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## 2D VAR single Doppler lidar vector retrieval and its application in offshore wind energy

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### Abstract

Remote sensors like Doppler lidars can map the winds with high accuracy and spatial resolution. One shortcoming of lidars is that the radial velocity measured by the lidar does not give a complete picture of the windfield necessitating additional data processing to reconstruct the windfield. Most of the popular vector retrieval algorithms rely on the homogenous wind field assumption which plays a vital role in reducing the indeterminacy of the inverse problem of obtaining Cartesian velocity from radial velocity measurements. Consequently, these methods fail in situations where the flow is heterogeneous e.g., Turbine wakes. Alternate methods are based either on statistical models (e.g., optimal interpolation [1]) or computationally intensive four dimensional variational methods [2]. This study deals with a 2D variational vector retrieval for Doppler lidar that uses the radial velocity advection equation as an additional constraint along with a tangential velocity constraint derived from a new formulation with gradients of radial velocity. The retrieval was applied on lidar data from a wind farm and preliminary analysis revealed that the algorithm was able to retrieve the mean wind field while preserving the small scale flow structure.

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**Keywords:** Doppler Wind lidars; Vector wind retrieval; 2D-VAR; Offshore Wind Energy; Wind turbine wakes; Wind turbine control; Optimization

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## 1. Introduction

Doppler wind lidars are becoming popular for remote wind measurements and have seen applications in atmospheric science, aviation safety and wind energy to name a few. One of the potential applications of Doppler wind lidars is in wind farm control techniques. The wind data from a lidar can be used to control individual turbines to maximize the power output of the entire wind farm. However, lidars can only measure radial velocity i.e., component of velocity along the lidar beam, requiring additional post processing steps to retrieve the full wind vector before being passed into a wind farm control algorithm. Majority of the vector retrieval algorithms in use today are based on the homogeneous wind field assumption (within the retrieval domain) and perform poorly in complex flow conditions (e.g. Wind turbine Wakes). Alternate methods based on 4D-VAR are prohibitively expensive and are often impractical to be employed in applications requiring real-time vector retrievals. To address this issue, a new computationally efficient 2D-VAR vector retrieval for low elevation PPI scans was developed and tested on data from an offshore wind farm. The following sections describe the formulation of this retrieval and present a preliminary validation of the retrieval using data from an offshore wind farm.

### Nomenclature

u	Component of wind velocity in X-direction (East-West direction)
v	Component of wind velocity in Y-direction (North- South direction)
J	Cost function
LAT	Lowest Astronomical Tide
PPI	Plan position indicator
RHI	Range height indicator
VAD	Velocity Azimuth Display
VVP	Volume Velocity Processing
CFD	Computational Fluid Dynamics

## 2. Relevant work

Variational retrieval methods [3,4,5] can be broadly classified into two types [6]: a) Parameter identification techniques (PI) and b) 4D-VAR based methods. In the former, data from the radar/lidar is used to estimate the unknowns (e.g., Cartesian velocity) by fitting them to a set of control equations pertaining to reflectivity/ radial velocity conservation equation [7]. The resulting retrieval could be considered as a time-mean estimate over the acquisition time period. The latter method (i.e., 4D-VAR based) relies on a forecast model to obtain the wind field along with thermodynamic variables [8]. 4D-VAR methods have been known to be computationally expensive to implement and often limited by the underlying assumptions in the forecast model.

The PI techniques involve finding the best time-mean estimate of the control variables ( $\mathbf{X} = [u,v,w,\dots]$ ) by minimizing a cost function ( $J(\mathbf{X})$ ) of the form:

$$J(\mathbf{X}) = \frac{1}{2\Omega} \int \left( \sum W_i C_i^2 \right) d\Omega \quad (1)$$

where,  $W_i$  are the weights pertaining to the relative importance of the constraints  $C_i$ , corresponding to the various control equations in a weak sense. Although, the control equations could be specified as strong constraints [9] or weak constraints [4,5] previous works [10,11] have shown that the weak constraint formulations perform better in the presence of model errors, especially with the reflectivity/radial velocity conservation equation.

