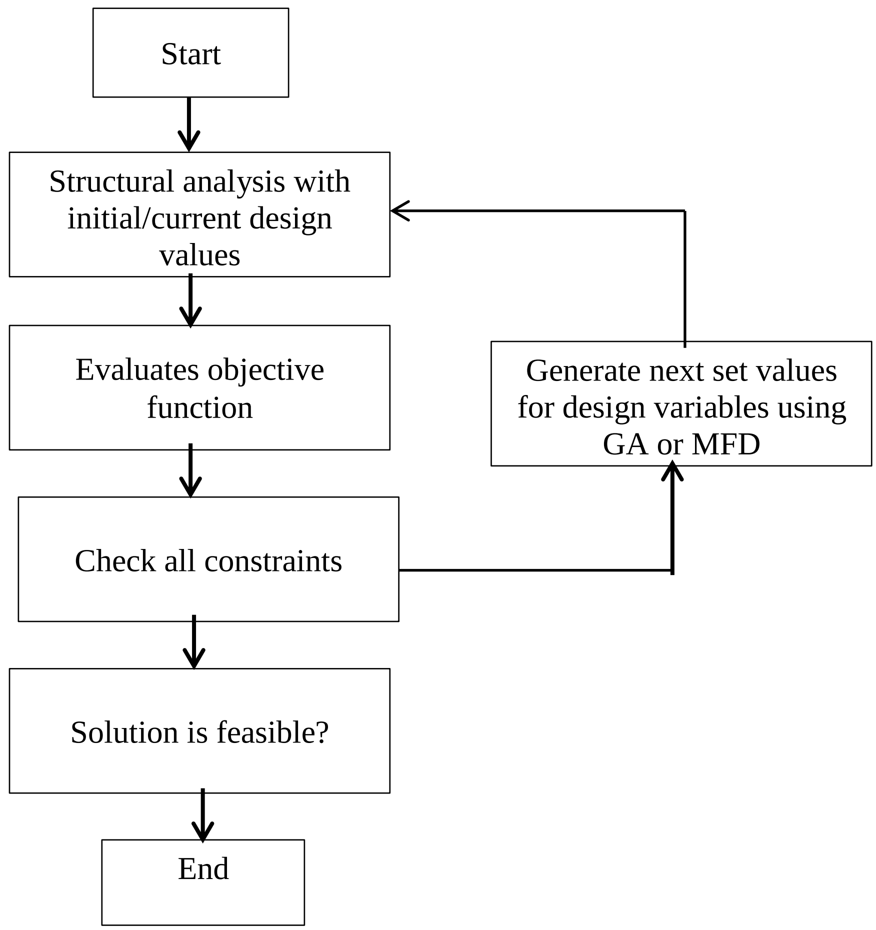


Figure 3-1 Flow chart in Frame3D program for design optimization

3.2 Building Layout

The 180ft x 90ft rectangular floor layout as shown in fig1.1 is used for the design optimization model of three buildings with six, sixteen and forty stories. The height from the basement to the lobby is 16 ft. All the other floor heights are 13 ft each. The floor system consists of 3" cellular steel deck with 2.5 inch concrete slab, supported on the steel joists. The unique layout is of irregular building, with large unsupported spans in both N-S (40') and E-W (30'). The first floor is taller than other floors and leads to a soft story effect under seismic loads.

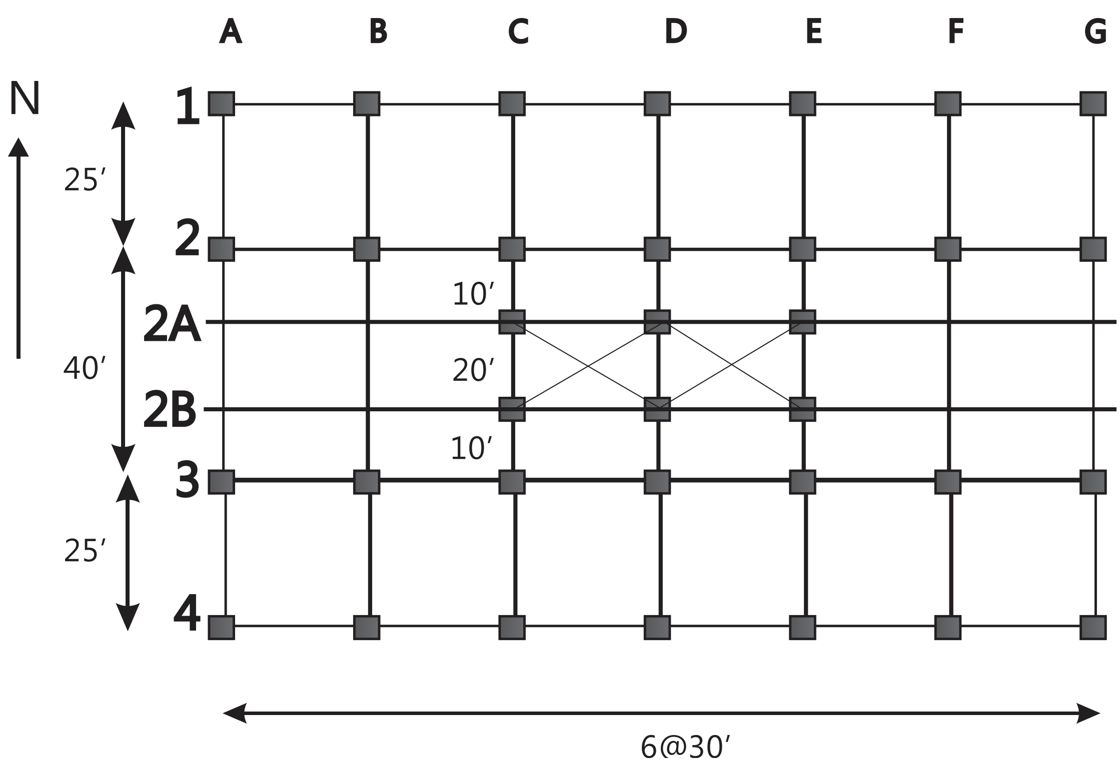


Figure 3-2 Typical floor plan

3.2.1 Elevator shaft

The elevator shaft is located in between grids 2A and 2B and along girds C, D and E. There are two elevators surrounded by reinforced concrete walls on all the three sides with the common wall located along grid D. The walls are 10 in. thick and are rigidly connected with the columns from top to bottom of the building.

3.2.2 Bracing Systems

The bracing system provides additional lateral stiffness to the structure, and especially in tall buildings, it effectively controls lateral deflections and increases torsional resistance due to wind loading. Typical bracing systems are placed symmetrically in order to be safe in design for both directions of wind loads. All the models with bracing elements are assumed to be symmetrical in this study.

3.3 Material Properties

All the columns, beams and bracings of the building are made of steel. The floor slabs and elevator shaft walls are made of concrete. The grade of structural steel is assumed to be A992/A992M and steel properties are as per ASTM specifications.

3.4 Loads

3.4.1 Gravity Loads

Apart from the assumed dead loads due to the steel deck and floor finishes, the live load on the roof is as per the specification in Table 4-1, ASCE-7-10.

Table 3-1 Material Properties

Type of material	Structural element	Material Property	Value
Steel, Grade A992/A992M	Columns, beams, and bracings	Mass density Elastic Modulus Yield stress	$slugs/in^3$ psi ksi
Concrete	Slab & elevator shaft walls	Mass density Elastic Modulus	$slugs/in^3$ psi

Table3-2 Dead and Live Loads

Location	Item	Load (psf)	Load (psi)	Total Load (psi)
Floor	3” Steel deck	125	0.87	0.87+0.11 =1.0
	Floor finishes	16	0.11	
Roof	Decking	42	0.29	0.29+0.04 = 0.3
	Felt +Gravel	6	0.04	
Lobby	Live load	100	0.7	0.7
Floors	Live load	80	0.56	0.56
Roof	Live load	20	0.14	0.14

3.4.2 Wind Loads

The assumed layout is flat enclosed building and is considered as the Main Wind-Force Resisting System (MWFRS). As per ASCE 7-2010,

- If the building is rigid then assumed value for gust factor is , G=0.85
- If the building is flexible or dynamically sensitive, the gust effect factor G_f is computed using Eq. 26.9-10 in Sec.26.9.5 ASCE7-2010.

Wind pressure loads are computed using directional procedure with positive internal pressure condition. The wind is assumed to be blowing in both N-S and E-W directions.

The building is designed to be symmetrical and hence wind blowing from S-N and W-E directions was ignored. The detailed wind pressure calculations for the six, sixteen and forty floor buildings are provided in Table A-1, A-2 and A-3 in Appendix A.

Table 3-3 Summary of Wind Load Parameters

Basic wind speed, V	120 mph
Risk category.	III
Exposure Category	C
Importance factor, I	1.0
Directionality Factor, K_d	0.85
Topographic Factor, K_{zt}	1.00
Gust Factor, G (for rigid structures)	0.85
Internal Pressure Coefficient, GC_{pi}	0.18
C_p(Windward)	0.8
C_p(Leeward)	0.5

Table 3-4 Wind Load Parameters

Parameter	**Six- Floor**	**Sixteen- Floor**	**Forty- Floor**
Height of building (ft)	81	211	510
Mean roof height(ft)	81	211	510
Velocity Pressure Coefficient (K_z)	1.22	1.48	1.77
Velocity Pressure at mean roof height q_h (psf)	38.01	46.23	55.46

3.4.3 Dynamic Response

Wind is a dynamic and random phenomenon in both time and space (Boggs and Dragovich, 2008). The dynamic response of a low-rise building is likely insignificant, assuming it has been adequately designed for both strength and stiffness limits states. However, it is important to quantify the case where dynamic response may be neglected. The building's fundamental natural frequency of vibration f , is the most widely accepted property of a structure used to determine whether dynamic response will be significant under wind loading. The ASCE Standard [ASCE 7-2010] classifies a structure as dynamically sensitive, or "flexible" if f < 1 Hz, otherwise it is considered to be "rigid." The classification used by the ASCE Standard is widely accepted as a reasonable boundary between dynamic and rigid behavior. If the structure is flexible, it is essential that the designer determine the natural frequency and mode shapes of the first few modes of vibration accurately using eigenvalue analysis. ASCE Standard specifies that for flexible or dynamically sensitive the gust effect factor G_f shall be calculated using the Eq. 26.9.10 which contains certain parameters that are specified as a function of the fundamental natural frequency of the structure.

The dynamic response in terms of serviceability with respect to occupier perception of lateral vibrations can become the governing the design issue and requires the purpose-designed damping systems to reduce the vibrations to acceptable level (Mendis et.al, 2007) Dynamic responses also play an important role in the detailed design of façade systems.

3.5 Finite Element Model

3.5.1 Element Type

The beam element uses a three-dimensional two-node formulation with six independent degrees of freedom (three translations and three rotations) at each end of the element.

Table 3-5 Element Properties

Type of finite element	Structural elements
Beam2	Beams , Columns & Bracings
QB4	Slab & Walls

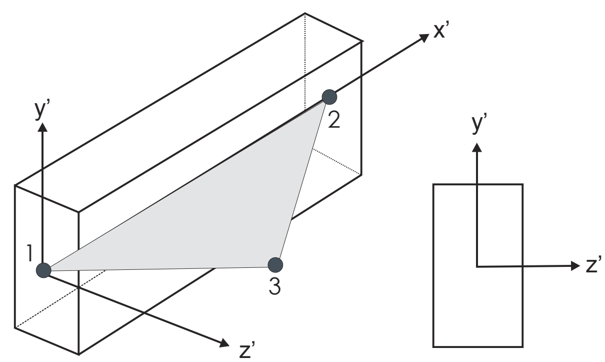


Figure 3-3 Beam2 element for beams, columns and bracing members

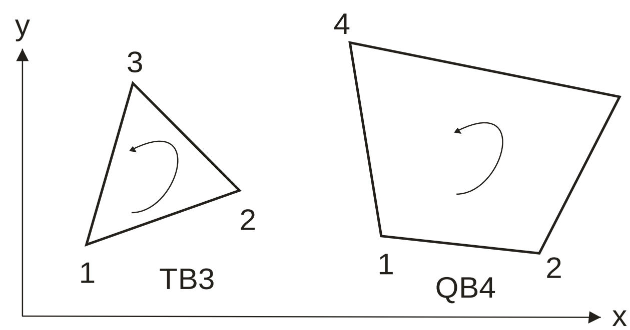


Figure 3-4 QB4 shell element for slab and walls

In Frame 3D, the stresses in space beam elements are computed using the formulae given below. The beam orientation is assumed such that the origin of the coordinate system is at the centroid of the cross-section and the axial direction is *x*, and the cross-section is oriented in the y-*z* plane. Assumptions in stress computations

Normal stress (Tension is positive)

$$\sigma_{\max}^{t} = \max\left(\frac{N_x}{A} + \frac{|M_y|}{S_y} + \frac{|M_z|}{S_z}, 0\right) \tag{1}$$

$$\sigma_{\max}^{c} = \min\left(\frac{N_x}{A} - \frac{|M_y|}{S_y} - \frac{|M_z|}{S_z}, 0\right) \tag{2}$$

Shear stress

$$\tau^{y} = \frac{|V_y|}{SF_y} \tag{3}$$

$$\tau^{z} = \frac{|V_z|}{SF_z} \tag{4}$$

$$\tau^{T} = \frac{|T_x|}{T_J} \tag{5}$$

$$\tau_{\max}^{conservative} = \max\left\{\tau^{y} + \tau^{T}, \tau^{z} + \tau^{T}\right\} \tag{6}$$

3.5.2 Element Properties

The element type and boundary conditions remain same for all the three building models. The cross sectional properties however vary in each model. The longitudinal beams in all floors are assumed to have the same cross section. Similarly the transverse beams in all floors have the same cross section. The cross section properties of columns are varied with height.

