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The variable stripe-length method revisited: Improved analysis

C. Lange,[a)] M. Schwalm, S. Chatterjee, and W. W. Rühle
Materials Sciences Center, Faculty of Physics and Materials Sciences Center, Philipps-Universität Marburg, Renthof 5, D-35032 Marburg, Germany

N. C. Gerhardt
AG Optoelektronische Bauelemente und Werkstoffe, Ruhr-Universität Bochum, D-44780 Bochum, Germany

S. R. Johnson, J.-B. Wang, and Y.-H. Zhang
Center for Solid State Electronics Research and Department of Electrical Engineering, Arizona State University, Tempe, Arizona 85287-6206, USA

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The variable stripe length method described by Shaklee and Leheny, [Appl. Phys. Lett. **18**, 475 (1971)] is a straightforward way to determine the steady-state gain spectrum of laser material. Here, common sources of error are identified and several new, robust ways of calculating the gain from the data are presented. The advantages of these methods are underlined by applying them to data obtained from a Ga(AsSb)/GaAs/(AlGa)As heterostructure. *© 2007 American Institute of Physics*.
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Several ways have been used to measure material gain, e.g., the Hakki-Paoli method,[1] time-resolved,[2,3] and cw[4] transmission gain spectroscopy. A relatively straightforward technique to determine the steady-state gain spectrum for an optically active medium is the variable stripe length method, originally introduced in Ref. 5. Here, the emission from a homogeneously illuminated stripe of varying length is collected out of a cleaved edge as a function of the stripe length. Usually, the gain value is then computed using the $\langle l/2l\rangle$ method[6] or by fitting an exponential function to the data.[7] In this letter, we present several significantly improved, robust ways of calculating the gain from typical data sets.

In the experimental setup, 200 ps pulses at 527 nm and 10 kHz repetition rate were used for a quasicontinuous excitation. The beam is focused to a homogeneous stripe on the sample of 16 μm width, passing its cleaved edge on one side. A slit aperture after the lens controls the length of the stripe on the sample. Amplified spontaneous emission (ASE) along the pump channel to the sample edge is analyzed using a spectrometer and a liquid nitrogen cooled InGaAs linear array detector without discriminating between polarizations. The width of the stripe should be small in comparison to the typical length that is used to calculate the gain value in order to reduce the leakage of ASE to the side of the pump channel. In our case, this length is in the order of 300 μm, about 20 times larger than the width. At both edges of the stripe, the step in refractive index reflects a certain portion of the ASE which then round-trips the pumped region and causes an additional carrier depletion and premature gain saturation. This was avoided by a 5° angle between the pump stripe and sample edge normal.[8,9] Details of the sample may be found in Ref. 10.

The experimental data are analyzed as follows. Assuming that from every unit volume along the stripe, spontaneous emission emerges and is amplified along the stripe, the following differential is found for the resulting intensity $I(l)$:[11] $\partial_l I(l)=[\Gamma g_m(l)-\alpha]I(l)+A(l)$. Here, the material gain g_m is modified with the confinement factor Γ and the propagation loss coefficient α. Spontaneous emission is represented by $A(l)$. For a homogeneous sample, the solution is

$$I(l)=\frac{A_0}{g_{\mathrm{mod}}}[\exp(g_{\mathrm{mod}}l)-1], \quad (1)$$

with the modal gain $g_{\mathrm{mod}}=\Gamma g_m-\alpha$, the length l of the stripe, and a scaling factor A_0. The latter factor is depending on the Einstein coefficient for spontaneous emission, pump intensity, and, most indefinite, geometrical form factors. If desired, the photoluminescence can also extracted from these data.[9] The $\langle l/2l\rangle$ method uses the intensity values at a certain stripe length l and at $2l$ to calculate the gain value. One obtains $g_{\mathrm{mod}}=1/l\ \ln[I(2l)/I(l)-1]$. The derivation of this result can be found in Ref. 12 or easily retraced along the derivation of the $\langle l/xl\rangle$ method below. One of the drawbacks of the $\langle l/2l\rangle$ method is that it requires two data points eventually too far apart from each other, which will be discussed below. This often leads to gain saturation,[11,13] and the gain value is underestimated. On the other hand, the advantage of this method is the existence of an analytical solution to the equation. We will show in the following that a numerical solution using two data points at the positions l and xl for $x>1$ does not require mentionable computational effort but eliminates the saturation issue.