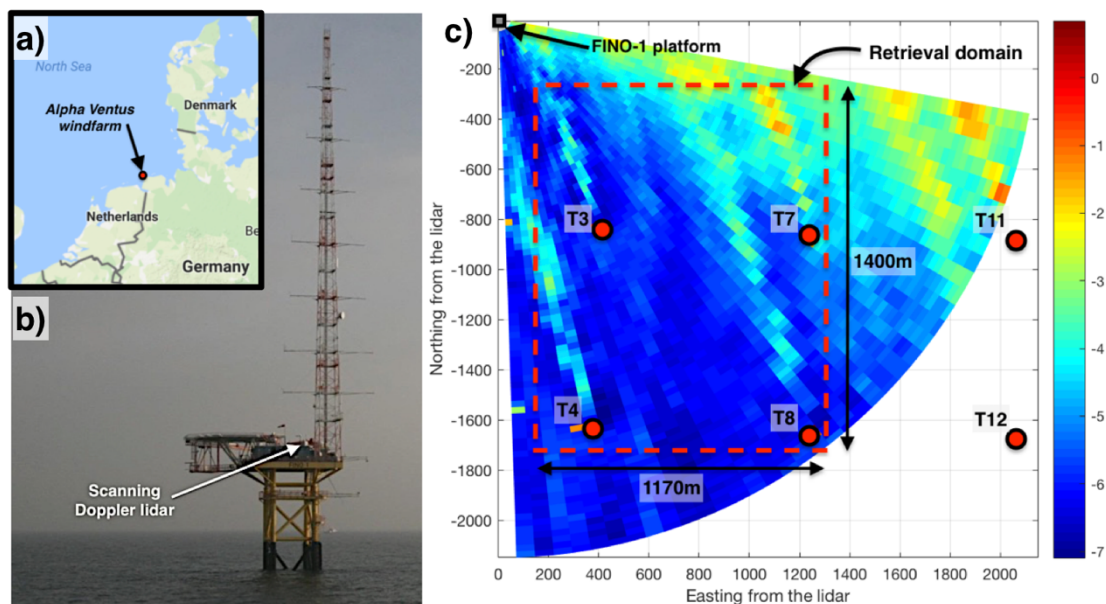


Fig. 1. (a) Location of Alpha Ventus wind farm (b) FINO-1 research platform with the meteorological mast and lidar (c) lidar scan with retrieval domain. T3,T4,T7,T8,T11 and T12 are wind turbines

Previous PI works mainly differed in the type of constraints and method used for the minimization of the cost function. The work presented in [7] first proposed a “simple adjoint method (SA)”, to retrieve the time mean winds of artificial data, using only the Lagrangian conservation of radar reflectivity as a strong constraint in the cost function. The SA method was later upgraded to include the eddy diffusion & residual forcing terms in the reflectivity conservation equation in [12], continuity equation as a strong constraint in [11] and radial velocity equation with algorithmic improvements in [13]. Following the SA method, [14] developed a computationally efficient least-squares formulation with weak zero horizontal divergence and vorticity constraint. A single Doppler radar wind retrieval intercomparison study by [15] showed that the least squares formulation performed better than the other retrieval methods for the given test case. When compared with the SA method, the least squares formulation was found to be robust and yielded similar retrievals for short scan periods. In addition, [16] added a background constraint to reduce the noise arising from the finite difference calculations of the gradients and to facilitate a smooth transition to fill the data void regions with wind field from the background. In an attempt to preserve local structure, [17] developed a two step variational method in which a proxy background was obtained from a second order expansion of Legendre polynomials. Some of the above methods have been extended for dual Doppler radar by [18].

Methods based on 4D-VAR of [8] have been tested on Doppler lidar by studies referenced in [19, 2, 20, 21]. A two-step variational retrieval method based on [17] was implemented for the Hong Kong International airport lidar dataset to detect flow hazards for airplanes [22] and was later used in Lagrangean coherent structure analysis by [23]. The same method was implemented for a plume dispersion and air quality study by [24].