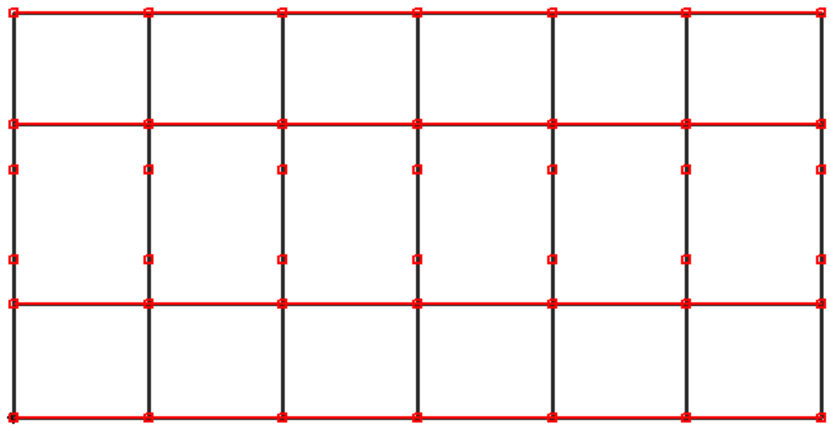


Figure 3-5 Typical floor plan with longitudinal beams highlighted in red

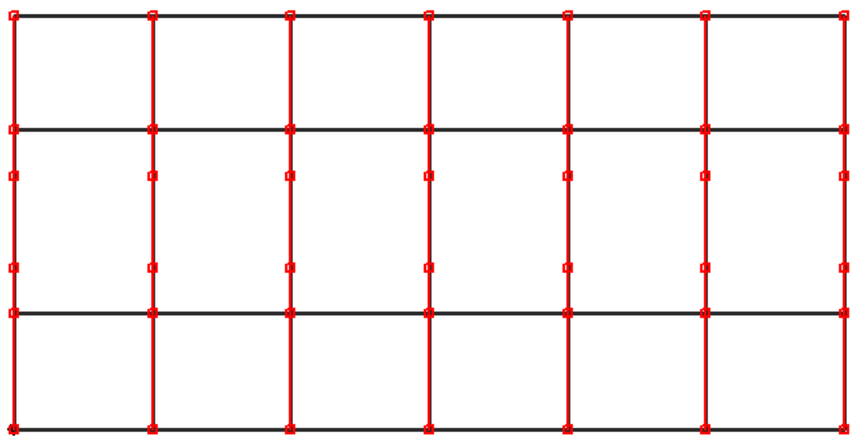


Figure 3-6 Typical floor plan with transverse beams highlighted in red

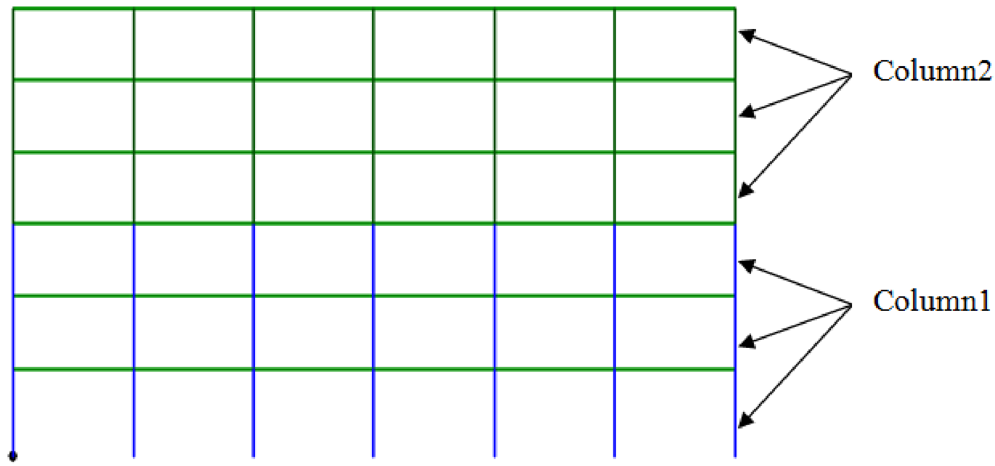


Figure 3-7 Plot showing different column groups for six floor model

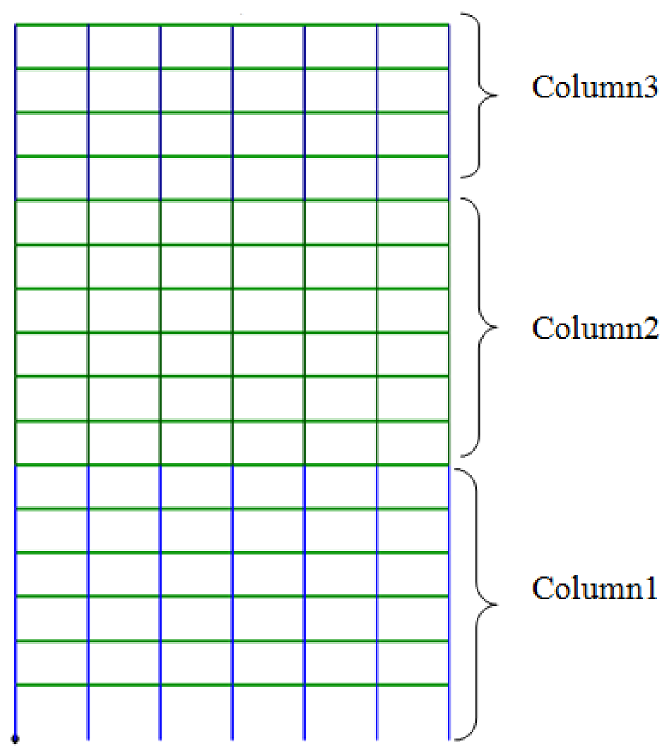


Figure 3-8 Plot showing different column groups for sixteen floor model

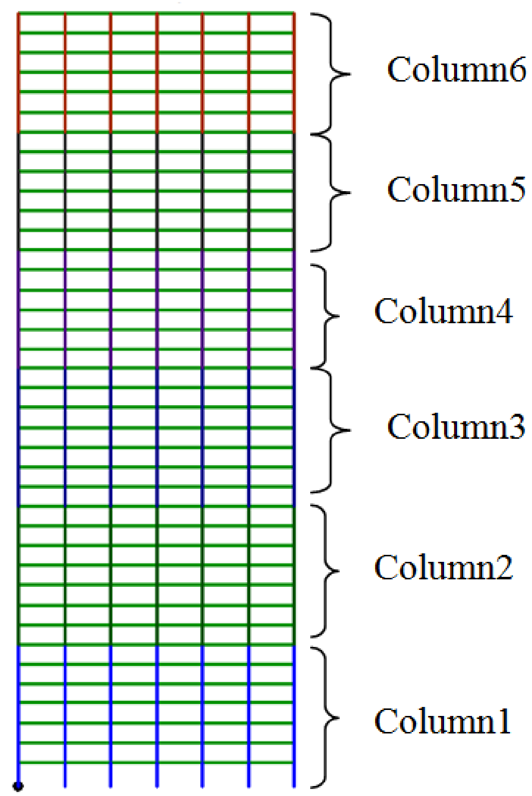


Figure 3-9 Plot showing different column groups for forty floor model

Table 3-6 Classification of Column Property Groups

Case	Floor numbers	Column group
1-Six story	1-3	Column 1
	4-6	Column 2
2-Sixteen story	1-6	Column 1
	7-12	Column 2
	13-16	Column 3
3-Forty story	1-7	Column 1
	8-14	Column 2
	15-21	Column 3
	22-27	Column 4
	28-33	Column 5
	34-40	Column 6

3.6 Naming Convention for members

A naming convention is specified in order to identify the location of members and nodes in the finite element models.

Table 3-7 Naming Convention for Nodes and Elements

Item	Naming
Node	N-Floor#-grid tag(E-W) –grid tag (N-S)
Column	C - Floor # – grid tag (E-W)- grid tag (N-S)
Longitudinal beam	LB- grid tag (N-S)- Floor #- grid1(E-W) –grid2(E-W)(*)
Transverse beam	LB- grid tag (E-W)- Floor # - grid1(N-S)- grid2 (N-S)(*)

(*) Beam spans between grid1 and grid 2

A column in the 4th floor at the intersection of grids A and 3 will be written as C-4-A-3.

A beam in the first floor between grids A and B and along grid 2 will be named as LB-2-1-A-B.

4 DESIGN OPTIMIZATION

4.1 Constrained Optimization Problem

In general, a constrained optimization can be formulated as,

Find $\mathbf{x} \in R^n$

To minimize $f(\mathbf{x})$ (7)

Subject to $g_i(\mathbf{x}) \leq 0 \qquad i = 1, 2, .., m$ (8)

$$x_j^L \leq x_j \leq x_j^U \qquad j = 1, 2....n \tag{9}$$

$\mathbf{x}$ is an n-dimensional design vector, $f(\mathbf{x})$ is the objective function, and Eq.2 and Eq.3 are inequality constraints and design variable bounds, respectively. In order to define a structural design optimization problem following are the required assumptions.

1. All the structural elements are straight and prismatic.
2. Materials are homogeneous and isotropic.
3. Material properties are assumed to be constant throughout the design process.
4. Response from small strains, small displacement linear elastic finite element analysis
   i. Nodal displacements
   ii. (Beam) Element nodal forces
   iii. (Beam + shell) Element max. tensile, compressive and shear stresses
   iv. Support reactions
   v. Fundamental frequency

4.2 Design Variables

The variables in a design optimization problem are the element sizing variables and topological variables. If only sizing optimization is considered in a particular case, then the topology is presumed to be fixed. Element sizing variables vary the cross-sectional properties of structural elements. They can be continuous, or discrete. In practical structural design, steel structures use standard AISC sections which are discrete variables or built–up sections which are continuous variables. Discrete and simple-continuous variables have been considered in this study. In order to solve the problem with continuous design variables, we assumed all the members are assigned wide flange sections. The wide flange sections from the AISC steel data base were examined for approximate relationship between the four dimensions, web height h_w, flange width b_f, web thickness t_w and flange thickness t_f. The custom wide flange sections are thus based on the relative dimensions of standard wide flange sections. Using the relationships from Fig 4-1, 4-2 and 4-3 and web height, as the independent variable, the following equations are derived.

$$b_f = 0.22h_w + 7 \tag{10}$$

$$t_f = 0.01b_f^2 - 0.05b_f + 0.38 \tag{11}$$

$$t_w = 0.58t_f + 0.035 \tag{12}$$

Since in a building with thousands of members, each member cannot be designed uniquely, we group the members into property groups. If using the discrete sizing variables $\underline{\mathbf{x}} = \{s_1, s_2....s_m\}$ and $\{s_1, s_2....s_m\}$ belong to DDVs set of standard steel sections from AISC steel data base where s_i is the wide flange section tag for the i^{th} property group. Similarly for the optimization with CDVs $\underline{\mathbf{x}} = \{h_1, h_2....h_m\}$, where h_i is the web height for the i^{th} property group. The cross sectional properties are evaluated using Eq. 4 to 6. In this study, we used the set of 267 discrete AISC wide flange sections for sizing variables. The list of AISC wide flange sections used in this study can be found in Appendix B. Hence the bounds for DDVs are 1 and 267. The bounds for CDVs have been specified based on engineering judgment.

$$1 \le s_i \le 267 \qquad i = 1,....n \tag{13}$$

$$h_i^L \le h_i \le h_i^U \qquad i = 1,....n \tag{14}$$

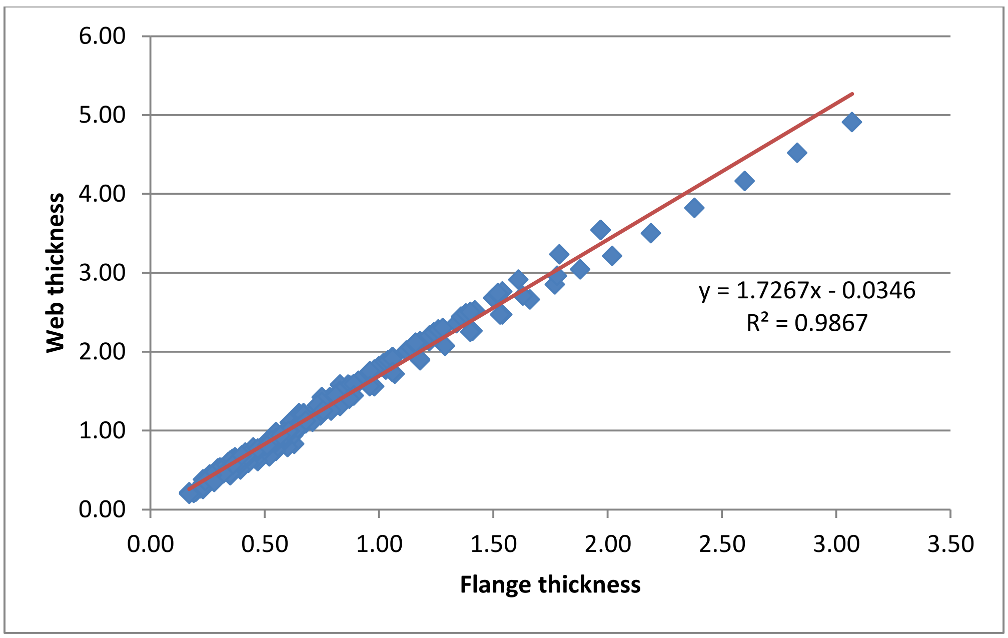


Figure 4-1 Relationship between flange thickness and web thickness for AISC W sections

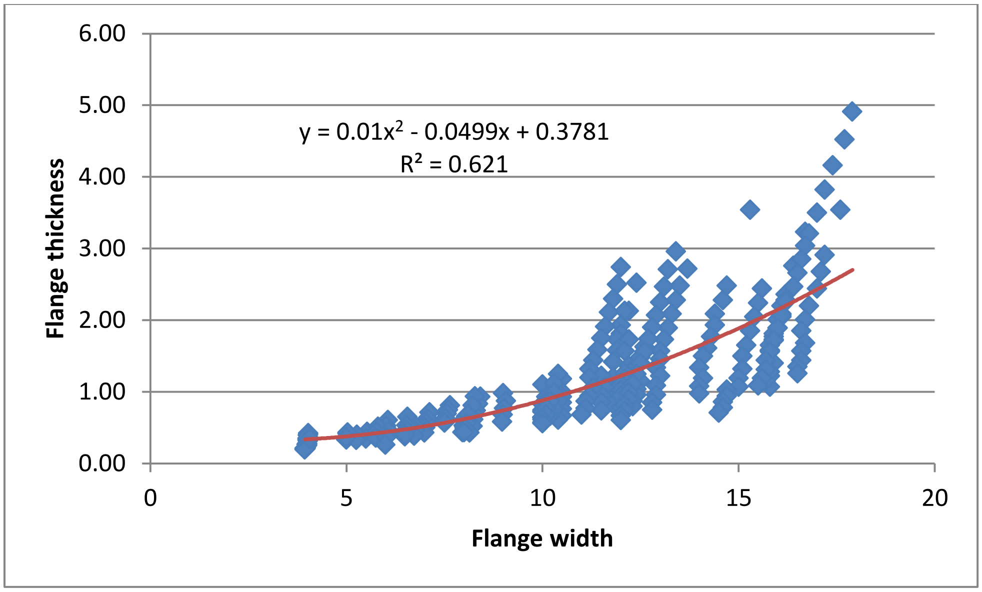


Figure 4-2 Relationship between flange width and flange thickness for AISC W sections

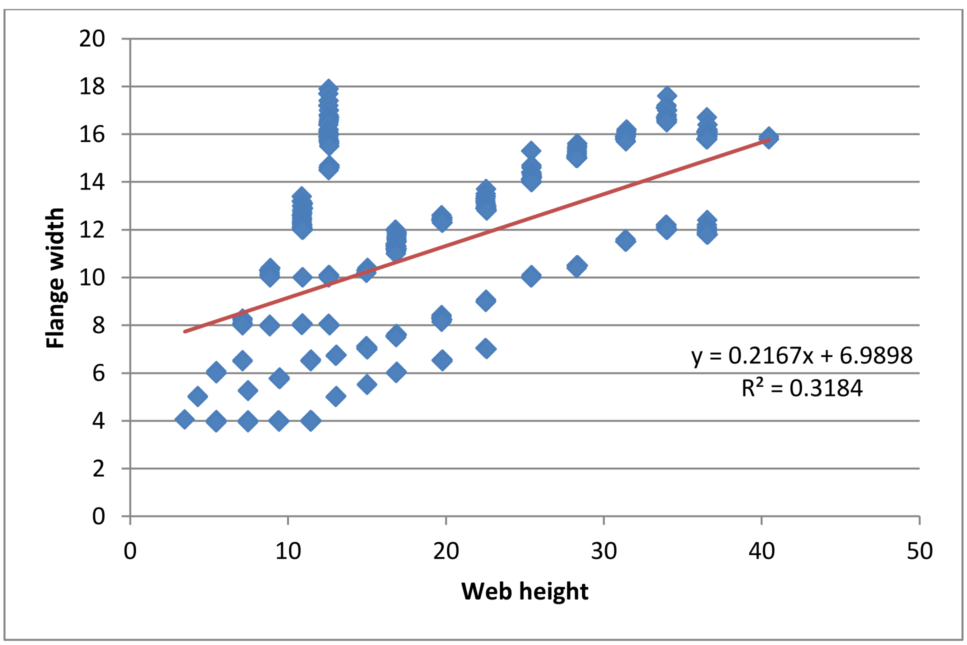


Figure 4-3 Relationship between web height and flange width for AISC W sections

4.3 Topology Design Variables

Pure rigid frame systems alone are not efficient in resisting lateral loads for tall steel buildings due to associated high costs. Truss members such as diagonals are often used to brace steel frameworks to maintain lateral drifts within acceptable limits. The optimal layout design of bracing systems is a challenging task for structural designers, because it involves a large number of possibilities for the arrangement of bracing members. In the absence of an efficient optimization technique, the selection of lateral bracing systems for multistory steel frameworks is usually undertaken by the designer based on a trial-and-error process or previous design experiences.

In topology optimization of a building structure, Boolean design variables are defined to structural elements. The value of variable is "1" if the element exists in the model and "0" if it does not. Frame3D program generates several topology designs for the structure to yield the optimized solution. In Chapter 6, test cases are presented for topology optimization using bracing elements of a building model as topological design variables.

4.4 Stress Constraints

The strength of structural members is the primary factor to be considered in structural design, because loads on a structure are transferred through structural elements to the foundation. The failure of these elements due to excessive stresses can lead to the collapse of the entire building. The DO problem formulation is based on Allowable Strength Design (ASD). ASD requires that the allowable strength of each structural component equals or exceeds the required strength. According to Eq. B3-1, in AISC Specification (2005),

The allowable tensile stress for gross steel cross section,

$$\sigma_a^t = 0.6 f_y \qquad (15)$$

The allowable compressive stress for gross steel cross section,

$$\sigma_a^c = 0.6 f_y \qquad (16)$$

The allowable shear stress for gross steel cross section

$$\tau_a = 0.4 f_y \qquad (17)$$

As per Table 3-1, the yield stress for grade A992 steel f_y = 50000 psi. Therefore, the stress constraints in the design problem are specified as:

$$\sigma_{\max,i}^t \le 30000\,psi \qquad (18)$$

$$\sigma_{\max,i}^t \le 30000\,psi \qquad (19)$$

$$\tau_{\max,i} \le 20000\,psi \qquad (20)$$

4.5 Displacement Constraints

The design of a multistory steel building under lateral loads is usually governed by overall stiffness rather than strength criteria (Liang et al., 2000). The stiffness of a structure can be evaluated based on allowable serviceability limits. In the ultimate limit state design, the second-order P-Delta effect is prevented by limiting the lateral deflections of the building (Ng & Lam, 2005). Drifts (lateral deflections) of concern in serviceability checking arise primarily from the effects of wind (ASCE 7-10). The drift at the top of the building should be less than 1/600 to 1/400 of the total building height (ASCE Task Committee on Drift Control of Steel Building Structures 1988 and Griffis 1993). The allowable limits for longitudinal and transverse drifts vary with the height of the building.

$$\delta_{\max}^{L} \le \delta_{a}^{L} \tag{21}$$

$$\delta_{\max}^{T} \le \delta_{a}^{T} \tag{22}$$

Table 4-1 Lateral Drift Limits

Parameter	**Six Floor**	**Sixteen Floor**	**Forty Floor**
Total Height (ft)	81	211	510
δ_a^L **(in)**	1.9	5.0	12.2
δ_a^T **(in)**	1.9	5.0	12.2

4.5.1 Vertical Deflection

The common deflection limits for horizontal members have been 1/360 of the span for floors subjected to full nominal live load. This is applicable for checking deflection in slabs and beams. The longest span of the beams in the building layout is 40 ft. and the post processing analysis results show the vertical deflection of these beams was critical for design. Thus the vertical deflection constraint is specified as:

$$\delta_{\max}^{V} \le 1.33 \tag{23}$$

4.5.2 Inter story Drift

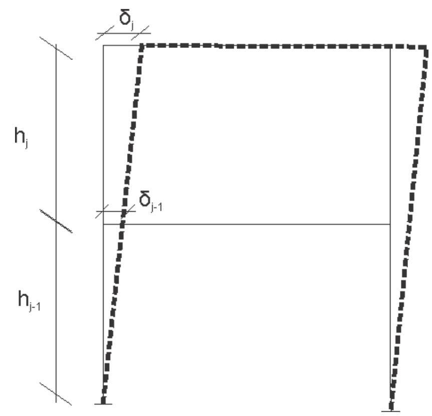


Figure 4-4 Inter-story drift in a planar frame

The inter-story drift is another serviceability criterion for design requirements. According to BS 8110-2:1997 (British Standards Institution 1997), the inter-story drift should be less than 1/500 of the story height. For a story height h_j, the drift of j^{th} story and $(j-1)^{th}$ are δ_j and δ_{j-1} respectively then,

$$\frac{\delta_j - \delta_{j-1}}{h_j} \le 0.002 \tag{24}$$

As per the building layout description the typical story height is 13 ft. Therefore, the inter-story drift constraints in the design problem are specified as (in inches):

$$\delta_j - \delta_{j-1} \le 0.31 \tag{25}$$

4.6 Frequency Constraints

We are designing for rigid building and hence the design constraint for frequency is specified as:

$$f_{lowest} \geq 1\ Hz \tag{26}$$

4.7 Optimization Problem Formulation - Minimum Weight Design (MWD)

As seen from literature review, the minimization of weight is the main goal for the optimization of steel buildings. For a general building structure with n structural elements having cross sectional area of members as A_i, for $i = 1.........n$, the weight of structure is calculated as

$$W = \sum_{i=1}^{n} A_i L_i \gamma_i \tag{27}$$

W is the total weight of structure

γ_i is the weight density of material for the i^{th} structural member and

L_i is the length of i^{th} structural members.