We start with the intensity values at these two stripe lengths and introduce $z=\exp(gl)$, obtaining for their ratio r_x,

$$r_x=\frac{I(xl)}{I(l)}=\frac{\exp(gxl)-1}{\exp(gl)-1}\equiv\frac{z^x-1}{z-1}. \quad (2)$$

At this point, the analytical solution for the $\langle l/2l\rangle$ method is found for $x=2$. Actually, another analytical solution can be found introducing a new variable $y^2=z$,

$$g=\frac{2}{l}\ln\left[\frac{r_{1.5}-1}{2}+\sqrt{\frac{(r_{1.5}-1)^2}{4}+(r_{1.5}-1)}\right], \quad (3)$$

where the unphysical result with negative values of y has been dropped. This result is obtained solving the resulting third order polynomial in y with the trivial zero $y=1$. For arbitrary x, Eq. (2) has to be solved numerically for z. Introducing $f(z)=(z^x-1)-r_x(z-1)$, the problem is relocated to

[a)]Electronic mail: christoph.lange@physik.uni-marburg.de

finding the zero of $f(z)$ for $z>0$, which can be done using the midpoint method.[14]

The function has the trivial zero $z=1$, which corresponds to $gl=0$. The first and second derivative of $f(z)$ show that the function has its minimum at $z_{\min}=(r_x/x)^{1/x-1}$, and has to have another zero if $z_{\min}\neq 1$, which is the nontrivial solution to the equation. In particular, for $z_{\min}<1$, the zero has to be searched for in $[0,z_{\min}]$. The result is again $z=1$ for $z_{\min}=1$. The zero is located in $[z_{\min},\infty]$ for $z_{\min}>1$. A practical upper searching boundary is $z_\infty=\exp(l_{\max}g_{\max})$, where $l_{\max}$ is the maximum length of the stripe and $g_{\max}$ is the maximum gain value expected. In our calculation, $z_\infty=10^{250}$ was used. Despite the huge search interval, the calculation time for a spectrally resolved measurement (512 wavelengths times 300 intervals for the length of the stripe) was only 4 s on a standard 3 GHz PENTIUM 4 office personal computer.

An alternative to using the integrated emission over the whole stripe is taking into account the differential values. Considering the first and second derivatives of Eq. (1), the gain g can be extracted,

$$g=\frac{\partial_l^2 I(l)}{\partial_l I(l)}. \tag{4}$$

This equation, further referred to as the $\langle d^2/d\rangle$ method, requires the data at exactly one position l_0 while the equation itself does not explicitly include the experimental value for l_0. Therefore, it is not susceptible to errors in determining the stripe offset. However, determining the second derivative from experimental data is rather sensitive to noise. In our calculation, a local polynomial fit of sixth order was used. Here, each data point around l is assigned a weight according to a Gaussian distribution, which has been proven to show the best results.

Another method to determine the gain value from the derivative uses the data at two different positions l and xl, similarly to the $\langle l/xl\rangle$ method discussed above. Since the derivative removes the additive constant in Eq. (1), the following analytical expression is found: $g=1/[l(x-1)]\ln[\partial_l I(xl)/\partial_l I(l)]$. Due to the implicit dependence on l, this analysis is sensitive to errors in determining the absolute stripe length. In contrast to the previous one, however, this method is less susceptible to noise because only the first derivative is used. This method will be referred to as the $\langle dl/xdl\rangle$ method from now on. In order to investigate the effect of noise on the stability of the methods, we perform an error estimation. Here, synthetic gain data (i.e., numerical ASE data generated according to Eq. (1) for g_{mod} and A_0 such that the resulting data is comparable to the experiment) are used. The data are scaled by Gaussian-distributed noise centered at unity to account for the variations of the excitation power. Furthermore, a constant noise floor representing detector noise is added,

$$I_{\text{noise}}(l)=I_{\text{syn}}(l)\times[1+n_{\text{laser}}(l)]+n_{\text{detector}}(l). \tag{5}$$