The method described in this study is based on [16] with different terms in the cost function. These terms are specifically applicable for low elevation PPI scans and facilitate a fast vector retrieval with real-time application capabilities.

### 3. Formulation

Let  $u$ ,  $v$  and  $w$  be the three components of velocity in Cartesian coordinate system. These Cartesian velocity components and their counterparts in spherical coordinates are related by:

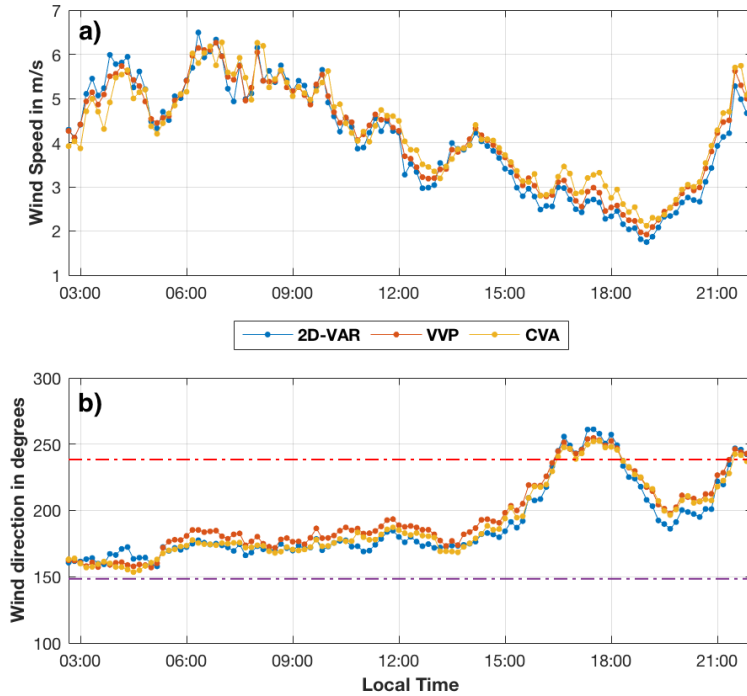


Fig. 2. Comparisons of (a) wind speed and (b) wind direction, retrievals from 2D-VAR, VVP and CVA

$$V_r = u \cdot \cos\theta \cos\varphi + v \cdot \sin\theta \cos\varphi + w \cdot \sin\varphi \tag{2}$$

$$V_{th} = -u \cdot \sin\theta + v \cdot \cos\theta \tag{3}$$

$$V_{tv} = -u \cdot \cos\theta \sin\varphi - v \cdot \sin\theta \sin\varphi + w \cdot \cos\varphi \tag{4}$$

where,  $V_r$  is the radial velocity,  $V_{th}$  is the tangential velocity in the horizontal plane,  $V_{tv}$  is the tangential velocity in the vertical and  $(\theta, \varphi)$  are the azimuth and elevation angles respectively. As mentioned previously, a Doppler lidar can measure only  $V_r$ .

Let us consider a repeated PPI scan at low elevation angles and attempt to retrieve the horizontal components of the velocity vector. At low elevation angles ( $\varphi \approx 0$ ), Eq. (2) reduces to

$$V_r = u \cdot \cos\theta + v \cdot \sin\theta \tag{5}$$

Differentiating Eq. (5) along the azimuth direction and using Eq. (3) we get

$$\frac{\partial \tilde{V}_r}{\partial \theta} = (-u \cdot \sin\theta + v \cdot \cos\theta) + \frac{\partial u}{\partial \theta} \cos\theta + \frac{\partial v}{\partial \theta} \sin\theta \tag{6}$$

$$= V_{th} + P \quad , \text{ where } P = \left( \frac{\partial u}{\partial \theta} \cos\theta + \frac{\partial v}{\partial \theta} \sin\theta \right) \tag{7}$$

The term ‘P’ is identically zero when the simplified constant wind assumption is true (e.g. VAD, VVP). ‘P’ could be understood as a measure of deviation from this assumption. We now have two equations (Eq. (5) and Eq. (7)) and three unknown variables ( $u, v$  and  $P$ ). These equations can be closed by considering the radial velocity advection equation. Assuming that radial velocity patterns advect with the flow, we get