To minimize

$$W = \sum_{i=1}^{n} A_i L_i \gamma_i \tag{28}$$

Subject to

$$\sigma_{\max,i}^{t} \leq 30000\, psi \tag{29}$$

$$\sigma_{\max,i}^{c} \leq 30000\, psi \tag{30}$$

$$\tau_{\max,i} \leq 20000\ psi \quad (31)$$

$$\delta_{\max}^{T} \leq \delta_{a}^{T} \quad (32)$$

$$\delta_{\max}^{L} \leq \delta_{a}^{L} \quad (33)$$

$$\delta_{\max}^{V} \leq \delta_{a}^{V} \quad (34)$$

$$\frac{\delta_j - \delta_{j-1}}{h_j} \leq 0.002 \quad (35)$$

$$f_{lowest} \geq 1\ Hz \quad (36)$$

4.8 Optimization Problem Formulation - Maximum Frequency Design (MFD)

In some cases, where dynamic response plays more significant role than the weight of the structure, maximization of natural frequency can be the desirable goal subjected to stress, displacement and a maximum weight limitation $W_{specified}$.

To minimize

$$-f_{lowest} \quad (37)$$

Subject to

$$\sigma_{\max,i}^{t} \leq 30000\,psi \quad (38)$$

$$\sigma_{\max,i}^{c} \leq 30000\,psi \quad (39)$$

$$\tau_{\max,i} \leq 20000\ psi \quad (40)$$

$$\delta_{\max}^{T} \leq \delta_{a}^{T} \quad (41)$$

$$\delta_{\max}^{L} \leq \delta_{a}^{L} \quad (42)$$

$$\delta_{\max}^{V} \le \delta_{a}^{V} \tag{43}$$

$$W \le W_{specified} \tag{44}$$

$$\frac{\delta_{j} - \delta_{j-1}}{h_{j}} \le 0.002 \tag{45}$$

5 CASE STUDIES

The building layout shown in Fig 3-2 is used to in this study to create three different test cases of six, sixteen and forty floor models for design optimization. These test cases show the effect of stresses and displacements in buildings of different heights. The results enumerate the efficiency of Frame3D but one may not expect to generate similar results for all types of buildings because load resisting capacity of the system depends on a various factors such as architectural features, layout of columns and beams. However, the tool is applicable to all building models and allows generating several design alternatives as required by the designer.

5.1 Naming Convention for Test Cases

A naming convention is specified in order to denote the problem formulation for the different test cases presented in this chapter. It is designated using:

Number of floors - Design variable type - Objective function - Design constraints

6F-DDV-W-S+D+F denotes the DO problem for minimizing total weight of the six floor building with discrete design variables subjected to stress ,displacement and frequency constraints. 6F-DDV-F-S+D+W –denotes the DO problem for maximizing lowest frequency of the six floor building with discrete design variables subjected to stress ,displacement and total weight constraints.

5.2 Six Floor: Sizing Optimization

5.2.1 Case 1- Effect of RC Elevator Shaft core (MWD)

The two models considered in Case 1 are:

a) Six story model with steel framework and floor slabs

b) Six story model with steel framework, floor slabs and elevator shaft

Both the models are subjected to all the performance design constraints for minimizing the total weight of the structure using discrete design variables from the set of AISC W sections for cross section groups.

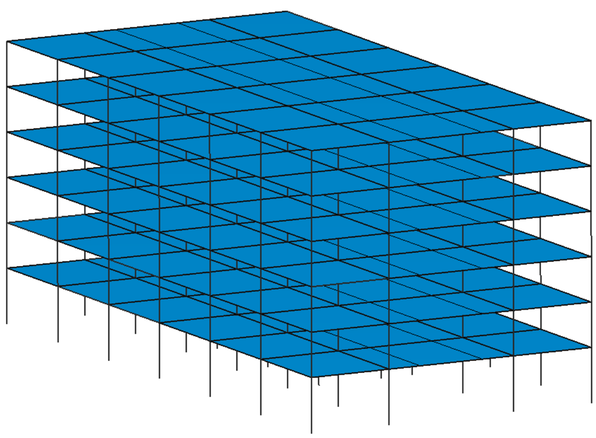


Figure 5-1 Six floor model with steel framework and floor slabs

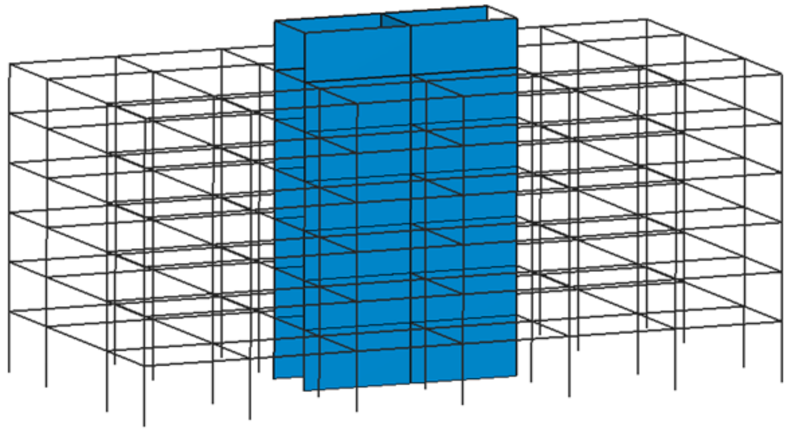


Figure 5-2 Six floor model with steel framework, floor slabs and elevator shaft

Table 5-1 Maximum Element Stresses and Location

Model Description	**Tensile stress (psi)**	**Compressive stress (psi)**	**Shear stress (psi)**
6F-DDV-W-S	27583	27773	8308
	TB-1-B-2-2A	C-1-F-2	TB-5-F-2B-3
6F-DDV-W-S+D	27583	27773	8308
	TB-1-B-2-2A	C-1-F-2	TB-5-F-2B-3
6F-DDV-W-S+D+F	28095	28051	8683
	TB-1-F-2B-3	TB-2-B-2-2A	TB-5-B-2-2A
6F-ES-DDV-W-S	18932	27586	6315
	TB-2-B-2-2A	C-1-B-3	7-C-2A
6F-ES-DDV-W-S+D	18932	27586	6315
	TB-2-B-2-2A	C-1-B-3	7-C-2A
6F-ES-DDV-W-S+D+F	18932	27586	6315
	TB-2-B-2-2A	C-1-B-3	7-C-2A

The design results for element stresses and nodal displacements show that:

1. Stress constraints govern the design in both models
2. Largest compressive stress values are in the lower most columns of the building
3. The total weight of frame work increases by 3.4 % with frequency constraint

4. The elevator shaft provides a lot of stiffness to the structure leading to an increase in the natural frequency with a total weight increase of 39% when compared to the model without elevator shaft.

Table 5-2 Maximum Nodal Displacements and Location

Model Description	Longitudinal Displacement (in)	Transverse Displacement (in)	Vertical Displacement (in)
6F-DDV-W-S	0.68	0.15	0.66
	6-F-2A	6-G-1	5-F-2A
6F-DDV-W-S+D	0.68	0.15	0.66
	6-F-2A	6-G-1	6-G-1
6F-DDV-W-S+D+F	0.44	0.14	0.63
	6-F-2A	6-G-1	6-G-1
6F-ES-DDV-W-S	0.01	0.05	0.64
	7-C-2A	6-A-1	5-B-2B
6F-ES-DDV-W-S+D	0.01	0.05	0.64
	7-C-2A	6-A-1	5-B-2B
6F-ES-DDV-W-S+D+F	0.01	0.05	0.64
	7-C-2A	6-A-1	5-B-2B

Table 5-3 Total Weight and Lowest frequency

Model Description	Weight (Mlb)	Lowest frequency (Hz)	Constraint	Location
6F-DDV-W-S	4.72	0.88	Compressive stress	First floor column
6F-DDV-W-S+D	4.72	0.88	Compressive stress	First floor column
6F-DDV-W-S+D+F	4.88	1.1	Tensile stress	First floor transverse beam
6F-ES-DDV-W-S	6.54	3.2	Compressive stress	First floor column
6F-ES-DDV-W-S+D	6.54	3.2	Compressive stress	First floor column
6F-ES-DDV-W-S+D+F	6.54	3.2	Compressive stress	First floor column

5.3 Six Floor: Sizing + Topology Optimization

5.3.1 Case 2- Optimization of Bracing system (MWD)

The models considered in Case 2 are:

a) Six story model with framework with no bracing members and floor slabs

b) Six story model with framework with X-bracing on periphery and floor slabs.

Two types of braced models are created, a model with initial design in which all bracing members are assigned Boolean design variable values of “1” and another model, with initial design in which all the Boolean design variable values are “0”. . The bracing members of each story and on either sides of the building were grouped together for symmetry. The optimization tool is then allowed to remove or retain the members in generating several topology alternatives. All the models are subjected to stress and displacement design constraints for minimizing the total weight of the structure using discrete design variables from the set of AISC W sections for cross section groups.

The design results for element stresses and nodal displacements show that:

1. Both the braced models had the same final topology even though the initial design was different.
2. Compressive stress governs the design in all three models.
3. Largest compressive stress values are in the lower most columns of the building
4. Simultaneous topology and sizing optimization yields a reduction in the total weight of steel frame work by 3.5 %.

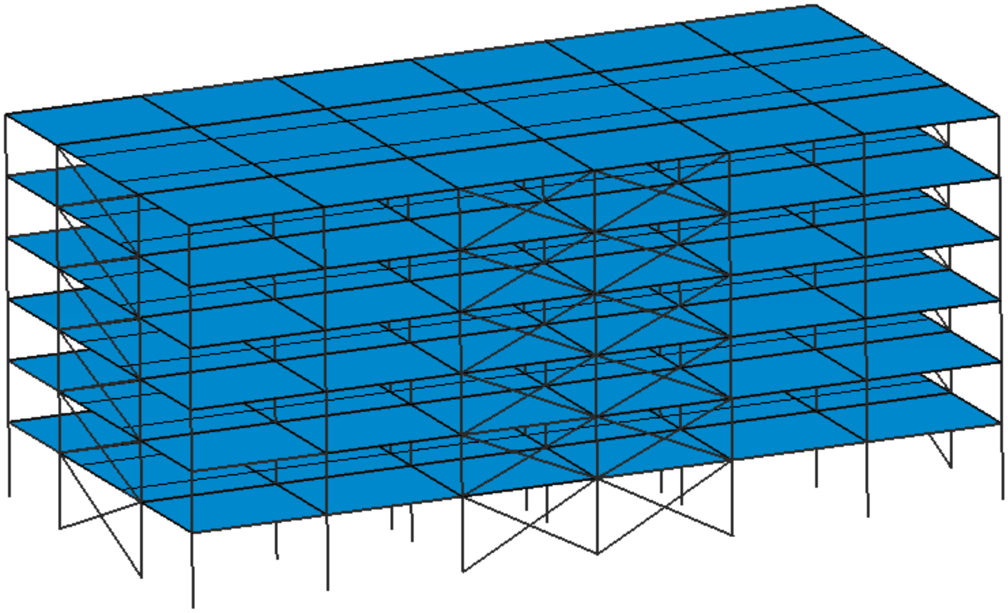


Figure 5-3 Six story model with framework with X-bracing

Table 5-4 Total Weight and Lowest frequency

Model description	Weight (Mlb)	Constraint	Location
6F-DDV-W-S+D	4.72	Compressive stress	First floor column
6F-BDV+DDV-W-S+D (BDVs =1)	4.56	Compressive stress	First floor column
6F-BDV+DDV-W-S+D (BDVs =0)	4.56	Compressive stress	First floor column

Table 5-5 Maximum Element Stresses and Location

Model Description	Tensile stress (psi)	Compressive stress (psi)	Shear stress (psi)
6F-DDV-W-S+D	27583	27773	8308
	TB-1-B-2-2A	C-1-F-2	TB-5-F-2B-3
6F-DDV+BDV-W-S+D	22000	29412.3	7710
	TB-4-B-22-A	C-1-F-2B	TB-5-F-2B-3
6F-DDV+BDV-W-S+D	22000	29412.3	7710
	TB-4-B-22-A	C-1-F-2B	TB-5-F-2B-3

Table 5-6 Six Floor - Topology of Elevation View

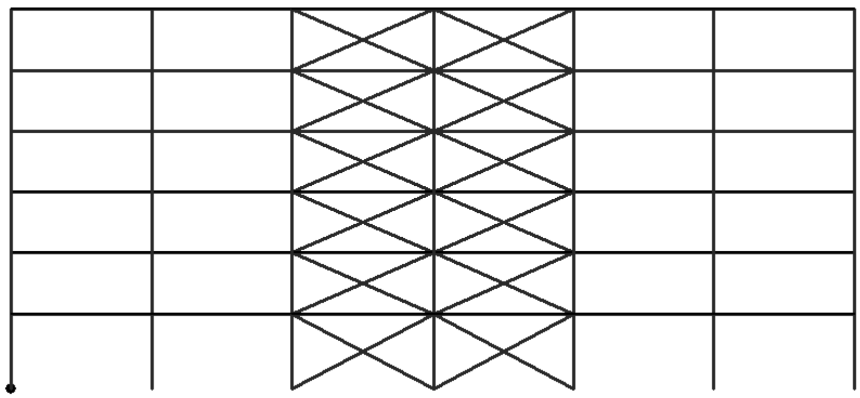
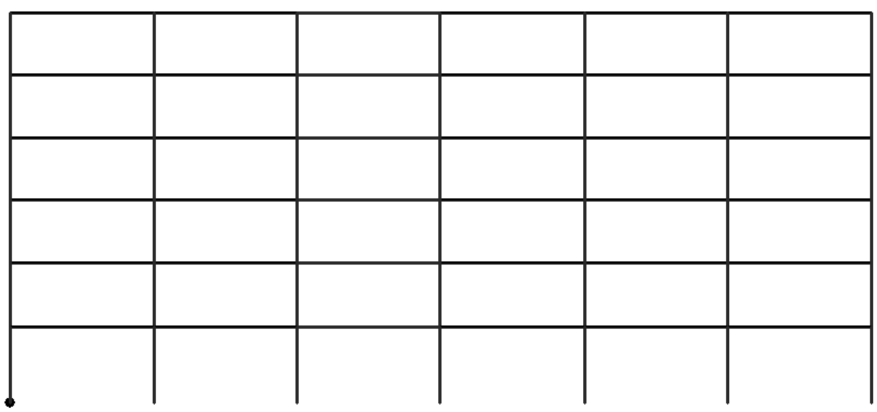
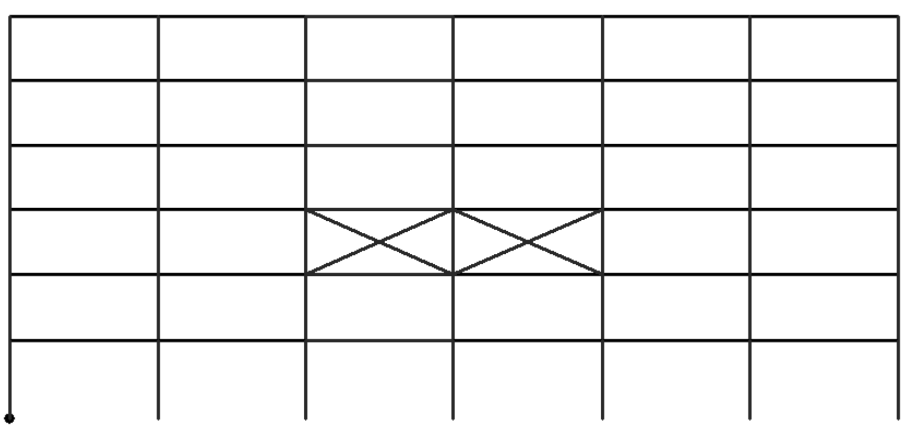
Topology of elevation view	**Description**
	Initial topology with full bracing
	Initial topology with no bracing
	Final topology for the braced models