The noise is scaled such that its amplitude is about 5% of the signal amplitude at the typical optimum stripe length for this sample of 0.25 mm. The dependence of the noise terms n_{laser} and n_{detector} on l is stochastic. Derivatives by l are treated as the difference of random variables scaled with $1/l$. The standard deviation increases by a factor of $2/\Delta l$ and is dependent on the experimental resolution. Both Eqs. (2) and (4) are applied to the synthetic data and, by means of error propagation, the resulting noise in the gain value is expressed as $\sigma_{\text{gain}}=\gamma\sigma_{\text{syn}}$. Both methods show very similar, negligible dependencies for γ on detector noise. Regarding laser noise, the $\langle l/xl\rangle$ method turns out to be less sensitive, since γ quickly drops below 10 as l becomes greater than 0.2 mm. For the $\langle dl/xdl\rangle$ method, γ is more than two orders of magnitude larger and is merely constant for all values of g and l. The $\langle l/xl\rangle$ method is especially suitable to analyze experimental data in the absorptive regime, as γ decreases for lower gain values. These numerical findings coincide with those from the evaluation of experimental data, as shown below.

Next, optical sources of error are discussed. Diffraction effects cause spatially inhomogeneously illuminated stripe edges for small stripe lengths and thereby produce artificial modulations in the emission spectrum.[11,15] Diffraction itself can only be reduced, e.g., by placing the sample as close as possible behind the apertures that constrain the stripe. Therefore, a method insensitive to diffraction is presented, referred to as the $\langle$difflog$\rangle$ method. A standard solving scheme for the differential version of Eq. (1) and separation of the integral at a midpoint x_0 yields

$$\begin{aligned}I(l)=&\left\{\int_0^{x_0}A(x')\exp\left[-\int_0^{x'}g(x'')dx''\right]dx'\right.\\&\left.+\int_{x_0}^{l}A(x')\exp\left[-\int_0^{x'}g(x'')dx''\right]dx'\right\}\\&\times\left[\exp\left(\int_0^{x_0}g(x')dx'\right)\exp\left(\int_{x_0}^{l}g(x')dx'\right)\right].\end{aligned} \tag{6}$$

Here, $g(x)$ and $A(x)$ are the gain and the spontaneous emission form factor, both depending on the position in the sample. Dropping these dependencies for large x_0 yields

$$I_{x_0}(l)=\frac{A_0}{g}\{\exp[g(l-x_0)]-1\}+I_i(x_0)\exp[g(l-x_0)]. \tag{7}$$

The first term is the amplified spontaneous emission over the homogeneous part of the stripe from x_0 to l [Eq. (1)]. The contribution $I_i(x_0)$ accounts for the ASE collected from 0 to x_0. It is emitted in the region of inhomogeneous pumping conditions and passes the homogeneous region before being detected. The latter is the reason for the amplification factor $\exp[g(l-x_0)]$. Simple math yields

$$\ln[\partial_l I(l)]=\{\ln[A_0+gI_i(x_0)]-gx_0\}+gl, \tag{8}$$

where the term in braces is a constant, so that g can be calculated by determining the slope of this function. This method does not require the experimental setup to be perfectly free of diffraction effects, as long as the stripe is homogeneous along the region behind x_0. Similar to Eq. (4), it is insensitive to errors in determining the absolute length of the stripe.

Now, these techniques are applied to experimental data acquired as described above. A three-dimensional plot of the raw data can be seen in the inset of Fig. 1. The pump intensity on the sample during excitation was 1.3 W/cm^2. As mentioned in the introduction, a proper calculation of the gain value is delicately dependent on the right choice of the stripe length interval. If a too low starting value is chosen, edge effects due to an improperly cleaved sample and a greater emission angle will cause deviations from Eq. (1).

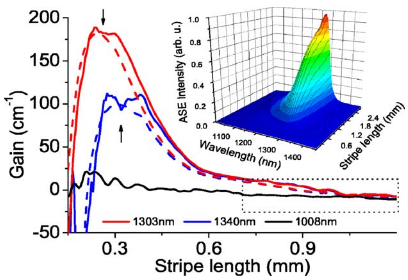

FIG. 1. (Color online) Gain value as a function of the interval of stripe length considered for the $\langle l/xl \rangle$ method (solid: $x=1.2$; dashed: $x=1.4$). The inset shows the raw experimental data.