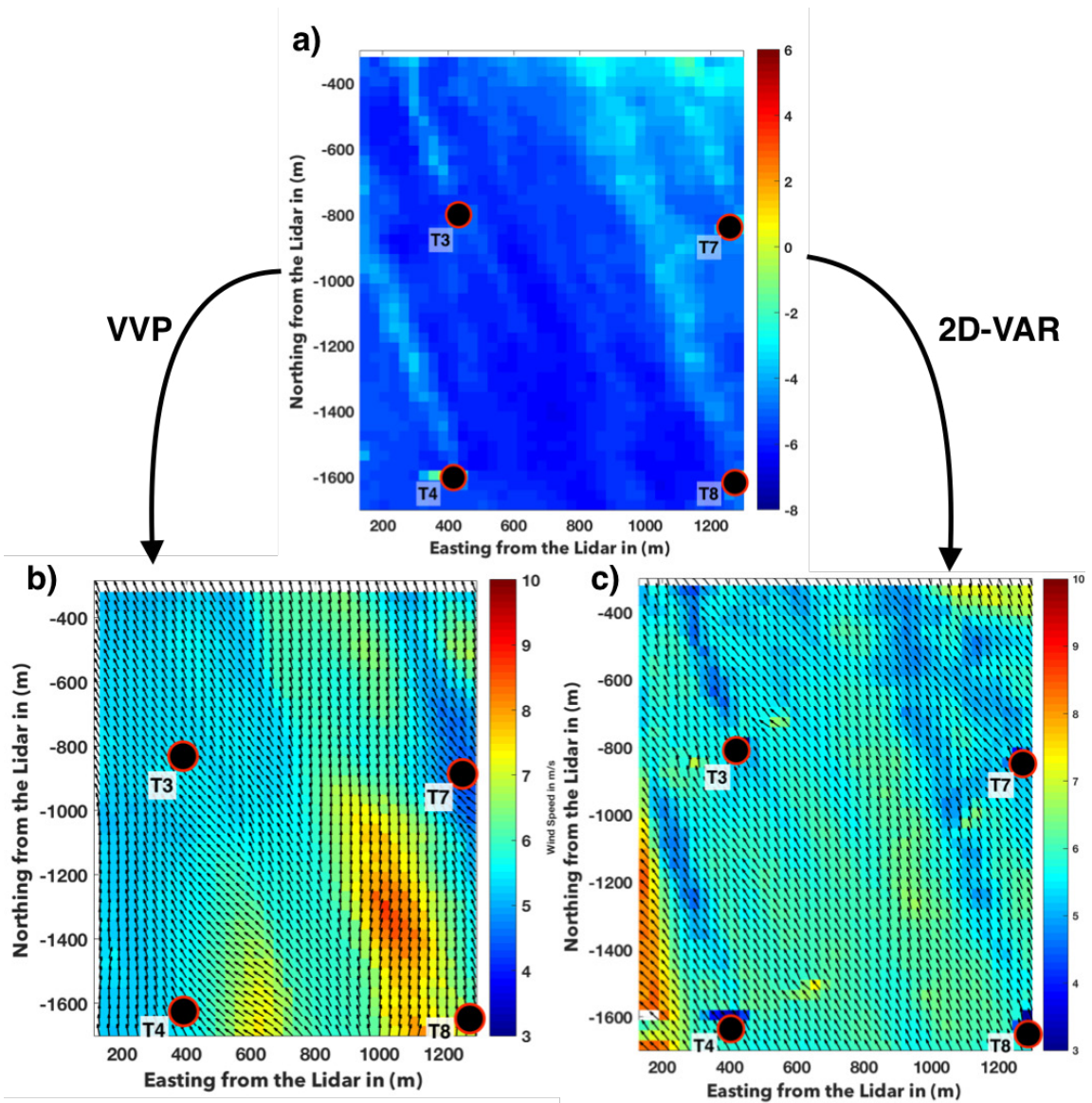


Fig. 3. (a) Radial velocity, (b) VVP retrieved wind field (c) 2D-VAR retrieved wind field