Table 5-7 Six Floor - Topology of Side View

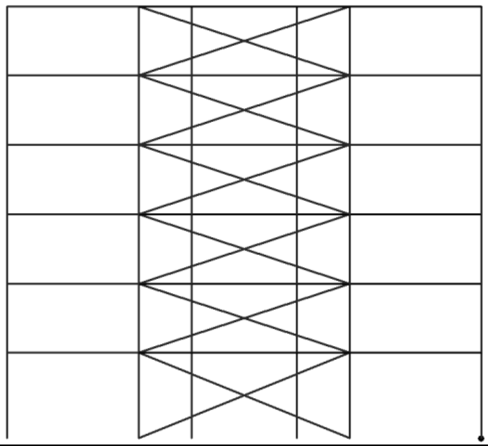
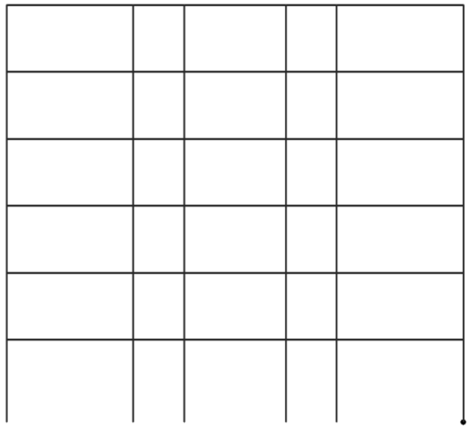
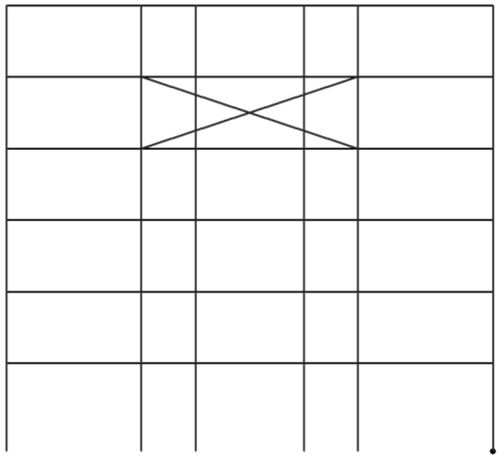
Topology of side view	Description
	Initial topology with full bracing
	Initial topology with no bracing
	Final topology for the braced models

Table 5-8 Maximum Nodal Displacements and Location

Model Description	Longitudinal Displacement (in)	Transverse Displacement (in)	Vertical Displacement (in)
6F-DDV-W-S+D	0.68	0.15	0.66
	6-F-2A	6-G-1	6-G-1
6F-DDV+BDV-W-S+D (BDVs=1)	1.77	0.13	0.63
	6-B-1	6-F-1	5-F-2A
6F-DDV+BDV-W-S+D (BDVs=0)	1.77	0.13	0.63
	6-B-1	6-F-1	5-F-2A

5.3.2 Case 3- Optimization of Bracing system (MLF)

The models used in Case 3 are same in Case 2. The models are subjected to stress, displacement and weight design constraints for maximizing the lowest frequency of the structure using discrete design variables from the set of AISC W sections for cross section groups. The design results for element stresses and nodal displacements show that:

1. Both the braced models had the same final topology even though the initial design was different when weight constraint was imposed.
2. Weight constraint was governing constraint in the braced models.
3. Sizing optimization yields a reduction in the total weight of steel frame work by 32 % in the model with weight constraint.
4. Simultaneous topology and sizing optimization gives a 15.6 % reduction in total weight in the braced models.

Table 5-9 Total Weight and Lowest frequency

Model description	Weight (Mlb)	Lowest Frequency (Hz)	Constraint Value	Location
6F-DDV-F-S+D	6.83	1.2	Compressive stress	First floor column
6F-DDV-F-S+D+W	5.16	1.19	Tensile stress	First floor transverse beams
6F-BDV+DDV-F-S+D (Initial guess :full bracing)	5.99	1.6	Tensile stress	First floor transverse beams
6F-BDV+DDV-F-S+D+W (Initial guess :full bracing))	5.71	1.32	Total Weight	-
6F-BDV+DDV-F-S+D (Initial guess :no bracing))	6.6	1.45	Compressive stress	First floor column
6F-BDV+DDV-F-S+D+W (Initial guess :no bracing)	5.71	1.32	Total Weight	-

Table 5-10 Maximum Element Stresses and Location

Model Description	Tensile stress (psi)	Compressive stress (psi)	Shear stress (psi)
6F-DDV-F-S+D	27583	27773	8308
	TB-1-B-2-2A	C-1-F-2	TB-5-F-2B-3
6F-DDV-F-S+D+W	27730	27616	8307
	TB-1-B-2-2A	TB-2-F-2B-3	TB-5-F-2B-3
6F-BDV+DDV-F-S+D (Initial guess :full bracing))	28099	28056	8682
	TB-1-B-2-2A	TB-2-F-2B-3	TB-5-F-2B-3
6F-BDV+DDV-F-S+D+W (Initial guess :full bracing))	21706	24977	6614
	TB-1-B-2-2A	C-1-F-2	TB-5-F-2B-3
6F-BDV+DDV-F-S+D (Initial guess :no bracing))	11000	16650	4579
	TB-1-B-2-2A	C-1-F-2	TB-5-B-2-2A
6F-BDV+DDV-F-S+D+W (Initial guess :no bracing))	21706	24977	6614
	TB-1-B-2-2A	C-1-F-2	TB-5-F-2B-3

Table 5-11 Six Floor - Topology of Elevation View

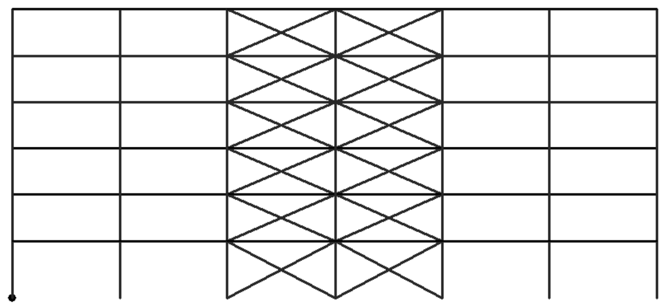
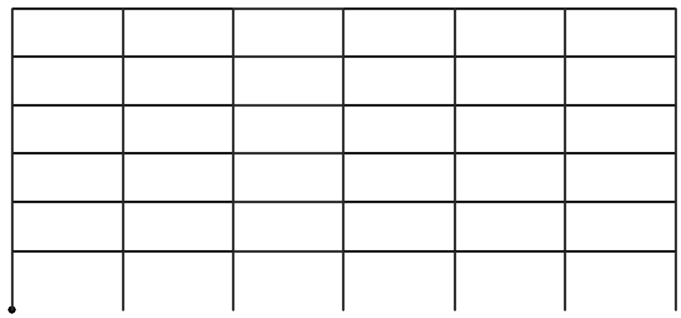
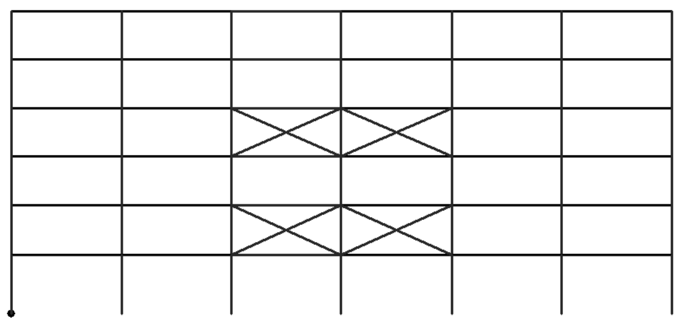
Topology of elevation view	**Description**
	Initial topology with full bracing
	Initial topology with no bracing
	Best design topology in both cases

Table 5-12 Six Floor - Topology of Side View

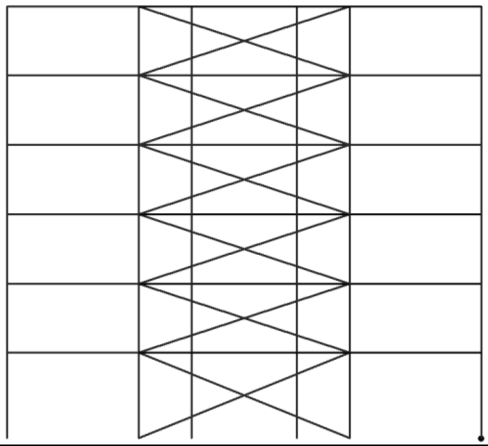
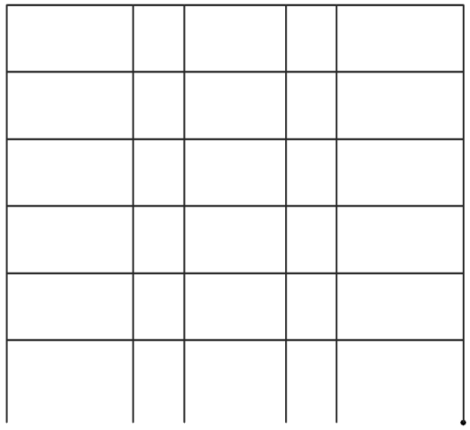
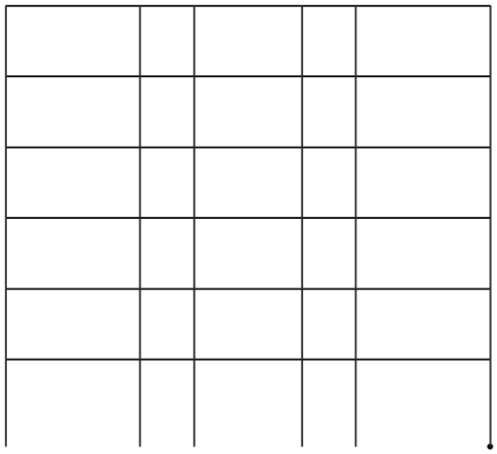
Topology of side view	Description
	Initial topology with full bracing
	Initial topology with no bracing
	Best design topology in both models

Table 5-13 Maximum Nodal Displacements and Location

Model Description	Longitudinal Displacement (in)	Transverse Displacement (in)	Vertical Displacement (in)
6F-DDV-F-S+D	0.68	0.15	0.66
	6-F-2A	6-G-1	6-G-1
6F-DDV-F-S+D+W	0.34	0.14	0.64
	6-G-2	6-G-1	5-B-2B
6F-BDV+DDV-F-S+D (Initial guess :full bracing)	0.17	0.13	0.63
	6-F-2A	6-G-1	5-F-2A
6F-BDV+DDV-F-S+D+W (Initial guess :full bracing)	0.25	0.13	0.62
	6-G-3	6-G-1	5-F-2A
6F-BDV+DDV-F-S+D (Initial guess :full bracing)	0.19	0.06	0.36
	6-G-3	6-G-1	5-F-2A
6F-BDV+DDV-F-S+D+W (Initial guess :full bracing)	0.25	0.13	0.62
	6-G-3	6-G-1	5-F-2A

5.4 Sixteen Floor: Sizing Optimization

5.4.1 Case 4 - Effect of RC Elevator Shaft core (MWD)

The two models considered in Case 4 are:

a) Sixteen story model with steel framework and floor slabs

b) Sixteen story model with steel framework, floor slabs and elevator shaft

Models are subjected to different types of design constraints for minimizing the total weight of the structure using web height as continuous design variable for each of cross section groups. The flange width, web thickness and flange thickness are defined using Eq.4 to Eq.6

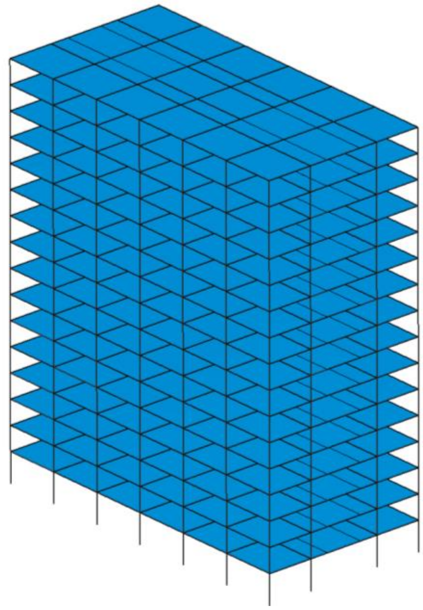


Figure 5-4 Sixteen story model with steel framework and floor slabs

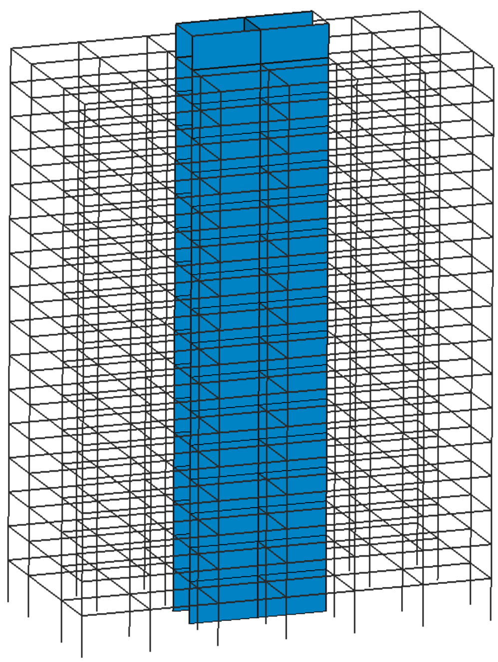


Figure 5-5 Sixteen story model with steel framework, floor slabs and elevator shaft

The design results for element stresses and nodal displacements show that:

1. Total weight of frame work increases 163% with frequency constraint when elevator shaft is not included in the model.
2. Total weight of frame work increases 3% with frequency constraint when elevator shaft is included in the model.

Table 5-14 Maximum Element Stresses and Location

Model Description	**Tensile stress (psi)**	**Compressive stress (psi)**	**Shear stress (psi)**
16F-CDV-W-S+D	25353	25221	5707
	TB-1-B-2B-3	C-1-F-3	TB-14-F-2B-3
16F-CDV-W-S+D+F	22505	22523	5109
	TB-1-B-2B-3	TB-12-B-2B-3	TB-15-F-2B-3
16F-ES- CDV-W-S+D	23890	23685	5461
	TB-1-B-2-2A	TB-2-B-2-2A	TB-15-B-2-2A
16F-ES-CDV-W-S+D+F	15395	24598	3721
	TB-2-B-2-2A	C-7-B-3	TB-15-B-2-2A

Table 5-15 Maximum Nodal Displacements and Location

Model Description	**Longitudinal Displacement (in)**	**Transverse Displacement (in)**	**Vertical Displacement (in)**
16F-CDV-W-S+D	4.98	1.48	1.32
	16-G-2	16-B-1	15-B-2B
16F-CDV-W-S+D+F	0.27	0.63	0.54
	16-G-3	16-A-1	15-F-2A
16F-ES-CDV-W-S+D	0.17	0.77	1.29
	17-C-2B	17-D-2B	15-B-2A
16F-ES-CDV-W-S+D+F	0.17	0.62	1.29
	17-C-2A	17-D-2B	15-B-2A

Table 5-16 Total Weight and Lowest frequency

Model description	Weight (Mlb)	Lowest frequency (Hz)	Constraint	Location
16F-CDV-W-S+D	18.51	0.32	Vertical Displacement	15th floor slab
16F-CDV-W-S+D+F	48.82	1.0	Lowest Frequency	-
16F-ES-CDV-W-S+D	22.07	0.32	Vertical Displacement	15th floor slab
16F-ES-CDV-W-S+D+F	22.73	1.0	Vertical Displacement	15th floor slab

5.5 Sixteen Floor: Sizing + Topology Optimization

5.5.1 Case 5 -Optimization of Bracing system (MWD)

The models considered in Case 5 are:

a) Sixteen story with steel framework with no bracing members and floor slabs

b) Sixteen story with steel framework with X-bracing and floor slabs.

The braced model is with initial design in which all bracing members are assigned Boolean design variable values of "1". The bracing members of each story and on either sides of the building were grouped together for symmetry. The optimization tool in Frame3D is then allowed to remove or retain the members in generating several topology alternatives. All the models are subjected to stress displacement and weight design constraints for minimizing the total weight of the structure using web height as continuous design variables for cross section groups. The design results for element stresses and nodal displacements show that:

1. Vertical deflection in floor slab governs the design in all three models.
2. Simultaneous topology and sizing optimization yields a reduction in the total weight of steel frame work by 3.2%.
3. By adding the weight constraint yields a reduction in the total weight of steel frame work by 5.3%.