Noise from the detector and shot noise from the sample emission play a considerable role as the signal is still small. The maximum stripe length on the other hand must not exceed the saturation length.[11]

To illustrate this issue, Fig. 1 shows the gain calculated with the $\langle l/xl \rangle$ method for $x=1.4$ and $x=1.2$ as a function of l. The datapoints for each gain value $g(l)$ are thus taken at l and $1.2l$ or $1.4l$, respectively. For $\lambda=1340$ nm and $\lambda=1303$ nm, the gain value is the maximum of the curve, located at $l \approx 0.25$ mm and $l \approx 0.30$ mm, respectively. It is important to see that the gain maximum is located at different stripe lengths for different wavelengths. This demonstrates that if the spectra are calculated for a fixed stripe length l, they only avoid saturation for a part of the spectral range. Note that for $x=1.2$, there is a plateau where the gain value is constant for a certain range of stripe lengths. Here, the true gain value is not decreased, neither by noise in the low emission regime nor by saturation effects. For both values of separation x, gain values are very similar, where the higher gain value is more accurate since it suffers less from saturation effects. Despite the smaller distance of the data points for $x=1.2$, the noise level is still very low, which demonstrates the utility of this method. For the absorptive regime at $\lambda=1008$ nm, the gain value is best estimated by the average over a range of large stripe lengths which is indicated by the dashed box. Here, no saturation effects are expected, and therefore values for large l are most reliable. Figure 2 (left) shows the derived gain spectra for a selection of the methods. Results obtained with the conventional $\langle l/2l \rangle$ method may be misleading, as illustrated by the black line, where a nonoptimal stripelength was intentionally chosen. Although the spectrum looks convincingly smooth and consistent, it is actually wrong. For optimum l, gain values are much higher. The $\langle l/1.2l \rangle$ method clearly shows that even the optimum choice of l for the $\langle l/2l \rangle$ method does not yield proper results due to saturation effects. Lower values of x down to $x=1.02$ show that the gain value converges for $x \rightarrow 1.0$ without introducing a critically high noise level, as can be seen in Fig. 2 (right). Here, a series of gain spectra for different values of x are shown to illustrate the convergence. The conventional $\langle l/2l \rangle$ method turns out to underestimate the proper gain values by about 20%. With the $\langle dl/xdl \rangle$ method and the $\langle d^2/d \rangle$ method (not shown, similar to $\langle dl/xdl \rangle$), the analysis was only possible on a limited spectral range within the gain region due to noise effects, as predicted by the estimation above. Similarly, the $\langle$difflog$\rangle$ method does not cover a broad spectral range, but delivers results comparable with the $\langle l/1.2l \rangle$ method. For this particular material system, the advantages of the latter three methods do not countervail the drawback of their sensitivity to noise. For the absorptive regime below 1000 nm where saturation is not an issue, both $\langle l/xl \rangle$ and $\langle l/2l \rangle$ are suitable and deliver exactly the same result. The overall gain bandwith of this material system of roughly 300 nm is very large. From the measurement, a bandgap of about 0.88 eV was determined.

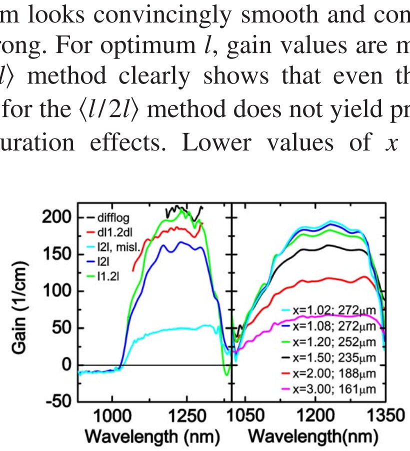

FIG. 2. (Color online) Left: gain spectra for the different methods. Right: gain spectra computed using the $\langle l/xl \rangle$ method with different spacings from $x=3.0$ down to $x=1.02$ and corresponding positions of the maximum gain.

In summary, four methods for calculating the gain for a variable stripe-length experiment were presented. Their sensitivity to noise was discussed theoretically and verified experimentally. A model insensitive to inhomogeneities at the stripe edges was presented. The standard $\langle l/2l \rangle$ method has been extended with a numerical technique in order to drastically improve the insensitivity of this class of methods against saturation effects, requiring data over a range as little as only 10% apart in stripe length. A GaAsSb-based sample has been investigated with these techniques, showing that the sample is suitable for laser emission over a broad spectral range of approximately 300 nm in width.

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