$$\frac{\partial \tilde{V}_r}{\partial t} + u \frac{\partial \tilde{V}_r}{\partial x} + v \frac{\partial \tilde{V}_r}{\partial y} = 0 \tag{8}$$

where,  $\tilde{V}_r$  is the filtered radial velocity (e.g., Gaussian filter - introduced to reduce the effect of noise on numerical derivatives). By substituting the partial derivatives with finite differences, and solving Eq. (5), (7) and (8), the horizontal vector field can be determined. Since gradients are prone to become unreliable in regions with high noise levels, a background constraint similar to [16] is included. The background constraint equation is formulated as the departure of the vector field (u,v) from the vector field derived using VVP or sector VAD ( $u_b, v_b$ ). The vector field (u,v) can be estimated by minimizing a cost function derived from the above mentioned constraints i.e.,

$$J(u, v, P) = \frac{1}{2\Omega} \int (W_a A^2 + W_b B^2 + W_c C^2 + W_d D_a^2 + W_d D_b^2) d\Omega \quad (9)$$

$$\text{where, } A = \left(\frac{ux}{r} + \frac{vy}{r}\right) - V_r^{obs}, \quad B = \left(-\frac{uy}{r} + \frac{vx}{r}\right) - \frac{\partial V_r^{obs}}{\partial \theta} + P,$$

$$C = \frac{\partial \tilde{V}_r}{\partial t} + u \frac{\partial \tilde{V}_r}{\partial x} + v \frac{\partial \tilde{V}_r}{\partial y}, \quad D_a = u - u_b, \quad D_b = v - v_b,$$

$$x = r \cdot \cos\theta, \quad y = r \cdot \sin\theta, \quad r = \sqrt{x^2 + y^2}$$

The weights for each term in the cost function were chosen such that the terms have same order of magnitude. The objective is to find  $u$ ,  $v$  and  $P$  at which the cost function is minimized or the gradients given in Eq. (10), (11) and (12) vanish.

$$\left(\frac{\partial J}{\partial u}\right)_{ij} = W_a A \frac{x}{r} - W_b B \frac{y}{r} + W_c \frac{\partial \tilde{V}_r}{\partial x} + W_d D_a \quad (10)$$

$$\left(\frac{\partial J}{\partial v}\right)_{ij} = W_a A \frac{y}{r} + W_b B \frac{x}{r} + W_c \frac{\partial \tilde{V}_r}{\partial y} + W_d D_b \quad (11)$$

$$\left(\frac{\partial J}{\partial P}\right)_{ij} = W_a \cdot B \cdot P \quad (12)$$

The solution which minimizes Eq. (9) can be determined by means of a quasi-Newton method. In addition, the radial velocity values from the lidar's spherical coordinate system were transformed to a Cartesian coordinate system, which enabled restructuring of the matrices for computational efficiency.

#### 4. Test case and validation

FINO-1 (Forschungsplattformen in Nord- und Ostsee Nr.1) is a German offshore wind energy research platform located close to the Alpha Ventus wind farm in the North Sea. As a part of an extensive offshore measurement campaign to improve the understanding of marine boundary layer, offshore wake propagation effects and air-sea interaction, two scanning Doppler wind lidars (Leosphere's Windcube 100S) were installed on the FINO-1 platform to perform various Dual-doppler and vertical profiling scans. On 31<sup>st</sup>, August, 2016, one of the lidars was configured to perform repeated low elevation angle (0.5°) PPI scans, primarily for the validation of the new 2D-VAR vector retrieval. The lidar scanned a 90° azimuthal sector in the direction of the wind farm (Fig. 1), with a 2°/s scan speed, accumulation time of 1 second and a 25m range resolution, the lidar could produce one scan product approximately every 45 seconds. With a good atmospheric aerosol content, the returns were clean and the lidar was able to capture winds at distance exceeding 2.5km at times.