Table 5-17 Total Weight and Lowest Frequency

Model description	**Weight (Mlb)**	**Constraint**	**Location**
16F-CDV-W-S+D	18.51	Vertical Displacement	15^{th} floor slab
16F-BDV+CDV-W-S+D (BDVs =1)	17.91	Vertical Displacement	15^{th} floor slab
16F-BDV+CDV-W-S+D+W (BDVs =1)	17.52	Vertical Displacement	15^{th} floor slab

Table 5-18 Maximum Element Stresses and Location

Model Description	**Tensile stress (psi)**	**Compressive stress (psi)**	**Shear stress (psi)**
16F-CDV-W-S+D	25315	25138	5698
	TB-1-B-2-2A	TB-2-B-2-2A	TB-14-B-2-2A
16F-BDV+CDV-W-S+D (Initial guess: full bracing)	20450	25790	4603
	TB-4-F-2B-3	C-6-F-2	TB-15-F-2-2A
16F-BDV+CDV-W-S+D+W (Initial guess: full bracing)	20452	25697	4665
	TB-4-B-2B-3	C-1-F-3	TB-2-F-2B-3

Table 5-19 Maximum Nodal Displacements and Location

Model Description	**Longitudinal displacement (in)**	**Transverse displacement (in)**	**Vertical displacement (in)**
16F-CDV-W-S+D	4.98	1.47	1.32
	16-G-2	16-B-1	15-B-2B
16F-BDV+CDV-W-S+D (Initial guess: full bracing)	3.29	1.48	1.27
	16-F-2B	16-F-1	15-F-2A
16F-BDV+CDV-W-S+D+W (Initial guess: full bracing)	4.78	1.74	1.25
	16-F-2B	16-B-1	15-F-2A

Table 5-20 Sixteen Floor - Topology of Elevation View

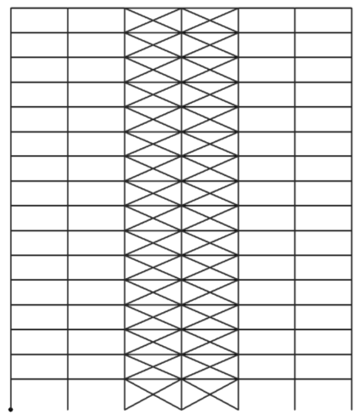
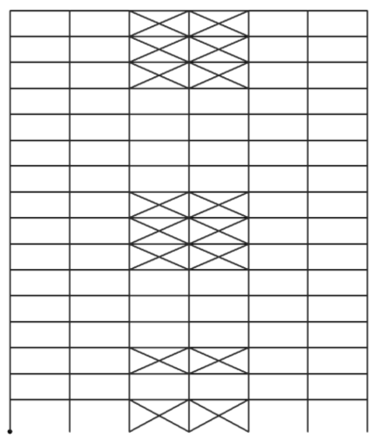
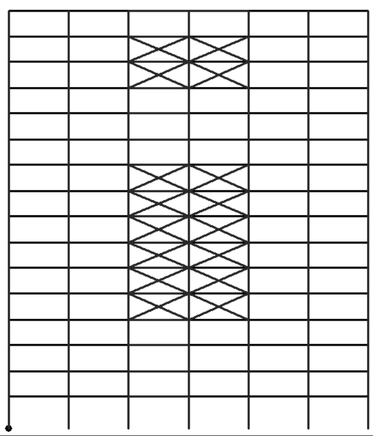
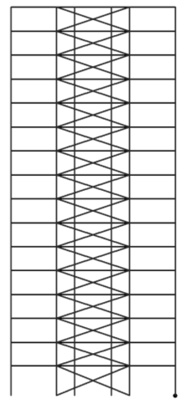
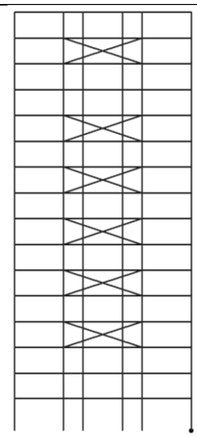
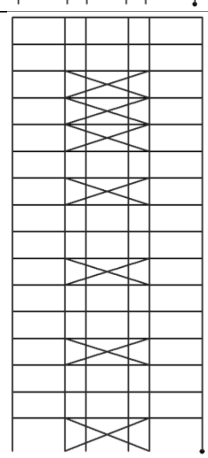
Topology of elevation view	Description
	Initial topology with full bracing
	Best design topology without weight constraint
	Best design topology with weight constraint

Table 5-21 Sixteen Floor - Topology of Side View

Topology of side view	Description
	Initial topology with full bracing
	Best design topology without weight constraint
	Best design topology with weight constraint

5.5.2 Case 6 -Optimization of Bracing system (MLF)

The models considered in Case 6 are:

a) Sixteen story model with steel framework with no bracing members and floor slabs

b) Sixteen story model with steel framework with X-bracing on periphery and floor slabs.

The models are subjected to stress displacement and weight design constraints for maximizing the lowest frequency of the structure using web height as continuous design variables for cross section groups. The design results for element stresses and nodal displacements show that:

1. With stress and displacement constraints a highly feasible solution was obtained with lowest frequency of 2.0Hz
2. When the weight constraint restricted the weight to be less than 50 Mlb, the solution was governed by tensile stress in transverse beams.
3. Simultaneous topology and sizing optimization yields a reduction in the total weight of steel frame work by 50% to achieve the lowest frequency of nearly 1.0Hz
4. Simultaneous topology and sizing optimization with the weight constraint did not yield a favorable design.

Table 5-22 Total Weight and Lowest frequency

Model description	Weight (Mlb)	Lowest Frequency (Hz)	Constraint	Location
16F-CDV-F-S+D	240	2.0	No Active constraints	-
16F-CDV-F-S+D+W	43.6	0.9	Tensile stress	First floor Transverse beam
16F-BDV+CDV-F-S+D	23.3	1.08	Vertical displacement	15th floor slab
16F-BDV+CDV-F-S+D+W	23.0	0.75	Tensile stress	First floor Transverse beam

Table 5-23 Maximum Element Stresses and Location

Model Description	Tensile stress (psi)	Compressive stress (psi)	Shear stress (psi)
16F-CDV-F-S+D	2043	2559	712
	TB-1-B-2B-3	C-1-B-3	TB-12-C-2-2A
16F-CDV-F-S+D+W	24940	24907	5597
	TB-1-F-2-2A	TB-6-B-2-2A	TB-15-B-2B-3
16F-BDV+CDV-F-S+D	18740	18582	4346
	TB-1-F-2-2A	TB-6-B-2B-3	TB-14-B-2-2A
16F-BDV+CDV-F-S+D+W	29973	29813	6577
	TB-1-F-2B-3	TB-6-B-2-2A	TB-15-B-2-2A

Table 5-24 Maximum Nodal Displacements and Location

Model Description	Longitudinal displacement (in)	Transverse displacement (in)	Vertical displacement (in)
16F-CDV-F-S+D	0.012	0.03	0.12
	16-G-1	16-A-1	15-B-2B
16F-CDV-F-S+D+W	0.32	0.79	0.61
	16-G-2	16-A-1	15-B-2B
16F-BDV+CDV-F-S+D	0.4	0.67	0.95
	16-G-2	16-F-1	15-B-2A
16F-BDV+CDV-F-S+D+W	0.79	0.82	1.31
	16-G-2	16-B-1	15-F-2A

Table 5-25 Sixteen Floor - Topology of Elevation View

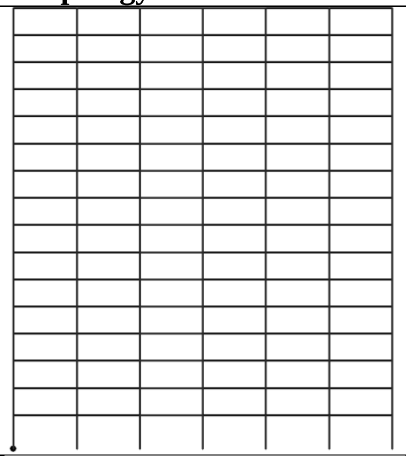
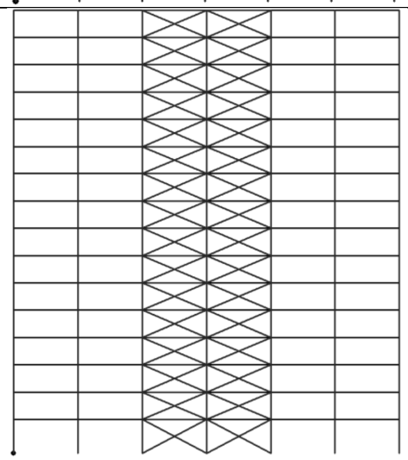
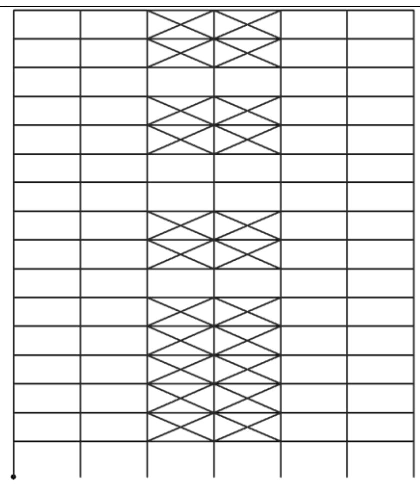
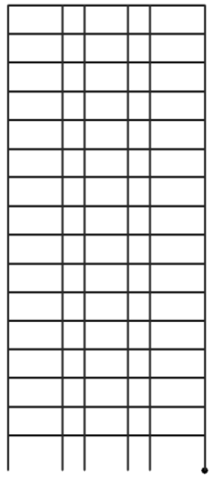
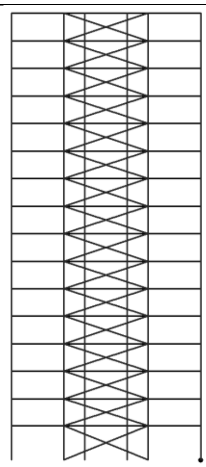
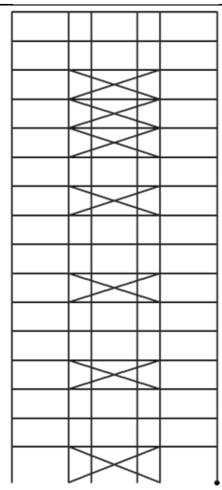
Topology of elevation view	Description
	Initial topology with full bracing
	Final topology for the braced model without weight constraint
	Final topology for the braced model with weight constraint

Table 5-26 Sixteen Floor - Topology of Side View

Topology of side view	**Description**
	Initial topology with full bracing
	Best design topology without weight constraint
	Best design topology with weight constraint

5.6 Forty Floor: Sizing Optimization (MWD)

The forty story model with steel frame work, floor slabs, and elevator shaft is subjected to different types of design constraints for minimizing the total weight of the structure using web height as continuous design variable for each cross section group. The flange width, web thickness and flange thickness are defined using Eq.4 to Eq.6.

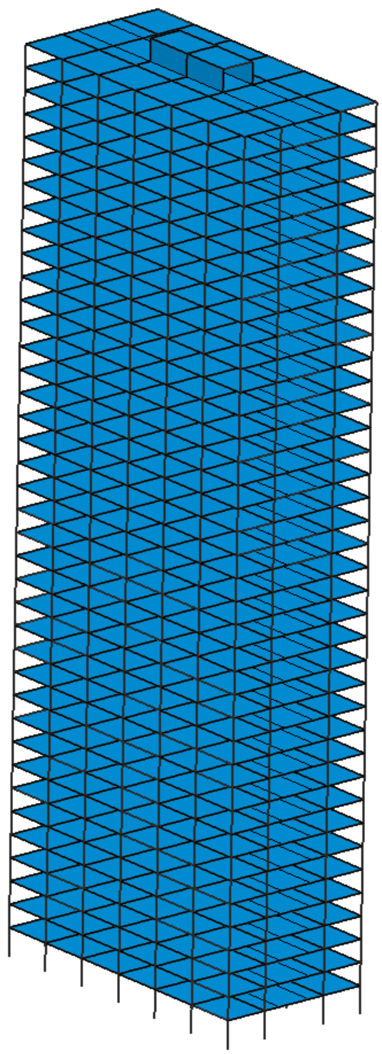


Figure 5-6 Forty story model with steel framework, floor slabs and elevator shaft

The design results for element stresses and nodal displacements show that:

1. When subjected to stress constraints alone, the design is governed by tensile stresses in transverse floor beams.
2. When subjected to stress and displacement constraints, vertical deflection in floor slab governs the design.
3. There is more than 100% increase in the total weight of steel frame work with the displacement constraints.
4. The maximum vertical displacement decreased from 4.5 in. to 1.29 in.
5. The maximum transverse displacement decreases form 10 in. to 3 in.

Table 5-27 Maximum Element Stresses and Location

Model Description	**Tensile stress (psi)**	**Compressive stress (psi)**	**Shear stress (psi)**
40F-ES-CDV-W-S	29996	29869	7430
	TB-17-F-2B-3	TB-15-F-2B-3	TB-6-D-2-2A
40F-ES-CDV-W-S+D	11981	11980	3127
	TB-13-F-2B-3	TB-15-F-2B-3	TB-7-D-2B-3

Table 5-28 Maximum Nodal Displacements and Location

Model Description	**Longitudinal Displacement (in)**	**Transverse Displacement (in)**	**Vertical Displacement (in)**
40F-ES-CDV-W-S	3.75	10	4.63
	41-C-2A	41-C-2B	41-B-2B
40F-ES-CDV-W-S+D	1.36	2.68	1.29
	41-C-2A	41-C-2A	40-F-2A

Table 5-29 Total Weight and Lowest frequency

Model description	Weight (Mlb)	Lowest frequency (Hz)	Constraint	Location
40F-ES-CDV-W-S	55.9	0.3	Tensile stress	Transverse floor beam 17th floor
40F-ES-CDV-W-S+D	118	0.42	Vertical Displacement	40th Floor Slab

5.7 Forty Floor: Sizing + Topology Optimization (MWD)

The models considered are:

a) Forty story model with steel framework with no bracing members and floor slabs.

b) Forty story model with steel framework with belt trusses and floor slabs.

Two types of braced models are created, a model with initial design in which all bracing members are assigned Boolean design variable values of "1" and another model, with initial design in which all the Boolean design variable values are "0". The bracing members of four stories and on either sides of the building were grouped together for symmetry. The optimization tool in Frame3D is then allowed to remove or retain the members in generating several topology alternatives. All the models are subjected to stress and displacement design constraints for maximizing the lowest frequency of the structure using web height as continuous design variables for cross section groups

Table 5-30 Total Weight and Lowest Frequency

Model description	**Weight (Mlb)**	**Lowest Frequency (Hz)**	**Constraint**
40F-ES-CDV-F-S+D	180.8	0.63	No active constraints
40F-ES-BDV+CDV-F-S+D (Initial guess: No bracing)	420.7	0.68	No active constraints
40F-ES-BDV+CDV-F-S+D (Initial guess: full bracing)	561.1	0.79	No active constraints

Table 5-31 Maximum Element Stresses and Location

Model Description	**Tensile stress (psi)**	**Compressive stress (psi)**	**Shear stress (psi)**
40F-ES-CDV-F-S+D	4941	5550	940
	BB-36-4-E-F	BB-35-4-E-F	TB-39-E-2B-3
40F-ES-BDV+CDV-F-S+D (Initial guess: No bracing)	4845.7	5673.7	1484
	C-15-B-3	C-40-E-2	TB-39-E-2-2A
40F-ES-BDV+CDV-F-S+D (Initial guess: Full bracing)	6697.8	7009.1	928.3
	BB-29-4-E-F	BB-28-4-E-F	TB-7-D-2-2A

Table 5-32 Maximum Nodal Displacements and Location

Model Description	**Longitudinal displacement (in)**	**Transverse displacement (in)**	**Vertical displacement (in)**
40F-ES-CDV-F-S+D	0.15	0.34	0.21
	41-C-2A	41-C-2B	40-F-2A
40F-ES-BDV+CDV-F-S+D (Initial guess: No bracing)	0.07	0.27	0.69
	41-C-2A	41-C-2B	40-G-2A
40F-ES-BDV+CDV-F-S+D (Initial guess: full bracing)	0.21	0.35	0.49
	41-C-2A	41-C-2B	40-F-2A

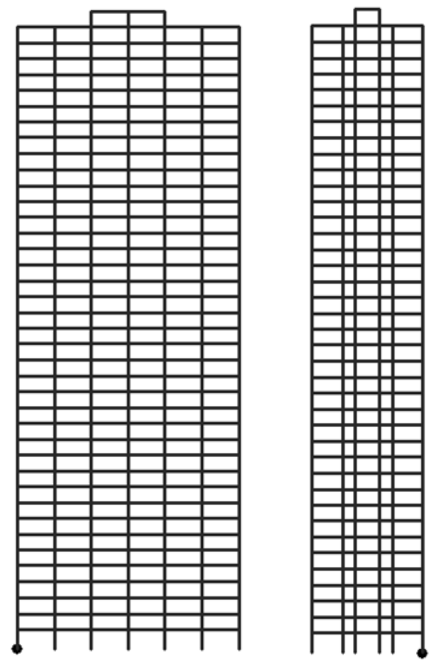


Figure 5-7 Initial topology with no bracing

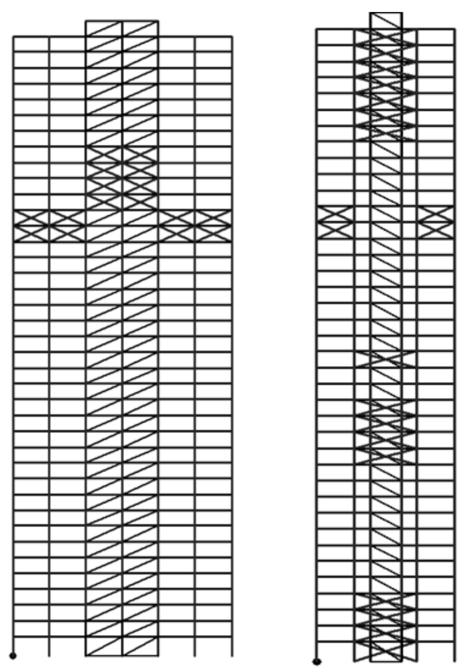


Figure 5-8 Best design topology with initial guess- no bracing

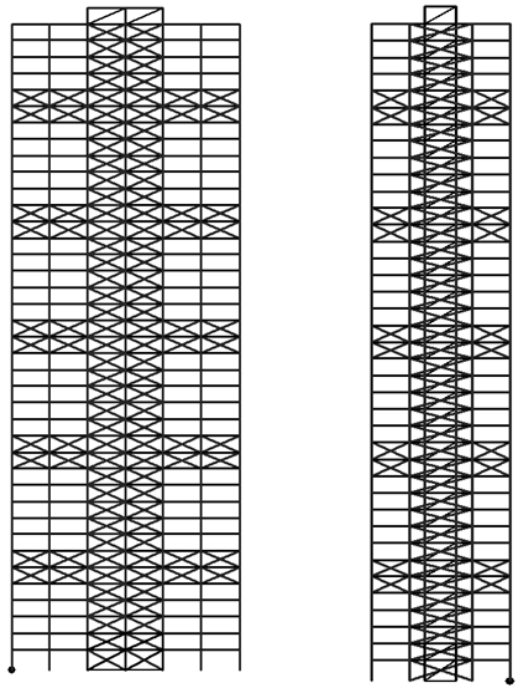


Figure 5-9 Initial topology with full bracing

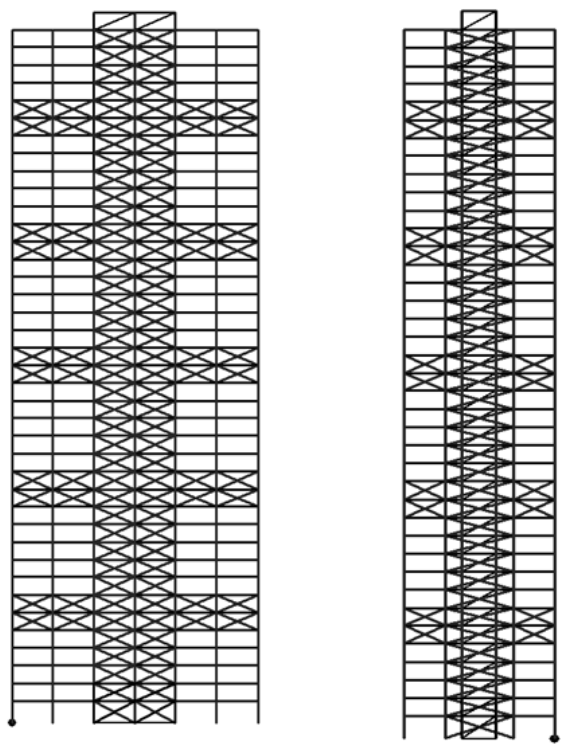


Figure 5-10 Best design topology with initial guess- full bracing

5.8 Forty Floor: Mesh Convergence

In order to understand the effect of finite element size on member stresses and nodal displacements, three forty floor models with different element sizes were analyzed. The models for analysis were the best design obtained from sizing optimization.

Table 5-33 Forty Floor: Model Details for Mesh Convergence

Model	Mesh #1	Mesh #2	Mesh #3
Nodes	2512	9913	31055
Shell elements	1200	6880	26560
Smallest size (ft.)	30x10	10x6.5	5x3.25
Largest size (ft.)	30x30	10x12.5	5x6.25
Beam elements	3816	6918	13440
Smallest size (ft.)	10	6.5	3.25
Largest size (ft.)	30	16	16
Total DOF	9864	43740	168456

The mesh convergence study with the three models –Mesh#1, Mesh#2, and Mesh #3 show that:

1. Nodal displacements are accurate even with a coarse mesh.
2. Accuracy of beam and plate element stresses increases with mesh refinement.
3. Both displacements and stresses converge from below.

Table 5-34 Forty Floor: Maximum Beam Element Stresses

Model Description	Tensile stress (psi)	Compressive stress (psi)	Shear stress (psi)
Mesh #1	7214	15260	1715
Mesh #2	19427	19249	10260
Mesh #3	20044	19847	12673

Table 5-35 Forty Floor: Maximum Stresses in Slab Elements

Model Description	Tensile stress (psi)	Compressive stress (psi)
Mesh #1	67	82
Mesh #2	12457	0
Mesh #3	17195	465

Table 5-36 Forty Floor: Maximum Stresses in Wall Elements

Model Description	Tensile stress (psi)	Compressive stress (psi)
Mesh #1	103	779
Mesh #2	275	771
Mesh #3	499	833

Table 5-37 Forty Floor: Maximum Nodal Displacements

Model Description	Longitudinal Displacement (in)	Transverse Displacement (in)	Vertical Displacement (in)
Mesh #1	0.767	1.33	1.03
Mesh #2	0.778	1.34	14
Mesh #3	0.778	1.34	16

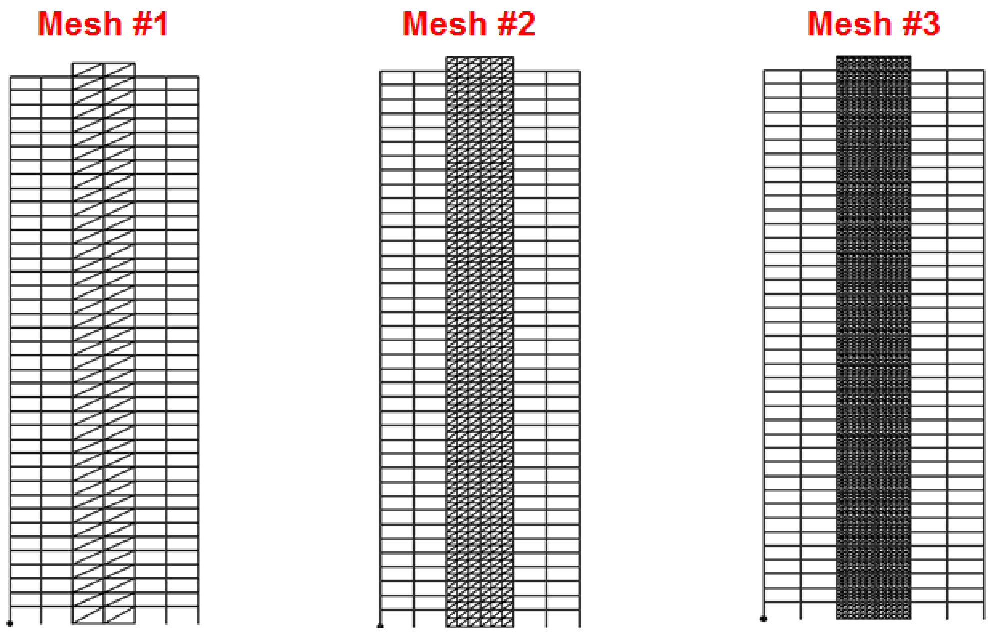


Figure 5-11 Forty Floor: FE mesh sizes of wall elements

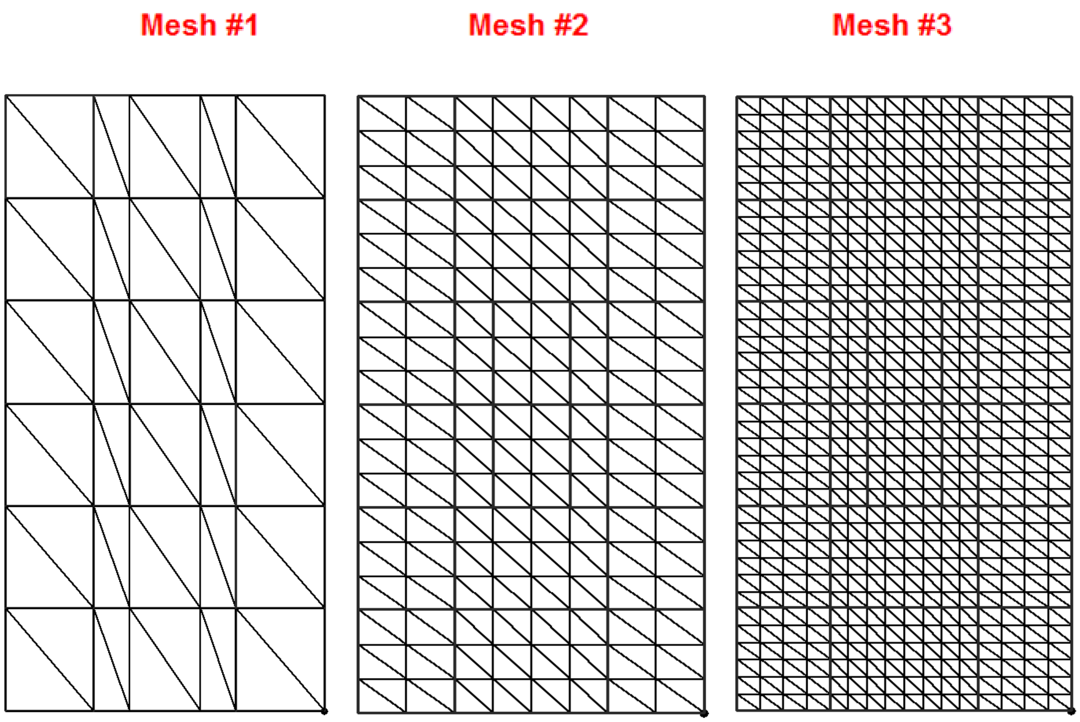


Figure 5-12 Forty Floor: FE mesh sizes of slab/beam elements

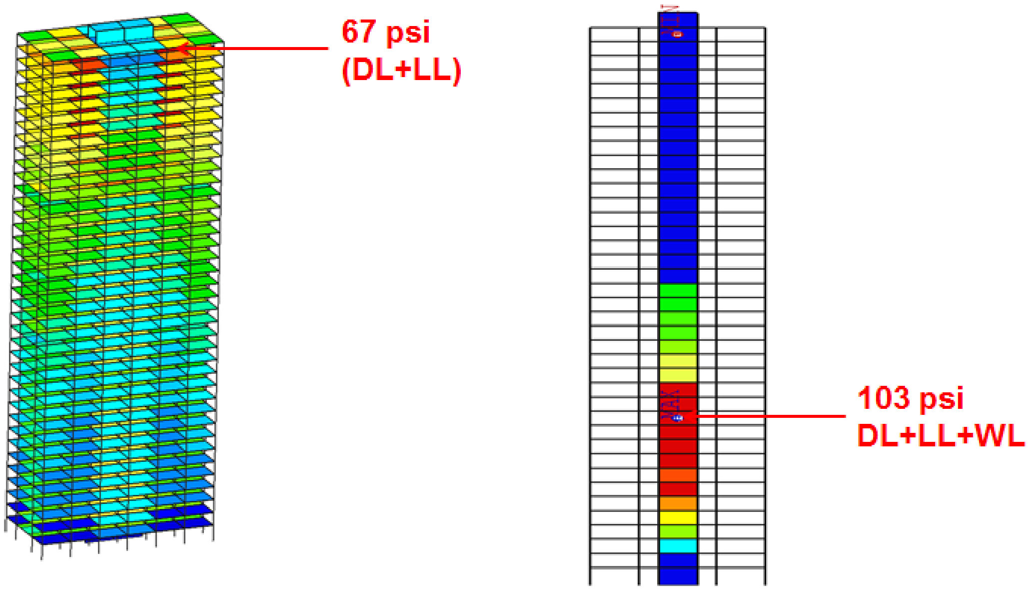


Figure 5-13 Mesh #1: Contour plot showing maximum element stresses in plate elements

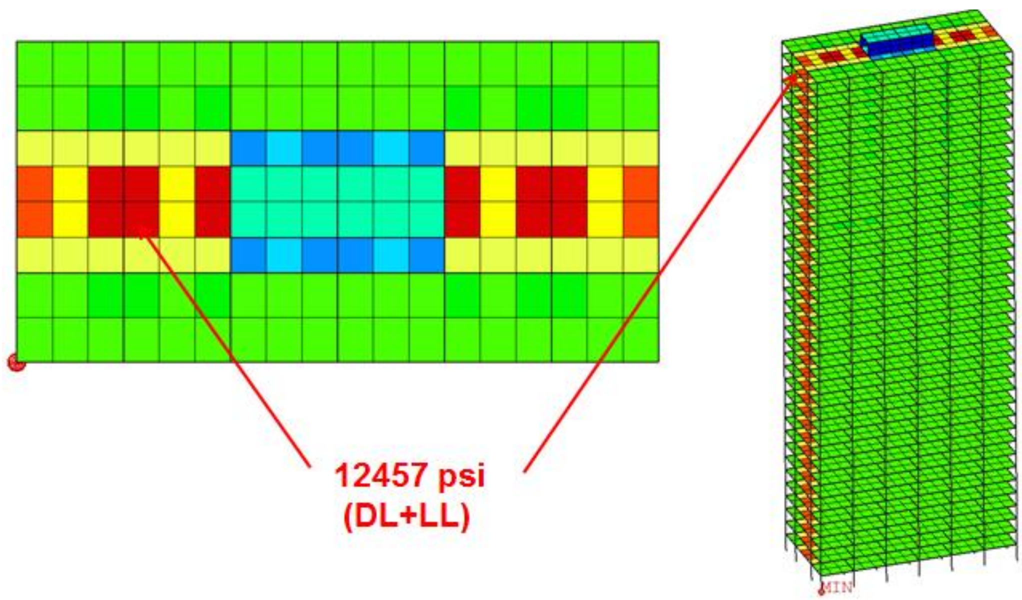


Figure 5-14 Mesh #2: Contour plot showing maximum element stresses in plate elements

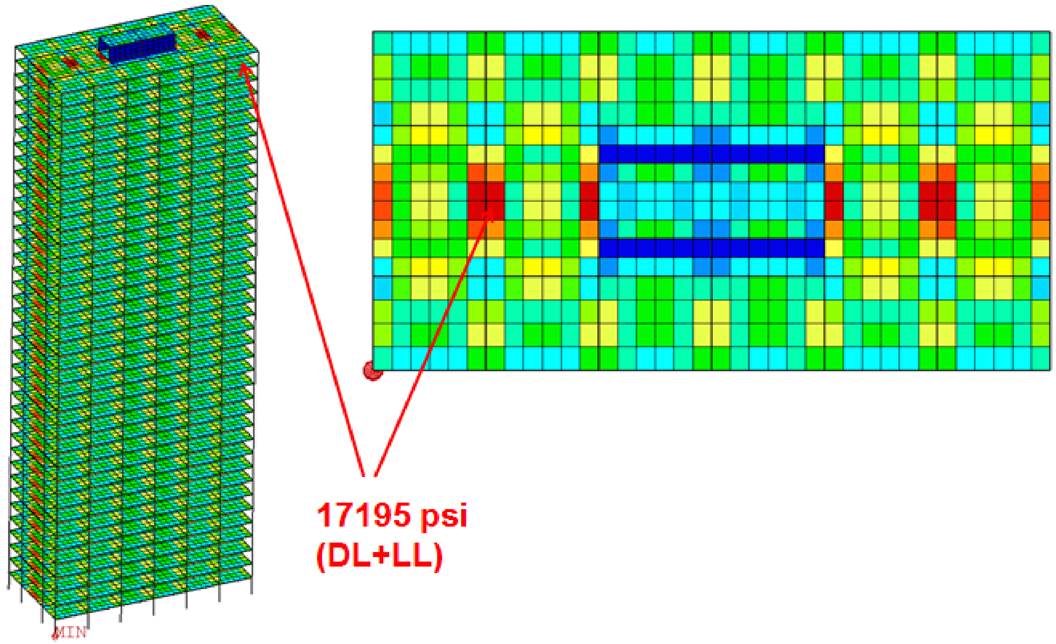


Figure 5-15 Mesh #3: Contour plot showing maximum element stresses in plate elements

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APPENDIX A

DESIGN WIND PRESSURE CALCULATIONS

Table A-1 Six Floor Building

Surface	Height (ft)	q_z (psf)	q_h (psf)	Cp	+Gcpi	p(+internal) (psi)
LW wall	81	38.01	38.01	-0.5	0.18	-0.065
Side wall	81	38.01	38.01	-0.7	0.18	-0.109
WW wall	16	26.95	38.01	0.8	0.18	0.175
	29	30.46	38.01	0.8	0.18	0.191
	42	32.90	38.01	0.8	0.18	0.203
	55	34.78	38.01	0.8	0.18	0.212
	68	36.41	38.01	0.8	0.18	0.219
	81	38.01	38.01	0.8	0.18	0.227