By combining two successive scans for each time step, the wind field in a 1170m x 1400m domain, with a grid spacing of 30m was retrieved using the 2D-VAR algorithm and the traditional Volume Velocity Processing (VVP) algorithm. The VVP estimates were obtained by pooling all the radial velocity measurements within a 200 m region around each grid point in the domain and estimating the velocity components that best fit the measurements in a least squares sense. The 200m search space for VVP is required to obtain reliable estimates of (u,v) such that the variation in radial velocity due to orientation of the line of sight is greater than turbulent fluctuations in the wind [25]. The downside of this implementation is that the VVP estimates become less reliable at grid points far away from the lidar. However, this wasn't an issue in the present study since only the grid point closest to the lidar were considered for obtaining the validation statistics with the anemometer data. The 10-minute averaged wind data from the cup and vane anemometer (CVA) situated at 33m LAT on the meteorological mast was used for corroborating and validating the wind retrieval from both 2D-VAR and VVP algorithms. Since the lidar and the met mast were both located on the FINO-1 platform, retrieved wind vector from the nearest grid point (excluding the points along

the boundary) were considered. In addition, the temporal profiles of wind speed and direction from the lidar were obtained by taking the mean of the retrieved  $u$  and  $v$  component of velocity within a 10-minute window around the CVA measurement time. The error in wind speed ( $\Delta U$ ), wind direction ( $\Delta\phi$ ) and Pearson correlation coefficient ( $R$ ) were calculated according to Eq. (13), (14) and (15).

$$\Delta U = \sqrt{\frac{1}{N} \sum_{i=1}^N (U_{lidar} - U_{CVA})^2} ; U = \sqrt{(u)^2 + (v)^2} \quad (13)$$

$$\Delta\phi = \sqrt{\frac{1}{N} \sum_{i=1}^N \left[ \tan^{-1} \left( \frac{u_{lidar} v_{CVA} - v_{lidar} u_{CVA}}{u_{lidar} u_{CVA} + v_{lidar} v_{CVA}} \right) \right]^2} \quad (14)$$

$$R = \frac{COV(U_{lidar}, U_{CVA})}{\sigma_{lidar} \sigma_{CVA}} \quad (15)$$

where, ‘ $COV()$ ’ is the covariance and ‘ $\sigma$ ’ is the standard deviation. From Table 1 and Fig. 2, it is evident that both VVP and the new 2D-VAR methods accurately estimate the mean flow although VVP performs slightly better primarily due to its underlying formulation which is designed to obtain the mean quantities. The downside of this is the loss of local flow structure as seen in Fig. 3. It is evident from this figure that the wind vectors estimated by the 2D-VAR algorithm (Fig. 3c) corroborate well with the radial velocity values, at least qualitatively (Fig. 3a) especially in capturing small scale flow structures, including what appear to be wakes behind the wind turbines. All this small scale information is essentially lost in the VVP retrieval (Fig. 3b).

Table 1. Validation of 2D-VAR and VVP with 10-minute averaged CVA data

Algorithm/Variable	Wind speed error	Wind speed correlation coefficient	Wind direction error	Wind direction correlation coefficient
2D-VAR	0.383 m/s (5.04%)	0.96	-1.4°	0.98
VVP	0.29 m/s (2.01%)	0.98	4.3°	0.99

## 5. Conclusions

A new 2D-VAR algorithm based on the parameter identification technique was devised to retrieve 2D- horizontal wind vectors from low elevation PPI scans. The algorithm determines the vector components of the wind field by minimizing a cost function formed by the radial velocity advection equation, radial velocity equation, tangential velocity equation and deviation from the background determined from a VVP algorithm. The retrieval was applied on data from a lidar installed on an offshore research platform, scanning a wind farm and the results were validated with measurements from a cup and vane anemometer. One limitation of this study was that, the true accuracy of the retrieval based on instantaneous measurements, could not be quantified from this dataset due to the lack of instrumentation in the lidar scan region. However, preliminary analysis from this exploratory study showed that the algorithm while being computationally efficient with fast runtime, was able to capture local structure in the flow including possible wakes from the wind turbines, which the VVP failed to capture while performing almost as well as VVP in capture the mean quantities.



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