Table A-2 Sixteen Floor Building

Surface	Height (ft)	q_z (psf)	q_h (psf)	Cp	+Gcpi	p(+internal) (psi)
LW wall	211	46.230	46.230	-0.5	0.18	-0.079
Side wall	211	46.230	46.230	-0.7	0.18	-0.133
WW wall	16	26.947	46.230	0.8	0.18	0.185
	29	30.457	46.230	0.8	0.18	0.202
	42	32.901	46.230	0.8	0.18	0.213
	55	34.781	46.230	0.8	0.18	0.222
	68	36.410	46.230	0.8	0.18	0.230
	81	38.008	46.230	0.8	0.18	0.237
	94	39.105	46.230	0.8	0.18	0.242
	107	40.029	46.230	0.8	0.18	0.247
	120	41.048	46.230	0.8	0.18	0.252
	133	42.066	46.230	0.8	0.18	0.256
	146	42.896	46.230	0.8	0.18	0.260
	159	43.507	46.230	0.8	0.18	0.263
	172	44.306	46.230	0.8	0.18	0.267
	185	45.043	46.230	0.8	0.18	0.271
	198	45.654	46.230	0.8	0.18	0.273
	211	46.230	46.230	0.8	0.18	0.276

Table A-3 Forty Floor Building

Surface	Height (ft)	q_z (psf)	q_h (psf)	Cp	+Gcpi	p(+internal) (psi)
LW wall	510	55.460	55.460	-0.5	0.18	-0.0944
Side wall	510	55.460	55.460	-0.7	0.18	-0.1598
WW wall	16	26.947	55.460	0.8	0.18	0.1966
	29	30.457	55.460	0.8	0.18	0.2131
	42	32.901	55.460	0.8	0.18	0.2247
	55	34.781	55.460	0.8	0.18	0.2336
	68	36.410	55.460	0.8	0.18	0.2413
	81	38.008	55.460	0.8	0.18	0.2488
	94	39.105	55.460	0.8	0.18	0.2540
	107	40.029	55.460	0.8	0.18	0.2584
	120	41.048	55.460	0.8	0.18	0.2632
	133	42.066	55.460	0.8	0.18	0.2680
	146	42.896	55.460	0.8	0.18	0.2719
	159	43.507	55.460	0.8	0.18	0.2748
	172	44.306	55.460	0.8	0.18	0.2785
	185	45.043	55.460	0.8	0.18	0.2820
	198	45.654	55.460	0.8	0.18	0.2849
	211	46.230	55.460	0.8	0.18	0.2876
	224	46.800	55.460	0.8	0.18	0.2903
	237	47.37074	55.460	0.8	0.18	0.2930
	250	47.94102	55.460	0.8	0.18	0.2957
	263	48.42983	55.460	0.8	0.18	0.2980
	276	48.91864	55.460	0.8	0.18	0.3003
	289	49.40745	55.460	0.8	0.18	0.3026
	302	49.82106	55.460	0.8	0.18	0.3046
	315	50.29107	55.460	0.8	0.18	0.3068
	328	50.69841	55.460	0.8	0.18	0.3087
	341	51.10575	55.460	0.8	0.18	0.3107
	354	51.5131	55.460	0.8	0.18	0.3126
	367	51.92044	55.460	0.8	0.18	0.3145
	380	52.32778	55.460	0.8	0.18	0.3164
	393	52.73512	55.460	0.8	0.18	0.3184
	406	53.10486	55.460	0.8	0.18	0.3201

Surface	Height (ft)	q_z (psf)	q_h (psf)	Cp	+Gcpi	p(+internal) (psi)
WW Wall	419	53.43074	55.460	0.8	0.18	0.3216
	432	53.75661	55.460	0.8	0.18	0.3232
	445	54.08248	55.460	0.8	0.18	0.3247
	458	54.40836	55.460	0.8	0.18	0.3263
	471	54.73423	55.460	0.8	0.18	0.3278
	484	55.0601	55.460	0.8	0.18	0.3293
	497	55.38598	55.460	0.8	0.18	0.3309
	510	55.46118	55.460	0.8	0.18	0.3312

APPENDIX B

AISC WIDE FLANGE SECTIONS

Symbol	**Property description**
$tag	Cross section tag for wide flange section
Area	Cross sectional area
Iz	Moment of inertia about local *z* axis
Iy	Moment of inertia about *y* axis
J	Torsional constant
Az	0.833*Cross sectional area
Ay	1.0*Az
Sz	Section modulus about the local *z* axis
Sy	Section modulus about the local *y* axis
SFz	"Shear factor" for computing shear stress (shear force acting in the local *z* direction)
SFy	"Shear factor" for computing shear stress (shear force acting in the local *y* direction)
Js	"Torsional factor" for computing the shear stress due to torsional moment

$ tag	area	Iz	Iy	J	Az	Ay	Sz	Sy	SFz	SFy	Js
WF44X335	98.3	31100	1200	74.0874	81.917	1	1410	151	17.9177	39.3234	48.063
WF44X290	85.8	27100	1050	50.7702	71.5	1	1240	132	16.1958	33.5691	36.897
WF44X262	77.2	24200	927	37.0428	64.333	1	1120	118	14.3426	30.2756	29.952
WF44X230	67.2	20800	796	24.0995	56	1	971	101	12.3411	26.9983	22.681
WF40X593	174	50400	2520	451.2055	145	1	2340	302	33.9379	65.7857	158.629
WF40X503	148	41700	2050	277.7621	123.333	1	1980	250	28.8047	55.9426	114.385
WF40X431	127	34800	1690	173.1892	105.833	1	1690	208	24.4612	48.1483	83.459
WF40X397	117	3200	1540	137.7735	97.5	1	1560	191	22.6812	4.3795	71.199
WF40X372	109	29600	1420	112.5265	90.833	1	1460	177	20.9448	41.4012	62.398
WF40X362	107	28900	1380	104.6918	89.167	1	1420	173	20.6421	40.0663	59.23
WF40X324	95.3	25600	1220	75.6519	79.417	1	1280	153	18.5611	35.5716	47.523
WF40X297	87.4	23200	1090	57.5458	72.833	1	1170	138	16.8206	33.0839	39.676
WF40X277	81.4	21900	1040	48.8123	67.833	1	1100	132	16.1487	29.588	35.049
WF40X249	73.3	19600	926	35.5009	61.083	1	993	118	14.4191	26.6609	28.361
WF40X215	63.4	16700	796	22.5854	52.833	1	859	101	12.4389	22.9626	20.998
WF40X199	58.5	14900	695	16.3485	48.75	1	770	88.2	10.8231	22.6981	17.359
WF40X392	115	29900	803	169.5906	95.833	1	1440	130	19.0751	50.0779	78.764
WF40X331	97.5	24700	644	102.0035	81.25	1	1210	106	15.9399	42.6795	56.086
WF40X327	96	24500	640	99.1317	80	1	1200	105	16.1664	41.5816	54.546
WF40X278	81.8	20500	521	61.0179	68.167	1	1020	87.1	13.4875	35.5901	39.522
WF40X264	77.6	19400	493	55.312	64.667	1	971	82.6	12.8047	24.6094	38.572
WF40X235	69	17400	444	38.557	57.5	1	875	74.6	11.8736	29.0648	28.558
WF40X211	62	15500	390	27.8655	51.667	1	786	66.1	10.6498	26.2116	22.984
WF40X183	53.8	13300	3336	17.7431	44.833	1	683	56.9	91.6669	22.7176	17.029

$ tag	area	Iz	Iy	J	Az	Ay	Sz	Sy	SFz	SFy	Js
WF40X167	49.2	11600	283	11.7947	41	1	600	47.9	7.7098	22.4675	13.487
WF40X149	43.8	9800	229	7.6128	36.5	1	513	38.8	6.1902	21.4267	10.363
WF36X798	235	62600	4200	1116.903	195.833	1	2980	467	48.546	83.3145	292.051
WF36X650	191	48900	3230	614.7035	159.167	1	2420	367	39.3807	67.5351	194.85
WF36X527	155	38300	2490	333.0463	129.167	1	1950	289	32.056	54.4519	128.457
WF36X439	129	31000	1990	173.5195	107.5	1	1620	235	26.3269	46.5977	83.704
WF36X393	116	27500	1750	140.8057	96.667	1	1450	208	23.847	40.5188	71.87
WF36X359	105	24800	1570	106.9828	87.5	1	1320	188	21.7071	37.0367	59.778
WF36X328	96.4	22500	1420	82.5392	80.333	1	1210	171	19.9332	33.5773	50.1
WF36X300	88.3	20300	1300	62.6414	73.583	1	1110	156	18.0712	30.8137	41.847
WF36X280	88.4	18900	1200	50.8974	73.667	1	130	144	16.9085	28.9243	36.398
WF36X260	76.5	17300	1090	39.9321	63.75	1	953	132	15.3658	27.2175	31.147
WF36X245	72.1	16100	1010	32.9948	60.083	1	895	123	14.42	25.8332	27.461
WF36X230	67.6	15000	940	27.0728	56.333	1	837	114	13.4333	24.467	24.133
WF36X256	75.4	16800	528	52.6316	62.833	1	895	86.5	13.3772	31.3374	35.3
WF36X232	68.1	15000	468	39.016	56.75	1	809	77.2	12.109	28.1918	28.848
WF36X210	61.8	13200	411	25.7329	51.5	1	719	67.5	10.5283	25.0202	21.937
WF36X194	57	12100	375	20.6128	47.5	1	664	61.9	9.7721	23.5015	18.981
WF36X182	53.6	11300	347	16.9347	44.667	1	623	57.6	9.0683	22.0402	16.645
WF36X170	50.1	10500	320	13.8611	41.75	1	581	53.2	8.5033	20.9623	14.623
WF36X160	47	9760	295	11.3363	39.167	1	542	49.1	7.8404	20.0541	12.878
WF36X150	44.2	9040	270	9.1618	36.833	1	504	45.1	7.1754	19.1002	11.264
WF36X135	39.7	7800	225	6.4506	33.083	1	439	37.7	5.9308	18.5861	9.17
WF33X387	114	24300	1620	150.4896	95	1	1350	200	23.7027	39.3332	73.987
WF33X354	104	22000	1460	115.4233	86.667	1	1240	181	21.6839	36.0995	61.915
WF33X318	93.6	19500	1290	84.5033	78	1	1110	161	19.4724	32.0894	50.112
WF33X291	85.7	17700	1160	64.6365	71.417	1	1020	146	17.767	29.6704	41.884
WF33X263	77.5	15900	1040	47.991	64.583	1	919	131	16.1742	26.8403	34.272
WF33X241	71	14200	933	35.3379	59.167	1	831	118	14.3243	25.2184	28.308
WF33X221	65	12800	840	26.8989	54.167	1	759	106	13.0723	23.3791	23.661
WF33X201	59.1	11500	749	19.882	49.25	1	686	95.2	11.8202	21.5433	19.386
WF33X169	33.8	9290	310	14.9333	28.167	1	549	53.9	8.9597	20.8171	15.025
WF33X152	44.7	8160	273	11.8256	37.25	1	487	47.2	7.8077	18.7659	12.945
WF33X141	41.6	7450	246	9.1937	34.667	1	448	42.7	7.0713	17.7726	11.036
WF33X130	38.3	6710	218	6.8925	31.917	1	406	37.9	6.2891	16.9206	9.224
WF33X118	34.7	5900	187	4.8826	28.917	1	359	32.6	5.4123	15.8966	7.43
WF30X391	115	20700	1550	176.8704	95.833	1	1250	198	24.4004	39.0042	80.882
WF30X357	105	18700	1390	135.5629	87.5	1	1140	179	22.2427	35.4104	67.512
WF30X326	95.8	16800	1240	103.4369	79.833	1	1040	162	20.1519	32.2858	56.293
WF30X292	85.9	14900	1100	75.2477	71.583	1	930	144	18.1782	28.7399	45.366
WF30X261	76.9	13100	959	53.5504	64.083	1	829	127	16.0867	25.9945	36.223
WF30X235	69.2	11700	855	39.6548	57.667	1	748	114	14.5837	23.0897	29.493
WF30X211	62.2	10300	757	27.4876	51.833	1	665	100	12.9139	21.4719	23.339
WF30X191	56.3	9200	673	20.1589	46.917	1	600	89.5	11.6526	19.6	18.998
WF30X173	51	8230	598	14.8311	42.5	1	541	79.8	10.3683	17.9538	15.543
WF30X148	43.5	6680	227	14.2035	36.25	1	436	43.3	7.8735	17.4987	13.904
WF30X132	38.9	5770	196	9.3059	32.417	1	380	37.2	6.7206	16.4116	10.729
WF30X124	36.5	5360	181	7.5925	30.417	1	355	34.4	6.2479	15.5503	9.401
WF30X116	34.2	4930	164	6.052	28.5	1	329	31.3	5.6764	14.917	8.16

$ tag	area	Iz	Iy	J	Az	Ay	Sz	Sy	SFz	SFy	Js
WF30X108	31.7	4470	146	4.6356	26.417	1	299	27.9	5.0623	14.2994	6.914
WF30X99	29.1	3990	128	3.4536	24.25	1	269	24.5	4.4555	13.5125	5.739
WF30X90	26.4	3610	115	2.5736	22	1	245	22.1	4.0607	12.2395	4.707
WF27X539	159	25600	2110	526.2943	132.5	1	1570	277	34.029	53.4585	165.286
WF27X368	108	16200	1310	173.9379	90	1	1060	179	23.2004	36.0251	77.998
WF27X336	98.9	14600	1180	133.8464	82.417	1	972	162	21.2596	32.6573	65.267
WF27X307	90.4	13100	1050	101.955	75.333	1	887	146	19.4857	29.9331	54.273
WF27X281	82.9	11900	953	79.881	69.083	1	814	133	17.7496	27.0636	46.01
WF27X258	76	10800	859	61.4074	63.333	1	745	120	16.2538	24.9614	38.584
WF27X235	69.4	9700	769	46.3118	57.833	1	677	108	14.7748	23.016	32.016
WF27X217	64	8910	704	36.852	53.333	1	627	99.8	13.7606	21.0451	27.327
WF27X194	57.2	7860	619	26.2201	47.667	1	559	88.1	12.2979	18.9192	21.776
WF27X178	52.5	7020	555	19.1096	43.75	1	505	78.8	11.011	18.1541	17.881
WF27X161	47.6	6310	497	14.3146	39.667	1	458	70.9	9.8568	16.3926	14.751
WF27X146	43.1	5660	443	10.5812	35.917	1	414	63.5	8.8629	14.9707	12.075
WF27X129	37.8	4760	184	10.8783	31.5	1	345	36.8	7.0568	14.8946	11.354
WF27X114	33.5	4080	159	6.9896	27.917	1	299	31.5	6.0895	13.7895	8.621
WF27X102	30	3620	139	5.0075	25	1	267	27.8	5.3429	12.3784	6.914
WF27X94	27.7	3270	124	3.7804	23.083	1	243	24.8	4.7739	11.6944	5.791
WF27X84	24.8	2850	106	2.5865	20.667	1	213	21.2	4.1004	10.9	4.559
WF24X370	109	13400	1160	213.3888	90.833	1	957	170	23.5209	35.877	87.041
WF24X335	98.4	11900	1030	159.195	82	1	864	152	21.5815	32.4242	71.236
WF24X306	89.8	10700	919	122.4308	74.833	1	789	137	19.615	29.3618	59.574
WF24X279	82	9600	823	93.1426	68.333	1	718	124	18.1388	26.9632	49.478
WF24X250	73.5	8490	724	68.5638	61.25	1	644	110	16.028	23.8367	40.235
WF24X229	67.2	7650	651	52.3762	56	1	588	99.4	14.661	21.8583	33.594
WF24X207	60.7	6820	578	38.8356	50.583	1	531	88.8	13.254	19.6734	27.45
WF24X192	56.3	6260	530	31.2305	46.917	1	491	81.8	12.1788	18.1247	23.731
WF24X176	51.7	5680	479	24.0478	43.083	1	450	74.3	11.1957	16.787	19.916
WF24X162	47.7	5170	443	18.5149	39.75	1	414	68.4	10.2076	15.6145	16.839
WF24X146	43	4580	391	13.3022	35.833	1	371	60.5	9.1579	14.3384	13.548
WF24X131	38.5	4020	340	9.3213	32.083	1	329	53	8.0151	13.2414	10.771
WF24X117	34.4	3540	297	6.5388	28.667	1	291	46.5	7.0776	11.9933	8.526
WF24X104	30.6	3100	259	4.5572	25.5	1	258	40.7	6.2289	10.8194	6.724
WF24X103	30.3	3000	119	6.9515	25.25	1	245	26.5	5.6346	11.9003	8.134
WF24X94	27.7	2700	109	5.1156	23.083	1	222	24	5.0939	11.0616	6.699
WF24X84	24.7	2370	94.4	3.5527	20.583	1	196	20.9	4.4635	10.0425	5.283
WF24X76	22.4	2100	82.5	2.5444	18.667	1	176	18.4	3.9271	9.3307	4.271
WF24X68	20.1	1830	70.4	1.748	16.75	1	154	15.7	3.3647	8.7199	3.374
WF24X62	18.2	1550	34.5	1.5774	15.167	1	132	9.8	2.599	8.8355	3.061
WF24X55	16.2	1360	29.1	1.0753	13.5	1	115	8.3	2.2148	8.1485	2.391
WF21X201	59.2	5310	542	41.7462	49.333	1	461	56.1	13.2377	18.3191	28.217
WF21X182	53.6	4730	483	31.0594	44.667	1	417	77.2	12.0116	16.5851	23.126
WF21X166	48.8	4280	435	23.7672	40.667	1	380	70	11.0231	14.935	19.254
WF21X147	43.2	3630	376	15.2856	36	1	329	60.1	9.3443	14.0855	14.643
WF21X132	38.8	3220	333	11.0986	32.333	1	295	53.5	8.3886	12.6391	11.813
WF21X122	35.9	2960	305	8.7998	29.917	1	273	49.2	7.76	11.6451	10.099
WF21X111	32.7	2670	274	6.6556	27.25	1	249	44.5	7.0396	10.6223	8.379
WF21X101	29.8	2420	248	5.0517	24.833	1	227	40.3	6.4361	9.6432	6.957

$ tag	area	Iz	Iy	J	Az	Ay	Sz	Sy	SFz	SFy	Js
WF21X93	27.3	2070	92.9	5.8607	22.75	1	192	22.1	4.9899	10.9829	7.175
WF21X83	24.3	1830	81.4	4.1805	20.25	1	171	19.5	4.4637	9.7184	5.704
WF21X73	21.5	1600	70.6	2.8846	17.917	1	151	17	3.9457	8.5521	4.443
WF21X68	20	1480	64.7	2.3139	16.667	1	140	15.7	3.642	8.0662	3.847
WF21X62	18.3	1330	57.5	1.7125	15.25	1	127	14	3.2639	7.4773	3.164
WF21X55	16.2	1140	48.4	0.9757	13.5	1	110	11.8	4.2735	8.0881	2.137
WF21X48	14.1	959	38.7	0.6328	11.75	1	93	9.52	3.4353	7.4201	1.625
WF21X57	16.7	1170	30.6	1.6521	13.917	1	111	9.35	2.6924	7.492	2.962
WF21X50	14.7	984	24.9	1.0378	12.25	1	94.5	7.64	2.1983	6.9248	2.223
WF21X44	13	843	20.7	0.6837	10.833	1	81.6	6.37	1.8425	6.3262	1.703
WF18X175	51.3	3450	391	34.8757	42.75	1	344	68.8	11.6586	15.4823	24.074
WF18X158	46.3	3060	347	25.7291	38.583	1	310	61.4	10.5534	13.9869	19.615
WF18X143	42.1	2750	311	19.5305	35.083	1	282	55.5	9.6551	12.5214	16.239
WF18X130	38.2	2460	278	14.717	31.833	1	256	49.9	8.6469	11.3238	13.46
WF18X119	35.1	2190	253	10.6261	29.25	1	231	44.9	7.7594	11.0152	11.003
WF18X106	31.1	1910	220	7.4196	25.917	1	204	39.4	6.8451	9.8345	8.662
WF18X97	28.5	1750	201	5.7994	23.75	1	188	36.1	6.326	8.9223	7.317
WF18X86	25.3	1530	175	4.0248	21.083	1	166	31.6	5.5773	7.9595	5.737
WF18X76	22.3	1330	152	2.7628	18.583	1	146	27.6	4.8985	6.9884	4.459
WF18X71	20.8	1170	60.3	3.419	17.333	1	127	15.8	3.9644	8.0242	4.782
WF18X65	19.1	1070	54.8	2.6693	15.917	1	117	14.4	3.6605	7.2883	4.034
WF18X60	17.6	984	50.1	2.1088	14.667	1	108	13.3	3.3871	6.7175	3.44
WF18X55	16.2	890	44.9	1.6009	13.5	1	98.3	11.9	3.0578	6.2763	2.879
WF18X50	14.7	800	40.1	1.1851	12.25	1	88.9	10.7	2.7637	5.7017	2.355
WF18X46	13.5	712	22.5	1.1661	11.25	1	78.8	7.43	2.3359	5.7247	2.233
WF18X40	11.8	612	19.1	0.7613	9.833	1	68.4	6.35	2.0226	4.9926	1.68
WF18X35	10.3	510	15.3	0.4625	8.583	1	57.6	5.12	1.6197	4.6828	1.241
WF16X100	29.4	1490	186	7.7118	24.5	1	177	35.7	6.6792	8.8435	8.57
WF16X89	26.2	1300	163	5.3971	21.833	1	157	31.4	5.9327	7.8639	6.752
WF16X77	22.6	1110	138	3.5091	18.833	1	136	26.9	5.1048	6.7809	5.054
WF16X67	19.7	954	119	2.3294	16.417	1	119	23.2	4.4646	5.8473	3.834
WF16X57	16.8	758	43.1	2.1515	14	1	92.2	12.1	3.2755	6.2567	3.395
WF16X50	14.7	659	37.2	1.4644	12.25	1	81	10.5	2.8779	5.5042	2.623
WF16X45	13.3	586	32.8	1.059	11.083	1	72.7	9.34	2.569	4.975	2.115
WF16X40	11.8	518	28.9	0.7472	9.833	1	64.7	8.25	2.2977	4.393	1.67
WF16X36	10.6	448	24.5	0.5023	8.833	1	56.5	7	1.948	4.1997	1.309
WF16X31	9.12	375	12.4	0.4208	7.6	1	47.2	4.49	1.559	3.8913	1.102
WF16X26	7.68	301	9.59	0.2305	6.4	1	38.4	3.49	1.2136	3.4854	0.756
WF14X808	237	16000	5510	1972.602	197.5	1	1400	594	60.6967	65.356	407.492
WF14X730	215	14300	4720	1580.649	179.167	1	1280	527	56.8426	52.8952	342.539
WF14X665	196	12400	4170	1215.94	163.333	1	1150	472	51.6912	47.4819	286.102
WF14X605	178	10800	3680	933.6848	148.333	1	1040	423	46.9306	42.5427	238.596
WF14X550	162	9430	3250	712.9735	135	1	931	378	42.5976	38.2562	198.329
WF14X500	147	8210	2880	542.5711	122.5	1	838	339	38.6612	34.397	164.654
WF14X455	134	7190	2560	414.4508	111.667	1	756	304	35.1221	31.0031	137.078
WF14X426	125	6600	2360	345.5985	104.167	1	706	283	32.9853	28.551	120.874
WF14X398	117	6000	2170	283.2329	97.5	1	656	262	30.7621	26.5755	105.649
WF14X370	109	5440	1990	229.8394	90.833	1	607	241	28.6251	24.5106	91.67
WF14X342	101	4900	1810	182.7015	84.167	1	558	221	26.4525	22.5224	78.454

$ tag	area	Iz	Iy	J	Az	Ay	Sz	Sy	SFz	SFy	Js
WF14X311	91.4	4330	1610	138.7825	76.167	1	506	199	23.9445	20.3163	65.112
WF14X283	83.3	3840	1440	105.7583	69.417	1	459	179	21.769	18.3327	54.157
WF14X257	75.6	3400	1290	79.8489	63	1	415	161	19.8003	16.4806	44.771
WF14X233	68.5	3010	1150	59.7512	57.083	1	375	145	17.9208	14.8472	36.804
WF14X211	62	2660	1030	44.4313	51.667	1	338	130	16.2513	13.4402	30.167
WF14X193	56.8	2400	931	34.5725	47.333	1	310	119	14.88	12.104	25.425
WF14X176	51.8	2140	838	26.1063	43.167	1	291	107	13.5033	11.1669	21.099
WF14X159	46.7	1900	748	19.3926	38.917	1	254	96.2	12.1942	9.9307	17.25
WF14X145	42.7	1710	677	14.8168	35.583	1	232	87.3	11.1476	9.0041	14.387
WF14X132	38.8	1530	548	11.9497	32.333	1	209	74.5	9.9855	8.4929	12.308
WF14X120	35.3	1380	495	9.0501	29.417	1	190	67.5	9.1017	7.7495	10.213
WF14X109	32	1240	447	6.8444	26.667	1	173	61.2	8.2981	6.8542	8.44
WF14X99	29.1	1110	402	5.1183	24.25	1	157	55.2	7.5075	6.2852	6.959
WF14X90	26.5	999	362	3.8425	22.083	1	143	49.9	6.8126	5.6835	5.739
WF14X82	24.1	882	148	4.816	20.083	1	123	29.3	5.6633	6.5774	6.103
WF14X74	21.8	796	134	3.6541	18.167	1	112	26.6	5.2024	5.7929	5.04
WF14X68	20	723	121	2.8156	16.667	1	103	24.2	4.7308	5.3219	4.235
WF14X61	17.9	640	107	2.0208	14.917	1	92.1	21.5	4.2262	4.787	3.393
WF14X53	15.6	541	57.7	1.7687	13	1	77.8	14.3	3.4827	4.6975	2.946
WF14X48	14.1	485	51.4	1.3005	11.75	1	70.2	12.8	3.1291	4.309	2.404
WF14X43	12.6	428	45.2	0.9177	10.5	1	62.6	11.3	2.7804	3.854	1.904
WF14X38	11.2	385	26.7	0.7514	9.333	1	54.6	7.88	2.2698	3.9387	1.632
WF14X34	10	340	23.3	0.5279	8.333	1	48.6	6.91	1.9974	3.6103	1.297
WF14X30	8.85	291	19.6	0.3443	7.375	1	42	5.82	1.6849	3.3871	0.992
WF14X26	7.69	245	8.91	0.3228	6.408	1	35.3	3.55	1.3571	3.1745	0.883
WF14X22	6.49	199	7	0.1797	5.408	1	29	2.8	1.0756	2.8362	0.61
WF12X336	98.8	4060	1190	256.182	82.333	1	483	177	25.7147	23.9649	92.509
WF12X305	89.6	3550	1050	194.1763	74.667	1	435	159	23.2529	21.5266	76.569
WF12X279	81.9	3110	937	147.9802	68.25	1	393	143	21.0764	19.8351	63.884
WF12X252	74.1	2720	828	110.7032	61.75	1	353	127	19.0369	17.7833	52.445
WF12X230	67.7	2420	742	85.4602	56.417	1	321	115	17.381	16.1609	43.994
WF12X210	61.8	2140	664	65.5002	51.5	1	292	104	15.849	14.5793	36.727
WF12X190	55.8	1890	589	49.1349	46.5	1	263	93	14.3605	12.9456	30.162
WF12X170	50	1650	517	35.4915	41.667	1	235	82.3	12.8266	11.6004	24.224
WF12X152	44.7	1430	454	25.5321	37.25	1	209	72.8	11.443	10.3258	19.413
WF12X136	39.9	1240	398	18.1443	33.25	1	186	64.2	10.1736	9.242	15.446
WF12X120	35.3	1070	345	12.5151	29.417	1	163	56	8.9453	8.2265	12.048
WF12X106	31.2	933	301	8.8051	26	1	145	49.3	7.9536	7.0213	9.461
WF12X96	28.2	833	270	6.5647	23.5	1	131	44.4	7.2145	6.2926	7.757
WF12X87	25.6	740	241	4.8312	21.333	1	118	39.7	6.4731	5.8416	6.342
WF12X79	23.2	662	216	3.6007	19.333	1	107	35.8	5.855	5.3014	5.208
WF12X72	21.1	597	195	2.721	17.583	1	97.4	32.4	5.3254	4.8341	4.317
WF12X65	19.1	533	174	1.9992	15.917	1	87.9	29.1	4.7877	4.3668	3.512
WF12X58	17	475	107	1.929	14.167	1	78	21.4	4.2249	4.0375	3.232
WF12X53	15.6	425	95.8	1.424	13	1	70.6	19.2	3.793	3.8448	2.659
WF12X50	14.7	394	56.3	1.6071	12.25	1	64.2	13.2	3.3888	4.1217	2.733
WF12X45	13.2	350	50	1.1635	11	1	57.7	12.4	3.0401	3.7165	2.203
WF12X40	11.8	310	44.1	0.8267	9.833	1	51.5	11	2.7138	3.2732	1.747
WF12X35	10.3	285	24.5	0.7227	8.583	1	45.6	7.47	2.226	3.3717	1.542

$ tag	area	Iz	Iy	J	Az	Ay	Sz	Sy	SFz	SFy	Js
WF12X30	8.79	238	20.3	0.44	7.325	1	38.6	6.24	1.8714	2.9	1.11
WF12X26	7.65	204	17.3	0.2854	6.375	1	33.4	5.34	1.6124	2.5534	0.834
WF12X22	6.48	156	4.66	0.2759	5.4	1	25.4	2.31	1.0867	2.8076	0.753
WF12X19	5.57	130	3.76	0.1656	4.642	1	21.3	1.88	0.8876	2.5175	0.544
WF12X16	4.71	103	2.82	0.0911	3.925	1	17.1	1.41	0.6648	2.3098	0.376
WF12X14	4.16	88.6	2.36	0.0613	3.467	1	14.9	1.19	0.5626	2.0842	0.29
WF10X112	32.9	716	236	15.0181	27.417	1	126	45.3	8.5355	7.38	12.775
WF10X100	29.4	623	207	10.7306	24.5	1	112	40	7.6142	6.5721	10.185
WF10X88	25.9	534	179	7.3704	21.583	1	98.5	34.8	6.6847	5.768	7.912
WF10X77	22.6	455	154	4.9563	18.833	1	85.9	30.1	5.8518	4.9896	6.053
WF10X68	20	394	134	3.4164	16.667	1	75.7	26.4	5.1594	4.3913	4.713
WF10X60	17.6	341	116	2.3486	14.667	1	66.7	23	4.5156	3.8901	3.668
WF10X54	15.8	303	103	1.7154	13.167	1	60	20.6	4.0556	3.4124	2.961
WF10X49	14.4	272	93.4	1.2942	12	1	54.6	18.7	3.7021	3.1125	2.454
WF10X45	13.3	248	53.4	1.4097	11.083	1	49.1	13.3	3.2763	3.2144	2.442
WF10X39	11.5	209	45	0.8904	9.583	1	42.1	11.3	2.7868	2.8673	1.806
WF10X33	9.71	170	36.6	0.5124	8.092	1	35	9.2	2.2797	2.6032	1.265
WF10X30	8.84	170	16.7	0.6034	7.367	1	32.4	5.75	1.9312	2.8169	1.306
WF10X26	7.61	144	14.1	0.3856	6.342	1	27.9	4.89	1.6579	2.4223	0.968
WF10X22	6.49	118	11.4	0.2241	5.408	1	23.2	3.97	1.3484	2.2069	0.685
WF10X19	5.62	96.3	4.29	0.2164	4.683	1	18.8	2.14	1.0149	2.2697	0.623
WF10X17	4.99	81.9	3.56	0.1411	4.158	1	16.2	17.78	0.8424	2.1482	0.479
WF10X15	4.41	68.9	2.89	0.0919	3.675	1	13.8	1.45	0.683	2.0274	0.366
WF10X12	3.54	53.8	2.18	0.0465	2.95	1	10.9	1.1	0.5287	1.6655	0.233
WF8X67	19.7	272	88.6	5.0099	16.417	1	60.4	21.4	5.0776	4.4499	5.699
WF8X58	17.1	228	75.1	3.2634	14.25	1	52	18.3	4.3718	3.9186	4.284
WF8X48	14.1	184	60.9	1.9045	11.75	1	43.2	15	3.6574	3.0351	2.954
WF8X40	11.7	146	49.1	1.0644	9.75	1	35.5	12.2	2.978	2.6729	2.019
WF8X35	10.3	127	42.6	0.7242	8.583	1	31.2	10.6	2.621	2.3016	1.554
WF8X31	9.13	110	37.1	0.4971	7.608	1	27.5	9.27	2.2977	2.0948	1.213
WF8X28	8.25	98	21.7	0.4966	6.875	1	24.3	6.63	2.0033	2.092	1.148
WF8X24	7.08	82.8	18.3	0.314	5.9	1	20.9	5.63	1.7135	1.7891	0.843
WF8X21	6.16	75.3	9.77	0.2659	5.133	1	18.2	3.71	1.3781	1.8724	0.726
WF8X18	5.26	61.9	7.97	0.1575	4.383	1	15.2	3.04	1.132	1.7001	0.519
WF8X15	4.44	48	3.41	0.1219	3.7	1	11.8	1.7	0.8103	1.7704	0.422
WF8X13	3.84	39.6	2.73	0.0756	3.2	1	9.91	1.37	0.6509	1.6401	0.31
WF8X10	2.96	30.8	2.09	0.0352	2.467	1	7.81	1.06	0.5208	1.2198	0.184
WF6X25	7.34	53.4	17.1	0.4465	6.117	1	16.8	5.61	1.82	1.8194	1.041
WF6X20	5.87	41.4	13.3	0.2293	4.892	1	13.4	4.41	1.4477	1.4579	0.666
WF6X15	4.43	29.1	9.32	0.0934	3.692	1	9.77	3.11	1.0231	1.2576	0.371
WF6X16	4.74	32.1	4.43	0.2129	3.95	1	10.2	2.2	1.0612	1.4472	0.573
WF6X12	3.55	22.1	2.99	0.0819	2.958	1	7.31	1.5	0.7241	1.2458	0.31
WF6X9	2.68	16.4	2.19	0.0354	2.233	1	5.56	1.11	0.5512	0.916	0.176
WF6X8.5	2.51	14.8	1.989	0.0284	2.092	1	5.08	1.01	0.4995	0.904	0.153
WF5X19	5.54	26.2	9.13	0.2976	4.617	1	10.2	3.63	1.423	1.2355	0.735
WF5X16	4.68	21.3	7.51	0.1769	3.9	1	8.55	3	1.1853	1.0791	0.521
WF4X13	3.83	11.3	3.86	0.1391	3.192	1	5.46	1.9	0.9148	1.0229	0.422